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Signatures of quantum vacuum non-linearity in laser-pulse collisions at finite focal offsets

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T1 project at FOR2783

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Overview

- Introduction
- Setup & Parameter
- Results
- Summary & Outlook

Introduction

Motivation

- Recent publication [1] studied influence of beam curvature on optical signatures at SACLA laboratory
- Analytical framework involves several ad-hoc assumptions, more thorough quantitative analysis seemed beneficial
- Setup: Pump-probe beam collision of Gaussian beams including focal offsets, study of their effect on signal profile

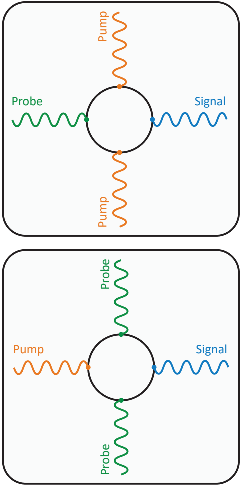
[1] Y. Seino, T. Inada, T. Yamazaki, T. Namba and S. Asai
New estimation of the curvature effect for the X-ray vacuum diffraction induced by an intense laser field,
Prog. Theor. Exp. Phys. **2020**, 073C02 (2020)

- Heisenberg-Euler effective Lagrangian [2] (Heaviside-Lorentz units, $c = 1 = \hbar$)

$$\mathcal{L}_{\text{int}} \simeq \frac{2\alpha^2}{45 m_e^4} \left((F^{\mu\nu} F_{\mu\nu})^2 + \frac{7}{4} ({}^*F_{\mu\nu} F^{\mu\nu})^2 \right)$$

- Leading coupling: Four-photon interaction
- Experimental measurement using a probe-pump setup
- Signal photons considered plane waves

[2] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936)



Interaction process

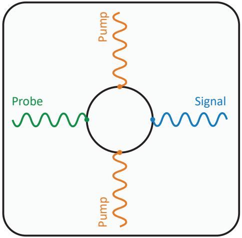
- Field profiles embedded inside corresponding S-matrix element:

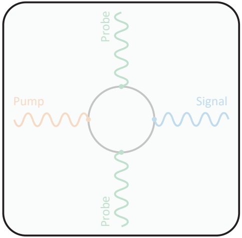
$$S(\mathbf{k}) \propto \int \mathrm{d}^4x\, \mathfrak{E}(x)\, E^2(x)\, \mathrm{e}^{\mathrm{i}k(\hat{\mathbf{k}}\cdot\boldsymbol{x}-t)}$$

- Fermi's golden rule allows for a determination of signal density:

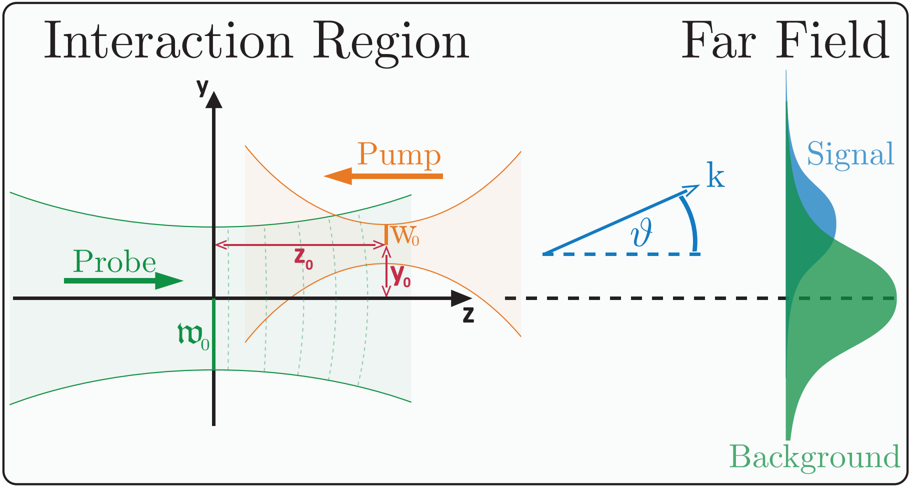
$$\mathrm{d}^3N(\mathbf{k}) = \frac{\mathrm{d}^3k}{(2\pi)^3}\,|S(\mathbf{k})|^2$$

- Polar coordinates $(\mathrm{k}, \varphi, \vartheta)$ chosen for signal distribution
- Radial integration (photon energy k) yields $\mathrm{d}^2N(\varphi, \vartheta)$





Setup & Parameter



- XFEL probe, intense optical pump, pump focus shifted.
- Resulting angular signal density will be shifted as consequence.

