Disorder-free localization in lattice gauge theories

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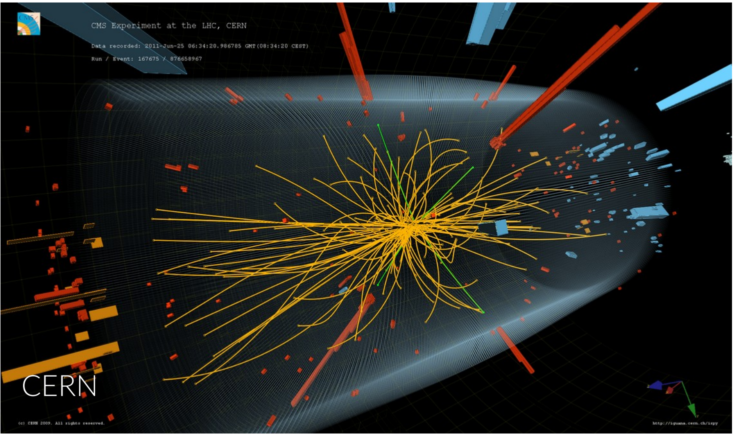
MCQST Gauge Workshop

Work together with

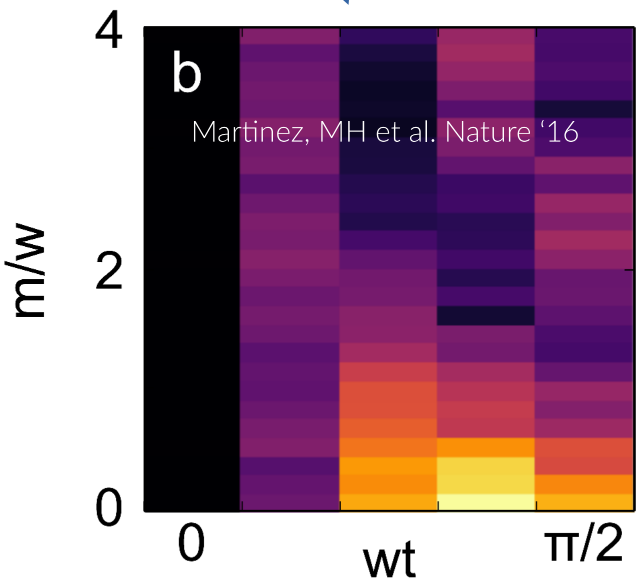
ICTP & SISSA: M. Dalmonte, M. Brenes, A. Scardicchio, F. Surace, R. Fazio, S. Notarnicola
MPI-PKS: P. Karpov, R. Moessner, N. Chakraborty, G.-Y. Zhu, R. Verdel, A. Russomanno, Y.-P. Huang
Cologne: M. Schmitt

