On the origin of species thermodynamics and the black hole-tower correspondence

Alvaro Herraez

Based on [arXiv:2406.xxxxx] with D. Lüst, J. Masías, M.Scalisi

MAX-PLANCK-INSTITUT FÜR PHYSIK

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On the origin of species thermodynamics and the black hole-tower correspondence

1. Species Scale
2. Covariant Entropy Bound and Gravitational Collapse
3. Field Theory Entropy vs. Species Entropy
4. Black Hole-Tower Correspondence

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The species Scale

- Maximum UV cut-off in QG in the presence of N light species [Dvali '07] [Dvali, Reedi '08] [Dvali, Lüst '10] [Dvali, Gómez '10]

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- Tower of states with $m_n = n^{1/p}\, m_t$

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$$\Lambda_{\rm sp} \simeq m_t \left(\frac{m_t}{M_{{\rm Pl},d}}\right)^{\dfrac{d-2}{d+p-2}} \qquad N_{\rm sp} \simeq \left(\frac{M_{{\rm Pl},d}}{m_t}\right)^{\dfrac{p(d-2)}{d+p-2}}$$

[Castellano, AH, Ibáñez ´21]

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Species Scale of the d-dim EFT = $M_{{\rm Pl},\, d+p}$ (decompactification of p dimensions)
$M_{\rm str}$ (weakly coupled string limits)

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[Castellano, AH, Ibáñez ´21]

- Radius of the smallest BH *in the EFT*

$$S = \frac{A}{4G_{N,d}} \sim \left(\frac{M_{{\rm Pl},d}}{\Lambda_{\rm sp}}\right)^{d-2} \simeq N_{\rm sp}$$

Gravitational Collapse and the Covariant Entropy Bound

- Configuration of energy E in a box of size L can collapse gravitationally unless

$$L \geq R_{\mathrm{BH}}(E) = \left(\frac{E}{M_{\mathrm{Pl},d}}\right)^{\frac{1}{d-3}} M_{\mathrm{Pl},d}^{-1}$$

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- Covariant Entropy Bound

[Bousso´99]

$$S \leq \frac{A}{4G_{N,d}} \sim (L M_{\mathrm{Pl},d})^{d-2}$$

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- Covariant Entropy Bound

[Bousso´99]

$$S \leq \frac{A}{4G_{N,d}} \sim (LM_{\mathrm{Pl},d})^{d-2} \xrightarrow{S \sim \frac{E}{T}} \quad E \lesssim TL^{d-2} \qquad S \lesssim L^{d-2}$$

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Can we approach the maximum entropy for some non-black hole configuration?

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- Coincide for $T \simeq 1/L$

[Castellano, AH, Ibáñez ´21] [AH, Lüst, Masías, Scalisi ´24]

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- Coincide for $T \simeq 1/L$ $\longrightarrow$ $E_{\rm max}(T) \simeq \dfrac{1}{T^{d-3}} \geq \dfrac{1}{\Lambda_{\rm sp}^{d-3}} \simeq \dfrac{N_{\rm sp}}{\Lambda_{\rm sp}}$

[Castellano, AH, Ibáñez ´21] [AH, Lüst, Masías, Scalisi ´24]

All one-particle states in the box with non-zero momentum have a large Boltzmann suppression $\sim e^{-E_m/T}$

$E_m = \sqrt{m_m + p_m}$ $\qquad p = n^2/L^2$

$$S_{\rm max}(T) \simeq \frac{1}{T^{d-2}} \geq \frac{1}{\Lambda_{\rm sp}^{d-2}} \simeq N_{\rm sp}$$

Field Theory Entropy

[AH, Lüst, Masias, Scalisi '24]

- Configuration of particles in a box of size L with a spectrum of species $m_n = n^{1/p} m_t$ all at a common T (neglect energy in the interactions $M_{\text{Pl},d} \gg \Lambda_{\text{sp}} \geq T$)

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- Partition function $$Z_{\mathrm{TOT}} = \frac{\prod_{n=1}^{N_T} \left(Z_{1,n}\right)^{N_n}}{\prod_{n=1}^{N_T} N_n!}$$

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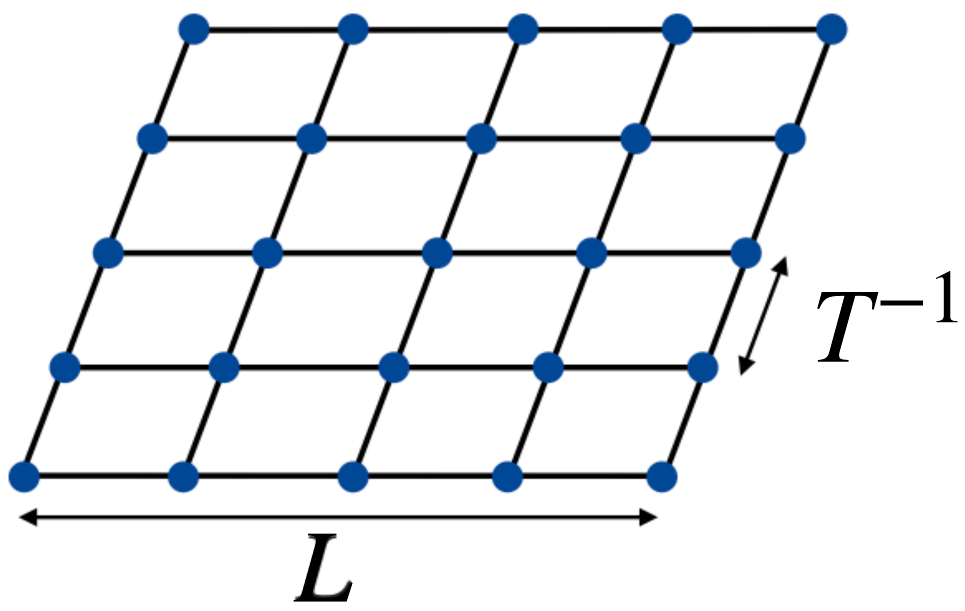
- Phase space (momentum states per *active* particle) $\sim (TL)^{d-1}$