Pulse Profile

- Helmholtz' equation in slowly varying envelope approximation yields transversal modes → Gaussian beam

$$\mathfrak{E}(x) = \mathfrak{E}_0\, \mathrm{e}^{-\left(\frac{\mathrm{z}-t}{T/2}\right)^2} \frac{\mathfrak{w}_0}{\mathfrak{w}(\mathrm{z})}\, \mathrm{e}^{-\frac{\mathrm{x}^2+\mathrm{y}^2}{\mathfrak{w}^2(\mathrm{z})}} \times \cos\left(\omega(\mathrm{z}-t) + \frac{\omega(\mathrm{x}^2+\mathrm{y}^2)}{2R(\mathrm{z})} - \arctan\left(\frac{\mathrm{z}}{\mathfrak{z}_R}\right)\right)$$

- Characterized by pulse duration T, waist size $\mathfrak{w}_0$, frequency ω and pulse energy $\mathfrak{W}$ ($\propto\ \mathfrak{E}_0^2$).
- Beam curvature effected by value of Rayleigh range $\mathfrak{z}_R$:

$$\mathfrak{z}_R = \frac{\omega\mathfrak{w}_0^2}{2M^2} \qquad \mathfrak{w}(\mathrm{z}) = \mathfrak{w}_0\sqrt{1+\left(\frac{\mathrm{z}}{\mathfrak{z}_R}\right)^2}$$

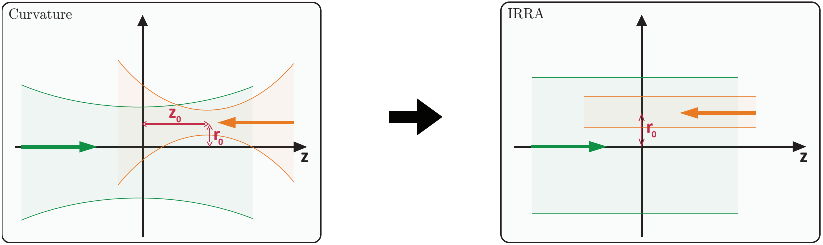
- M^2 beam quality factor, widely employed; field no longer solution to Helmholtz' eq. for $M^2 \neq 1$

Infinite Rayleigh range approximation

- For close foci ($z_0 \ll \mathfrak{z}_R$) curvature usually negligible
  $\rightarrow$ Infinite Rayleigh range approximation (IRRA)
- Focal shifts result in waist-dependent exponential damping:

$$d^3 N_{r_0} = \exp\left(-\frac{4\, r_0^2}{2\, \mathfrak{w}_0^2 + w_0^2}\right)\, d^3 N_{\text{no shift}}\,,$$

with $r_0^2 = x_0^2 + y_0^2$.



Considered Laser Parameter

- Consider two scenarios: SACLA test parameter set (A), the other a possible future HIBEF one (B).

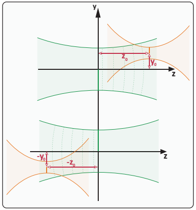
	Setup A [1]	**Setup B**
Probe Parameters:		
waist $\mathfrak{w}_0$ [µm]	6	3
pulse duration T [fs]	17	17, 220
frequency ω [keV]	9.8	12.914
pulse energy $\mathfrak{W}$ [mJ]	0.47	2.07
Pump Parameters:		
waist w_0 [µm]	9.8	1.0
pulse duration τ [fs]	40	40
Rayleigh range z_R [µm]	377.15	3.93
pulse energy W [J]	2.1×10^{-4}	12.5

Considered Focal Offsets

- Focal offsets range in orders $y_0 \simeq \mathcal{O}(1)\mathfrak{w}_0$ and $z_0 \simeq \mathcal{O}(1)\mathfrak{z}_R$.