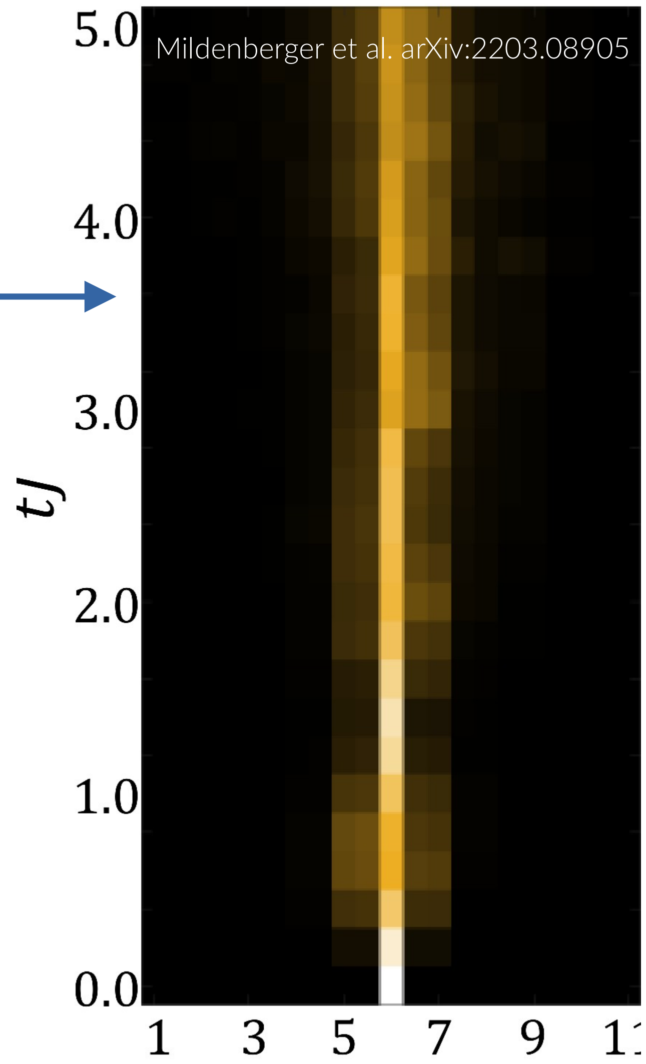
Dynamics in gauge theories

Particle collisions



Quantum simulators





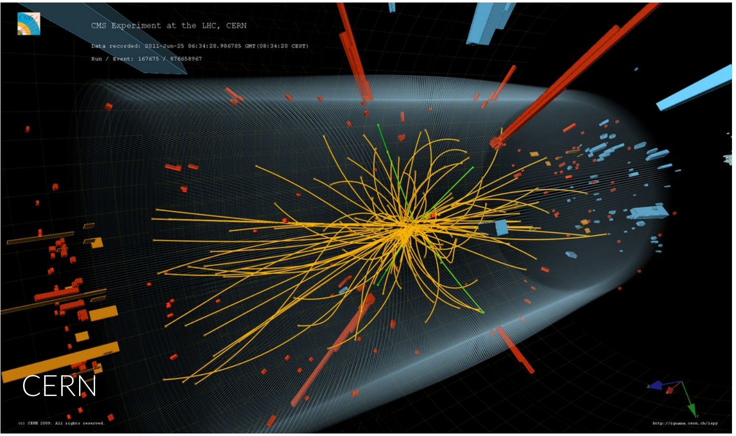
Dynamics in gaug

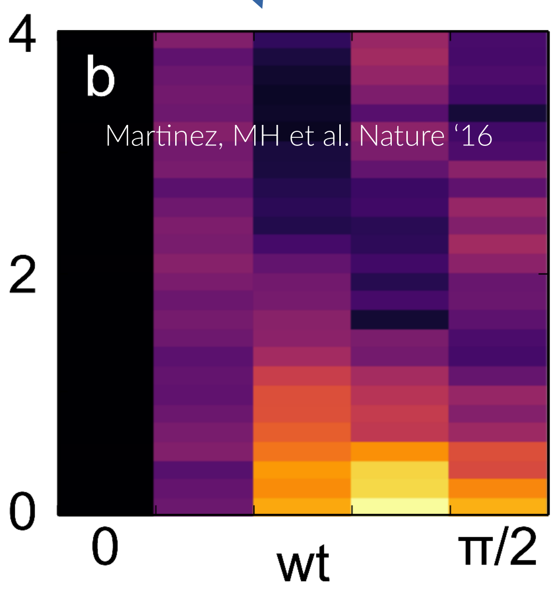
Challenge: calculate dynamics theoretically

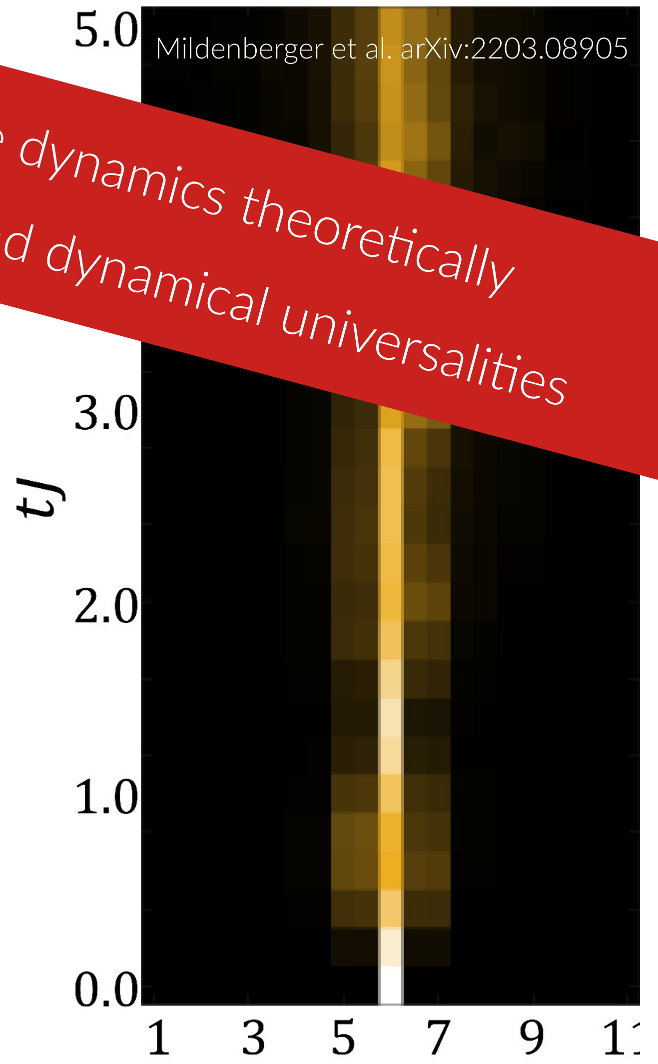
Goal: explore dynamical principles and dynamical universalities

