T-duality for non-critical heterotic strings

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Work to appear

@ Swamplandia 2024, Abby Seeon

Overview

[Kaidi, Ohmori, Tachikawa, Yonekura'23]

- Recently, **non-supersymmetric p-branes** constructed in heterotic string:

  0-brane and 6-brane in $Spin(32)/Z_2$ string

  4-brane and 7-brane in $E_8 \times E_8$ string

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- Described at near-horizon by linear dilaton background given by **tachyon-free subcritical heterotic strings**: [Kaidi '21]

$p = 0$:	$Spin(24)/Z_2$	$D = 2$
$p = 4$:	$E_7 \times E_7/Z_2$	$D = 6$
$p = 6$:	$SU(16)/Z_2$	$D = 8$
$p = 7$:	E_8	$D = 9$

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  [Hellerman, Swanson '06]

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$D>10$ strings (N tachyons) → Critical strings ($N-(D-10)$ tachyons)

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Why?

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- Regions of moduli space of critical nonSUSY on S^1 have tree-level tachyons. Where do they condense to?

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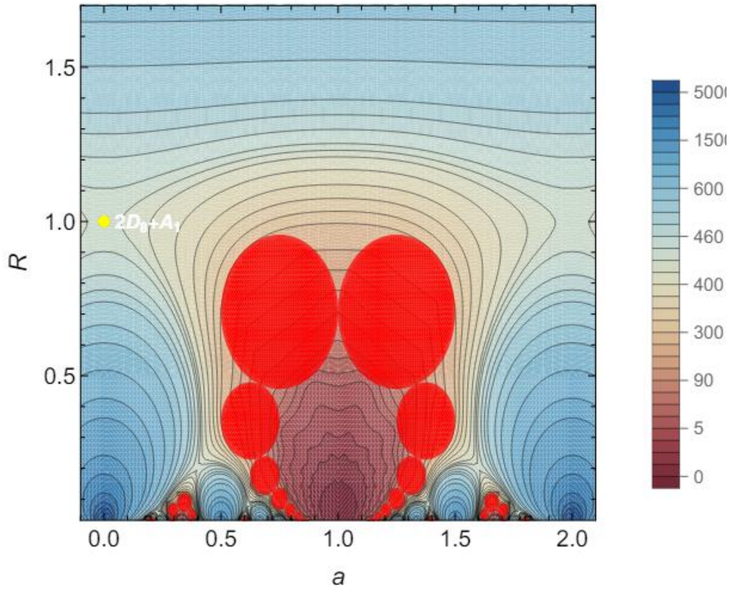

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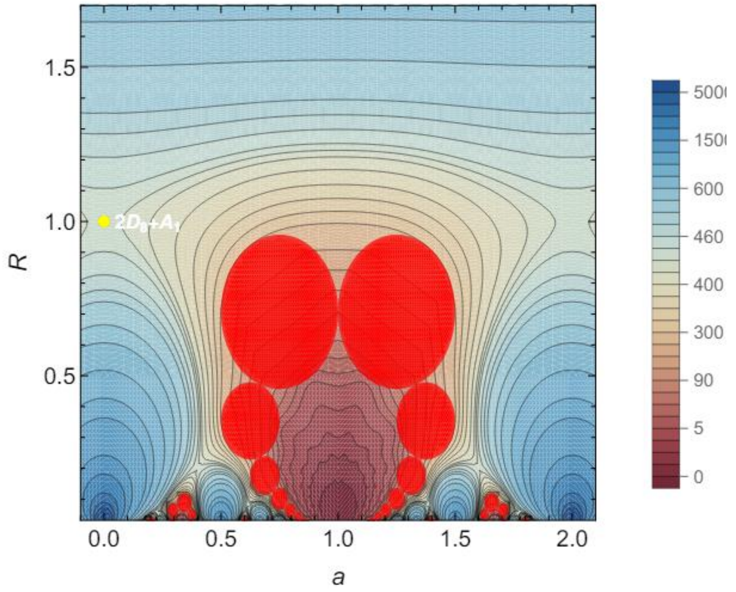

<u>Example:</u> Scherk-Schwarz reduction of SUSY theories leads to a pair of tachyons as the circle shrinks.
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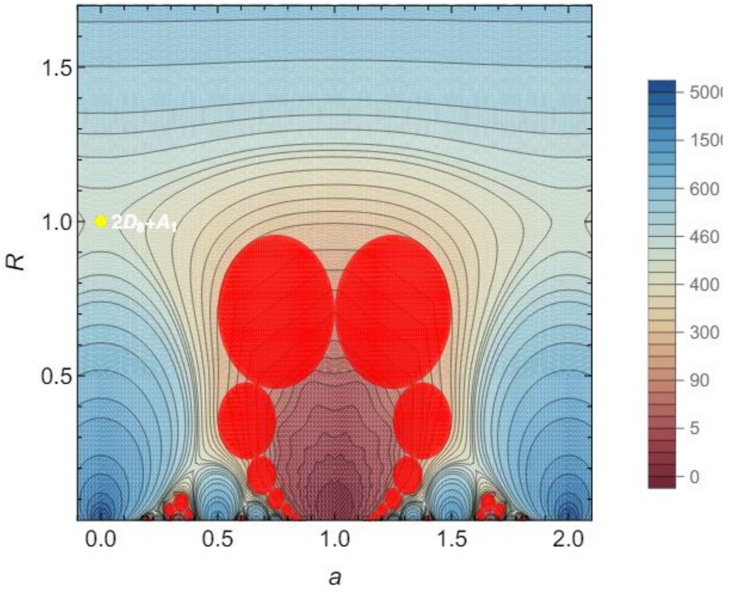

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- **More generally:** why are nonSUSY compactifications plagued with tree-level tachyons? *Perhaps we can learn something…*

Moduli spaces: Basics

Heterotic strings on S^1 have radius and Wilson line moduli thanks to the gauge bundle. Locally, they span the coset

$$M_{local} = O(1,1+r)/O(1+r), \qquad r = rank(G)$$

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Critical strings have $r = 16$, $\Gamma = O(II_{1,17})$ for SUSY and $\Gamma = O\left(I_{1,17}\right)$ for nonSUSY.
(Automorphisms of even self-dual lattice)

$II_{1,17}$ = lattice of all electric charges in the SUSY theory.

$I_{1,17}$ = lattice of electric charges *for bosons* in the nonSUSY theory.

