The Minimal Weak Gravity Conjecture

Alessandro **Mininno**
based on arXiv:2312.04619 [hep-th]
with C. F. Cota, T. Weigand and M. Wiesner

II. Institut für Theoretische Physik
Universität Hamburg

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Swamplandia 2024
in
Bavaria

May 28, 2024

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Weak Gravity Conjecture

Weak Gravity Conjecture

Every gauge theory coupled to gravity has a particle with

$$\frac{g^2_{\mathrm{U(1),\,D}}q^2}{m_D^2} \geq \left.\frac{g^2_{\mathrm{U(1),\,D}}Q^2}{M^2}\right|_{\mathrm{B.H.}} \equiv \gamma\,\frac{1}{M^{D-2}_{\mathrm{Pl,\,D}}}\,.$$ [1]

Motivation

Every charged black holes can decay: **No remnants**.[2]

How to satisfy the Weak Gravity Conjecture

1. The super-extremal state is a **particle**: $m \leq M_{\mathrm{BH,\,min.}}$.
2. The super-extremal state is a **black hole**.[3]

[1] N. Arkani-Hamed, L. Motl, A. Nicolis, C. Vafa, *JHEP* **06**, 060, arXiv: hep-th/0601001 (hep-th)

[2] N. Arkani-Hamed, L. Motl, A. Nicolis, C. Vafa, *JHEP* **06**, 060, arXiv: hep-th/0601001 (hep-th); L. Susskind, arXiv: hep-th/9501106 (hep-th)

[3] Y. Kats, L. Motl, M. Padi, *JHEP* **12**, 068, arXiv: hep-th/0606100; C. Cheung, J. Liu, G. N. Remmen, *JHEP* **10**, 004, arXiv: 1801.08546 (hep-th); Y. Hamada, T. Noumi, G. Shiu, *Phys. Rev. Lett.* **123**, 051601, arXiv: 1810.03637 (hep-th)

Tower Weak Gravity Conjecture

Tower Weak Gravity Conjecture

Every U(1) gauge theory coupled to gravity has a **tower** with **infinitely** many super-extremal states such that

$$\frac{g^2_{\mathrm{U(1),\,D}}\,q^2}{m^2_D} \geq \gamma \frac{1}{M^{D-2}_{\mathrm{Pl,\,D}}}\,.^4$$

Motivations

1. Consistency under **dimensional reduction**.[4]
2. **Absence** of global symmetries in limit $g_{\mathrm{U(1),\,D}} \longrightarrow 0$.

How to satisfy the Weak Gravity Conjecture

1. Tower of super-extremal **particles**: $m_n \leq M_{\mathrm{BH,\,min.}}$ with charge nq.
2. Tower of super-extremal state **above** the BH threshold.

[4] B. Heidenreich, M. Reece, T. Rudelius, *JHEP* **02**, 140, arXiv: 1509.06374 (hep-th); M. Montero, G. Shiu, P. Soler, *JHEP* **10**, 159, arXiv: 1606.08438 (hep-th); S. Andriolo, D. Junghans, T. Noumi, G. Shiu, *Fortsch. Phys.* **66**, 1800020, arXiv: 1802.04287 (hep-th)

Evidence for Tower and Asymptotic Weak Gravity Conjecture

Weak Coupling

- KK towers,[5]
- String towers.

Asymptotically Weak Coupling

- Type IIB[6] or M-theory[7] on CY 3-folds.
- Perturbative heterotic string theory.[5]
- F-theory for weakly-coupled $\mathrm{U}(1)$s.[8]

Away from Weak Coupling

- BPS black hole towers in M-theory on CY 3-folds.[9]
- BPS SCFT sectors in M-theory on CY 3-folds.