Quantum simulators

Particle collisions

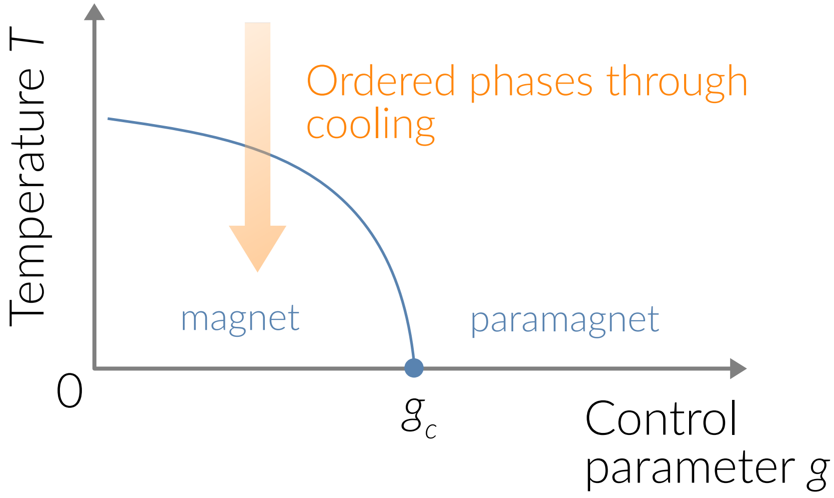






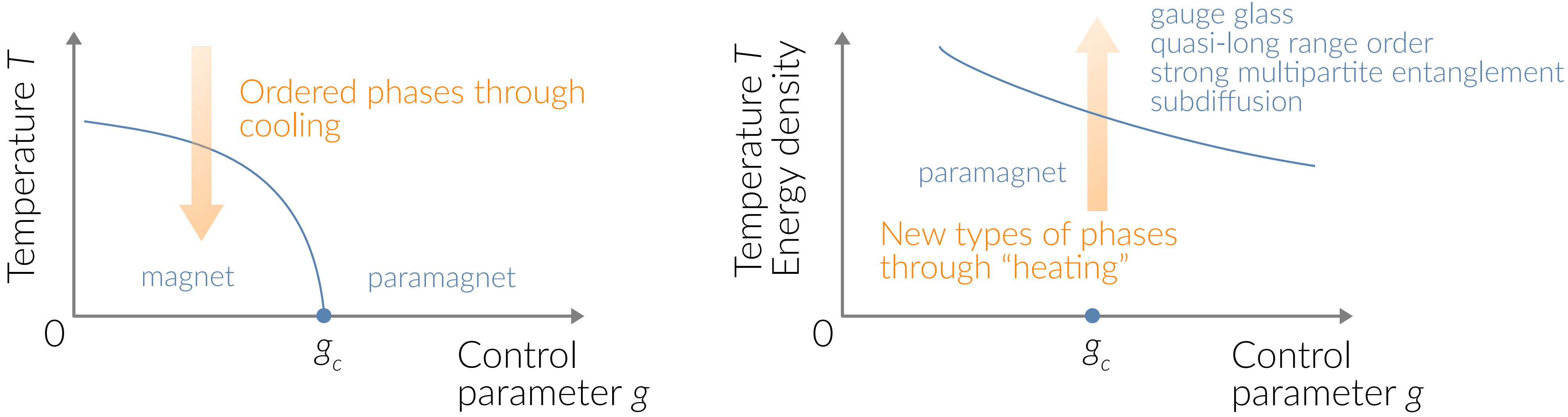
New kinds of order

Conventional matter



New kinds of order

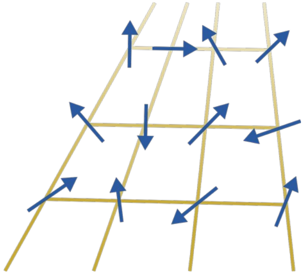
Conventional matter | Matter with gauge constraints



Not happening (generically) in thermodynamic systems → requires **new mechanism**

Quantum thermalization

Isolated quantum magnet



?

Realistic (homogeneous) systems thermalize

Ensemble equivalence: long-time steady state ↔ thermal Gibbs state

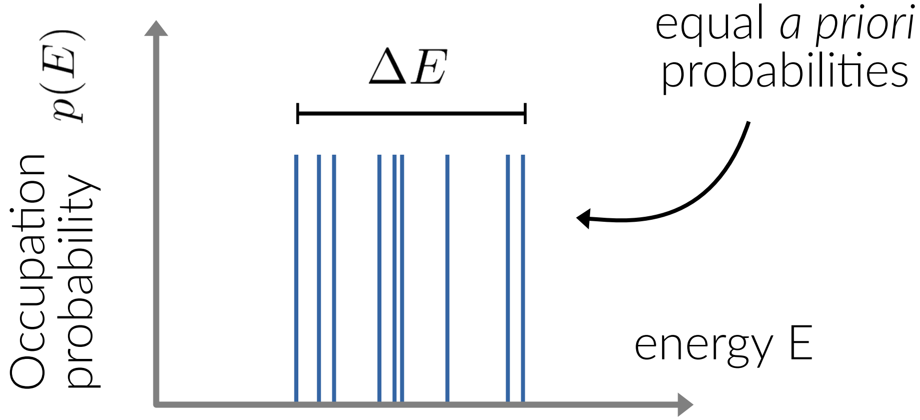
$$\rho_A = \mathrm{tr}_B \rho(t \to \infty) = \mathrm{tr}_B \frac{1}{Z} e^{-\beta H}$$

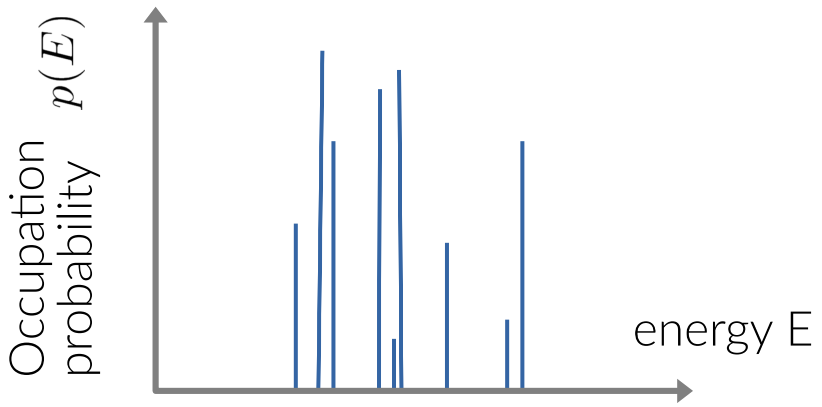
subsystems thermalize

(remainder acting as an effective bath)

Breaking ergodicity

No thermalization: **avoid equipartition**





New quantum states ***beyond thermodynamic constraints***

Breaking ergodicity

Integrability

- Fine-tuning
- *Not robust*
  → no stable nonthermal states

Polkovnikov *et al.* RMP '11

Disorder

- (Many-body) Localization
- Robust in 1D
- But *unstable beyond 1D!*

Vosk & Altman, Nandkishore & Huse, Annu. Rev. '15

Local constraints

- Fractons
- Quantum many-body scars
- *Gauge invariance*
  → *Disorder-free localization*

Smith et al. PRL'17
Brenes, **MH** et al. PRL '18

Breaking ergodicity

Robust ergodicity breaking in any dimension → Disorder-free localization

Integrability

- Fine-tuning
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Polkovnikov *et al.* RMP '11

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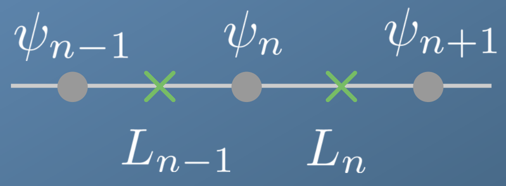
Disorder-free localization

Gauge invariance & conservation laws

Lattice gauge theory (LGT)



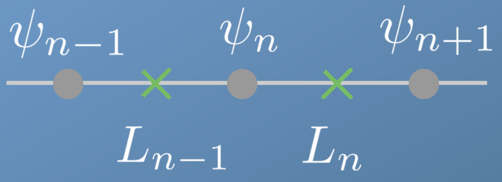
Local gauge symmetry → Generators: $[G_n, H] = 0 \quad \forall n$

