χ^2 fitter for Acts

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für Bildung
und Forschung

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Part 1

Acts

Acts — a common tracking software

- based on ATLAS tracking software
- design goals
  - detector independent
  - parallelised
  - abstract
  - modern (C++17, CI)
- open source: github.com/acts-project

Track propagation

- extrapolation of track parameters (+covariance matrices) through detector geometry
- interactions with material, magnetic field

```
Propagator<Stepper<Magnetic Field>,Navigator>
PropagatorOptions<Actors,Aborters> opt(geoCtx, magCtx)
                  ↑
                  ActionList<Chi2Actor>
```

Part 2

track reconstruction

Track reconstruction

track finding

- group hits which are believed to originate from the same particle

track fitting

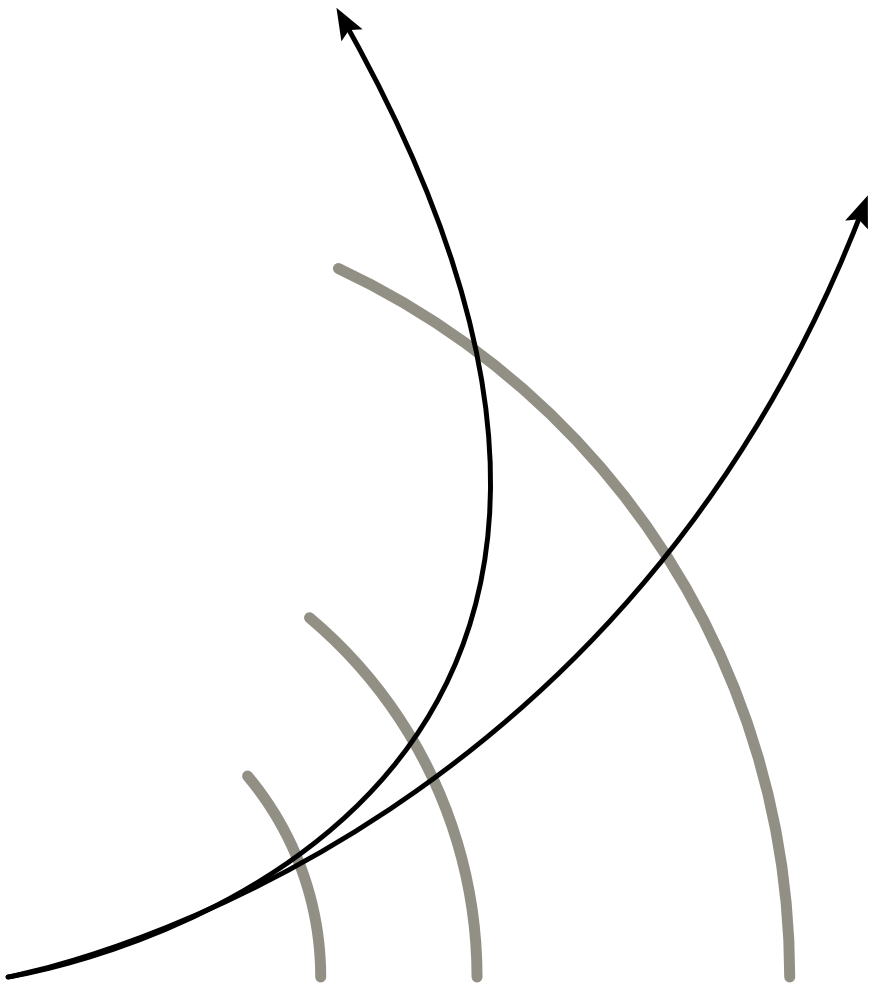
- usually process subsets created by *track finding*
- estimate track parameters
- e.g. Kálmán Filter (KF) → iterative approach
- e.g. global χ^2 fitter

combined approach

- Combinatorial Kálmán Filter (CKF)

