

The boundary of moduli spaces: between algebras and geometry

Veronica Collazuol

IPhT CEA/Saclay

Based on

- Work in progress with S. Baines, B. Fraiman, M. Graña and D. Waldram
 - `arXiv:2203.01341`, `arXiv:2210.13471` with M. Graña, A. Herraez and H. Parra De Freitas
 - `arXiv:2402.01606` with I. V. Melnikov



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May 28, 2024

Motivation: the Distance Conjecture

From string compactifications: universal patterns appearing at infinite distance in moduli spaces.

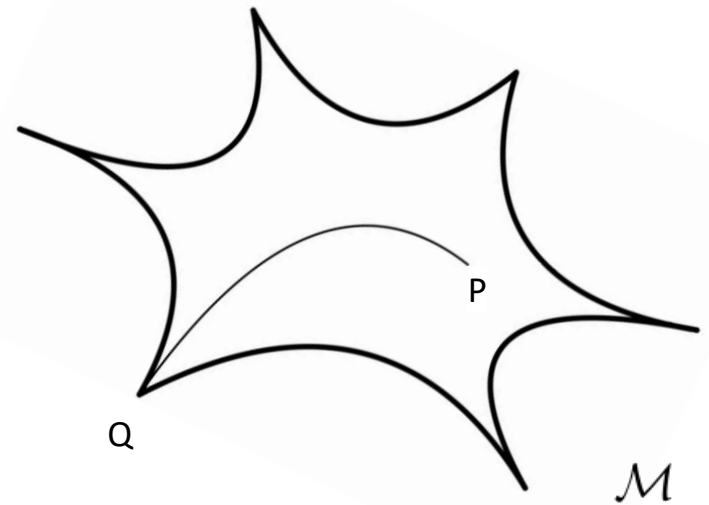
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Moving in moduli space from a point P towards a point Q an infinite geodesic distance away, an infinite tower of states becomes exponentially light (in Planck units) as

$$M(Q) \sim M(P)e^{-\alpha d_{P,Q}}$$

[Ooguri, Vafa, '06]



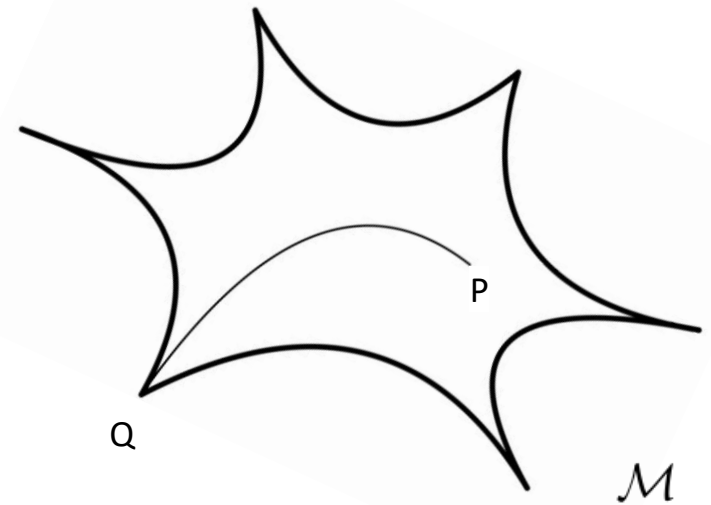
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[Baume, Blumenhagen, Buratti, Calderon-Infante, Castellano, Cecotti, Corvilain, Cribiori, Etheredge, Font, Gendler, Grimm, Heidenreich, Herraez, Ibañez, Joshi, Kaya, Klaewer, Lee, Lerche, Li, Lockhart, Lust, McNamara, Marchesano, Martucci, Montella, Montero, Ooguri, Palti, Perlmutter, Qiu, Rastelli, Rudelius, Ruiz, Stout, Uranga, Vafa, Valenzuela, van de Heisteeg, Weigand, Wiesner, Wolf, ...]

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Geometry of moduli spaces \longleftrightarrow **Spectrum** of the theory

- Geodesics
- Structure of the boundary

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- M theory on T^d : $G = E_{d(d)}$

- Type II on T^d (no RR): $G = O(d, d)$

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[Link, '08]

Definition

- **Geometric:** manifold \mathcal{M} with an inversion symmetry at any point p

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We will focus on symmetric spaces of **non-compact** type.