Noether theorem: *local symmetry → local conservation law*

Extensive # local conserved operators/quantities
(almost like *integrable* systems, but not equal to the # DOF)

LGTs have *builtin constrained* dynamics

Fragmentation & Superselection sectors



Abelian LGT → Generators:

$$[G_n, H] = 0 \quad [G_n, G_{n'}] = 0 \quad \forall n$$

Hamiltonian eigenstates can be labeled by the ***eigenvalues of the generators***

Superselection sector

$$\{q_\alpha\}_{\alpha=1,\dots,N}$$

$$G_n|\Psi_{\{q_\alpha\}}\rangle = q_n|\Psi_{\{q_\alpha\}}\rangle$$

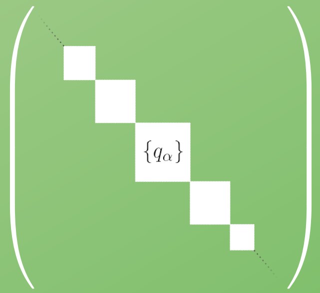
$$\Rightarrow [\nabla E_n - \rho_n - q_n]|\Psi_{\{q_\alpha\}}\rangle = 0$$

'static background charge'

Hamiltonian decomposes into disconnected blocks

(fragmentation builtin automatically)

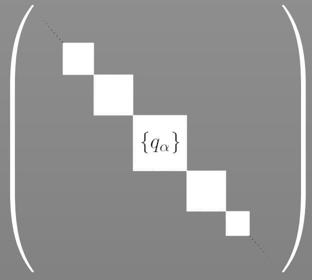
$H =$



Background charges & dynamics

$$e^{-iHt}|\Psi_{\{q_\alpha\}}\rangle = e^{-iH_{\{q_\alpha\}}t}|\Psi_{\{q_\alpha\}}\rangle$$

$H =$



Dynamics for superposition states:

$$|\Psi\rangle = \sum_{\{q_\alpha\}} C_{\{q_\alpha\}}|\Psi_{\{q_\alpha\}}\rangle$$

Diagonal observable
(gauge invariant)

$$\langle\mathcal{O}(t)\rangle = \sum_{\{q_\alpha\}} |C_{\{q_\alpha\}}|^2 \langle\Psi_{\{q_\alpha\}}|e^{iH_{\{q_\alpha\}}t}\mathcal{O}e^{-iH_{\{q_\alpha\}}t}|\Psi_{\{q_\alpha\}}\rangle$$

"**disorder average**" *even though the state might be* ***homogeneous***

Disorder-free localization from strong ***random background charges***

Smith et al. PRL '17 & Brenes, **MH** et al. PRL '18

DFL Mechanisms

Localization through interference

Disorder landscape generated by the background charges
(disorder often spatially correlated)

→ Anderson or many-body localization

→ Johannes' talks on Friday

Localization through kinetic constraints

Fragmentation in real & Hilbert space

→ Quantum/classical percolation problem

→ this talk

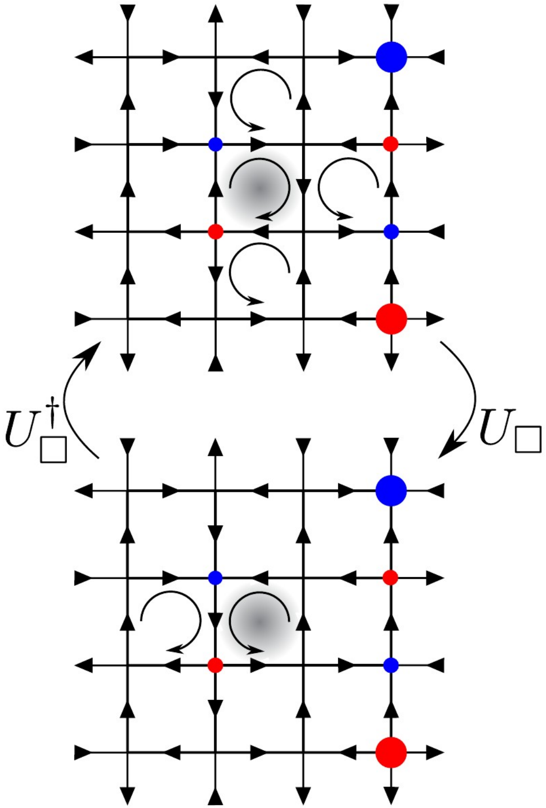
DFL in interacting 2D LGTs

U(1) quantum link model (QLM)

$$H = H_0 + V \equiv \lambda \sum_{\square} (U_\square + U_\square^\dagger)^2 - J \sum_{\square} (U_\square + U_\square^\dagger)$$

Potential energy counting # flippable plaquettes

Correlated spin flip on full plaquette



Local symmetry: $G_n = \frac{1}{2}\left[\# \text{ out - } \# \text{ in}\right]$

Quantum spin ice (G_n=0) → Emergent strong-coupling QED, quantum dimer model ($G_n=(-1)^n$), …

