Cold-atom regularizations of relativistic 4-Fermi QFTs:

Exploring correlated topological phases

Alejandro Bermúdez,

Gauge Workshop, Munich 2022

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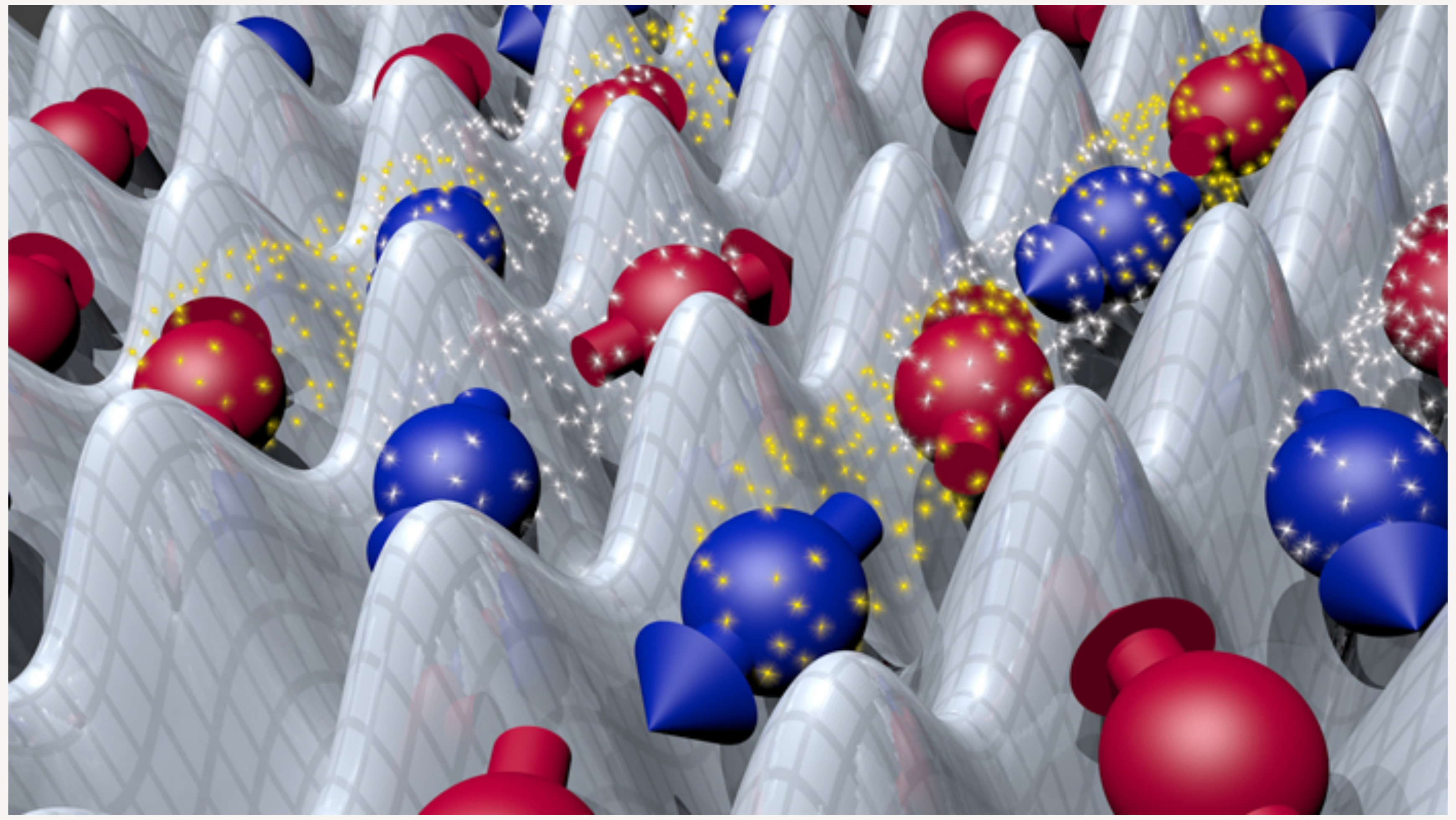
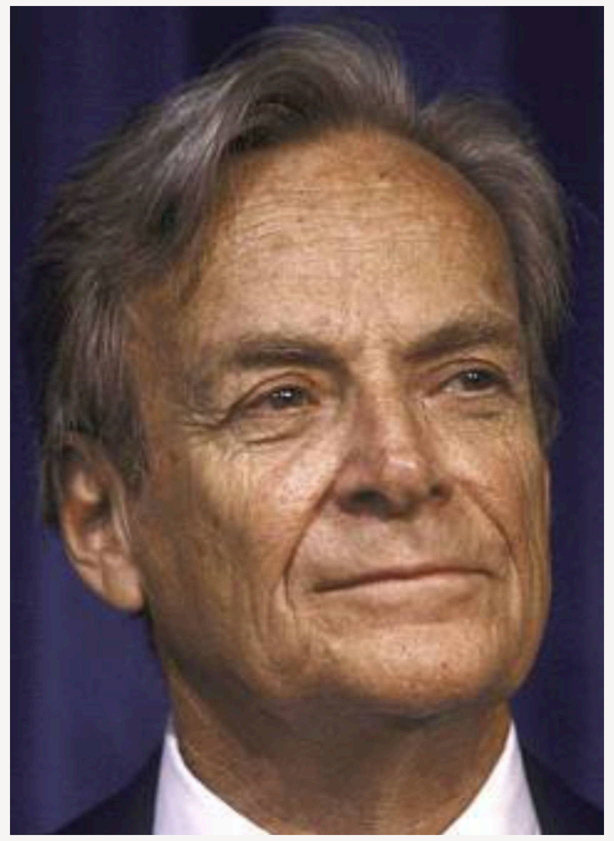
Instituto de Física Teórica
UAM-CSIC

MOTIVATION

Can we exploit **cold-atom** quantum simulators for **high-energy physics**?





R. Feynman, Int.J.Th.P. **21,** 467 (1982).

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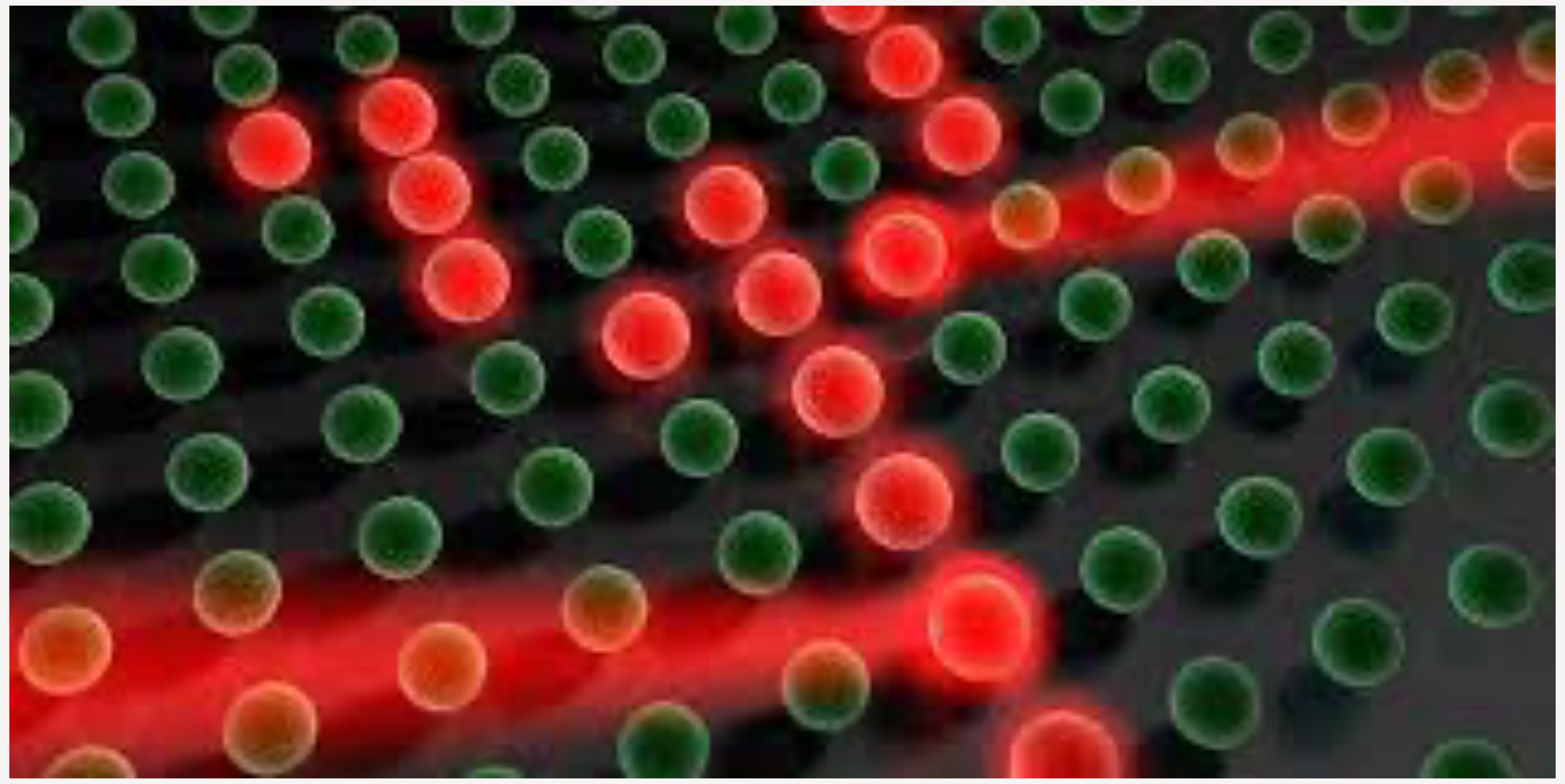

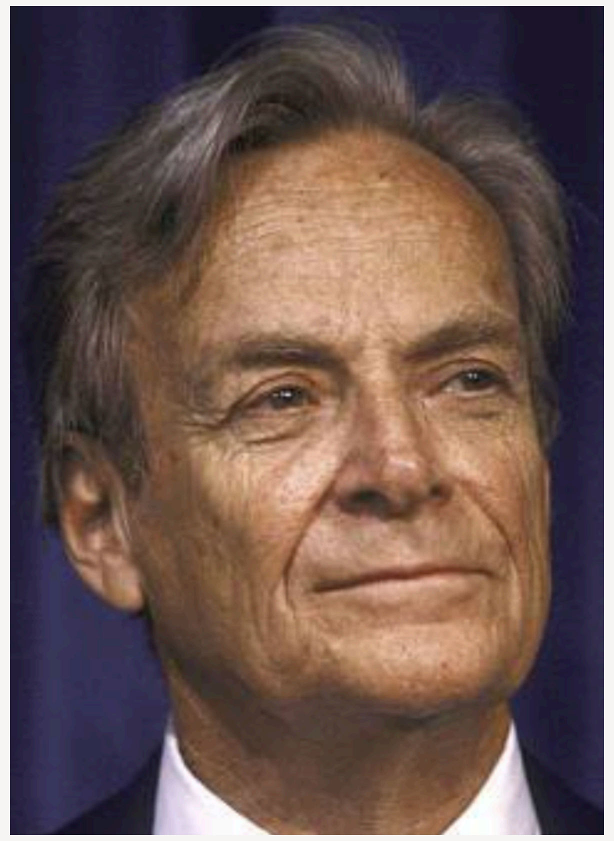

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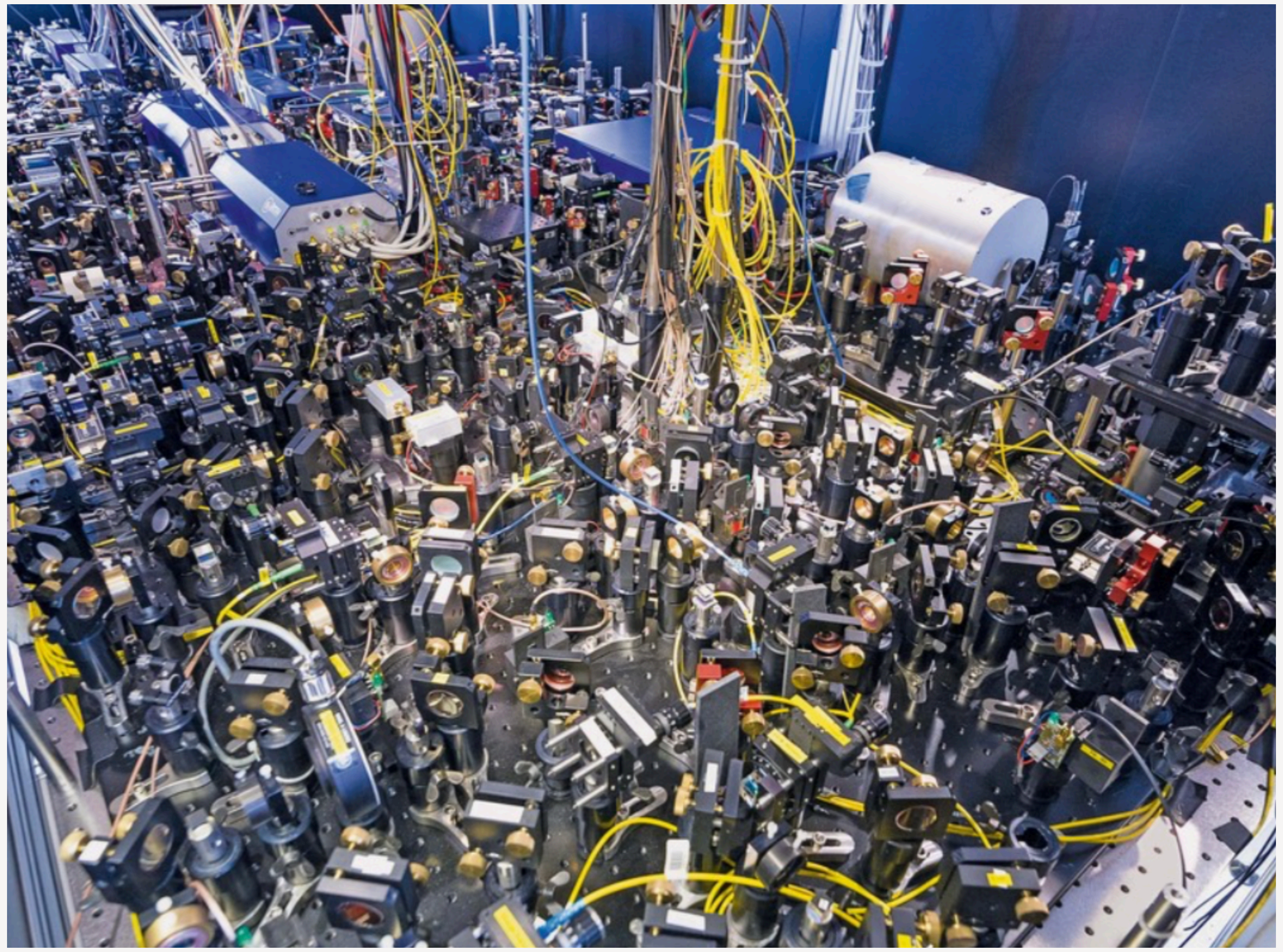

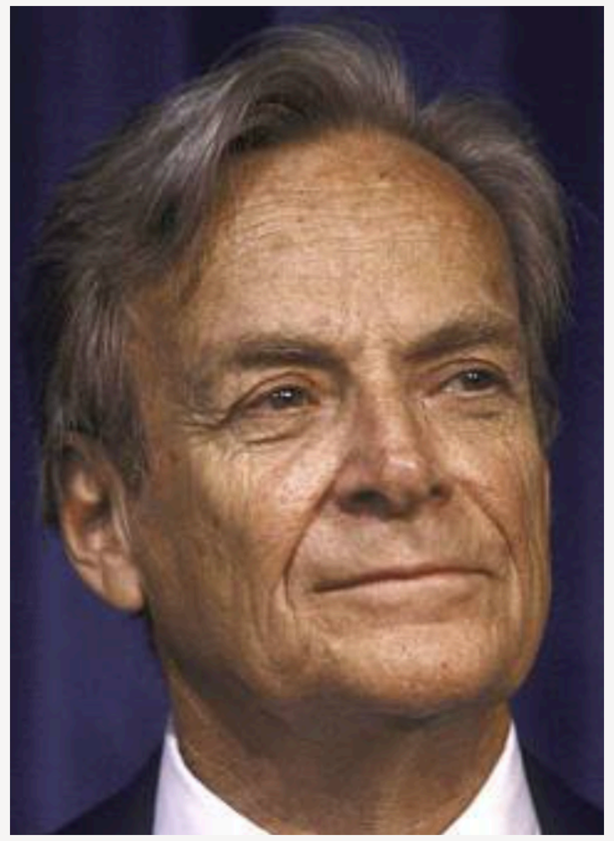

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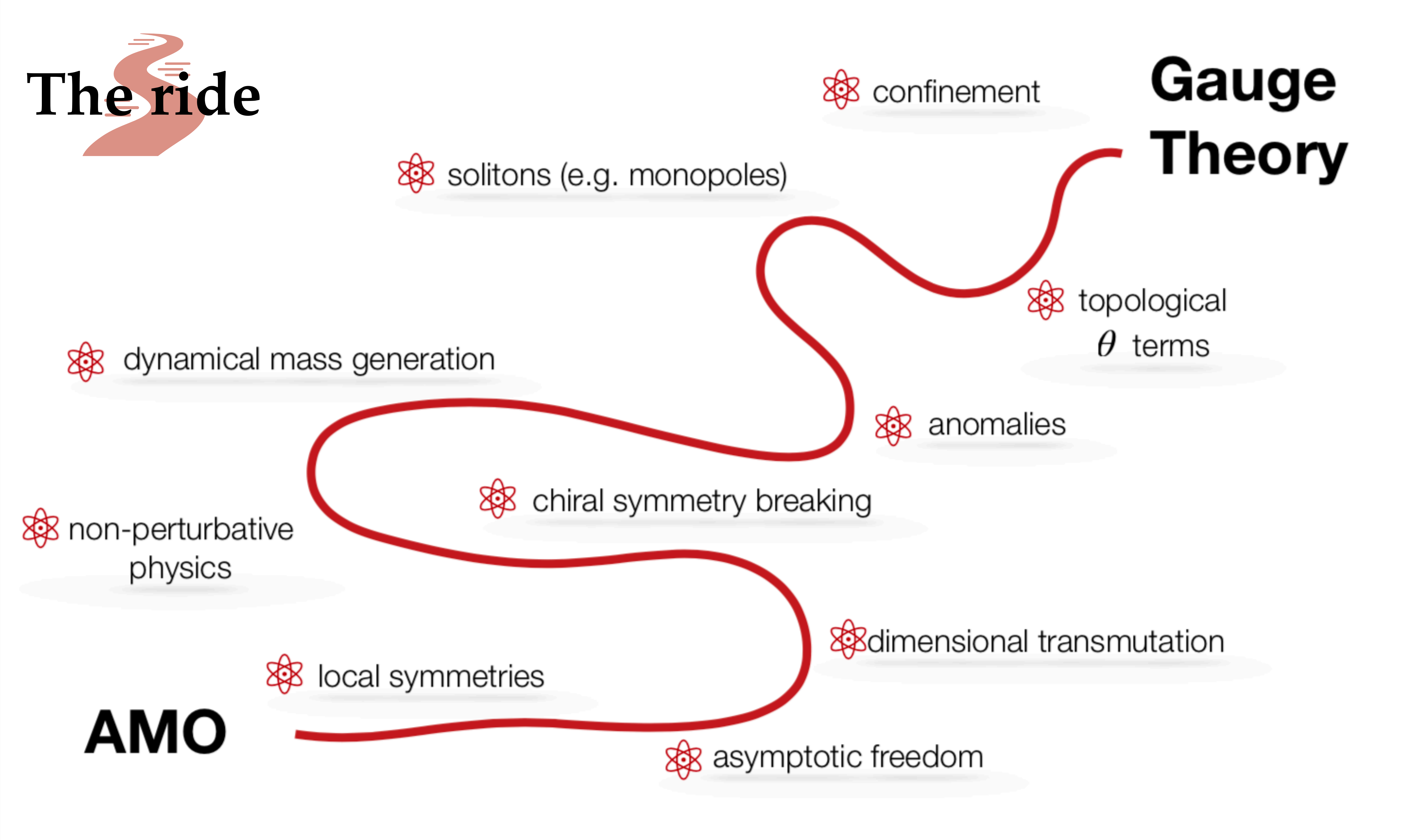
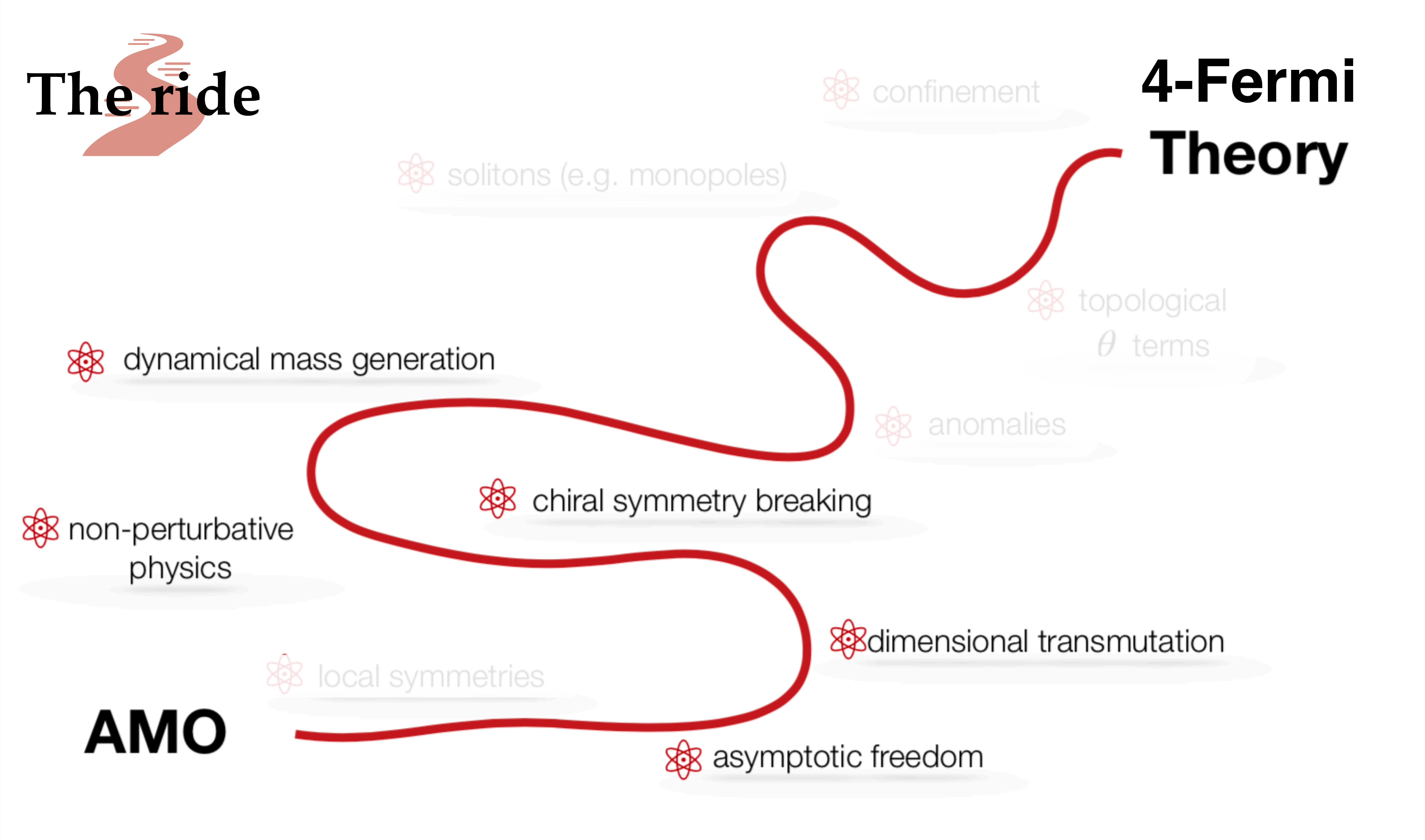


The ride
confinement
Gauge Theory
solitons (e.g. monopoles)
topological θ terms
dynamical mass generation
anomalies
chiral symmetry breaking
non-perturbative physics
dimensional transmutation
local symmetries
AMO
asymptotic freedom