→ Explicit parametrization of the boundaries

- Geodesics
- Parabolic subgroups

Geodesics and boundaries

[Borel, Ji '06]

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Geodesics (distance induced from the Killing form on \mathfrak{g})

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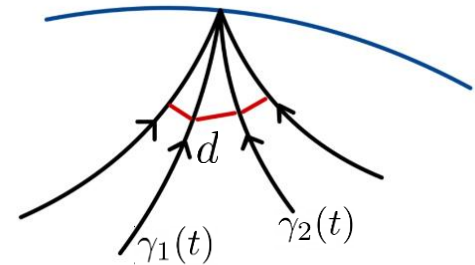
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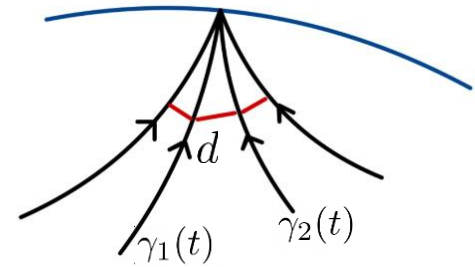
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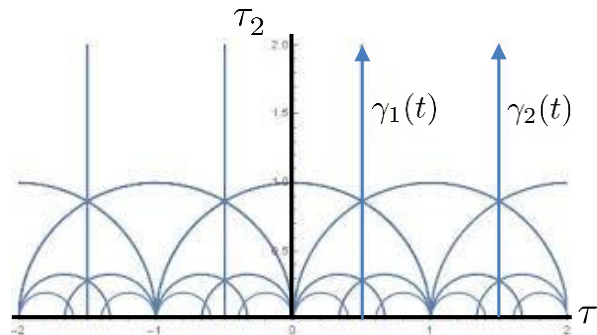
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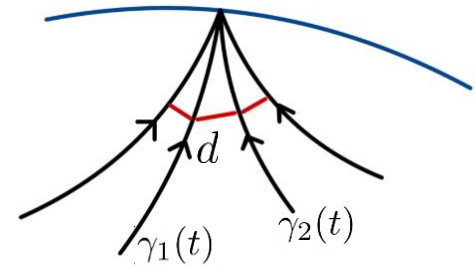
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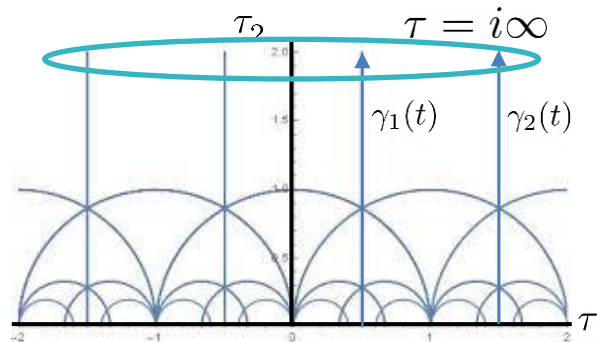
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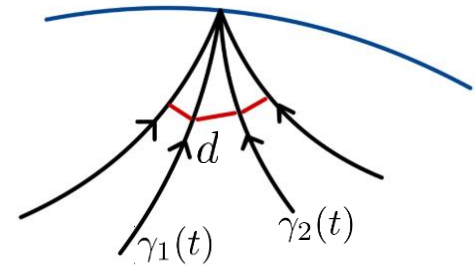
