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Enrique Rico Ortega Friday, 13/05/2022

Max Planck Institute of Quantum Optics, Munich

Gauge Workshop Munich 2022

May 9 – 13, 2022 Max Planck Institute of Quantum Optics



# A fruitful dialogue (two-way communication)







### Quantum Information Science and Technology



# A fruitful dialogue (two-way communication)







### Quantum Information Science and Technology

#### Qubit alive thanks to the anomaly

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#### arXiv:2109.11824v1





# Simulating lattice gauge theories within quantum technologies





Quantum simulation of light-front parton correlators





# Simulating lattice gauge theories within quantum technologies



Collaborators: M. Dalmonte, S. Montangero, U.-J. Wiese, P. Zoller...

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Colloquium

THE EUROPEAN PHYSICAL JOURNAL D

### Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls<sup>1,2</sup>, Rainer Blatt<sup>3,4</sup>, Jacopo Catani<sup>5,6,7</sup>, Alessio Celi<sup>3,8</sup>, Juan Ignacio Cirac<sup>1,2</sup>, Marcello Dalmonte<sup>9,10</sup>, Leonardo Fallani<sup>5,6,7</sup>, Karl Jansen<sup>11</sup>, Maciej Lewenstein<sup>8,12,13</sup>, Simone Montangero<sup>14,15,a</sup>, Christine A. Muschik<sup>3</sup>, Benni Reznik<sup>16</sup>, Enrique Rico<sup>17,18</sup>, Luca Tagliacozzo<sup>19</sup>, Karel Van Acoleyen<sup>20</sup>, Frank Verstraete<sup>20,21</sup>, Uwe-Jens Wiese<sup>22</sup>, Matthew Wingate<sup>23</sup>, Jakub Zakrzewski<sup>24,25</sup>, and Peter Zoller<sup>3</sup>



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#### **Quantum simulation of light-front parton correlators**

M. G. Echevarria<sup>(D)</sup>,<sup>1,\*</sup> I. L. Egusquiza,<sup>2,†</sup> E. Rico<sup>(D)</sup>,<sup>3,4,‡</sup> and G. Schnell<sup>(D)</sup>,<sup>2,4,§</sup>

arXiv:2011.01275 Phys. Rev. D 104, 014512 (2021)

Project in progress with: M.G. Echevarria, I.L. Egusquiza, G. Schnell



Quantum simulation of light-front parton correlators





### Three ingredients to describe Nature





### Three ingredients to describe Nature

 Quantum matter as the basic building block





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### Three ingredients to describe Nature

 Quantum matter as the basic building block

• Gauge symmetry as a fundamental principle and at the origin of every force





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### Three ingredients to describe Nature

Quantum matter as the basic building block

• Gauge symmetry as a fundamental principle and at the origin of every force

• Renormalisation group as a tool to study Nature at different scales





Memory

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## Quantum matter as the basic building block



R.P. Feynman, Int. J. Theor. Phys. (1982)



Preparation of a general quantum state

 $|\psi\rangle = c_1 |\uparrow\uparrow\cdots\uparrow\rangle + c_2 |\uparrow\uparrow\cdots\downarrow\rangle + \cdots + c_{2^N} |\downarrow\downarrow\cdots\downarrow\rangle$ 





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### Quantum matter as the basic building block



R.P. Feynman, Int. J. Theor. Phys. (1982)



Preparation of a general quantum state

S. Lloyd, Science (1996)

Evolution of a general quantum state

 $|\psi(t)\rangle = U(t)|\psi\rangle$ 







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Implementing the gauge invariant dynamics





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### modern microscopes

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(semi-inclusive) deep-inelastic lepton scattering



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#### modern microscopes

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(semi-inclusive) deep-inelastic lepton scattering highly virtual photons resolve inner (partonic) structure



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#### modern microscopes

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(semi-inclusive) deep-inelastic lepton scattering highly virtual photons resolve inner (partonic) structure factorization theorems separate non-calculable from calculable parts



