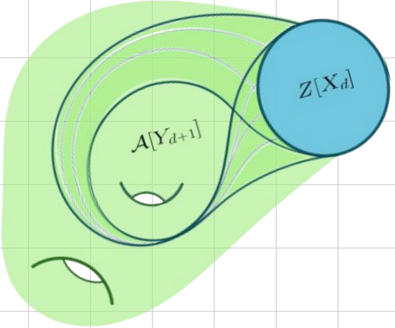
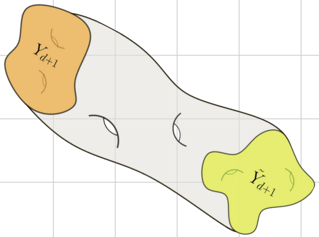


ANOMALIES & BORDISMS
OF NON-SUPERSYMMETRIC STRINGS

Matilda Delgado



Based on:
[2310.06895] I. Basile, A. Debray, M.D., M. Montero

BIG PICTURE

Our world is non-supersymmetric

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(**at least** at low energies)

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It is crucial for phenomenology to understand String Theory (QG) in setups without supersymmetry!

BIG PICTURE

On top of that: on our quest to understand the set of EFTs that come from QG,

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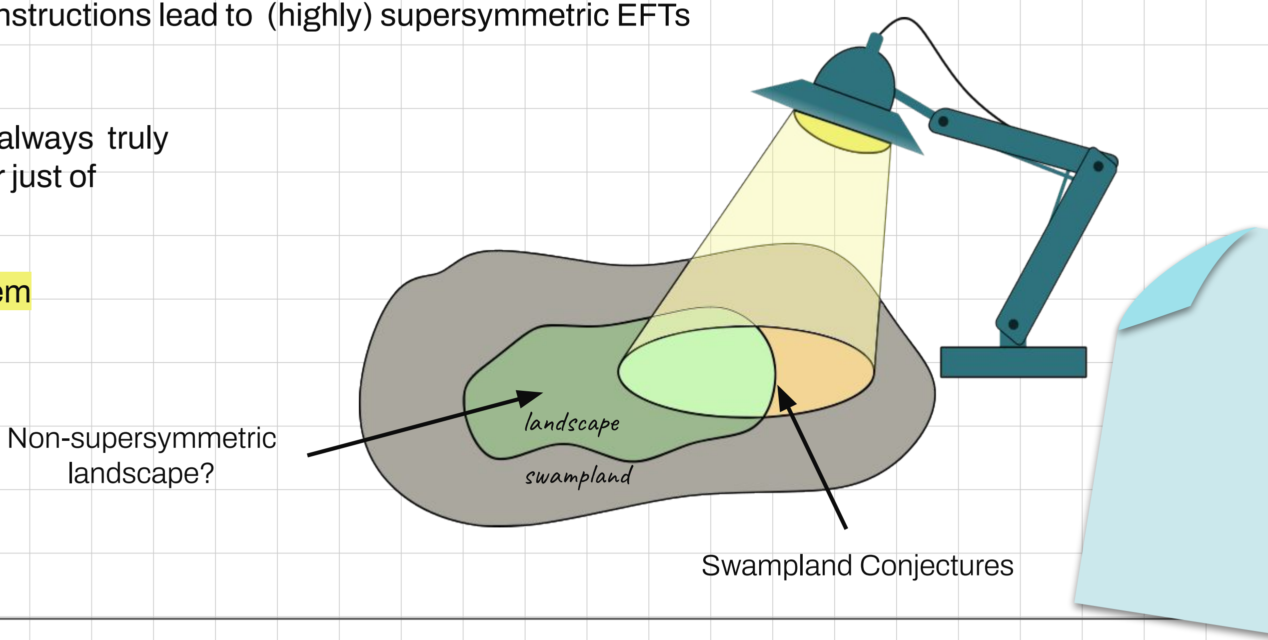
Q: Are our Swampland conjectures always truly representative of quantum gravity or just of supersymmetry?

"Supersymmetric Lamppost" problem



BIG PICTURE

“Understand QG away from the supersymmetric lamppost”

HOW?

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[1] From the **top-down?**

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In reality: easier said than done...

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- > as soon as we do not have (a lot of) **supersymmetry**, we lose (a lot of) computational control

BIG PICTURE

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What about from the **bottom up**?

This could work because bottom-up arguments in principle need not rely on SUSY at all

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⇒ Find bottom-up black hole arguments (exploit their UV/IR mixing properties)
Ex: **no global symmetries in QG** [T. Banks and L. J. Dixon, ‘93]

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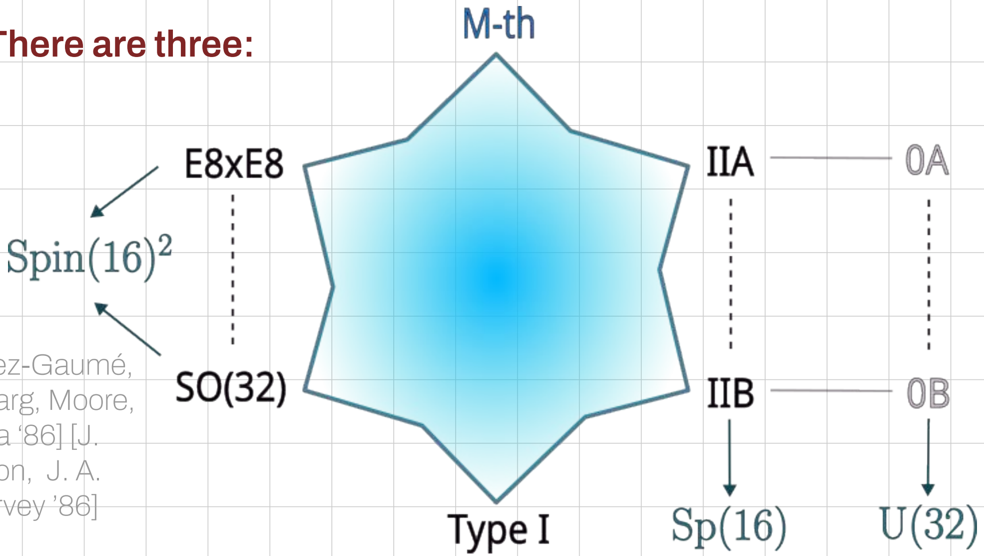
Today: Apply this to the three 10D non-supersymmetric, non-tachyonic string theories

10D NON-SUPERSYMMETRIC STRING THEORIES

There are three:



[Alvarez-Gaumé, Ginsparg, Moore, Vafa '86] [J. Dixon, J. A. Harvey '86]

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Most direct way of studying Quantum Gravity away from SUSY !!

