Quantum Many Body Scars in Abelian Lattice Gauge Theories

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Work done with

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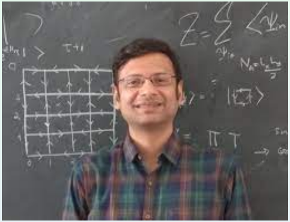
Northwestern
University, USA



Arnab Sen
IACS, Kolkata

Based on:

- DB and Sen; Phys. Rev. Lett. 126, 220601 (2021).
- Biswas, DB, Sen; SciPost Phys. 12, 148 (2022).

Outline

Thermalization and gauge theories

Microscopic models

Results

Outlook

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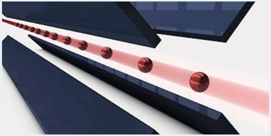
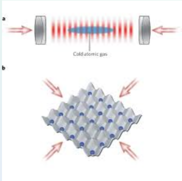
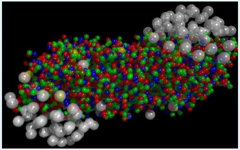
Thermalization

- Quantum systems with many degrees of freedom (mostly) thermalize.
- How fast or slow? Can it be evaded?
- Tremendous (theoretical and experimental) progress in the last decades.



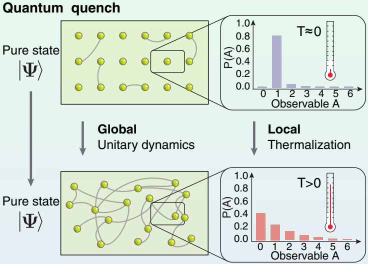




- As relevant for high-energy physics as condensed matter physics.
- E.g. formation of a quark gluon plasma in a collider laboratory depends on how fast the initial states thermalize.

(Self-) Thermalization: how?

- Unitary evolution: $|\psi(t)\rangle = \exp(-iHt)\,|\psi(0)\rangle$,
  causes (product) states to develop long-range quantum correlations.



Kaufman, (Science, 2016).

Information about the initial state converted into (non-local) correlations through spreading of quantum entanglement.

Nandkishore, Huse (Ann. Rev. of CMP, 2015).

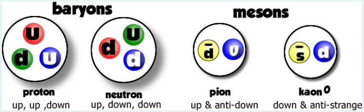
- Approach to equilibrium generally guided via Eigenstate Thermalization Hypothesis (ETH).

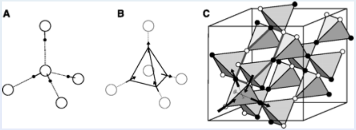
  Deutsch (PRA 1991), Srednicki (PRE, 1994).

- Increasing examples of translational-invariant interacting systems showing (weak-) ergodicity breaking: quantum many-body scars.

Gauge theories: quantum link models

- Many strongly interacting systems in Nature admit a description via microscopic models with extensive number of local conservation laws.
- E.g. Quantum chromodynamics, quantum spin ice.





QLM: realize continuous gauge symmetries with discrete link operators

$\rightarrow$ finite dimensional Hilbert space

$\rightarrow$ extension of Wilson formulation of gauge theories.

$\rightarrow$ possibility of new physics scenarios.

Horn (PLB, 1981); Orland, Rohrlich (NPB, 1990); Wiese, Chandrasekharan (NPB, 1997).

Rokhsar, Kivelson (PRL, 1988); Moessner, Sondhi, Fradkin (PRB, 2002).

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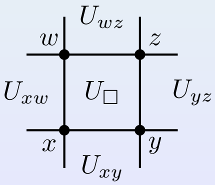
Abelian Links in $(2+1)$-d