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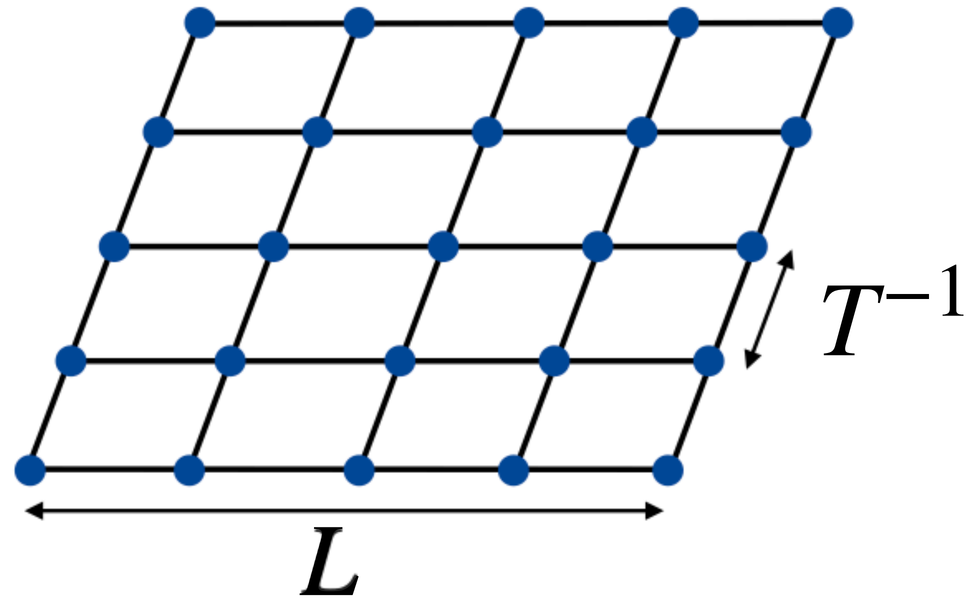

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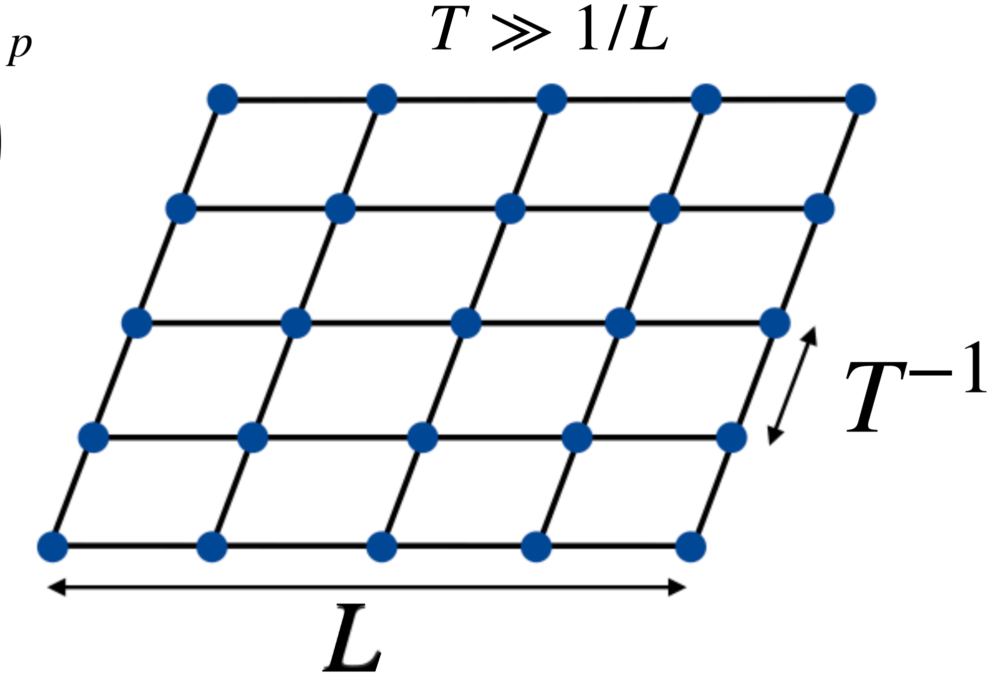

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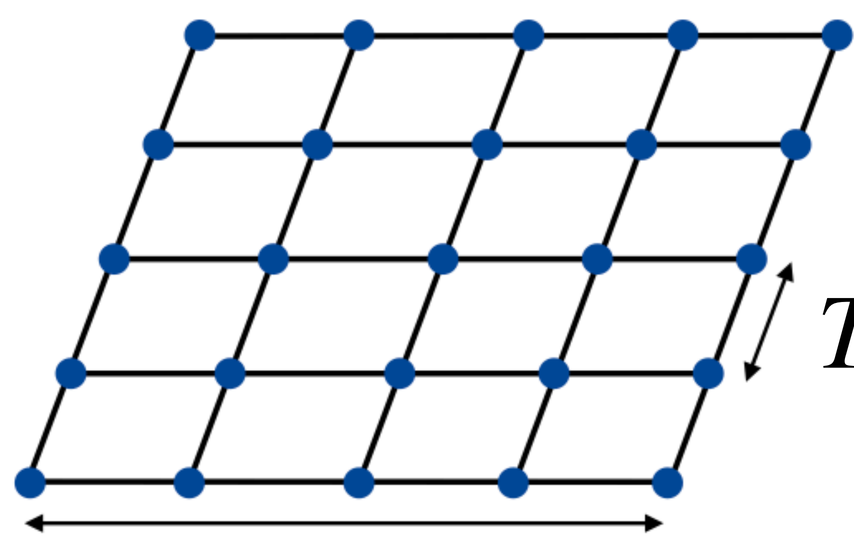

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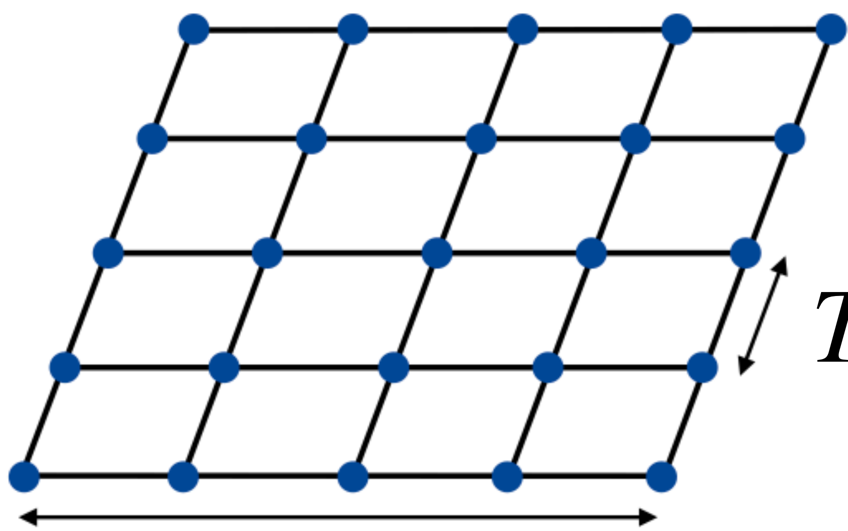