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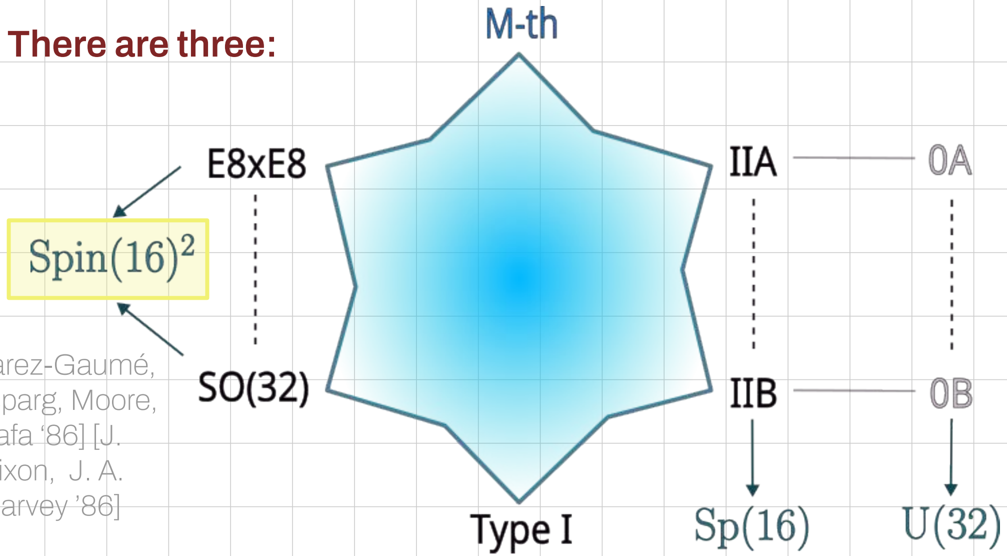

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Spin(16)²

a.k.a. SO(16)xSO(16)

Can be obtained from the SO(32) and E_8 x E_8 theories by a projection on the worldsheet.

Chiral dof:
Dilatino
Gaugini
..

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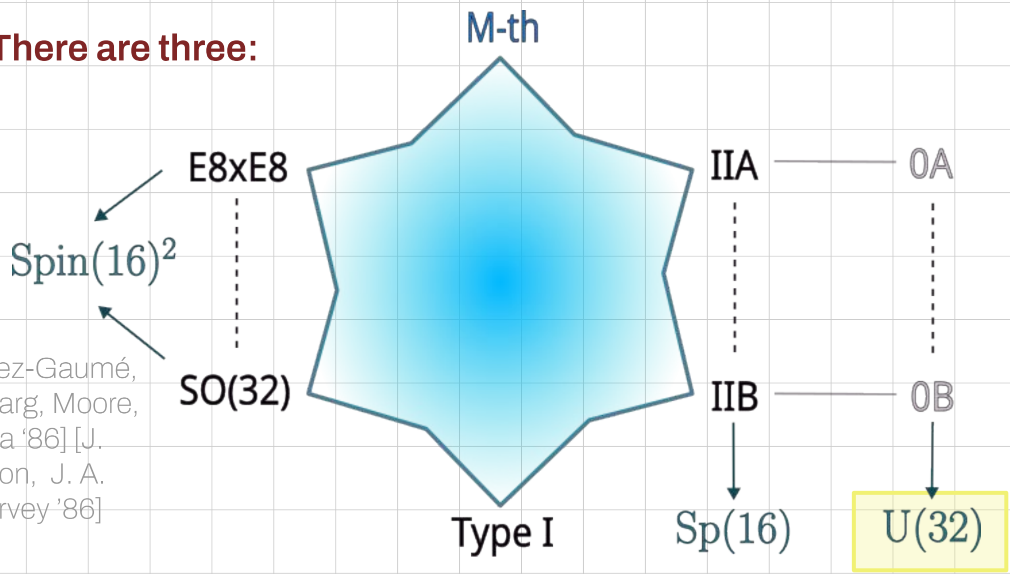

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U(32)

a.k.a. the Sagnotti string

Can be obtained from an orientifold of Type 0B that removes the tachyon.

Chiral dof:
Self-dual gauge field, Gaugini..

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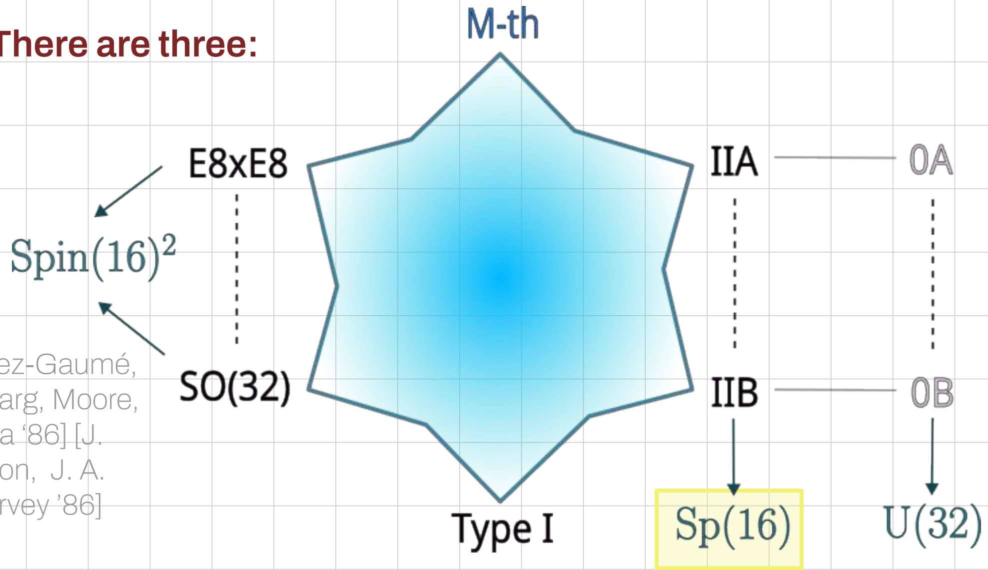

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Sp(16)

a.k.a. USp(32) or the Sugimoto string

Can be obtained from a supersymmetry breaking orientifold of IIB

"non-supersymmetric sister of Type I"

Chiral dof: Gravitino, Dilatino, Gaugini..