[5] B. Heidenreich, M. Reece, T. Rudelius, *JHEP* **02**, 140, arXiv: 1509.06374 (hep-th)

[6] T. W. Grimm, E. Palti, I. Valenzuela, *JHEP* **08**, 143, arXiv: 1802.08264 (hep-th); B. Bastian, T. W. Grimm, D. van de Heisteeg, *JHEP* **06**, 162, arXiv: 2011.08854 (hep-th); N. Gendler, I. Valenzuela, *JHEP* **01**, 176, arXiv: 2004.10768 (hep-th)

[7] S.-J. Lee, W. Lerche, T. Weigand, *Nucl. Phys. B* **938**, 321–350, arXiv: 1810.05169 (hep-th); C. F. Cota, A. Mininno, T. Weigand, M. Wiesner, *JHEP* **08**, 057, arXiv: 2212.09758 (hep-th)

[8] S.-J. Lee, W. Lerche, T. Weigand, *JHEP* **10**, 164, arXiv: 1808.05958 (hep-th); S.-J. Lee, W. Lerche, T. Weigand, *JHEP* **08**, 104, arXiv: 1901.08065 (hep-th); D. Klaewer, S.-J. Lee, T. Weigand, M. Wiesner, *JHEP* **03**, 252, arXiv: 2011.00024 (hep-th)

[9] M. Alim, B. Heidenreich, T. Rudelius, *Fortsch. Phys.* **69**, 2100125, arXiv: 2108.08309 (hep-th); N. Gendler, B. Heidenreich, L. McAllister, J. Moritz, T. Rudelius, arXiv: 2212.10573 (hep-th)

Evidence for the Absence of Towers: Open String U(1)s

Asymptotic WGC with Minimal Supersymmetry

F-theory compactification:[10]

1. Only **non-BPS excitations** of a weakly-coupled BPS string are available, as candidate.
2. Super-extremal states exists in emergent string limits.[11]

Absence of a Tower in 4d

No particle-like excitations of BPS strings can be a tower of super-extremal tower of states when[12]

$$\frac{\Lambda^2_{\text{WGC}}}{\Lambda^2_{\text{QG}}} \equiv \frac{g^2_{\text{U(1), D}} M^{D-2}_{\text{Pl, D}}}{\Lambda^2_{\text{QG}}} \to \infty \,.$$

[10] S.-J. Lee, W. Lerche, T. Weigand, *JHEP* **10**, 164, arXiv: 1808.05958 (hep-th); S.-J. Lee, W. Lerche, T. Weigand, *Nucl. Phys. B* **938**, 321–350, arXiv: 1810.05169 (hep-th)

[11] S.-J. Lee, W. Lerche, T. Weigand, *JHEP* **08**, 104, arXiv: 1901.08065 (hep-th); D. Klaewer, S.-J. Lee, T. Weigand, M. Wiesner, *JHEP* **03**, 252, arXiv: 2011.00024 (hep-th); C. F. Cota, A. Mininno, T. Weigand, M. Wiesner, *JHEP* **11**, 058, arXiv: 2208.00009 (hep-th)

[12] C. F. Cota, A. Mininno, T. Weigand, M. Wiesner, *JHEP* **11**, 058, arXiv: 2208.00009 (hep-th)

The Asymptotic WGC in M-theory

Absence of a Tower in 5d

1. There are directions in the charge lattice where there is **no** BPS tower.
2. There are **BPS** and **non-BPS towers** in **all** directions that admit a weak-coupling limit.
3. If there is **no** tower of BPS states either there is **no** a weak-coupling limit, or there are super-extremal non-BPS states arising as excitations of a **critical string**.[13]

Classifying Weak-coupling Limits

U(1)s are weakly-coupled in a given infinite distance limit if C_3 is reduced over[13]

1. a curve contained in the **generic fiber** or
2. a curve in a degenerate fiber at **finite** distance in the deformation space of the fiber.

[13] C. F. Cota, A. Mininno, T. Weigand, M. Wiesner, *JHEP* **08**, 057, arXiv: 2212.09758 (hep-th)

The Minimal Weak Gravity Conjecture

The Minimal Weak Gravity Conjecture

Towers of (super-)extremal particle states exist **if and only if** they are required by the WGC under dimensional reduction:

1. Emergent string limits,
2. Kaluza–Klein reductions with KK gauge bosons.
3. Strongly coupled limits with exactly extremal states. (**?**)

Absent Towers

All known cases without established super-extremal tower are **consistent**:

1. Perturbative open string $U(1)$s.
2. F-theory away from emergent string limits.
3. Conifold $U(1)$ in M-theory.
4. ...