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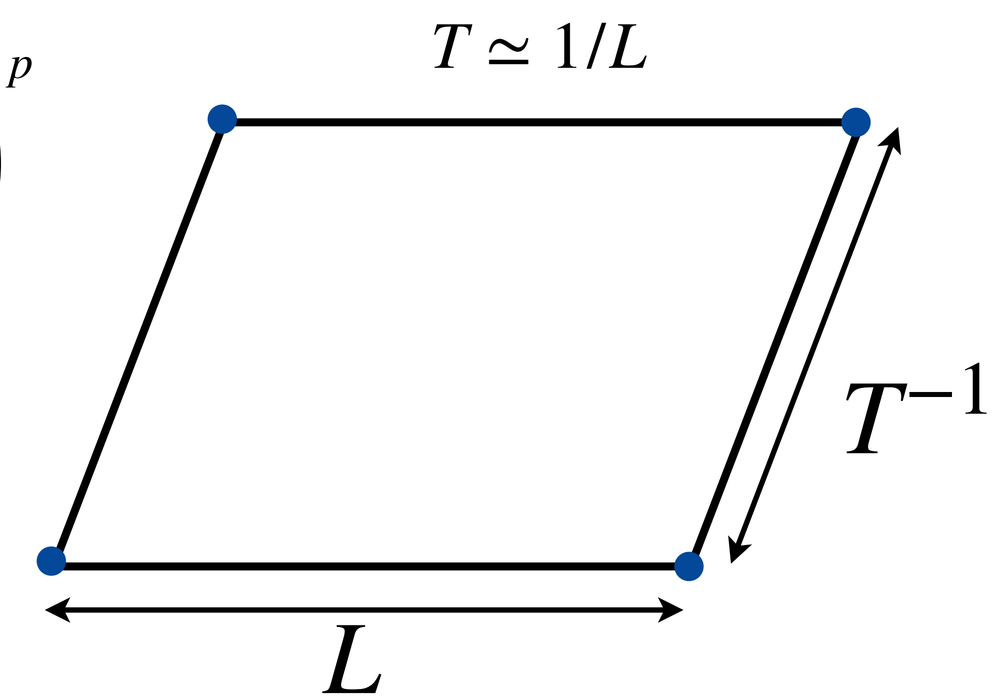

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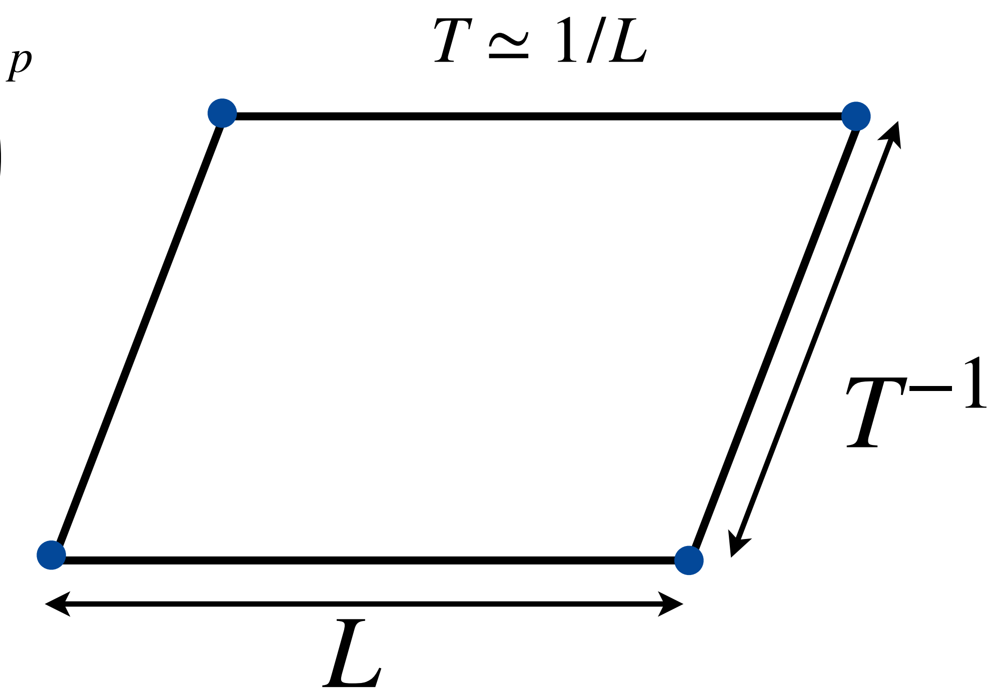

Species Entropy from EFT thermodynamics in the limit

- CEB and Gravitational Collapse Bound coincide for $T \simeq 1/L$
  [AH, Lüst, Masías, Scalisi ´24]

$$E(T) \simeq TN_T \leq \frac{1}{T^{d-3}} \qquad S(T) \simeq N_T \lesssim \frac{1}{T^{d-2}} \qquad S \sim \frac{E}{T}$$

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- Saturated when $T \simeq \frac{M_{\mathrm{Pl},d}}{N_T^{\frac{1}{d-2}}} \simeq N_T^{\frac{1}{p}} m_t \longrightarrow T \simeq \Lambda_{\mathrm{sp}} \qquad N_T \simeq N_{\mathrm{sp}}$

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$$E \simeq \Lambda_{\mathrm{sp}} N_{\mathrm{sp}} = \sum_i m_i \qquad S \simeq N_{\mathrm{sp}}$$

Species Thermodynamics

[Cribiori, Lüst, Montella, ´23]

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Black Hole-String Correspondence

[Susskind ´93]
[Horowitz, Polchinski '96 '97]

[Chen, Maldacena, Witten '21]
[Susskind '21]
[Ceplack, Emparan, Puhm, Tomasevic '22]
[Bedroya, Vafa, Wu '23]

Black Hole-String Correspondence

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Black Hole

$$\ell_{\mathrm{Pl,d}}^{d-2} = g_{s,d}^2 \ell_{\mathrm{str}}^{d-2}$$

$$M_{\mathrm{BH}} \sim \frac{R_{\mathrm{BH}}^{d-3}}{\ell_{\mathrm{Pl,d}}^{d-2}} \sim \frac{R_{\mathrm{BH}}^{d-3}}{g_{s,d}^2 \ell_{\mathrm{str}}^{d-2}}$$

$$S_{\mathrm{BH}} \sim \left(R_{\mathrm{BH}}/\ell_{\mathrm{Pl,d}}\right)^{d-2} \sim g_{s,d}^{\frac{2}{d-3}} \left(M_{\mathrm{BH}} \ell_{\mathrm{str}}\right)^{\frac{d-2}{d-3}}$$