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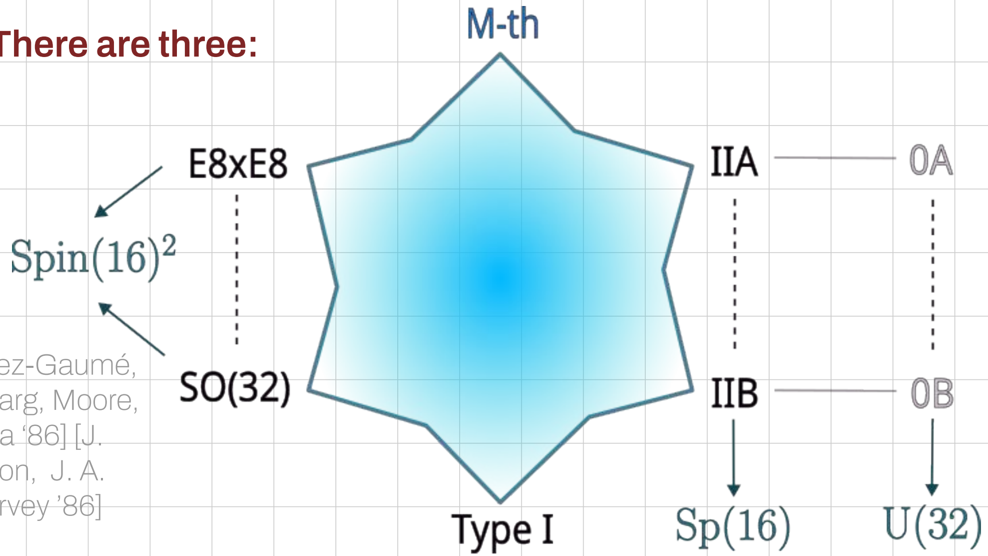

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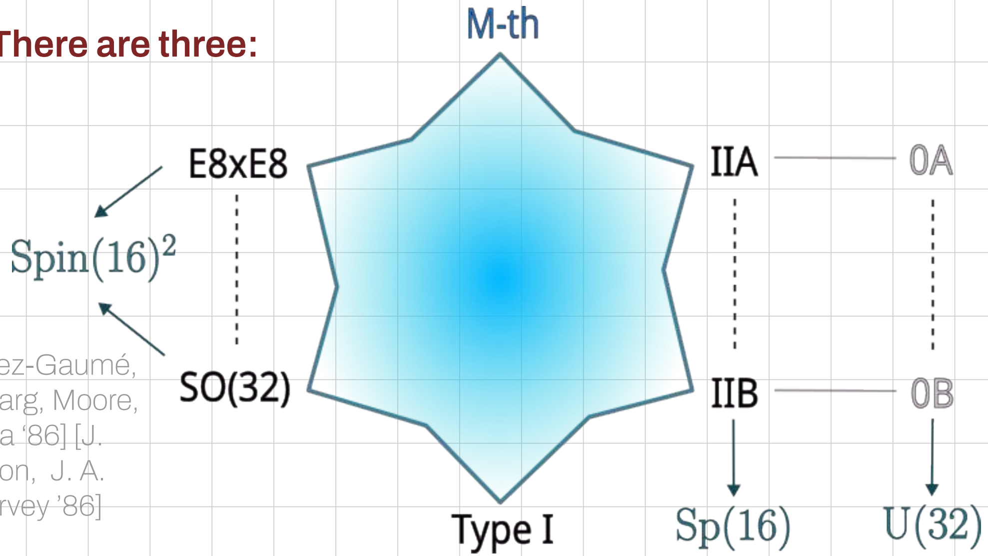

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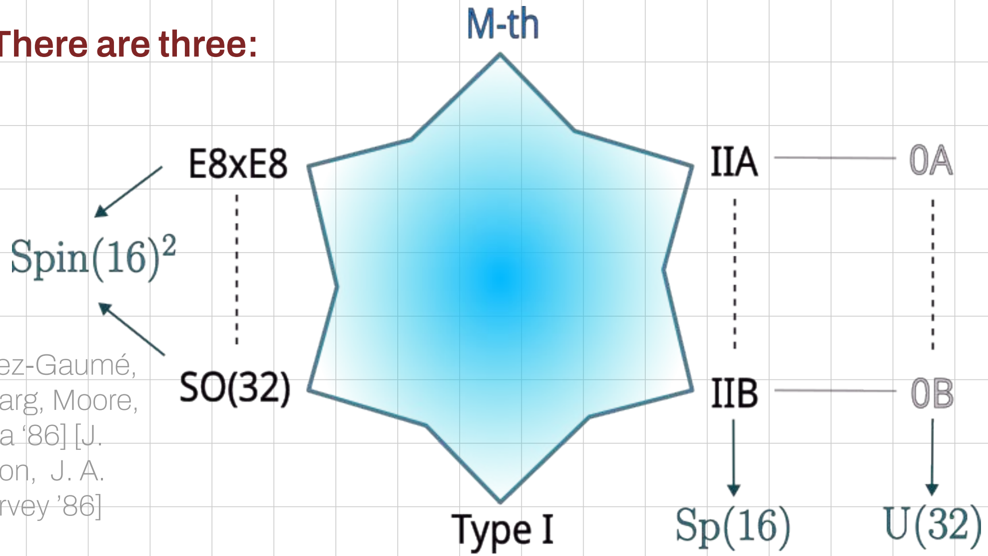

