

# TRIMER STATES WITH $Z_3$ TOPOLOGICAL ORDER IN RYDBERG ATOM ARRAYS

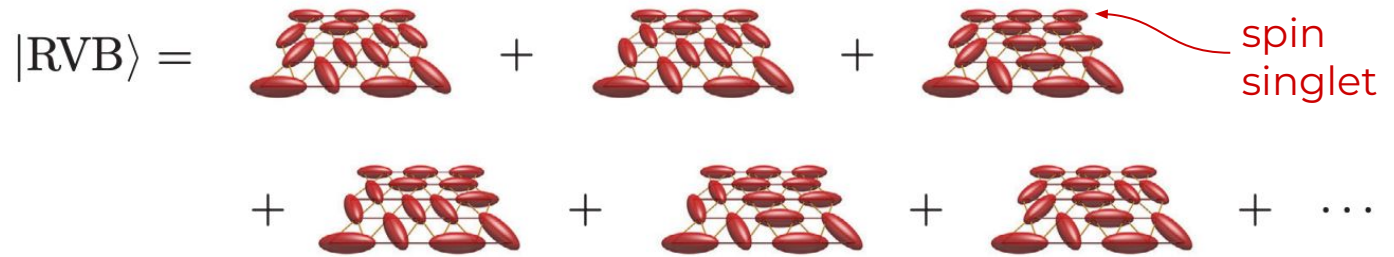
Federica Surace

Gauge Workshop Munich - May 11, 2022

Caltech

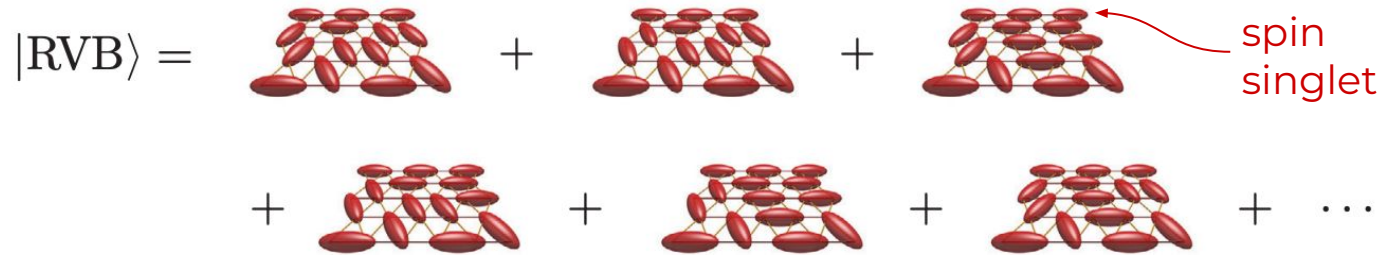
IQIM

## Anderson 1973: Quantum spin liquids



Savary, Rep. Prog. Phys 2017

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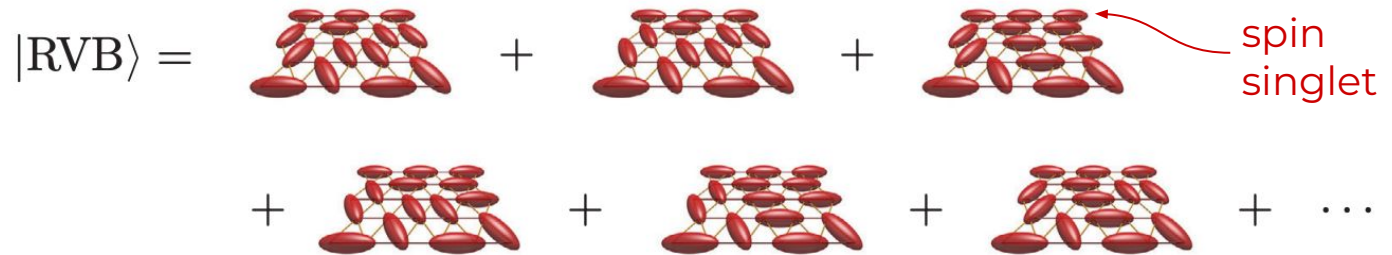


Savary, Rep. Prog. Phys 2017

## Rokhsar Kivelson 1988: Quantum dimer model

See also: 1987 Kivelson Rokhsar Sethna, 1988 Affleck Marston, 1991 Read Sachdev, 1991 Wen

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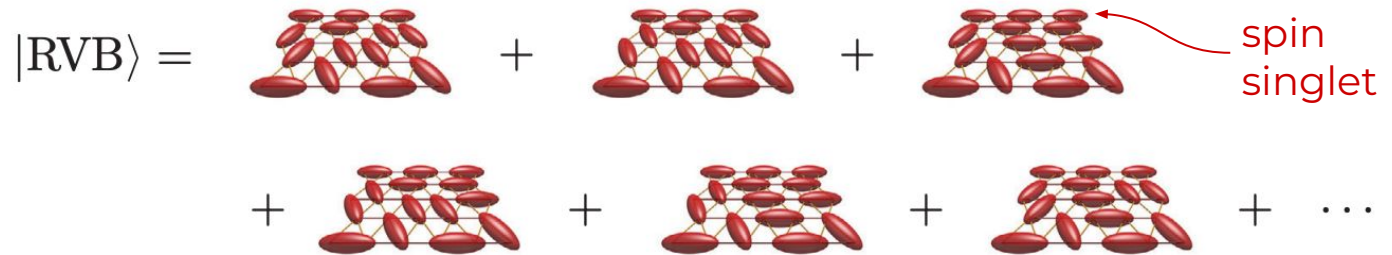
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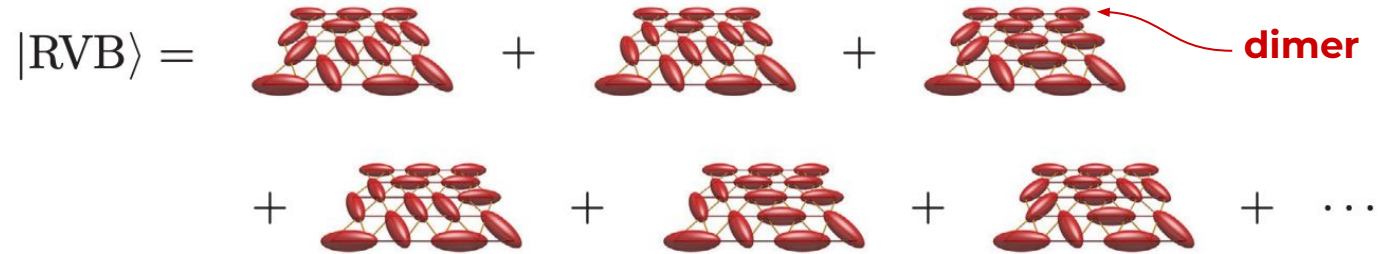
## Rokhsar Kivelson 1988: Quantum dimer model

## Senthil Fisher 2000: QSL and Ising lattice gauge theory

## Moessner Sondhi 2001: Stable QSL in the dimer model on triangular lattice

See also: 1987 Kivelson Rokhsar Sethna, 1988 Affleck Marston, 1991 Read Sachdev, 1991 Wen

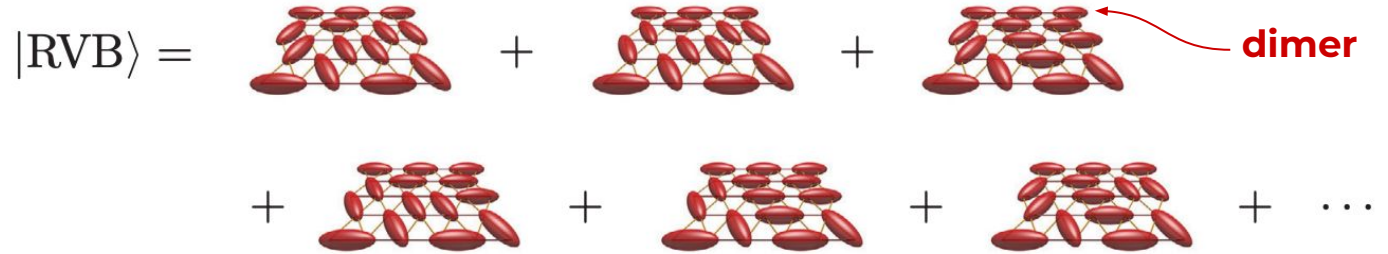
# TOPOLOGICAL ORDER IN DIMER MODELS



*is it a stable QSL?*

Savary, Rep. Prog. Phys 2017

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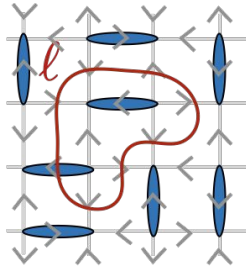
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## 2D bipartite lattices ✗

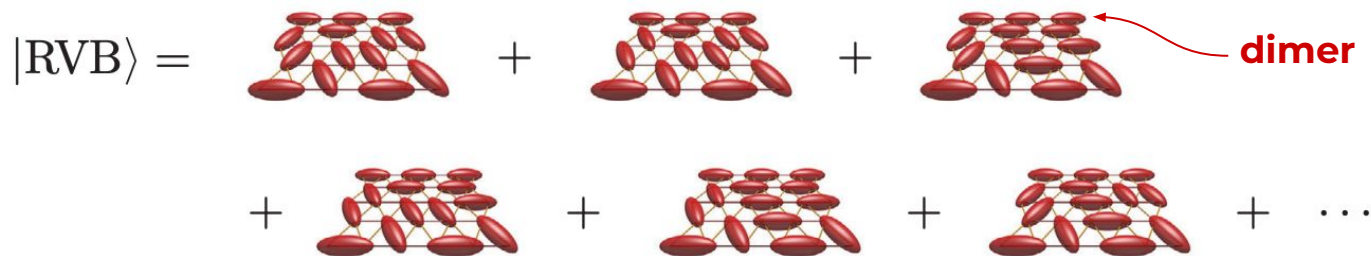
U(1) gauge symmetry:

$$\sum_{i \in \ell_{\text{in}}} Z_i - \sum_{i \in \ell_{\text{out}}} Z_i = N_v^A - N_v^B$$



- infinite correlation length
- no deconfined phase (Polyakov)

# TOPOLOGICAL ORDER IN DIMER MODELS



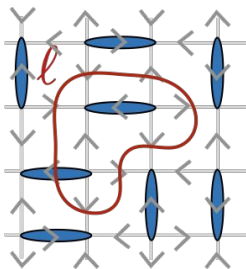
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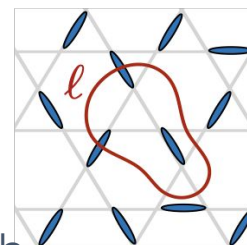


- infinite correlation length
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## 2D non-bipartite lattices ✓

$Z_2$  gauge symmetry:

$$\prod_{i \in \ell} Z_i = (-1)^{N_v}$$

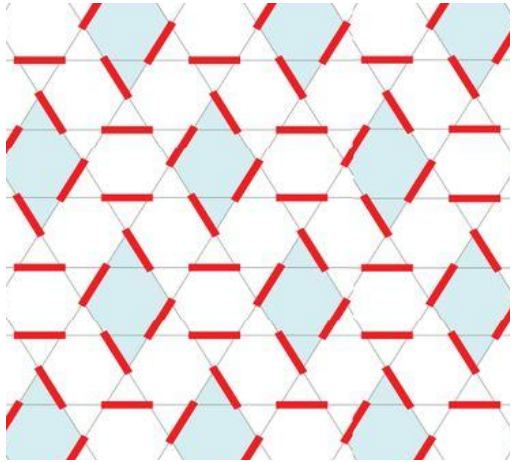


- finite correlation length
- gapped QSL with  $Z_2$  topological order



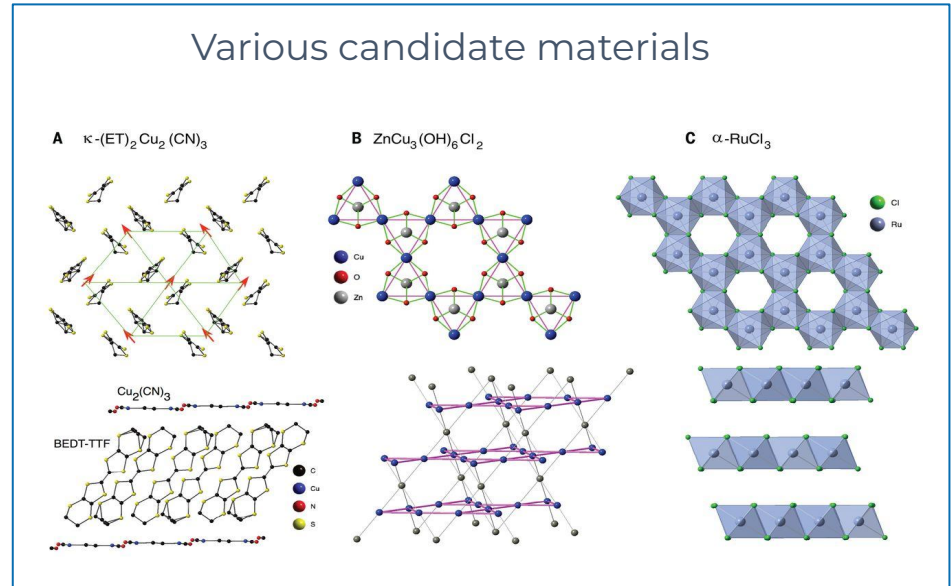
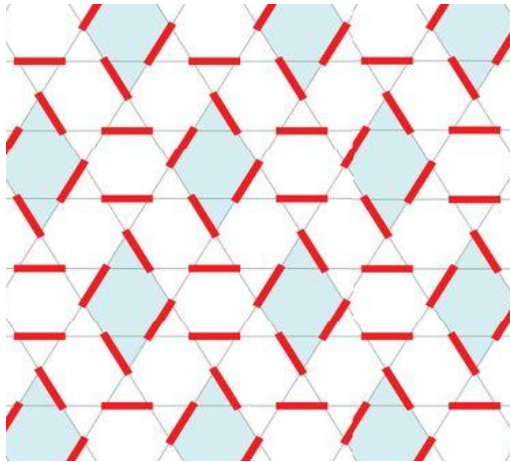
# TOPOLOGICAL ORDER IN DIMER MODELS

Dimer RVB state on triangular, kagome lattices have  $Z_2$  topological order



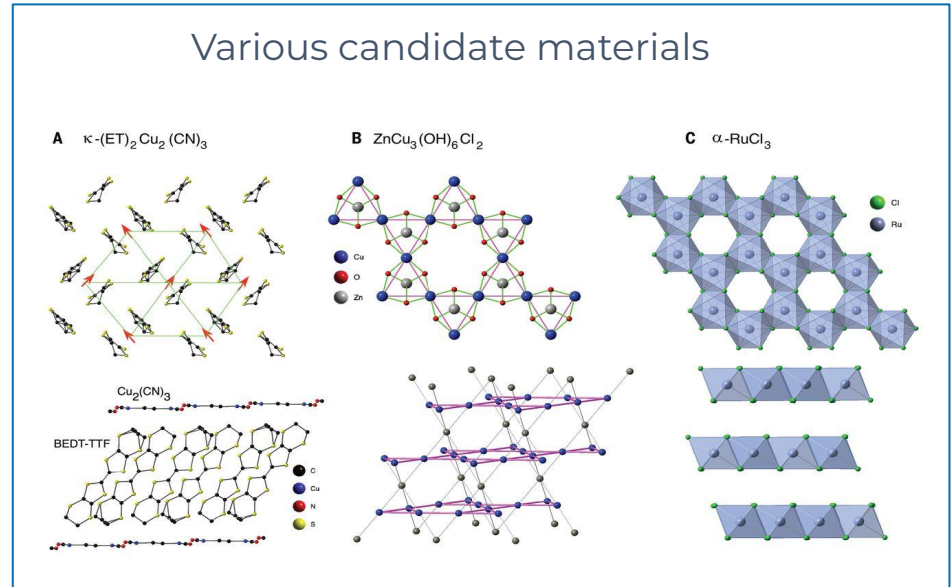
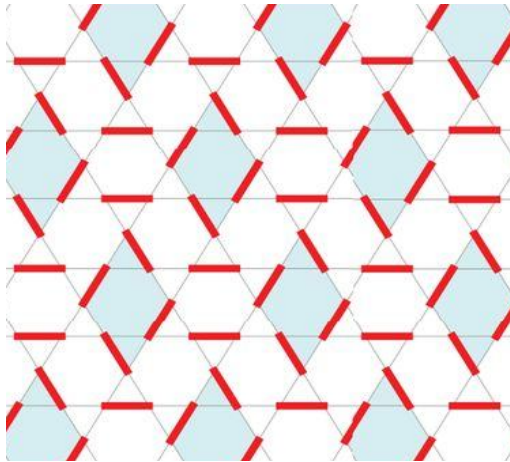
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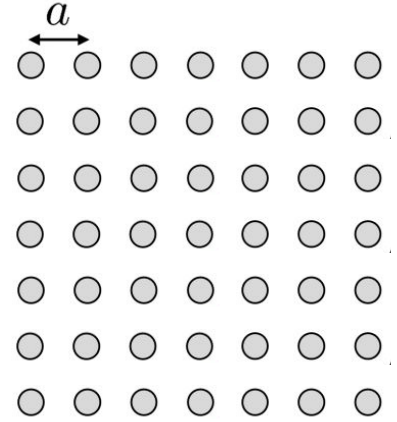
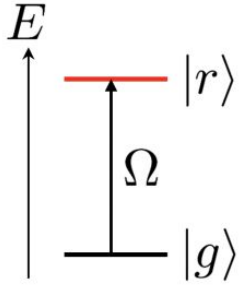