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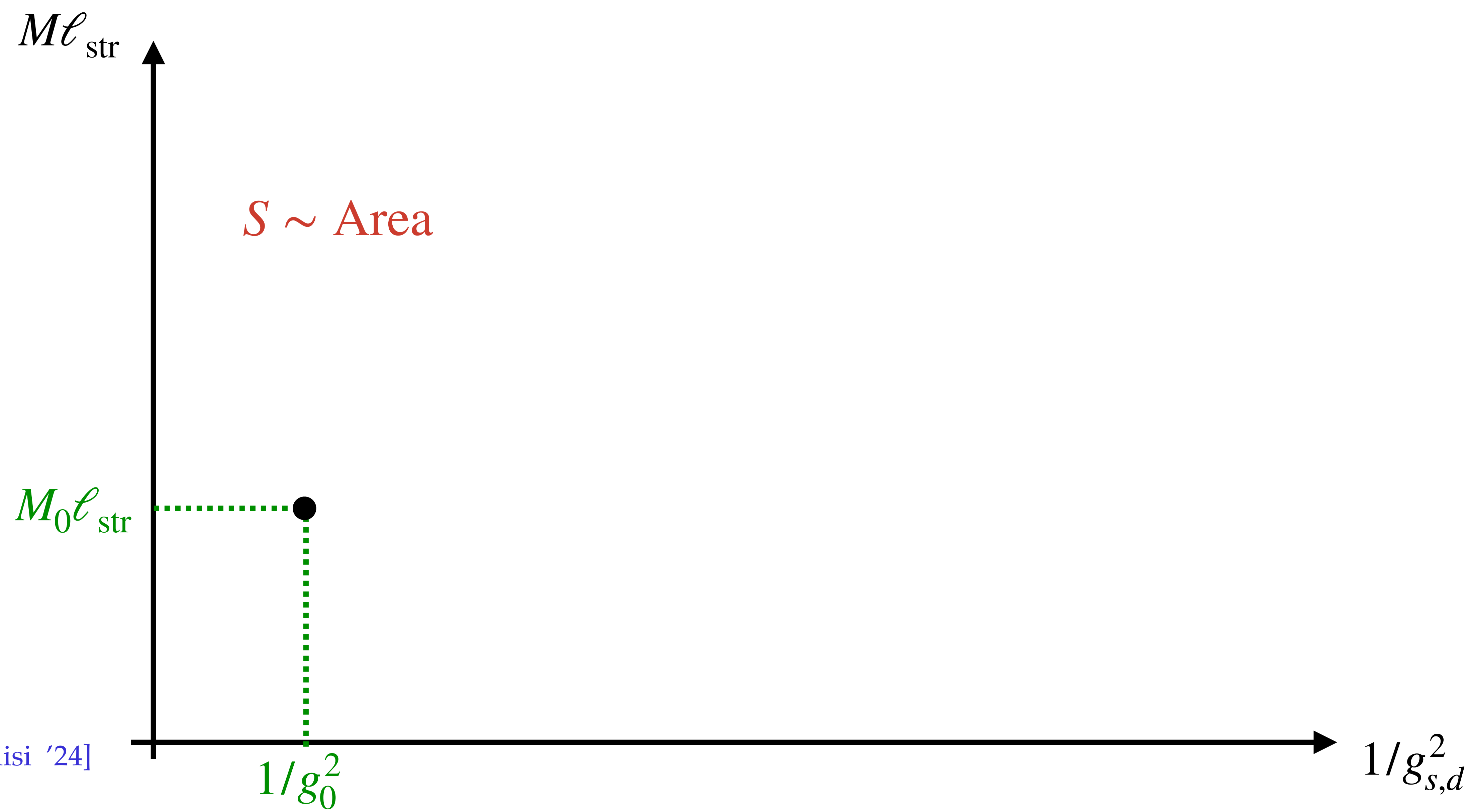

[AH, Lüst, Masias, Scalisi '24]

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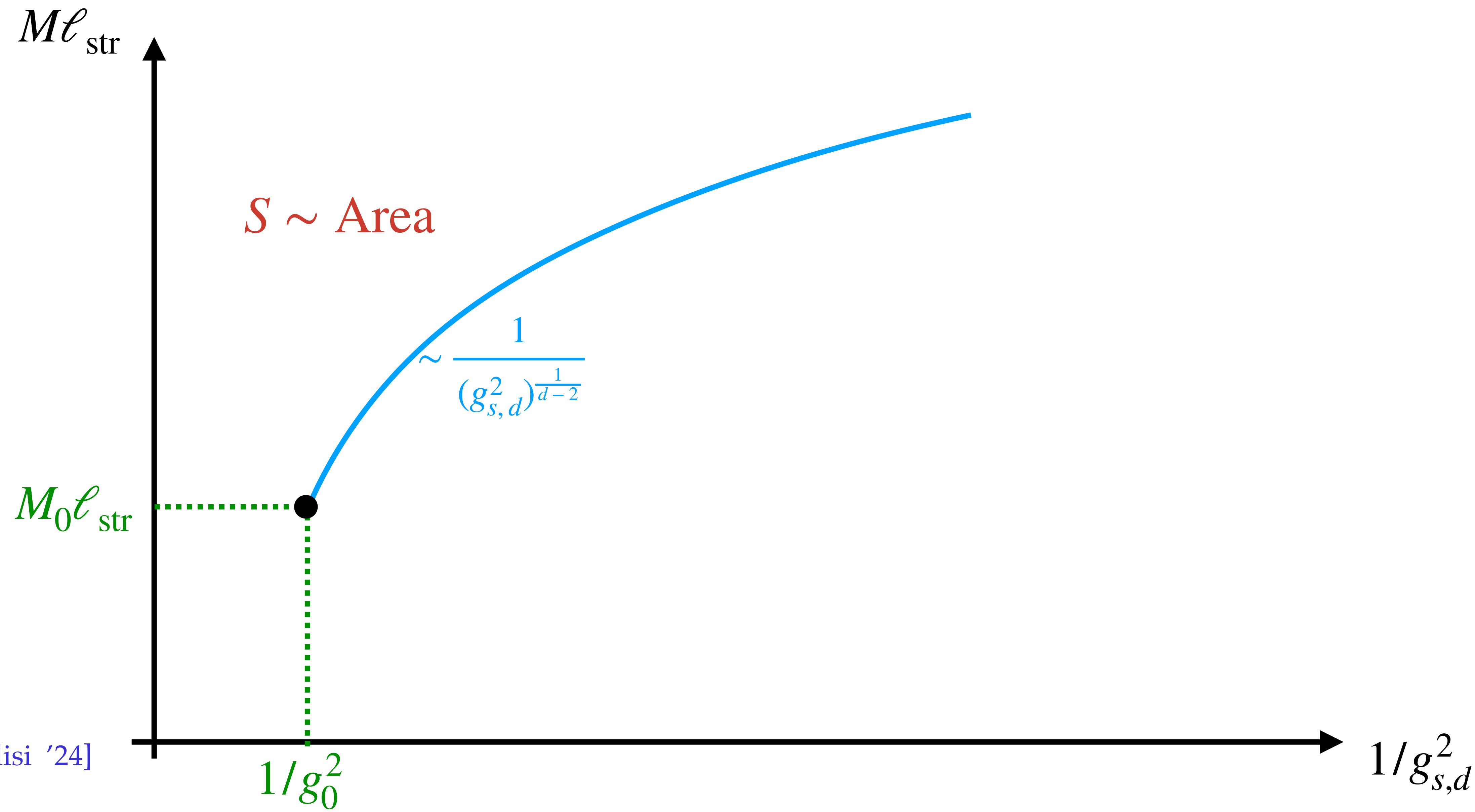