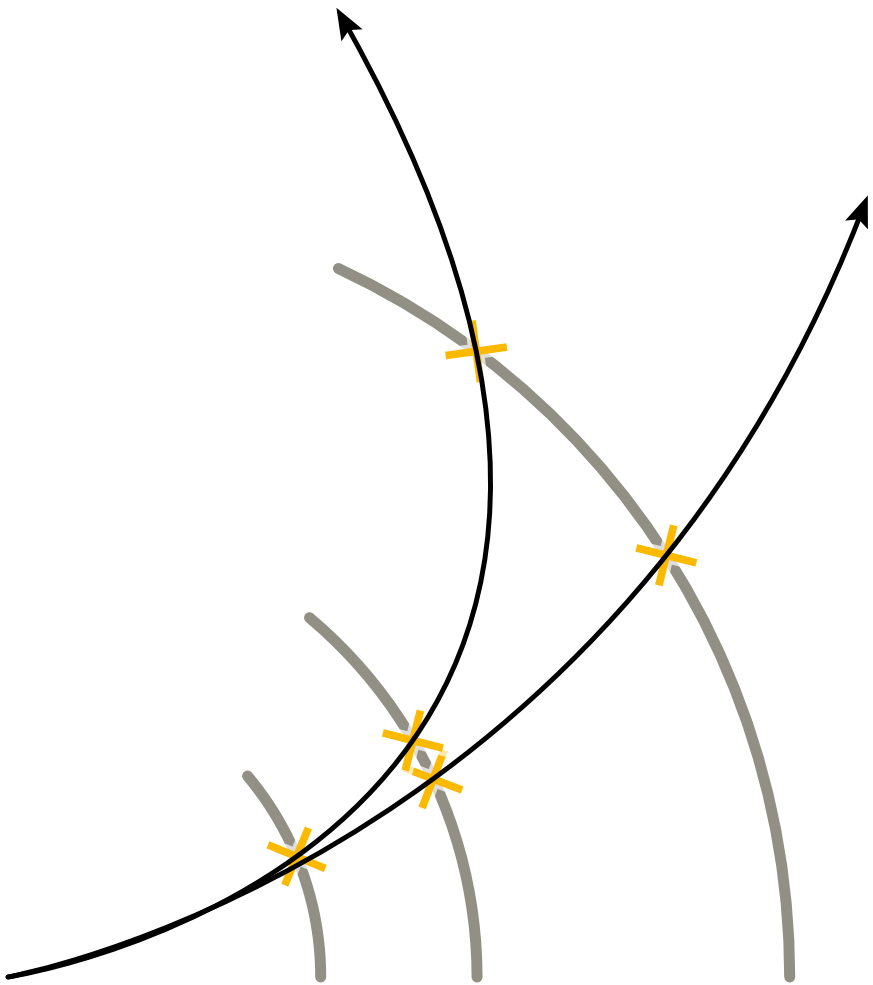
χ^2 fitter — mechanics: detector

- particle tracks



χ^2 fitter — mechanics: detector

- particle tracks
- detector hits



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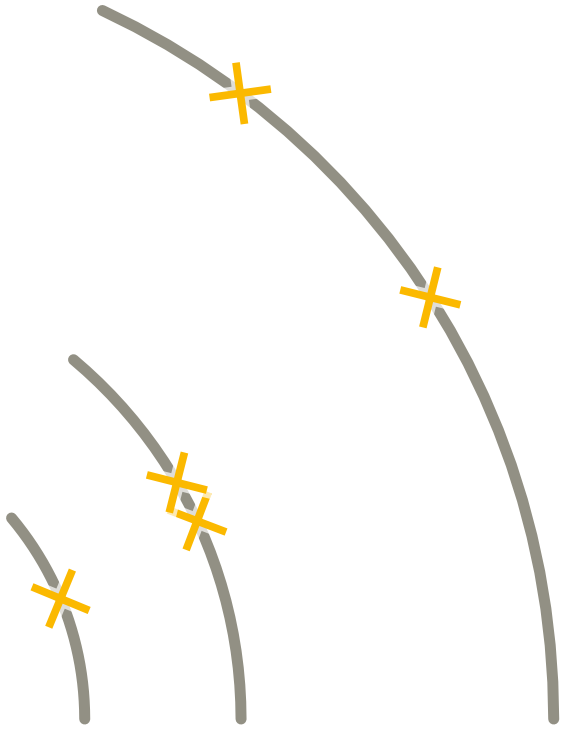