*Quantum simulators?*

Broholm, Science 2020

# RYDBERG ATOM ARRAYS

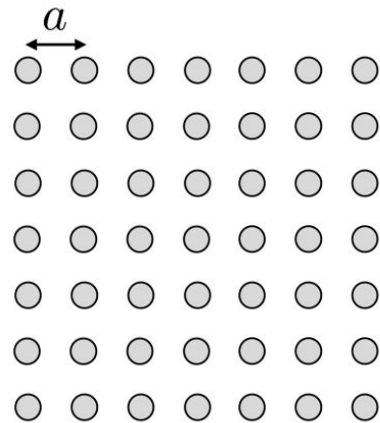
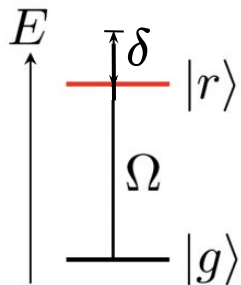
$$|r\rangle\langle r| = n \quad |r\rangle\langle g| = b^\dagger$$



$$H = \frac{\Omega}{2} \sum_i (b_i + b_i^\dagger)$$

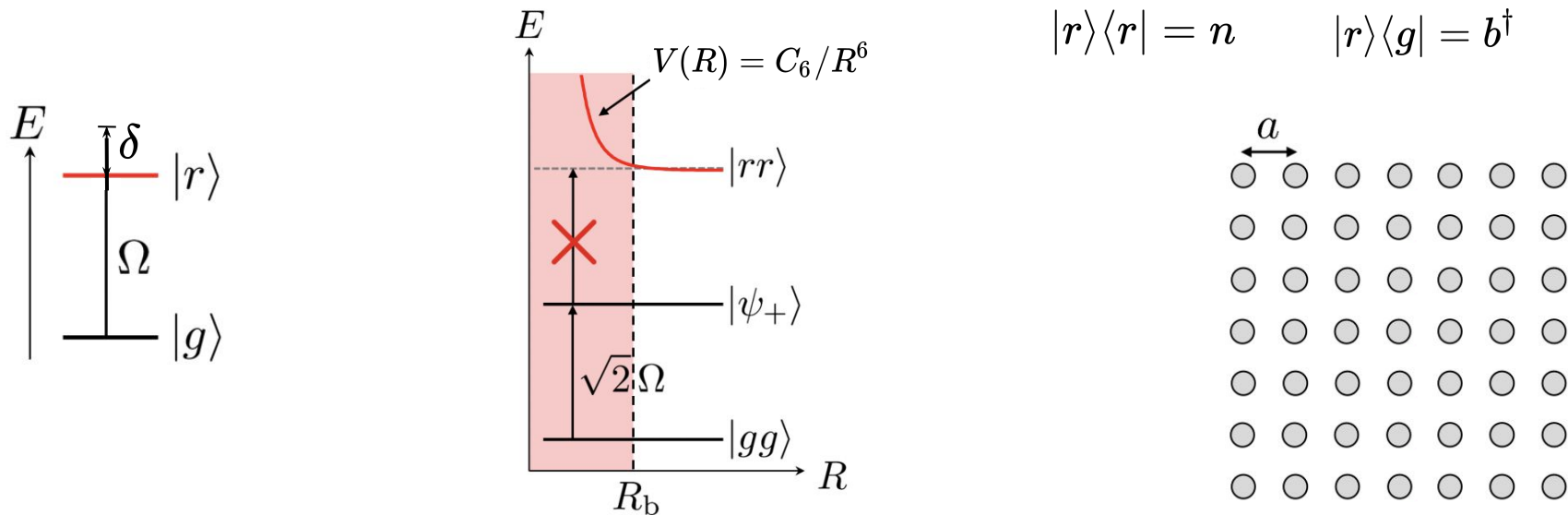
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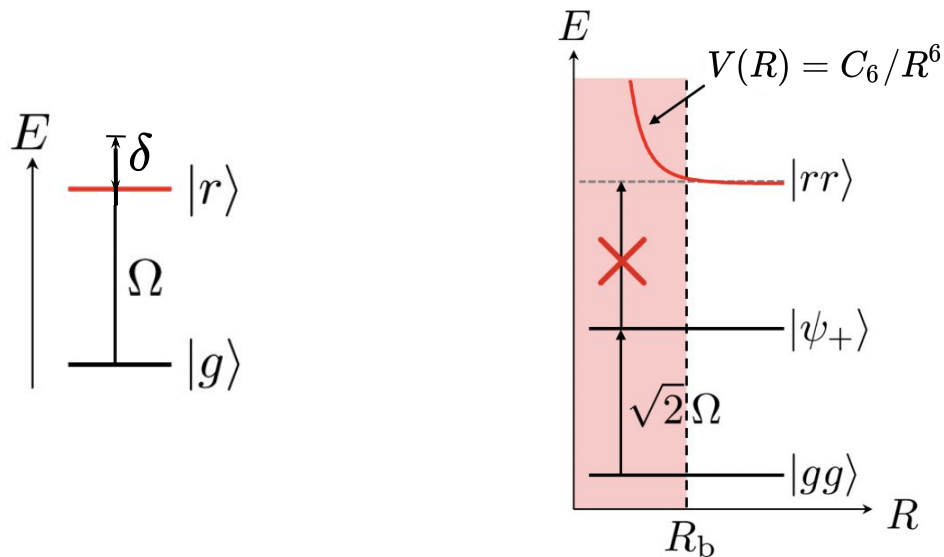
$$H = \frac{\Omega}{2} \sum_i (b_i + b_i^\dagger) - \delta \sum_i n_i$$

# RYDBERG ATOM ARRAYS

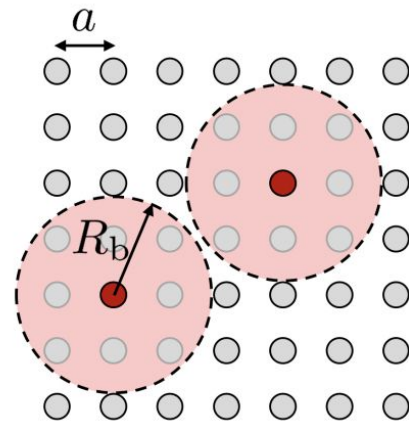


$$H = \frac{\Omega}{2} \sum_i (b_i + b_i^\dagger) - \delta \sum_i n_i + \frac{1}{2} \sum_{i,j} V(|i-j|) n_i n_j$$

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$$|r\rangle\langle r| = n \quad |r\rangle\langle g| = b^\dagger$$

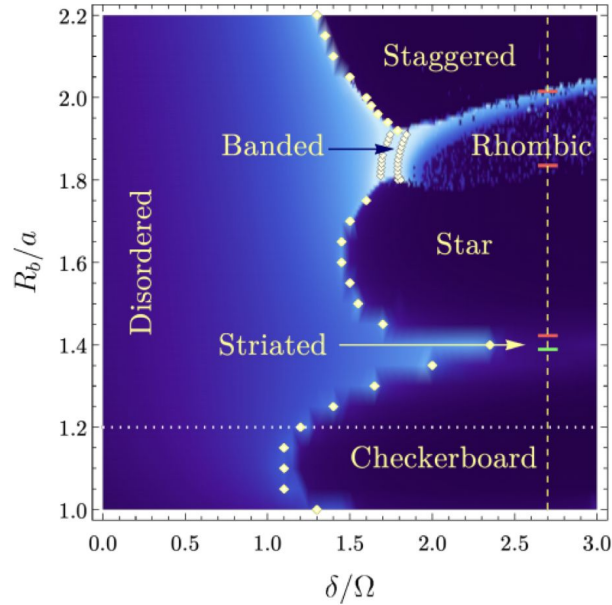


$$V(R_b) := \Omega$$

**Blockade  
radius**

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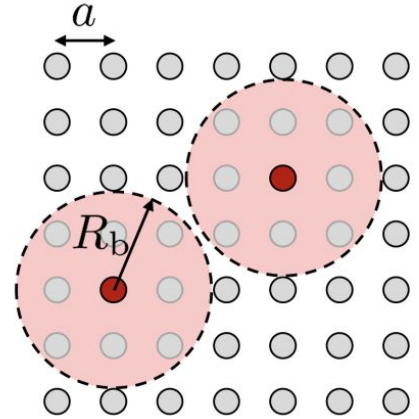
# RYDBERG ATOM ARRAYS



Square  
lattice

Samajdar, Ho, Pichler,  
Lukin, Sachdev, PRL **124**,  
103601 (2020)

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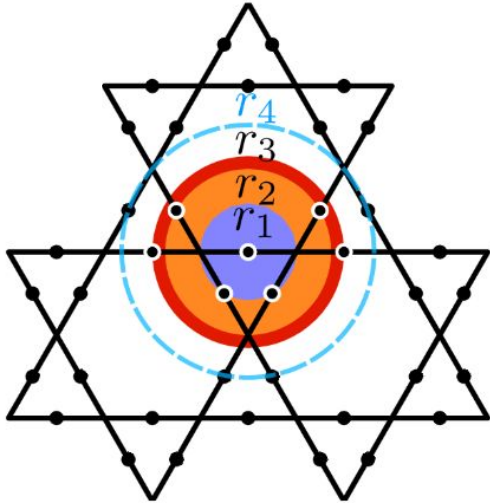
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Browaeys, Lahaye, *Nat. Phys.* 16, 132–142 (2020)



# DIMER MODELS FROM RYDBERG BLOCKADE

Ruby lattice



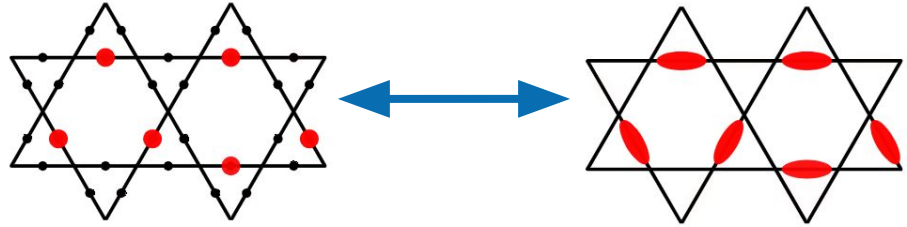
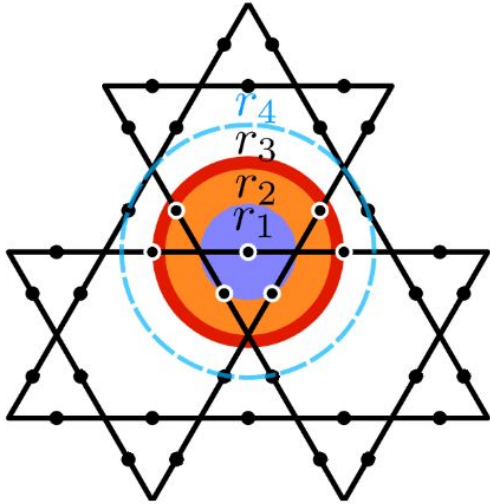
*Vishwanath's talk on Monday*

$$H = \frac{\Omega}{2} \sum_i (b_i + b_i^\dagger) - \delta \sum_i n_i + \frac{1}{2} \sum_{i,j} V(|i - j|) n_i n_j$$

Verresen, Lukin, Vishwanath, Phys. Rev. X 11, 031005 (2021)

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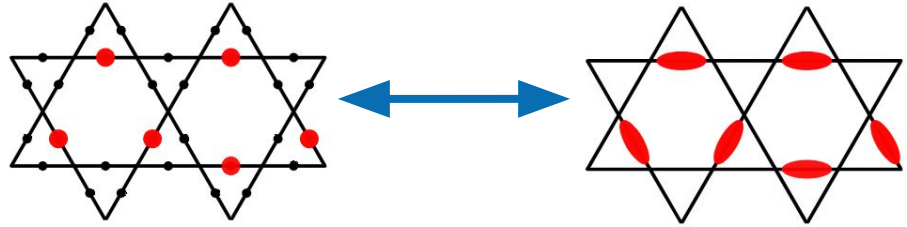
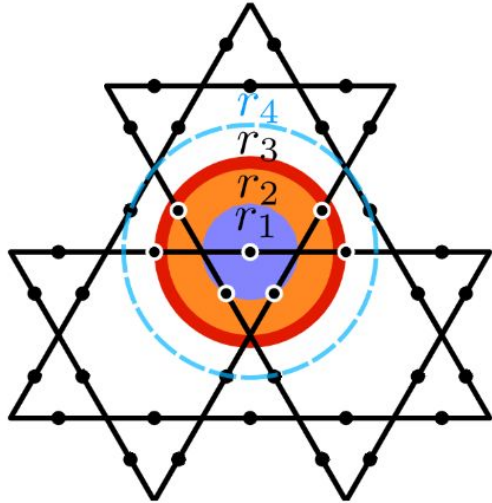
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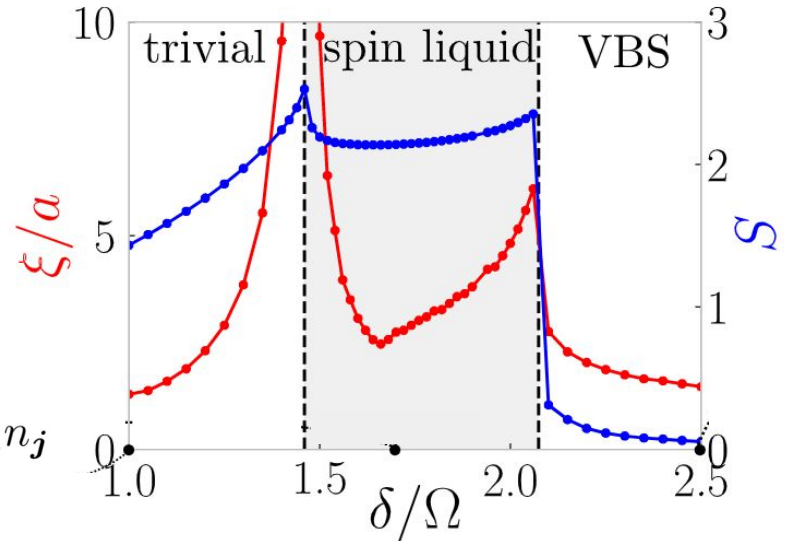
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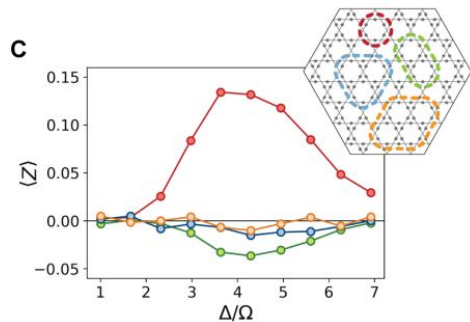