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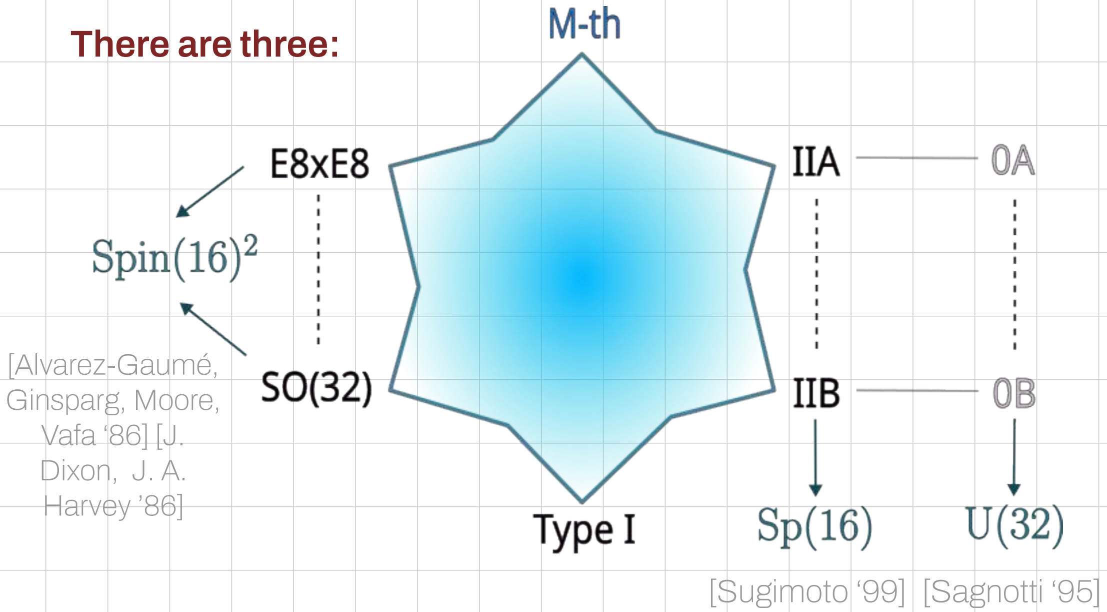

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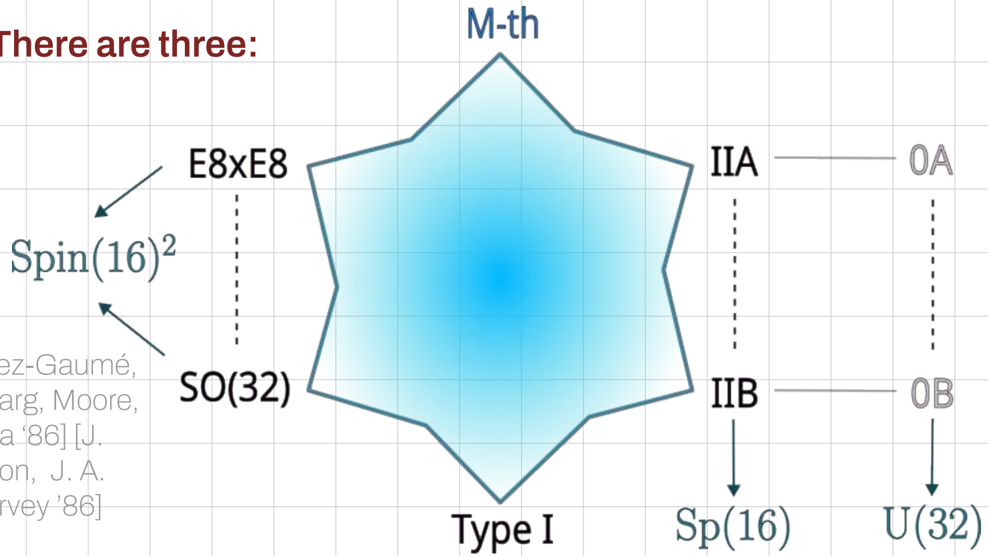

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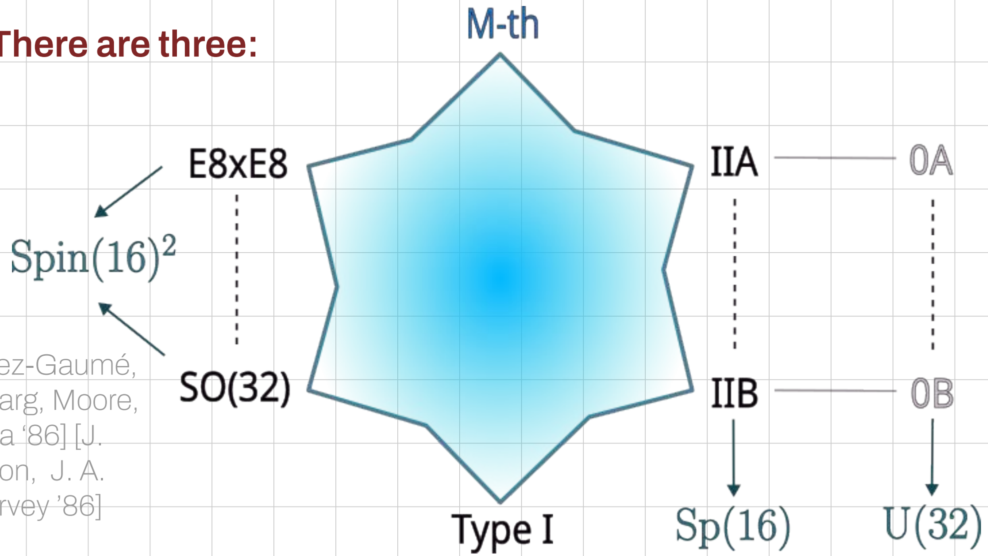

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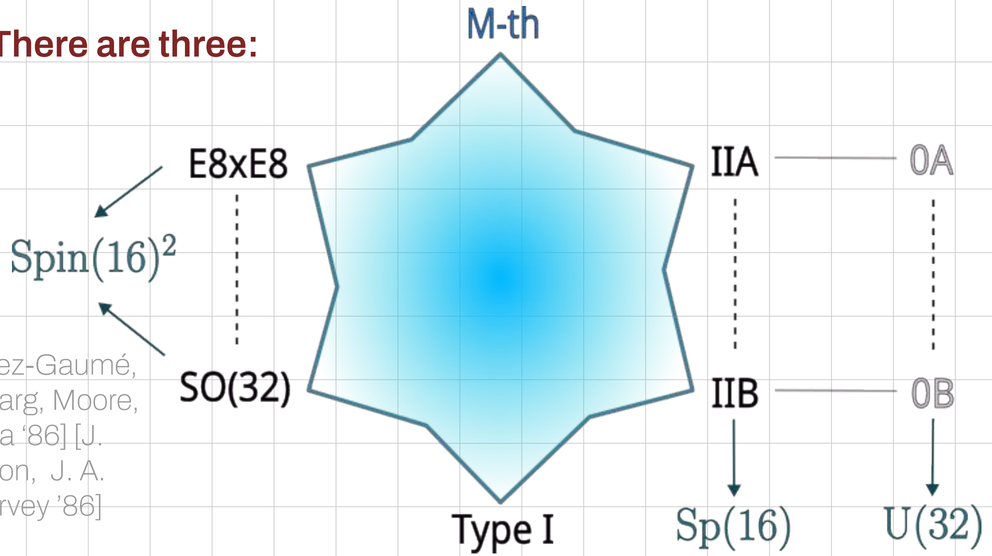