MOTIVATION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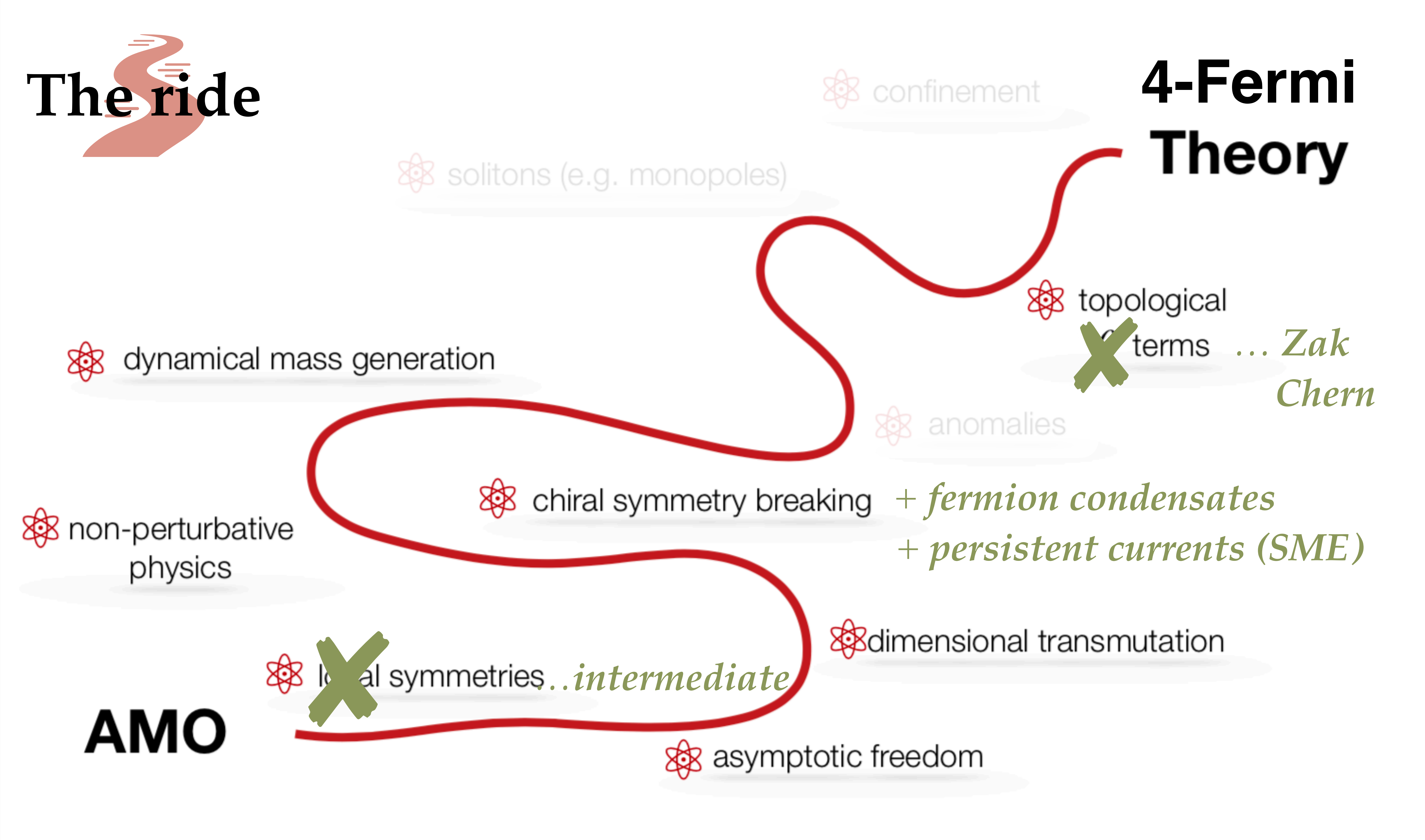
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… Zak
Chern
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+ fermion condensates
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…intermediate
AMO
asymptotic freedom

4-Fermi field theories in d+1 dimensions

We consider self-interacting Dirac fermions with N flavors in $D = d + 1$ spacetime dimensions

$x = (x^0, \vec{x})$ Minkowski spacetime

$\eta = \mathrm{diag}(1, -1, \ldots, -1)$ metric

$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ Dirac matrices

$$\mathscr{L} = \underbrace{\sum_{f=1}^{N} \overline{\psi}_{\mathrm{f}}(x)(\mathrm{i}\gamma^\mu \partial_\mu)\psi_{\mathrm{f}}(x)}_{\text{free Dirac QFT}} + \frac{g^2}{2N}\underbrace{\left(\sum_{\mathrm{f}=1}^{N} \overline{\psi}_{\mathrm{f}}(x)\psi_{\mathrm{f}}(x)\right)^2}_{\text{4-Fermi interactions}},$$

$\psi_f(x) = (\psi_{f,\uparrow}(x), \psi_{f,\downarrow}(x))^t, \quad \overline{\psi}_f(x) = \psi_f^\dagger(x)\gamma^0$

$d = 1, 2 \Rightarrow$ 2-component spinor $\longrightarrow$ $d = 1$, e.g. $\gamma^0 = \sigma^z, \ \gamma^1 = i\sigma^y, \ \gamma^5 = \gamma^0\gamma^1 = \sigma^x$

$\uparrow, \downarrow$

$d = 2$, e.g. $\gamma^0 = \sigma^z, \ \gamma^1 = i\sigma^y, \ \gamma^2 = -i\sigma^x, \ \cancel{\exists}\,\gamma^5$

4-Fermi term, e.g. $N=1 \equiv$ Hubbard $\frac{g^2}{2}\left(\overline{\psi}_f(\vec{x})\psi_f(\vec{x})\right)^2 \propto -g^2 n_{f,\uparrow}(x)\, n_{f,\downarrow}(x)$

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$D = 3 + 1$ non-renormalizable QFT, χ**SB** by dynamical mass generation

th Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

$D = 1 + 1$ renormalizable QFTs, χ**SB** by dynamical mass generation, asymptotic freedom, dimensional transmutation

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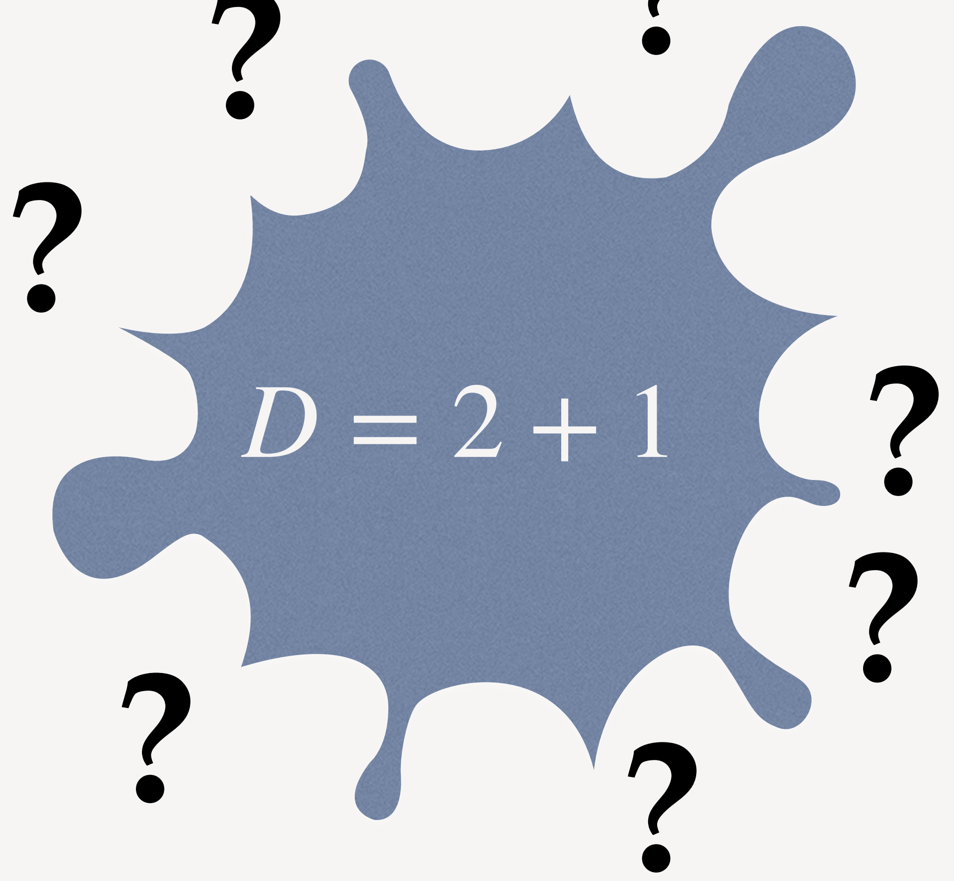
$N \lessgtr N_c$ $\overset{?}{=}$

Strong-coupling $\overset{?}{=}$ fixed point



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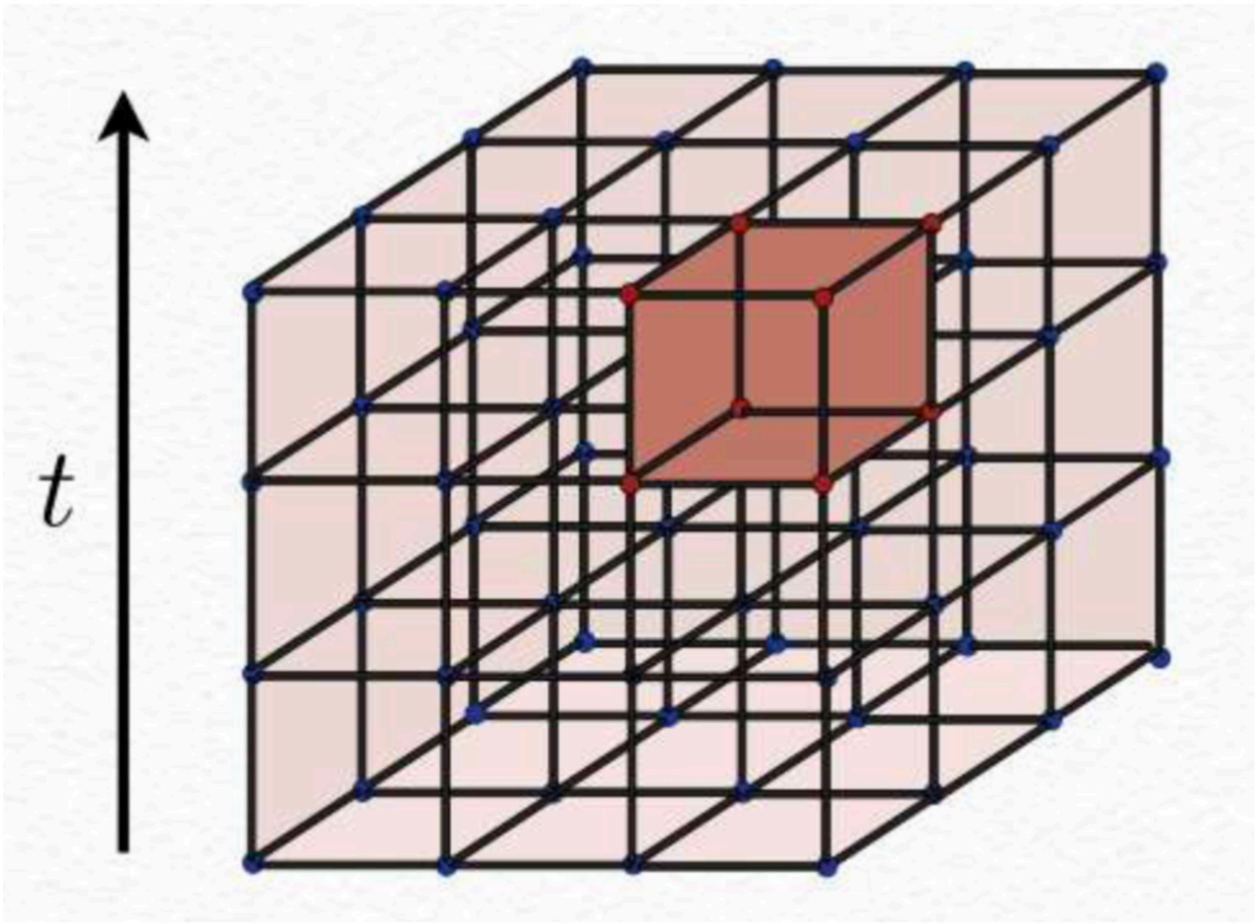
- S. Hands, arXiv:hep-lat/9706018.

4-Fermi lattice field theories in d+1 dimensions

Exploring non-perturbative effects (e.g. strong-coupling fixed point) by an **artificial lattice**

Euclidean LFTs vs Hamiltonian LFTs



$x^0 = t \rightsquigarrow$ continuous time — th L. Susskind, PRD **16**, 3031 (1977).

$\bar{x} = a\, n_j \hat{e}_j$, $\bar{x}/a \in \mathbb{Z}^d \rightsquigarrow$ discretized space $\Lambda_c = 2\pi/a$ ≡ UV cutoff

$\partial_j \psi(x) = \dfrac{\psi(\bar{x} + a\hat{e}_j) - \psi(\bar{x} - a\hat{e}_j)}{2a} \rightsquigarrow$ naïve discretization

$$S_D^{\Lambda_c} \xrightarrow[a\to 0]{} \sum_{\bar{n}_d} \bar{\Psi}_{\bar{n}_d}(x)\left(i\gamma^\mu_{\bar{n}_d}\partial_\mu - m\right)\Psi_{\bar{n}_d}(x) = \sum_{\bar{n}_d} S_{\bar{n}_d}$$

$\bar{k}_{\bar{n}_d} = \frac{\pi}{a}\bar{n}_d$, $\bar{n}_d \in \mathbb{Z}_2^d$, $\mathbb{Z}_2 = \{0,1\}$ → $N_D = 2^d$ fermion doublers $\gamma^5_{\bar{n}_d} \in \{-\gamma^5, +\gamma^5\}$ opposite chiralities $N_+ = N_D/2$, $N_- = N_D/2$

We recover Dirac QFT @ center & faces/corners BZ ≡ Γ, X, M points

Wilson lattice regularization in d+1 dimensions

There are alternative discretizations that **deal differently with the doublers**

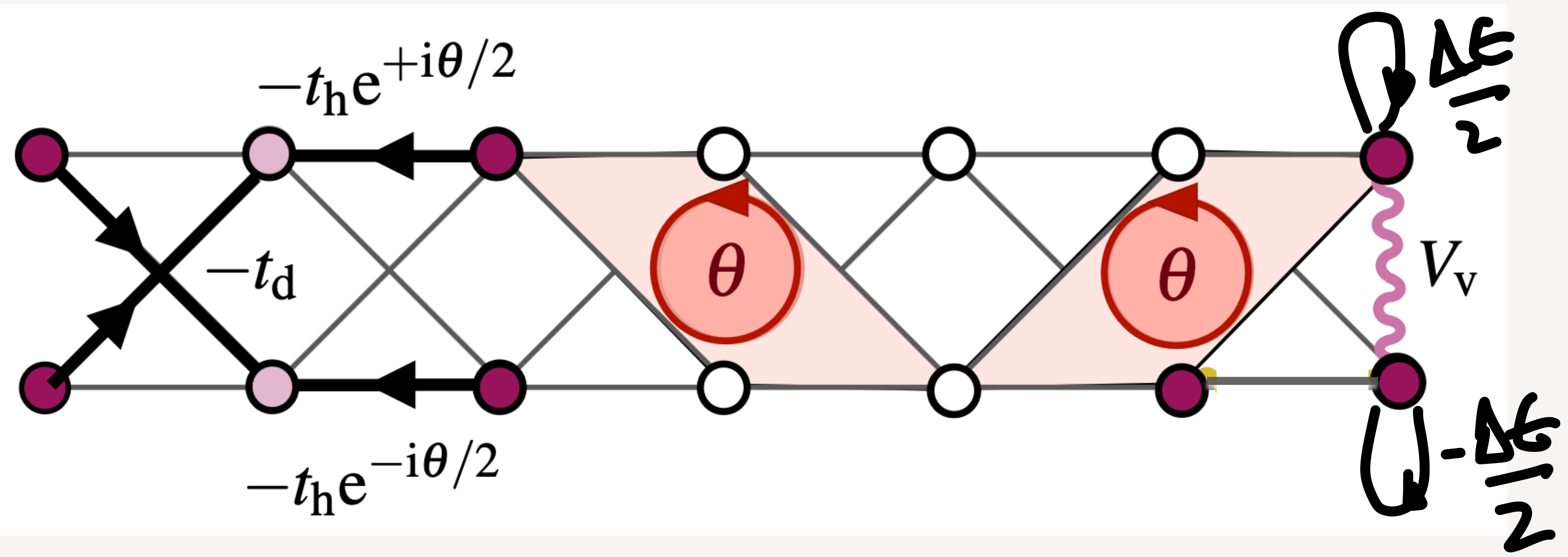
$$H = a_1\cdots a_d \sum_{\boldsymbol{x}\in\Lambda_{\rm s}}\left[\sum_{j=1}^{d}\left(-\overline{\Psi}(\boldsymbol{x})\left(\frac{{\rm i}\gamma^j}{2a_j}+\frac{r_j}{2a_j}\right)\Psi(\boldsymbol{x}+a_j\mathbf{e}_j)+\overline{\Psi}(\boldsymbol{x})\left(\frac{m}{4}+\frac{r_j}{2a_j}\right)\Psi(\boldsymbol{x})+{\rm H.c.}\right)-\frac{g^2}{2N}\left(\overline{\Psi}(\boldsymbol{x})\Psi(\boldsymbol{x})\right)^2\right],$$

Wilson-type discretization

$r_j \in (0,1)$

K. G. Wilson, in *New Phenomena in Subnuclear Physics* (1977)

d=1, N=1, Unitarily equivalent to an **imbalanced cross-linked ladder** pierced by a **U(1) field**

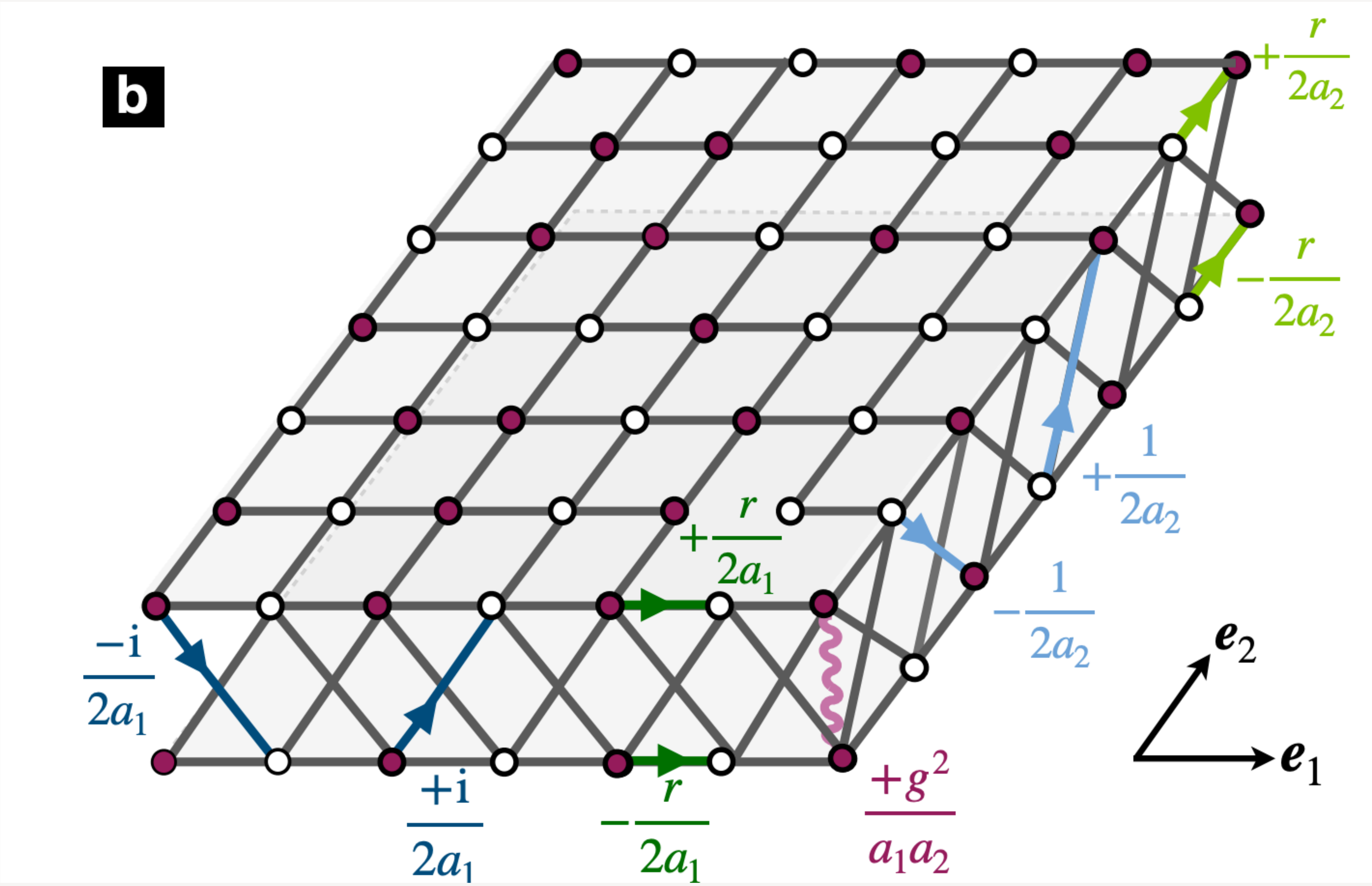


$N=1,\ \theta=\pi$ π-flux ladder

$t_d = \frac{t_h}{r} = \frac{1}{2a}$, $ma = \frac{\Delta\epsilon}{4t_h} - r$, $\frac{g^2}{2} = \frac{V_v}{4t_h}$

Wilson lattice regularization in d+1 dimensions

d=2, N=1, equivalent to an **imbalanced cross-linked bilayer** , again with **Hubbard interactions**



Intra-layer tunneling along $\hat{e}_j$

$t_j = \pm r/2a_j$

$+ \longrightarrow \uparrow$ layer
$- \longrightarrow \downarrow$ layer

Inter-layer tunneling along $\hat{e}_j$

$\hat{t}_1 = \pm \frac{i}{2a_1}$ $\hat{t}_2 = \pm \frac{1}{2a_2}$

Inter-layer Hubbard interaction

$g^2 = \frac{V_U}{4|\hat{t}_1 \hat{t}_2|}$ $ma = \frac{\Delta\varepsilon}{4|\hat{t}_1|} - (r_1 + r_2 \xi_2)$

$\xi_2 = a_1/a_2$

SPT phases in D=1+1 dimensions

This discretization can host **symmetry-protected topological groundstates**

$$H_{\mathrm{C}} = -\sum_{i,\ell}\left(t_{\mathrm{h}}\mathrm{e}^{-\mathrm{i}\frac{s_\ell\theta}{2}}c^\dagger_{i+1,\ell}c_{i,\ell} + t_{\mathrm{d}}c^\dagger_{i+1,\ell}c_{i,\bar{\ell}} - \frac{s_\ell\Delta\varepsilon}{2}c^\dagger_{i,\ell}c_{i,\ell} + \mathrm{H.c.}\right),$$

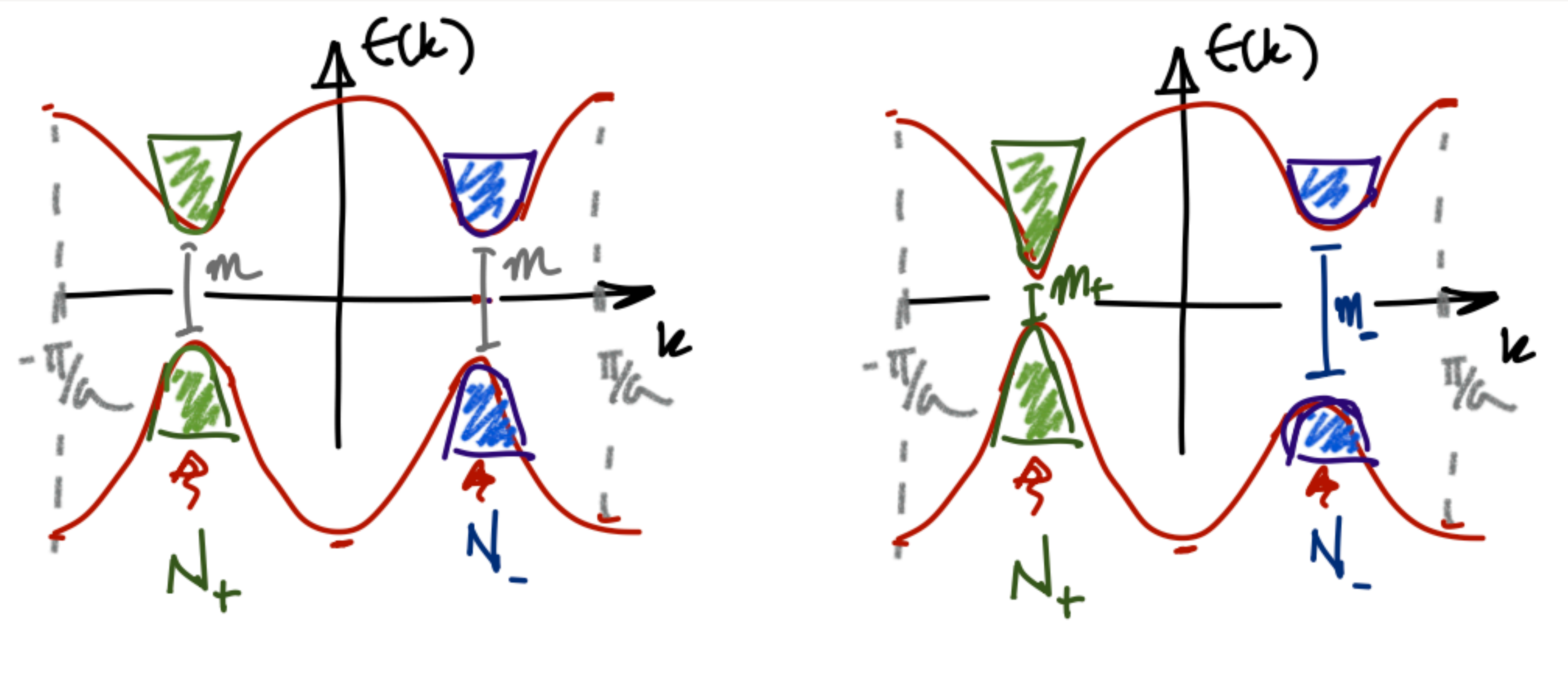
$$H_{\mathrm{CH}} = H_{\mathrm{C}} + \frac{V_{\mathrm{v}}}{2}\sum_{i,\ell}c^\dagger_{i,\bar{\ell}}c^\dagger_{i,\ell}c_{i,\ell}c_{i,\bar{\ell}},$$

Imbalanced & interacting version of Creutz's ladder

th M. Creutz, Phys. Rev. Lett. **83**, 2636 (1999).