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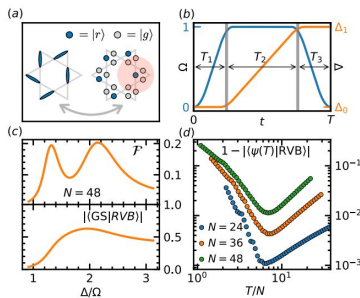


Verresen, Lukin, Vishwanath, Phys. Rev. X 11, 031005 (2021)

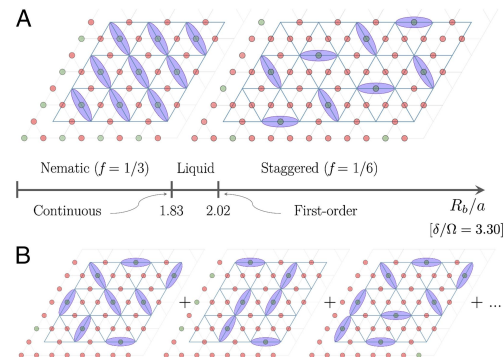
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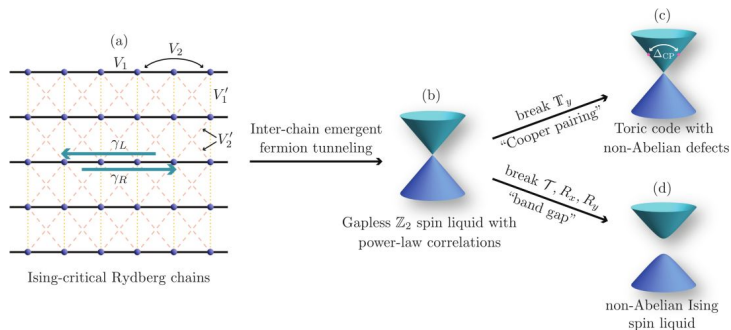
Semeghini et al, Science 2021



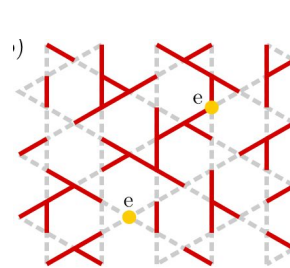
Giudici et al, arXiv:2201.04034



Samajdar et al, PNAS 2021, arXiv:2204.00632



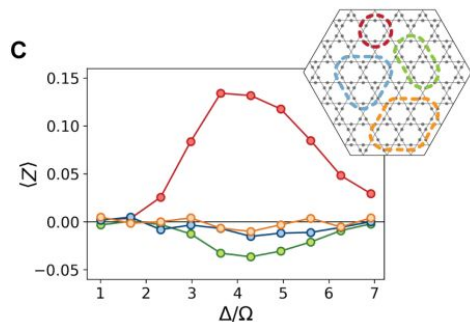
Slagle et al, arXiv:2204.00013



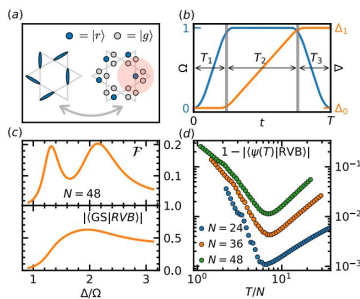
f)	Toric Code	Rydberg
$\frac{1}{\sqrt{2}}( \triangleright\rangle +  \triangleleft\rangle)$	$ \triangleright\rangle$	$ \triangleright\rangle$
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Tarabunga et al, (in preparation)

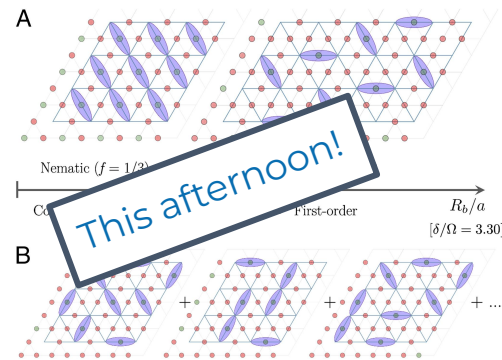
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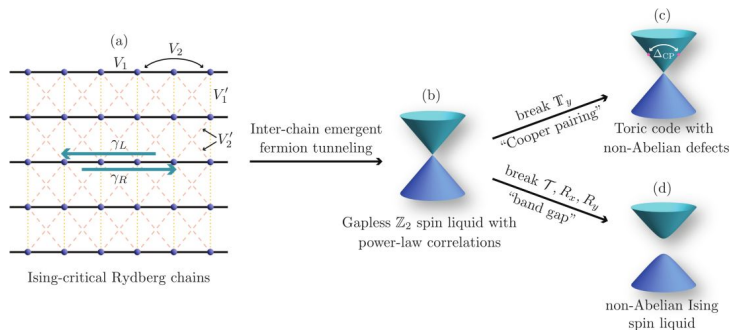
Semeghini et al, Science 2021



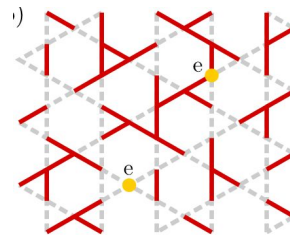
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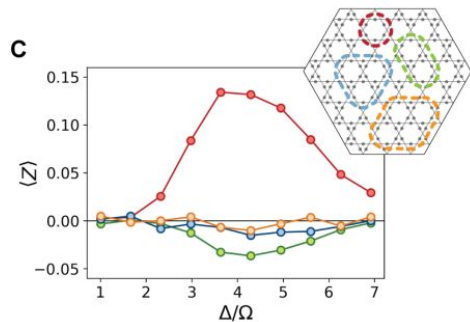
Slagle et al, arXiv:2204.00013



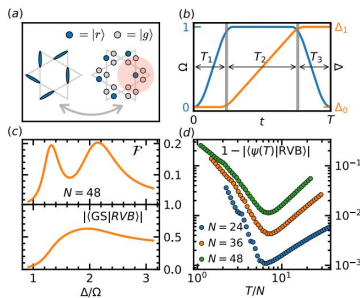
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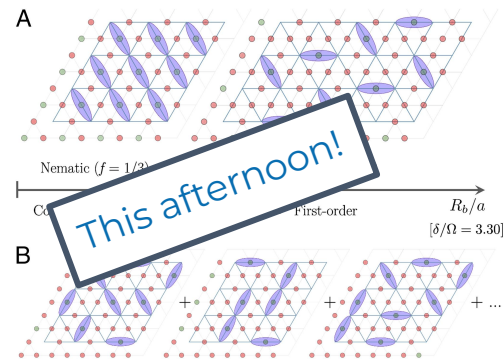
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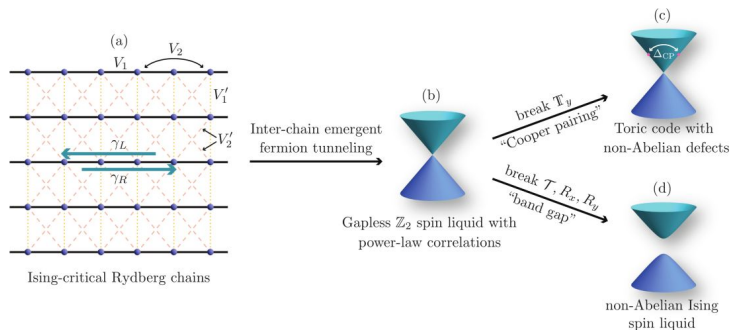
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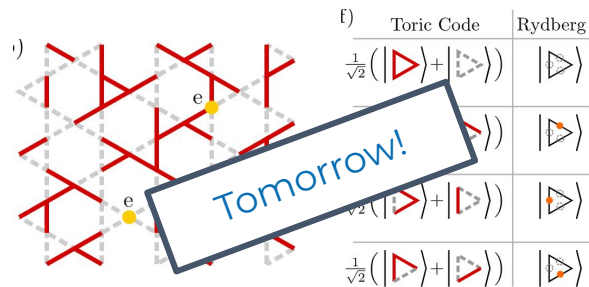
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Beyond dimer models:

*Can other types of topological order emerge in  
constrained models?*

Beyond dimer models:

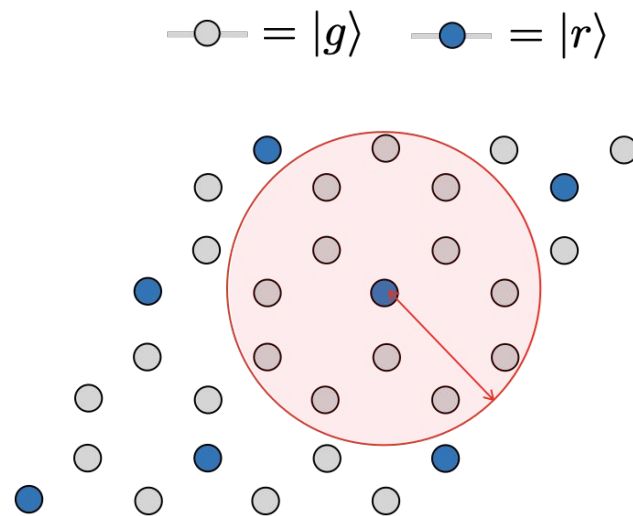
*Can other types of topological order emerge in  
constrained models?*

*Can they be probed with Rydberg atoms?*



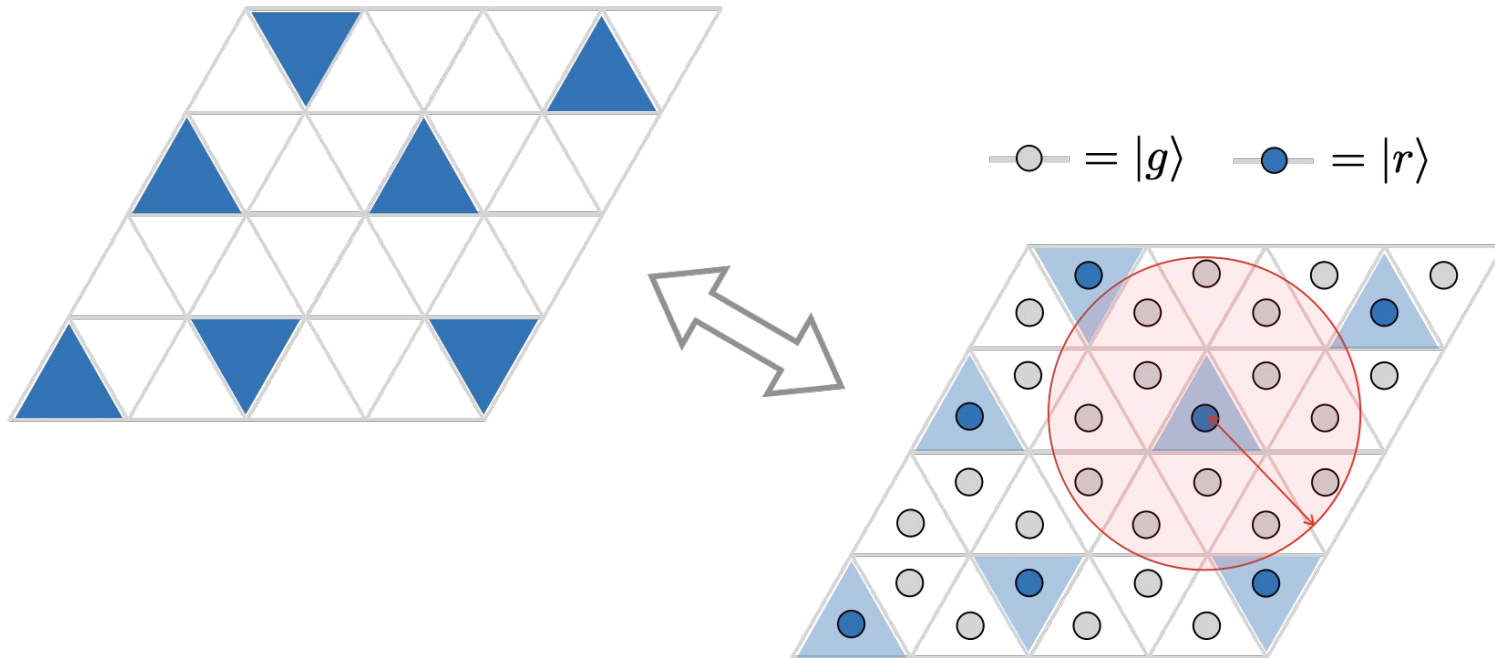
# BEYOND DIMER MODELS

Example: honeycomb lattice



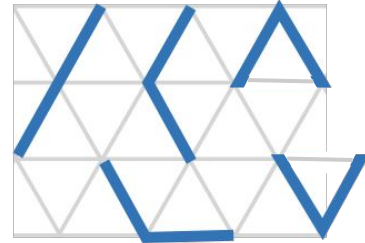
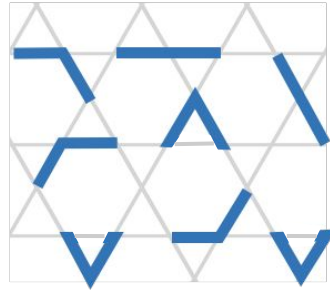
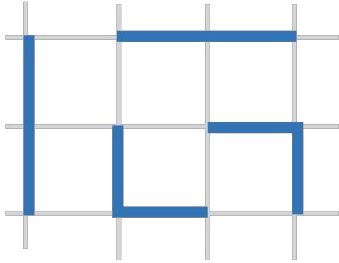
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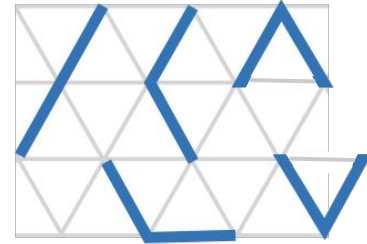
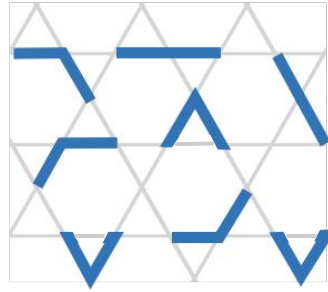
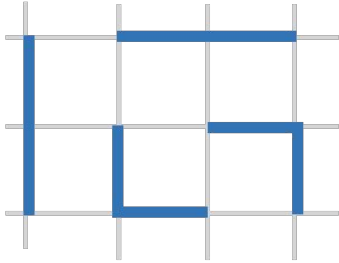
Trimer = two adjacent links



...

# TRIMER MODELS

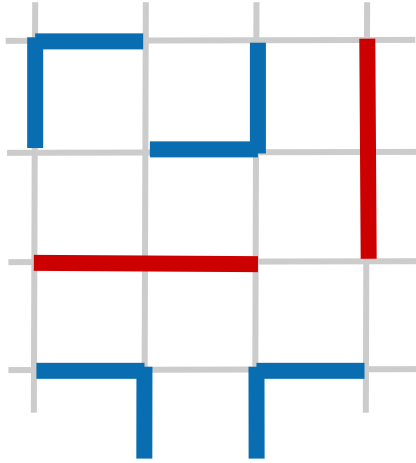
Trimer = two adjacent links



...

Much more freedom than with dimers...