Contents

Contents

The Convex Hull Condition

Weak Gravity Conjecture for Particles

Every $\mathrm{U}(1)_D$ gauge theory in D dimensions coupled to gravity, there **exists** a particle with mass m_D and charge q, such that[14]

$$\frac{g^2_{\mathrm{U(1),\,D}}q^2}{m_D^2} \geq \left.\frac{g^2_{\mathrm{U(1),\,D}}Q^2}{M^2}\right|_{\mathrm{B.H.}} \equiv \gamma\,\frac{1}{M_{\mathrm{Pl,\,D}}^{D-2}}\,,$$

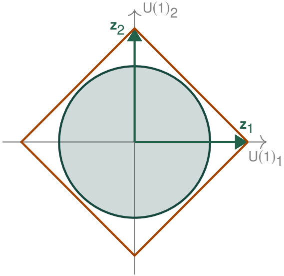
with $\gamma = \frac{D-3}{D-2}$ in the absence of massless scalars.

Convex Hull Condition (CHC)

In case of multiple $\mathrm{U}(1)_D$s, the convex hull formed by all states with charge-to-mass ratio

$$\mathbf{z}_i = \frac{M_{\mathrm{Pl,\,D}}^{\frac{D-2}{2}}}{m_i}\gamma^{-1/2}\left(g_{\mathrm{U(1),\,D,\,1}}q_1,\ldots,g_{\mathrm{U(1),\,D,\,N}}q_N\right) \equiv \frac{M_{\mathrm{Pl,\,D}}^{\frac{D-2}{2}}}{m_i}\mathbf{q}_i\,,$$

must include the **unit ball**.[15]



[14] N. Arkani-Hamed, L. Motl, A. Nicolis, C. Vafa, *JHEP* **06**, 060, arXiv: hep-th/0601001 (hep-th)

[15] C. Cheung, G. N. Remmen, *Phys. Rev. Lett.* **113**, 051601, arXiv: 1402.2287 (hep-ph)

The WGC under Dimensional Reduction

KK-U(1)$_{\text{KK}}$

Consider the circle compactification of a D-dimensional $\mathrm{U}(1)_D$ gauge theory:

1. $\mathrm{U}(1)_{\text{KK}}$ gauge theory with gauge coupling
$$\frac{1}{g^2_{\text{KK, D}-1}} = \frac{1}{2} r^2_{S^1} M^{D-3}_{\text{Pl, D}-1} \, .$$
2. The mass of the states charged under the KK-$\mathrm{U}(1)_{\text{KK}}$ is
$$m^2_{D-1} = m^2_D + \frac{1}{r^2_{S^1}} \left(q_{\text{KK}} - q\theta\right)^2 \, ,$$
where $0 \leq \theta < 1$ is the $\mathrm{U}(1)_D$ Wilson line parameter.

The CHC must be **satisfied** for $\mathrm{U}(1)_{\text{KK}} \times \mathrm{U}(1)_{D-1}$.[16]

[16] B. Heidenreich, M. Reece, T. Rudelius, *JHEP* **02**, 140, arXiv: 1509.06374 (hep-th). See also M. Montero, G. Shiu, P. Soler, *JHEP* **10**, 159, arXiv: 1606.08438 (hep-th); S. Andriolo, D. Junghans, T. Noumi, G. Shiu, *Fortsch. Phys.* **66**, 1800020, arXiv: 1802.04287 (hep-th)

The WGC under Dimensional Reduction

Requirement

There must exist in the D-dimensional gauge theory a state

$$z_D = g_{\mathrm{U(1),D}} M_{\mathrm{Pl,D}}^{\frac{D-2}{2}} \gamma^{-1/2} \frac{|q|}{m_D},$$

such that

$$(m_D r_{S^1})^2 \geq \frac{1}{4z_D^2(z_D^2-1)} + \frac{q\theta(1-q\theta)}{z_D^2},$$

for **all values** of r_{S^1} and θ.[17]

Bottom-Up Tower Weak Gravity Conjecture

To satisfy the CHC for every value of $r_{S^1} \longrightarrow 0$, in the D-dimensional theory there must **exist** a tower of super-extremal states.[17]