Naive

Wilson $\theta=\pi$



$$\left.\begin{aligned} m_+ a &= \frac{\Delta\varepsilon}{4t_{\mathrm{H}}} - r \\ m_- a &= \frac{\Delta\varepsilon}{4t_{\mathrm{H}}} + r \end{aligned}\right\}$$ momentum-dependent masses

AIII/BDI phases for non-vanishing **Berry phase**

$$\gamma_{\mathrm{B}} = \frac{\pi}{2}\left(\mathrm{sgn}(m_-) - \mathrm{sgn}(m_+)\right)$$

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$$H_{\rm C} = -\sum_{i,\ell}\left(t_{\rm h}e^{-{\rm i}\frac{s_\ell\theta}{2}}c^\dagger_{i+1,\ell}c_{i,\ell} + t_{\rm d}c^\dagger_{i+1,\ell}c_{i,\bar{\ell}} - \frac{s_\ell\Delta\varepsilon}{2}c^\dagger_{i,\ell}c_{i,\ell} + {\rm H.c.}\right),$$

$$H_{\rm CH} = H_{\rm C} + \frac{V_{\rm v}}{2}\sum_{i,\ell}c^\dagger_{i,\bar{\ell}}c^\dagger_{i,\ell}c_{i,\ell}c_{i,\bar{\ell}},$$

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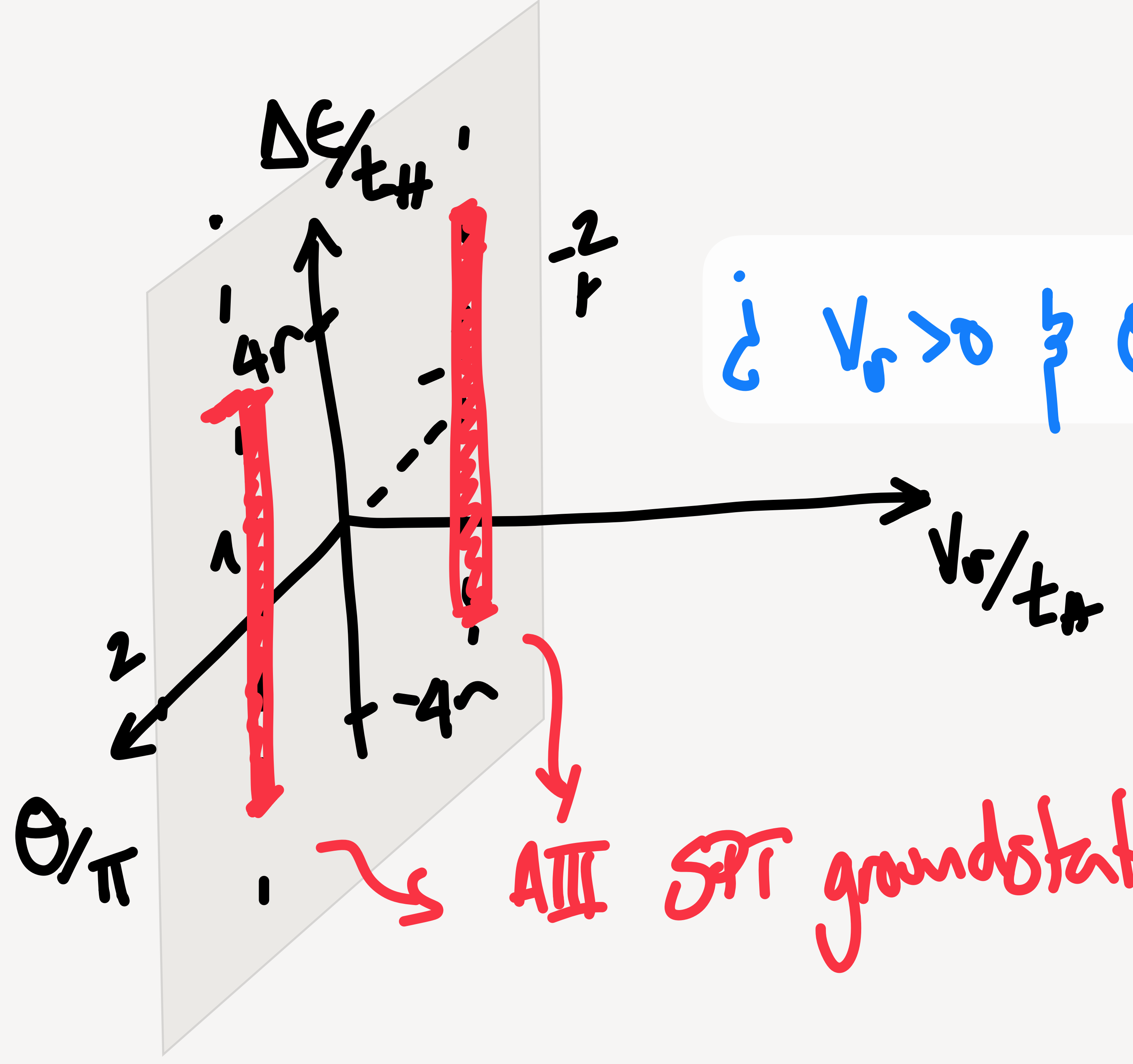


AIII SPT groundstate $\gamma_B = \pm\pi$

¿ $V_v > 0$ & $\theta \neq \pm\pi$?

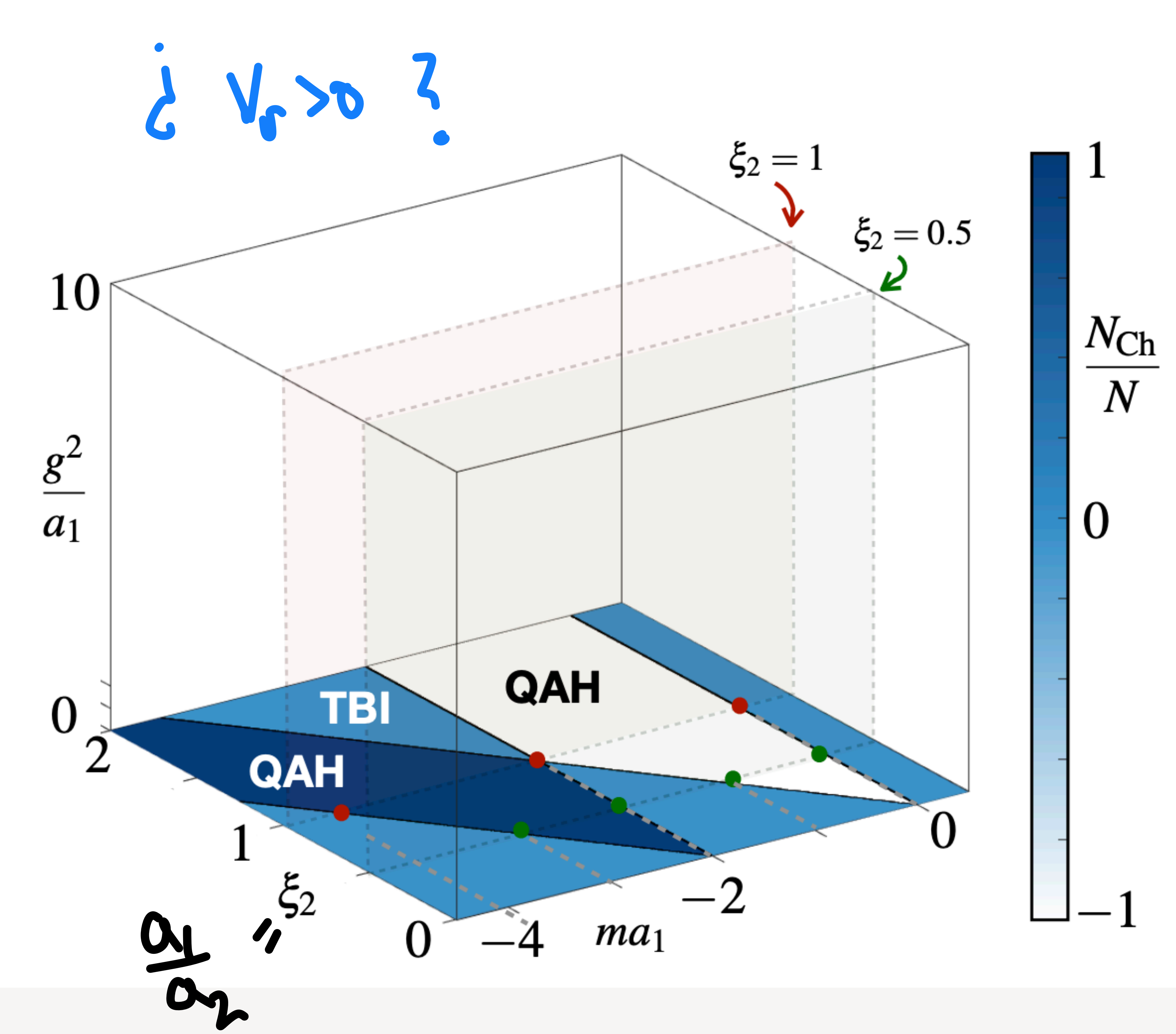
$$\left.\begin{aligned} m_+ a &= \frac{\Delta\epsilon}{4t_H} - r \\ m_- a &= \frac{\Delta\epsilon}{4t_H} + r \end{aligned}\right\} \text{momentum-dependent masses}$$

AIII/BDI phases for non-vanishing **Berry phase**

$$\gamma_{\rm B} = \frac{\pi}{2}\left({\rm sgn}(m_-) - {\rm sgn}(m_+)\right)$$

Chern insulators in D=2+1 dimensions

This discretization can host an **quantum anomalous Hall (QAH) effect**



momentum-dependent Wilson masses

$\bar{n}_d \in \mathbb{Z}_2 \times \mathbb{Z}_2$

$$m_{\boldsymbol{n}_d} = m + \frac{2r_1}{a_1} n_{d,1} + \frac{2r_2}{a_2} n_{d,2}.$$

bilayer version of Haldane's QAH

D. Haldane, Phys. Rev. Lett. **61,** 2015 (1988).

M. Golterman, K. Jansen, and D. Kaplan, PLB **301,** 219 (1993).

Non-vanishing **Chern numbers**

$$N_{\mathrm{Ch}} = \frac{N}{2} \sum_{\boldsymbol{n}_d} (-1)^{(n_{d,1}+n_{d,2})} \mathrm{sign}(m_{\boldsymbol{n}_d}).$$

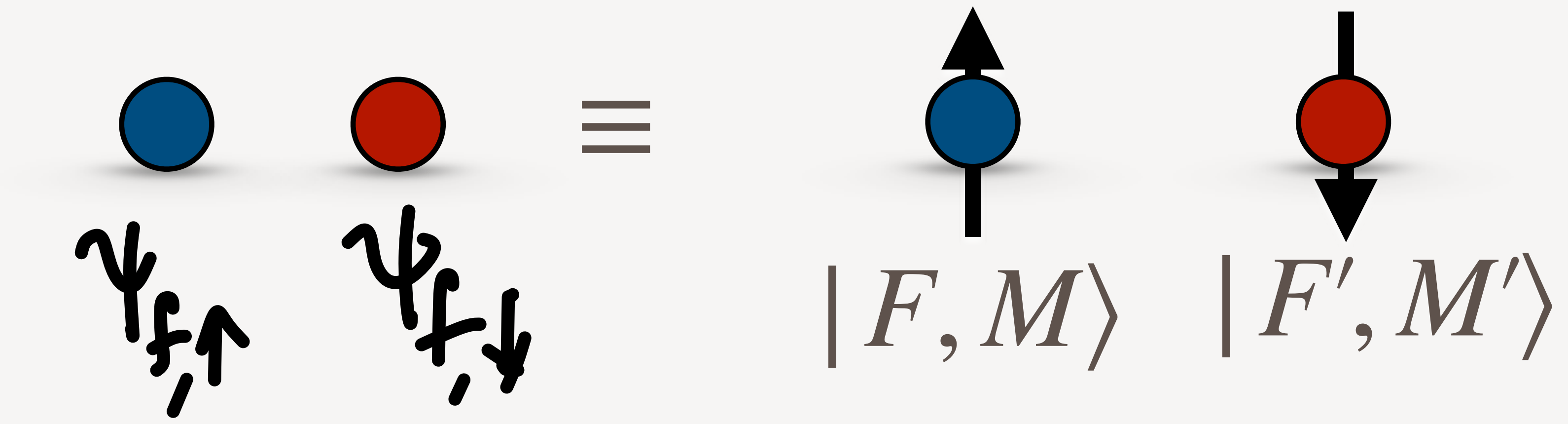
Cold-atom regularization: photon-assisted tunneling

Dirac spinor as two **hyperfine states** of a neutral Fermi gas, e.g. ^{87}Sr

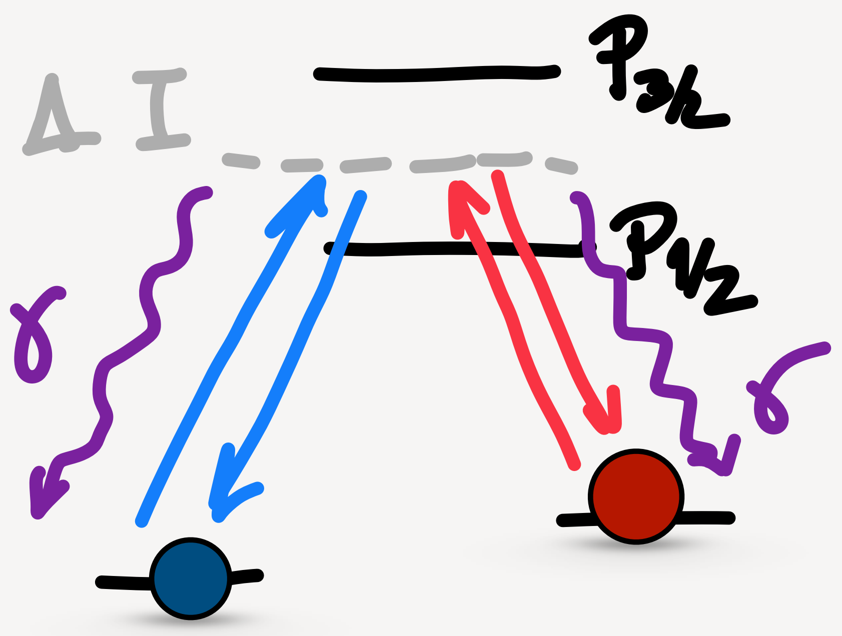
+

Raman-assisted tunneling in spin-dependent lattices

th D. Jaksch, P.Zoller, New J. Phys. **5**, 56 (2003).



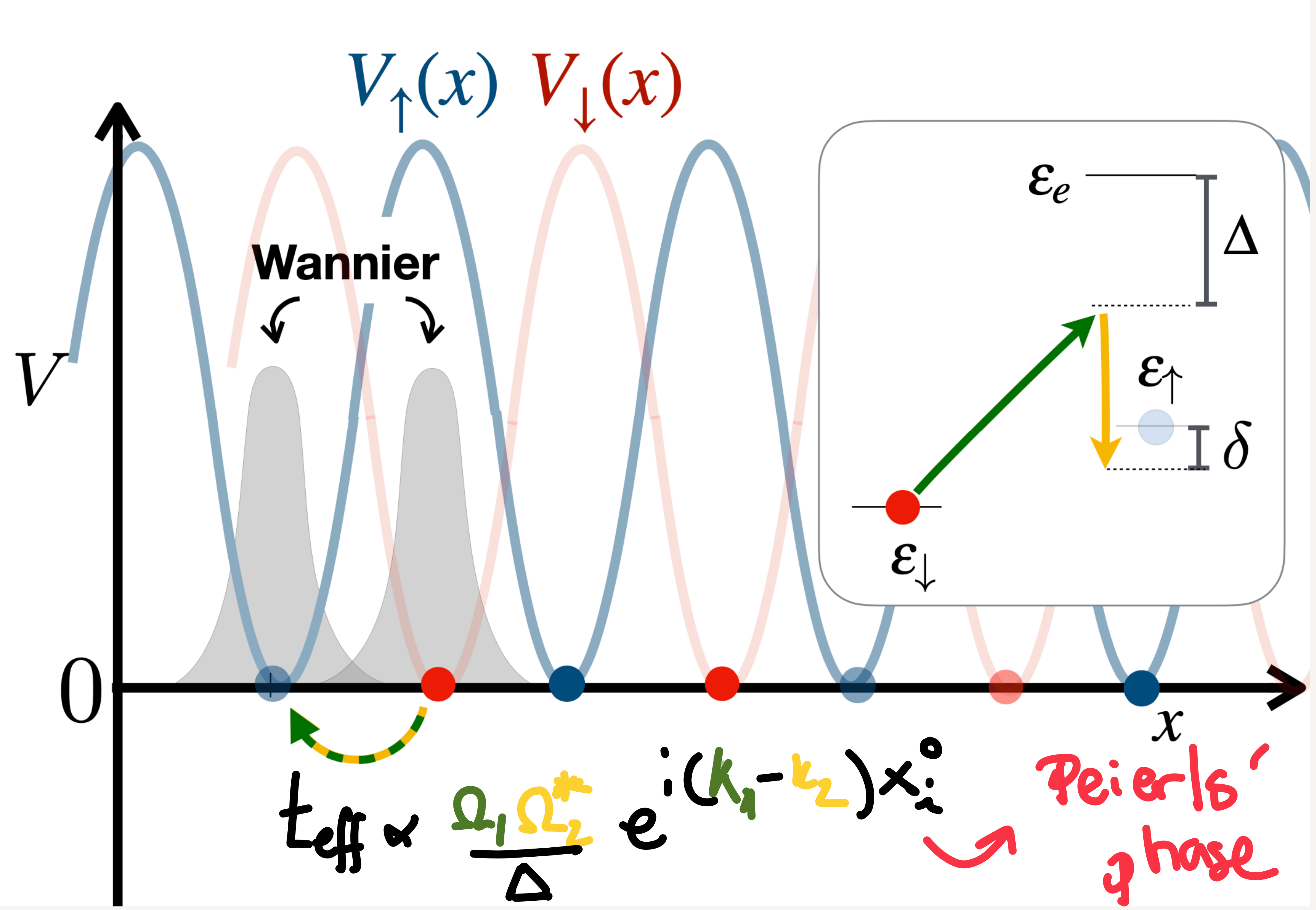
Problem with spin-dep lattices for fermions



Δ limited by fine structure

⇓

residual γ scattering



Alternatives based on e.g. **Floquet** instead of Raman

ex M. Aidelsburger, et al., PRL **111**, 185301 (2013).

Cold-atom regularization: Raman lattices

The advantage is that we need both spin-conserving and spin-flipping tunneling

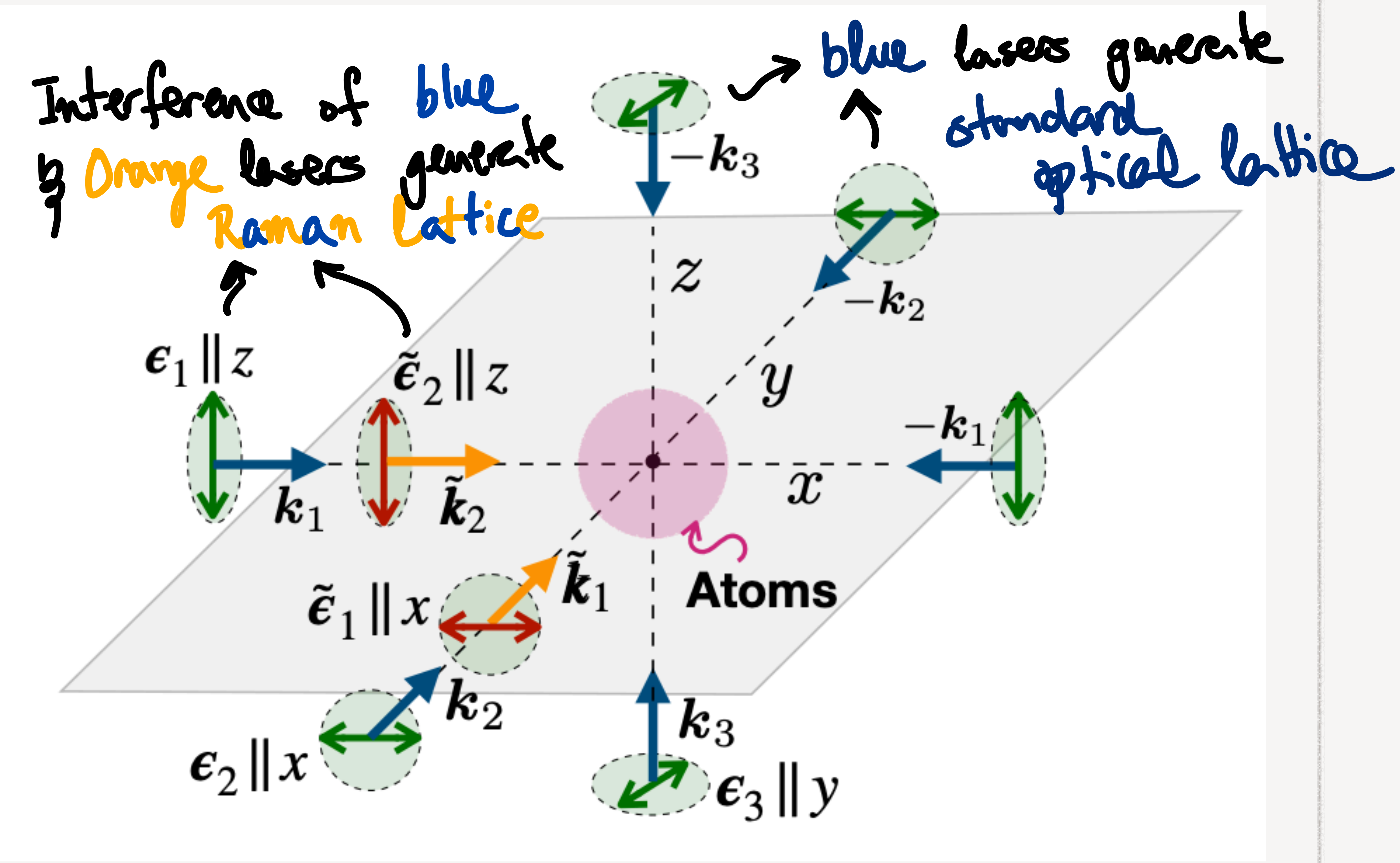
Avoid spin-dependent lattices

⇓

The **standard & cross-link** tunnelings provided by a **Raman optical lattice**

Recent experiment with fermions

ex M.-C. Liang., et al., **arXiv:2109.08885**



r L. Zhang and X.-J. Liu, Synthetic Spin-Orbit Coupling in Cold Atoms, pp. 1-87 (World Scientific 2018)