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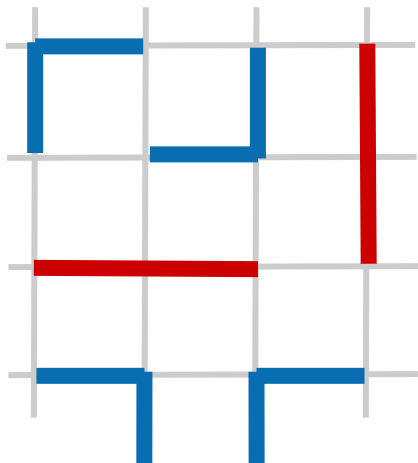


Lee, Ho, Han, Katsura, PRB 2017

Dong, Chen, Tu, PRB 2018

Jandura, Iqbal, Schuch, PRR 2020

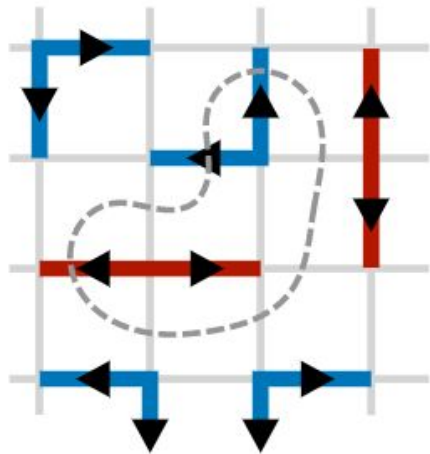
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Dong, Chen, Tu, PRB 2018  
Jandura, Iqbal, Schuch, PRR 2020

$\mathbb{Z}_3$  topological order can emerge

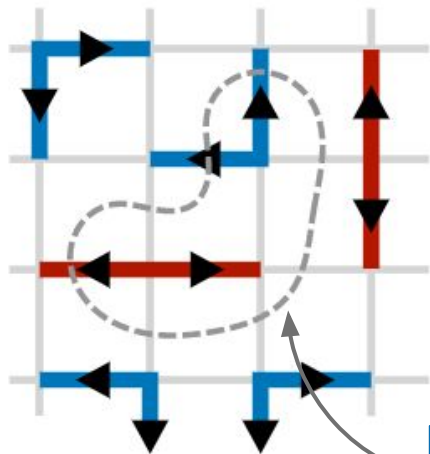
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Lee, Ho, Han, Katsura, PRB 2017  
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Jandura, Iqbal, Schuch, PRR 2020

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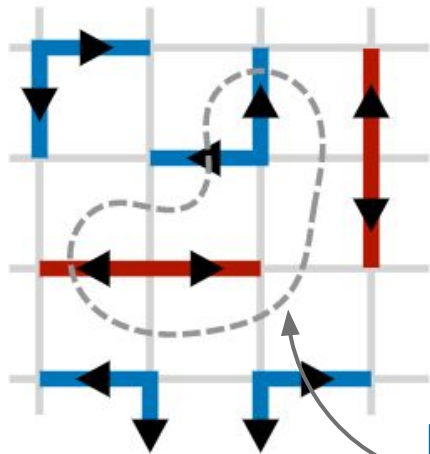
Lee, Ho, Han, Katsura, PRB 2017  
Dong, Chen, Tu, PRB 2018  
Jandura, Iqbal, Schuch, PRR 2020

$\mathbb{Z}_3$  topological order can emerge

Region with  $N_s$  vertices  $\longrightarrow$  Flux =  $2N_s \bmod 3$   
 **$\mathbb{Z}_3$  Gauss law**



# TRIMER MODELS



Lee, Ho, Han, Katsura, PRB 2017  
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Jandura, Iqbal, Schuch, PRR 2020

$\mathbb{Z}_3$  topological order can emerge

Region with  $N_s$  vertices  $\longrightarrow$  Flux =  $2N_s \bmod 3$   
 **$\mathbb{Z}_3$  Gauss law**

But some trimer models have a “larger” local symmetry and no gapped QSL



# MAIN RESULT

Trimer RVB states

$$|\text{tRVB}\rangle = \left| \begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{array} \right\rangle + \left| \begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{array} \right\rangle + \left| \begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{array} \right\rangle + \left| \begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{array} \right\rangle + \dots$$

G. Giudice, FMS, H. Pichler, G. Giudici, in preparation

# MAIN RESULT

Trimer RVB states

$$|tRVB\rangle = \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + \dots$$

## Tripartite

- tRVB has infinite correlation length
- $U(1) \times U(1)$  Gauss' law

# MAIN RESULT

Trimer RVB states

$$|\text{tRVB}\rangle = \left| \begin{array}{cc} \square & \square \\ \square & \square \end{array} \right\rangle + \left| \begin{array}{cc} \square & \square \\ \square & \square \end{array} \right\rangle + \left| \begin{array}{cc} \square & \square \\ \square & \square \end{array} \right\rangle + \left| \begin{array}{cc} \square & \square \\ \square & \square \end{array} \right\rangle + \dots$$

## Tripartite

- tRVB has infinite correlation length
- $U(1) \times U(1)$  Gauss' law

## Non-tripartite

- tRVB can have finite correlation length
- $Z_3$  Gauss' law

# OUTLINE

- **Resonating trimer states**
  - Definition (square lattice)
  - Tensor network representation
  - Gauge symmetry:  $Z_3$  vs  $U(1) \times U(1)$
  - Other lattices
- **Dilute trimer state**
  - Stability
  - Hamiltonian
- **Implementation with Rydberg atom**
  - Square lattice

# 1. RESONATING TRIMER STATES

# RESONATING TRIMER STATE

$$|t\text{RVB}\rangle = \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + t \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + t^2 \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + t^3 \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + \dots$$

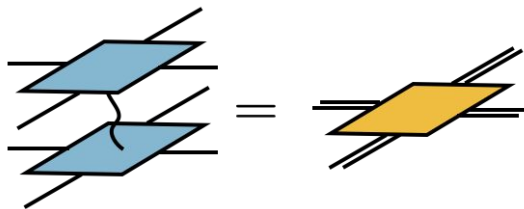
The diagram shows a 4x4 grid of sites. The first term is a blue trimer on the top row. The second term is a blue trimer on the bottom row with a red vertical line on the right. The third term is a blue trimer on the left column with a red vertical line on the right. The fourth term is a red trimer on the top row with a blue trimer on the bottom row and a red vertical line on the right.



# RESONATING TRIMER STATE

$$|t\text{RVB}\rangle = \left| \begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{array} \right\rangle + t \left| \begin{array}{ccc} \text{---} & \text{---} & | \\ \text{---} & \text{---} & | \\ \text{---} & \text{---} & | \end{array} \right\rangle + t^2 \left| \begin{array}{ccc} \text{---} & \text{---} & | \\ \text{---} & \text{---} & | \\ \text{---} & \text{---} & | \end{array} \right\rangle + t^3 \left| \begin{array}{ccc} \text{---} & \text{---} & | \\ \text{---} & \text{---} & | \\ \text{---} & \text{---} & | \end{array} \right\rangle + \dots$$

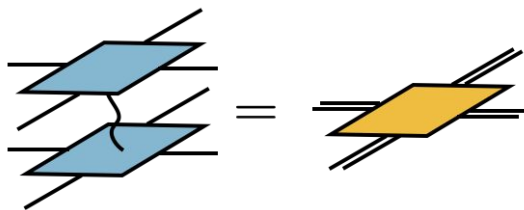
Tensor network representation



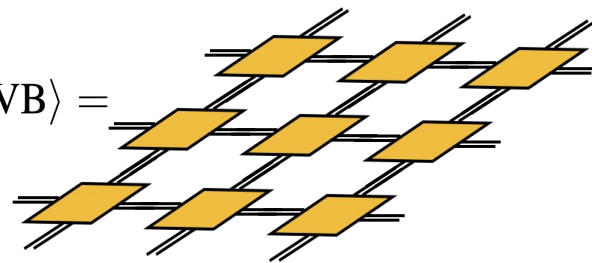
# RESONATING TRIMER STATE

$$|t\text{RVB}\rangle = \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + t \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + t^2 \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + t^3 \left| \begin{array}{cc} \text{---} & \text{---} \\ \text{---} & \text{---} \end{array} \right\rangle + \dots$$

Tensor network representation



$$Z = \langle t\text{RVB} | t\text{RVB} \rangle =$$

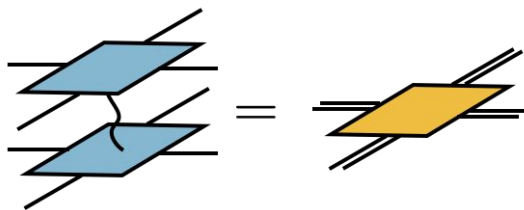


classical statmec  
partition function

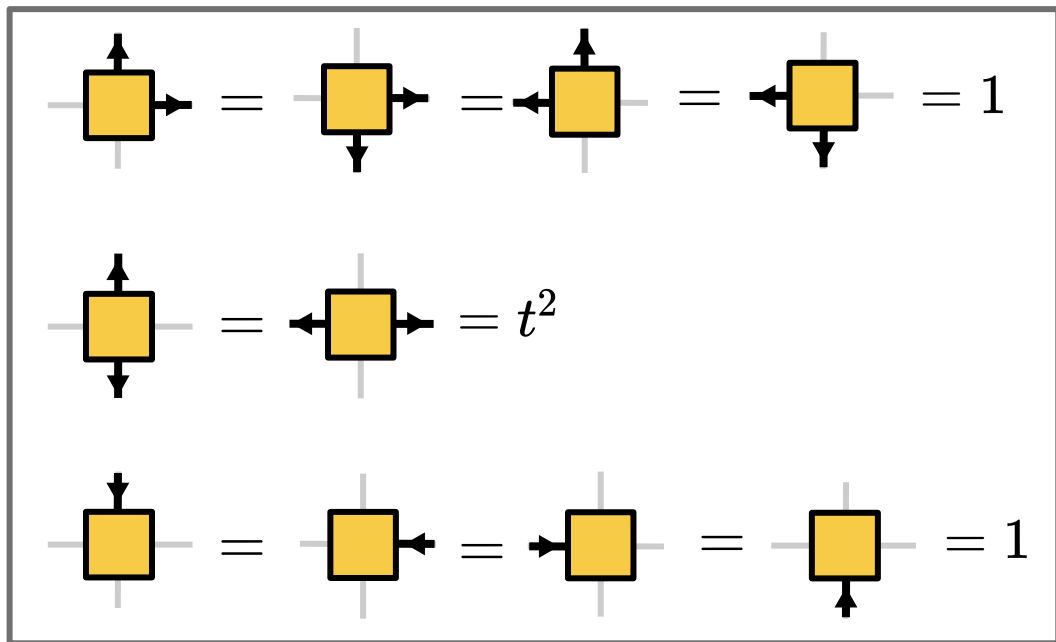
# RESONATING TRIMER STATE

$$|t\text{RVB}\rangle = \left| \begin{array}{c} \text{blue} \\ \text{blue} \end{array} \right\rangle + t \left| \begin{array}{c} \text{blue} \\ \text{red} \end{array} \right\rangle + t^2 \left| \begin{array}{c} \text{blue} \\ \text{blue} \\ \text{red} \end{array} \right\rangle + t^3 \left| \begin{array}{c} \text{red} \\ \text{red} \\ \text{blue} \end{array} \right\rangle + \dots$$

Tensor network representation

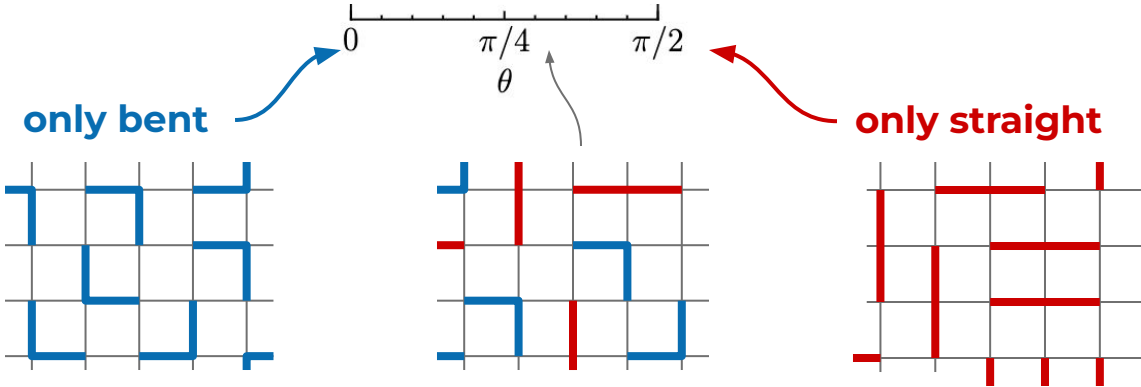
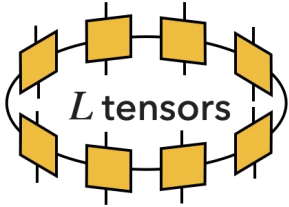


virtual indices  $\leftrightarrow$  gauge fields



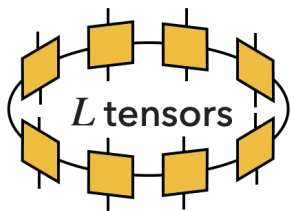
# NUMERICAL RESULTS

Cylinder transfer matrix

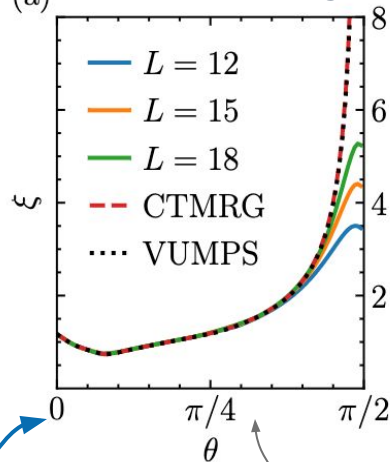


# NUMERICAL RESULTS

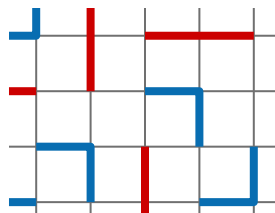
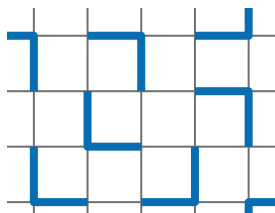
Cylinder transfer matrix