	Setup A	**Setup B**
Focal offset:		
x-shift x_0 [µm]	0	0
y-shift y_0 [µm]	3.7	3.0
z-shift z_0 [m]	0.85	0.29

- Cylindrical symmetry $\rightarrow x_0 \equiv 0$, thus profile of interest along ϑ ($y - z-$plane)
- Resulting signal symmetric to origin with respect to focal shifts



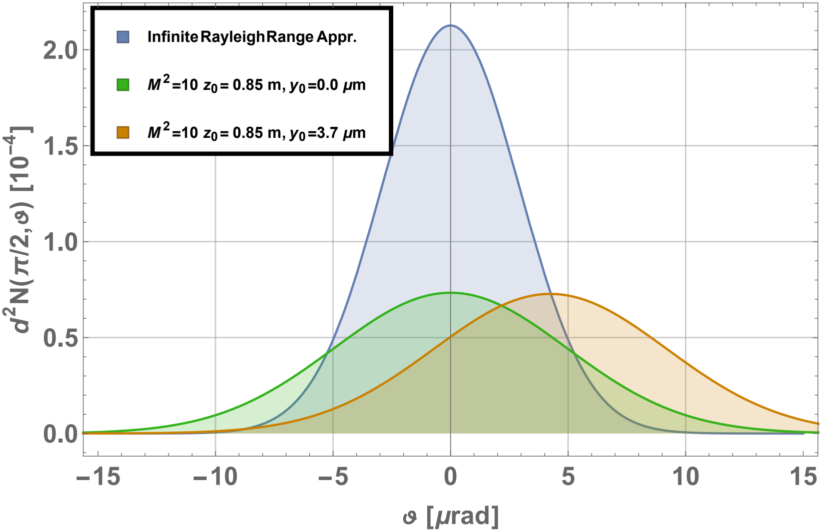
Results

Setup A

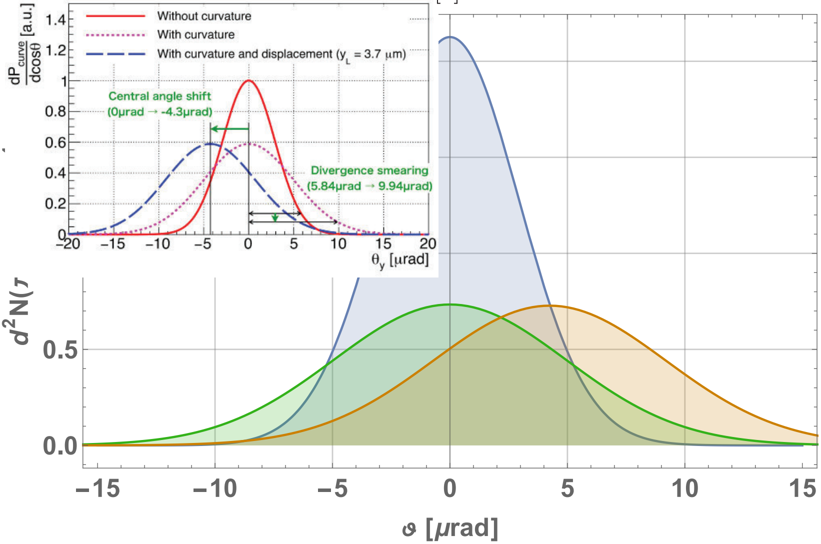
- Parameter set considered for prototype experiment at SACLA
- XFEL probe
- optical pump wavelength $\lambda = 800$ nm
- Probe Rayleigh range $\mathfrak{z}_R \simeq \frac{1}{M^2} \times 0.89$ m
- Beam quality factor of $M^2 = 10$ considered [1]

	Setup A
Probe:	
$\mathfrak{w}_0$ [µm]	6
T [fs]	17
ω [keV]	9.8
$\mathfrak{W}$ [mJ]	0.47
Pump:	
w_0 [µm]	9.8
τ [fs]	40
z_R [µm]	377.15
W [J]	2.1×10^{-4}