Cold-atom regularization: Raman lattices

The **spin-conserving tunnelings** stem from the standard lattice t_j

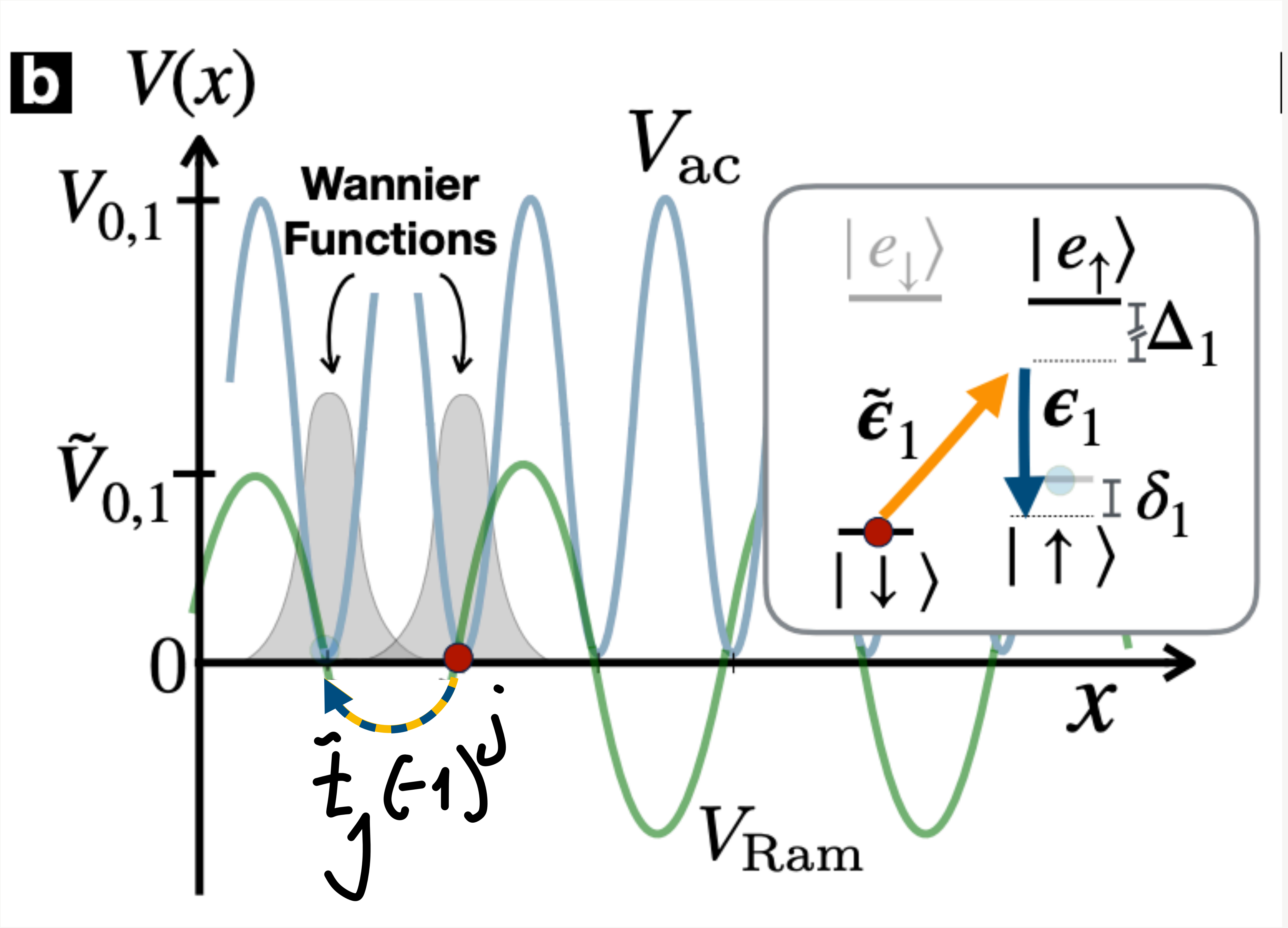
The **spin-flipping terms** benefit from the **doubled period** of the **Raman potential**

No local spin flips

$$\int \mathrm{d}^3x\, w(\boldsymbol{x}-\boldsymbol{x}_i^0)\frac{\tilde{V}_{0,1}}{2}\cos(\tilde{\boldsymbol{k}}_1\cdot\boldsymbol{x})w(\boldsymbol{x}-\boldsymbol{x}_i^0)=0.$$

Alternating spin-flip tunneling

$$\tilde{t}_j=\left|\int \mathrm{d}^3x\, w(\boldsymbol{x}-\boldsymbol{x}_i^0)\frac{\tilde{V}_{0,1}}{2}\cos(\tilde{\boldsymbol{k}}_1\cdot\boldsymbol{x})w(\boldsymbol{x}-\boldsymbol{x}_{i+\boldsymbol{e}_j}^0)\right|\neq 0$$



Cold-atom mapping $a_j=\frac{1}{2\tilde{t}_j}$, $r_j=\frac{t_j}{\tilde{t}_j}$, $m=\frac{\delta}{2}-2(t_1+t_2)$, $g^2=\frac{U_{\uparrow\downarrow}}{4\tilde{t}_1\tilde{t}_2}$, $U_{\uparrow\downarrow}\propto a_{s\text{-wave}}$

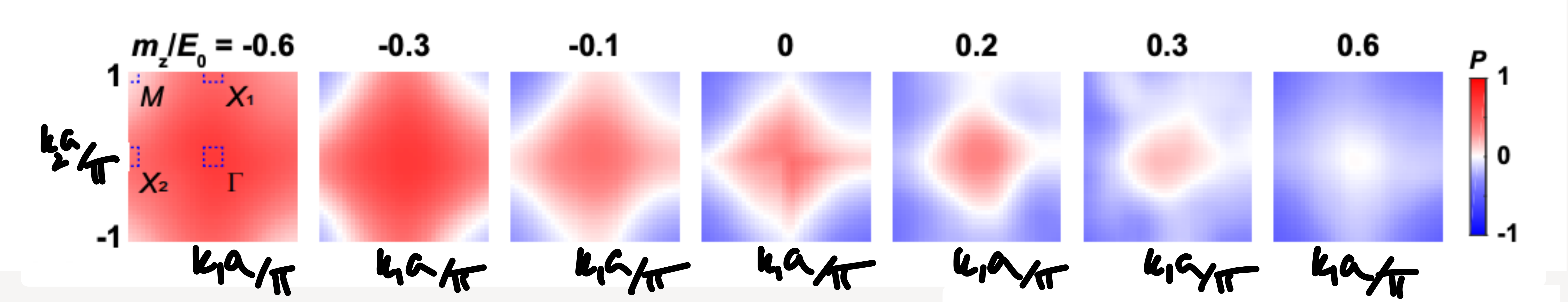
Cold-atom regularization: Raman lattices

The **non-interacting limit** has been explored in **cold-atom experiments** $N \sim 10^4$ ^{87}Sr @ $0.2\,\mu K$

ex M.-C. Liang, et al., arXiv:2109.08885

Spin-resolved ToF

$n_\uparrow(\bar{k})$, $n_\downarrow(\bar{k})$



Polarization

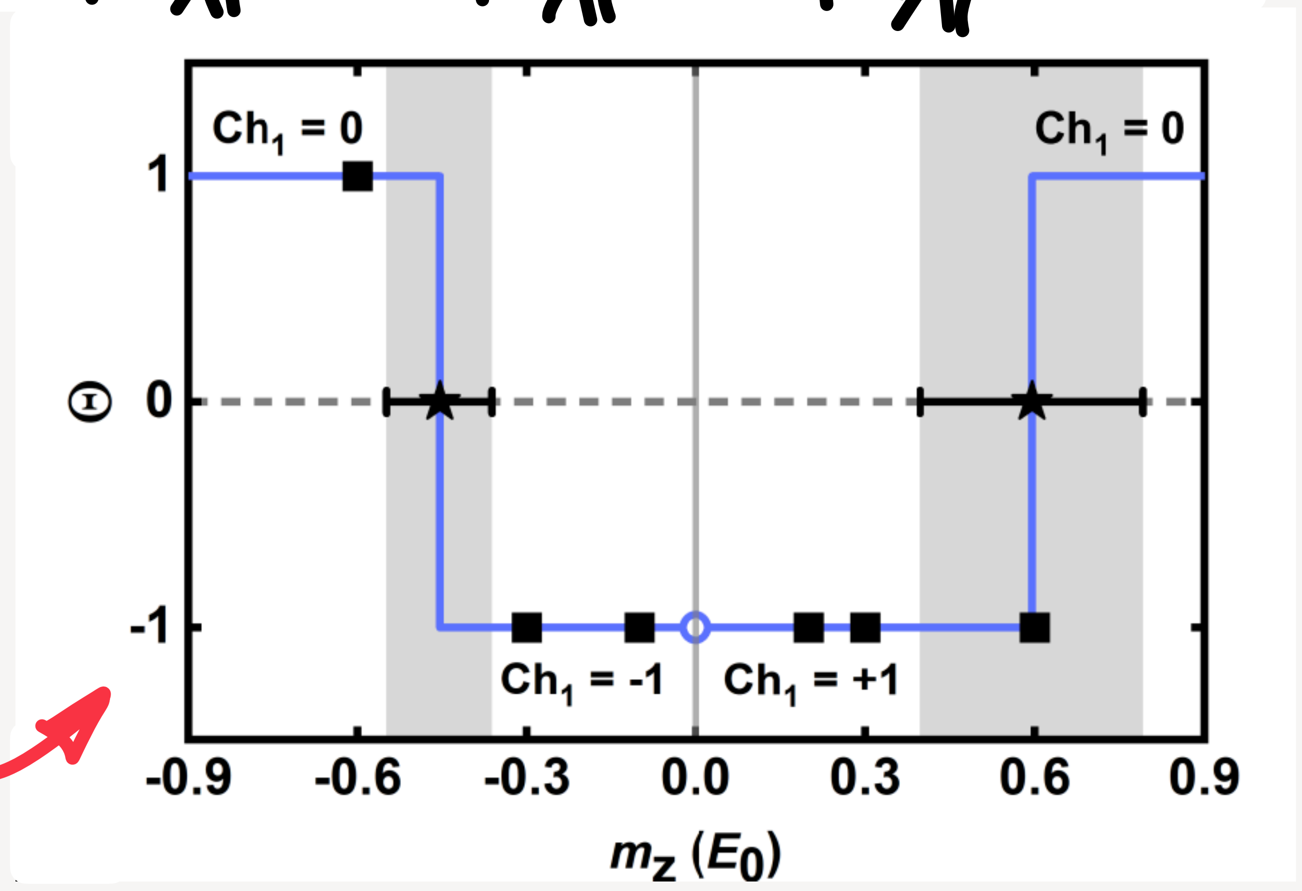
$$P_{\bar{k}} = \frac{n_\uparrow(\bar{k}) - n_\downarrow(\bar{k})}{n_\uparrow(\bar{k}) + n_\downarrow(\bar{k})}$$

$$\Rightarrow \quad \Theta = \mathrm{sgn}\left(P(\Gamma)\cdot P(X_2)\cdot P(X_1)\, P(M)\right)$$

center, corner, faces of BZ

"doublers"

equivalent to ch_1 number



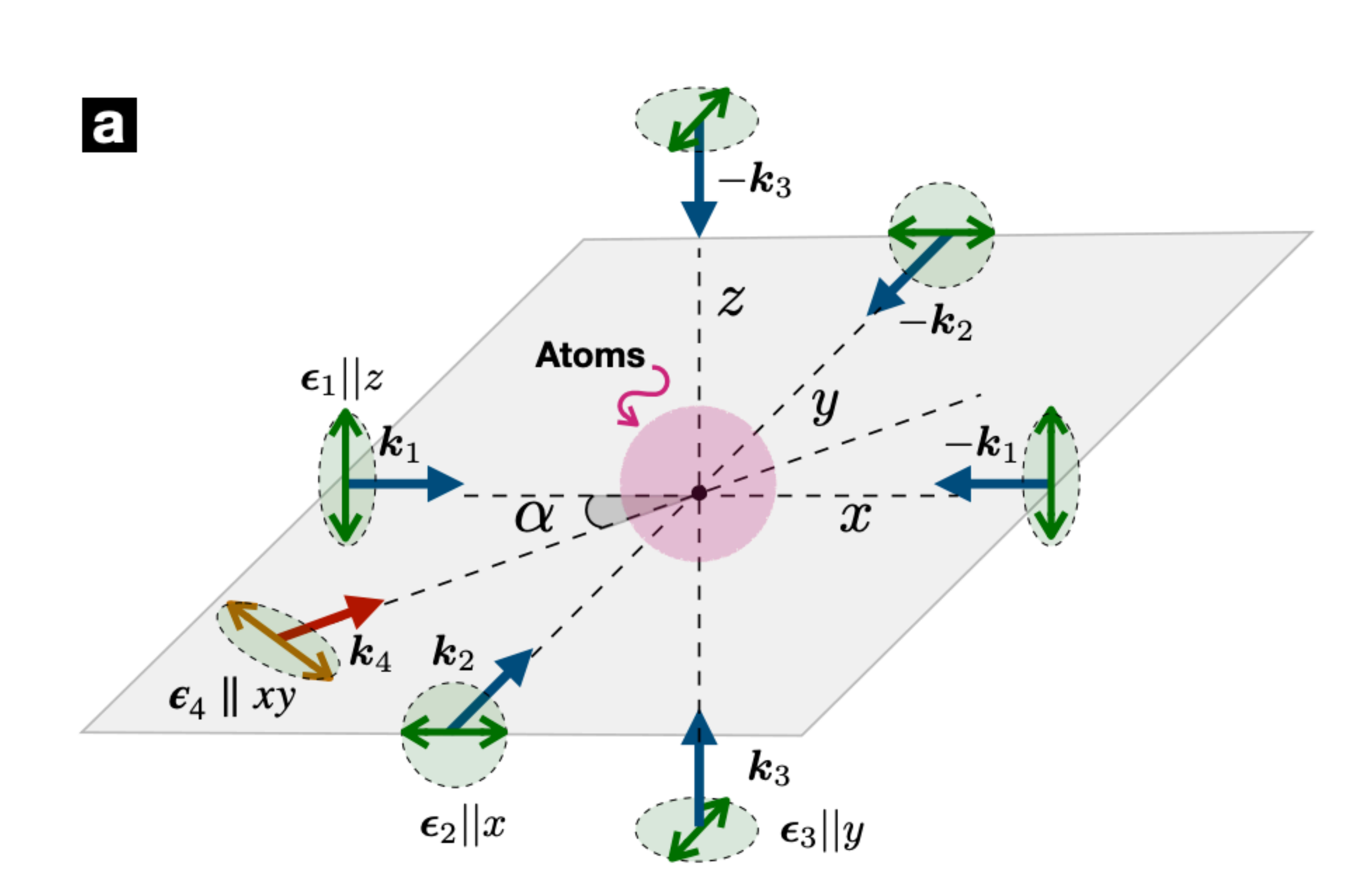
Cold-atom regularization: Raman lattices

The **interacting limit** could be explored by tuning **Feshbach resonances** $a_{s\text{-wave}}(B_{ext})$

$$g^2 \propto T_v \propto k_L a_{s\text{-wave}}(B_{ext}) \longrightarrow \boxed{V_r > 0 \;?}$$

One could **tilt** the **Raman beams**
to induce a **U(1) gauge field** (away from pi-flux)

$$\theta = \pi\left(1 + \frac{k_4}{k_1}\cos\alpha\right) \longrightarrow \boxed{\theta \neq \pm\pi \;?}$$



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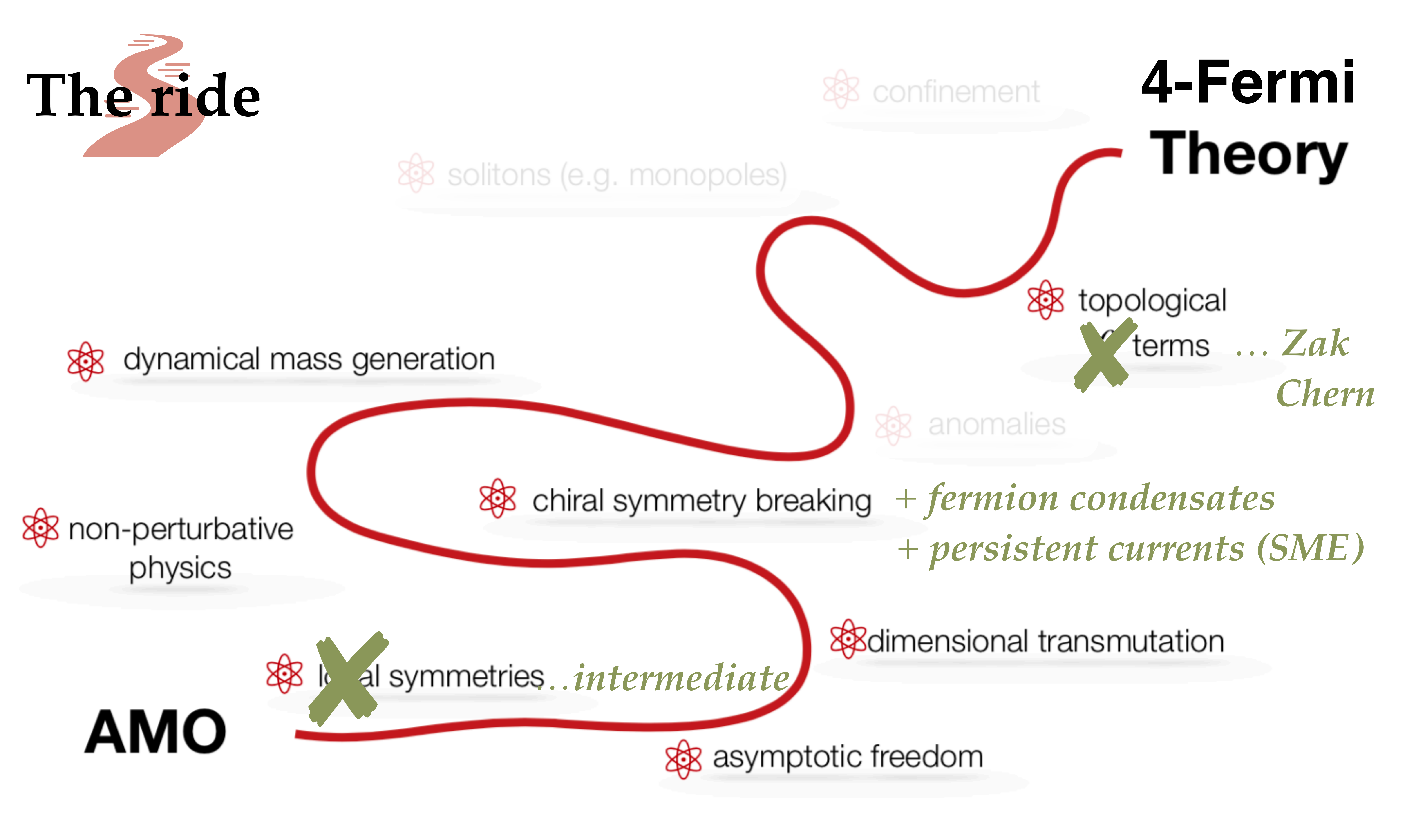

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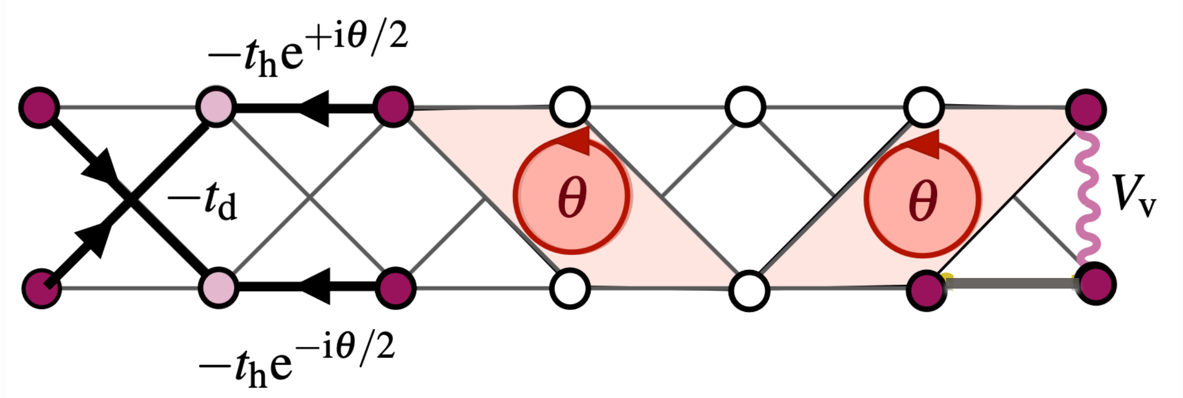
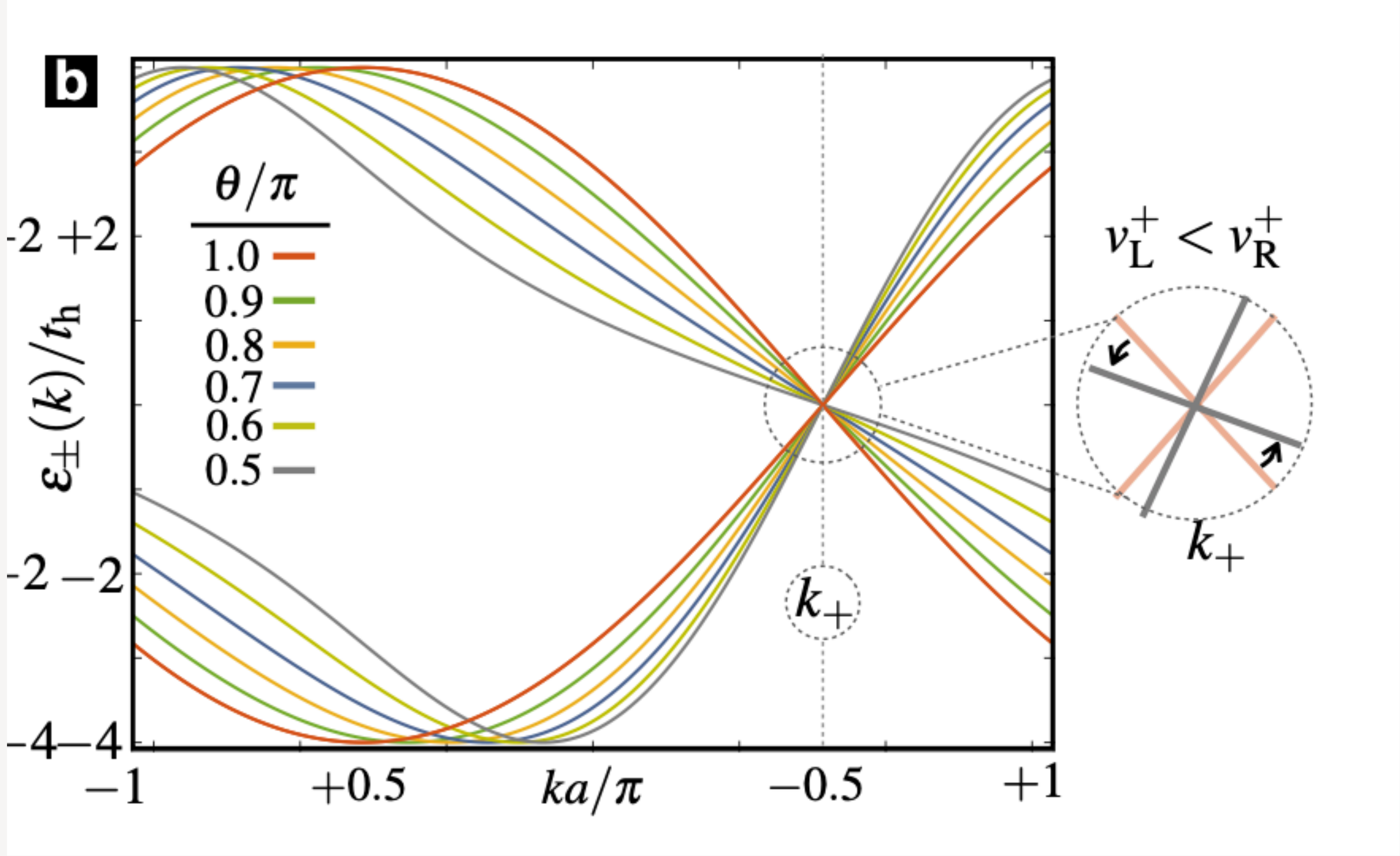


Lorentz violations @ generic θ

The continuum limit now described by Dirac QFT with Lorentz violation

th Colladay, Kostelecky, PRD 58, 116002 (1998)



$$S_{\mathrm{CH}} = \int d^2x \left(\sum_{\eta=+,-} \overline{\Psi}_\eta(x) \left(ic\Gamma^\mu_\eta \partial_\mu - m_\eta c^2 \right) \Psi_\eta(x) \right)$$

SME: Lorentz violation modifies Dirac γ^μ

$$\Gamma^\mu_\eta = \gamma^\mu_\eta + c^{\mu\nu}_\eta g_{\mu\tau} \gamma^\tau_\eta.$$