Nonequilibrium dynamics

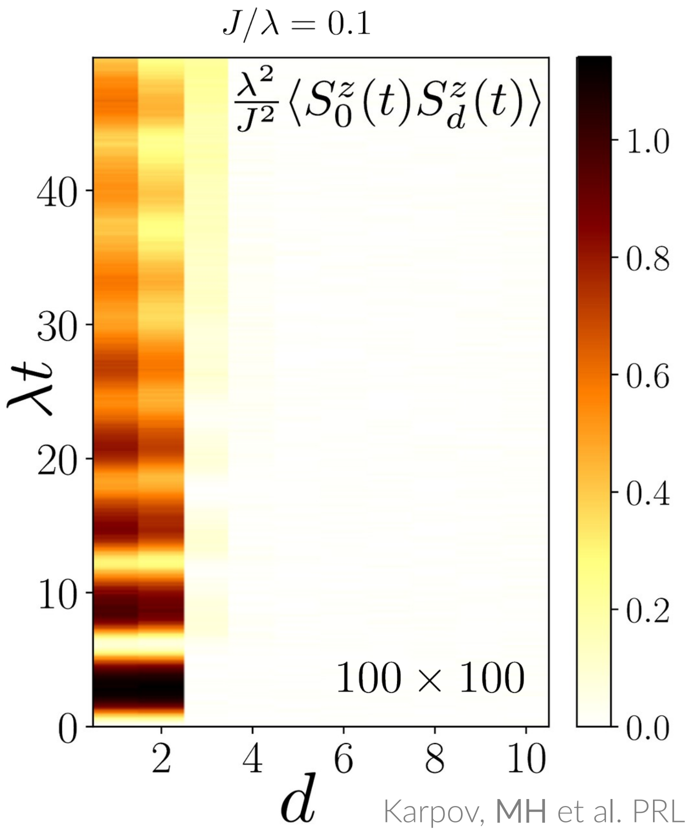
Initial condition:

$$|\psi_0\rangle = |\rightarrow\rangle = \frac{1}{\sqrt{\dim_H}} \sum_s |s\rangle$$

$\rightarrow$ superposition over all superselection sectors

No propagation of quantum correlations

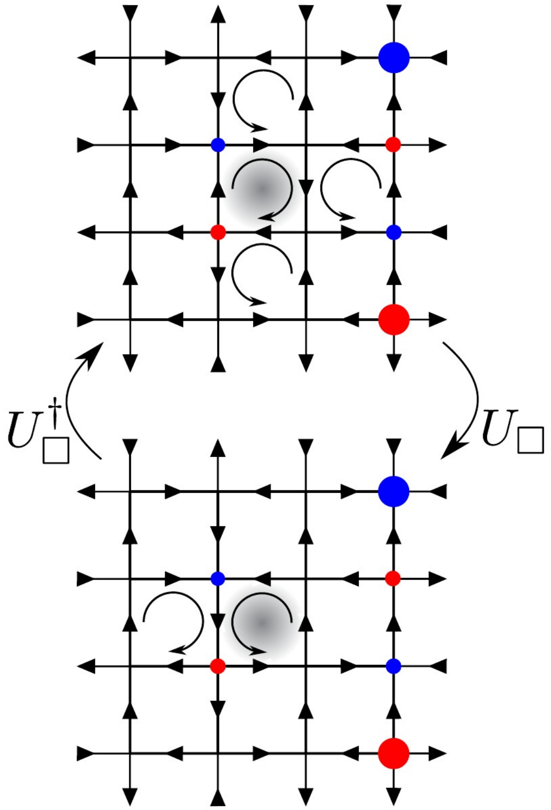
Is this just *slow dynamics* or really *nonergodic* behavior?

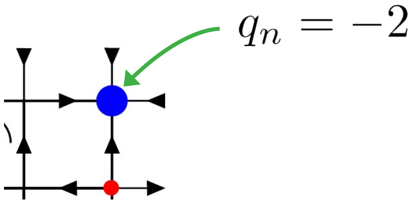


Karpov, MH et al. PRL '21

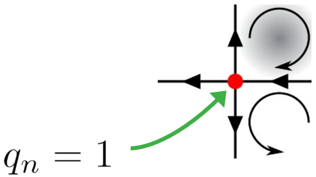
Solution via classical networks
Verdel, MH et al. PRB '21

Background charges & kinetic constraints





All 4 spins will be frozen ***forever***

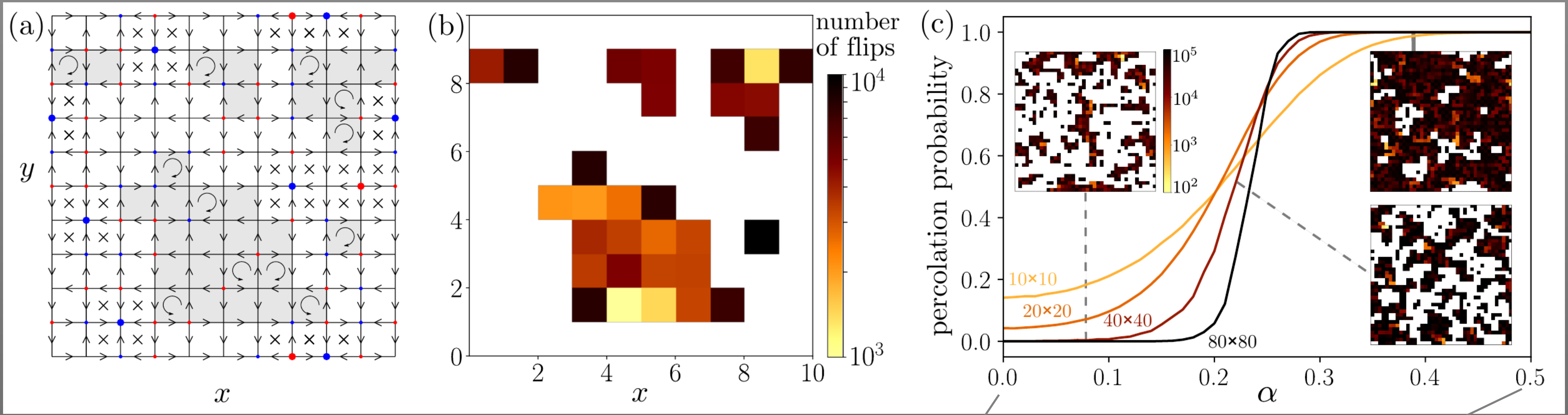


At least 2 neighboring plaquettes unflippable

Key question: kinetics local or can excitations still propagate?