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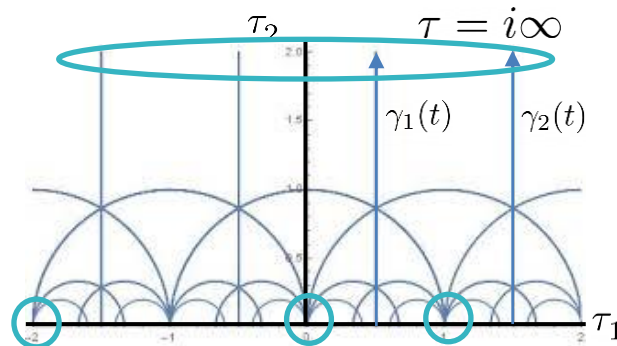
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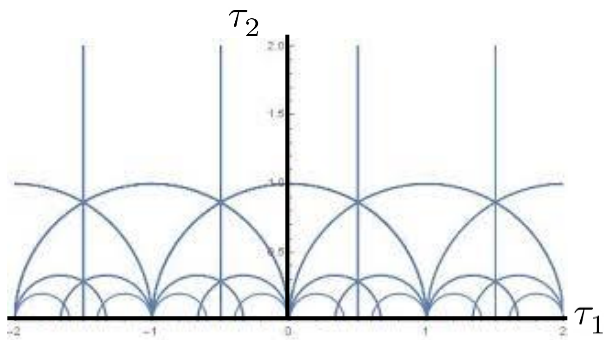
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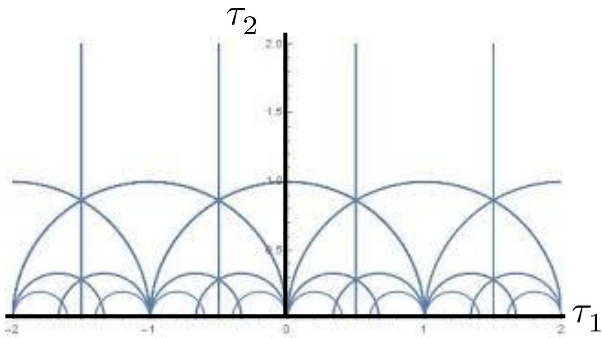
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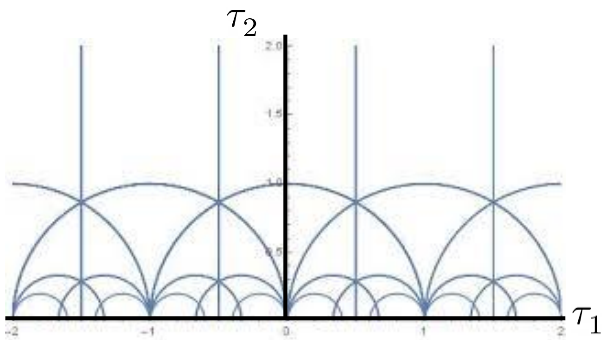
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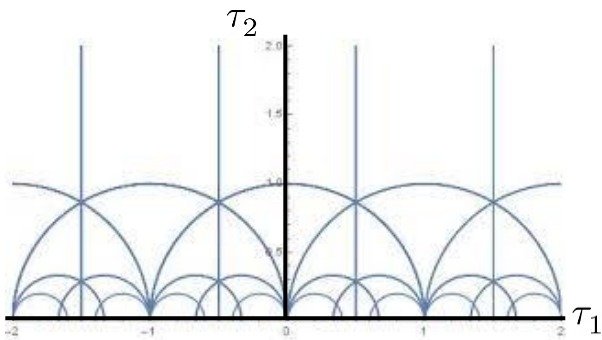
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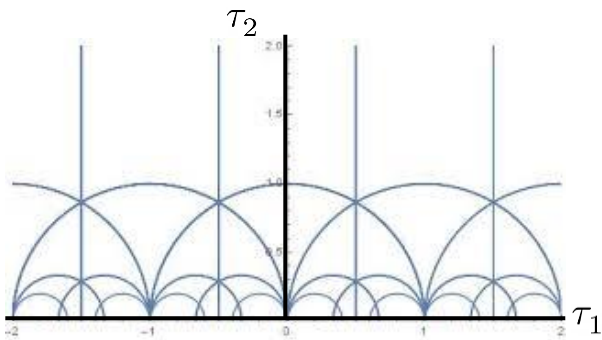
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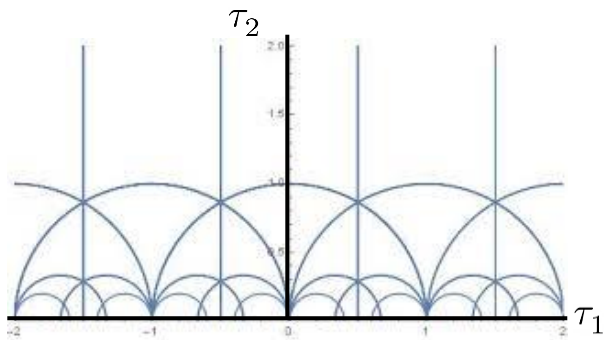
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$$p \in \partial \mathcal{M} \quad \longleftrightarrow \quad (P, h) + K \text{ action}$$