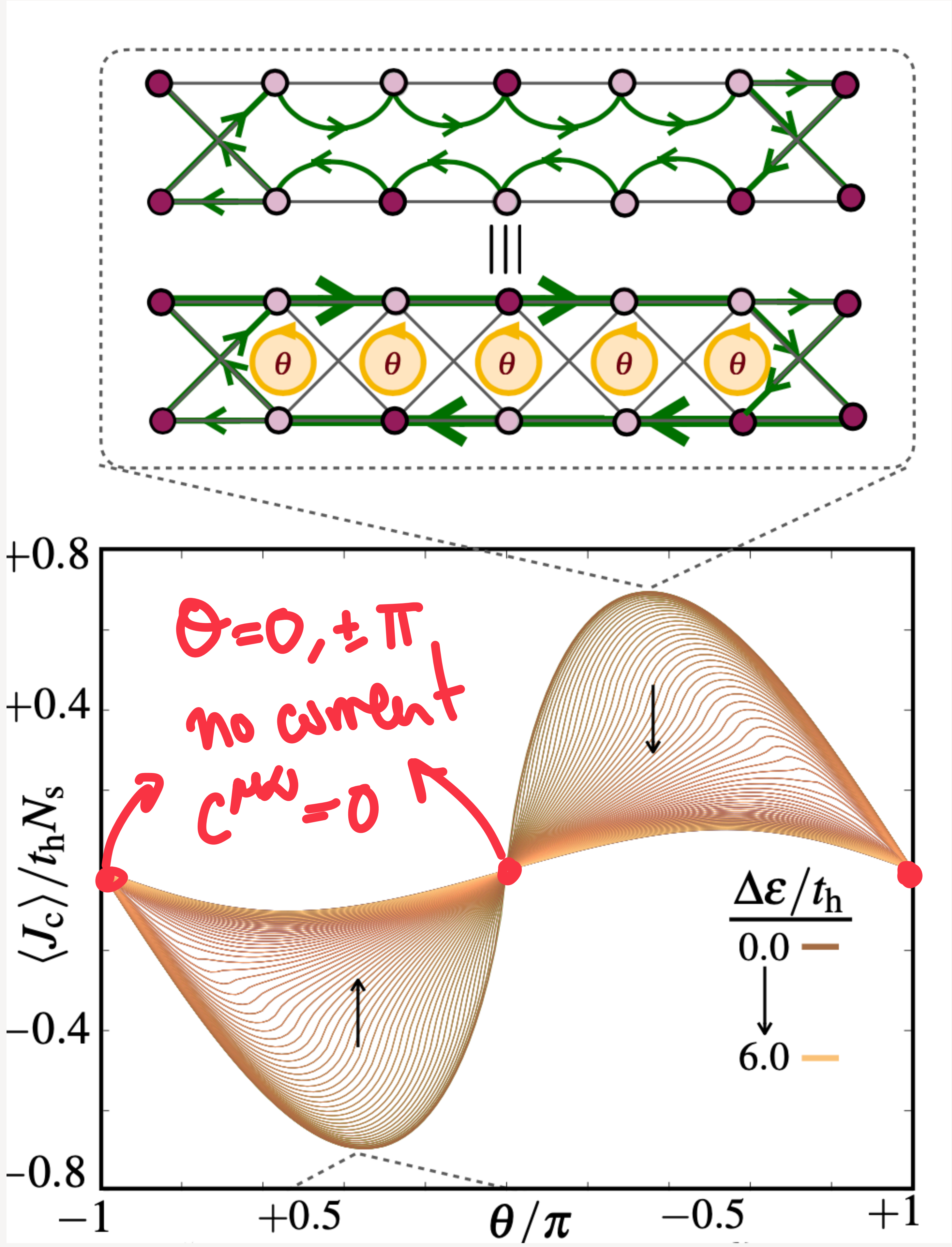
SME: tr-less tensor $c^{\mu\nu}$

$$c^{\mu\nu}_\pm = \begin{cases} \pm\cos(\frac{\theta}{2}) & \text{if } \mu = 1, \nu = 0 \\ 0 & \text{else,} \end{cases}$$

This specific form amounts to diff. v_R / v_L

$$v_R^\pm = 2t_h a \left(1 \pm \cos\left(\frac{\theta}{2}\right)\right), \quad v_L^\pm = 2t_h a \left(1 \mp \cos\left(\frac{\theta}{2}\right)\right)$$

Persistent circulating 'chiral' currents @ generic θ



Different R/L-moving velocities suggest possible net flow

"Chiral" current

$$J_c = \sum_j \left(\mathrm{i}t_h e^{\mathrm{i}\frac{\theta}{2}} c^\dagger_{j+1,\uparrow} c_{j,\uparrow} - \mathrm{i}t_h e^{-\mathrm{i}\frac{\theta}{2}} c^\dagger_{j+1,\downarrow} c_{j,\downarrow} + \mathrm{H.c.} \right).$$

$\wr$

skipping orbits in QHE / screening Meissner current bosonic SF

$\langle J_c \rangle$ has been measured in neutral-atom standard ladder

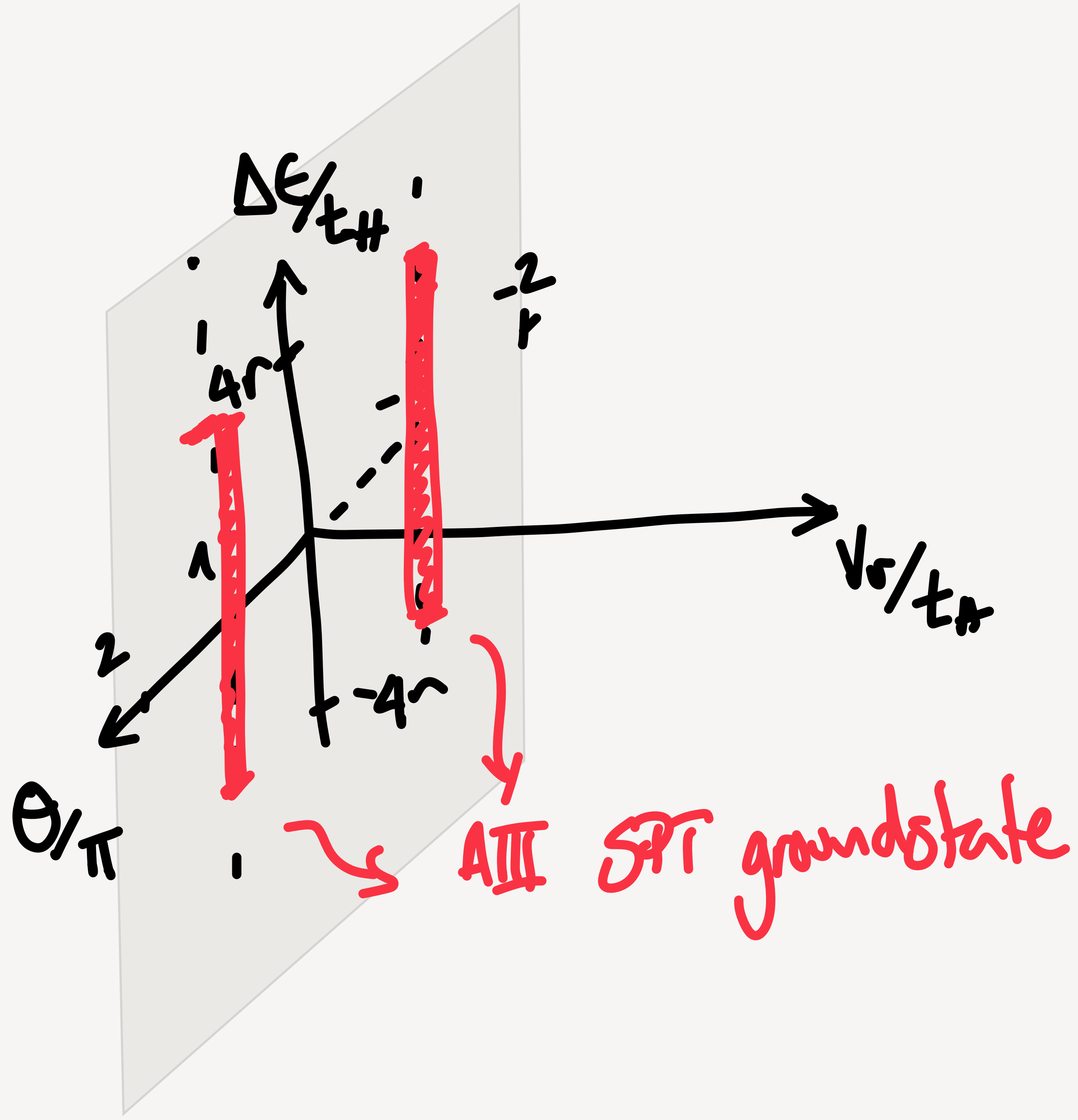
ex M. Atala, et al., Nat Phys **10,** 588 (2014).

The "chiral" susceptibility can diverge @ critical points

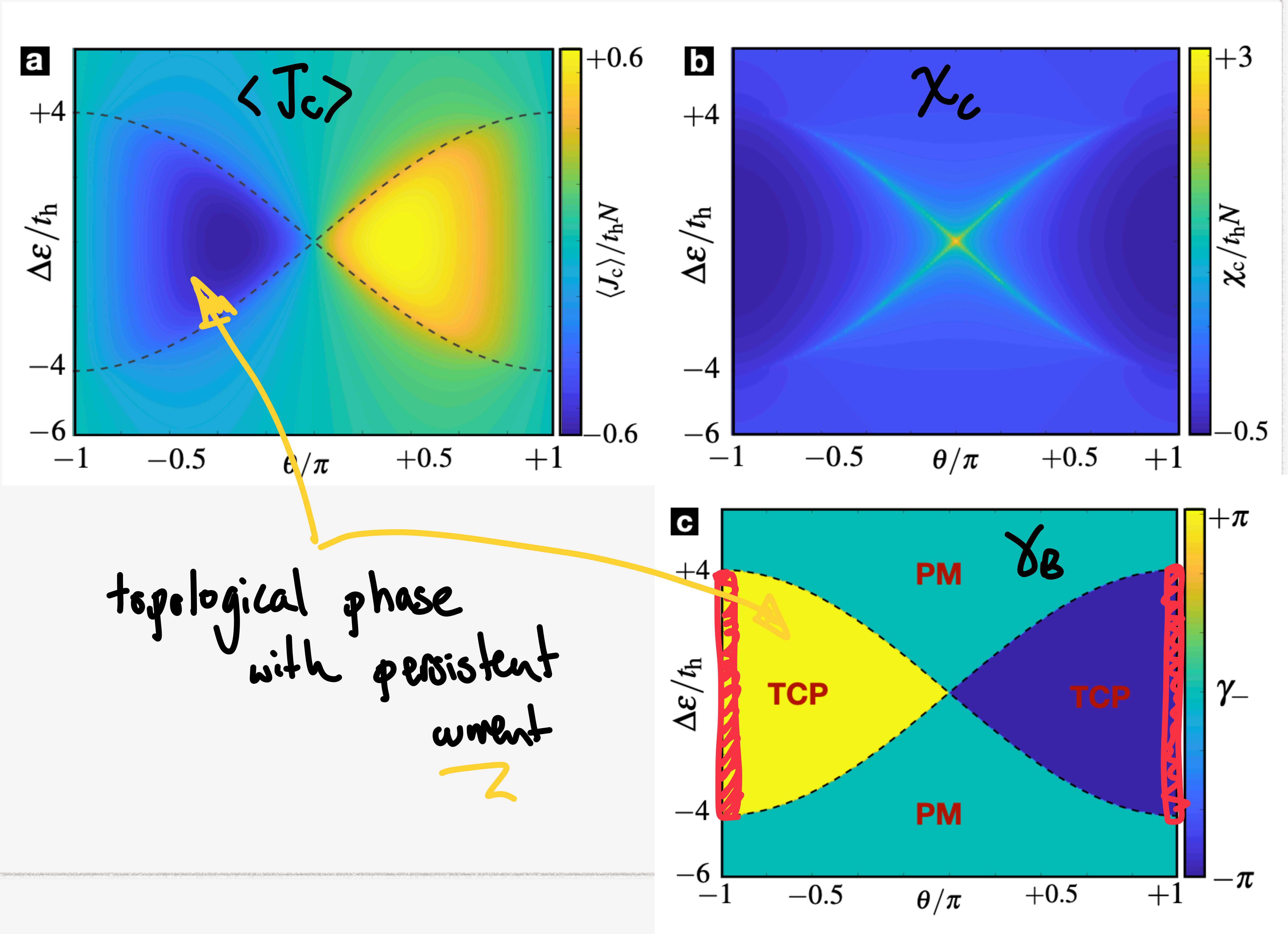
$$\chi_c = \left\langle \frac{\partial J_c}{\partial \theta} \right\rangle$$

Inversion-symmetry SPT @ generic θ

Our goal was to understand the fate of SPT for $\theta \neq \pm\pi$



AIII SPT groundstate $\gamma_B = \pm\pi$



topological phase with persistent current

Inversion-symmetry SPT @ generic θ

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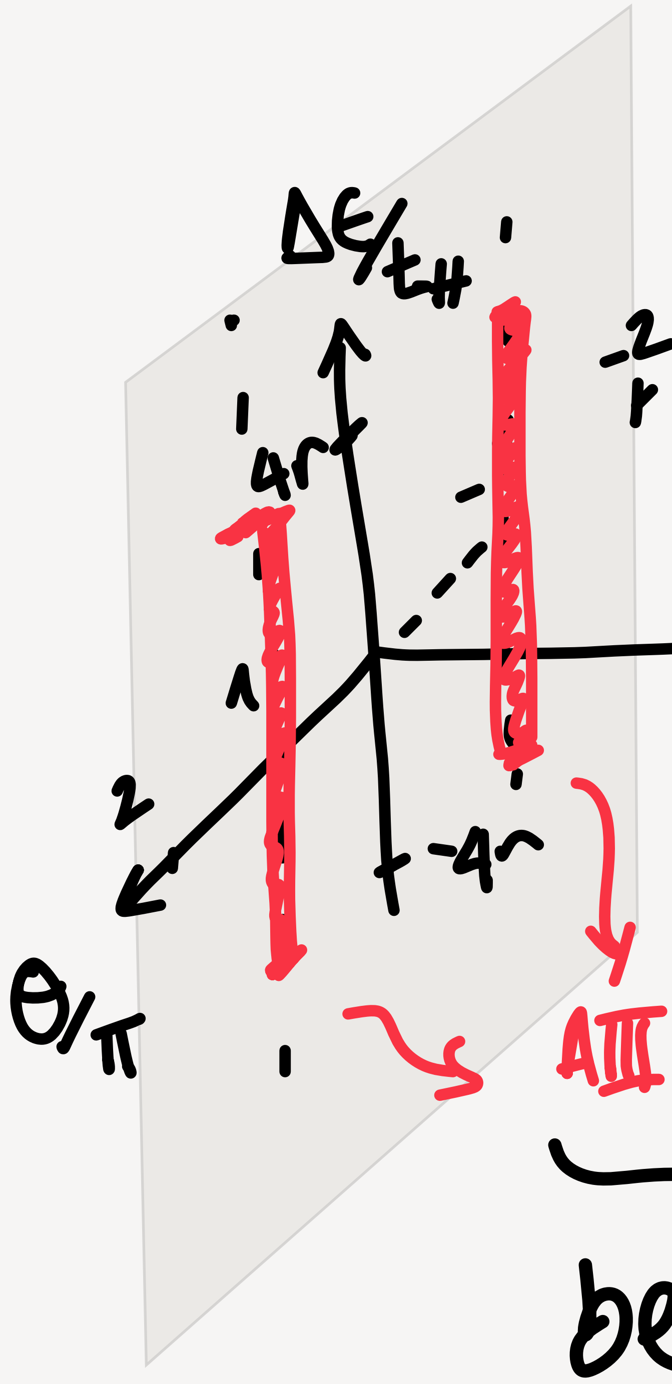
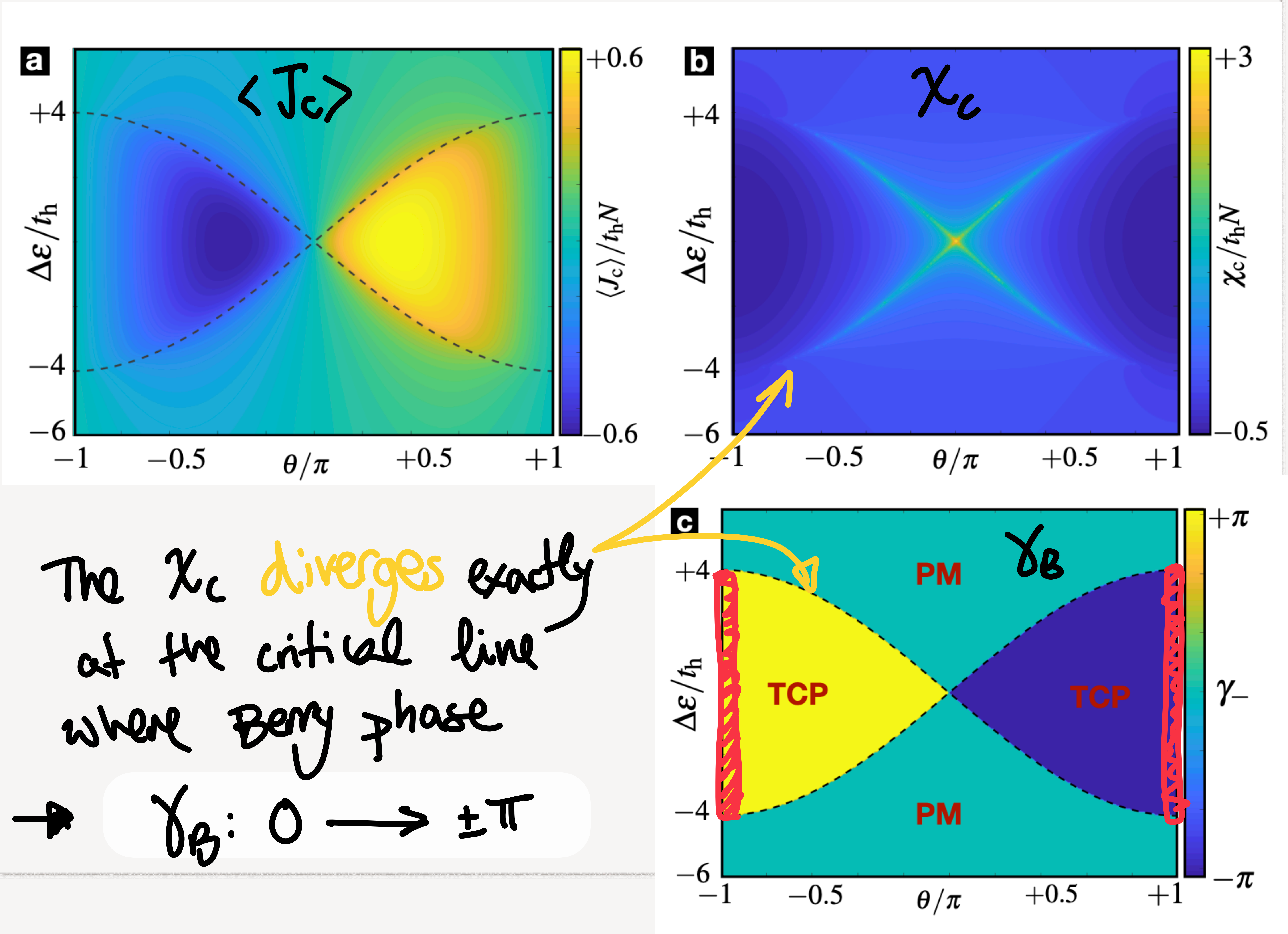


AIII SPT groundstate $\gamma_B = \pm\pi$

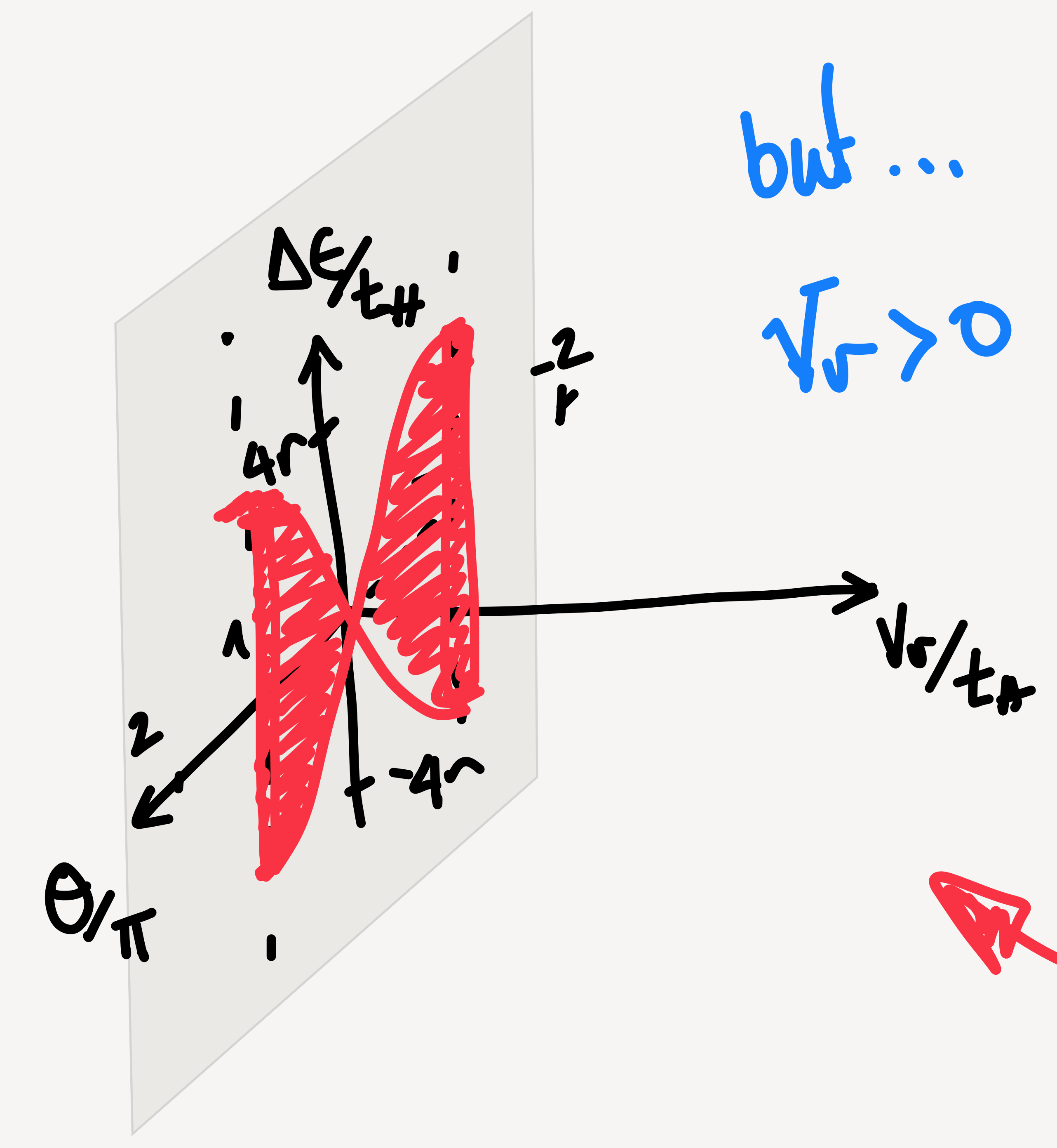
becomes inv-symm prot $\rightarrow$ $\gamma_B : 0 \longrightarrow \pm\pi$