- Quantum spins $\vec{S}_{xy}$ with $(2S+1)$-d Hilbert space at each link.
- Electric field: $E = S^z$; Gauge fields: $U = S^+$, $U^\dagger = S^-$.
- $[E, U] = U$; $[E, U^\dagger] = -U^\dagger$; $[U, U^\dagger] = 2E$.
- Minimal spin representation $S = 1/2$:

$$E|\rightarrow\rangle = \tfrac{1}{2}|\rightarrow\rangle; \quad U|\rightarrow\rangle = 0; \quad U^\dagger|\rightarrow\rangle = |\leftarrow\rangle;$$

$$E|\leftarrow\rangle = -\tfrac{1}{2}|\leftarrow\rangle; \quad U|\leftarrow\rangle = |\rightarrow\rangle; \quad U^\dagger|\leftarrow\rangle = 0;$$

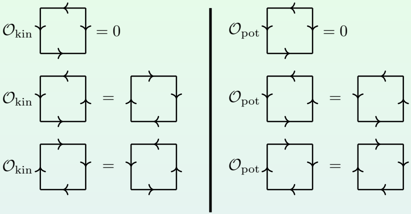


$U_\Box = U_{xy} U_{yz} U^\dagger_{wz} U^\dagger_{xw}$

$$\begin{aligned}
H &= \mathcal{O}_{\text{kin}} + \lambda \mathcal{O}_{\text{pot}} \\
\mathcal{O}_{\text{kin}} &= -\sum_\Box (U_\Box + U^\dagger_\Box) \\
\mathcal{O}_{\text{pot}} &= \sum_\Box (U_\Box + U^\dagger_\Box)^2
\end{aligned}$$

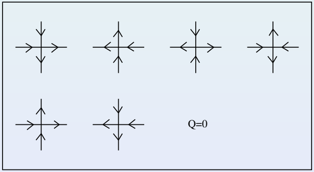
Gauge invariance

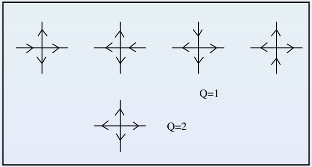
Action of $\mathcal{O}_{\text{kin}}$ and $\mathcal{O}_{\text{pot}}$:



- Gauss Law:
  $G_x = \sum_i (\mathrm{E}_{x,x+i} - \mathrm{E}_{x-i,x})$
- $[H, G_x] = 0; \quad V = \prod_r \exp(i\theta_r G_r)$
  $\rightarrow \tilde{H} = VHV^\dagger = H.$

- Hilbert space splits into in superselection sectors labelled by G_x.

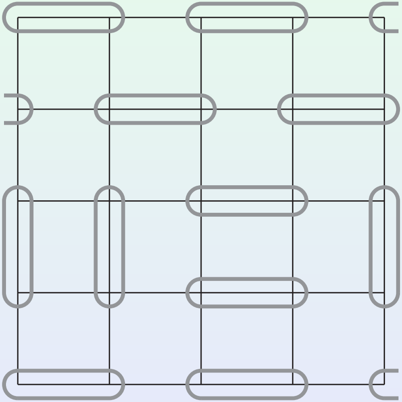




- For the $U(1)$ quantum link model QLM: $G_x |\psi\rangle = 0$ for all x.
- In addition: topological winding numbers (W_x, W_y).

Dimer models

Using a modification of the Gauss Law, the link Hamiltonian realizes a quantum dimer model QDM on the square lattice.



Gauss Law:

$$G_x|\Psi\rangle = (-1)^{x_1+x_2}|\Psi\rangle;$$

$$\mathrm{E}_{xy} = (-1)^{x_1+x_2}(\mathrm{D}_{xy} - \frac{1}{2})$$

$$\mathrm{D}_{xy} = 0, 1.$$

Candidate to describe non-Néel phases of anti-ferromagnets.

Rokhsar, Kivelson (PRL, 1988).