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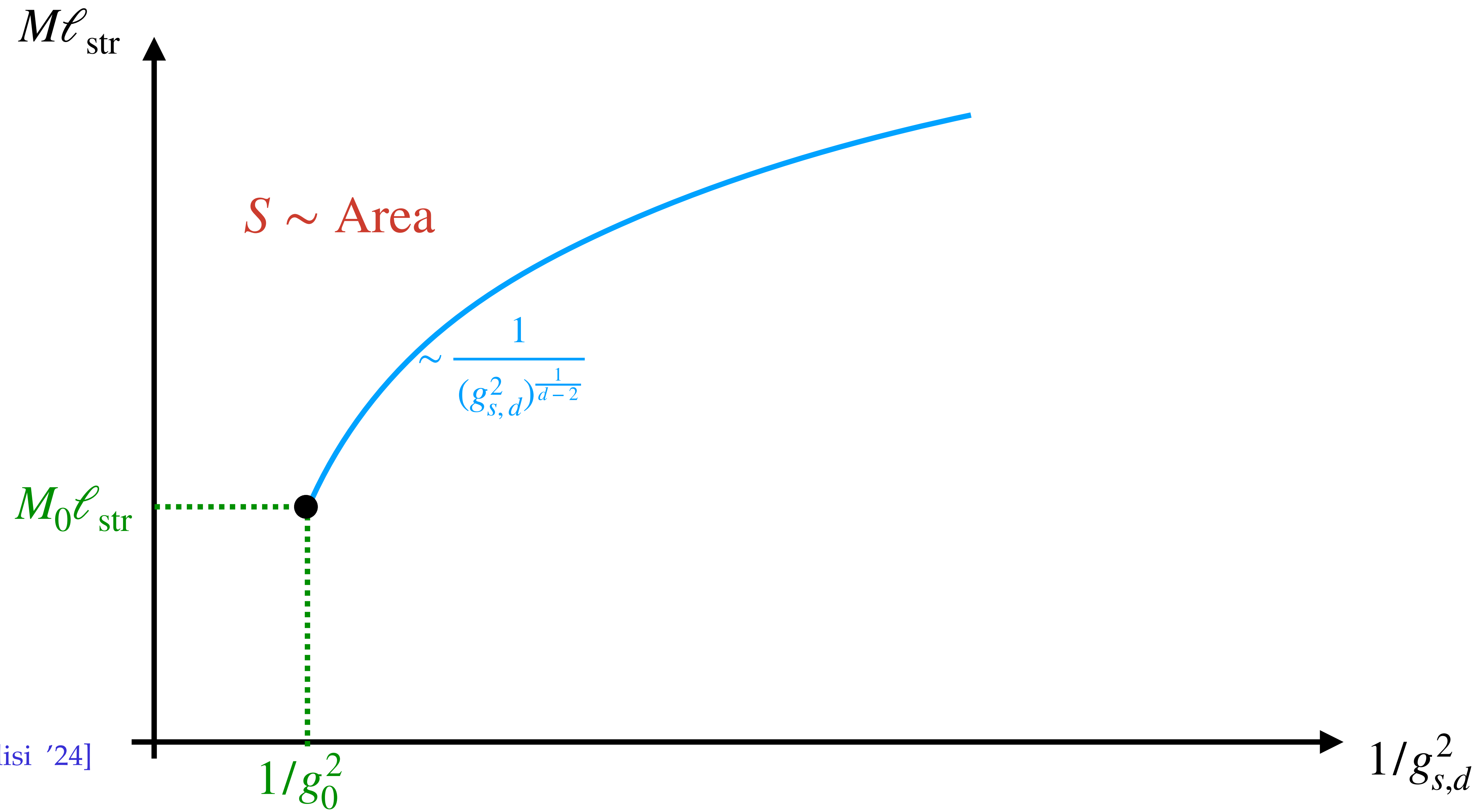
(Free) String

$$M_{\text{str}} \sim \frac{L_{\text{str}}}{\ell_{\text{str}}^2}$$

$$S_{\text{str}} \sim L_{\text{str}}/\ell_{\text{str}} \sim M_{\text{str}} \ell_{\text{str}}$$



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[Horowitz, Polchinski ’96 ’97]

$$\ell_{\rm Pl,d}^{d-2} = g_{s,d}^2 \ell_{\rm str}^{d-2}$$

Black Hole

$$M_{\rm BH} \sim \frac{R_{\rm BH}^{d-3}}{\ell_{\rm Pl,d}^{d-2}} \sim \frac{R_{\rm BH}^{d-3}}{g_{s,d}^2 \ell_{\rm str}^{d-2}}$$