Mapping onto percolation problem

Typical superselection sector | Kinetics & clusters | Percolation of clusters



Karpov, **MH** et al. PRL '21

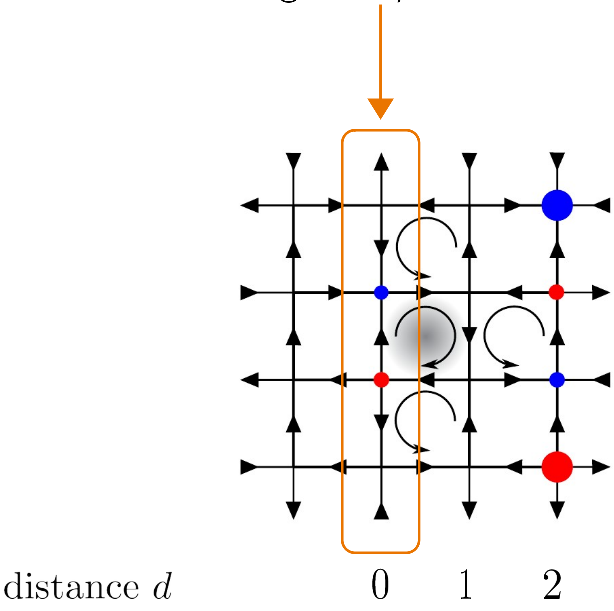
$|\rightarrow\rangle$

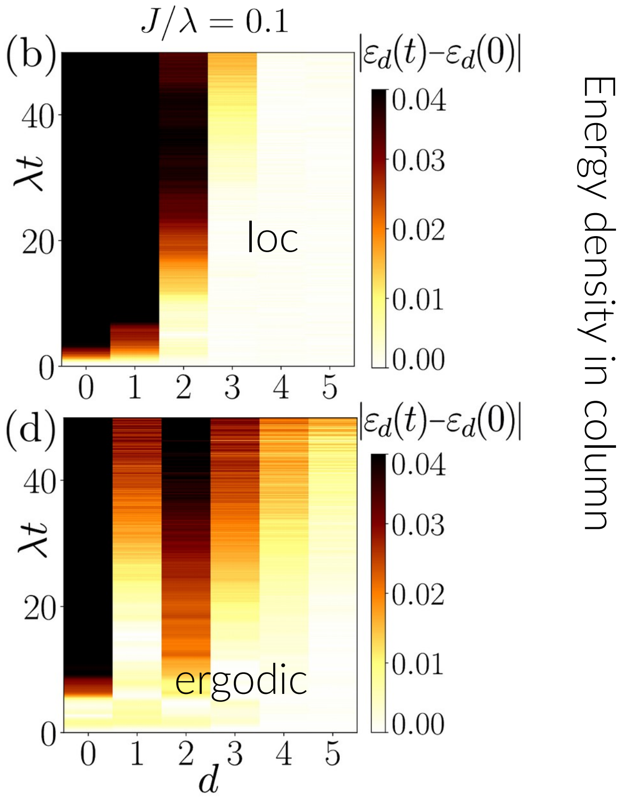
fully-flippable sector
(no background charges)

No percolation → Small clusters → **Ergodicity breaking**

Signatures in quantum dynamics

Create an energy inhomogeneity in column





Background charges & mixed states

$$Z = \mathrm{tr} e^{-\beta H} = \sum_{\{q_\alpha\}} \mathrm{tr}_{\{q_\alpha\}} e^{-\beta H_{\{q_\alpha\}}}$$

low temperatures: typically favoring homogeneous configurations (lower entropy)

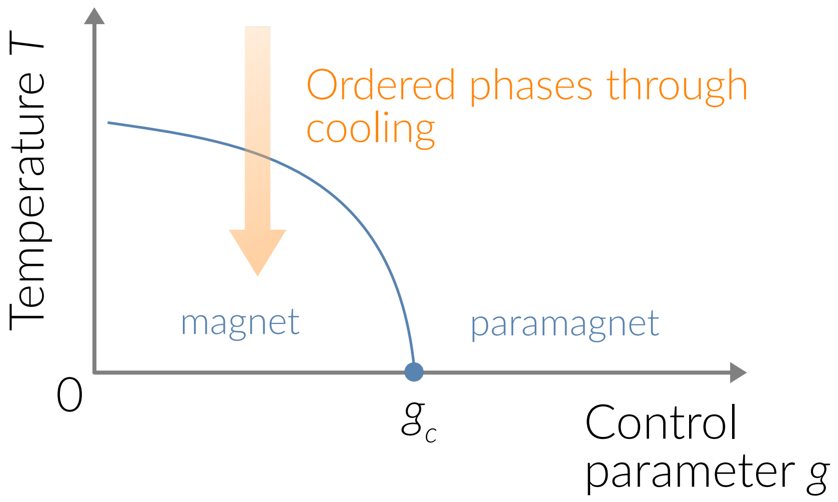
$$Z \to \mathrm{tr}_{\{q_\alpha = 0\}} e^{-\beta H_{\{q_\alpha = 0\}}}$$