[17] B. Heidenreich, M. Reece, T. Rudelius, *JHEP* **02**, 140, arXiv: 1509.06374 (hep-th). See also M. Montero, G. Shiu, P. Soler, *JHEP* **10**, 159, arXiv: 1606.08438 (hep-th); S. Andriolo, D. Junghans, T. Noumi, G. Shiu, *Fortsch. Phys.* **66**, 1800020, arXiv: 1802.04287 (hep-th)

Contents

Arguing for the Minimal Radius

KK Particles

A KK tower of mass $M_{\text{KK}} \sim \frac{1}{2\pi r_{S^1}}$ can be **detected** from the EFT if

$$M_{\text{KK}} \sim \frac{1}{2\pi r_{S^1}} \leq M_{\text{BH, min.}}$$

Species Scale and Minimal Black Hole

$$\frac{M_{\text{BH, min.}}}{M_{\text{Pl, D}}} = \left(\frac{M_{\text{Pl, D}}}{\Lambda_{\text{QG}}}\right)^{D-3} \quad \text{with } r^{-1}_{\text{BH, min.}} \sim \Lambda_{\text{QG}}.$$ [18]

Minimal Radius

1. Suppose $\Lambda_{\text{QG}} = M_{\text{Pl, D}-1}$, then we have
$$2\pi r^{\text{min.}}_{S^1} = M^{-1}_{\text{Pl, D}-1} \; : \text{ minimal radius}.$$
2. It exists every time the small radius limit is not associated with a weakly coupled tower.

[18] G. Dvali, *Fortsch. Phys.* **58**, 528–536, arXiv: 0706.2050 (hep-th); N. B. Agmon, A. Bedroya, M. J. Kang, C. Vafa, arXiv: 2212.06187 (hep-th)
See Max's talk at Strings and Geometry 2024.

More

Can the Minimal Radius Argument Fail?

The Species Scale

The Quantum Gravity cutoff can drop below $M_{\mathrm{Pl},\,D-1}$:

1. Decompactification Limits: Λ_{QG} is higher dimensional Planck mass.[19]
2. Emergent String Limits:[20] $\Lambda_{\mathrm{QG}} \sim M_s$.[21]

In both cases, weakly coupled **states appear**.[22]

Absence of Minimal Radius

1. S^1 reduction of M-theory from 11d to 10d. But, **minimal radius** in M-theory compactifications.
2. S^1 reduction of perturbative string theory, due to T-duality:
   - Heterotic string: the tower is necessary.
   - Open string theory: the tower is **not** necessary.

[19] C. Long, M. Montero, C. Vafa, I. Valenzuela, *JHEP* **03**, 109, arXiv: 2112.11467 (hep-th); F. Marchesano, L. Melotti, *JHEP* **02**, 112, arXiv: 2211.01409 (hep-th); A. Castellano, A. Herráez, L. E. Ibáñez, arXiv: 2310.07708 (hep-th); D. van de Heisteeg, C. Vafa, M. Wiesner, D. H. Wu, arXiv: 2212.06841 (hep-th); N. Cribiori, D. Lüst, G. Staudt, *Phys. Lett. B* **844**, 138113, arXiv: 2212.10286 (hep-th)

[20] S.-J. Lee, W. Lerche, T. Weigand, *JHEP* **10**, 164, arXiv: 1808.05958 (hep-th)

[21] G. Dvali, D. Lust, *Fortsch. Phys.* **58**, 505–527, arXiv: 0912.3167 (hep-th); G. Dvali, C. Gomez, arXiv: 1004.3744 (hep-th)

[22] H. Ooguri, C. Vafa, *Nucl. Phys. B* **766**, 21–33, arXiv: hep-th/0605264 (hep-th)

M-theory/Type IIA Duality

M-theory/Type IIA Duality

M-theory on S^1 is dual to Type IIA with

$$g_{\text{IIA}}^{2/3} = 2\pi M_{\text{11d}} r_{S^1} \,, \quad \frac{M_s}{M_{\text{11d}}} = g_{\text{IIA}}^{1/3}$$