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<u>For illustration</u> consider the bosonic string on T^2 ,
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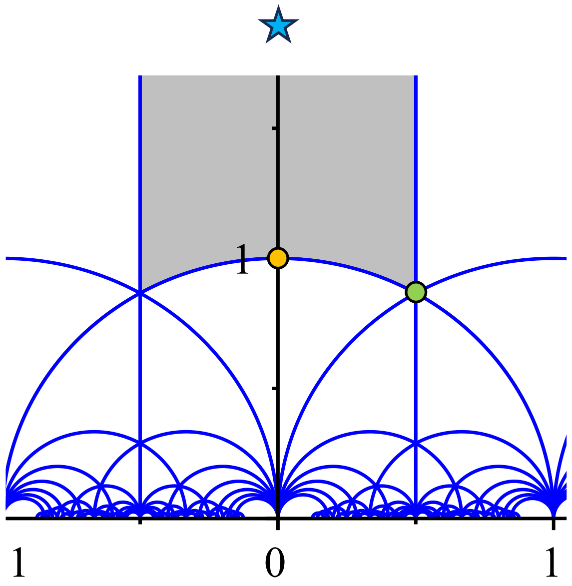
Infinite distance cusps ≈ Decompactification



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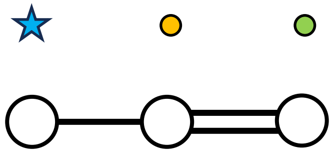
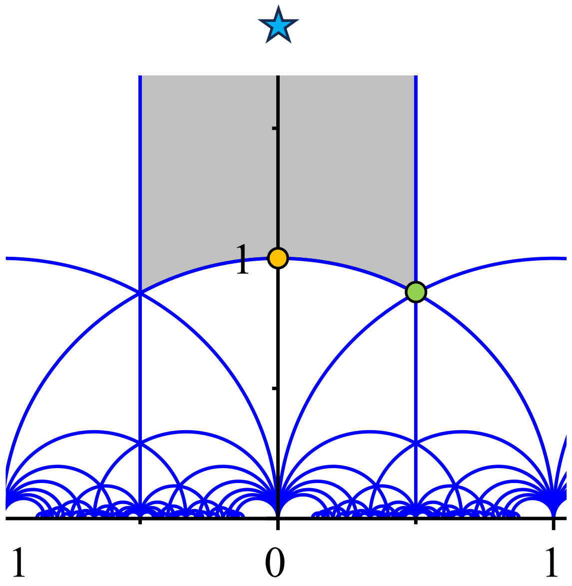
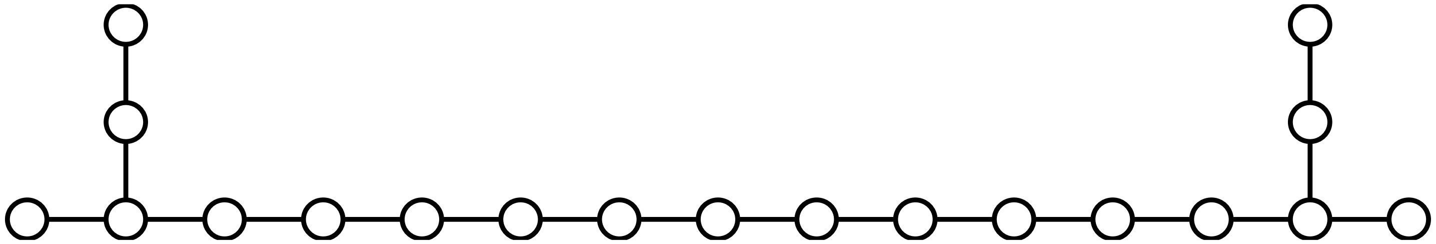
Represent walls as nodes in a **Coxeter diagram**



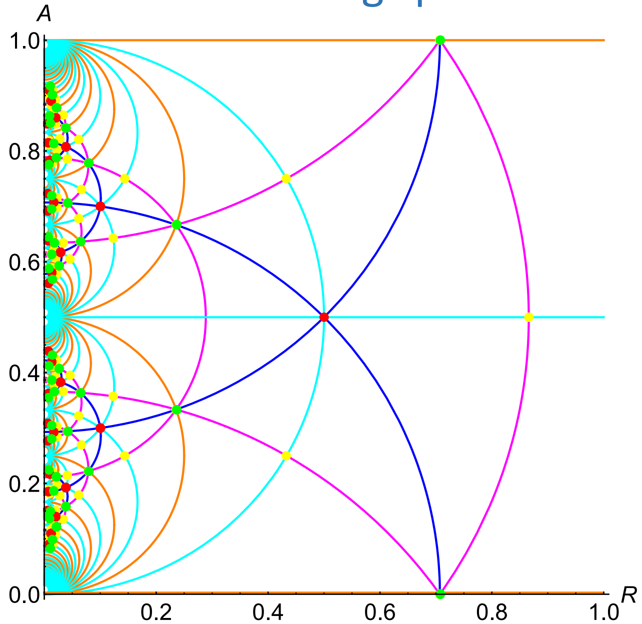


Moduli spaces: Diagrams, Symmetries, Enhancements

Vinberg (1972) determined the Coxeter diagrams for $II_{1,17}$ and $I_{1,17}$
Application to heterotic strings done in [Cachazo, Vafa '00], [Fraiman, Graña, HPF, Sethi '23]



Slice of covering space in 9D



[Fraiman, Graña, Nuñez '18]

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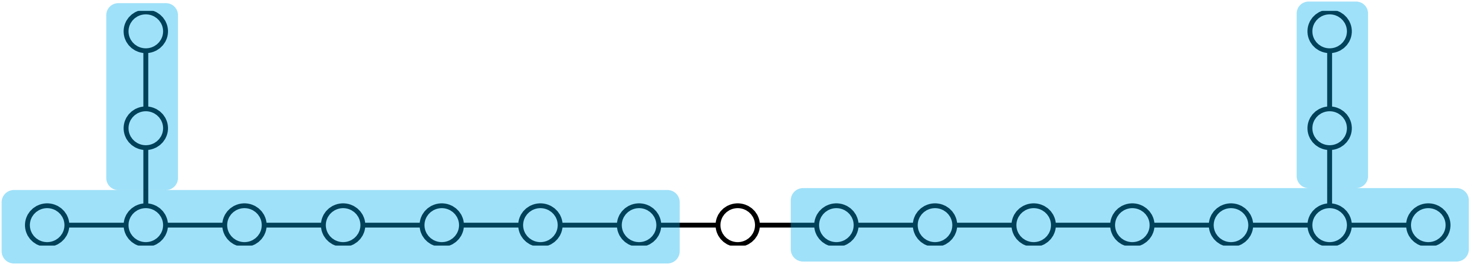