The χ_c diverges exactly at the critical line where Berry phase

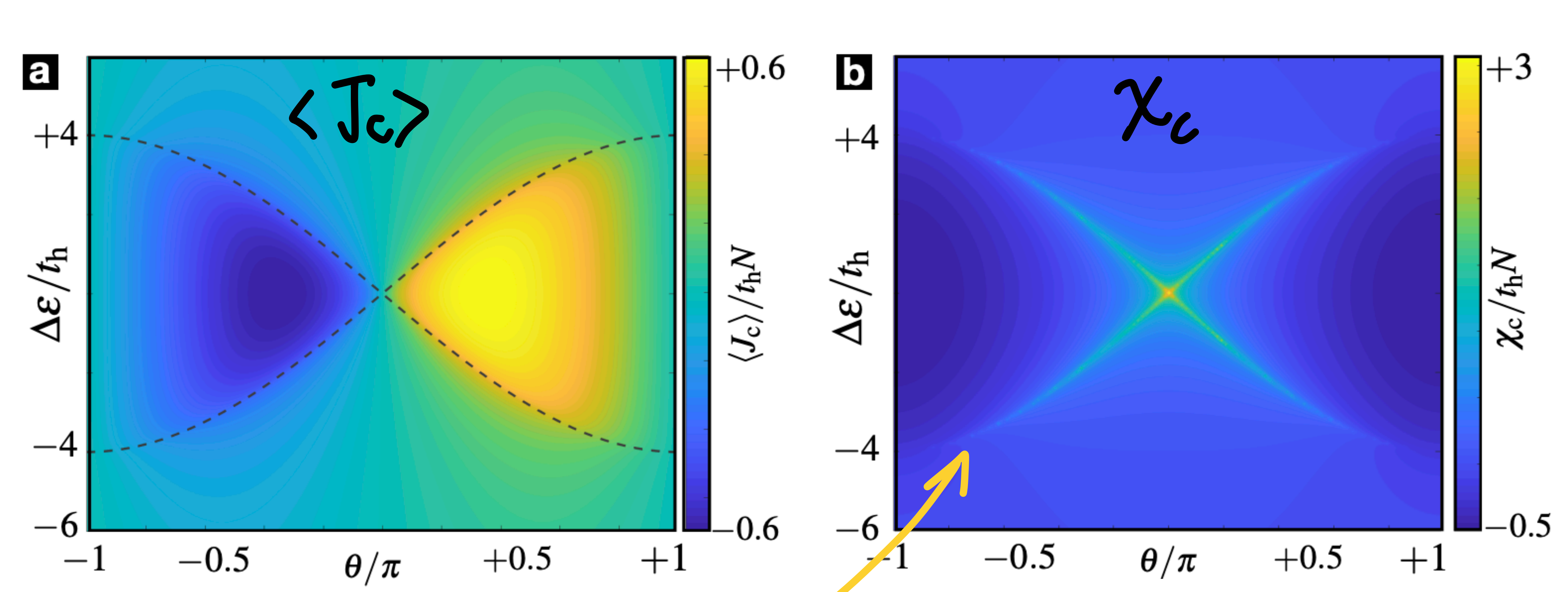
Inversion-symmetry SPT @ generic θ

Our goal was to understand the fate of SPT for $\theta \neq \pm\pi$



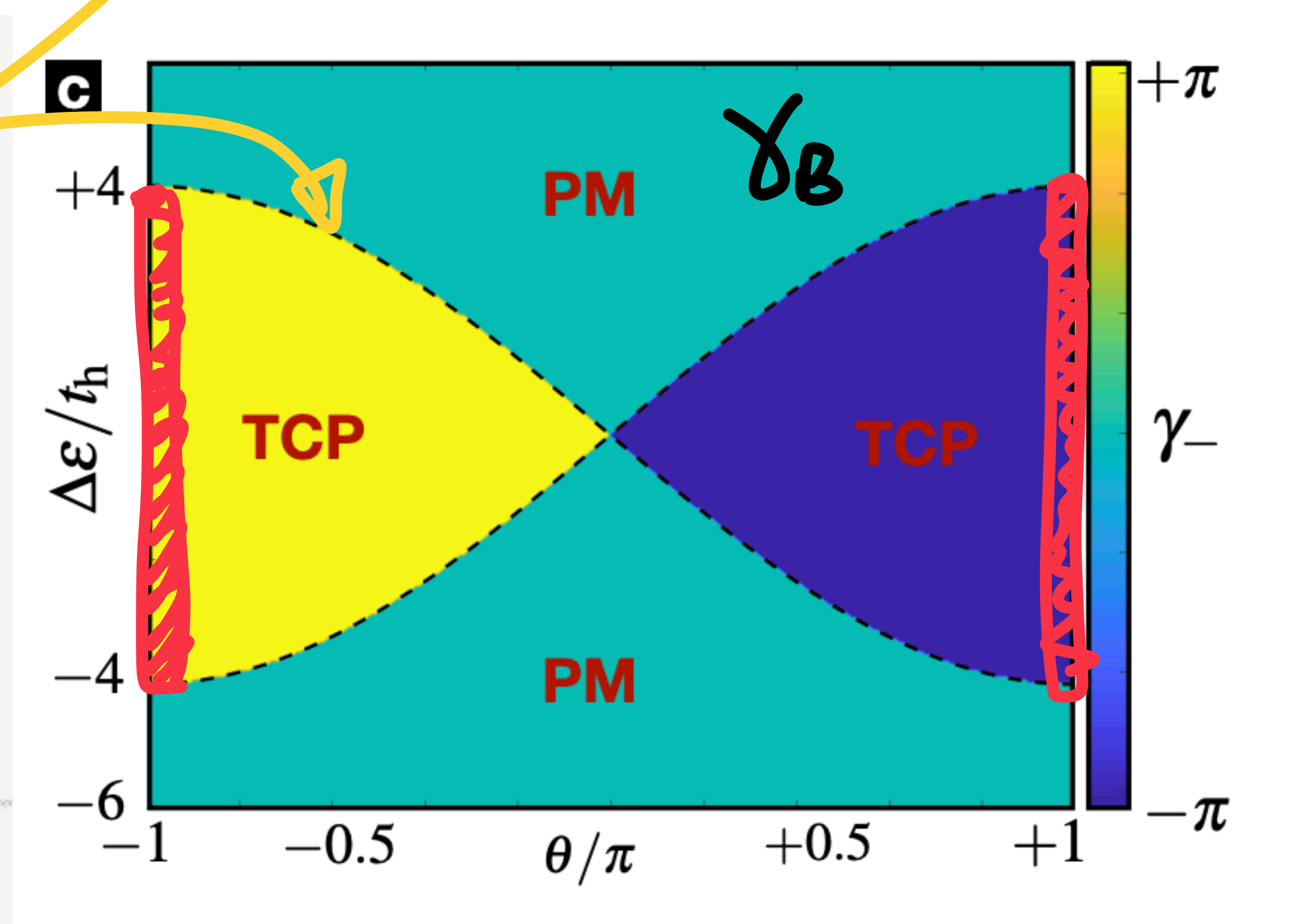
but ...

$V_r > 0$?



The χ_c diverges exactly at the critical line where Berry phase

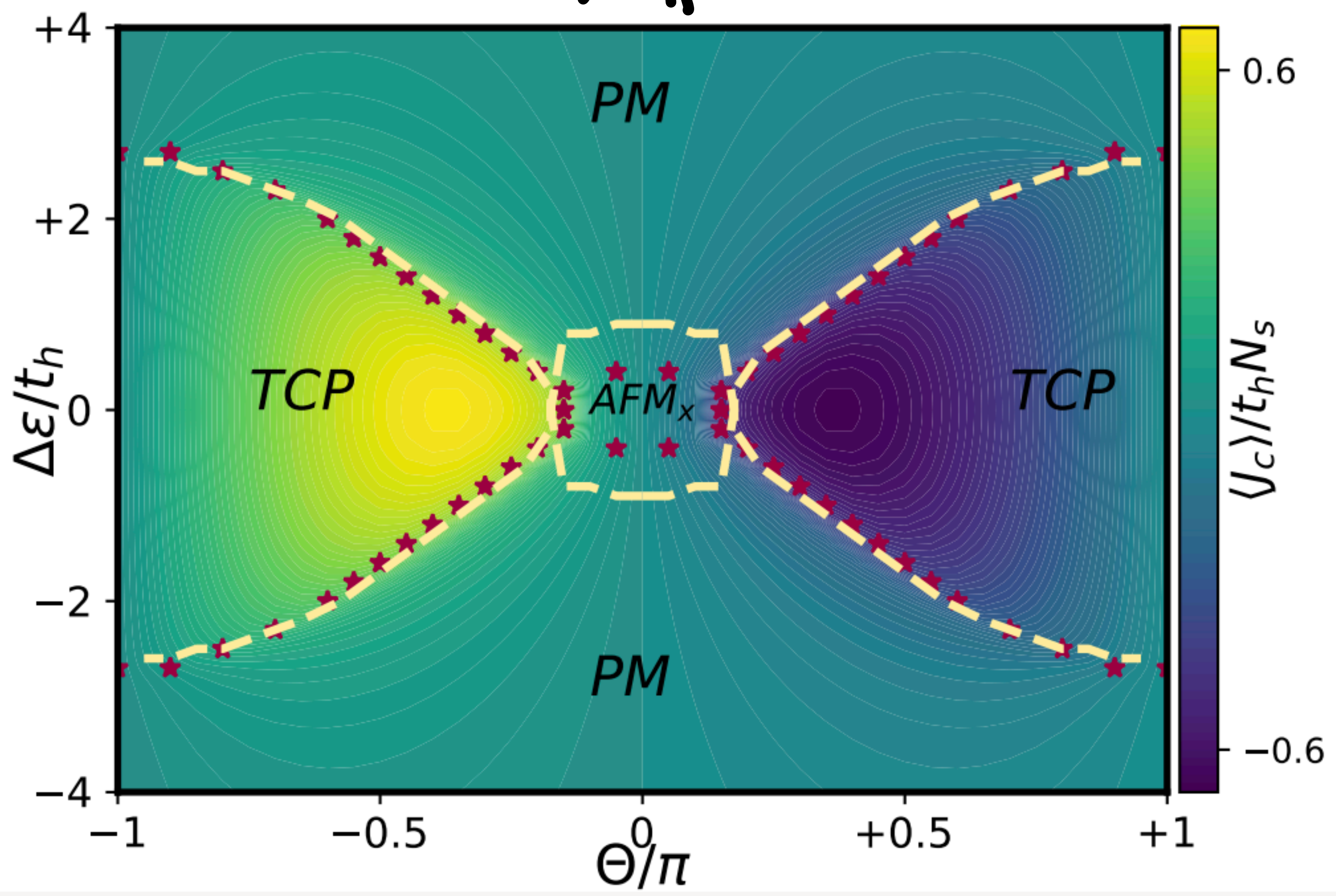
$\gamma_B : 0 \longrightarrow \pm\pi$



We use Hartree-Fock MFT ---
& DMRG ★

Orbital magnetism @ generic θ

$V_\sigma/t_H = 2$

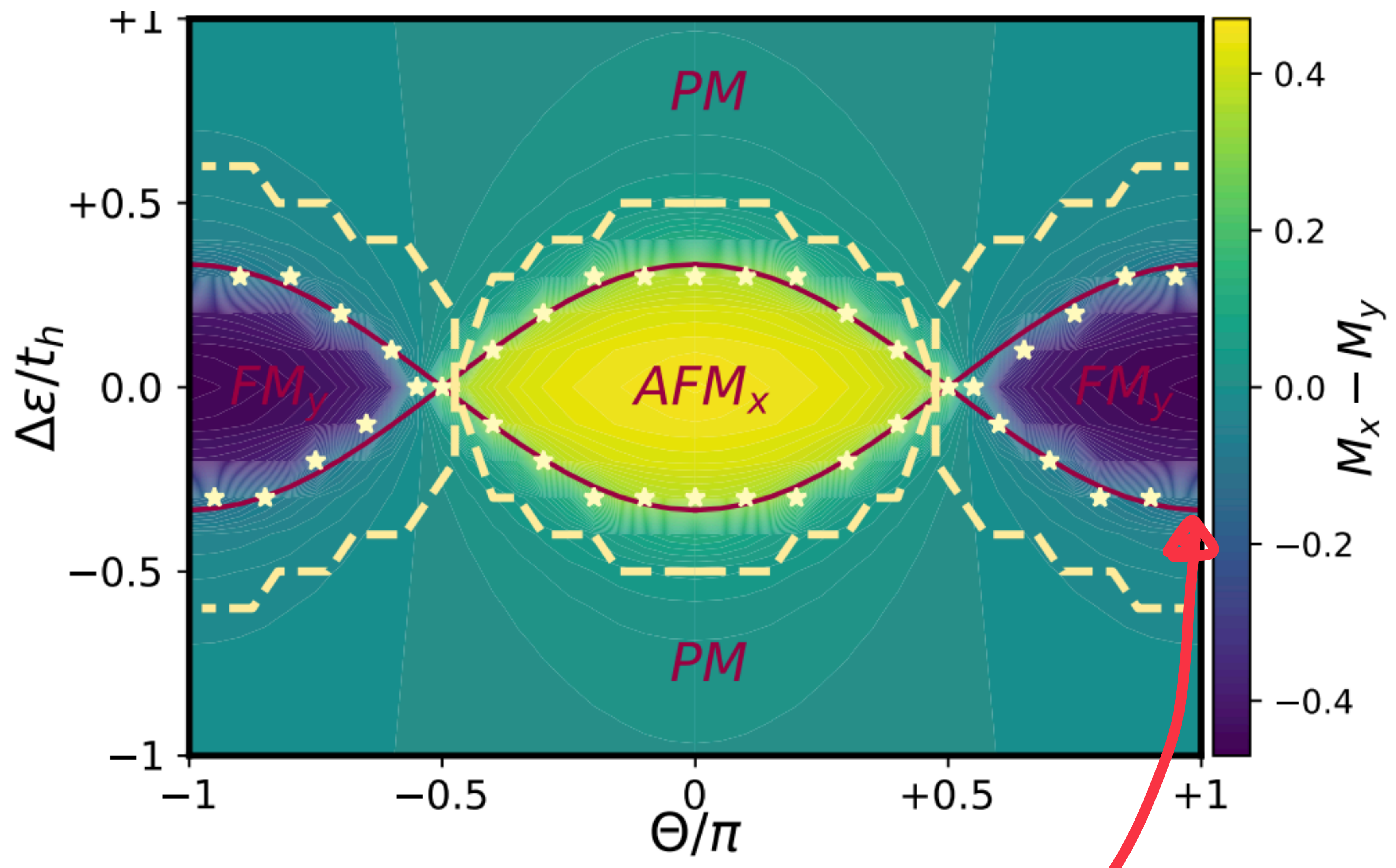


$V_\sigma \gg t_H$

Super exchange

Dzyaloshinskii Moriya

$V_\sigma/t_H = 12$

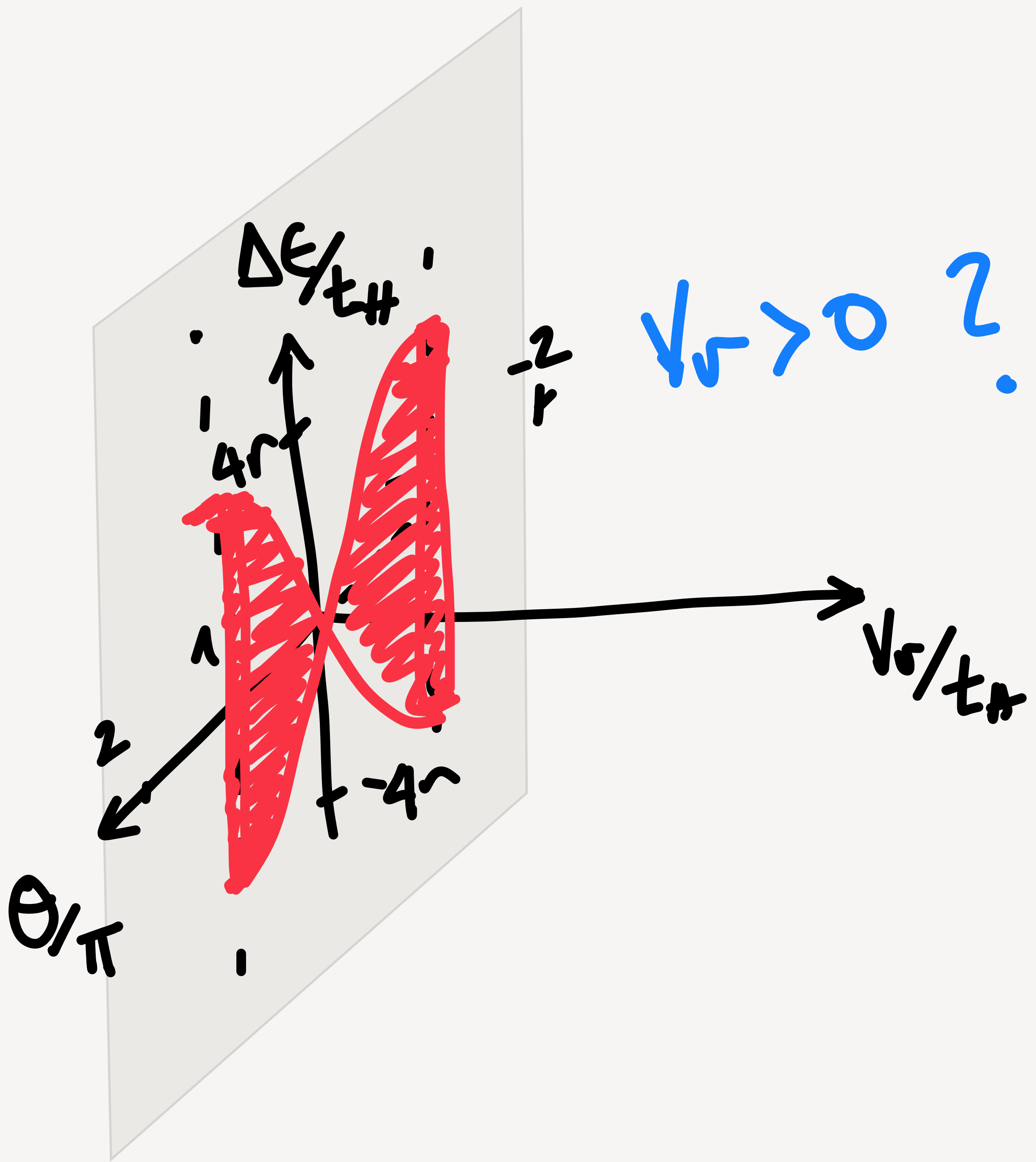


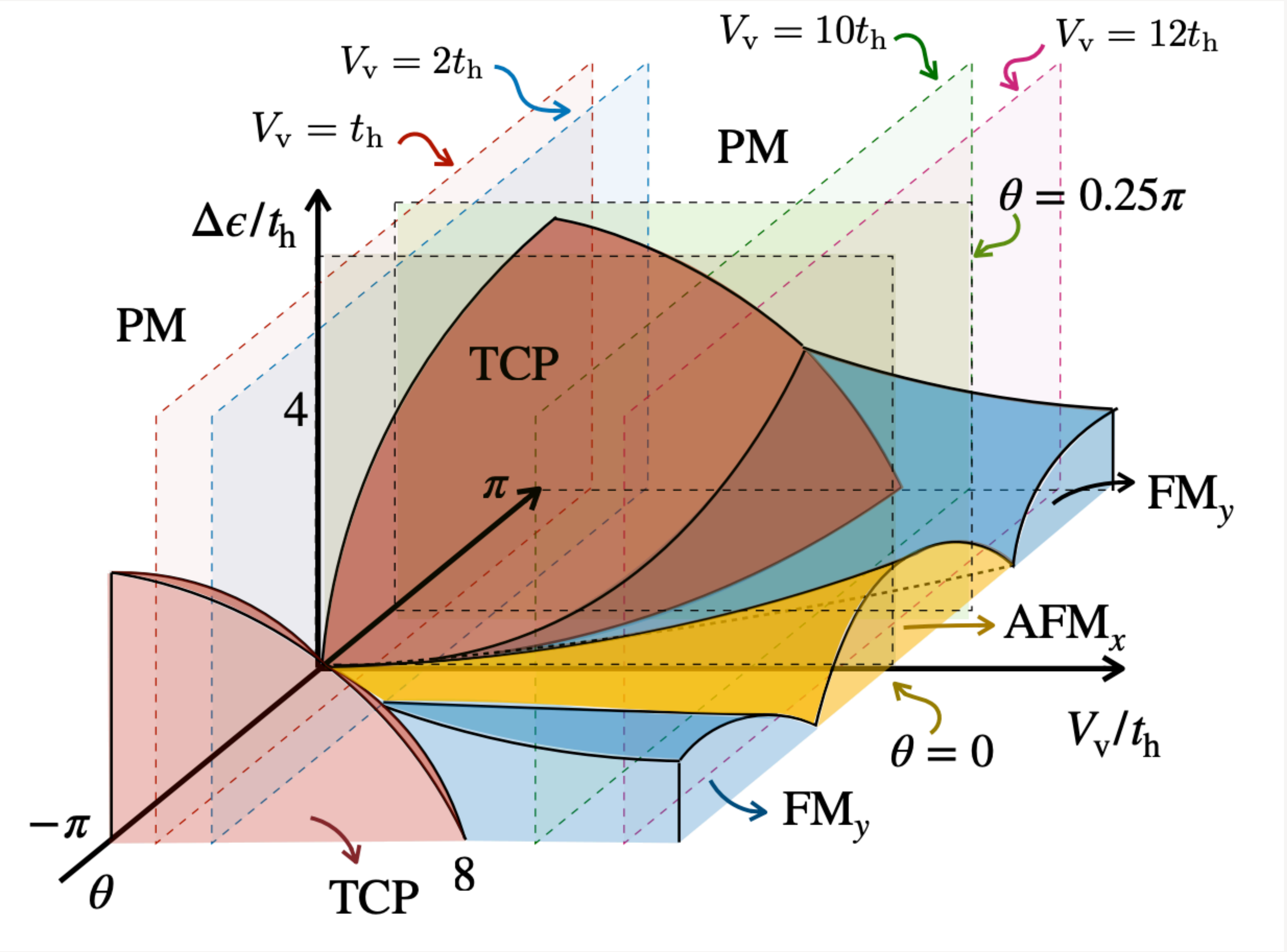
solid red lines are exact solution

$$H_{\mathrm{DM}} = \sum_i \left(J(1+\xi) T_i^x T_{i+1}^x + J(1-\xi) T_i^y T_{i+1}^y \right) + \sum_i \left(\boldsymbol{D} \cdot (\boldsymbol{T}_i \times \boldsymbol{T}_{i+1}) + h T_i^z \right),$$

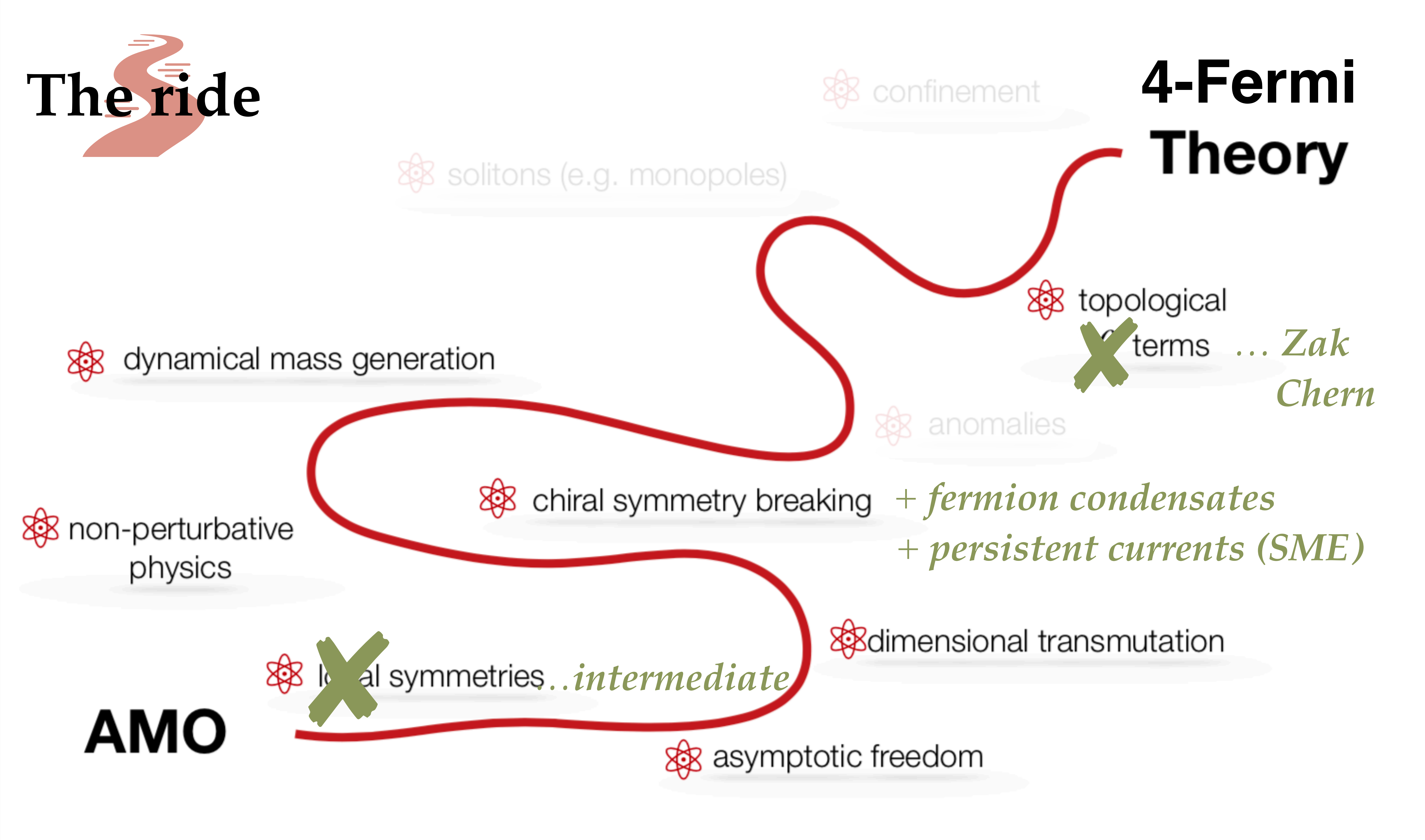
Phase diagram for abitrary θ and V_{v}

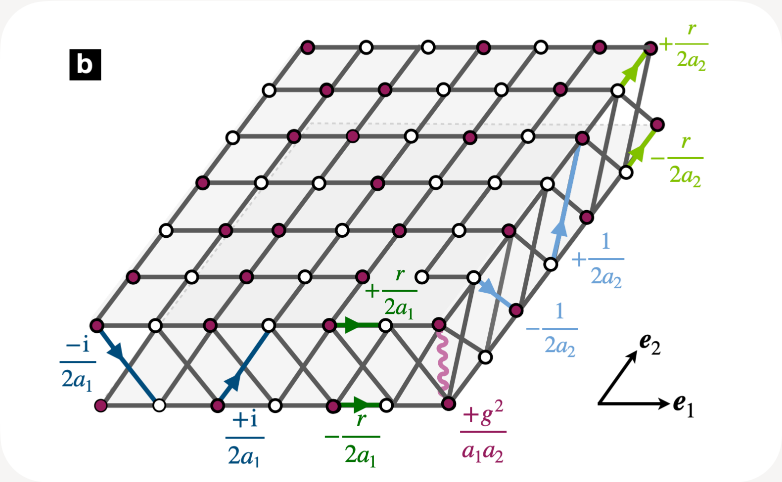
We have explored other planes for $V_v > 0$ via DMRG