Tower of States for Small Radius

1. In 11d/10d duality:

$$\frac{M_{\text{D0}}}{M_{\text{Pl, 10}}} \sim \frac{1}{g_{\text{IIA}}^{3/4}} \,.$$

2. For small radius, the species scale $\Lambda_{\text{QG}} = M_s$, and

$$\frac{M_{\text{D0}}}{M_{\text{Pl, 10}}} \ll \frac{1}{g_{\text{IIA}}^{7/4}} = \frac{M_{\text{Pl, 10}}^7}{M_s^7} = \frac{M_{\text{BH, min.}}}{M_{\text{Pl, 10}}} \,.$$

3. **No minimal radius**!

M-theory/Type IIA Duality on X_3

M-theory/Type IIA Duality

M-theory on $X_3 \times S^1$ is dual to Type IIA on X_3 with

$$g_{\text{IIA}}^{2/3} = 2\pi M_{\text{11d}} r_{S^1} \,, \quad \frac{M_s}{M_{\text{11d}}} = g_{\text{IIA}}^{1/3}$$

Small Radius Limit

1. We want to understand the limit of the KK circle,
$$r_{S^1} M_{\text{Pl},5} \to 0 \iff r_{S^1} M_{\text{11d}} \to 0 \text{ at } \mathcal{V}_{X_3} = \text{const.} \,.$$
2. Expressing the volume of X_3 in string units, we obtain that
$$\mathcal{V}_{X_3} = \frac{\mathcal{V}_{X_3,s}}{g_{\text{IIA}}^2} \,,$$
is the 4d dilaton.
3. Decreasing the radius, implies
$$\mathcal{V}_{X_3,s} \sim (2\pi)^3 (M_{\text{11d}} r_{S^1})^3 \to 0 \,.$$

Minimal Radius?

String Kähler Moduli Space

1. The total CY volume **cannot decrease** below the string scale due to Type IIA α'-corrections.
2. At the quantum level $\mathcal{V}_{X_3,s} = e^{-\mathcal{K}_{\text{K}}(X_3)} \longrightarrow \frac{1}{6}\int_{X_3} J \wedge J \wedge J - \frac{\chi(X_3)\zeta(3)}{4\pi^3}$.

Mirror Symmetry

1. Via mirror symmetry, there exist a CY Y_3, whose Kähler potential for the complex structure

$$e^{-\mathcal{K}_{\text{c.s}}(Y_3)} = i\int_{Y_3} \Omega \wedge \bar{\Omega}\,,$$

is identified with $e^{-\mathcal{K}_{\text{K}}(X_3)}$.

2. $\mathcal{V}_{X_3,s} \longrightarrow 0$ means there exist a regime in complex structure moduli space of Y_3 where

$$|X^0|^{-2}\int_{Y_3} \Omega \wedge \bar{\Omega} \longrightarrow 0\,.$$

See Max's talk at Strings and Geometry 2024.

The Minimal Radius via M-theory/Type IIA Duality

Minimal Volume

1. This limit leads to **singularities**:
   - Orbifold singularities: $\mathcal{K}_{\text{c.s}}(Y_3)$ is finite.
   - Conifold singularities: $|X^0|^{-2}e^{-\mathcal{K}_{\text{c.s.}}(Y_3)} = -|\mu|^2 \log|\mu|^2 + \text{const.}$
2. $\mathcal{V}_{X_3,s}$ is **finite** and with a minimum α, such that

$$\mathcal{V}_{X_3,s} \geq \alpha\,, \qquad \alpha \sim \mathcal{O}(1)\,.$$

Minimal Radius

1. Decreasing g_{IIA}, we leave the **vector multiplet** moduli space, from which we can define $r_{S^1}^{\text{min.}}$

$$g_{\text{IIA}}^{2/3} \sim (2\pi) r_{S^1}^{\text{min.}} M_{11\text{d}} = \frac{\alpha^{1/3}}{\left(\mathcal{V}_{X_3}\right)^{1/3}} \quad \Longrightarrow \quad r_{S^1}^{\text{min.}} M_{11\text{d}} = \frac{\alpha^{1/3}}{(2\pi)\left(\mathcal{V}_{X_3}\right)^{1/3}}\,.$$

2. The notion of a KK circle ceases to be appropriate: $\dfrac{M_{\text{D0}}}{M_{\text{Pl},4}} \sim \dfrac{1}{\left(\mathcal{V}_{X_3,s}\right)^{1/2}} \sim \dfrac{1}{r_{S^1}^{3/2} M_{11\text{d}}^{3/2}}\,.$
3. D0-branes tower **cannot** be interpreted as KK tower associated with a freely scalable circle.