$E_8 \times E_8$ string limit

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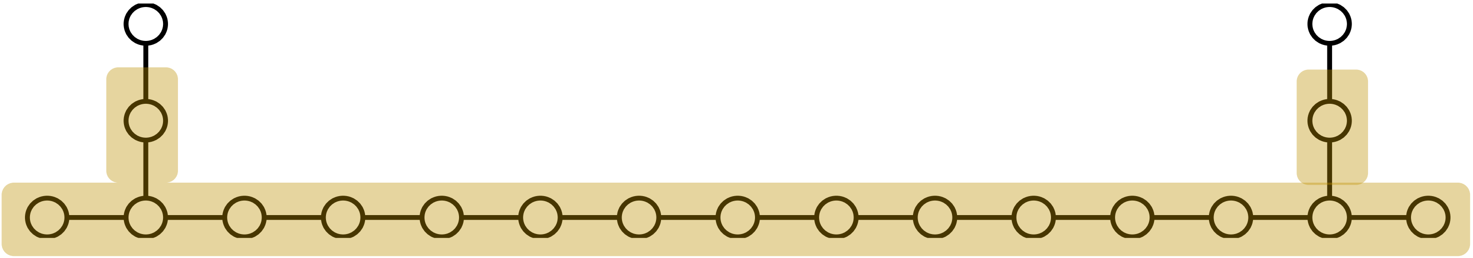

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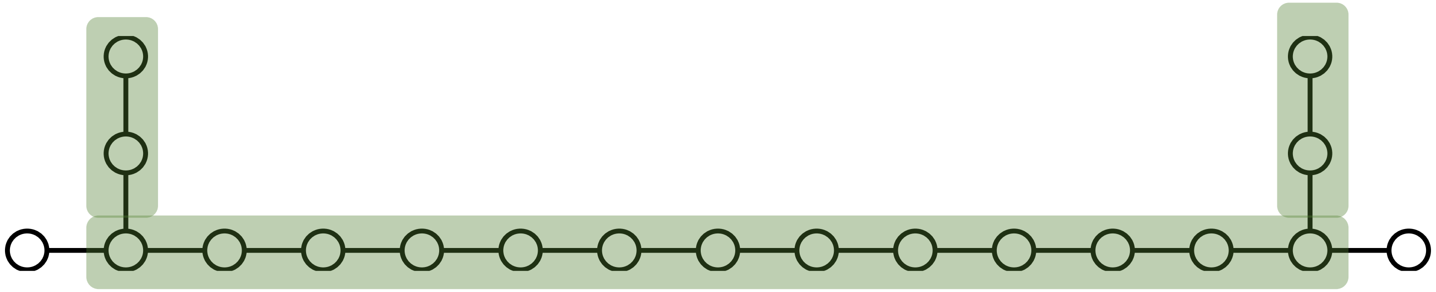

$SU(18)/Z_3$ enhancement

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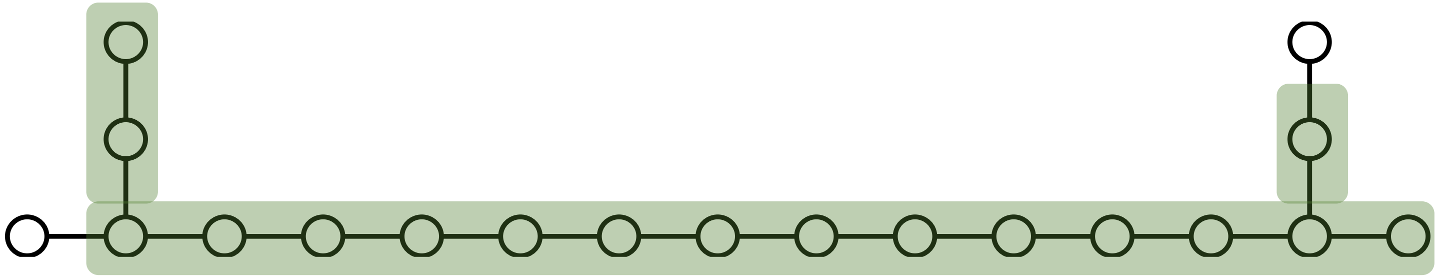

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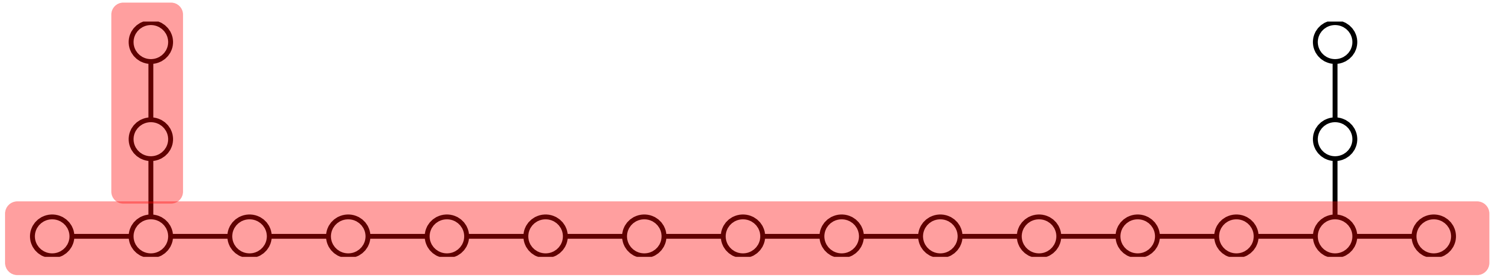

Not a vertex!