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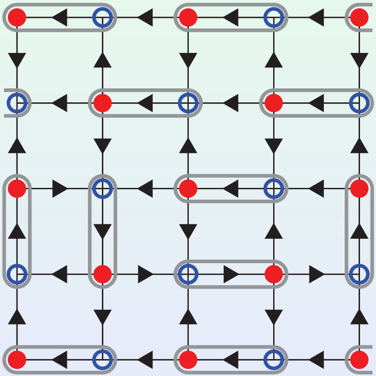

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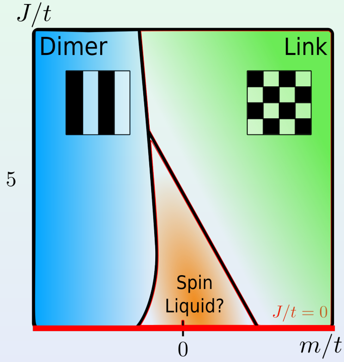
Link Electrodynamics

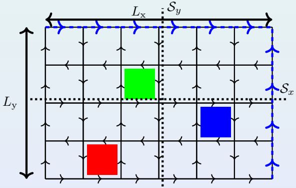
Dynamical matter can bridge the two limits.

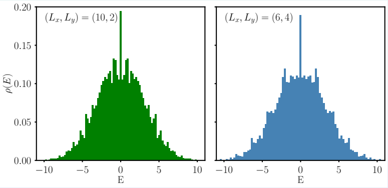


T. Hashizume, J. Halimeh, P. Hauke, D. Banerjee
(arXiv: 2112.00756).

Lattice symmetries and index theorem

- Point group symmetries: lattice translation, rotations, reflections.
- Charge conjugation: $(U, U^\dagger, E) \to (U^\dagger, U, -E)$.
- At $\lambda = 0$: $\{H, \mathbb{C}\} = 0, \qquad \mathbb{C} = \prod_{xy} E_{xy}$,
  only horizontal (vertical) links on even x(y) contribute.
- For any eigenstate E, we have $\mathbb{C}\,|E\rangle = |-E\rangle$.





$\rho(E) = \alpha\delta(E = 0) + \rho_{\text{reg}}$
$E = 0$ states protected by an index theorem and have a $\mathbb{C}$-charge.

Schechter and Iadecola (PRB, 2018).

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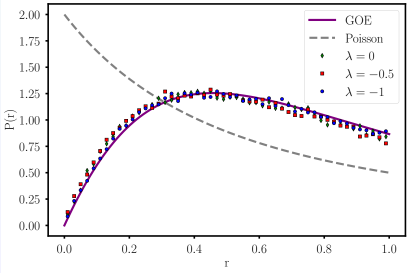
Non-integrablity

- Models are strongly interacting, non-integrable and model physical systems. No simple non-interacting limit.
- Ground-state and finite-T phase diagram $\longrightarrow$ cluster QMC methods. Crystalline confined phases in QLM and columnar phases in QDM.

  DB, Jiang, Widmer, Wiese (J. Stat. Mech, 2013);

  DB, Bögli, Hofmann, Jiang, Widmer, Wiese (PRB, 2014, 2016).
- For exploring excited state physics, use large-scale full ED. (64 spins (QLM); 96 spins (QDM); Matrix sizes $\sim 50,000 - 70,000$)



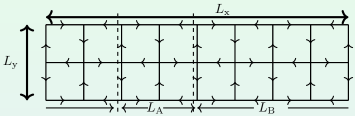
- Level spacing distribution follows GOE-distribution.
- $r = \frac{\min(s_n, s_{n+1})}{\max(s_n, s_{n+1})} \in [0, 1]$
- $s_n = E_{n+1} - E_n$.

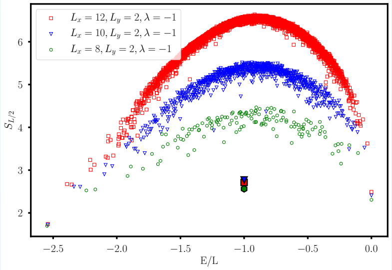
  Oganesyan and Huse (PRB, 2007).

Anomalous states in the spectrum

Entanglement measures as well as (local) correlation functions of certain excited (infinite temperature) states show anomalous behaviour.