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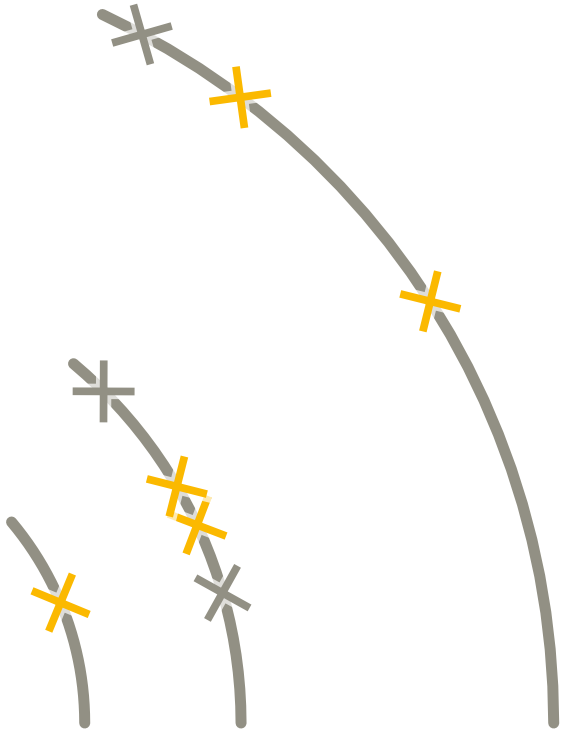
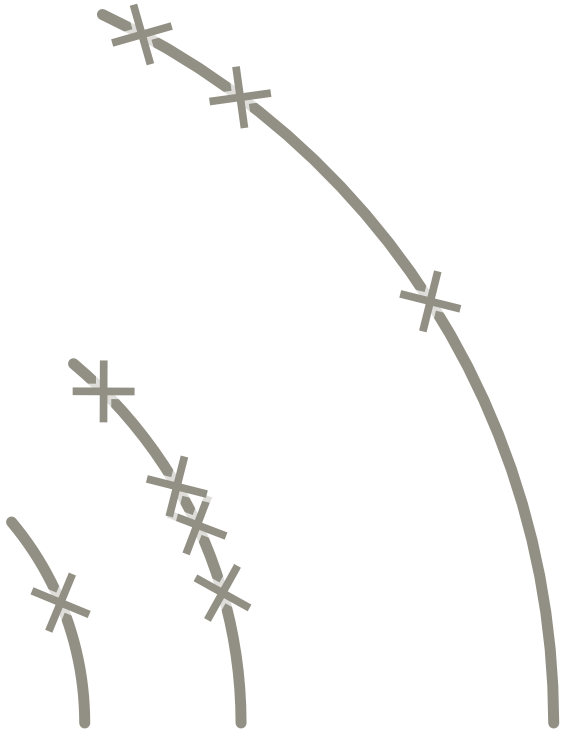