Cartan in the closure of the positive Weyl chamber (norm 1)

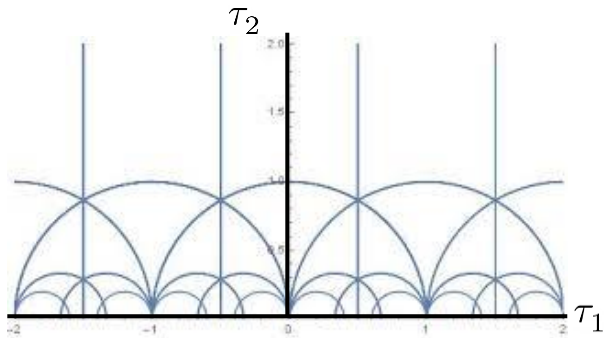
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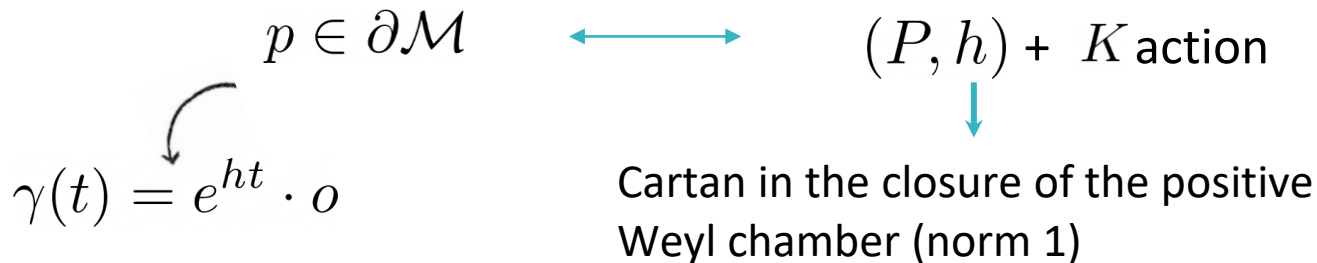
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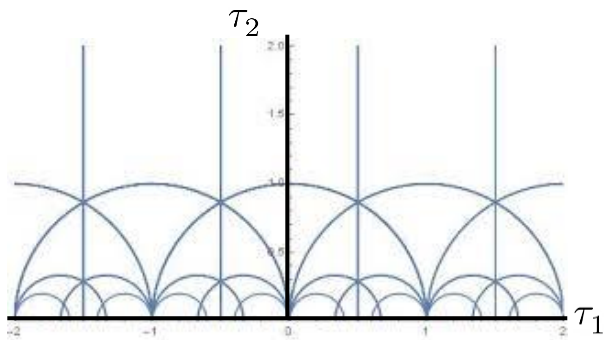
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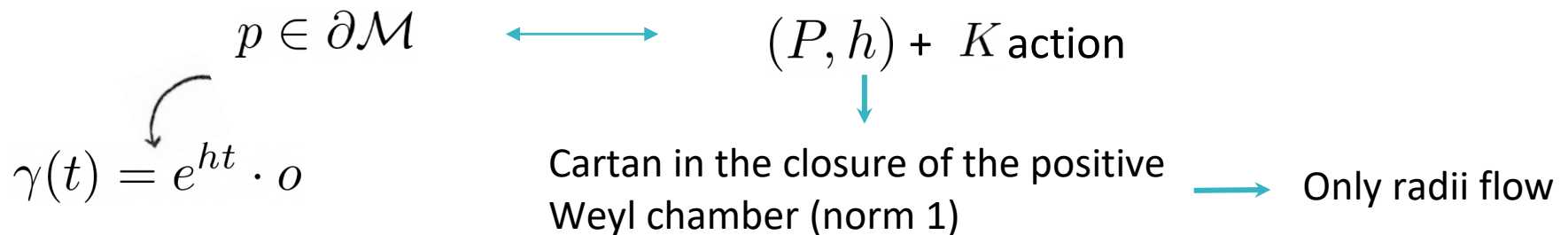
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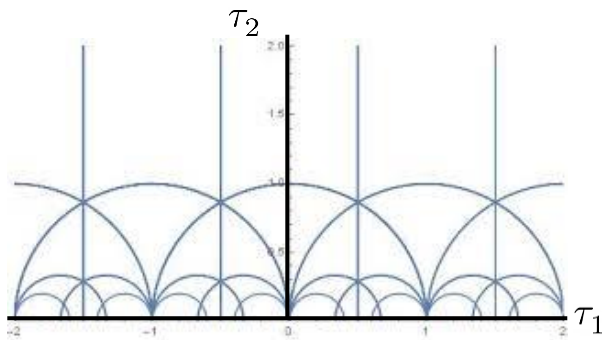
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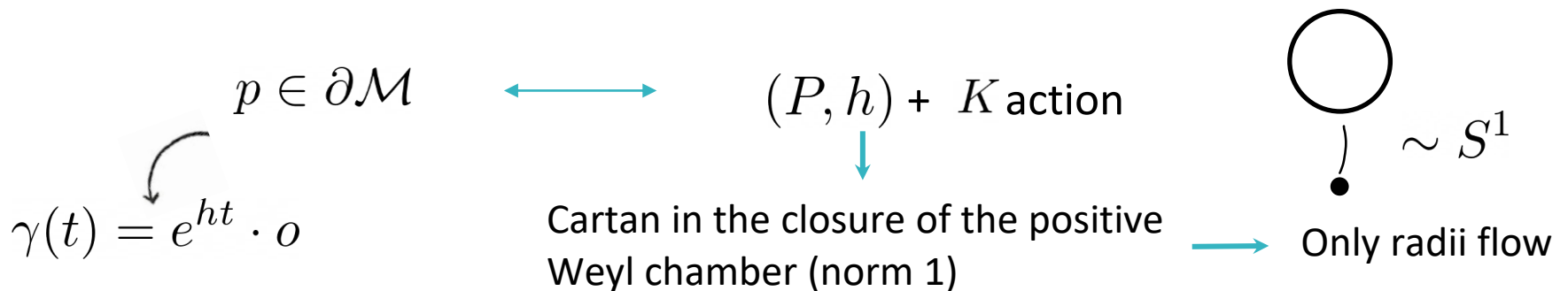
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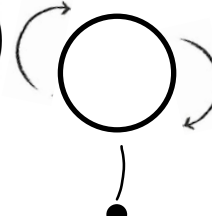
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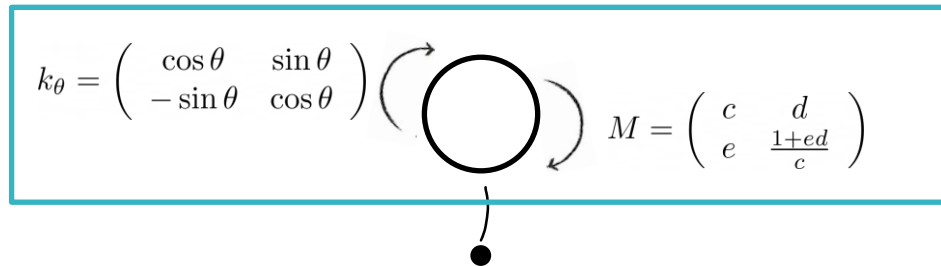
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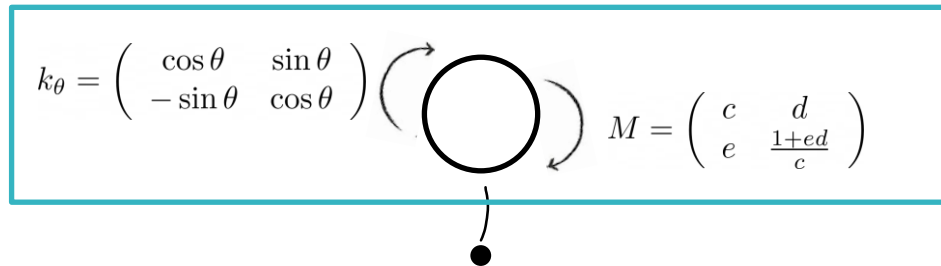
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- Only geodesics with rational compact moduli reach the boundary.