- von-Neumann entanglement entropy: $S_{L/2}$.
- Shannon entropy: $S_1 = \sum_\alpha |\psi_\alpha|^2 \ln |\psi_\alpha|^2$, $|\Psi\rangle = \sum_{\alpha=1}^{\mathcal{N}} \psi_\alpha |\alpha\rangle$.
- Flux correlator: $\frac{1}{L_x} \sum_x \langle E_{\hat{j}}(x) E_{\hat{j}}(x+\hat{i})\rangle$, where $E_{\hat{j}}(x) = \sum_y E_{\mathrm{r},\hat{j}}$





scars in QLM

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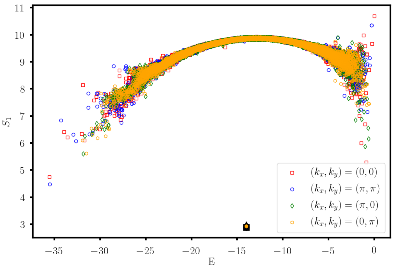
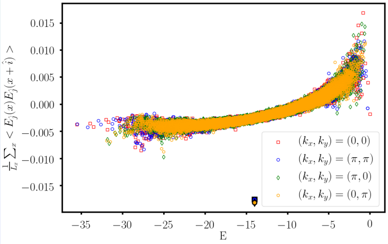
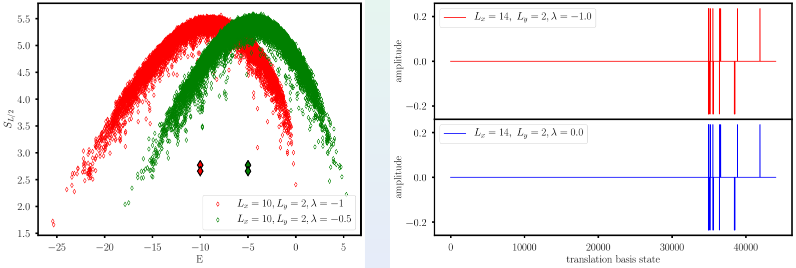




scars in QLM

Localization in the anomalous states

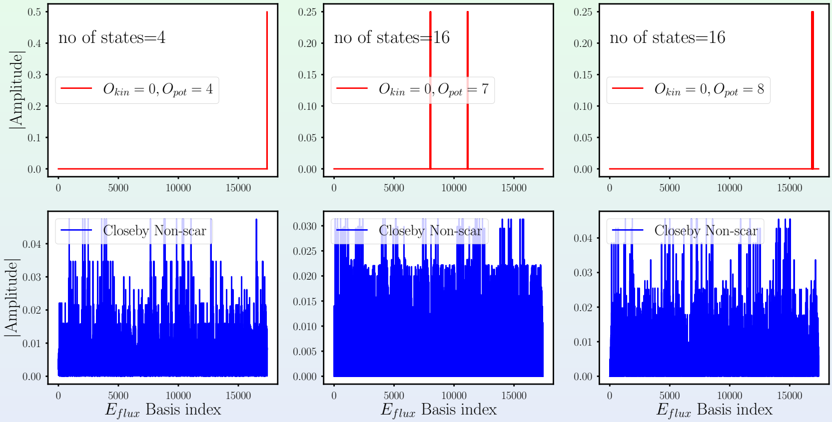
- $|\psi_{\rm QMBS}\rangle$ are eigenstates of both $\mathcal{O}_{\rm kin}$ and $\mathcal{O}_{\rm pot}$.
  $\mathcal{O}_{\rm kin}|\psi_{\rm QMBS}\rangle = 0$; $\mathcal{O}_{\rm pot}|\psi_{\rm QMBS}\rangle = N_{\rm f}|\psi_{\rm QMBS}\rangle$, $N_{\rm f}$ is an integer.
  $H(\lambda)|\psi_{\rm QMBS}\rangle = \lambda N_{\rm f}|\psi_{\rm QMBS}\rangle$.
- (type-I) Scar states determined at one λ are fixed for all $\lambda \neq 0$.



Type-I scars in QLM

- Indications of localization in Hilbert space.