High temperatures: background charges increasingly random

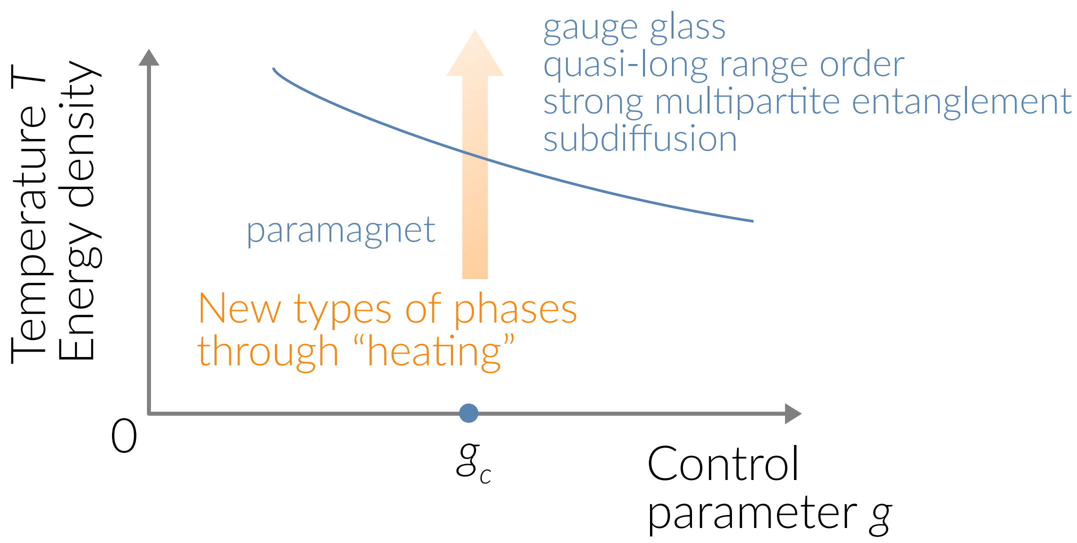
→ disorder-free localization

New kinds of order

Conventional matter



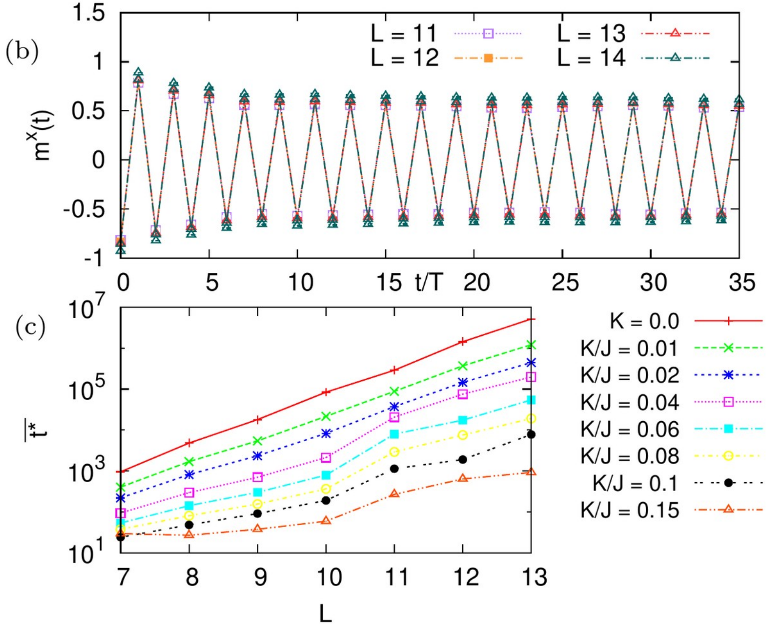
Matter with gauge constraints



New types of order

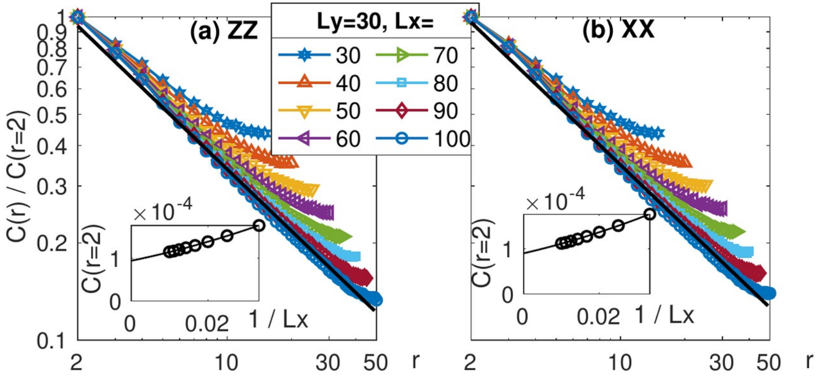
Localization protected quantum order

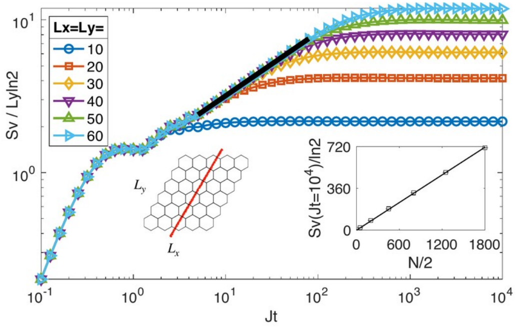
Gauge time crystal



Russomanno, **MH** et al. PRR '20

Quasi long-range order





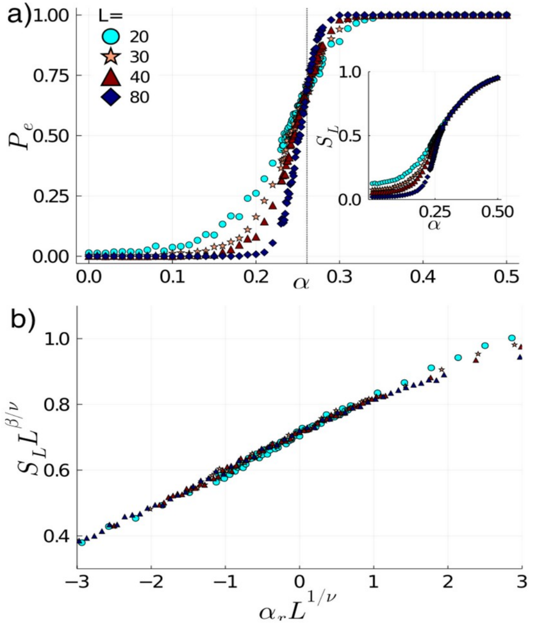
Zhu & **MH** PRR '21

Summary & outlook

- Disorder-free localization as a new mechanism for ergodicity breaking
- Robust ergodicity breaking even in 2D (provided gauge invariance is preserved)
- What new types of nonequilibrium phases are possible?
- What about 3D? Can there still be a nonergodic phase?
- What happens in the presence of matter in 2D?
- Implications on high-temperature spectral functions?