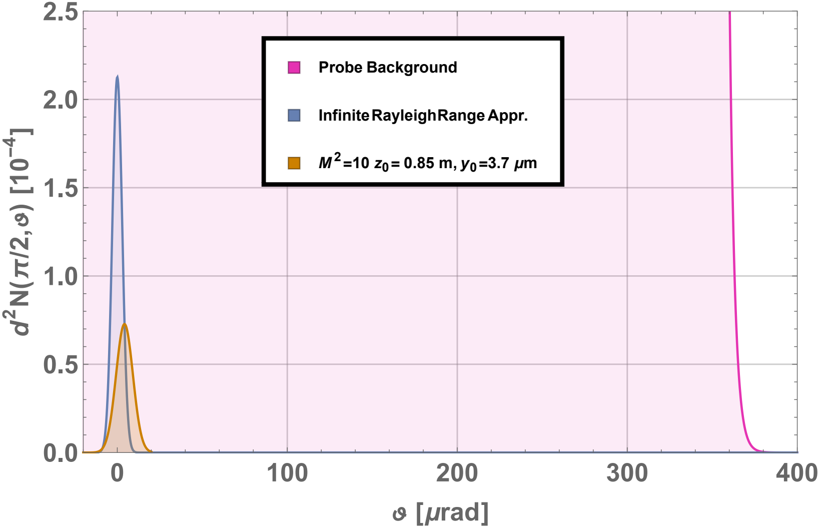
Setup A - Plot recreation



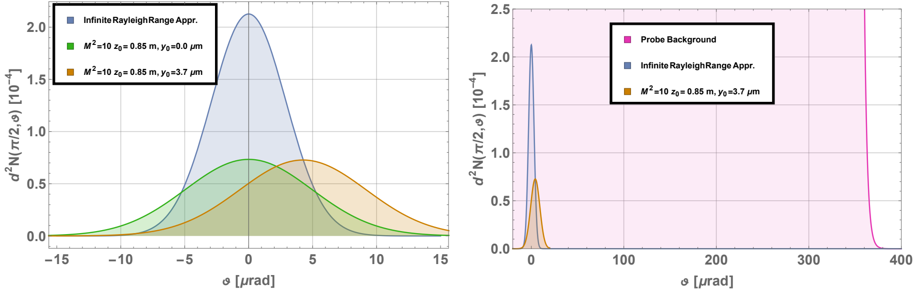
[1]


Without curvature
With curvature
With curvature and displacement (y_L = 3.7 μm)
Central angle shift
(0μrad → -4.3μrad)
Divergence smearing
(5.84μrad → 9.94μrad)
$\frac{dP_{curve}}{d\cos\theta}$ [a.u.]
θ_y [μrad]
$d^2N(\tau$
ϑ [μrad]

Setup A - Compared to background



Setup A - Summary findings



- Signal's divergence apparently widened by z_0 but unaffected by y_0
- Signal center moved by y_0 to a small degree
- Still: Weak total signal and dominant background
- Next step: Study of signal characteristics by focal shifts

- Comparing to Gaussian distribution allows for rough analytical approximations of signal divergence θ_s and offset $\Delta\vartheta_s$:

$$\theta_\mathrm{s} \simeq \frac{2}{\omega \mathfrak{w}(\mathrm{z}_0)} \sqrt{\frac{[1+2(\frac{\mathfrak{w}(\mathrm{z}_0)}{w_0})^2]^2 + (\frac{\mathrm{z}_0 M^2}{\mathfrak{z}_\mathrm{R}})^2}{1+2(\frac{\mathfrak{w}(\mathrm{z}_0)}{w_0})^2}}$$

$$\Delta\vartheta_\mathrm{s} \simeq \frac{4\,\mathrm{y}_0\,\mathrm{z}_0\,M^2}{\omega\,\mathfrak{z}_\mathrm{R}\,\left(w_0^2 + 2\mathfrak{w}(\mathrm{z}_0)^2\right)}$$

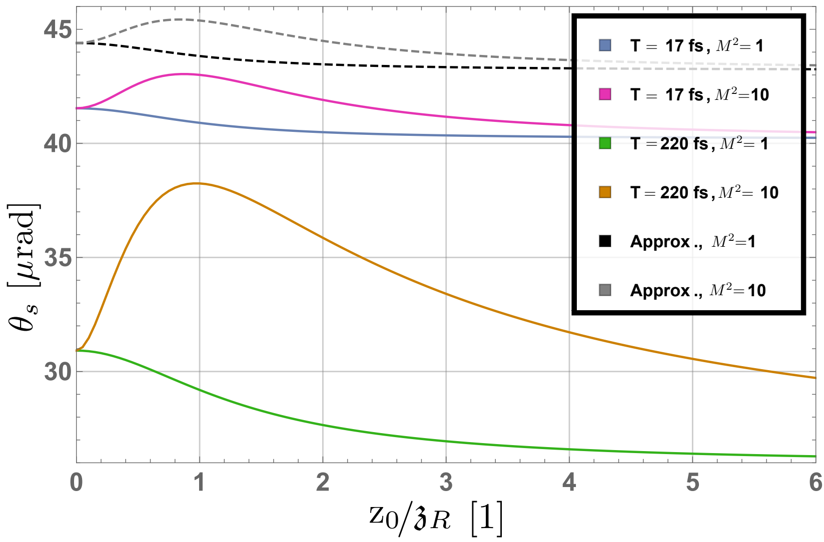
- Divergence θ_s unaffected by y_0. Also strange non-monotonous behaviour for large M^2
- Offset directly proportional $\Delta\vartheta_s \propto \mathrm{y}_0$; consistency $\Delta\vartheta_s\big|_{\mathrm{z}_0=0} \equiv 0$
- Approximations localised → lack predictions on influence by pulse durations