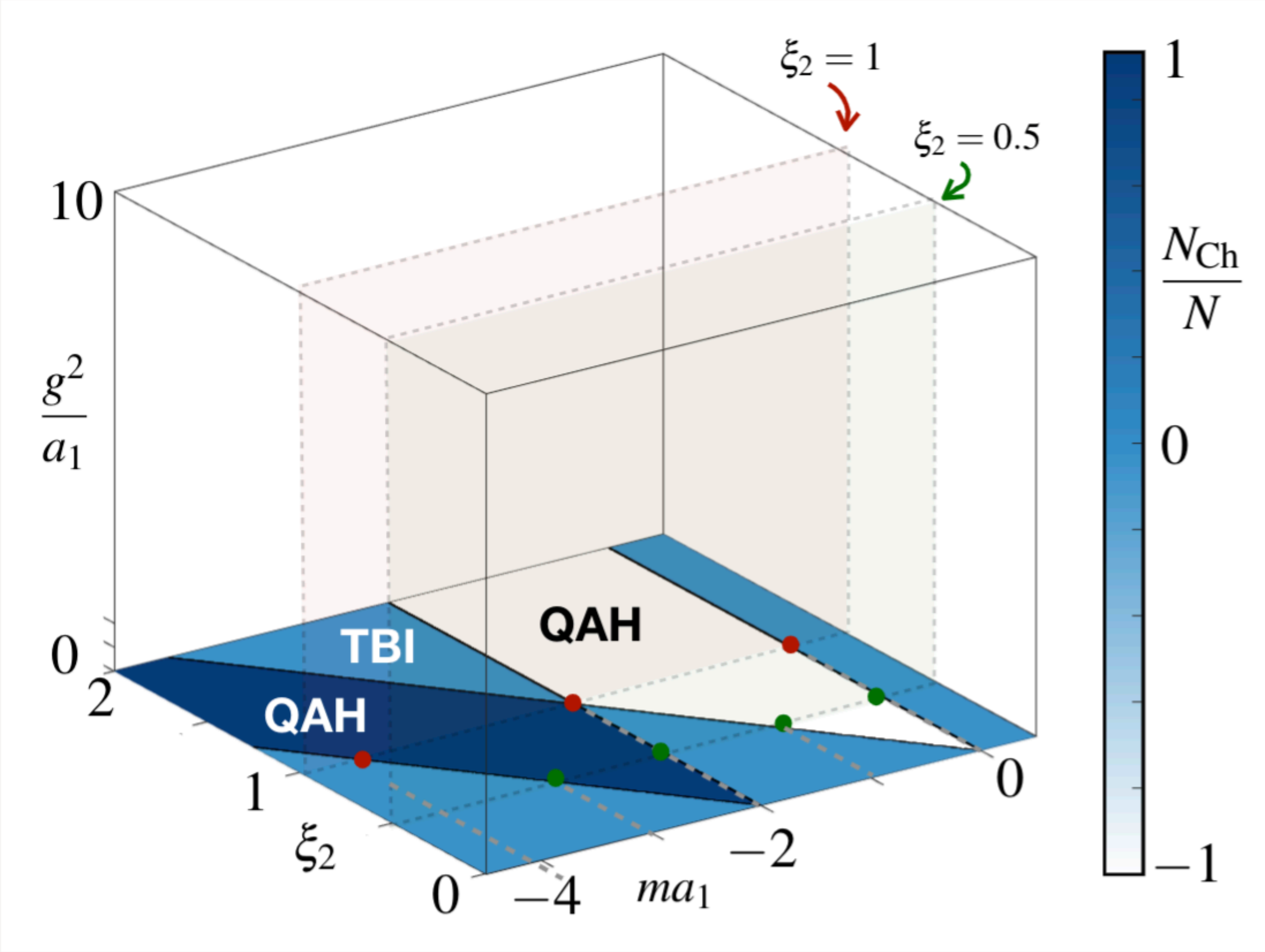
MOTIVATION


The ride
4-Fermi Theory
confinement
solitons (e.g. monopoles)
topological terms … Zak Chern
dynamical mass generation
anomalies
chiral symmetry breaking + fermion condensates + persistent currents (SME)
non-perturbative physics
dimensional transmutation
local symmetries … intermediate
AMO
asymptotic freedom



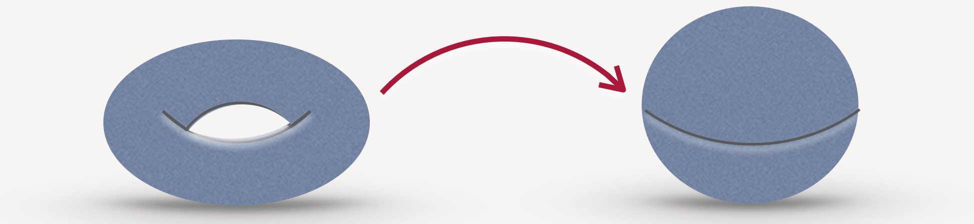
Fate of Chern insulators @ strong couplings

At **zero coupling** $g^2 = 0$, the groundstate develops a **quantum anomalous Hall effect**



Topological ~~θ~~ terms…Chern

$\hat{\boldsymbol{d}} : \mathbb{T}^2 \to \mathbb{S}^2,$

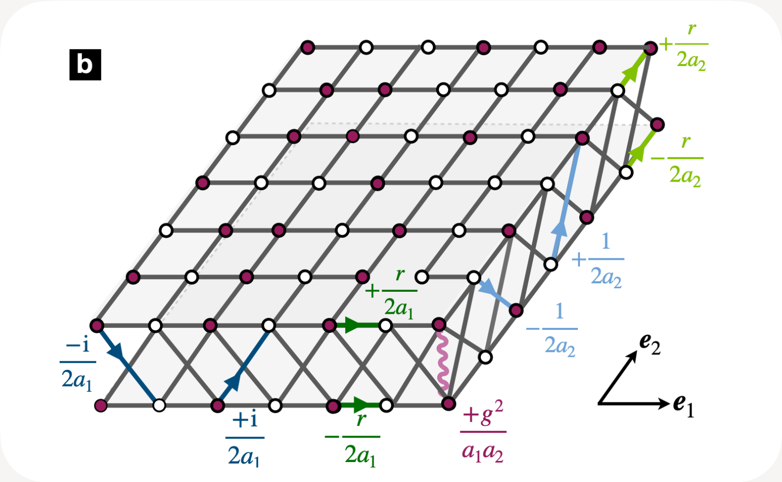


$\sigma_{xy} \propto$ Winding # Chern numbers

$$N_{\mathrm{Ch,b}} = \frac{1}{4\pi} \int_{\mathrm{BZ}} \mathrm{d}^2 k \, \hat{\boldsymbol{d}}_{\boldsymbol{k}}(m) \cdot (\partial_{k_1} \hat{\boldsymbol{d}}_{\boldsymbol{k}}(m) \wedge \partial_{k_2} \hat{\boldsymbol{d}}_{\boldsymbol{k}}(m)).$$

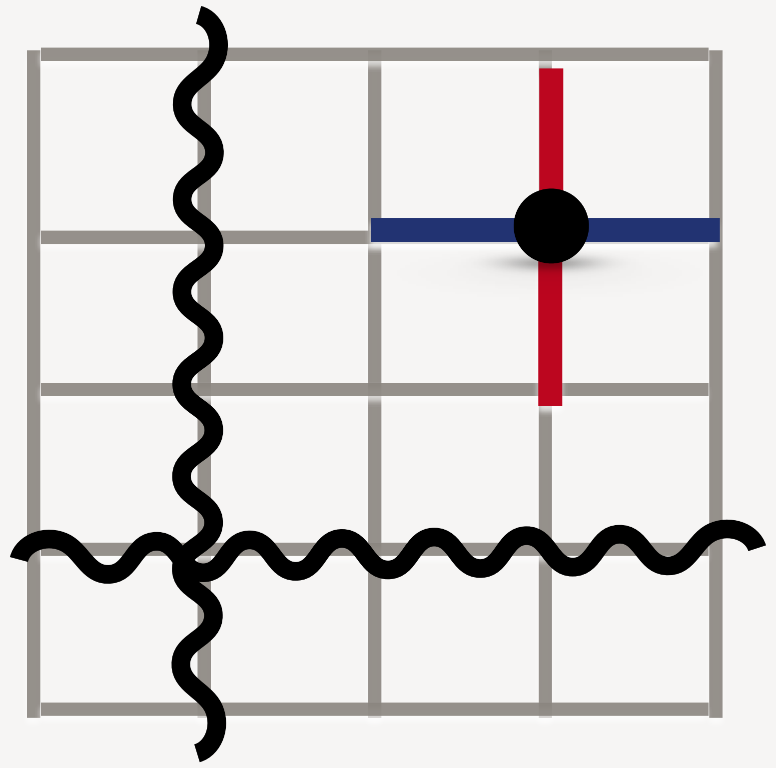
th

- F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)
- D. B. Kaplan, Physics Letters B **288**, 342 (1992).



Fate of Chern insulators @ strong couplings

At **strong couplings** $g^2/a_1 \gg 1$, we obtain a **quantum compass model**



$\tau^y\tau^y$ link

$\tau^x\tau^x$ link

$$H_{\text{eff}} = \sum_{\boldsymbol{n}} \left(J_x \tau^x_{\boldsymbol{n}} \tau^x_{\boldsymbol{n}+\boldsymbol{e}_2} + J_y \tau^y_{\boldsymbol{n}} \tau^y_{\boldsymbol{n}+\boldsymbol{e}_1} - h\tau^z_{\boldsymbol{n}} \right),$$

$$S^x_{i_0} = \prod_j \tau^x_{i_0,j}$$

~~local~~ symmetry…**intermediate**

similar to Dirac strings in $\mathbb{Z}_2$LGTs

th Z. Nussinov and E. Fradkin, Phys. Rev. B **71**, 195120 (2005)

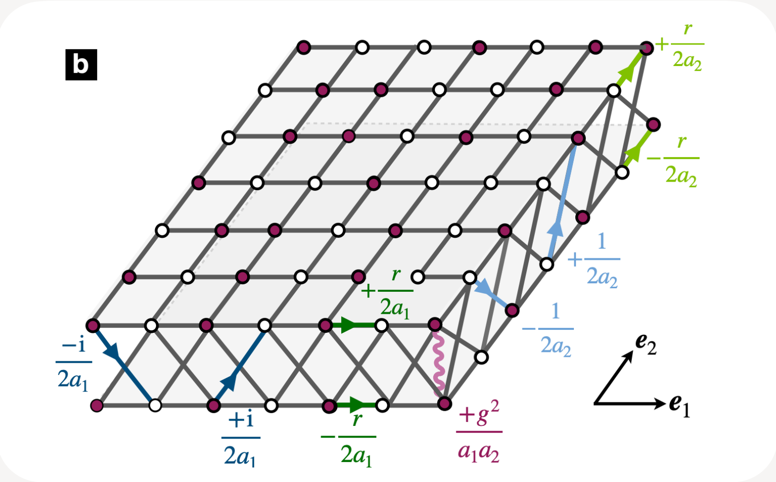
We obtained the critical lines with **tensor-network variational methods (iPEPs)**

~~chiral~~ symmetry breaking…**inversion**

$$\Psi(\boldsymbol{x}) \mapsto \mathbb{I}_N \otimes \gamma^0 \Psi(-\boldsymbol{x})$$

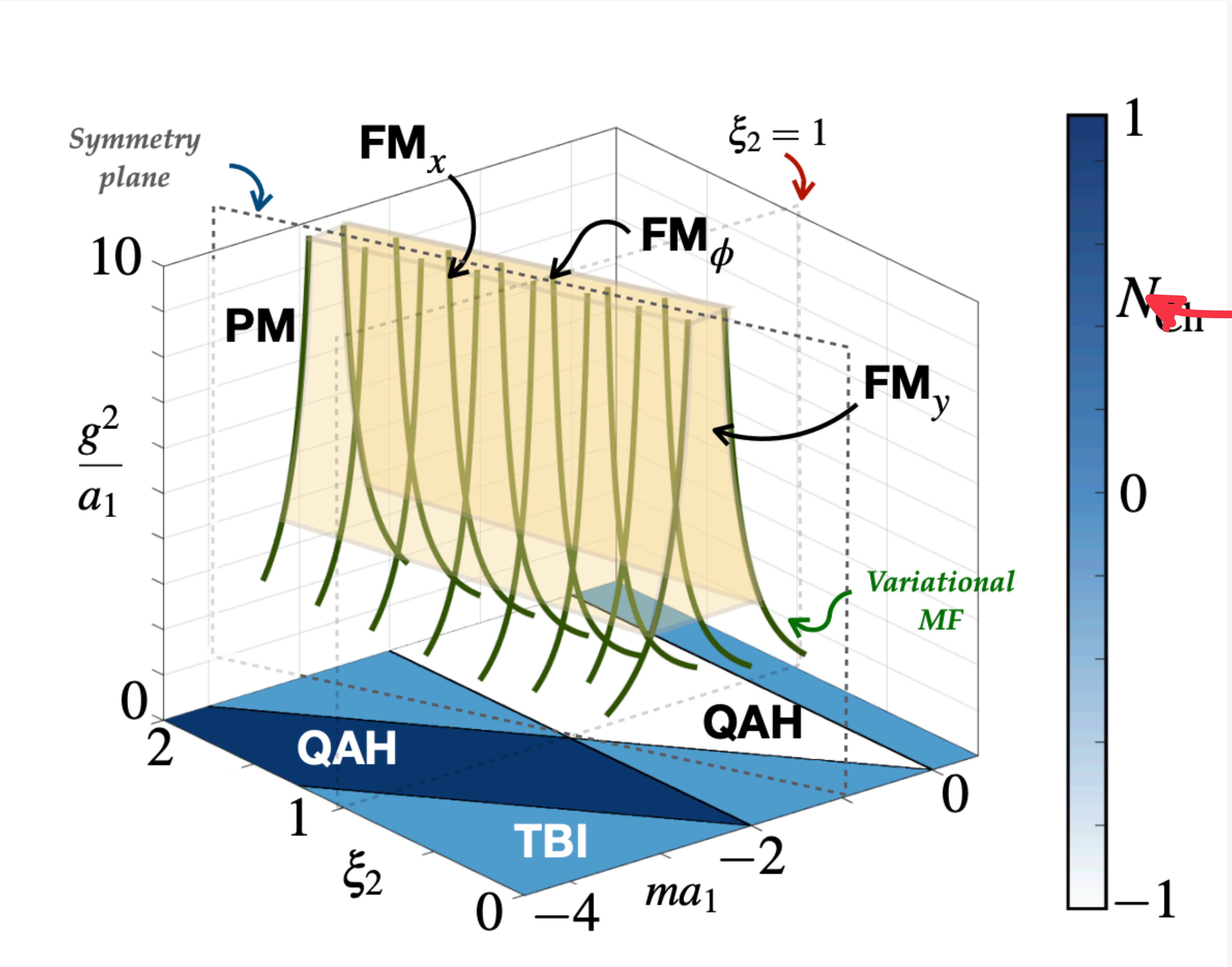
fermion condensates

$$\Pi_1 = \langle \overline{\Psi}\gamma^1\Psi \rangle \neq 0, \quad |J_x| \geq |h|, \; |J_y| < |J_x|,$$
$$\Pi_2 = \langle \overline{\Psi}\gamma^2\Psi \rangle \neq 0, \quad |J_y| \geq |h|, \; |J_x| < |J_y|,$$



Fate of Chern insulators @ strong couplings

At **strong couplings** $g^2/a_1 \gg 1$, we obtain a **quantum compass model**



$$H_{\rm eff} = \sum_{\boldsymbol{n}} \left(J_x \tau^x_{\boldsymbol{n}} \tau^x_{\boldsymbol{n}+\boldsymbol{e}_2} + J_y \tau^y_{\boldsymbol{n}} \tau^y_{\boldsymbol{n}+\boldsymbol{e}_1} - h \tau^z_{\boldsymbol{n}} \right),$$

We use a variational MF ansatz to obtain critical lines compass model in a tensverse field (iPEPs next slide)

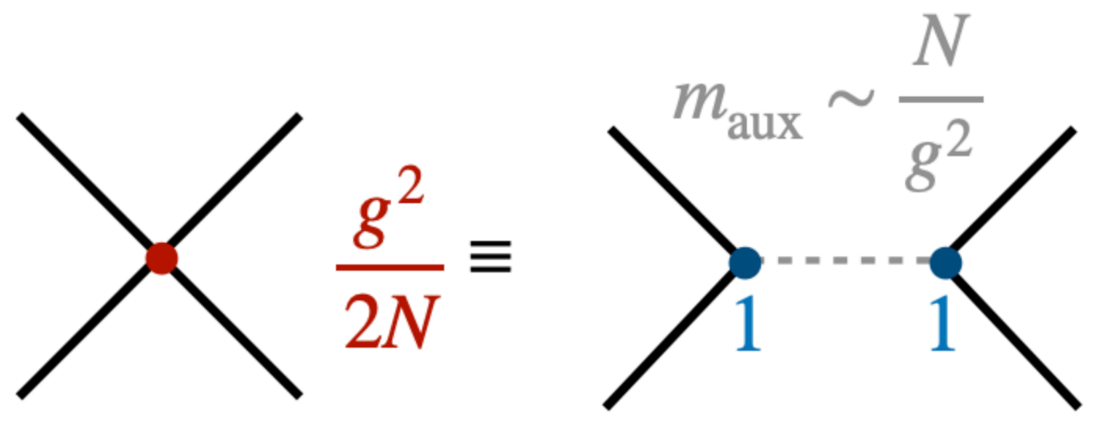
$$g^2 = \frac{a_1}{a_2 \left| m + \frac{1}{a_1} + \frac{1}{a_2} \right|}, \quad \text{if } a_1 > a_2,$$

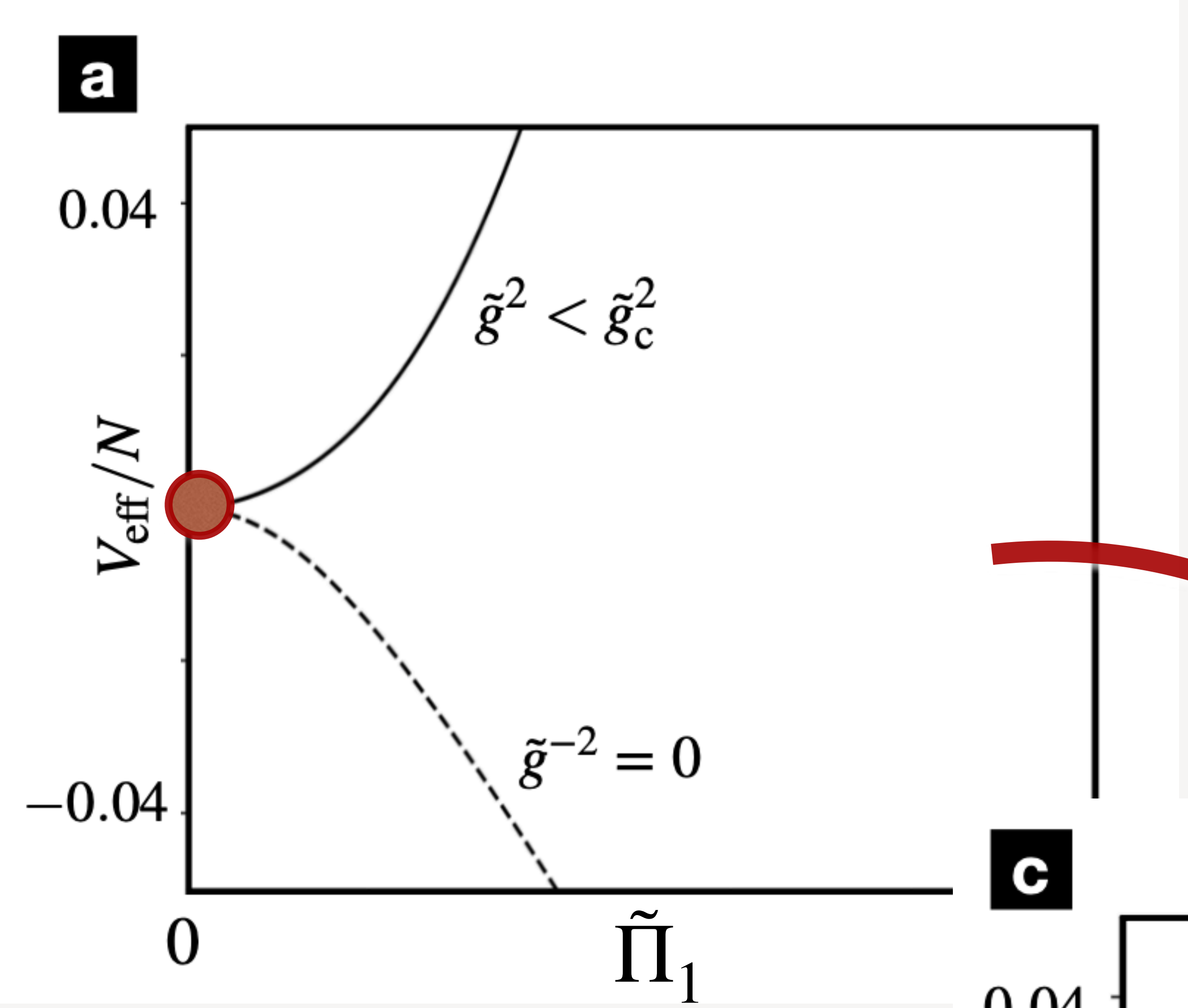
$$g^2 = \frac{a_2}{a_1 \left| m + \frac{1}{a_1} + \frac{1}{a_2} \right|}, \quad \text{if } a_1 < a_2.$$

Effective potential and large-N Chern insulators

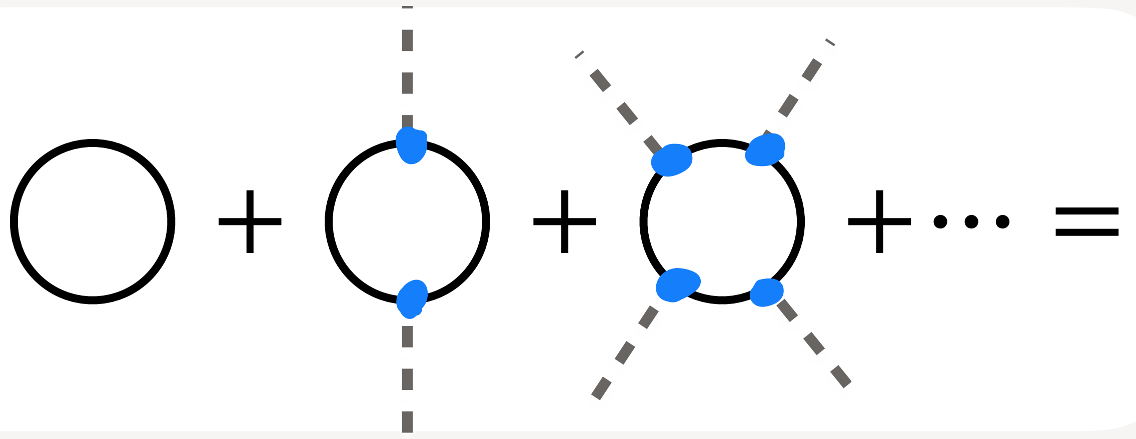
Auxiliary $\underbrace{\sigma, \pi \text{ fields}}_{}$ and resum the leading-order diagrams at **large** N