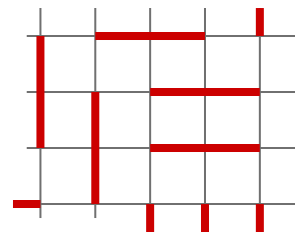
(a) correlation length



only bent

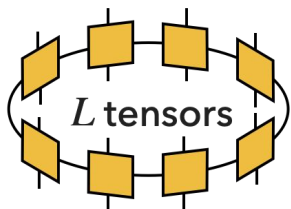


only straight

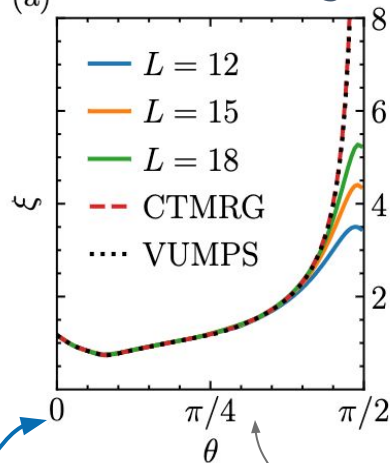


# NUMERICAL RESULTS

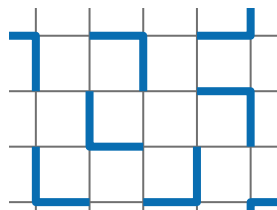
Cylinder transfer matrix



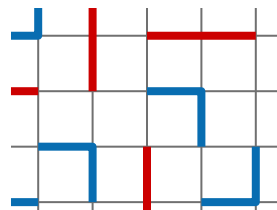
(a) correlation length



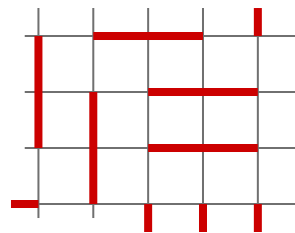
only bent



only straight



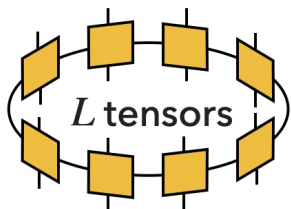
only straight



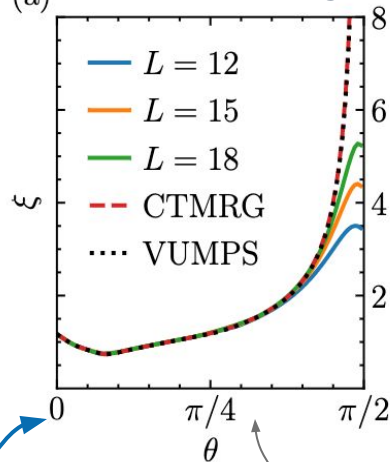
gapped

# NUMERICAL RESULTS

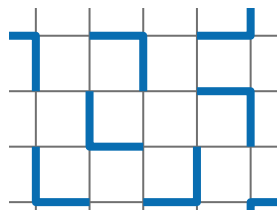
Cylinder transfer matrix



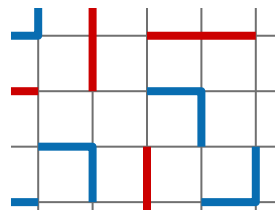
(a) correlation length



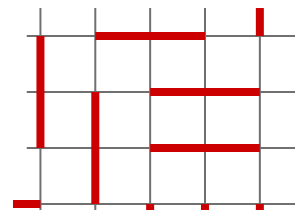
only bent



gapped



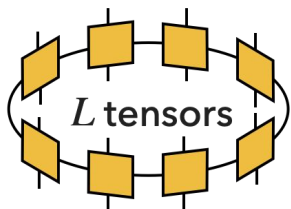
only straight



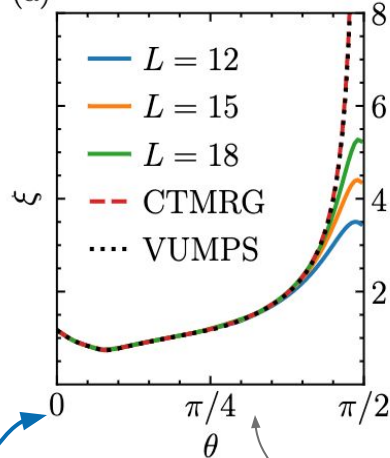
gapless

# NUMERICAL RESULTS

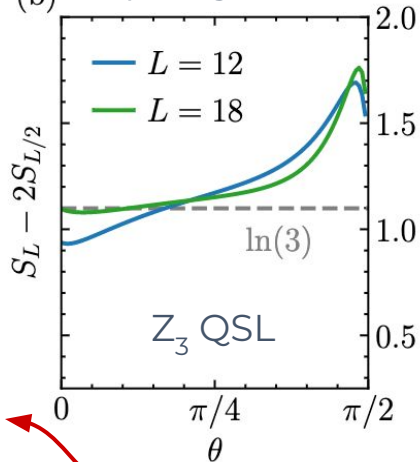
Cylinder transfer matrix



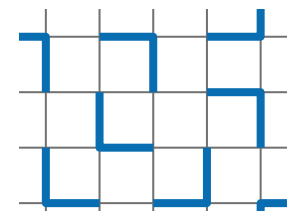
(a) correlation length



(b) topological EE

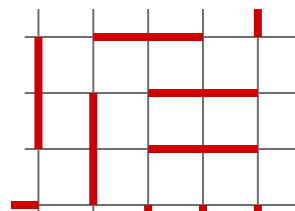
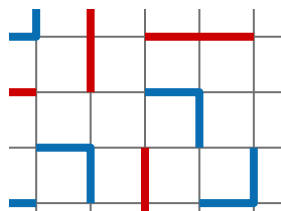


only bent



gapped

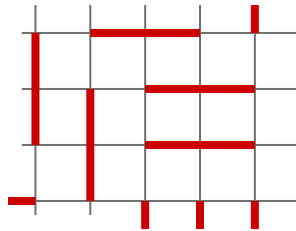
only straight



gapless

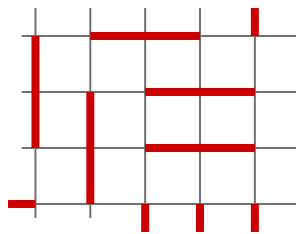


# GAPLESS CASE



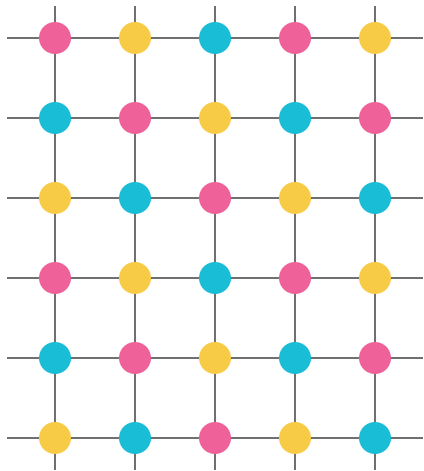
← Why is it gapless?

# GAPLESS CASE

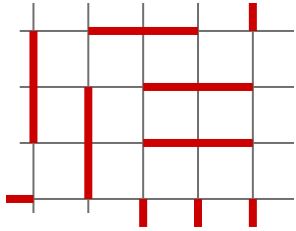


*Why is it gapless?*

3 sublattices: A, B, C

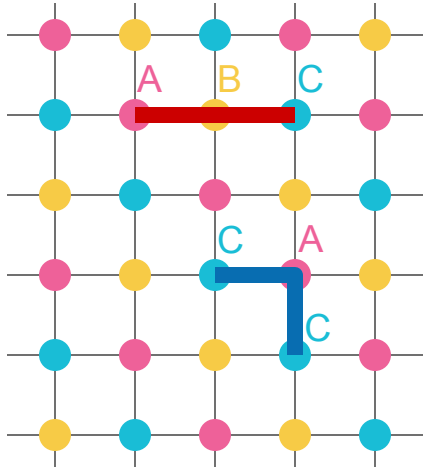


# GAPLESS CASE

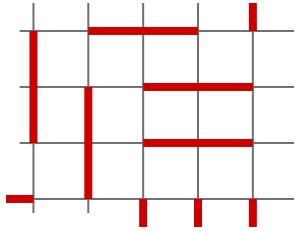


Why is it gapless?

3 sublattices: A, B, C

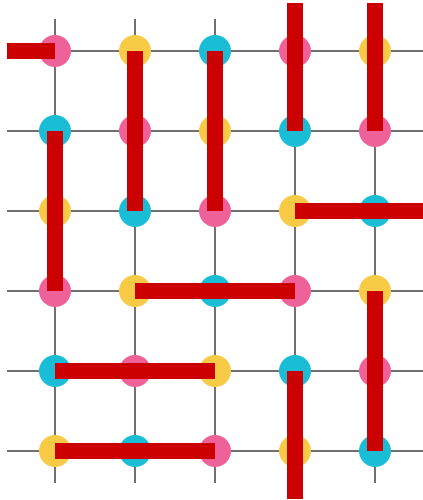


# GAPLESS CASE

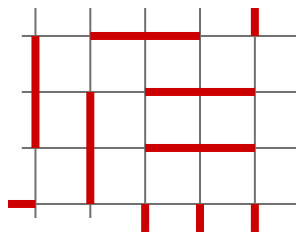


Why is it gapless?

3 sublattices: A, B, C

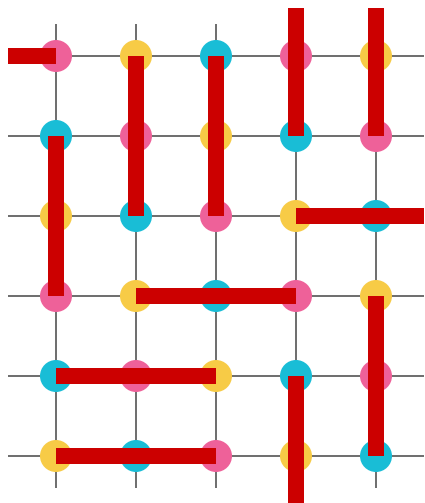


# GAPLESS CASE

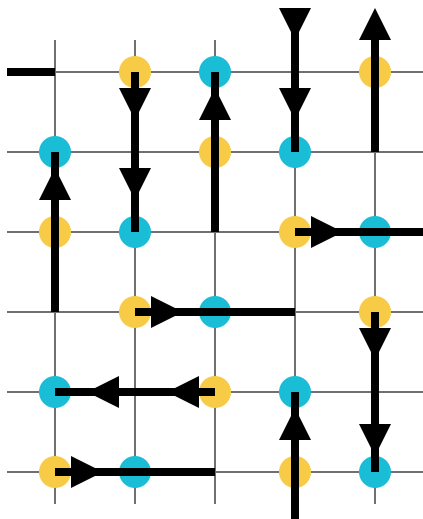


Why is it gapless?

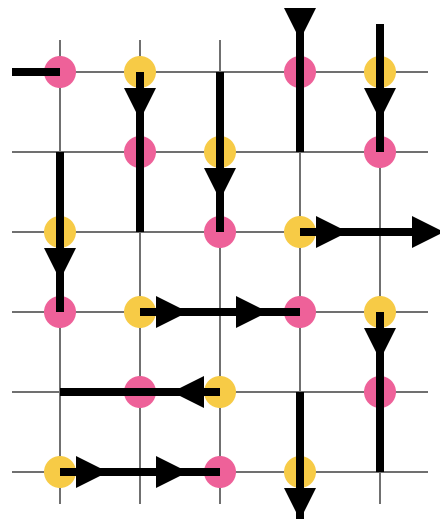
3 sublattices: A, B, C



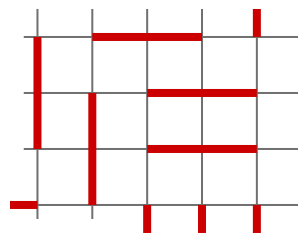
Electric field 1: B  $\rightarrow$  C



Electric field 2: B  $\rightarrow$  A

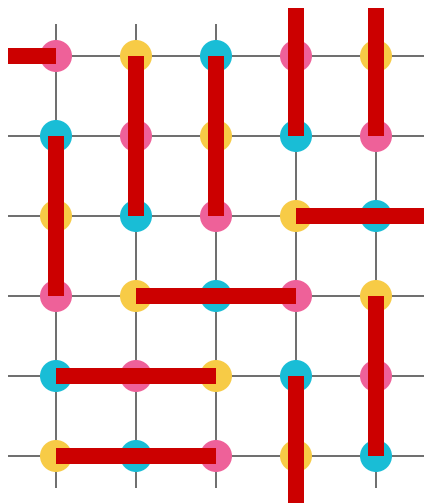


# GAPLESS CASE



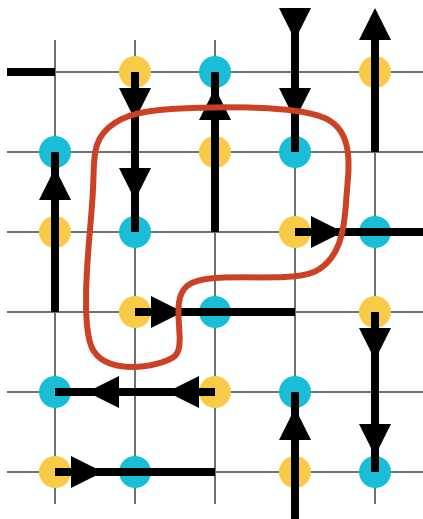
Why is it gapless?

3 sublattices: A, B, C



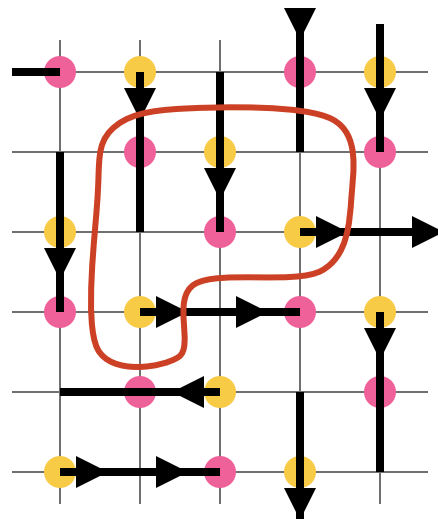
$$\text{Flux} = N_B - N_C$$

Electric field 1: B  $\rightarrow$  C

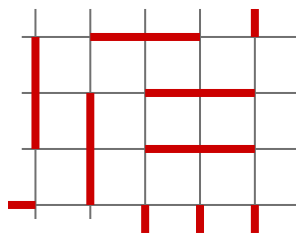


$$\text{Flux} = N_B - N_A$$

Electric field 2: B  $\rightarrow$  A



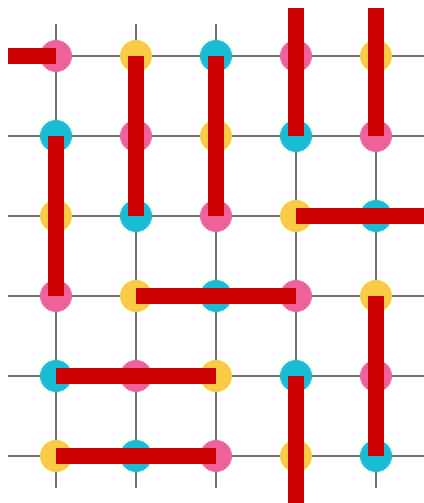
# GAPLESS CASE



Why is it gapless?

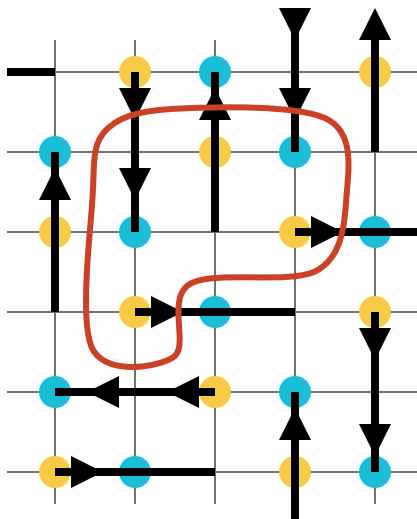
$U(1) \times U(1)$  gauge symmetry:  
no gapped QSL phase

3 sublattices: A, B, C



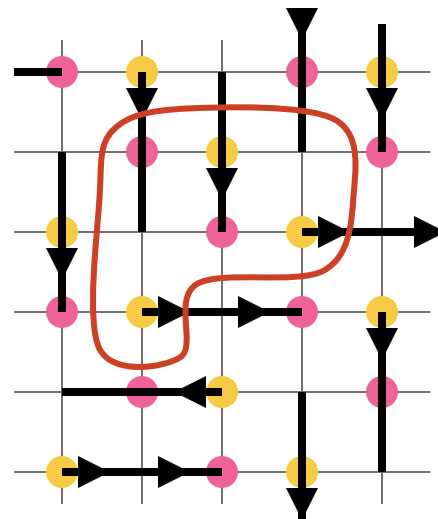
$$\text{Flux} = N_B - N_C$$

Electric field 1: B  $\rightarrow$  C



$$\text{Flux} = N_B - N_A$$

Electric field 2: B  $\rightarrow$  A



# GENERAL CRITERION

## Definition:

A trimer model is tripartite if there exist 3 sublattices (A, B, C) such that each trimer touches all 3 sublattices



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Depends both on the lattice and on the shapes of the trimers!

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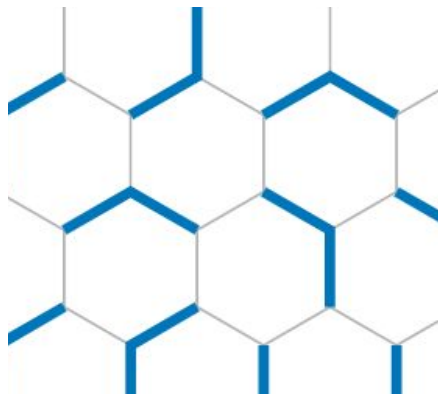


Depends both on the lattice and on the shapes of the trimers!