The CHC for M-theory/Type IIA Compactifications

Constraint from Minimal Radius

Using

$$r_{S^1}^{\text{min.}} M_{11d} = \frac{\alpha^{1/3}}{(2\pi)\left(\mathcal{V}_{X_3}\right)^{1/3}}\,, \quad m_5^2 = (2\pi)^2 \mathcal{V}_C M_{11d}^2\,,$$

one obtains

$$\alpha^{2/3} \geq \frac{\gamma}{2Q_\alpha f^{\alpha\beta} Q_\beta} \left(\frac{\gamma \mathcal{V}_C}{|q|^2 \left(2|q|^2 Q_\alpha f^{\alpha\beta} Q_\beta \mathcal{V}_{X_3}^{2/3} - \gamma \mathcal{V}_C\right)} + \frac{q\theta(1-\theta q)}{|q|^2} \right).$$

Examples

$$\begin{aligned} \alpha_{\mathbb{P}^4_{11222}[8]} &\simeq 2.83\,, \quad \text{RHS}_{\mathbb{P}^4_{11222}[8]} \simeq 0.17\,, \\ \alpha_{\mathbb{P}^4_{11226}[12]} &\simeq 6.00\,, \quad \text{RHS}_{\mathbb{P}^4_{11226}[12]} \simeq 0.10\,. \end{aligned}$$

Details

Tower for Heterotic String Limits

Perturbative String Theory

1. Consider heterotic string theory on K3. In EML

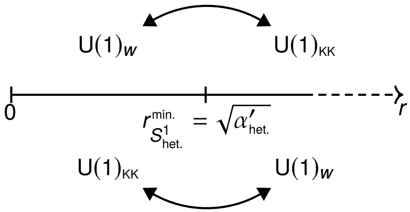
$$\frac{M_{\text{BH, min.}}}{M_{\text{Pl},6}} = \left(\frac{M_{\text{Pl},6}}{M_{\text{het.}}}\right)^3 \sim \frac{1}{g_{\text{het.}}^{3/2}} .$$

2. T-duality seems to impose a **minimal radius** at

$$r^{\text{min.}}_{S^1_{\text{het.}}} \sim \sqrt{\alpha'_{\text{het.}}} .$$

however, there exists a winding $U(1)_w$:

$$M_w \sim \frac{r_{S^1_{\text{het.}}}}{\alpha'_{\text{het.}}} \Longrightarrow \frac{M_w}{M_{\text{Pl},6}} \sim \frac{r_{S^1_{\text{het.}}}}{\sqrt{\alpha'_{\text{het.}}}} g_{\text{het.}}^{1/2} .$$



No Minimal Radius

One can consider any $r_{S^1_{\text{het.}}} \geq 0$ and apply the criterion:

$$M_{\text{KK}} \sim \frac{1}{2\pi r_{S^1_{\text{het.}}}} \leq M_{\text{BH, min.}} \Longrightarrow r^{\text{min.}}_{S^1_{\text{het.}}} \sim g^2_{\text{het.}} \sqrt{\alpha'_{\text{het.}}} \to 0 \text{ for } g_{\text{het.}} \to 0.$$

The CHC for Heterotic String Limits

Perturbative Gauge Theory

1. The gauge coupling for the perturbative U(1) is $g^2_{\mathrm{U}(1)_{\mathrm{pert.}},6} M^4_{\mathrm{Pl},6} \propto g^2_{\mathrm{het.}}$.
2. There exist super-extremal states of charges $q^2 = 4mn$, with n being the excitation level.[23]
3. After the circle reduction: $4\frac{r^2_{S^1_{\mathrm{het.}}}}{\alpha'_{\mathrm{het.}}} \geq \frac{1}{n}\left(\frac{n-1}{4} + \mathfrak{n}_6(1-\mathfrak{n}_6)\right)$, violated for $g_{\mathrm{het.}} \longrightarrow 0$.
4. A tower of super-extremal state is **required**.