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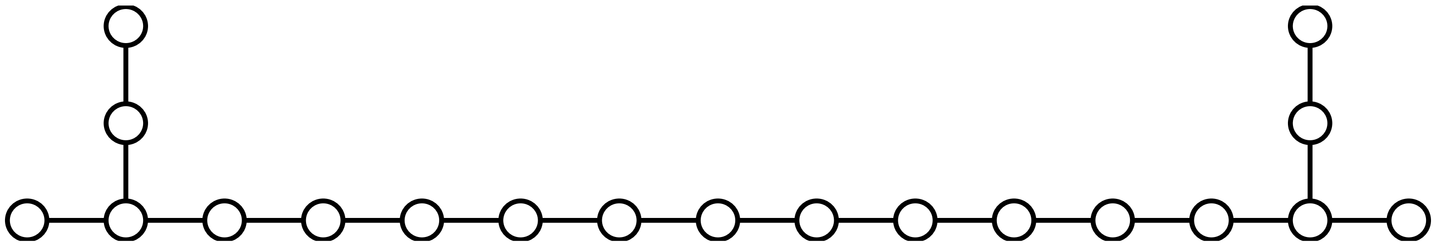
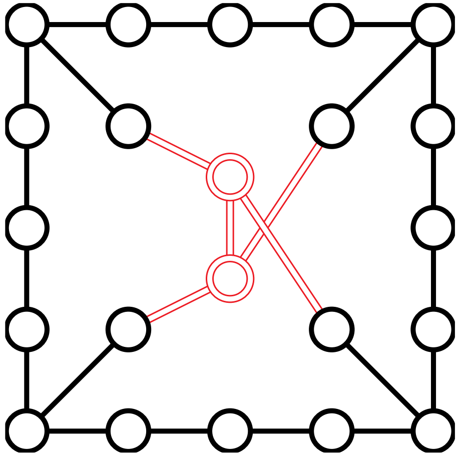


For $I_{1,17}$ there are special walls where two tachyons saturate lower bound for m^2.



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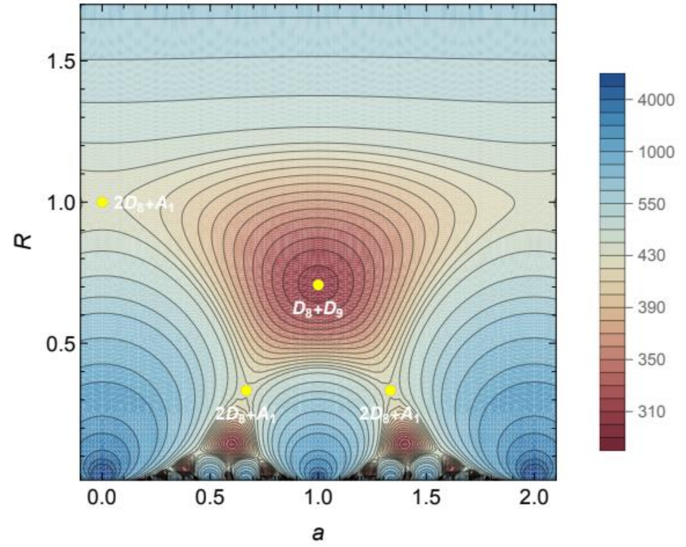

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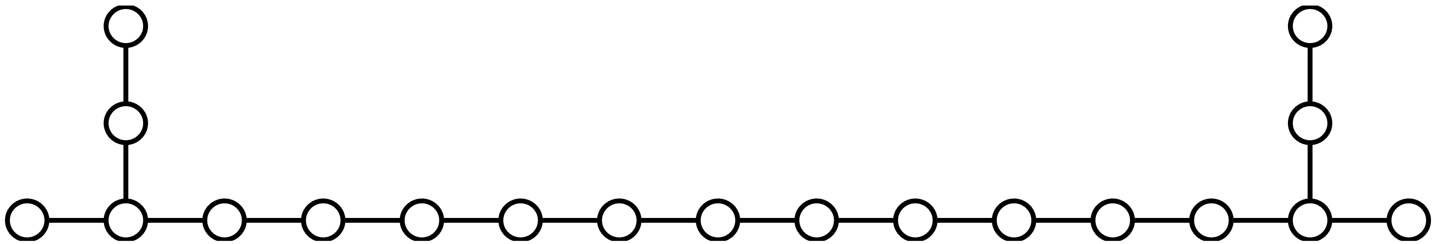

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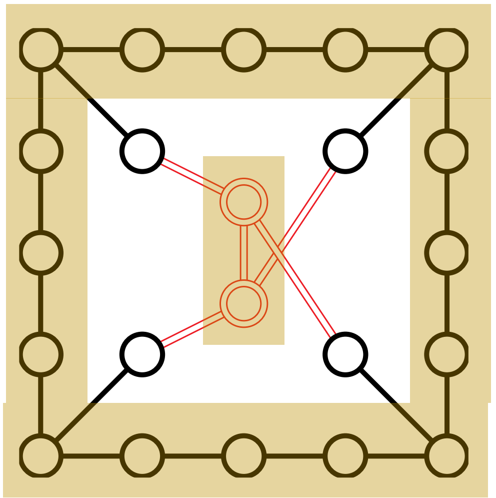

$SU(16)/Z_2 \times U(1)$ string limit

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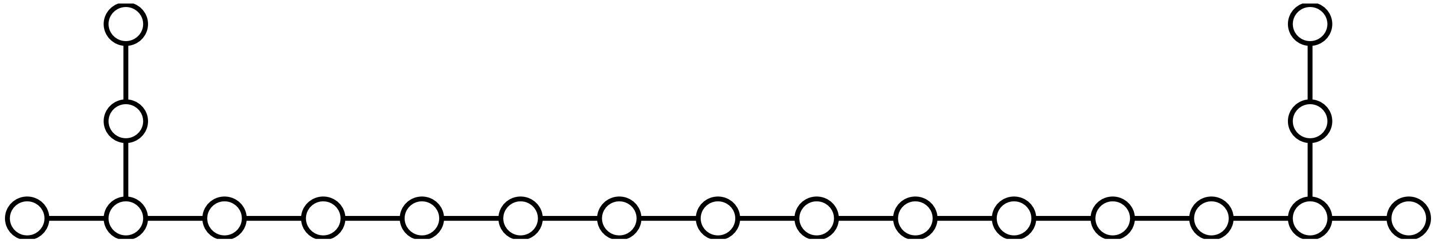

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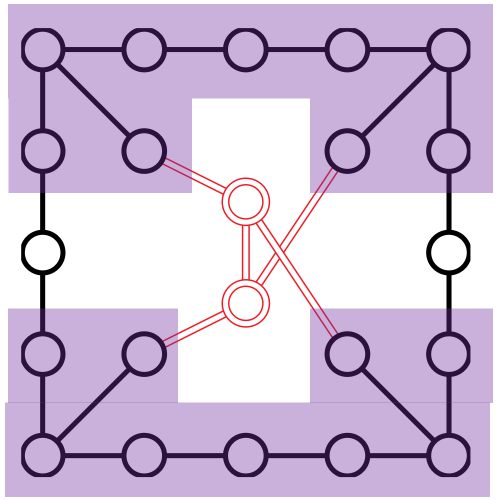