[Keurentjes, '06]

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- Assuming a lattice of charges: explicit expression for the string spectrum along the geodesic fixing the duality frame

→ SDC 

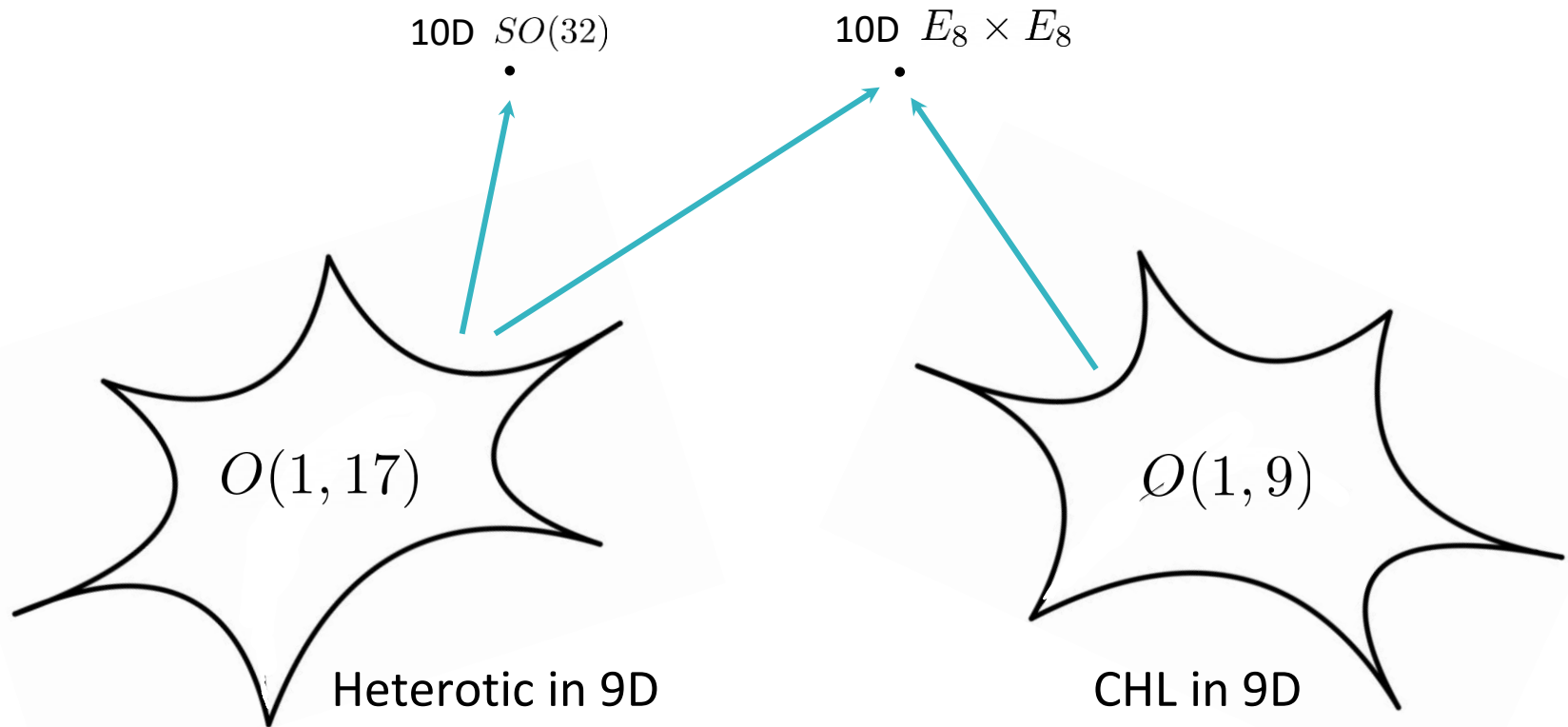
[Cecotti '15]