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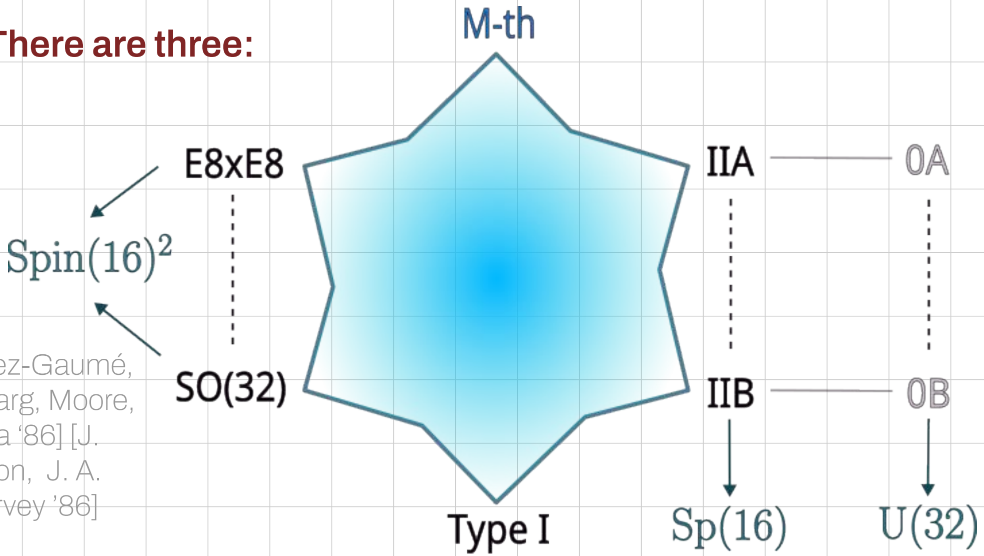

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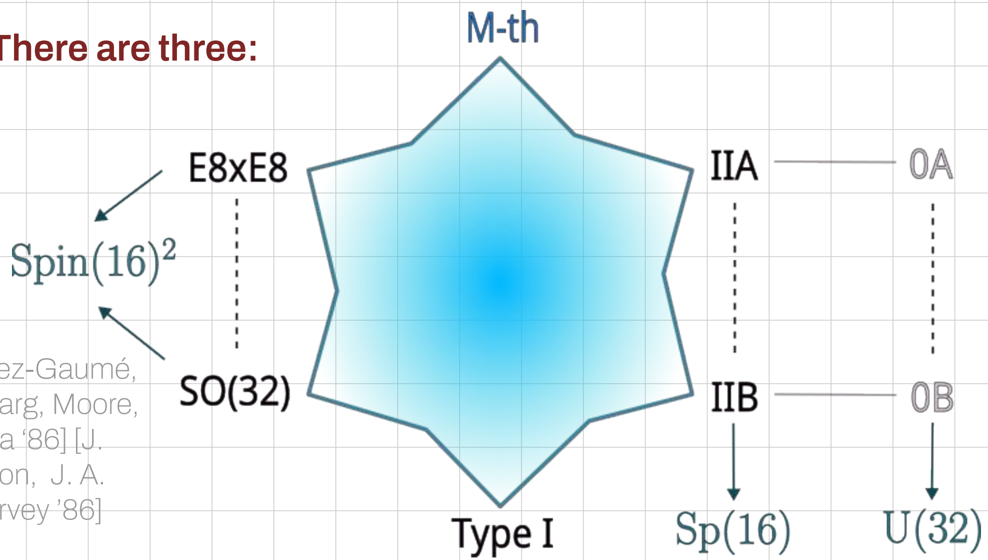

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TODAY:

shed light on these questions

PLAN

Goal: Bottom-up methods for understanding the 10D non-supersymmetric, non-tachyonic string theories

❶

Global Gauge Anomaly Cancelation
&
Bordisms

❷

No Global Symmetries
&
new extended objects

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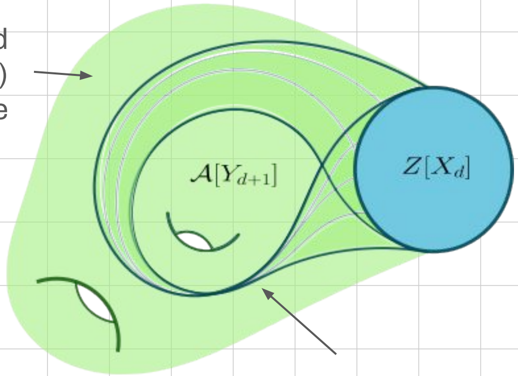
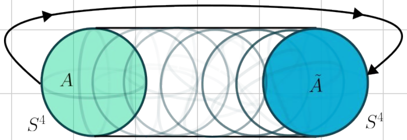
The modern way of computing **global gauge and gravitational** anomalies of a theory on X_d is through a (d+1)-dimensional *anomaly theory* on Y_{d+1} such that $\partial Y_{d+1} = X_d$.

All "structures" extend to the (d+1) dimensional space



Non-collapsible path in configuration space of gauge field / metric

~generalization of mapping torus



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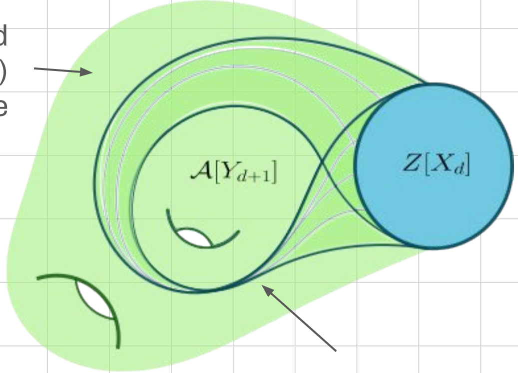

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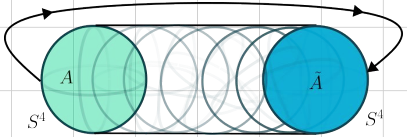