$O(16) \times O(16)$ string limit

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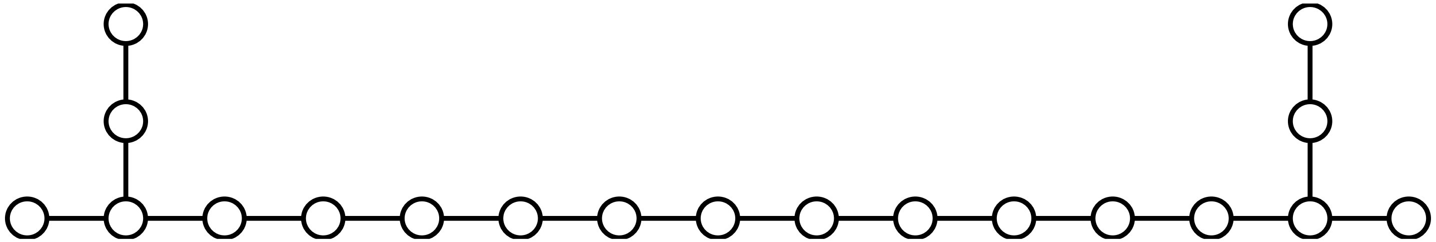

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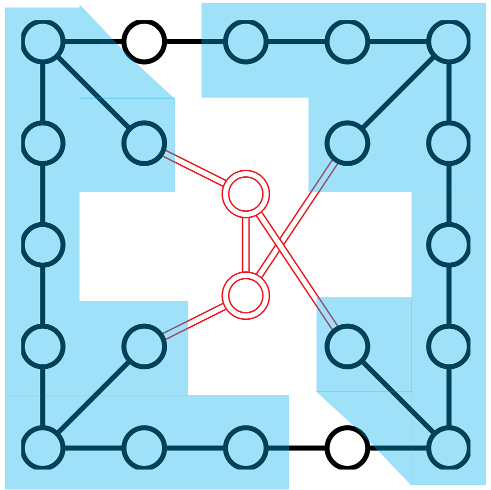

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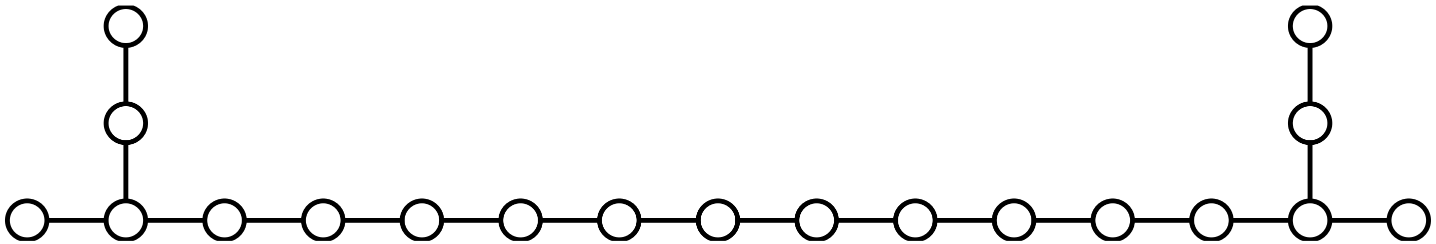
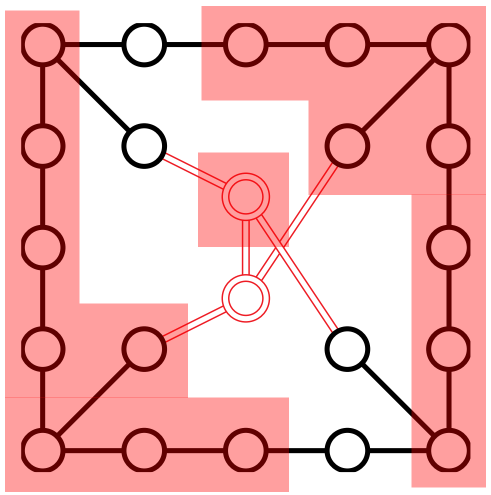


Scherk-Schwarz reductions also live in this moduli space.



$E_8 \times E_8$ string on Scherck-Schwarz circle at self-dual radius

Moduli spaces: Noncritical strings

Just as critical non-supersymmetric heterotic strings, non-critical ones are related to odd self-dual lattices. We can show that for even dimension D,

$$\Gamma = O\left(I_{1,1+r}\right) = O(1,1+r;Z)\,, \qquad r = 11 + D/2$$

(odd self-dual lattice)

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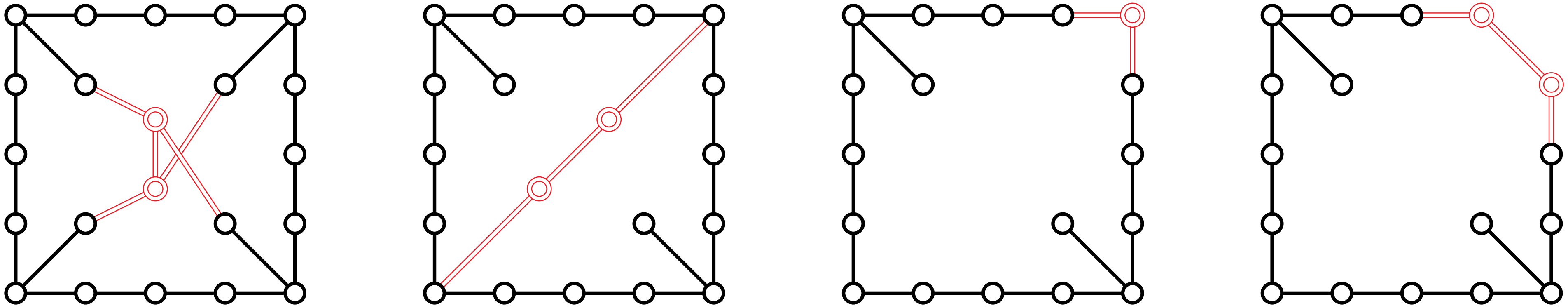