Non-perturbative Gauge Theory

1. The gauge coupling for the non-perturbative U(1) is $g^2_{\mathrm{U}(1)_{\mathrm{n.p.}},6} M^2_{\mathrm{het.}} \propto g^{-2}_{\mathrm{het.}}$.
2. After the circle reduction: $\frac{r^2_{S^1_{\mathrm{het.}}}}{\alpha'_{\mathrm{het.}}} \geq \frac{\mathfrak{n}_6(1-\mathfrak{n}_6)}{z_6^2} \propto g^4_{\mathrm{het.}}\mathfrak{n}_6(1-\mathfrak{n}_6)$, **not** violated for $g_{\mathrm{het.}} \longrightarrow 0$.
3. A tower of super-extremal state is **not** required.

[23] S.-J. Lee, W. Lerche, T. Weigand, *JHEP* **02**, 190, arXiv: `1910.01135 (hep-th)`

Contents

The Minimal Weak Gravity Conjecture

The Minimal Weak Gravity Conjecture

Towers of (super-)extremal particle states exist **if and only if** they are required by the WGC under dimensional reduction:

1. Emergent string limits,
2. Kaluza–Klein reductions with KK gauge bosons.
3. Strongly coupled limits with exactly extremal states. (**?**)

Thank you!

F-theory/M-theory Duality

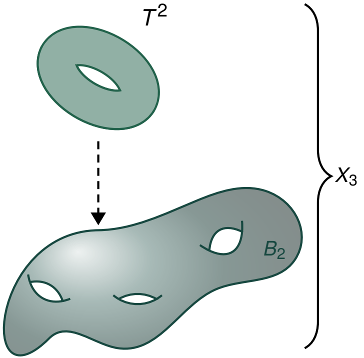
F-theory/M-theory Duality

F-theory on $X_3 \times S^1$ is dual to M-theory on X_3 with

$$r_{S^1} M_{11d} = \frac{1}{2\pi \mathrm{vol}(T^2) M_{11d}^2} \equiv \frac{1}{2\pi\tau} \, .$$

Small Radius

1. Naively $r_{S^1} M_{11d} \to 0 \Longrightarrow \tau \to \infty$.
2. Vector multiplet limit: $\mathrm{vol}(X_3) M_{11d}^6 \equiv \mathcal{V}_{X_3} \simeq \tau \mathcal{V}_{B_2} \simeq$ const.
3. Looking for **lower bound** on $r_{\mathrm{min.}}$.



The Maximal Fiber

The **maximal** fiber is given by

$$\tau_{\mathrm{max.}} = \max_{\tau \geq 0} \left(\tau \, \middle| \, \mathcal{V}_{X_3} = \alpha\tau^3 + \beta\tau^2 + \tau\mathcal{V}_{B_2} \overset{!}{=} 1 \right) .$$

Min. Rad.

How to Compute α

Computation of α

1. Consider CICY of hypersurfaces on products of (weighted) projective spaces.
2. Type IIB mirror dual, complex structure ϕ^i of Y_3, related to Kähler moduli t^i of X_3 by
$$t^i = \frac{i}{2\pi} \log \phi^i + \dots .$$
3. The models have a Landau–Ginzburg phase for small $|\phi^i|$, such that
$$e^{-\mathcal{K}_{\rm c.s}(Y_3)} = i \int_{Y_3} \Omega \wedge \bar{\Omega}$$
is **minimized** at $\phi^i = 0$.
4. Changing the radius of the compactification, corresponds to shrinking the curve volumes of X_3: from $\phi^i \to \infty$ to $\phi^i \to 0$ for all i.
5. α is obtained by evaluating $e^{-\mathcal{K}_{\rm c.s}(Y_3)}$ in the **Landau–Ginzburg phase**.

CHC