Quantum simulation of light-front parton correlators



leptonic part

non-perturbative

hard part

cross section:



factorization theorems separate non-calculable from calculable parts



Quantum simulation of light-front parton correlators





partonic cross section: calculable leptonic
part
part

/\*
hard
part

/\*
hard
part
part
/\*

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factorization theorems separate non-calculable from calculable parts



Quantum simulation of light-front parton correlators





partonic cross section: calculable

non-perturbative parametrization of nucleon: PDFs, TMDs etc.

factorization theorems separate non-calculable from calculable parts

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leptonic part

non-perturbative

hard part





$$f_{f/P}(\xi) = \sum_{S} \int \frac{dy}{2\pi} e^{-i\xi p^+ y^-} \langle PS | \left[\bar{\psi}\mathcal{U}\right] \left(y^-\right) \frac{\gamma}{2} \left[\mathcal{U}^{\dagger}\psi\right] (0) | PS \rangle$$



$$f_{f/P}(\xi) = \sum_{S} \int \frac{dy^{-}}{2\pi} e^{-i\xi p^{+}y^{-}} \langle PS | \left[\bar{\psi}\mathcal{U}\right] \left(y^{-}\right) \frac{\gamma^{+}}{2} \left[\mathcal{U}^{\dagger}\psi\right] (\mathbf{0}) | PS \rangle$$

Non-local (space-time) matrix elements require Wilson lines for gauge invariance We study the quantum simulation of Wilson loops in space and real-time



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Quantum simulation of light-front parton correlators



Non-local (space-time) matrix elements require Wilson lines for gauge invariance We study the quantum simulation of Wilson loops in space and real-time

$$f_{f/P}(\xi) = \sum_{S} \int \frac{dy^{-}}{2\pi} e^{-i\xi p^{+}y^{-}} \langle PS | \left[\bar{\psi}\mathcal{U}\right] \left(y^{-}\right) \frac{\gamma^{+}}{2} \left[\mathcal{U}^{\dagger}\psi\right](\mathbf{0}) | PS \rangle$$

Requirements for the quantum simulation of parton correlators:



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# Quantum simulation of light-front parton correlators



Non-local (space-time) matrix elements require Wilson lines for gauge invariance We study the quantum simulation of Wilson loops in space and real-time

$$f_{f/P}(\xi) = \sum_{S} \int \frac{dy^{-}}{2\pi} e^{-i\xi p^{+}y^{-}} \langle PS | \left[\bar{\psi}\mathcal{U}\right] \left(y^{-}\right) \frac{\gamma^{+}}{2} \left[\mathcal{U}^{\dagger}\psi\right](0) | PS \rangle$$

Requirements for the quantum simulation of parton correlators:

- encode in quantum degrees of freedom both matter and gauge fields
- preparation of a reference state, e.g., vacuum, proton, glue-ball
- simulate gauge-invariant quantities, e.g., minimal gauge-matter coupling
- real-time evolution, since the Wilson line is non-local in time
- carry out measurements after the evolution, i.e., quantum interferometer



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Digital simulation: Universal simulator



Decompose dynamics into sequence of quantum gates

Stroboscopic simulation in an analog simulator



# Quantum simulation of light-front parton correlators



Discretisation of space-time in a Hamiltonian formulation



the temporal gauge  $A_0=0$  is chosen





Quantum simulation of light-front parton correlators



Moving a single quark:

$$u_{12} = \exp\left\{\frac{-i\pi}{2}\sum_{\alpha\beta} \left[\psi_{\alpha,1}^{\dagger}U_{\alpha\beta}(e)\psi_{\beta,2} + h.c.\right]\right\}$$

$$\rightarrow (-i)\left[\psi_{\alpha,1}^{\dagger}U_{\alpha\beta}(e)\psi_{\beta,2} + h.c.\right],$$

$$(-i)\left[\psi_{\alpha,1}^{\dagger}U_{\alpha\beta}(e)\psi_{\beta,2} + h.c.\right],$$



Quantum simulation of light-front parton correlators



Moving a single quark:

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Quantum simulation of light-front parton correlators



Moving a single quark:

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Quantum simulation of light-front parton correlators



2

#### Moving a single quark:

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$$|\mathbf{m}\rangle \equiv \frac{1}{N^{1/2}} \sum_{\alpha=1}^{N} |\alpha(1), \bar{\alpha}(2)\rangle$$



Quantum simulation of light-front parton correlators



2

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Quantum simulation of light-front parton correlators



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Quantum simulation of light-front parton correlators



line

2

### Moving a single quark:

$$\begin{split} u_{12} &= \exp\left\{\frac{-i\pi}{2}\sum_{\alpha\beta}\left[\psi_{\alpha,1}^{\dagger}U_{\alpha\beta}(e)\,\psi_{\beta,2} + \mathrm{h.c.}\right]\right\}\\ &\to (-i)\left[\psi_{\alpha,1}^{\dagger}U_{\alpha\beta}(e)\,\psi_{\beta,2} + \mathrm{h.c.}\right], \end{split}$$

$$\begin{split} |\mathbf{m}\rangle &\equiv \frac{1}{N^{1/2}} \sum_{\alpha=1}^{N} |\alpha(1), \bar{\alpha}(2)\rangle \\ \mathscr{U}(A_1, B_L) &= \frac{1}{N^{1/2}} \sum_{\alpha\beta\cdots\mu\nu\omega\cdots\theta\phi} |\alpha(A_1)\rangle U_{\alpha\beta}(e_1)\cdots U_{\mu\nu}(e_{L/2-1}) U_{\omega\nu}^*(e_{L/2})\cdots U_{\phi\theta}^*(e_{L-1}) |\bar{\phi}(B_L)\rangle \\ &= \frac{1}{N^{1/2}} \sum_{\alpha\phi} |\alpha(A_1)\rangle \mathscr{U}_{\alpha\phi}(e_1, \cdots, e_{L-1}) |\bar{\phi}(B_L)\rangle \\ \end{split}$$
 we built a spatial Wilson



## Quantum simulation of light-front parton correlators



 $W(\tau,\lambda) = W_{C_1}W_{\tau_1}W_{C_2}W_{\tau_2}\cdots W_{C_k}W_{\tau_k}\cdots$ 

Time-evolution by a single time step

$$|\psi(0)\rangle \equiv e^{-iH\tau} \equiv |\psi(\tau)\rangle$$





Decompose dynamics induced by systems' Hamiltonian into sequence of quantum gates

Digital simulation can simulate any model but requires many gate operations

Stroboscopic simulation in an analog simulator

 $H = H_{\rm el} + H_{\rm mag}$ 

Efficient for local interactions

$$e^{-iH} \simeq \left[ e^{-iH_{\rm el}/2n_T} e^{-i\lambda H_{\rm mag}/n_T} e^{-iH_{\rm el}/2n_T} \right]^{n_T}$$

Trotter-Suzuki approximation

S. Lloyd, Science (1996)


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Quantum simulation of light-front parton correlators



Proof of principle: Z<sub>2</sub> pure gauge model



within a few Trotter steps a fidelity closed to one is achieved

operator norm:

 $\left| \operatorname{Tr} \left[ \mathscr{W}^{\dagger} \mathscr{W}_{\mathbf{n}_{\mathrm{T}}} \right] \right|$ 

ground state fidelity:

 $\langle g.s. | \mathcal{W}^{\dagger} \mathcal{W}_{n_{T}} | g.s. \rangle$ 





#### **Quantum simulation of light-front parton correlators**

M. G. Echevarria<sup>(D)</sup>,<sup>1,\*</sup> I. L. Egusquiza,<sup>2,†</sup> E. Rico<sup>(D)</sup>,<sup>3,4,‡</sup> and G. Schnell<sup>(D)</sup>,<sup>2,4,§</sup>

arXiv:2011.01275

Project in progress with: M.G. Echevarria, I.L. Egusquiza, G. Schnell

Collaborators: M. Dalmonte, S. Montangero, U.-J. Wiese, P. Zoller...