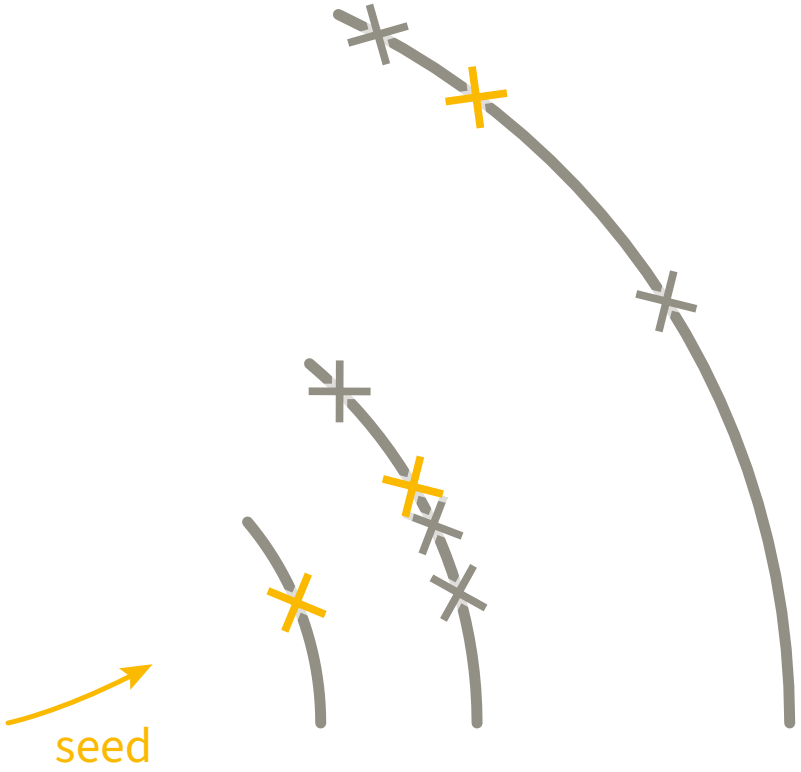
χ^2 fitter — mechanics: detector

- particle tracks
- detector hits
→ track finder



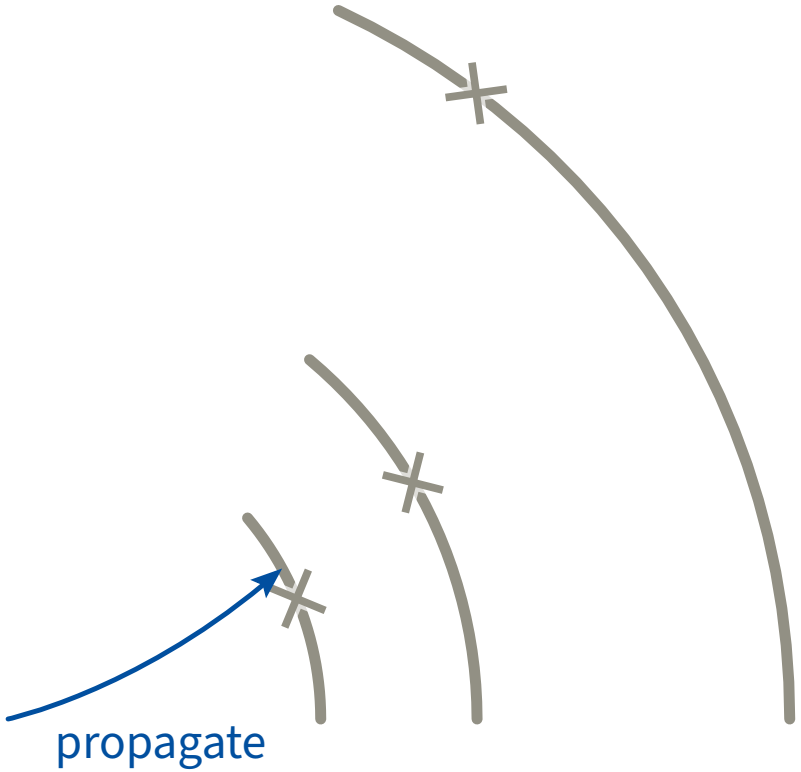
χ^2 fitter — mechanics: track fitter

- starting track parameters $\vec{x}_0$ (6-dim)



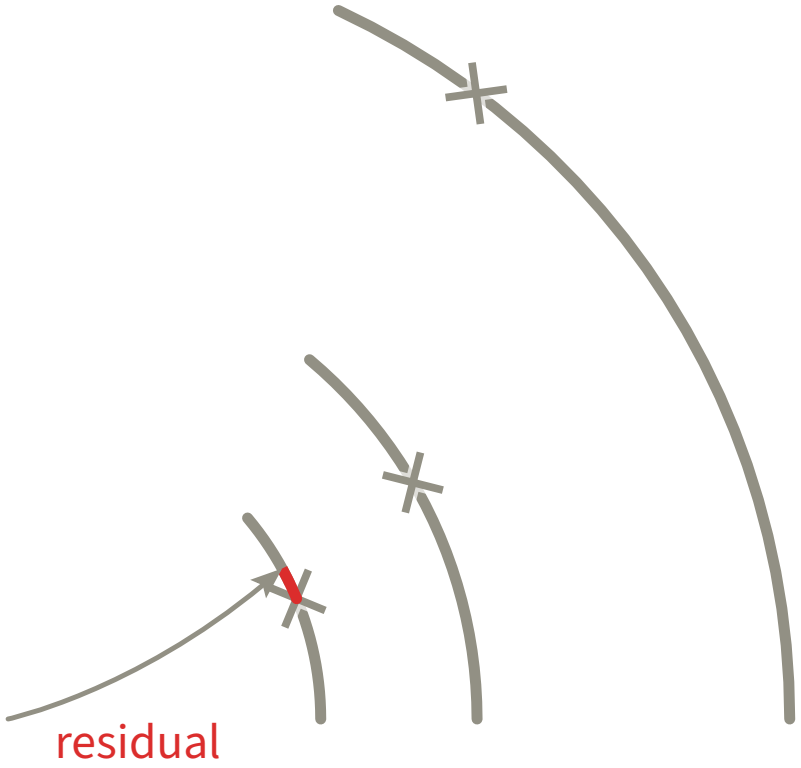
χ^2 fitter — mechanics: track fitter

- starting track parameters $\vec{x}_0$ (6-dim)
- propagation with transport matrix H