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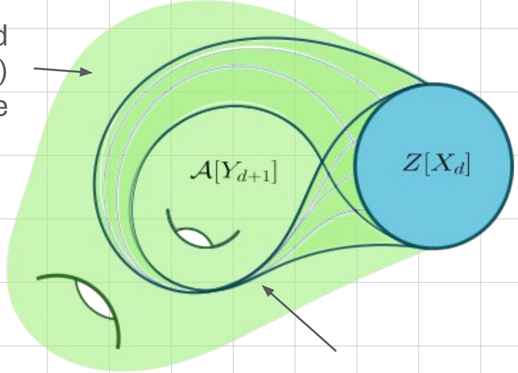

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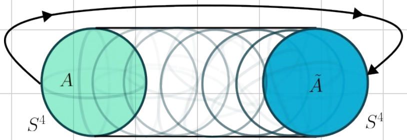

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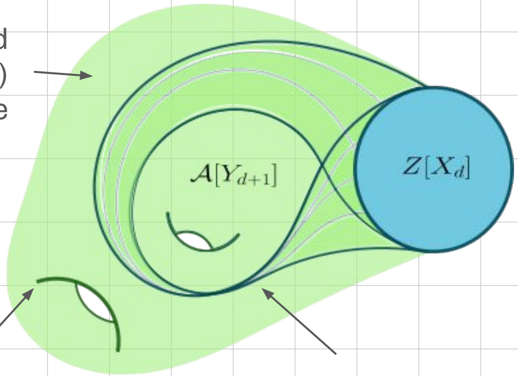

Non-collapsible path in configuration space of gauge field / metric

In QG, allow for topology-change

⇒ "Dai-Freed anomalies" Account for the possibility of a transformation that involves topology change

[García-Etxebarria, Montero '18]

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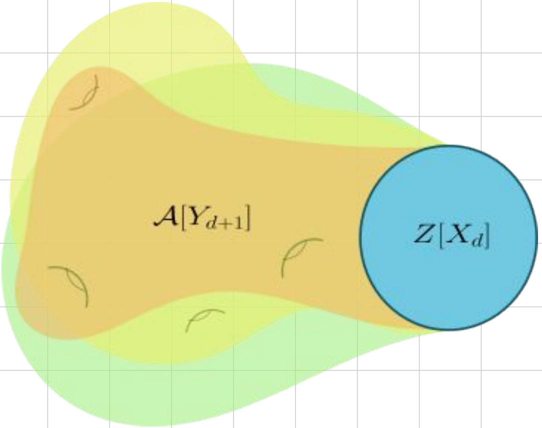

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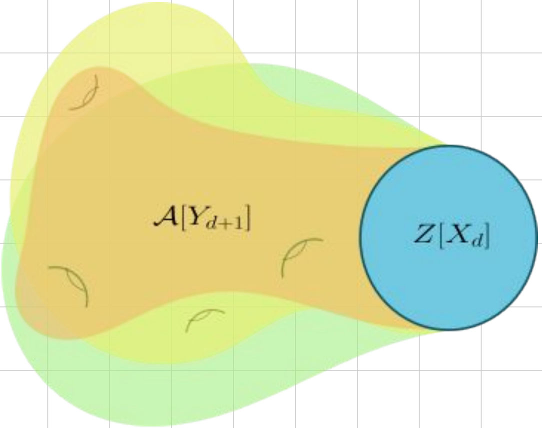

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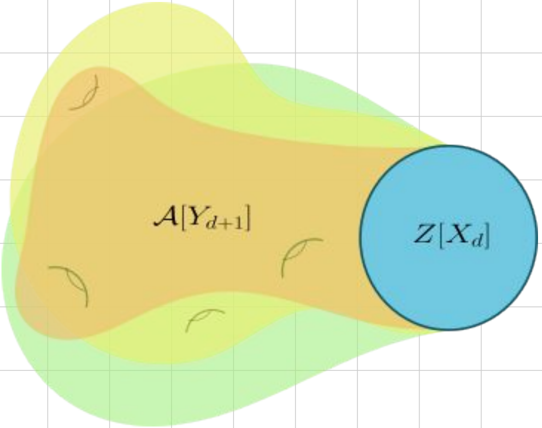

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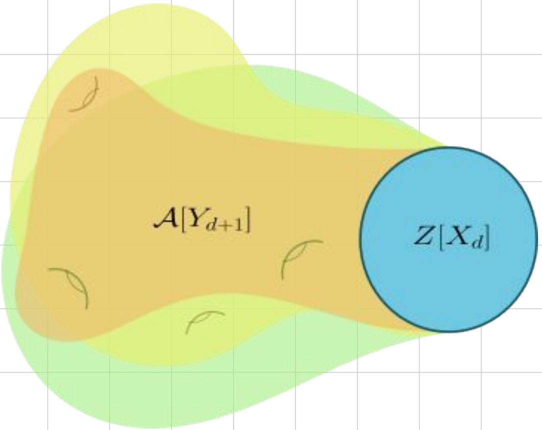

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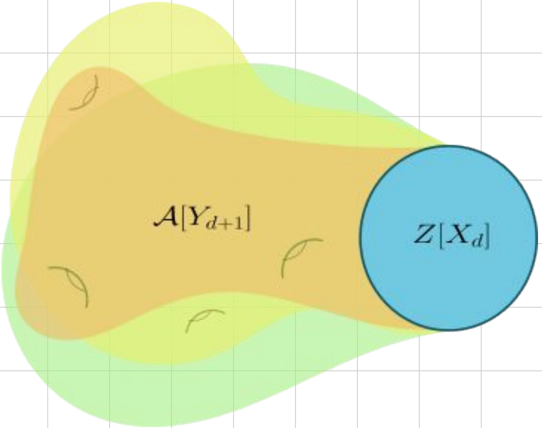

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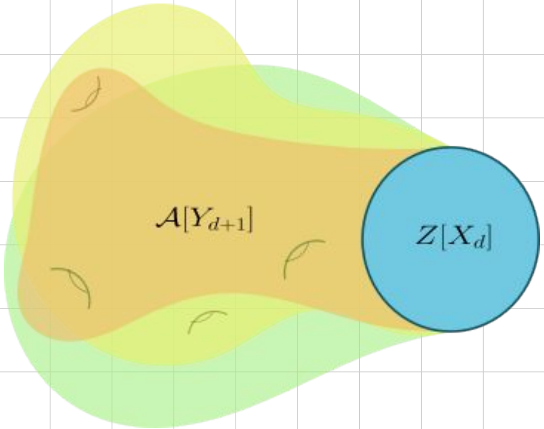