Setup B

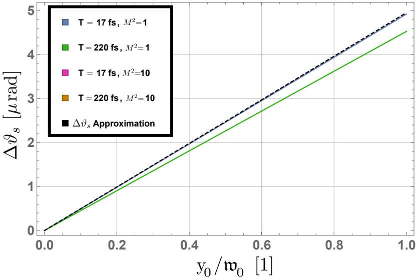
- Parameters proposed for future experiments at HIBEF
- Probe Rayleigh range: $\mathfrak{z}_R \simeq \frac{1}{M^2} \times 0.29$ m
- $\mathcal{N} = 1 \times 10^{12}$ probe photons per shot
- Two different probe pulse durations studied
- Polarisation purity of $P = 1.3 \times 10^{-11}$ taken into account

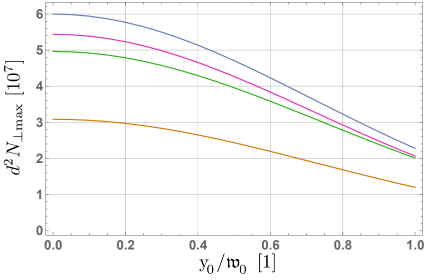
	Setup B
Probe:	
$\mathfrak{w}_0$ [µm]	3
T [fs]	17, 220
ω [keV]	12.914
$\mathfrak{W}$ [mJ]	2.07
Pump:	
w_0 [µm]	1.0
τ [fs]	40
z_R [µm]	3.93
W [J]	12.5