D = 10 D = 8 D = 6 D = 4

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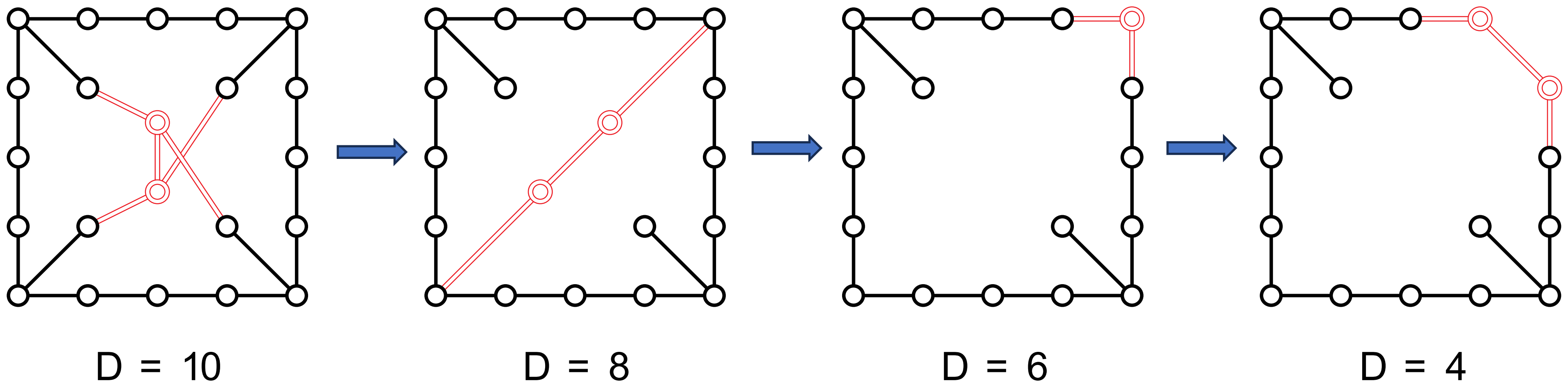

Changing one diagram to another corresponds to condensing two tachyons.

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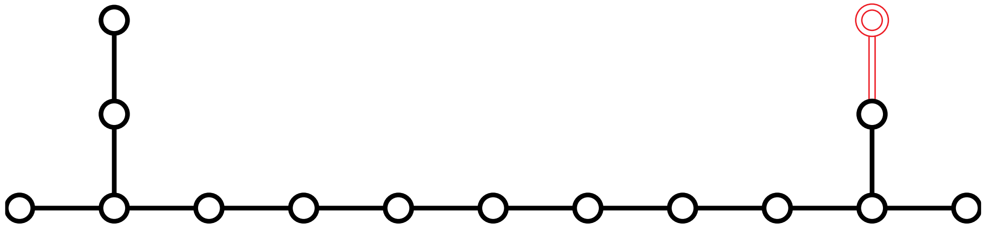


D = 2

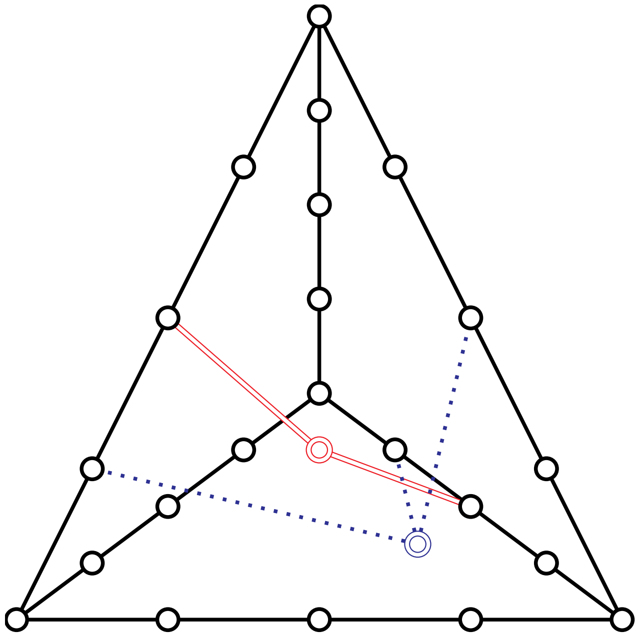
D = 2 theory cannot be compactified on spacelike circle due to linear dilaton.

Still, our results apply as well to thermal circles.

This case was anticipated in [Davis, Larsen, Seiberg '05] using covariant lattice approach.

Moduli spaces: Noncritical strings

Kaplinskaya and Vinberg (1978) worked out the diagrams relating to $D = 12,14$ supercritical heterotic strings:



D = 12

There are two kinds of “tachyonic” nodes. Condensation of the respective tachyons gives either a supersymmetric or a non-supersymmetric critical theory. Note also the outer automorphisms.

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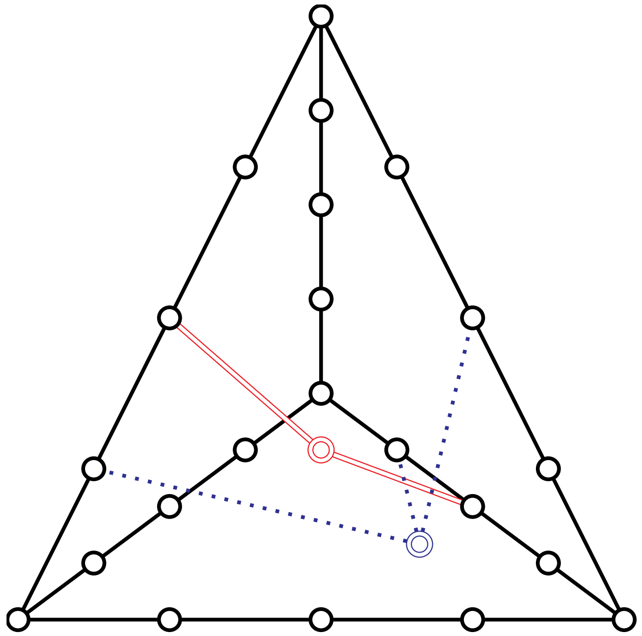