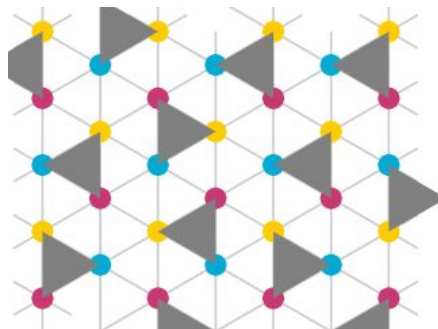
**Tripartite** trimer models: gapless tRVB

# OTHER LATTICES

honeycomb  
 $Z_3$  gauge

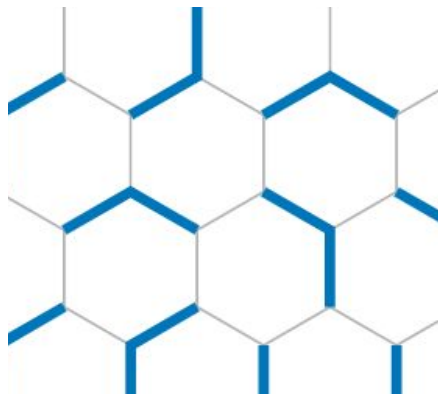


triangular  
 $U(1) \times U(1)$  gauge

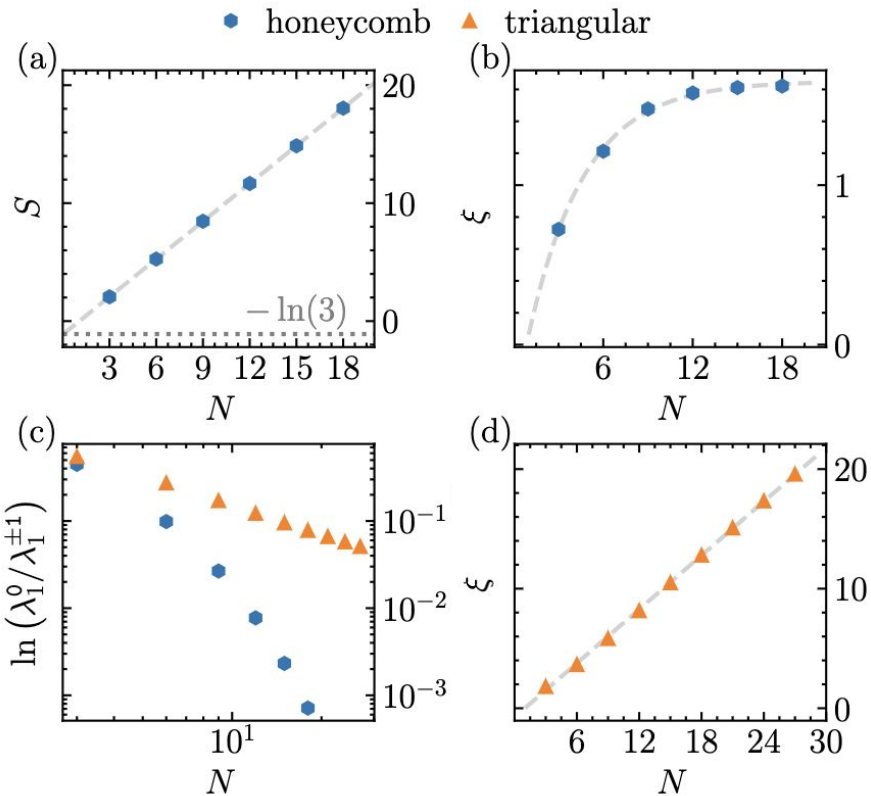
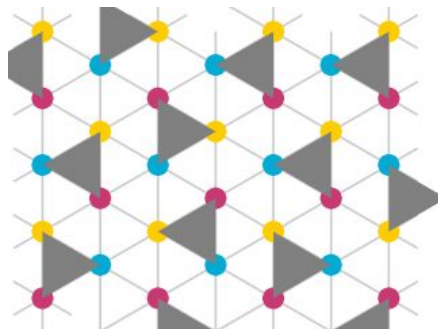


# OTHER LATTICES

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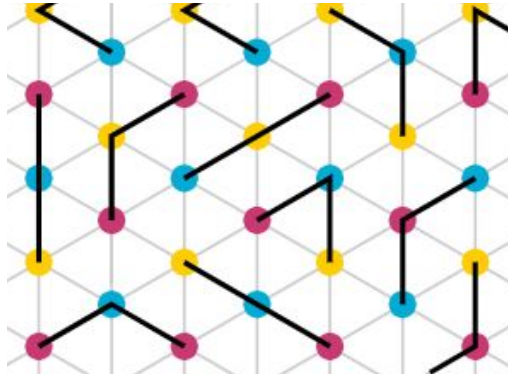


# OTHER LATTICES

**Tripartite** trimer models: gapless tRVB

Depends both on the lattice and on the shapes of the trimers!

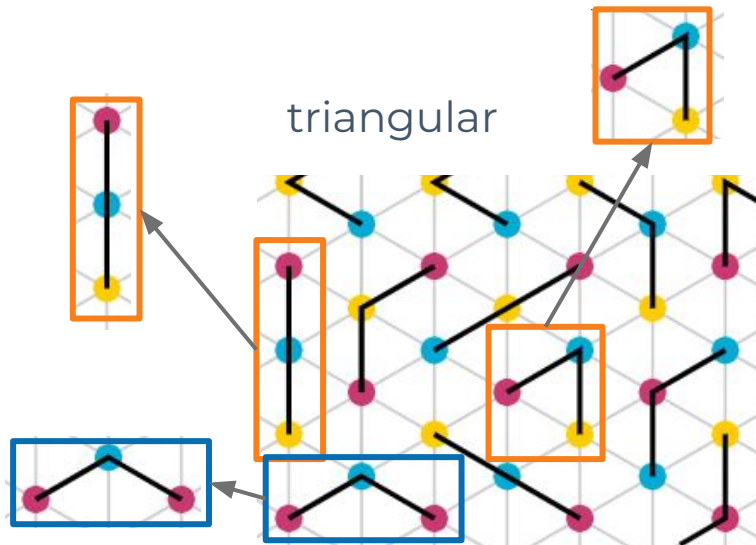
triangular



# OTHER LATTICES

*Tripartite* trimer models: gapless tRVB

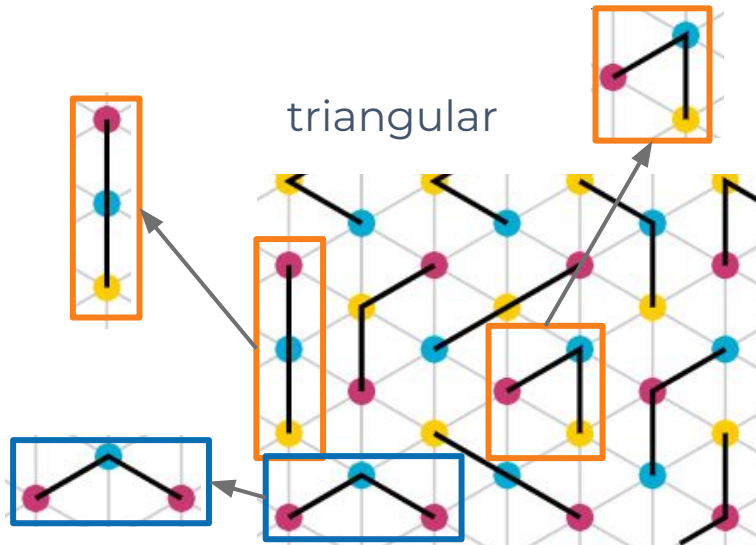
Depends both on the lattice and on the shapes of the trimers!



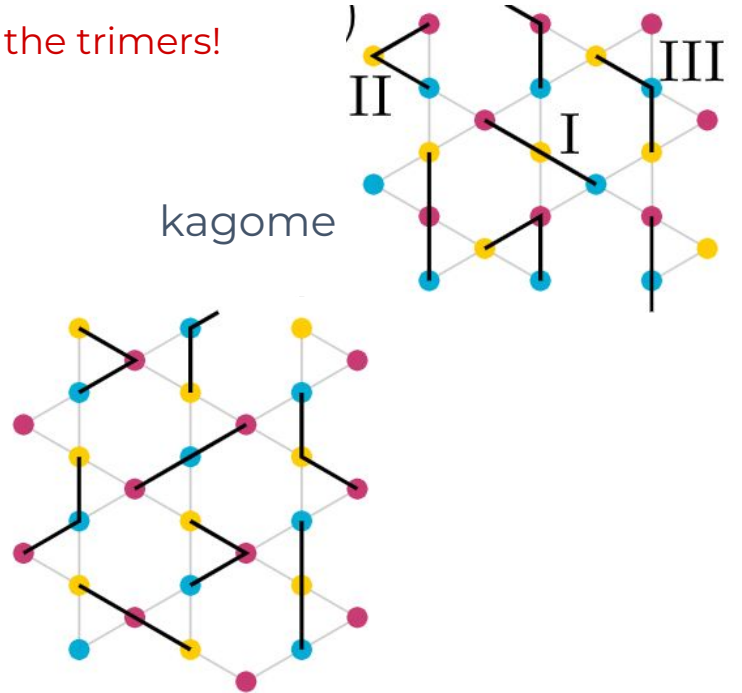
# OTHER LATTICES

**Tripartite** trimer models: gapless tRVB

Depends both on the lattice and on the shapes of the trimers!



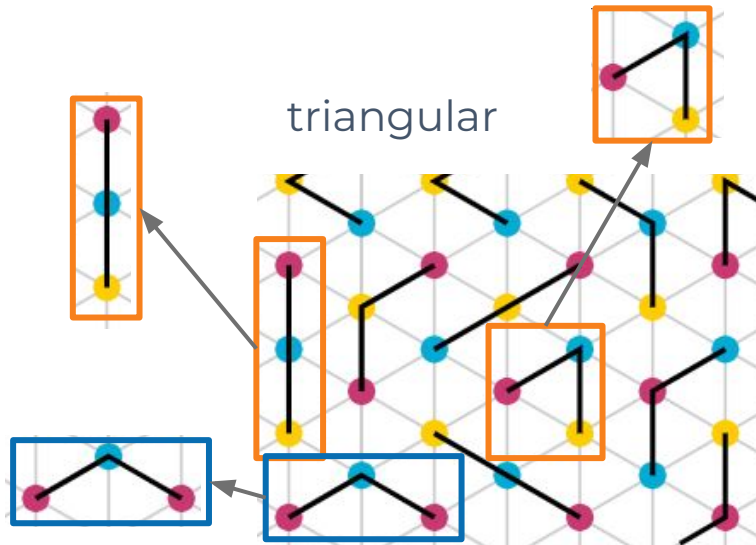
kagome



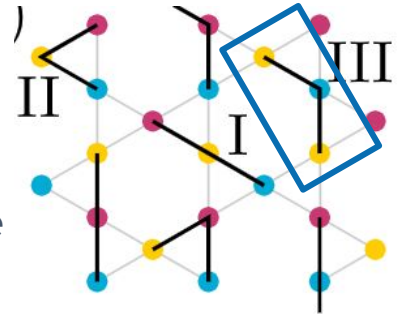
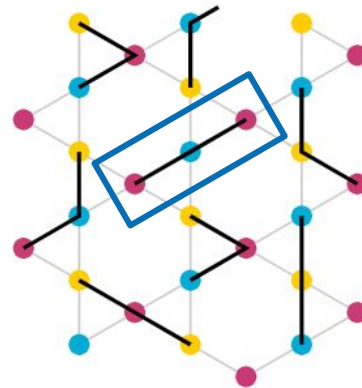
# OTHER LATTICES

**Tripartite** trimer models: gapless tRVB

Depends both on the lattice and on the shapes of the trimers!



kagome

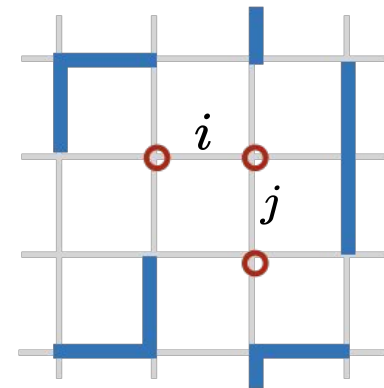




## 2. DILUTE TRIMER STATE

# DILUTE TRIMER STATES

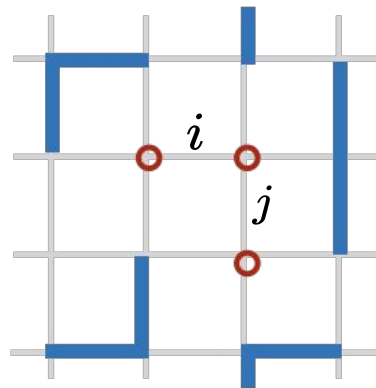
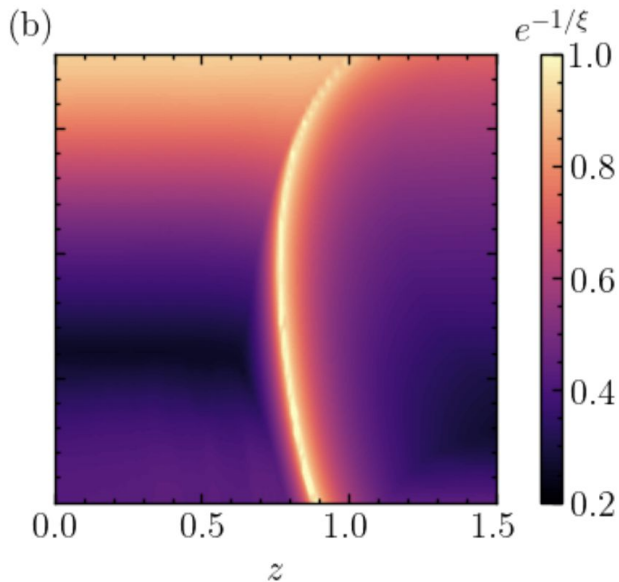
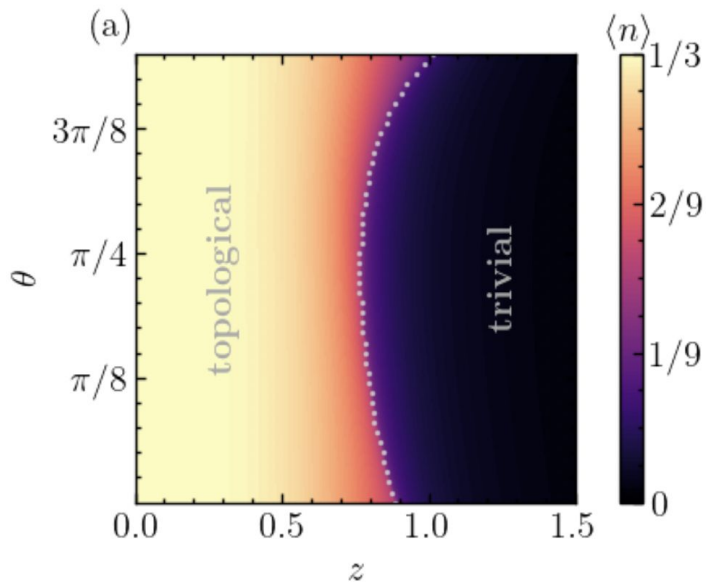
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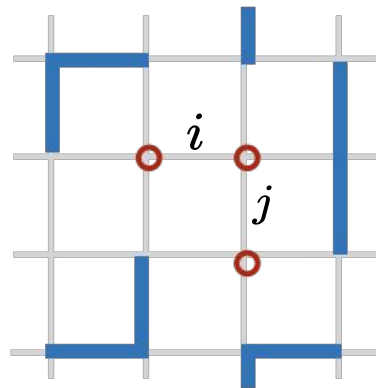
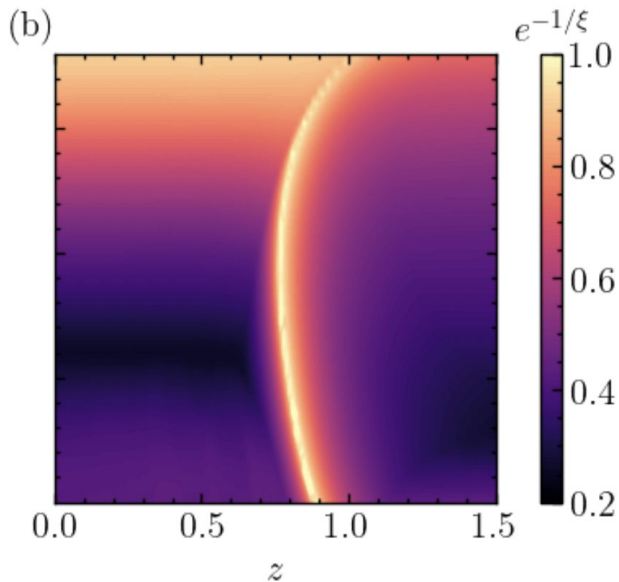
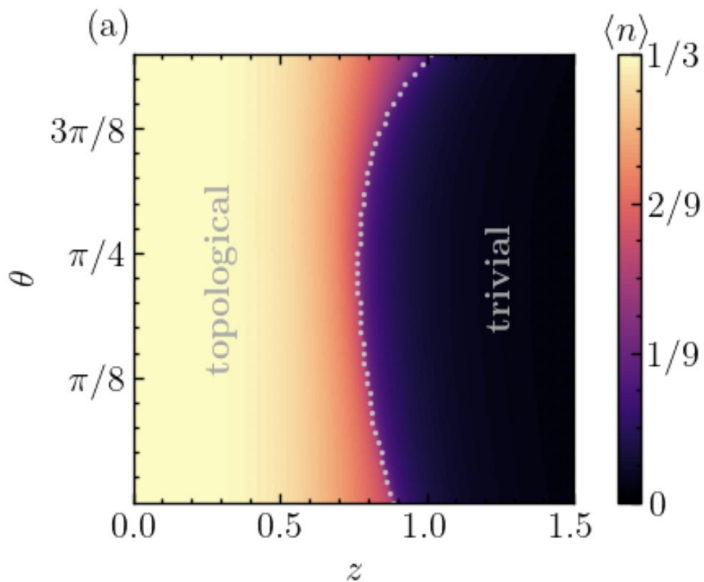
CTMRG results:



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CTMRG results:



$Z_3$  topological order  
is stable against  
dilution

# DILUTE TRIMER STATES

*Is there a simple Hamiltonian that has this tRVB as (approximate) ground state?*

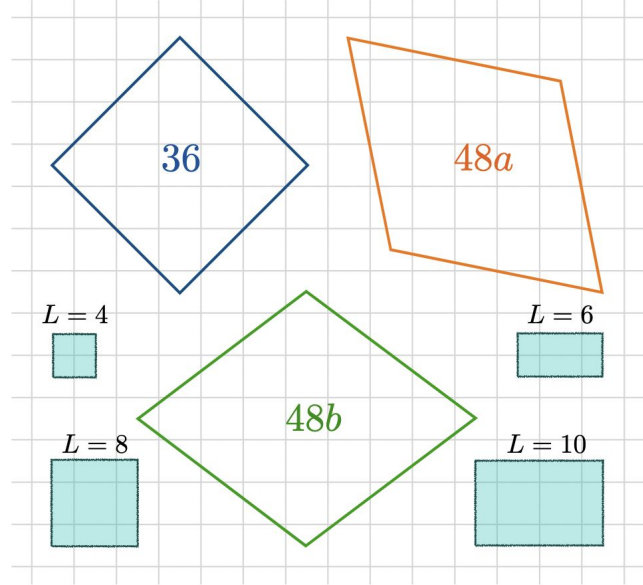
# DILUTE TRIMER STATES

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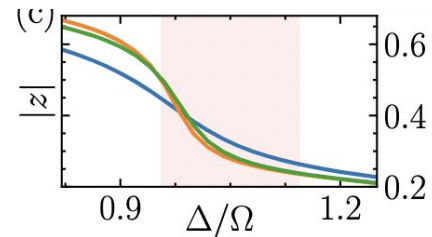
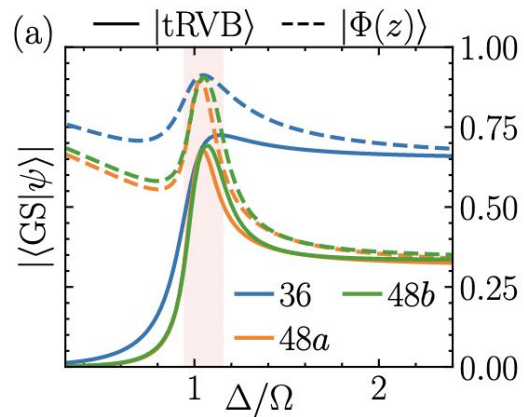
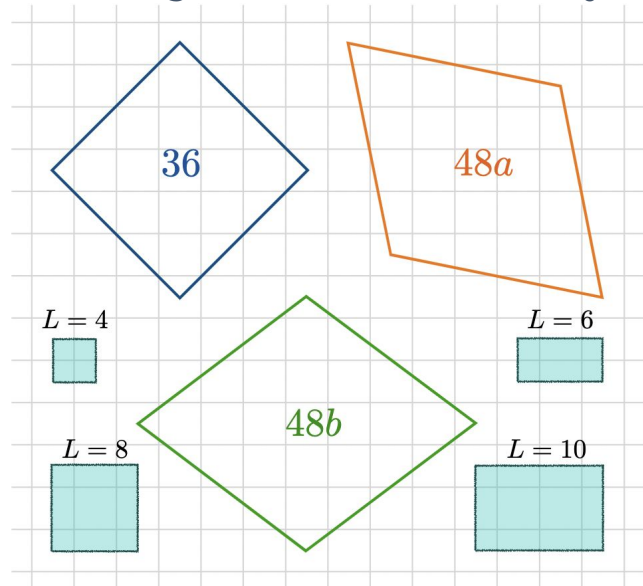
Exact diagonalization on finite systems



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Exact diagonalization on finite systems

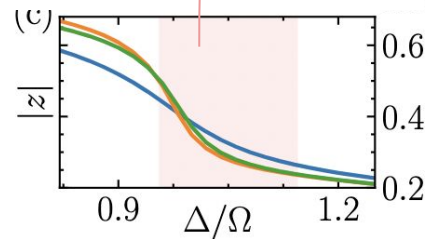
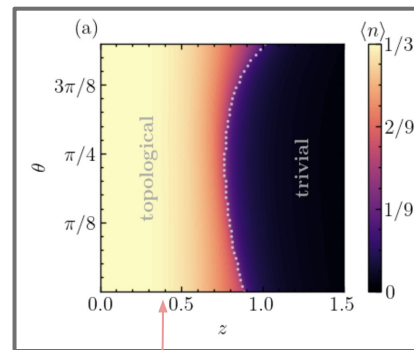
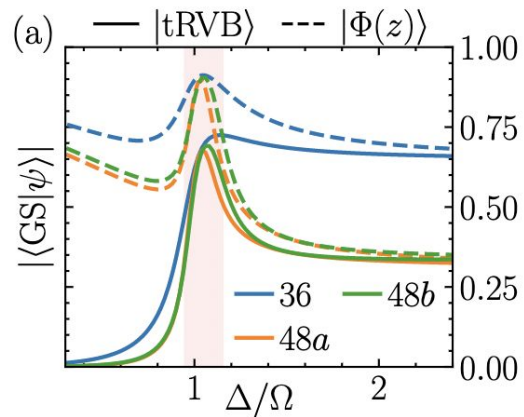
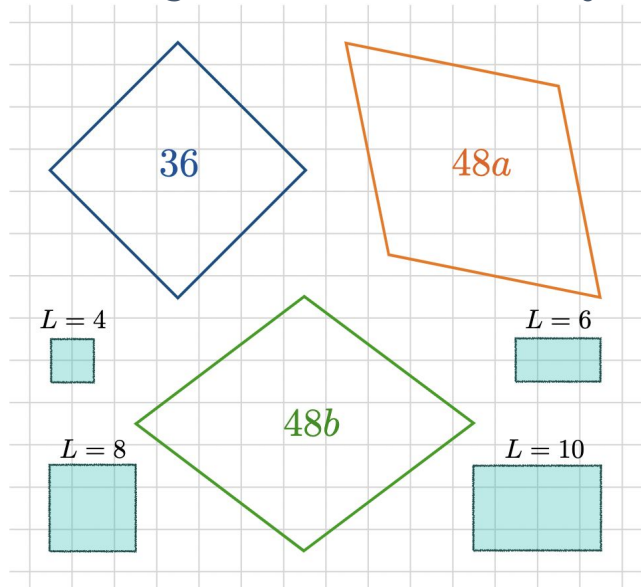


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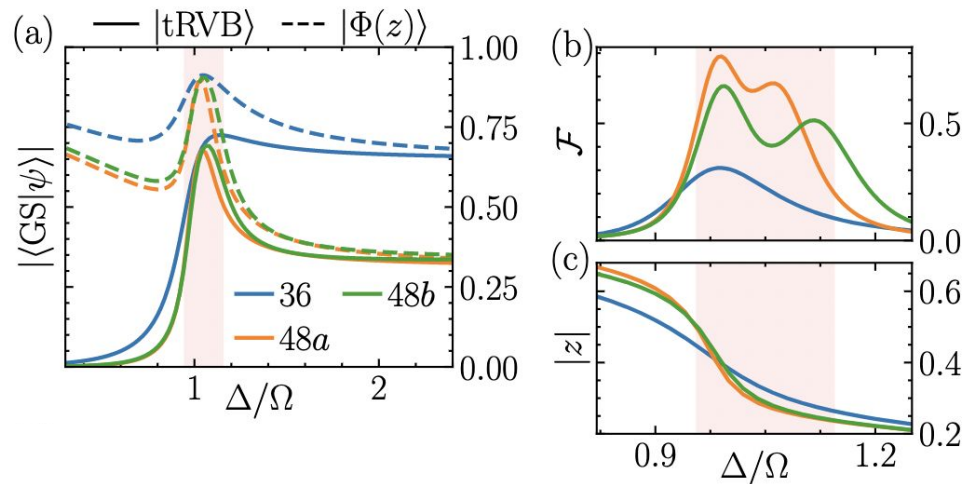
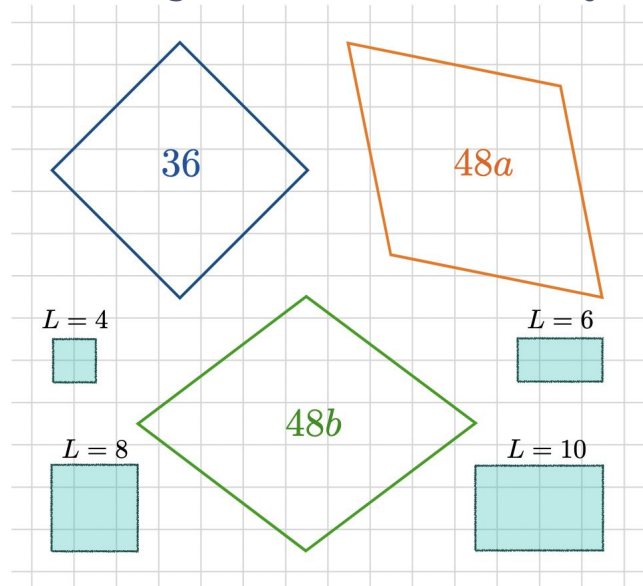
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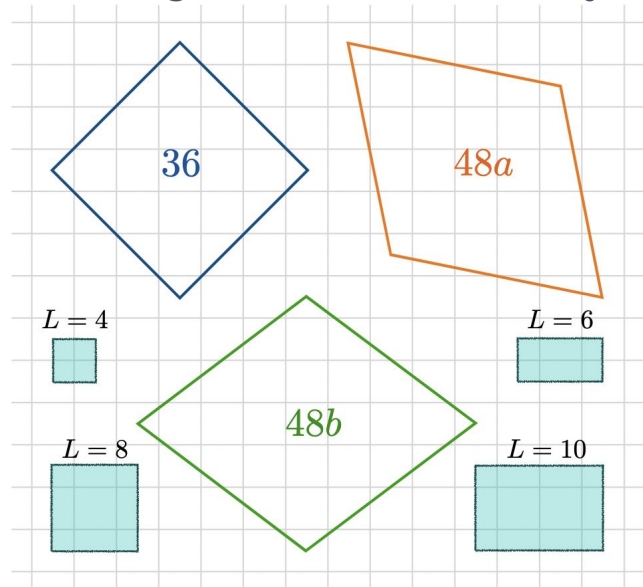
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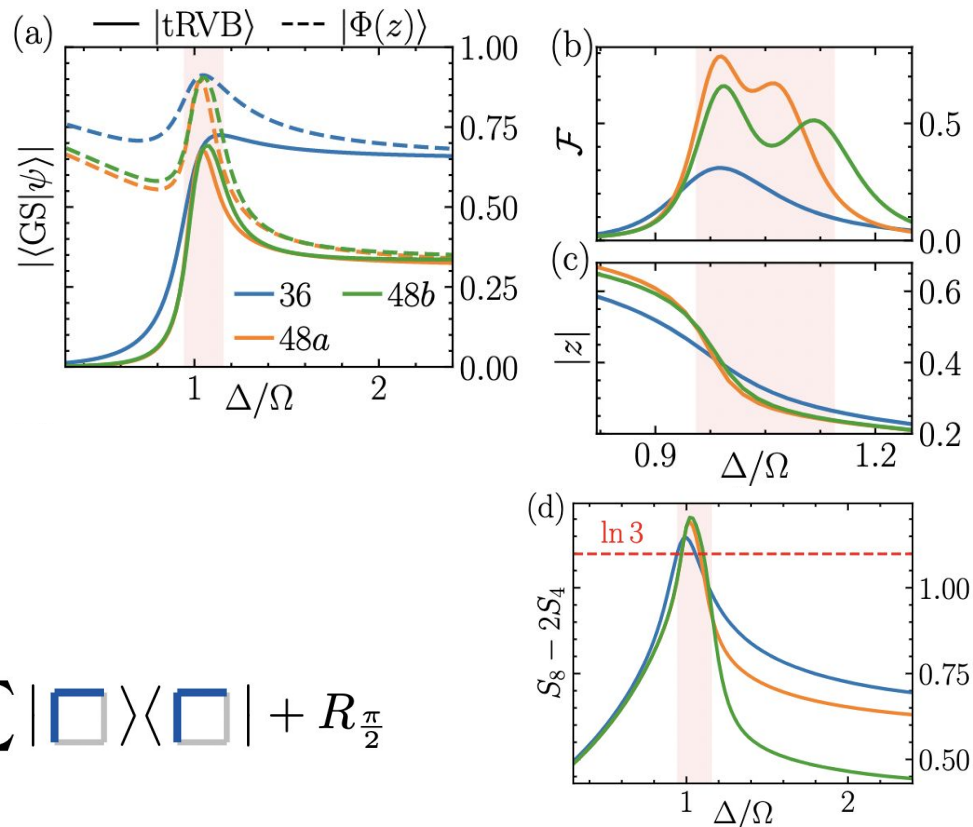
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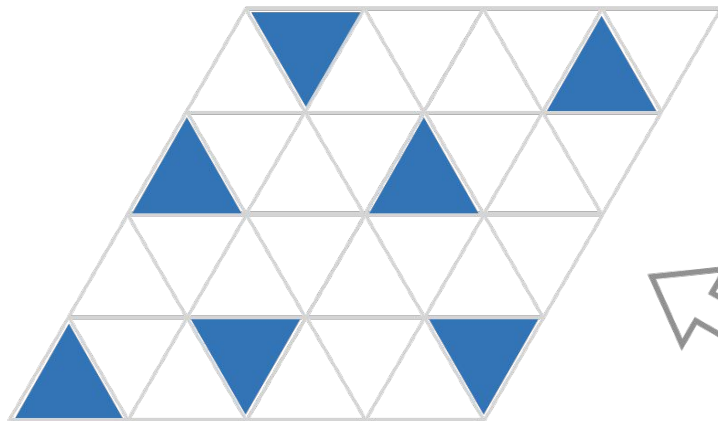
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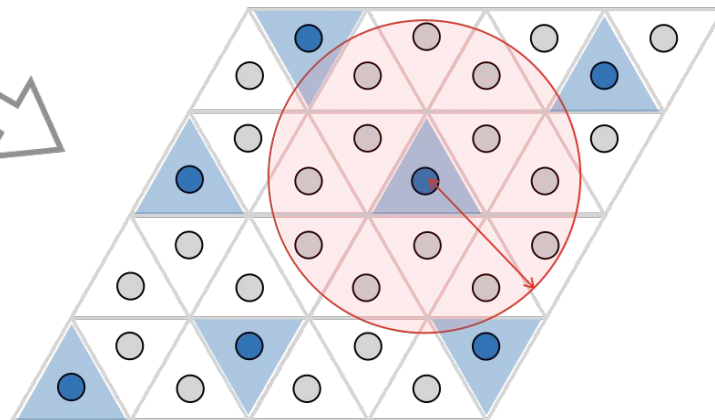
# **3. RYDBERG IMPLEMENTATION**