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### Qubit alive thanks to the anomaly

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### **Quantum anomaly**

In quantum physics an anomaly or quantum anomaly appears when the symmetry of a classical theory is not equally represented by the quantum theory.





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In quantum physics an anomaly or quantum anomaly appears when the symmetry of a classical theory is not equally represented by the quantum theory.

Classical group symmetry of the ring







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Classical group symmetry of the ring





Rotation by any angle

SO(2)





### **Quantum anomaly**

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In quantum physics an anomaly or quantum anomaly appears when the symmetry of a classical theory is not equally represented by the quantum theory.





# Free quantum particle on a ring





 $\hat{H} = E_c \left( \hat{n} - n_g \right)^2 \qquad \qquad \left[ \hat{\theta}, \hat{n} \right] = i$ 



## Free quantum particle on a ring









# Free quantum particle on a ring





 $\hat{H} = E_c \left( \hat{n} - n_g \right)^2$ vector potential or
magnetic flux

 $\left[\hat{\theta}, \hat{n}\right] = i$ 

conjugate variables

 $\theta \in (-\pi, +\pi]$ 

compact variable

spect  $(\hat{n}) \in \mathbb{Z}$ 



# Free quantum particle on a ring





Any rotation is a symmetry of the quantum Hamiltonian:  $\hat{U}_{\alpha} = e^{i\hat{n}\alpha}$   $SO(2) \sim U(1)$ 



# Free quantum particle on a ring



 $SO(2) \sim U(1)$ 



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spect  $(\hat{H})$ 

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## Free quantum particle on a ring



 $SO(2) \sim U(1)$ 

Any rotation is a symmetry of the quantum Hamiltonian:  $\hat{U}_{\alpha} = e^{i\hat{n}\alpha}$ 



$$\hat{U}_P \qquad \begin{array}{c} n \to -n \\ \theta \to -\theta \end{array}$$

$$\hat{U}_P \hat{U}_\alpha \hat{U}_P = \hat{U}_{-\alpha}$$
$$O(2) = SO(2) \times \mathbb{Z}_2$$



### Free quantum particle on a ring



Any rotation is a symmetry of the quantum Hamiltonian:  $\hat{U}_{\alpha} = e^{i\hat{n}\alpha}$   $SO(2) \sim U(1)$ About the reflexion symmetry...





$$\hat{V}_P \hat{U}_\alpha \hat{V}_P = e^{-i\alpha} \hat{U}_{-\alpha}$$

 $O(2) = SO(2) \times \mathbb{Z}_2$ 

 $\hat{U}_P \hat{U}_\alpha \hat{U}_P = \hat{U}_{-\alpha}$ 



### Free quantum particle on a ring



Any rotation is a symmetry of the quantum Hamiltonian:  $\hat{U}_{\alpha} = e^{i\hat{n}\alpha}$   $SO(2) \sim U(1)$ About the reflexion symmetry...











Minimal model with potential term that still keeps similar symmetry features

$$\hat{H} = E_c \left( \hat{n} - n_g \right)^2 - E_J \cos\left( 2\hat{\theta} \right)$$



### Quantum particle on a ring with a potential



Minimal model with potential term that still keeps similar symmetry features

$$\hat{H} = E_c \left( \hat{n} - n_g \right)^2 - E_J \cos\left(2\hat{\theta}\right)$$

$$n_g = 0 \qquad \qquad n_g = 1/2$$



### **Quantum particle on** a ring with a potential



Minimal model with potential term that still keeps similar symmetry features

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$$\begin{split} n_g &= 0 & n_g = 1/2 \\ \hat{U}_P & \begin{array}{c} n \to -n \\ \theta \to -\theta \end{array} & \hat{V}_P &= e^{-i\hat{\theta}} \hat{U}_P & \begin{array}{c} n \to 1-n \\ \theta \to -\theta \end{array} \end{split}$$