Heterotic theory decompactified

To understand the physics of decompactification limits (fixed dilaton):
massless states

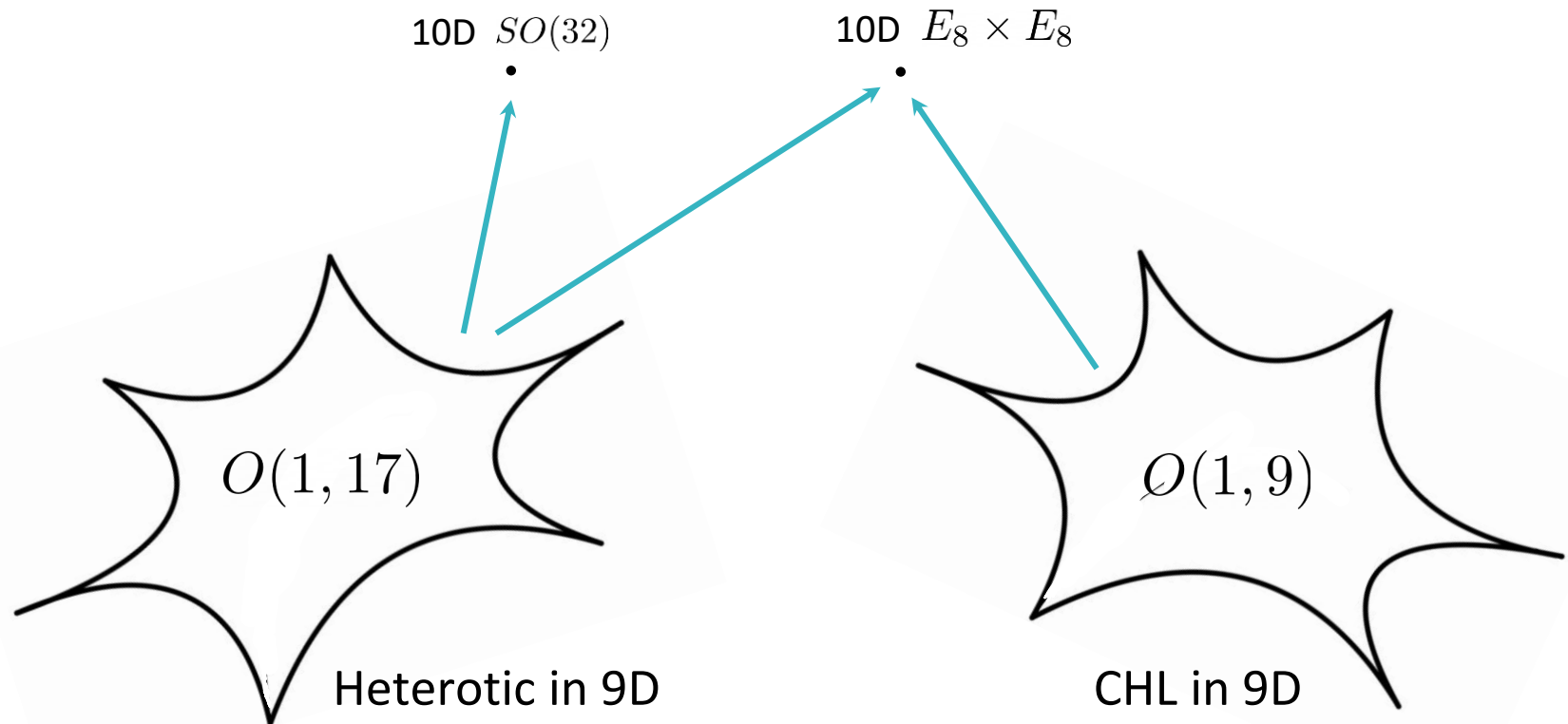
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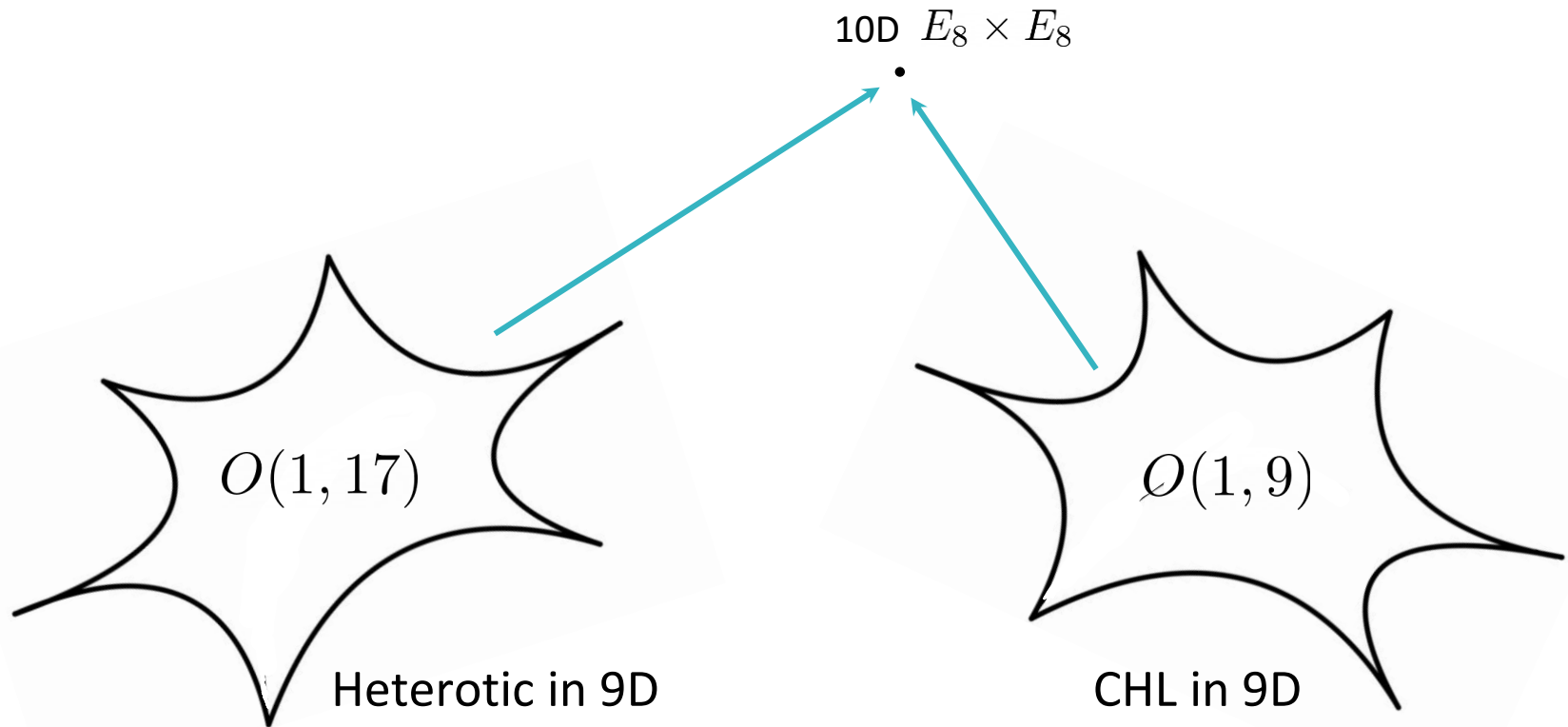