# RYDBERG IMPLEMENTATION

Honeycomb lattice

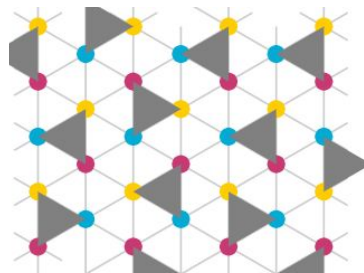
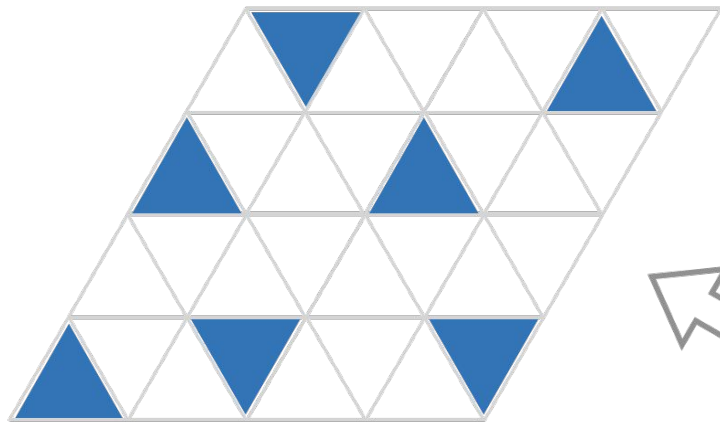


$\text{---}\circ\text{---} = |g\rangle$     $\text{---}\bullet\text{---} = |r\rangle$



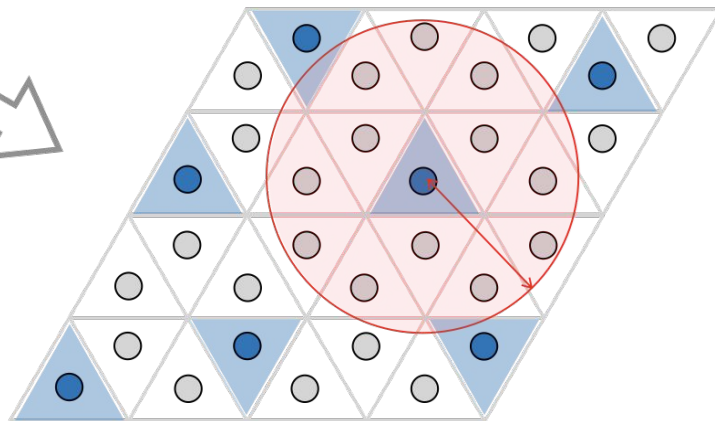
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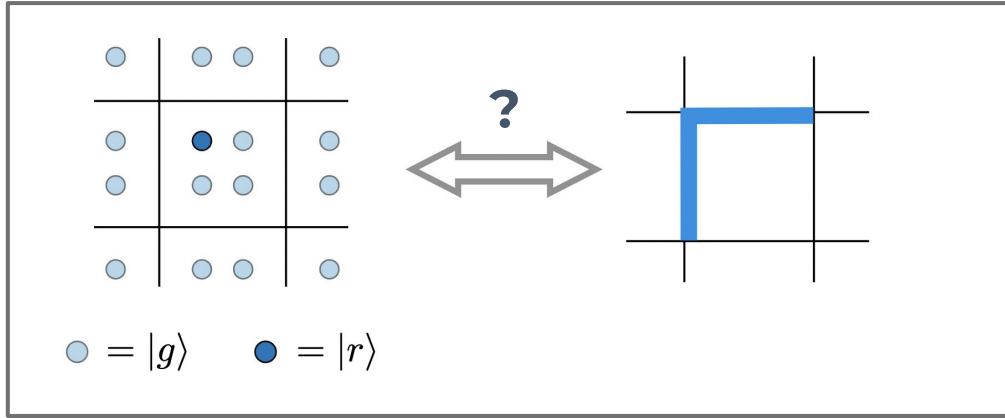


**X**  
Gapless tRVB

$\circ = |g\rangle$     $\bullet = |r\rangle$

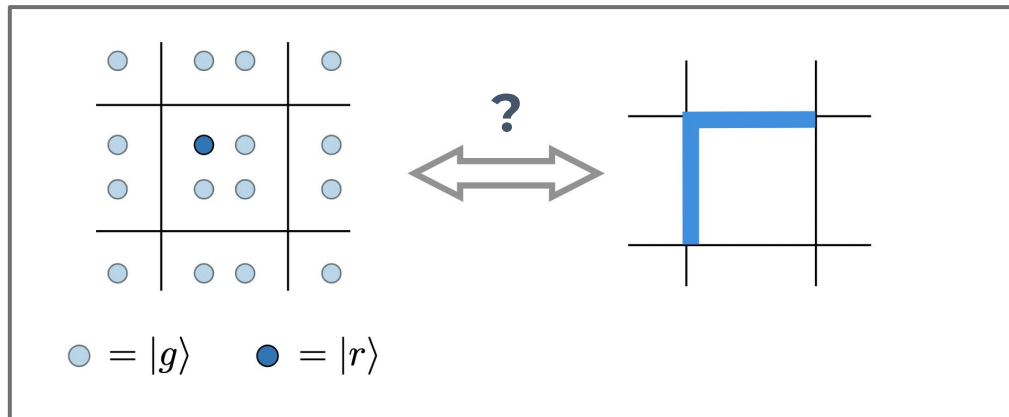


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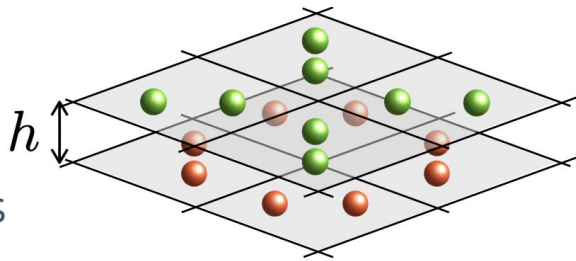
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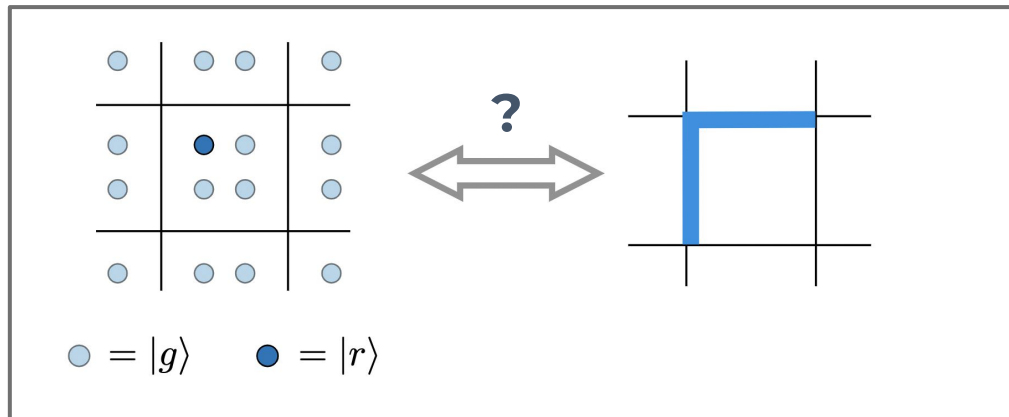
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Idea:  
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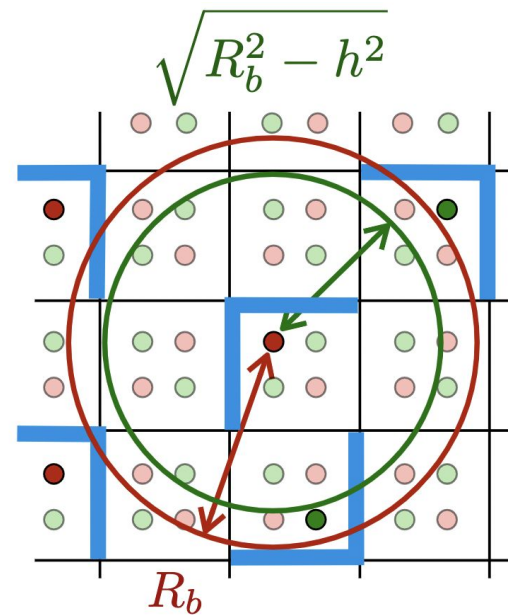
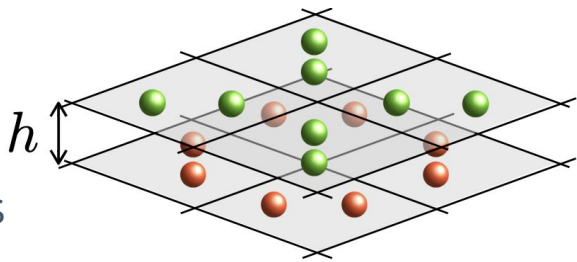


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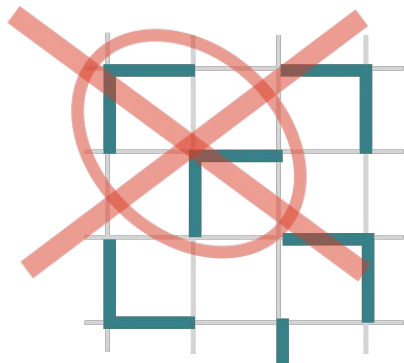
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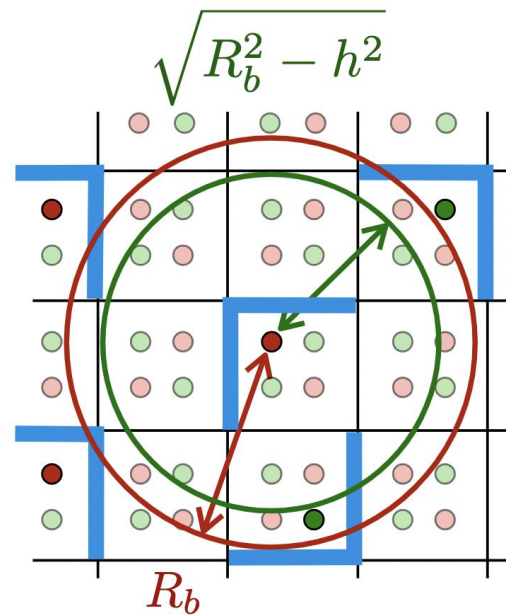
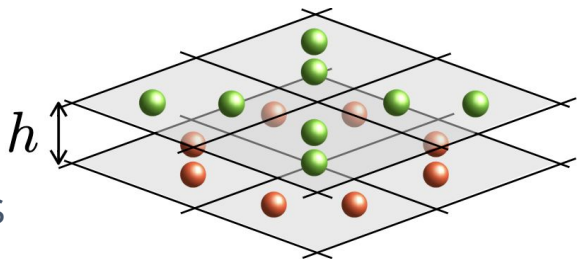


# RYDBERG IMPLEMENTATION

Not exactly the same as trimer constraint!



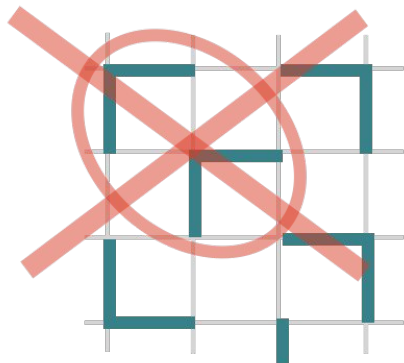
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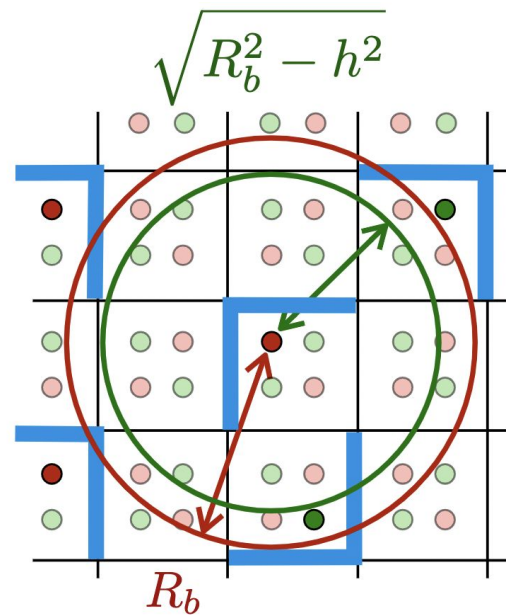
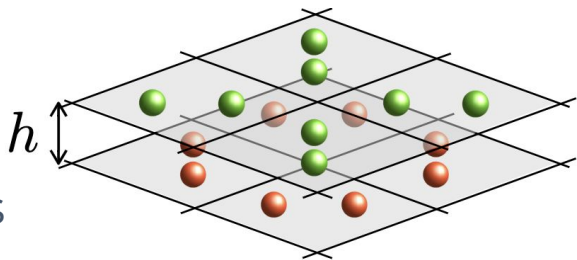
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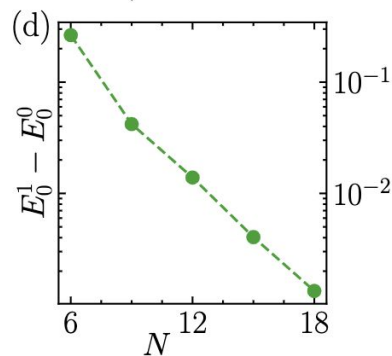
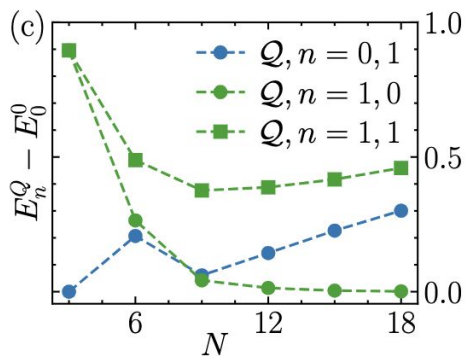
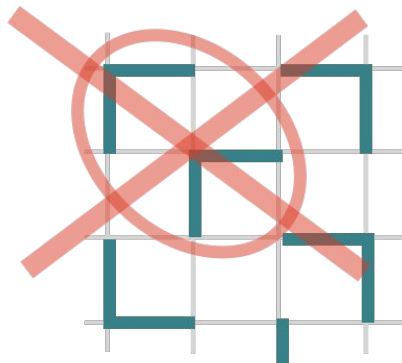
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Not exactly the same as trimer constraint!

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Yes! But with larger correlation length

$$\xi \simeq 1.7$$

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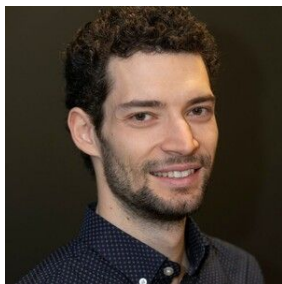
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G. Giudice



H. Pichler



G. Giudici

**Trimer states with  $Z_3$  topological order, *in preparation***

**THANK YOU FOR YOUR  
ATTENTION!**



# DYNAMICAL PREPARATION