ĺ About the (discrete) rotation symmetry:

$$\hat{U}_{\pi} = e^{i\hat{n}\pi} \qquad \mathbb{Z}_2$$



commute

### **Quantum particle on** a ring with a potential



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 $\hat{U}_{\pi} = e^{i\hat{n}\pi}$  $\mathbb{Z}_2$ About the (discrete) rotation symmetry:

$$\begin{array}{ll} n_g = 0 & parameter & n_g = 1/2 \\ \hat{U}_P \hat{U}_\pi = \hat{U}_\pi \hat{U}_P & multiplication relations & \hat{V}_P \hat{U}_\pi = - \hat{U}_\pi \hat{V}_P \\ \mathbb{Z}_2 \times \mathbb{Z}_2 & group \ symmetry & D_4 \end{array}$$

anti-commute

1/2



commute

## Quantum particle on a ring with a potential



degeneracy

Minimal model with potential term that still keeps similar symmetry features

$$\hat{H} = E_c \left( \hat{n} - n_g \right)^2 - E_J \cos\left(2\hat{\theta}\right)$$

About the reflexion symmetry...

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 $\begin{array}{ccc} n_g = 0 & parameter & n_g = 1/2 \\ \hat{U}_P \hat{U}_\pi = \hat{U}_\pi \hat{U}_P & multiplication relations & \hat{V}_P \hat{U}_\pi = - \hat{U}_\pi \hat{V}_P & \text{anti-commute} \\ \mathbb{Z}_2 \times \mathbb{Z}_2 & group symmetry & D_4 & \bigvee_{\text{two-fold}} \end{array}$ 



### **Superconducting qubits**

Example: the transmon



J. Koch et al., Phys. Rev. A (2007)





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### **Superconducting qubits**

Charging hamiltonian of the SC: Junction also acts as a capacitor





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### Superconducting qubits

Charging hamiltonian of the SC: Junction also acts as a capacitor

Josephson tunnelling:

• couple two superconductors via oxide layer

superconducting gap inhibits electron tunnelling

• oxide layer acts as tunnelling barrier





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Josephson tunnelling:

- couple two superconductors via oxide layer
- oxide layer acts as tunnelling barrier
- superconducting gap inhibits electron tunnelling

Josephson Hamiltonian:

$$\hat{H}_{J} = -\frac{E_{J}}{2} \sum_{n} |n\rangle \langle n+1| + |n+1\rangle \langle n|$$

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### **Superconducting qubits**

Charging hamiltonian of the SC: Junction also acts as a capacitor



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- superconducting gap inhibits electron tunnelling

Josephson Hamiltonian:

$$\hat{H}_{J} = -\frac{E_{J}}{2} \sum_{n} |n\rangle \langle n+1| + |n+1\rangle \langle n|$$
written in terms of the con

 $= -E_J \cos \hat{\theta}$ 

written in terms of the conjugate variable (Fourier transform) Physically: difference of the SC phases

$$\left[\hat{\theta},\hat{n}\right]=i$$







### **Energy parameters**

 $\hat{H} = 4E_C \hat{n}^2 - E_J \cos \hat{\theta}$ 

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 $E_J \gg E_C$  $\omega = \sqrt{8E_C E_J}$ 

 $C_{s}$   $C_{s}$   $C_{s}$ Designed regime: (Potential dominated)





 $L_{J}$  $C_{I}$ 

(a)

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 $C_{s}$ 

Designed regime:

(Potential dominated)

Φ

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#### **Energy parameters**

 $\hat{H} = 4E_C \hat{n}^2 - E_J \cos \hat{\theta}$ 

 $E_J \gg E_C$ 

 $\phi$ 

 $\omega = \sqrt{8E_C E_J}$ 

anharmonicity non-linear inductance

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P Krantz et al. App. Phys. Rev. (2019)



 $\pi$ 



(c)