χ^2 fitter — mechanics: track fitter

- starting track parameters $\vec{x}_0$ (6-dim)
- propagation with transport matrix H
- residuals $r = m - H\,\vec{x}_0$



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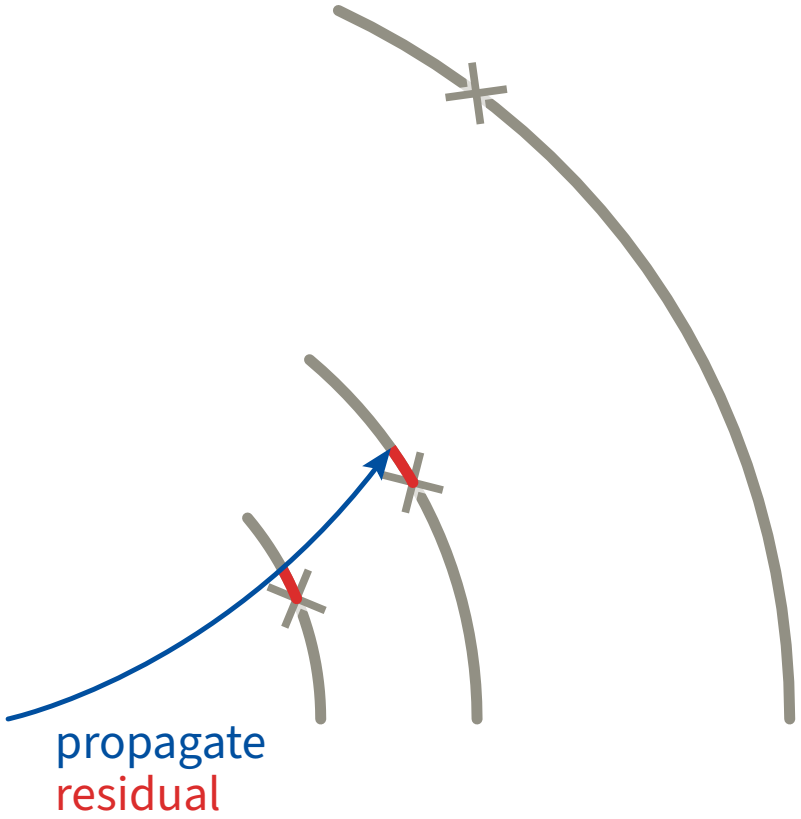

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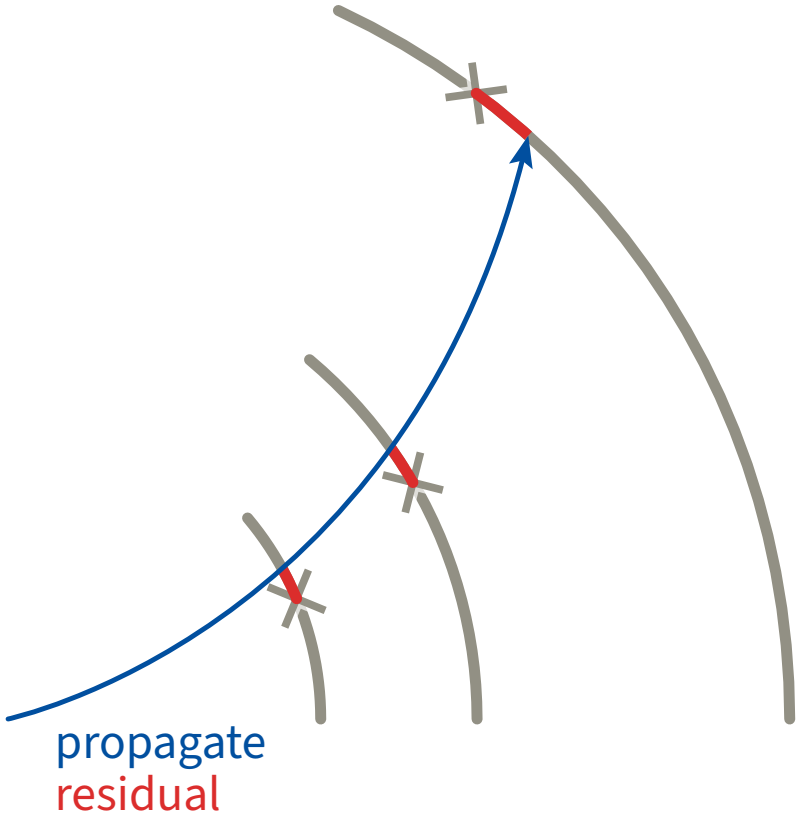
$$\chi^2 = r^T \,\mathrm{cov}^{-1}\, r$$