Localization in the anomalous states

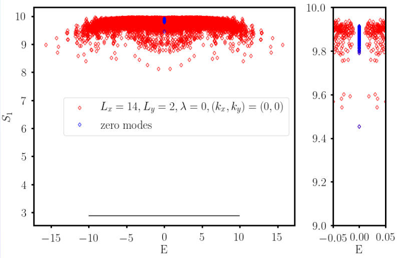


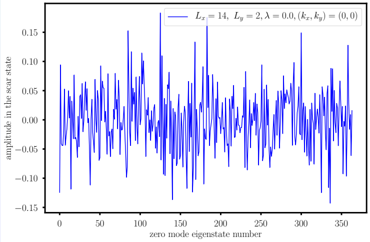
Type-I scars in QDM

- Indications of localization in Hilbert space.

Order by disorder in Hilbert space

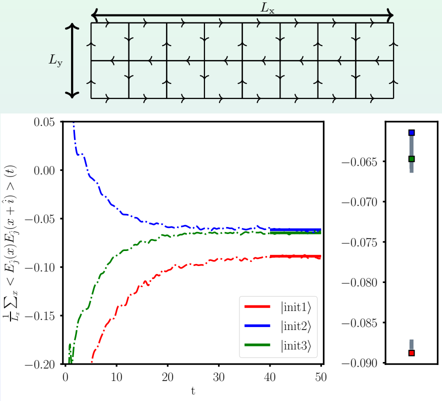
- $H(\lambda = 0) = \mathcal{O}_{\text{kin}}$, there are exponentially large number of exact zero modes: $\mathcal{O}_{\text{kin}} |\text{ZM}\rangle = 0$.
- $|\text{ZM}\rangle$ are typical excited states, locally thermal ('disordered').
- Rediagonalize $\langle \text{ZM}|\mathcal{O}_{\text{pot}}|\text{ZM}\rangle$: IF integer eigenvalues exist, we have (type-I) QMBS: $|\psi_{\text{QMBS}}\rangle$.
- For $\lambda \neq 0, \mathcal{O}_{\text{pot}}$ causes a order-by-disorder in Hilbert space by causing (pseudo-random) superposition of $|\text{ZM}\rangle$.





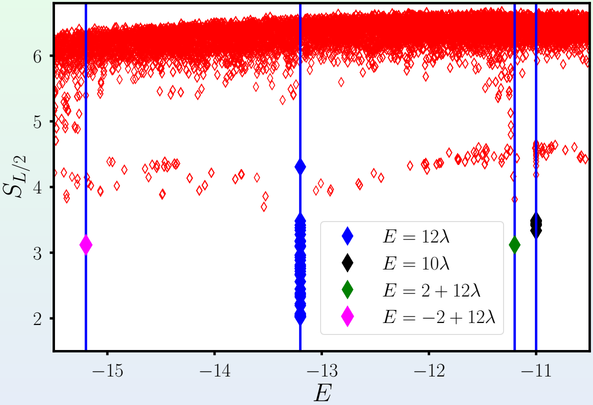
Slow thermalization

Out of 6433 initial states with average energy density $\lambda L_x L_y/2$, only 18 have overlap with the QMBS for $L_x = 14, L_y = 2$ (56 spins).



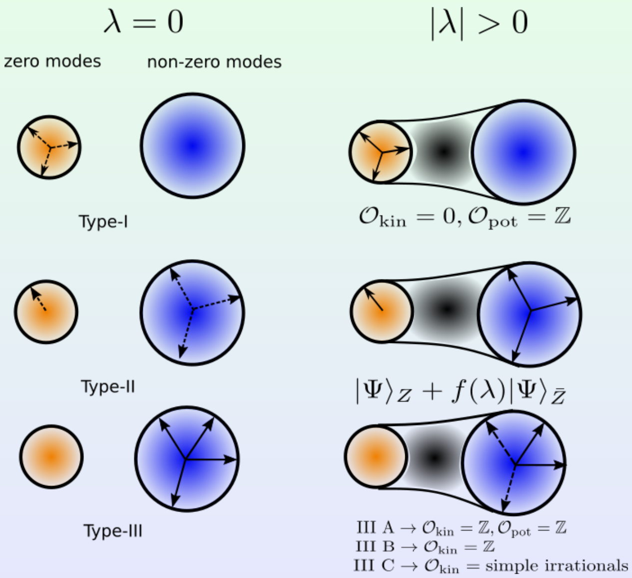
Clear memory effect in the real-time dynamics of these initial states.