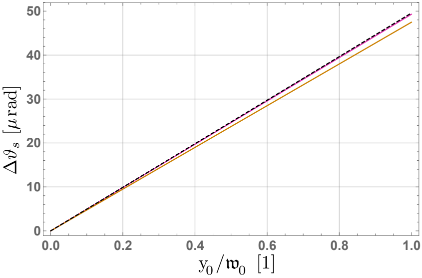
Setup B - Divergence and z-shift

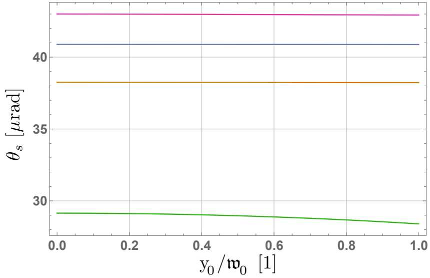


Setup B - Signal behaviour and y-shift

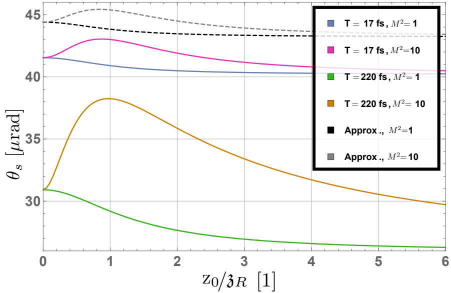


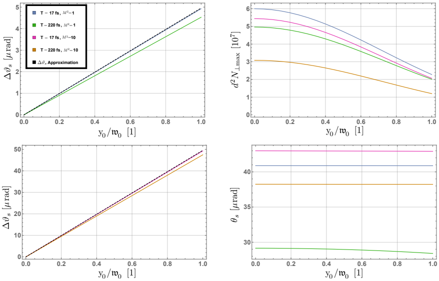






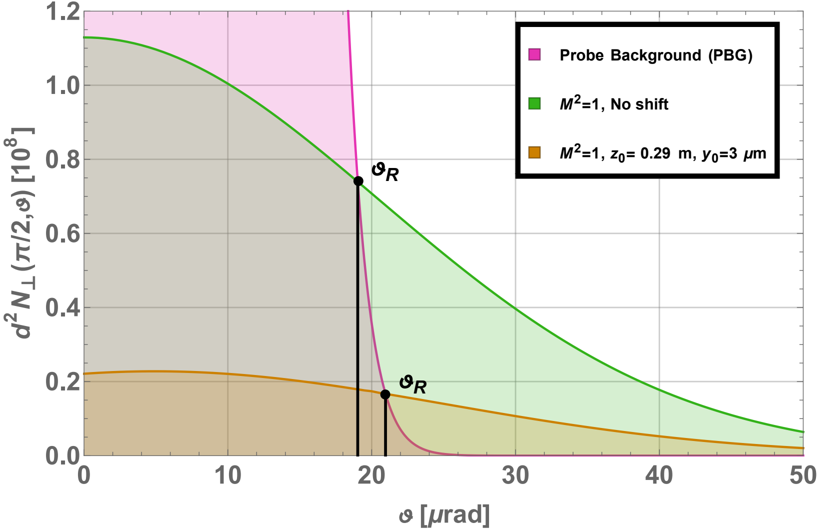
Setup B - Signal behaviour summary



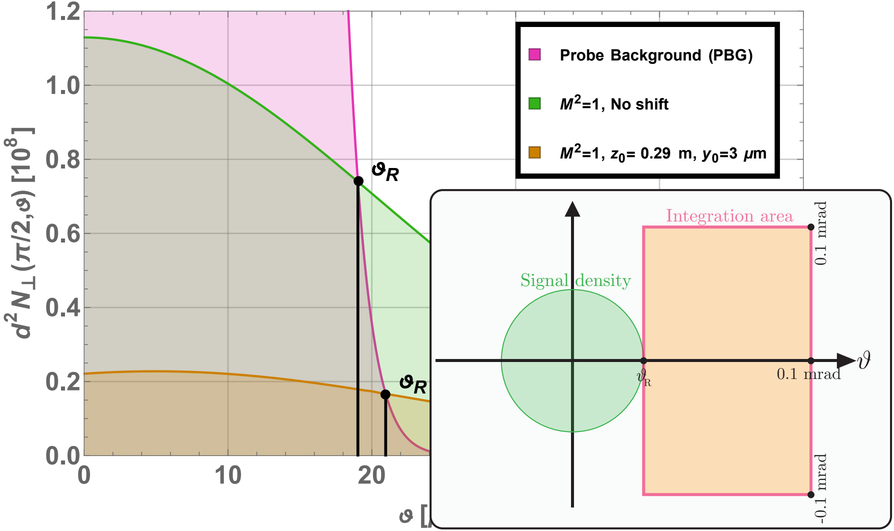


- Odd divergence maximum for $M^2 > 1$, possible artifact (violation of Helmholtz' eq.)
- Linear offset-increase well predicted by approximation
- Exponential damping of signal maximum
- Distribution deviates from Gaussian from → increased curtosis affects divergence.

Setup B - Compared to background



Setup B - Compared to background



Setup B - Signal Photon Numbers

- Polarisation-flipped photons $N_\perp$ by angular integration of $d^2N_\perp$
- Yield in area beyond cross-section with background ϑ_R?

$T = 17$ fs	$N_\perp$	ϑ_R [µrad]	$N_\perp^{\vartheta>\vartheta_\mathrm{R}}$ $[10^{-2}]$	$\frac{N_\perp^{\vartheta>\vartheta_\mathrm{R}}}{N_\perp^{\vartheta<\vartheta_\mathrm{L}}}$
No focal shift	0.31	**19.03**	5.55	1.00
$z_0 = 0.29$ m	0.16	19.95	2.62	1.00
$z_0 = 0.29$ m, $y_0 = 3$ µm	0.06	**20.96**	0.92	1.05
$T = 220$ fs				
No focal shift	0.14	20.01	1.71	1.00
$z_0 = 0.29$ m	0.07	20.95	0.79	1.00
$z_0 = 0.29$ m, $y_0 = 3$ µm	0.03	21.68	0.28	1.13

Summary & Outlook

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Summary

- Able to precisely simulate probe-pump collisions with focal offsets of Gaussian beams, recreating results of a previous publication
- Influence of focal shifts on signal photon distribution studied
- Offset increases linearly, while total signal is exponentially damped

Outlook

- Further usage of signal-to-background separation by focal shifting
- Measurement of distribution assymetry

[1] Y. Seino, T. Inada, T. Yamazaki, T. Namba and S. Asai
New estimation of the curvature effect for the X-ray vacuum diffraction induced by an intense laser field,
Prog. Theor. Exp. Phys. **2020**, 073C02 (2020)