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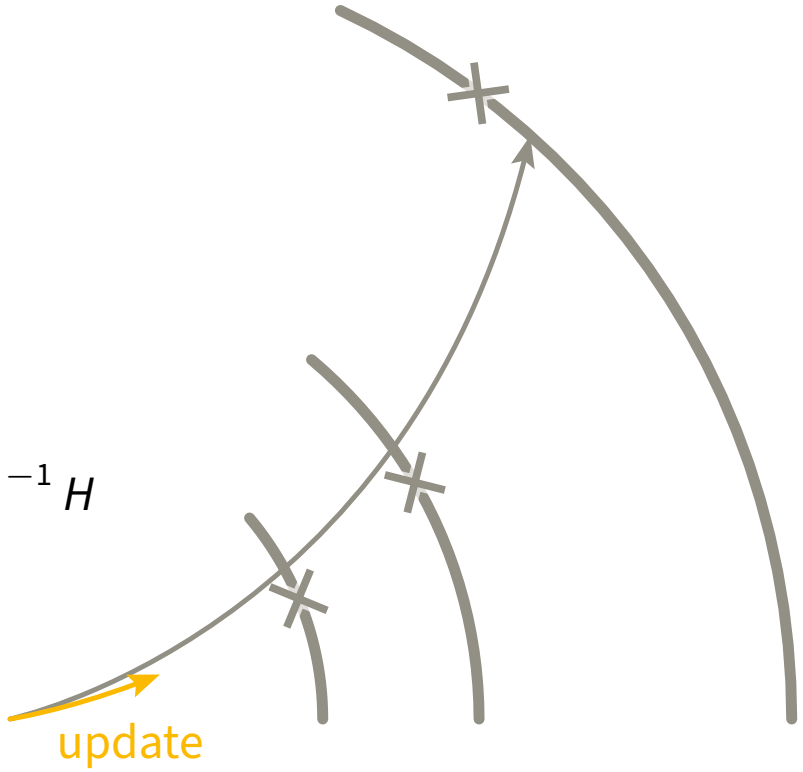
$$\chi^2 = r^T \operatorname{cov}^{-1} r$$

- update

$$\frac{\mathrm{d}\chi^2}{\mathrm{d}x} = -2H^T \operatorname{cov}^{-1} r \qquad \frac{\mathrm{d}^2\chi^2}{\mathrm{d}x^2} = 2H^T \operatorname{cov}^{-1} H$$

$$\vec{x}_1 = \vec{x}_0 - \left(\frac{\mathrm{d}^2\chi^2}{\mathrm{d}x^2}\right)^{-1} \frac{\mathrm{d}\chi^2}{\mathrm{d}x}$$