More scars

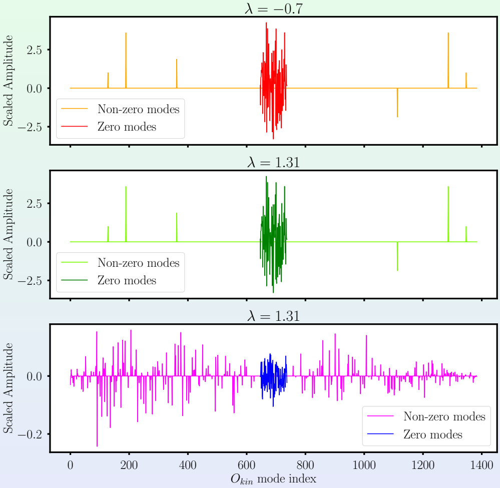


More (types + numbers) of QMBS appear on larger lattices.
46 (106) type-I scars with $(\mathcal{O}_{\text{kin}}, \mathcal{O}_{\text{pot}}) = (0, N_{\text{f}} = \frac{L_x L_y}{2})$ on $L_x = 6(8)$ and $L_y = 4$.

More scars: schematic illustration

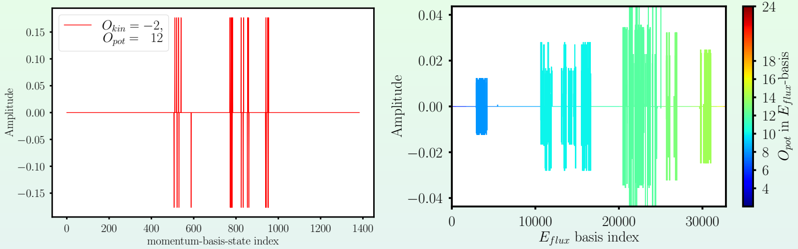


Type-II scar

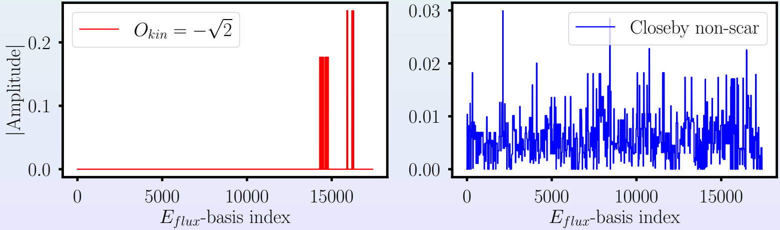


Type-II scar in the $(6, 4)$ QLM (48 spins).

Type-III scars



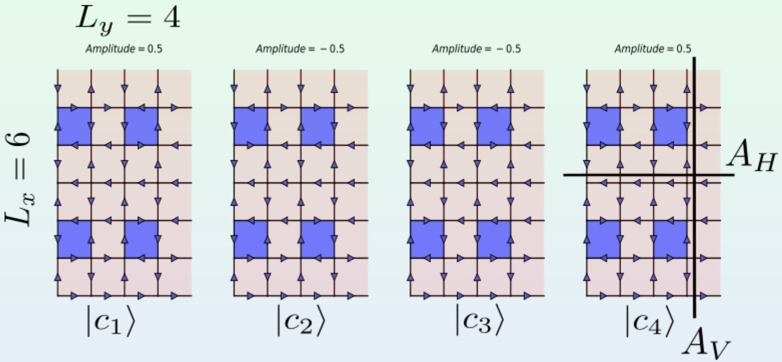
Type-IIIA (left) and Type-IIIB (right) scar in the $(6, 4)$ QLM, $\lambda = 0$.



Type-IIIC scar in the $(8, 4)$ QDM, $\lambda = 0$ (64 spins).

Lego Scars in QDM

Certain scars in the QDM are exceptionally simple: Lego scars.