How do you see these limits from the lower dimensional perspective?

→ compute the current algebras

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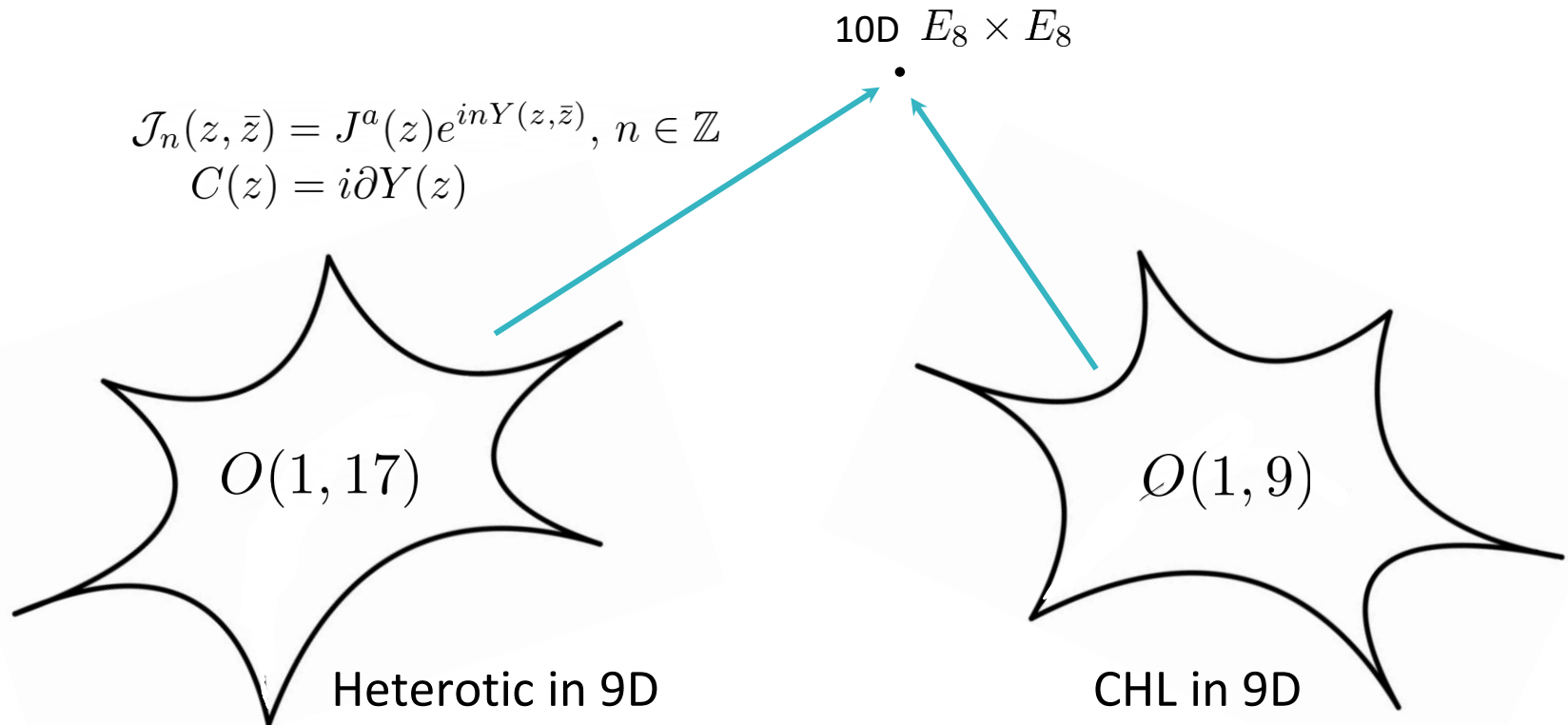