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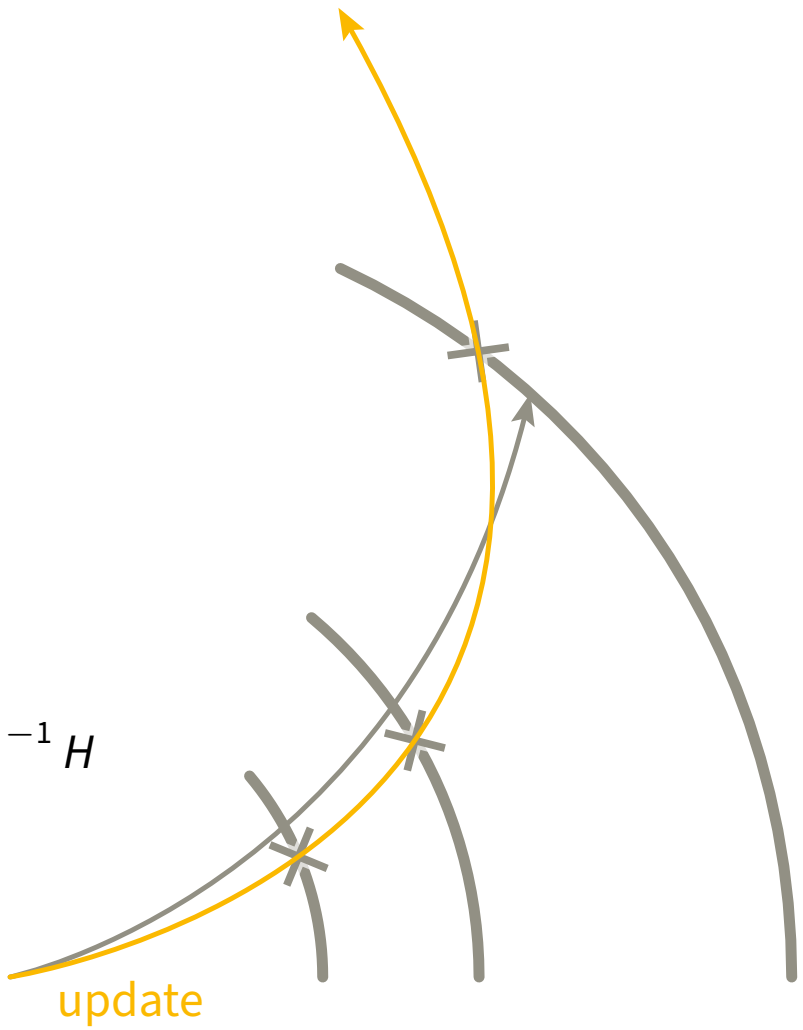

Part 3

outlook

current status and plans

- ✓ working prototype, unit test
- ☐ implement `Acts::Algorithm` interface for `Chi2Fitter`
  - allows testing of χ^2 fitter on real geometries, e.g. FASER
- ☐ add scattering effects, energy loss

$$\chi^2 = \sum_{\text{meas}} \frac{r^2_{\text{meas}}}{\sigma^2_{\text{meas}}} + \sum_{\text{scat}} \left(\frac{\theta^2_{\text{scat}}}{\sigma^2_{\text{scat}}} + \frac{(\sin\theta_{\text{loc}})^2 \phi^2_{\text{scat}}}{\sigma^2_{\text{scat}}} \right) + \sum_{\text{Eloss}} \frac{(\Delta E - \overline{\Delta E})^2}{\sigma^2_{\text{Eloss}}}$$

- ☐ benchmark χ^2 against (C)KF

backup

minimization

$$\text{res} = \begin{pmatrix} \text{px}\,x \\ \text{px}\,y \\ \text{strip} \end{pmatrix} \quad \text{cov} = \begin{pmatrix} \sigma\,\text{px}\,x & & \\ & \sigma\,\text{px}\,y & \\ & & \sigma\,\text{strip} \end{pmatrix} \quad H_{is} = \begin{cases} \begin{pmatrix} 1\,0 & 0\,0 & 0\,0 \\ 0\,1 & 0\,0 & 0\,0 \end{pmatrix} & \text{pixel} \\ \begin{pmatrix} 0\,1 & 0\,0 & 0\,0 \end{pmatrix} & \text{strip} \end{cases}$$

$$H_i = H_{is} \cdot J(x_{\text{start}} \to x_i) = H_{is} \prod J_{i-1\to i} \quad (i\text{-th surface})$$

$$\chi^2 = \text{res}^T\,\text{cov}^{-1}\,\text{res}$$

$$\frac{\mathrm{d}\chi^2}{\mathrm{d}x}(x_0) = -2H^T\,\text{cov}^{-1}\,\text{res} \qquad \frac{\mathrm{d}^2\chi^2}{\mathrm{d}x^2}(x_0) = 2H^T\,\text{cov}^{-1}\,H$$

$$x_1 = x_0 - \left(\frac{\mathrm{d}^2\chi^2}{\mathrm{d}x^2}(x_0)\right)^{-1} \frac{\mathrm{d}\chi^2}{\mathrm{d}x}(x_0)$$