- $|\Psi_{\text{QMBS}}\rangle = \frac{1}{2}(|c_1\rangle - |c_2\rangle - |c_3\rangle + |c_4\rangle)$;
- $(\mathcal{O}_{\text{kin}}, \mathcal{O}_{\text{pot}}) = (0, 4)$; $\quad S_{L/2;A_H} = 0$; $\quad S_{L/2;A_V} = 2\ln(2)$.
- Can be written as a tensor product state: $|\Psi_{\text{QMBS}}\rangle = |\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle$

Quantum Caging

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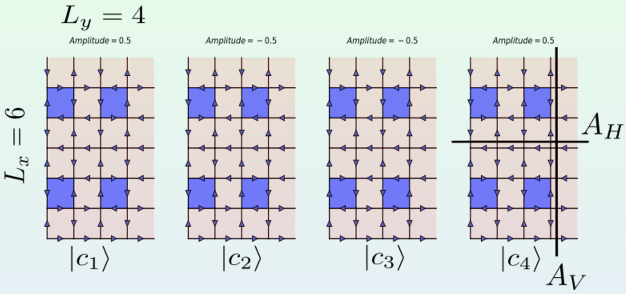

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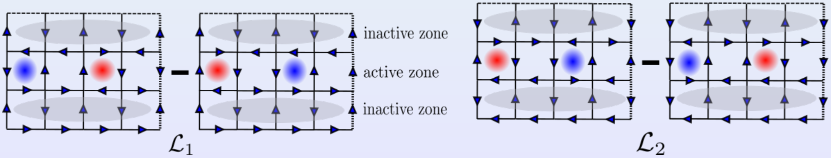
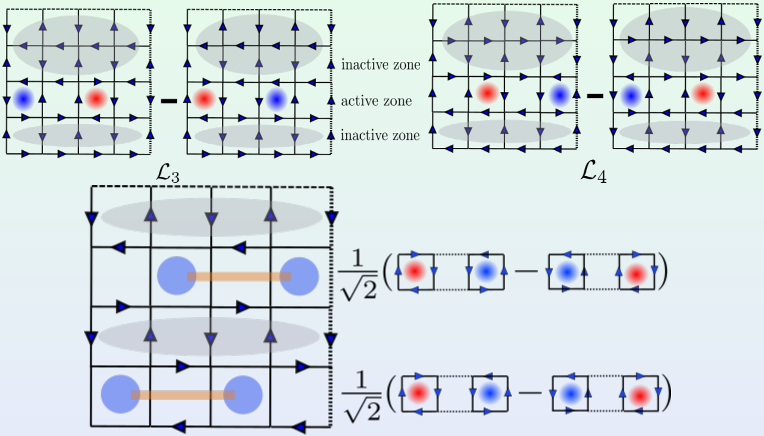
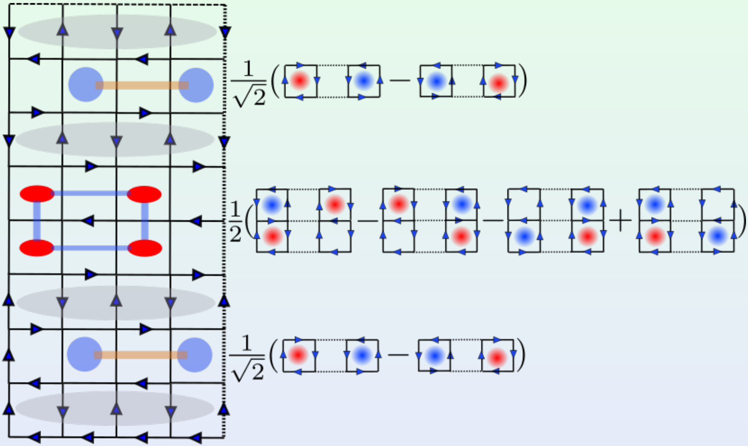


Welcome to the lego game



Various types of legos can be identified on the lattices we study.