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(Free) String

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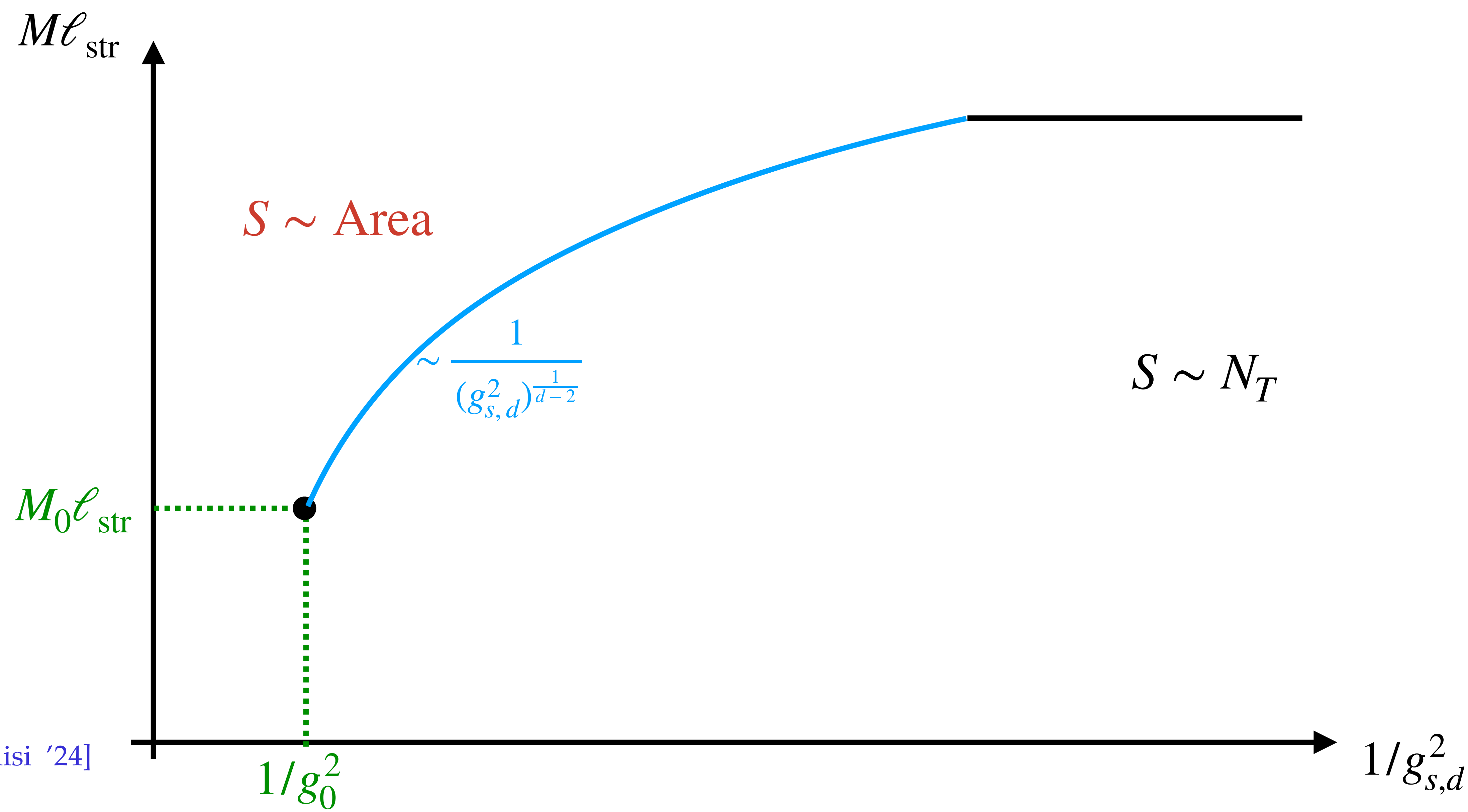

[AH, Lüst, Masias, Scalisi ’24]

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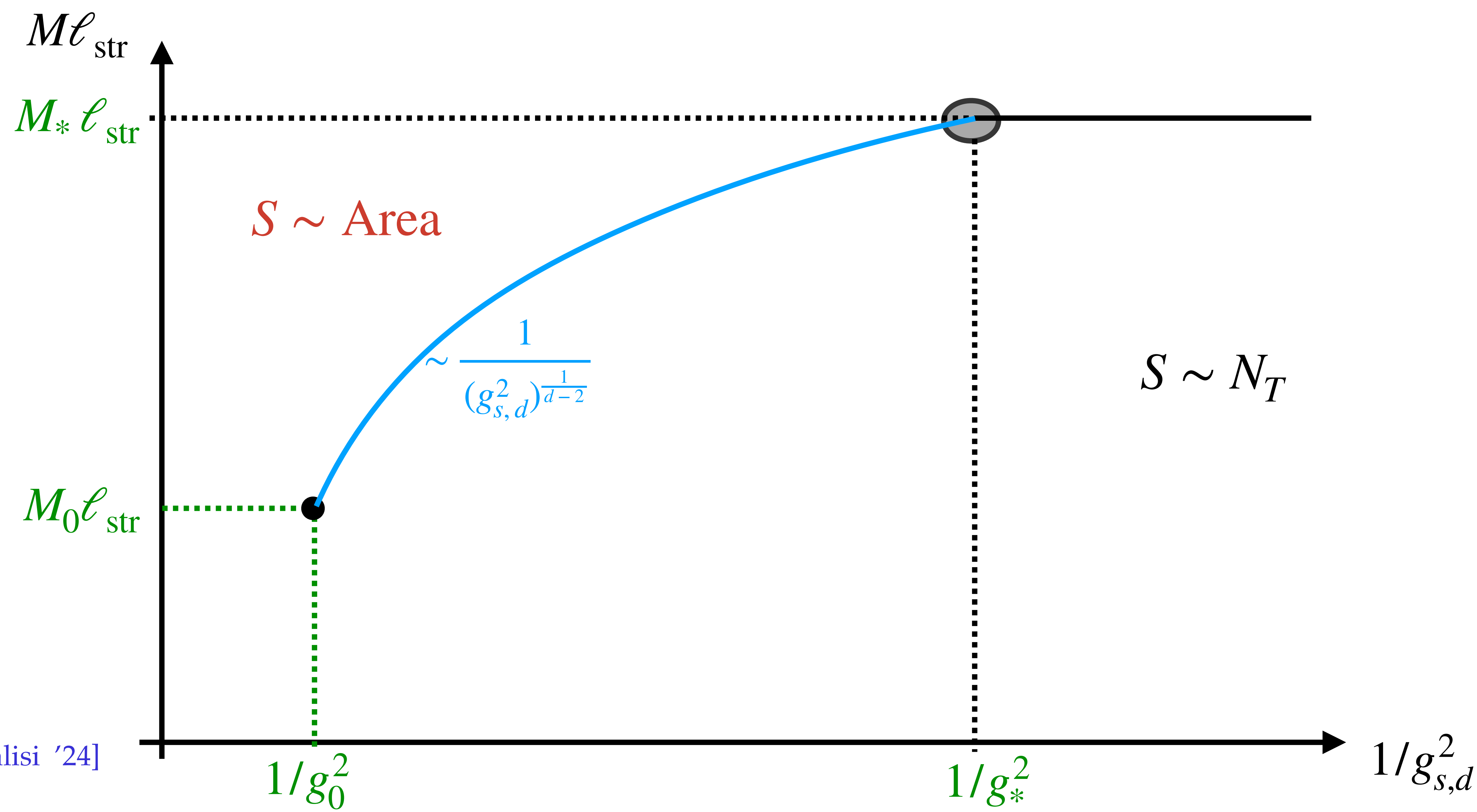