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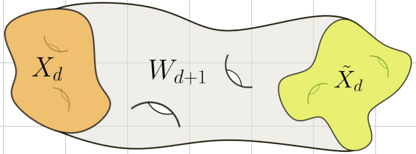
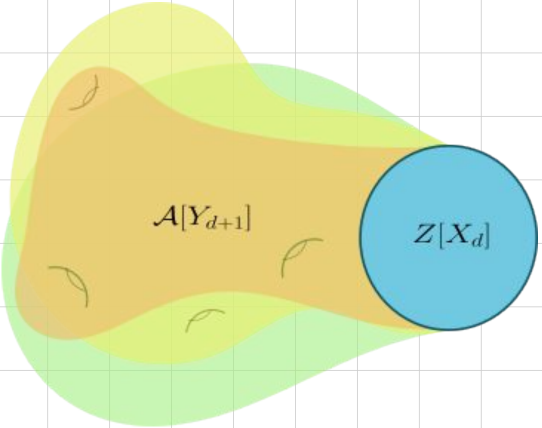


The two d-dimensional manifolds can be deformed into each other
-> They are in the same bordism class!

[García-Etxebarria, Montero '18]

GLOBAL ANOMALIES

⇒ the anomaly is a bordism invariant !



To determine the presence of a global anomaly:

Compute the (d+1)-dimensional bordism group for your theory

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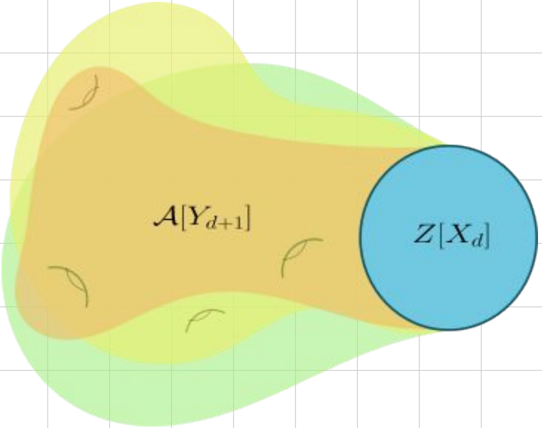

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1. If it is non-trivial:

    a. determine its bordism invariant(s) and the corresponding generating manifold(s)

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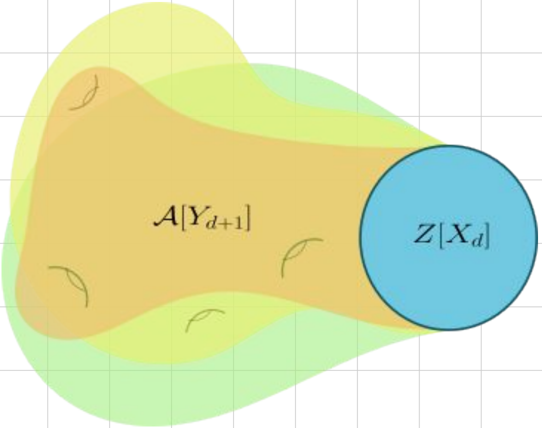

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1. If it is non-trivial:
   a. determine its bordism invariant(s) and the corresponding generating manifold(s)
   b. Evaluate the anomaly theory on the generating manifold to get the anomaly
2. If it is trivial,
   You're done! There are no anomalies

RELEVANT BORDISM GROUPS

So what **11D cobordism groups** are the relevant ones for these three theories?

All three theories only make sense on backgrounds that satisfy the non-trivial **Bianchi identity** associated to H :

$$dH \sim \mathrm{tr}F^2 - \mathrm{tr}R^2 = 0$$

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➡ **twisted string bordism**

Not many of them are known, we computed

$$\Omega_{11}^{string-\mathrm{Sp}(16)}, \quad \Omega_{11}^{string-\mathrm{Spin}(16)^2}, \quad \Omega_{11}^{string-U(32)}$$

using the Adams spectral sequence.

RESULTS

So what are the groups??

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Maybe you are more crafty than we are?

PLAN

Goal: Bottom-up methods for understanding the 10D non-supersymmetric, non-tachyonic string theories

❶

Global Gauge Anomaly Cancelation
&
Bordisms

❷

No Global Symmetries
&
new extended objects

NO GLOBAL SYMMETRIES

The fact that there should be no global symmetries in QG is well-tested (sometimes proven) conjecture

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$$Q \sim \int_{M_p} [\text{Topological OP}]_p$$

(can also be a lot more exotic than that)

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Gauge it: new consistency conditions for compactification of your theory → refine your notion of bordism

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So if we had non-vanishing lower-dimensional bordism groups, by the cobordism conjecture;

We’d either learn these theories have **new (D-p-1)-dimensional objects,**

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We computed the lower dimensional twisted-string bordism groups:

k	$\Omega_k^{\text{String}-Spin(16)^2}$	$\Omega_k^{\mathbb{G}_{16,16}}$	$\Omega_k^{\text{String}-Sp(16)}$	$\Omega_k^{\text{String-}SU(32)\langle c_3\rangle}$
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
2	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
3	0	$\mathbb{Z}_8$	0	0
4	$\mathbb{Z}^2$	$\mathbb{Z} \oplus \mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$
5	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
6	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0 or $\mathbb{Z}_2$
7	0	$\mathbb{Z}_{16}$	$\mathbb{Z}_4$	$\mathbb{Z}_2$ or $\mathbb{Z}_4 \oplus \mathbb{Z}_2$
8	$\mathbb{Z}^6$	$\mathbb{Z}^3 \oplus \mathbb{Z}_2^i$	$\mathbb{Z}^3 \oplus \mathbb{Z}_2$	$\mathbb{Z}^3 \oplus \mathbb{Z}_2$ or $\mathbb{Z}^3 \oplus \mathbb{Z}_2^2$
9	$\mathbb{Z}_2^5$	$\mathbb{Z}_2^j$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$
10	$\mathbb{Z}_2^7$	$\mathbb{Z}_2^k$	$\mathbb{Z}_2^3$	$\mathbb{Z} \oplus \mathbb{Z}_2^2$ or $\mathbb{Z} \oplus \mathbb{Z}_2^3$

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A ton of classes to kill !!