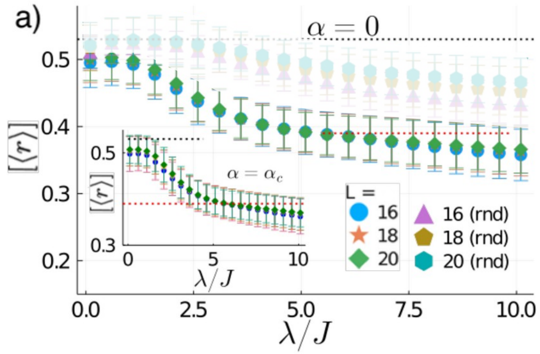
Quantum localization transition

Chakraborty, **MH** et al. '22



Universality class of 2D site percolation

Level-spacing statistics for individual clusters

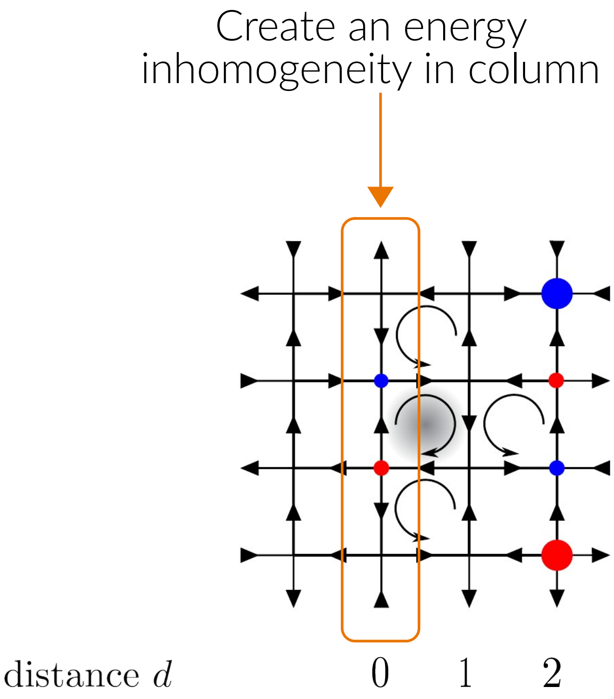


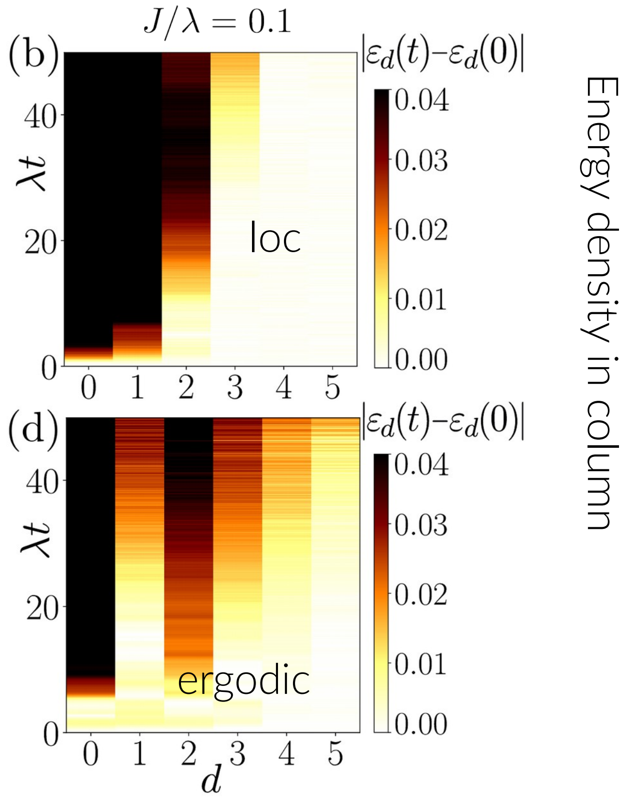
cluster size

Clusters ergodic

Quantum thermalization transition
= classical percolation transition

Signatures in quantum dynamics





Method: classical networks

Challenge: no *efficient* compression of quantum states available 2D

$$|\psi\rangle = \sum_s \psi(s)|s\rangle$$

amplitudes exponential in system size

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amplitudes exponential in system size

Classical networks: *"Don't store. Generate on the fly."*

Sample using MC techniques

Schmitt & MH SciPost '18

Method: classical networks

$|s\rangle$ → Classical network → $\tilde{\psi}(s,t) = e^{\mathcal{H}_{\text{eff}}(s,t)}$

Schmitt & MH SciPost '18

Effective classical Hamiltonian

- Structure obtained from cumulant expansion
  (around a classical limit)

$$\mathcal{H}_{\text{eff}}(s,t) = h_0(s,t) + \epsilon h_1(s,t) + \epsilon^2 h_2(s,t) + \dots$$

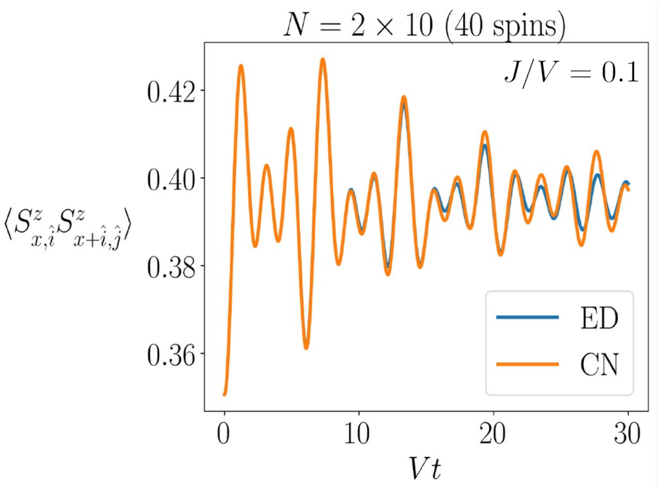
- Further variationally optimized
- "Simple artificial neural network"

Method: classical networks

$$|s\rangle \longrightarrow \text{Classical network} \longrightarrow \tilde{\psi}(s,t) = e^{\mathcal{H}_{\text{eff}}(s,t)}$$

Schmitt & MH SciPost '18

Dynamics in zero charge superselection sector (worst case for method)



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