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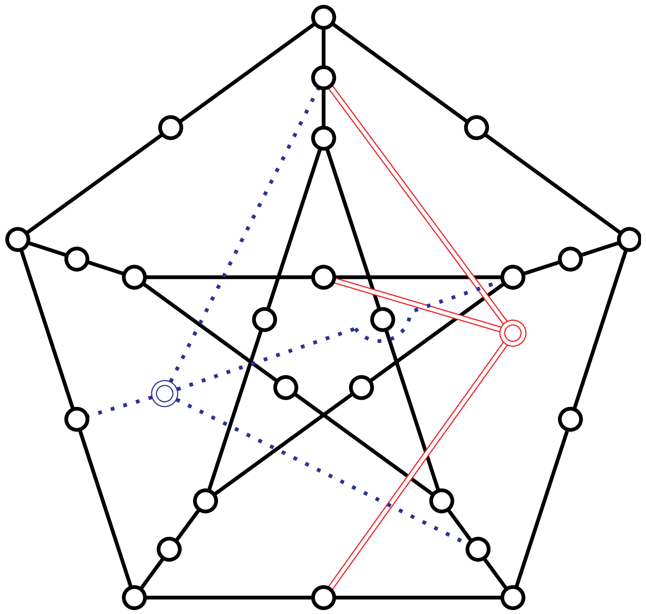

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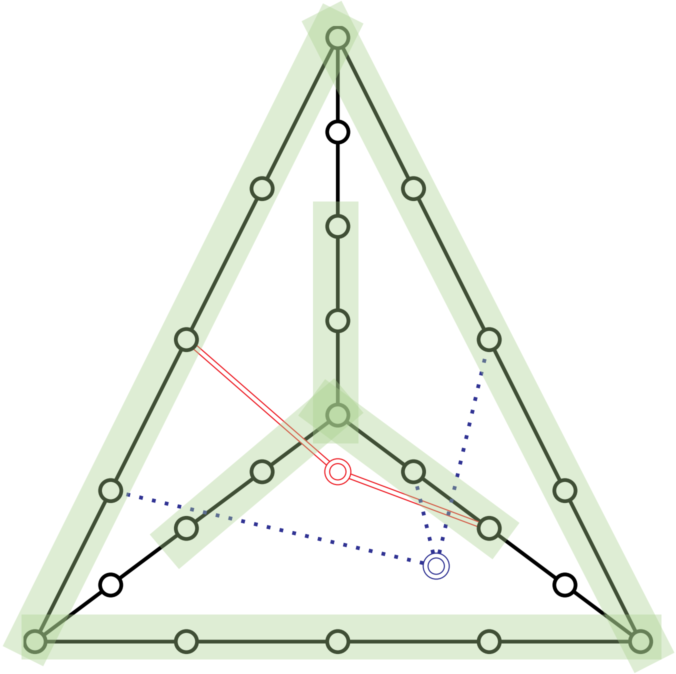

D = 12

$E_6 \times SU(12)/Z_3$ string limit

Unique tachyon-free $D = 12$ heterotic string does not condense to $D = 10$ theory, but 1-loop potential diverges due to timelike linear dilaton

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Tachyon Condensation and Brane T-duality

- Any heterotic string on a Scherck-Schwarz circle with self-dual radius has a worldsheet CFT factor consisting of free Majorana-Weyl fermions

$2 \times \lambda_L$

Tachyons

$3 \times \psi_R$

$SU(2)_2$

Tachyon Condensation and Brane T-duality

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$2 \times \lambda_L$ | $3 \times \psi_R$
Tachyons | $SU(2)_2$

- Condensing the tachyons reduces the overall dimension by 3, introduces spacelike linear dilaton and preserves the $SU(2)_2$.

Tachyon Condensation and Brane T-duality

- Any heterotic string on a Scherck-Schwarz circle with self-dual radius has a worldsheet CFT factor consisting of free Majorana-Weyl fermions

$2 \times \lambda_L$

Tachyons

$3 \times \psi_R$

$SU(2)_2$

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- The result is the linear dilaton background describing two coincident NS5 branes in the original theory.

For 10D heterotic strings: $R^{5,1} \times R_\phi \times SU(2)_2 \times G$

For SUSY strings, 2NS5 is also SUSY

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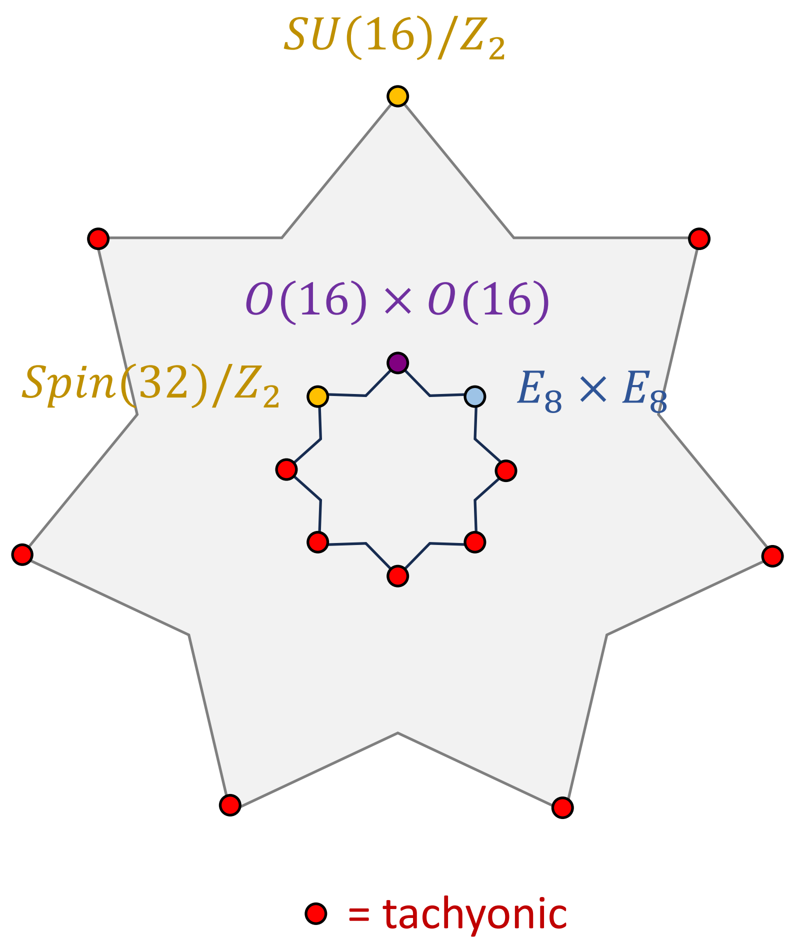