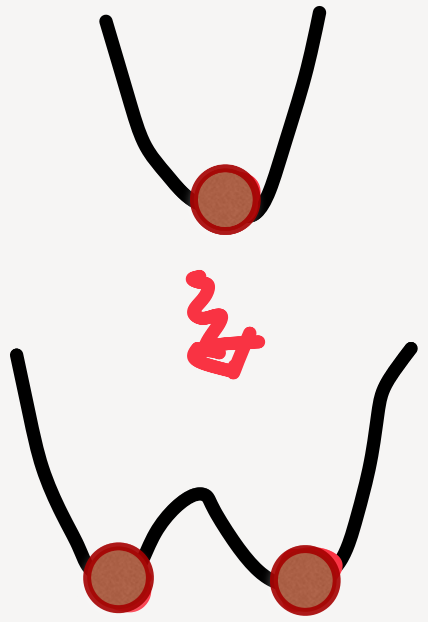
Hubbard-Stratonovich fields mediate interactions via internal •---• lines suppressed $\frac{1}{m_{aux}} \sim \frac{1}{N}$

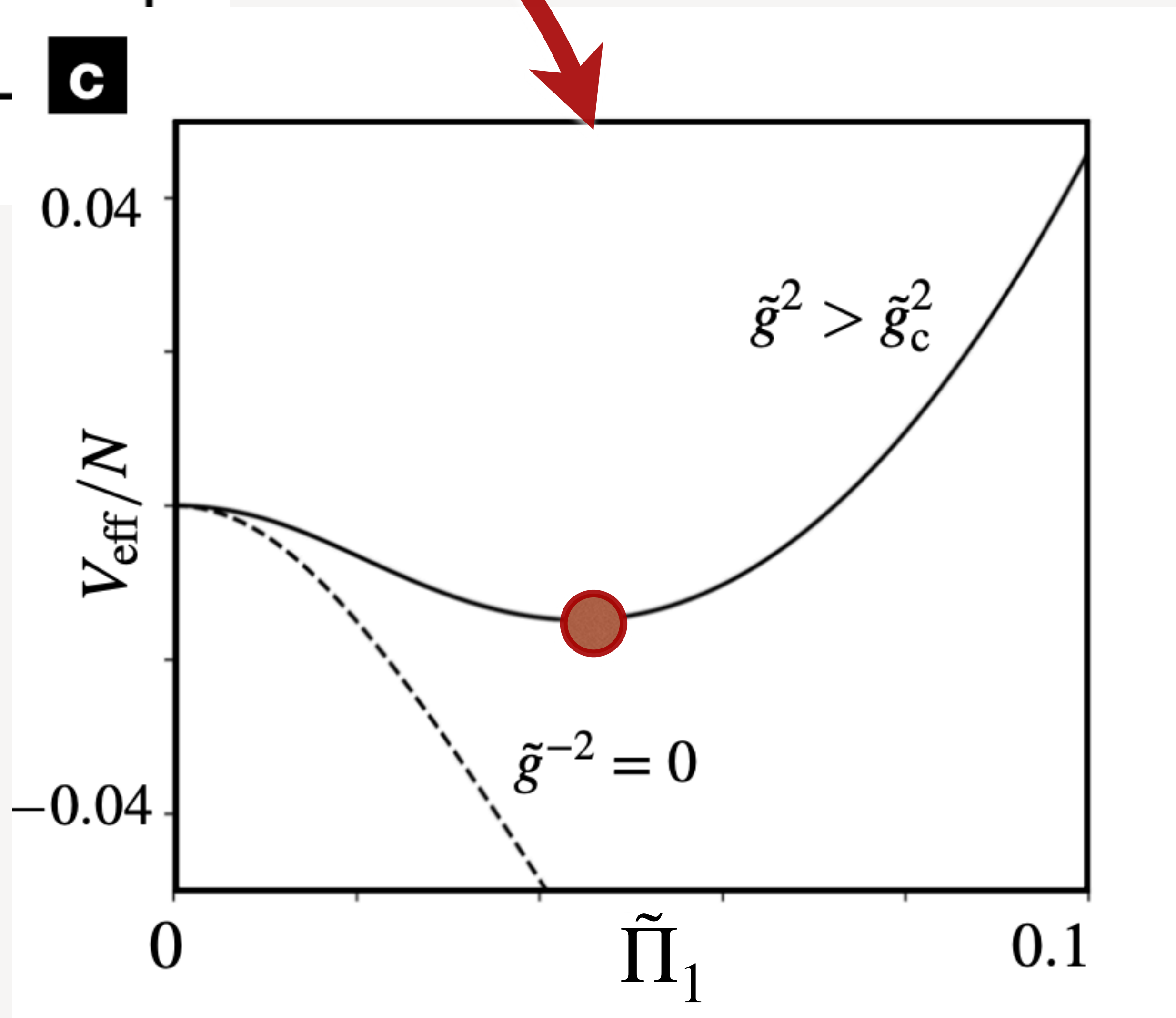




large-N V_{eff} by 1 fermion loop & $n \in \mathbb{Z}^+$ external auxiliary lines





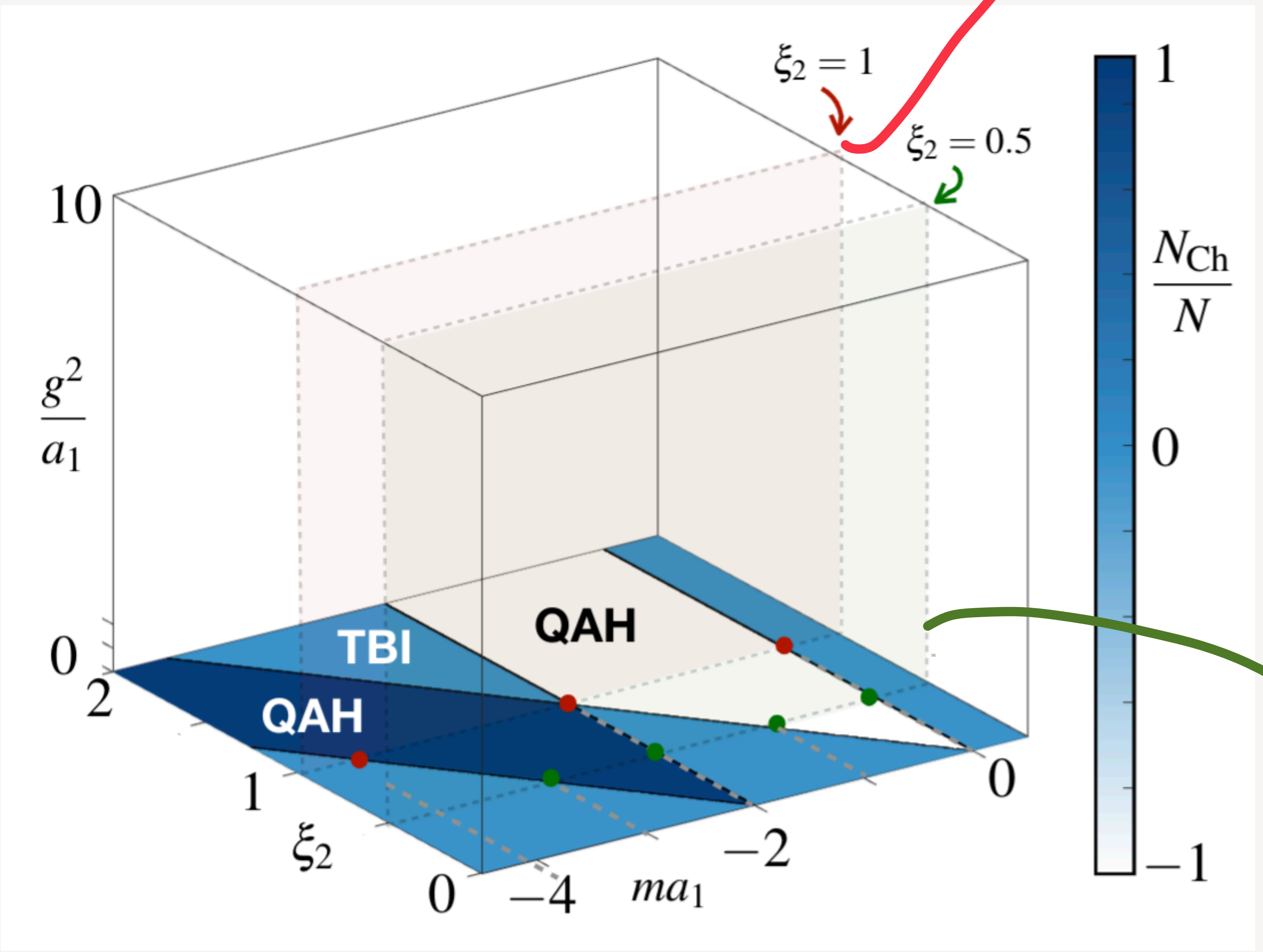


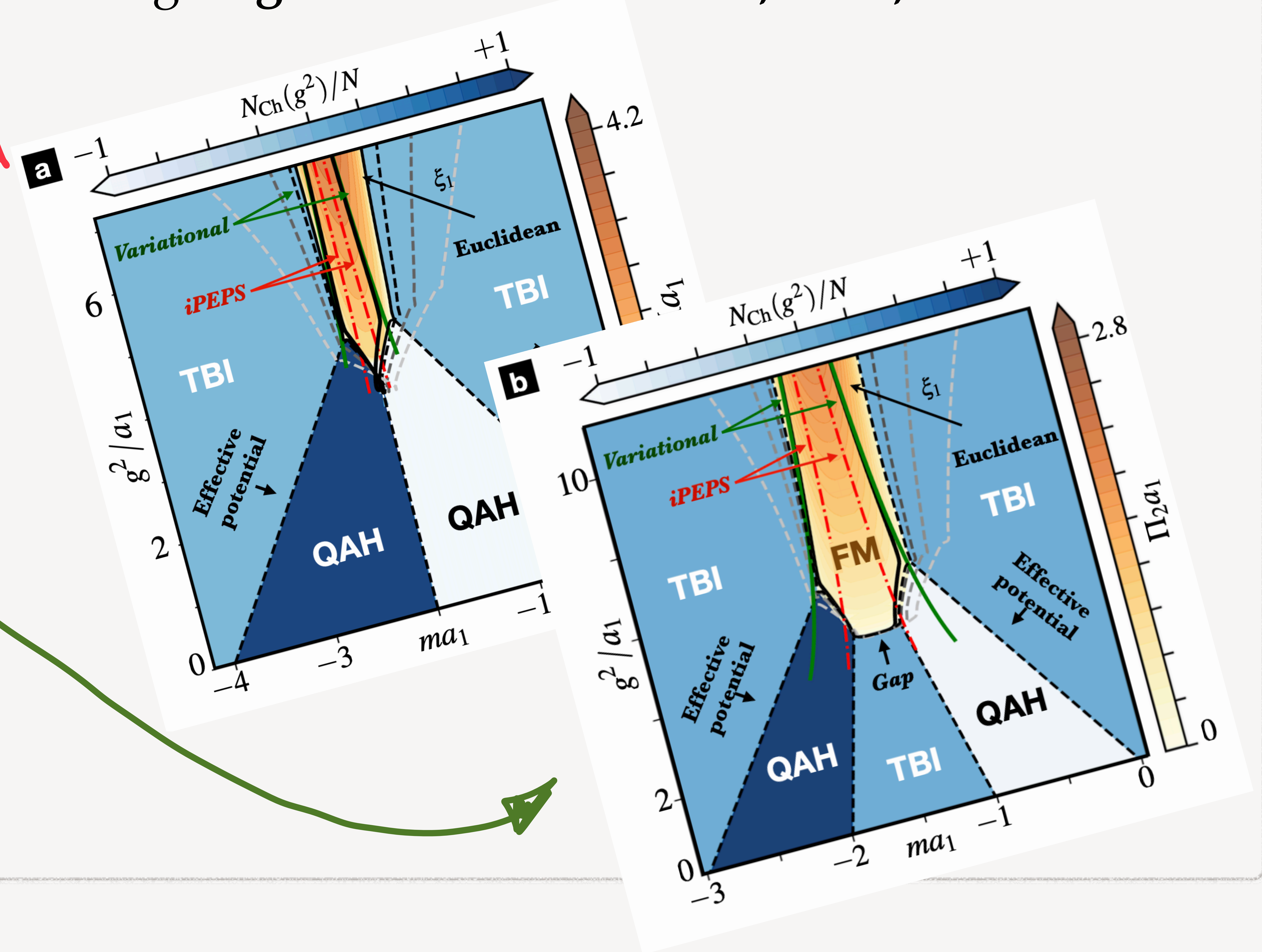
$$V_{\text{eff}}(\tilde{\Pi}_1) = \frac{N\tilde{\Pi}_1^2}{2\tilde{g}^2} + N \sum_{n=1}^{\infty} \frac{1}{n} \int_p \mathrm{Tr}\left(-\mathrm{i}\tilde{\gamma}_1 \frac{\tilde{\Pi}_1}{\mathrm{i}\not{p} + \tilde{m}}\right)^n,$$

Non-standard resummation & novel radiative corrections

Effective potential and large-N Chern insulators

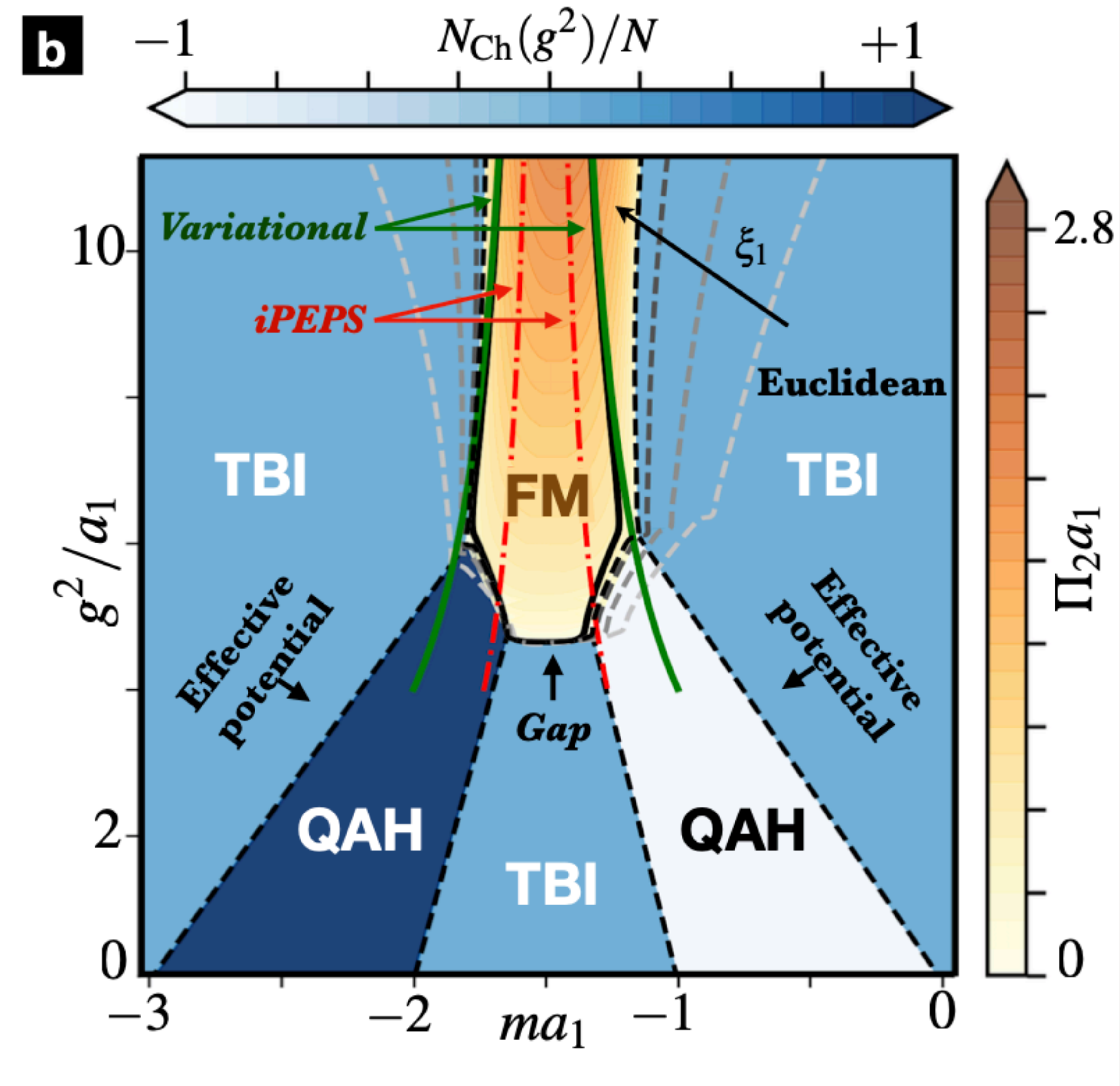
The piase diagram has regions hosting **large-N Chern insulators, TBIs,** and **FMs**





Effective potential and large-N Chern insulators

Large-N condensates give simple contributions to **self-energy**



$\Sigma_g(\omega, \vec{k}) = \mathbb{1}_N \otimes (\gamma^0 \Sigma + \gamma^0\gamma^1 \Pi_1)\, \delta(\omega - 0)\, \delta(\vec{k} - \vec{0})$

feed into topological Hamiltonian

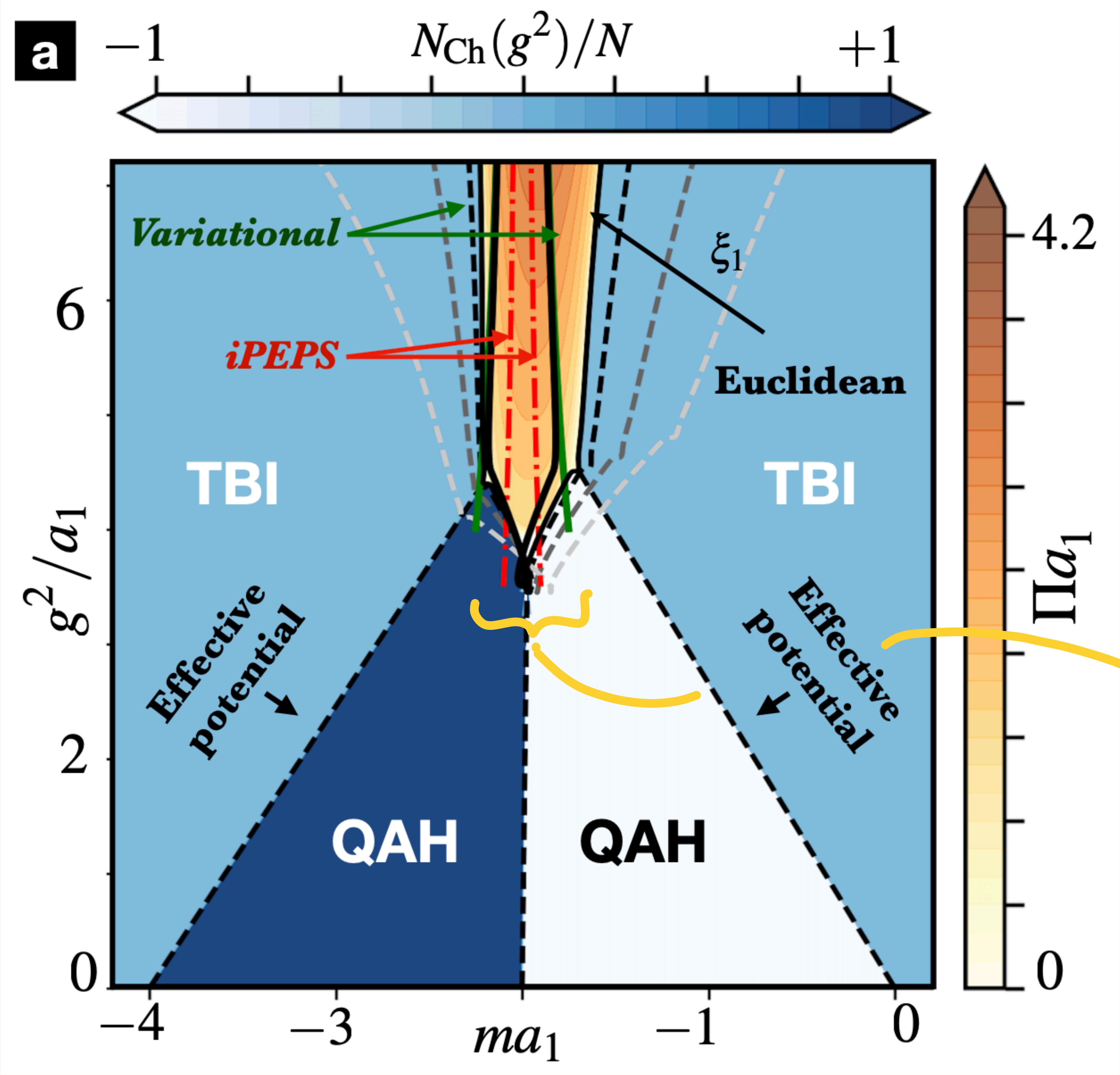
Z. Wang and S.-C. Zhang, PRX **2**, 031008 (2012)

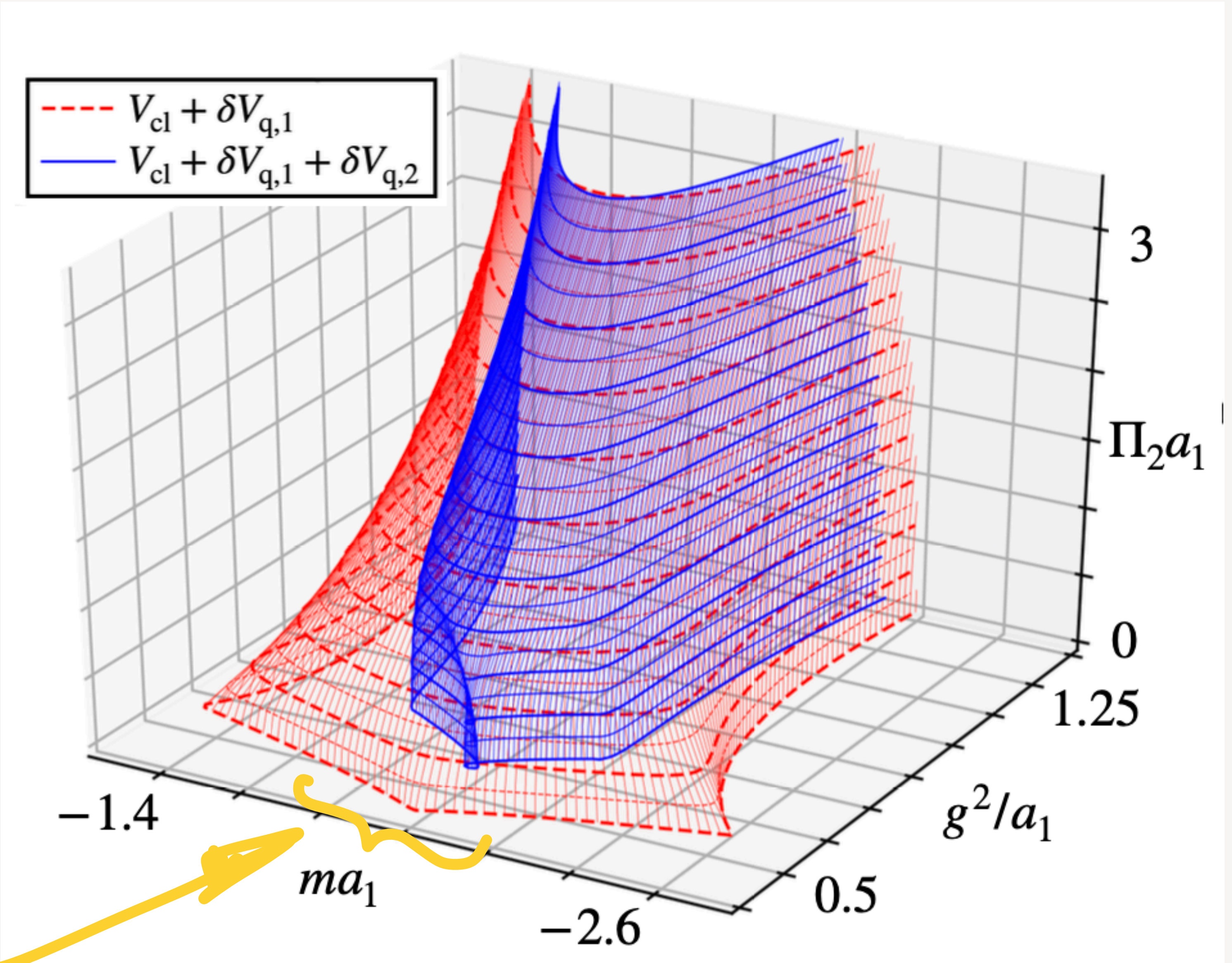
The σ condensate renormalizes bare m & changes the value of $N_{Ch}(g^2)$ via $\Sigma_g(0, \vec{k})$

$$\hat{d}_k(m) \to \hat{d}_k(m + \Sigma, \Pi_1)$$

Effective potential and large-N Chern insulators

New large-N **quantum corrections** lead to **1st-order phase transition**





reentrant condensate

Thanks for your attention

arXiv:2011.08744

arXiv:2111.04485

arXiv:2112.07654

E. Tirrito

L. Ziegler

M. Lewenstein

S. Hands