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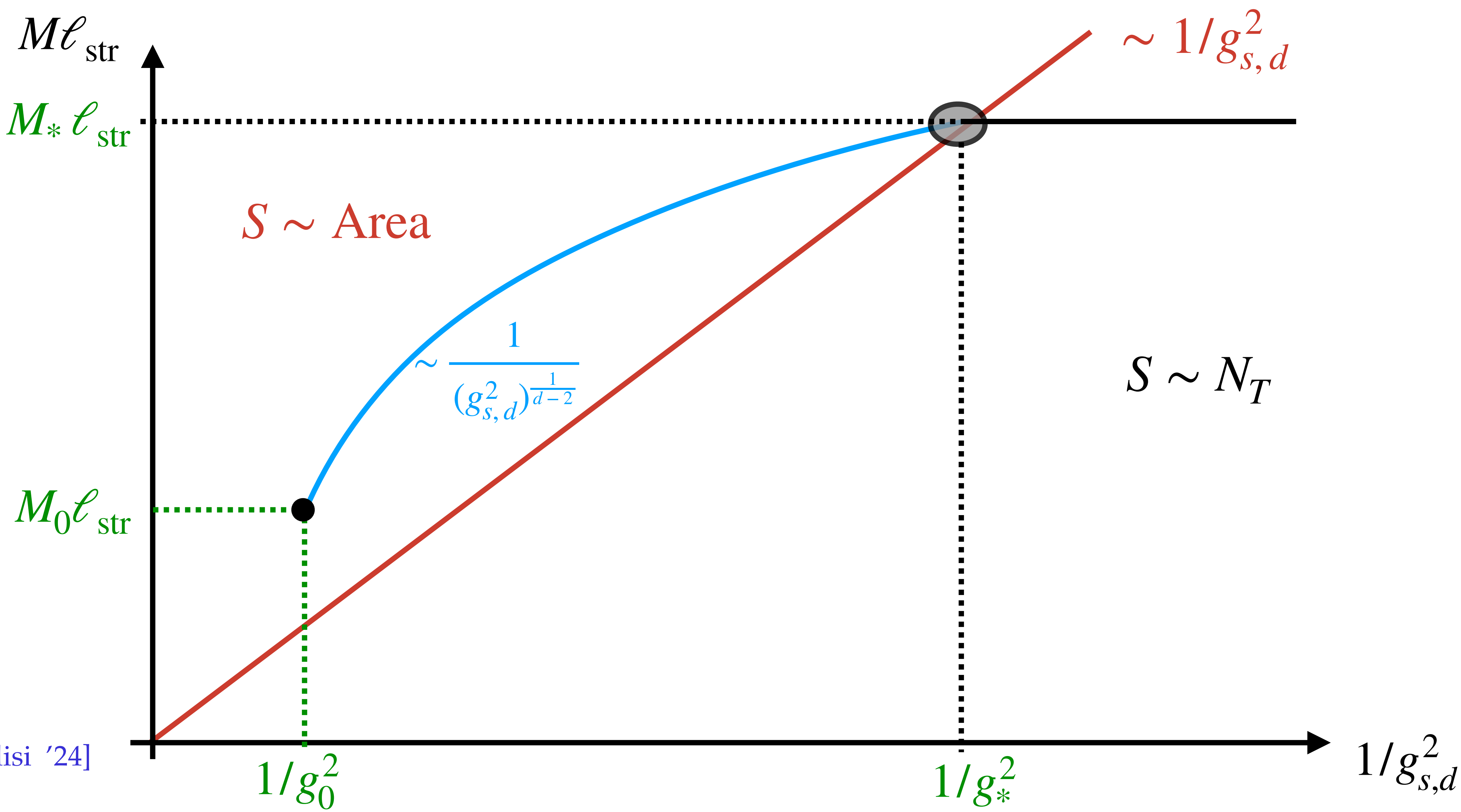

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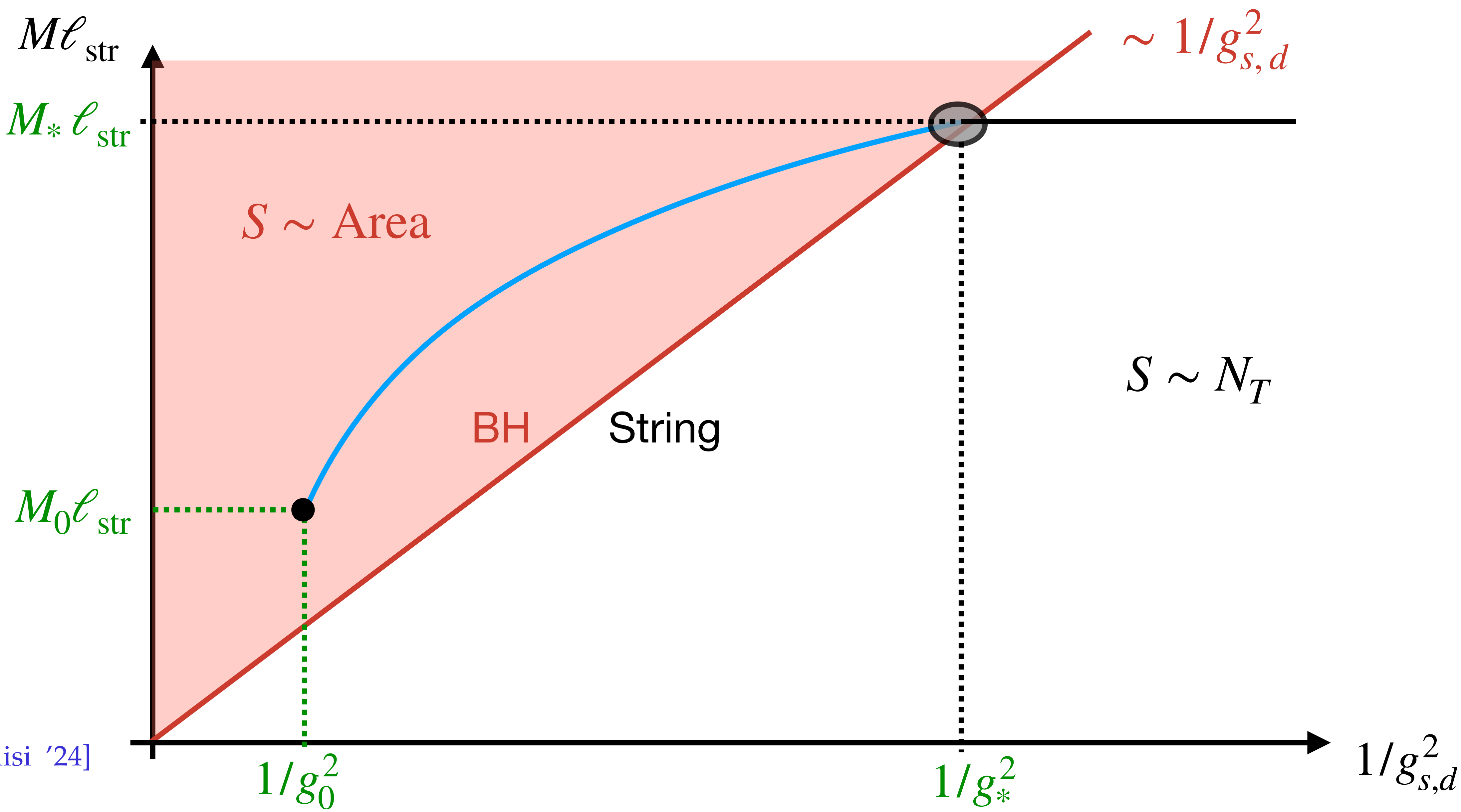