#### **Other (control) parameters**

$$\hat{H} = 4E_C \left(\hat{n} - n_g\right)^2 - E_J \cos\hat{\theta}$$

 $n_g \rightarrow$  Gate charge

Transmon: insensitive to charge noise

J. Koch et al., Phys. Rev. A (2007)











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$$H_{0-\pi} = 4E_{C_J}\hat{Q}_{\phi}^2 + E_L\hat{\phi}^2 + 4E_{C_s}\left(\hat{n}_{\theta} - n_g\right)^2 - 2E_J\cos\hat{\theta}\cos\left(\hat{\phi} - \frac{\varphi_{\text{ext}}}{2}\right)$$

#### Protected from both charge noise and flux noise

A. Di Paolo, et al. New J. Phys. (2019)

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 $0 - \pi$  qubit



W.C. Smith, et al. Quant. Info. (2020)

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 $0 - \pi$  qubit





W.C. Smith, et al. Quant. Info. (2020)



### $0 - \pi$ qubit



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#### $0 - \pi$ Hamiltonian





 $H_{0-\pi} = 4E_{C_J}\hat{Q}_{\phi}^2 + E_L\hat{\phi}^2 + 4E_{C_s}\left(\hat{n}_{\theta} - n_g\right)^2 - 2E_J\cos\hat{\theta}\cos\left(\hat{\phi} - \frac{\varphi_{\text{ext}}}{2}\right)$ 



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## $0 - \pi$ Hamiltonian





harmonic oscillator diagonal interaction  $H_{0-\pi} = 4E_{C_J}\hat{Q}_{\phi}^2 + E_L\hat{\phi}^2 + 4E_{C_s}\left(\hat{n}_{\theta} - n_g\right)^2 - 2E_J\cos\hat{\theta}\cos\left(\hat{\phi} - \frac{\varphi_{\text{ext}}}{2}\right)$ 

$$\begin{bmatrix} \hat{\theta}, \hat{n}_{\theta} \end{bmatrix} = i \qquad \theta \in (-\pi, +\pi] \qquad \text{spect} \left( \hat{n} \right) \in \mathbb{Z}$$
$$\begin{bmatrix} \hat{\phi}, \hat{Q}_{\phi} \end{bmatrix} = i \qquad \phi \in \mathbb{R} \qquad \hat{Q}_{\phi} \in \mathbb{R}$$



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## $0 - \pi$ Hamiltonian





harmonic oscillator diagonal interaction  $H_{0-\pi} = 4E_{C_J}\hat{Q}_{\phi}^2 + E_L\hat{\phi}^2 + 4E_{C_s}\left(\hat{n}_{\theta} - n_g\right)^2 - 2E_J\cos\hat{\theta}\cos\left(\hat{\phi} - \frac{\varphi_{\text{ext}}}{2}\right)$ 

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high symmetry point

$$n_g = 1/2, \ \varphi_{\text{ext}} = \pi$$

$$\hat{V}_P = e^{-i\hat{\theta}}\hat{U}_P \qquad \begin{array}{c} n_\theta \to 1 - n_\theta \\ \theta \to -\theta \end{array}$$

$$\hat{U}_{\pi} = e^{i\hat{n}_{\theta}\pi}\hat{P}_{\phi} \qquad \begin{array}{c} \theta \to \theta + \pi \\ \phi \to -\phi \end{array}$$

$$\hat{V}_P \hat{U}_\pi = - \hat{U}_\pi \hat{V}_P$$



# $0 - \pi$ Hamiltonian





high symmetry point

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$$\label{eq:V_P} \hat{V}_P = e^{-i\hat{\theta}} \hat{U}_P \qquad \begin{array}{c} n_\theta \to 1 - n_\theta \\ \theta \to -\theta \end{array}$$