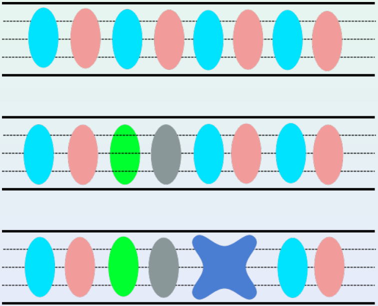
Welcome to the lego game



Various types of legos can be identified on the lattices we study.

Thermodynamic limit

- Consider a lattice with $L_y = 4$ but with infinite L_x.
- Construct exact eigenstates of H by tensoring (appropriate) legos.
- Locally there is more than one choice.



- Exponentially such number of states can be analytically constructed in the thermodynamic limit.

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Many open questions

Anomalous states on interacting physically interesting physical models reported. Novel mechanism: Order-by-disorder in Hilbert space.

- Robustness of the QMBS: can other operators (with point spectrum) hybridize zero-modes differently to form other kinds of scars?
- Classification: are all types of scars in these models identified?
- Description: Generalize lego scar descriptions to other types of scars in QLM+QDM?
- Extension: generate QMBS in other quantum systems (with exponentially large nullspaces) by adding specific interactions?
- Richer structures expected in two and three dimensions.

THANK YOU FOR YOUR ATTENTION

Numbers for the QLM

Hilbert space in Quantum Link Model			
(L_x, L_y)	Gauss law	$(W_x, W_y) = (0,0)$	$(k_x, k_y) = (0,0)$
(8, 2)	7074	2214	142
(10, 2)	61098	17906	902
(12, 2)	539634	147578	6166
(14, 2)	4815738	1232454	44046
(16, 2)	43177794	10393254	324862
(4, 4)	2970	990	70
(6, 4)	98466	32810	1384
(8, 4)	3500970	1159166	36360
(6, 6)	16448400	5482716	152416

Numbers for the QDM

Hilbert space in Quantum Dimer Model			
(L_x, L_y)	Gauss law	$(W_x, W_y) = (0, 0)$	$(k_x, k_y) = (0, 0)$
(8, 2)	1156	384	29
(10, 2)	6728	2004	106
(12, 2)	39204	10672	460
(14, 2)	228488	57628	2077
(6, 4)	3108	1456	71
(8, 4)	39952	17412	571
(10, 4)	537636	216016	5490
(12, 4)	7379216	2739588	57379
(6, 6)	90176	44176	1256
(8, 6)	3113860	1504896	31464

Numbers for the Scars

(L_x, L_y)	Type	Degeneracy	$(\mathcal{O}_{\mathrm{kin}}, \mathcal{O}_{\mathrm{pot}})$
Scars in QLM at $(W_x, W_y) = (0,0)$			
$(L, 2)$	Type I	4	$(0, N_p/2)$
(4,4)	Type I	26	$(0, 8)$
	Type I	12	$(0, 6)$
	Type IIIA	6	$(\pm 2, 8)$
	Type IIIB	12	$(\pm 2, \cdots)$
(6,4)	Type I	46	$(0, 12)$
	Type I	8	$(0, 10)$
	Type II	4	$(\cdots, \cdots)$
	Type IIIA	2	$(\pm 2, 12)$
	Type IIIB	5	$(\pm 2, \cdots)$
(8,4)	Type I	106	$(0, 16)$
	Type I	12	$(0, 14)$
	Type IIIA	2	$(\pm 2, 16)$
	Type IIIB	1	$(\pm 2, \cdots)$

Numbers for the Scars

(L_x, L_y)	Type	Degeneracy	$(\mathcal{O}_{\text{kin}}, \mathcal{O}_{\text{pot}})$
Scars in QDM at $(W_x, W_y) = (0,0)$			
(4,4)	Type I	9	$(0,4)$
	Type I	1	$(0,6)$
(6,4)	Type I	6	$(0,4)$
(8,4)	Type I	4	$(0,8)$
	Type I	16	$(0,7)$
	Type I	8	$(0,4)$
	Type IIIC	16	$(\pm\sqrt{2}, \cdots)$