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Black Hole-Species Tower Correspondence

$$\ell_{\rm Pl,d}^{d-2} = \frac{\ell_{\rm sp}^{d-2}}{\mathcal{V}}$$

$\ell_{\rm str} \to \ell_{sp} = \Lambda_{\rm sp}^{-1}$

$g_s^{-2} \to \mathcal{V}$

Black Hole

$$M_{\rm BH} \sim \frac{R_{\rm BH}^{d-3}}{\ell_{\rm Pl,d}^{d-2}} \sim \frac{R_{\rm BH}^{d-3}\,\mathcal{V}}{\ell_{\rm sp}^{d-2}}$$

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Box of species

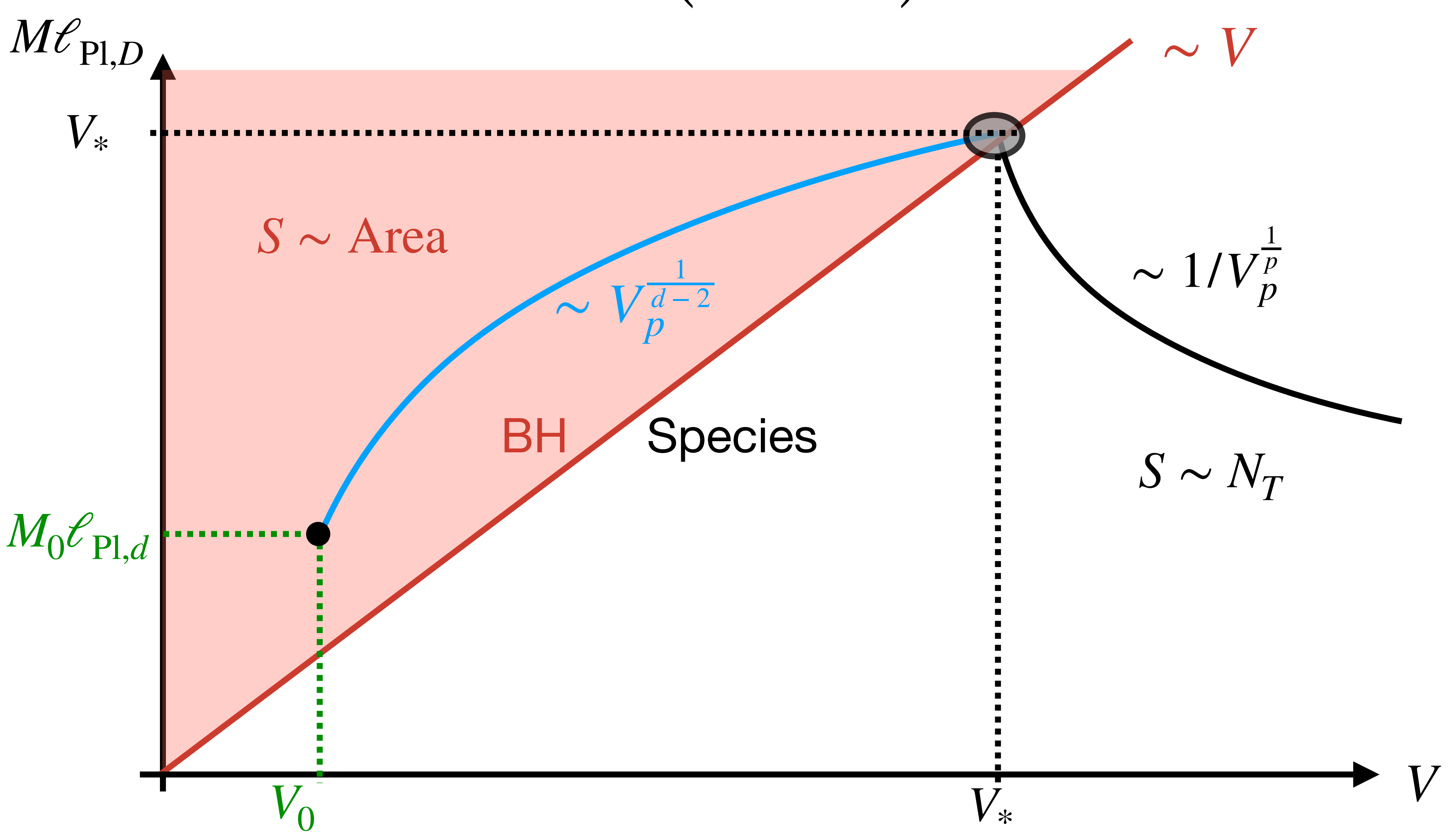
$$L^{-1} = T \leq \Lambda_{\rm sp}$$

$$S \sim N_T (TL)^{d-1}$$

Correspondence point (line)

$R_{\rm BH} \sim \ell_{\rm sp}$

$S = \mathcal{V}_* = N_{{\rm sp},*}$



[AH, Lüst, Masias, Scalisi '24]

Towers in the swampland

- What about *other kinds of towers*? ⟶ Emergent String Conjecture
  [Basile, (Cribiori), Montella, Lüst ´23 ´24] [Bedroya, Mishra, Wiesner ’24] [Lee, Lerche, Weigand ´19]

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  - ✳ Superexponential ⟶ Divergent Canonical Partition Function ✗

  - ✳ Subpolynomial: $p \to 0$ ⟶ $\dfrac{S}{N_T} \simeq \dfrac{p+1}{p} \to \infty$ ✗

On the origin of species thermodynamics and the black hole-tower correspondence

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- Study configurations that do not collapse gravitationally and approach maximum possible entropy ($T \simeq 1/L$)

- Recover volume scaling and species thermodynamics scalings in the right limits $S \sim (TL)^{d-1}$ for $T \ll m_t$ and $S \simeq N_T \to N_{\rm sp}$ for $T \simeq 1/L \to \Lambda_{\rm sp}$

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- These towers can give rise to a tower-black hole correspondence and the scaling of the entropy of black holes can be accounted by the entropy of the free system

- Polynomial and exponentially degenerate towers have the right scaling of entropy as $T \to \Lambda_{sp}$ (i.e. recover area law for small black holes) $\longrightarrow$ Emergent String Conjecture

Thank you!