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$$\hat{V}_P \hat{U}_\pi = - \hat{U}_\pi \hat{V}_P$$

harmonic oscillator  

$$\int_{-\pi} = 4E_{C_J}\hat{Q}_{\phi}^2 + E_L\hat{\phi}^2 + 4E_{C_s}\left(\hat{n}_{\theta} - n_g\right)^2 - 2E_J\cos\hat{\theta}\cos\left(\hat{\phi} - \frac{\varphi_{\text{ext}}}{2}\right)$$

$$\begin{bmatrix} \hat{\theta}, \hat{n}_{\theta} \end{bmatrix} = i \qquad \theta \in (-\pi, +\pi] \qquad \text{spect} \left( \hat{n} \right) \in \mathbb{Z}$$
$$\begin{bmatrix} \hat{\phi}, \hat{Q}_{\phi} \end{bmatrix} = i \qquad \phi \in \mathbb{R} \qquad \hat{Q}_{\phi} \in \mathbb{R}$$

Spectrum in units of 
$$E_L = E_{C_S} = E_{C_J}$$

the whole spectrum is two-fold degenerate independent of any energy scales



### $0 - \pi$ Hamiltonian



$$H_{0-\pi} = 4E_{C_J}\hat{Q}_{\phi}^2 + E_L\hat{\phi}^2 + 4E_{C_s}\left(\hat{n}_{\theta} - n_g\right)^2 - 2E_J\cos\hat{\theta}\cos\left(\hat{\phi} - \frac{\varphi_{\text{ext}}}{2}\right)$$

Sensitivity to charge and flux noise exponentially decreased in the regimen:

$$E_{C_J}^2 \gg E_{C_J} E_L \gg E_J^2 \gg E_{C_J} E_{C_s}$$



### $0 - \pi$ Hamiltonian



$$H_{0-\pi} = 4E_{C_J}\hat{Q}_{\phi}^2 + E_L\hat{\phi}^2 + 4E_{C_s}\left(\hat{n}_{\theta} - n_g\right)^2 - 2E_J\cos\hat{\theta}\cos\left(\hat{\phi} - \frac{\varphi_{\text{ext}}}{2}\right)$$

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Potential dominance for  $\boldsymbol{\theta}$ 

$$E_J \gg E_{C_s}$$

Transmon regime

**Protection against charge noise** 



## $0 - \pi$ Hamiltonian



$$H_{0-\pi} = 4E_{C_J}\hat{Q}_{\phi}^2 + E_L\hat{\phi}^2 + 4E_{C_s}\left(\hat{n}_{\theta} - n_g\right)^2 - 2E_J\cos\hat{\theta}\cos\left(\hat{\phi} - \frac{\varphi_{\text{ext}}}{2}\right)$$

Sensitivity to charge and flux noise exponentially decreased in the regimen:

$$E_{C_J}^2 \gg E_{C_J} E_L \gg E_J^2 \gg E_{C_J} E_{C_s}$$

Kinetic dominance

$$E_{C_J} \gg E_L$$

High delocalisation of  $\phi$ , no effect of  $\varphi_{ext}$ 

**Protection against flux noise** 

Potential dominance for  $\theta$ 

$$E_J \gg E_{C_s}$$

Transmon regime

**Protection against charge noise** 



### $0 - \pi$ Hamiltonian



Sensitivity to charge and flux noise exponentially decreased in the regimen:

 $E_{C_J}^2 \gg E_{C_J} E_L \gg E_J^2 \gg E_{C_J} E_{C_s}$ 





### $0 - \pi$ Hamiltonian



Sensitivity to charge and flux noise exponentially decreased in the regimen:

 $E_{C_J}^2 \gg E_{C_J} E_L \gg E_J^2 \gg E_{C_J} E_{C_s}$ 







# Qubit alive thanks to the anomaly



#### There exist anomalous symmetries: degeneracies independent of energy parameters

# The degeneracies in the protected regime of the $\,0$ – $\pi\,$ quit are a remnant of the anomalous symmetry

#### PHYSICAL REVIEW B 105, L201104 (2022)

#### Letter

#### Role of anomalous symmetry in $0-\pi$ qubits

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#### **Confirmed invited speakers**



# New trends in complex quantum systems dynamics 2022

In our Quantum Science era where fault-tolerant quantum devices are still not available but Noisy Intermediate-Scale Quantum (NISQ) devices are accessible, quantum information tools to guide their development play a fundamental role. With the foreseen increasing complexity of available NISQ

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