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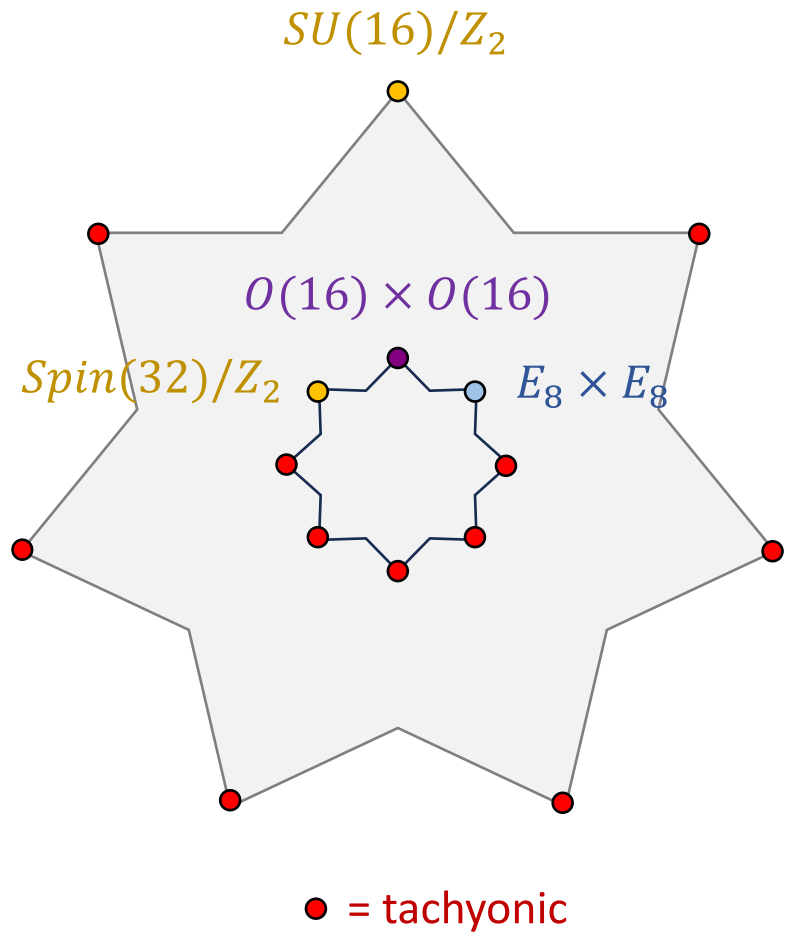

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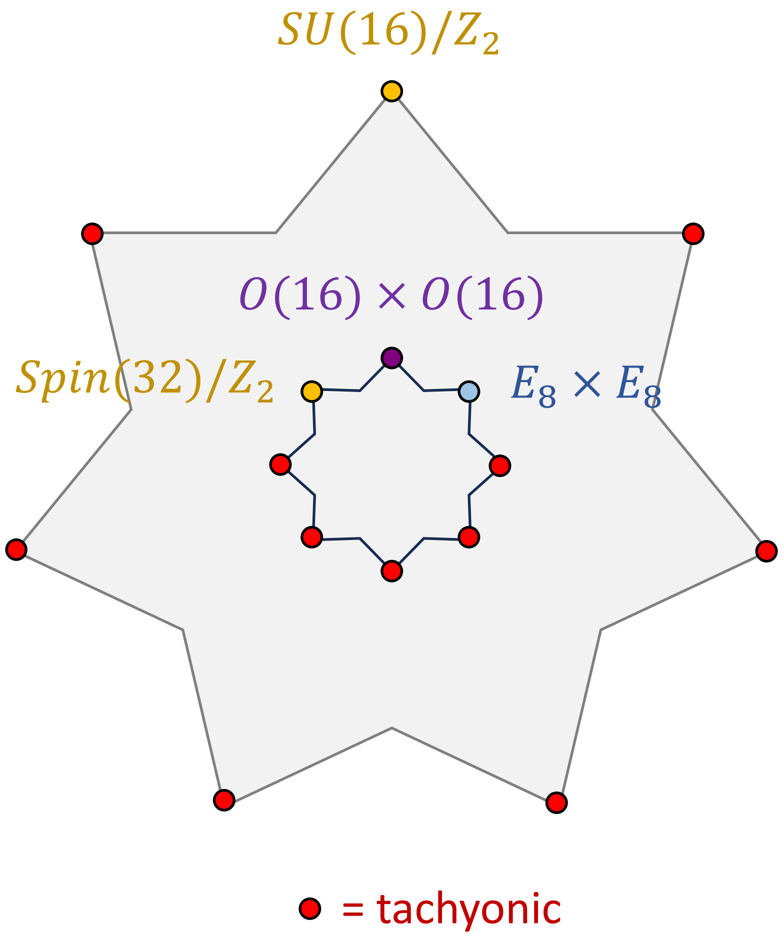
- Nonsupersymmetric 6-brane on S^1 might be T-dual to 2NS5. Topology changing T-duality at the level of transverse spheres and fluxes verifies this.



Tachyon Condensation and Brane T-duality

- Two Scherck-Schwarz circles with self-dual radii similarly give rise to a worldsheet CFT factor consisting of free Majorana-Weyl fermions

$4 \times \lambda_L$
Tachyons

$6 \times \psi_R$
$SU(2)_2 \times SU(2)_2$

- Condensing the tachyons result in the linear dilaton background describing an intersection of two pairs of two coincident NS5 branes in the original theory.

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For 10D heterotic strings: $R^{2,1} \times R_\phi \times SU(2)_2 \times SU(2)_2 \times G$

- These points lie in moduli space of 6D subcritical heterotic on T^2, equivalently torus compactification of $E_7 \times E_7/Z_2$ string with suitable radius and Wilson lines.
  → Suggests duality with non-supersymmetric 4-brane.

In conclusion…

- Symmetry enhancement data for noncritical strings on a circle are encoded in **Coxeter diagrams**. Relations among the diagrams reflect effect of tachyon condensation.

- Subcritical case corresponds to compactification of a non-supersymmetric brane background. In a special case there are 2NS5 points.

- *Scherck-Schwarz tachyons may condense to 2NS5*.

- Supercritical strings get progressively more complex, exhibiting interesting symmetries captured by the diagrams.

- Results suggest that 2NS5 is dual to nonsusy 6-brane on a circle. Similarly 2NS5 x 2NS5 may be dual to nonsusy 4-brane on a torus.

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For the future:

- Can these nonsupersymmetric linear dilaton backgrounds be regularized?
- Is the 1-loop potential a physically meaningful quantity?

Thanks for your attention!