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Homework Sheet

$$\begin{aligned}
&\Omega_0^{\text{String-}Sp(16)} \cong \mathbb{Z} && \Omega_6^{\text{String-}Sp(16)} \cong \mathbb{Z}_2 \\
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&\Omega_3^{\text{String-}Sp(16)} \cong 0 && \Omega_9^{\text{String-}Sp(16)} \cong (\mathbb{Z}_2)^{\oplus 3} \\
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\end{aligned}$$

Example: Sugimoto

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(Good luck 💅)

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Example: Sugimoto

On the quest to characterizing these new extended objects (a non-exhaustive list):
[Andriot, Angius, Blumenhagen, Buratti, Carqueville, Cribiori, Calderon-Infante, DeBiasio, Debray, Delgado, Dierigl, Friedrich, Garcia-Etxebarria, Hebecker, Heckman, Huertas, Kneissl, Makridou, Montero, McNamara, Lust, Torres, Uranga, Vafa, Valenzuela, Velazquez, Walcher, Wang…’19-’24]

PLAN

Goal: Bottom-up methods for understanding the 10D non-supersymmetric, non-tachyonic string theories

❶

Global Gauge Anomaly Cancelation

was proven for these 3 theories* , through the computation the **11D bordism groups**

✅

❷

No Global Symmetries (Cobordism Conjecture)

Gave us new consistency conditions and extended objects to look for, through the computation of **lower-dimensional bordism groups**

✅

LAST REMARK

These three 10D non-supersymmetric string theories are a great arena to **test QG away from the supersymmetric lamppost**

They pass every consistency check so far!

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(beyond the homework sheet)

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There might be other accessible things to discover about them!

For example: now that we know all anomalies cancel on all backgrounds
➥ use anomaly inflow to gain insight into the worldvolume theory of branes in these theories

➥ we implemented this for the NS5 brane in SO(16)xSO(16),
as did [M. Blaszczyk, S. Groot Nibbelink, O. Loukas, F. Ruehle '15]

THANKS!

XTRA SLIDES

LOCAL ANOMALIES

In theories coupled to gauge fields and dynamical gravity, there can generally be gauge/gravitational anomalies.

Anomalies in gauge symmetries are a BIG problem (unlike for anomalies in global symmetries)

An anomaly is a lack of invariance of the path integral under a gauge transformation or diffeomorphism:

$$Z[X_d] \Longrightarrow \tilde{Z}[X_d] \neq Z[X_d]$$

- Local anomalies = "*usual ones*" , associated to gauge transformations that can be made **arbitrarily small**
  Think triangle (n-gon) diagrams

[e.g. Álvarez-Gaumé, Vázquez-Mozo '22]

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LOCAL ANOMALIES

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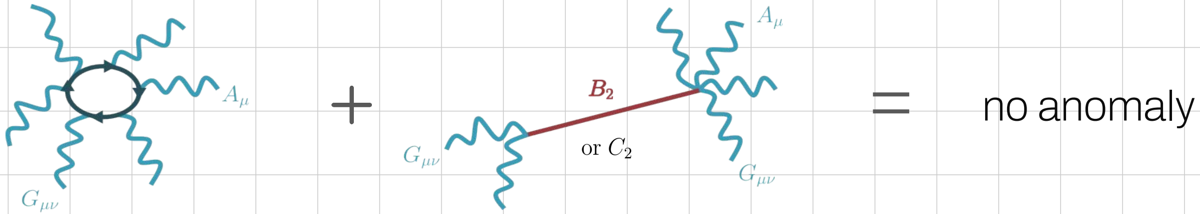
These are the anomalies that are killed by the Green-Schwarz mechanism in all three theories:



The added Green-Schwarz term cancels the anomaly by coupling the B-field to gravity and the gauge field:

$$S_{GS} = -\int_{M^{10}} B_2 \wedge X_8 \qquad X_8 = \mathrm{tr}F^4 + \mathrm{tr}R^4 + \mathrm{tr}F^2\mathrm{tr}R^2 + (\mathrm{tr}R^2)^2 + (\mathrm{tr}F^2)^2$$

or C_2 (for example)

Gauge field contribution

Modified Bianchi identity → $dH \sim \mathrm{tr}F^2 - \mathrm{tr}R^2$

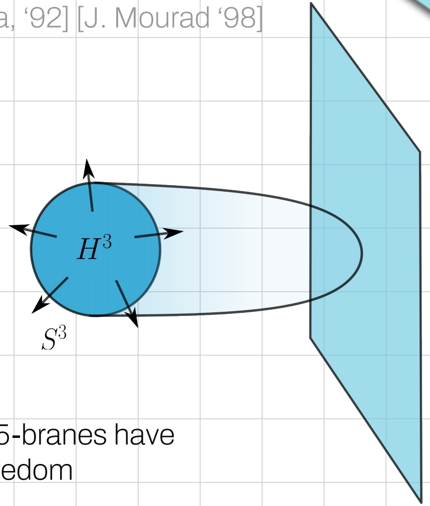
Gravity contribution

[J. A. Dixon, M. J. Duff, J. C. Plefka, '92] [J. Mourad '98]

ANOMALY INFLOW

Now let's add in 5 branes that source magnetic H-flux:

$$S = S_{bulk} + S_{GS} + \underbrace{\mu \int B_6}_{} + \underbrace{S_{worldvolume}}_{}$$



Since the 5-brane sources H-flux, it participates in the modified Bianchi identity for H:

There is a new contribution to the bulk anomaly

The worldvolume theory of 5-branes have chiral degrees of freedom

They source an anomaly in the worldvolume theory

Consistency of the theory (all anomalies vanish) requires that the two contributions cancel eachother out

This is anomaly inflow

NON-SUPERSYMMETRIC 5-BRANES

The three 10D non-supersymmetric models have 5-branes that source magnetic H-flux.

For the Sugimoto and Sagnotti string: these are the D5 branes.

Because they are Dirichlet, their worldvolume theory can be computed from the worldsheet

The anomaly inflow mechanism works

[E. Dudas and J. Mourad '00] [J. A. Dixon, M. J. Duff, J. C. Plefka, '93] [J. Mourad '97]

For SO(16)xSO(16): this is the NS5 brane whose worldvolume dofs are unknown!

We can reverse the anomaly inflow argument to shed light on the chiral field content of this NS5 brane

I.e. find a chiral field content that gives the right anomalous contribution to the inflow mechanism

We get:

- Four fermion singlets
- A fermion in the (16,1) + (1,16) of SO(16)xSO(16)
- A self-dual 2-form

} Non-supersymmetric interacting CFT? (just wishful thinking)