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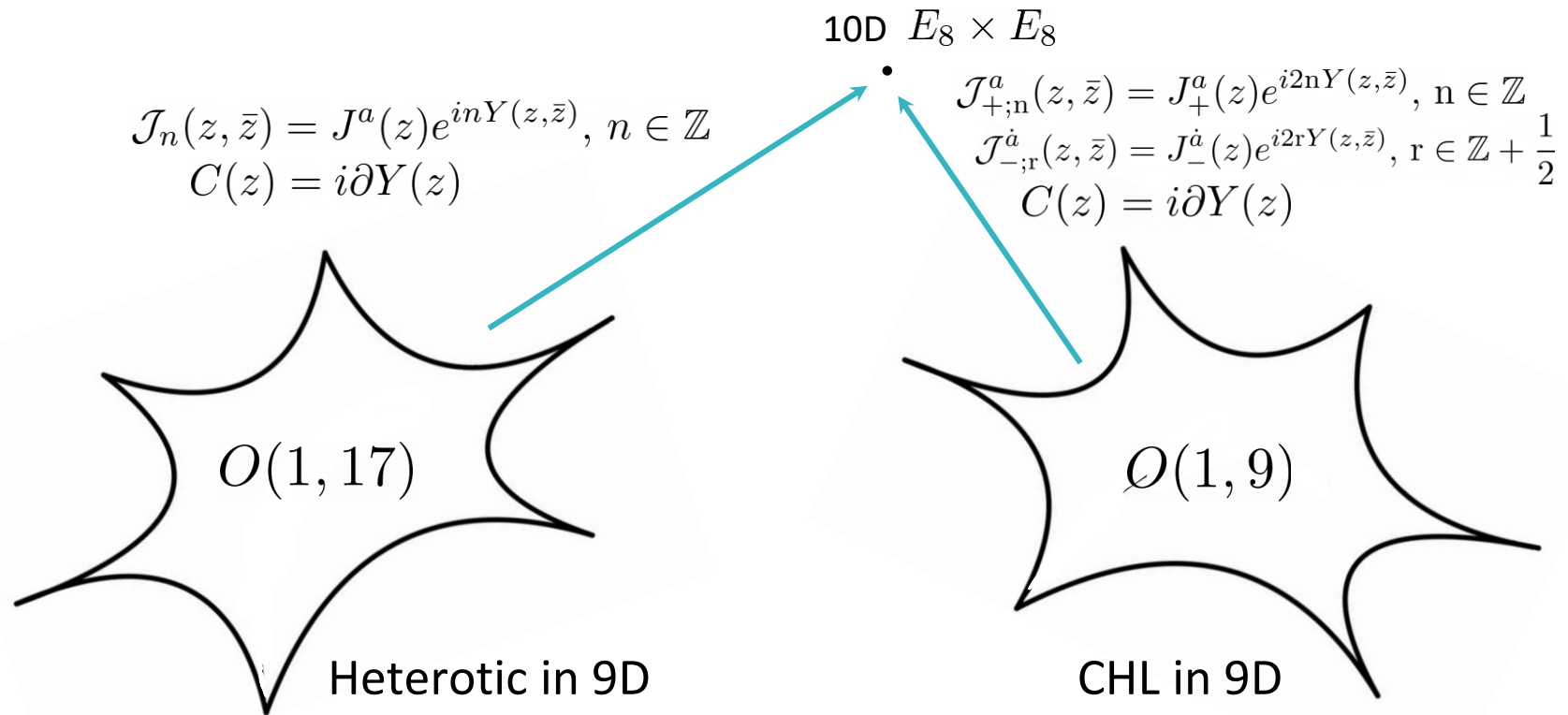


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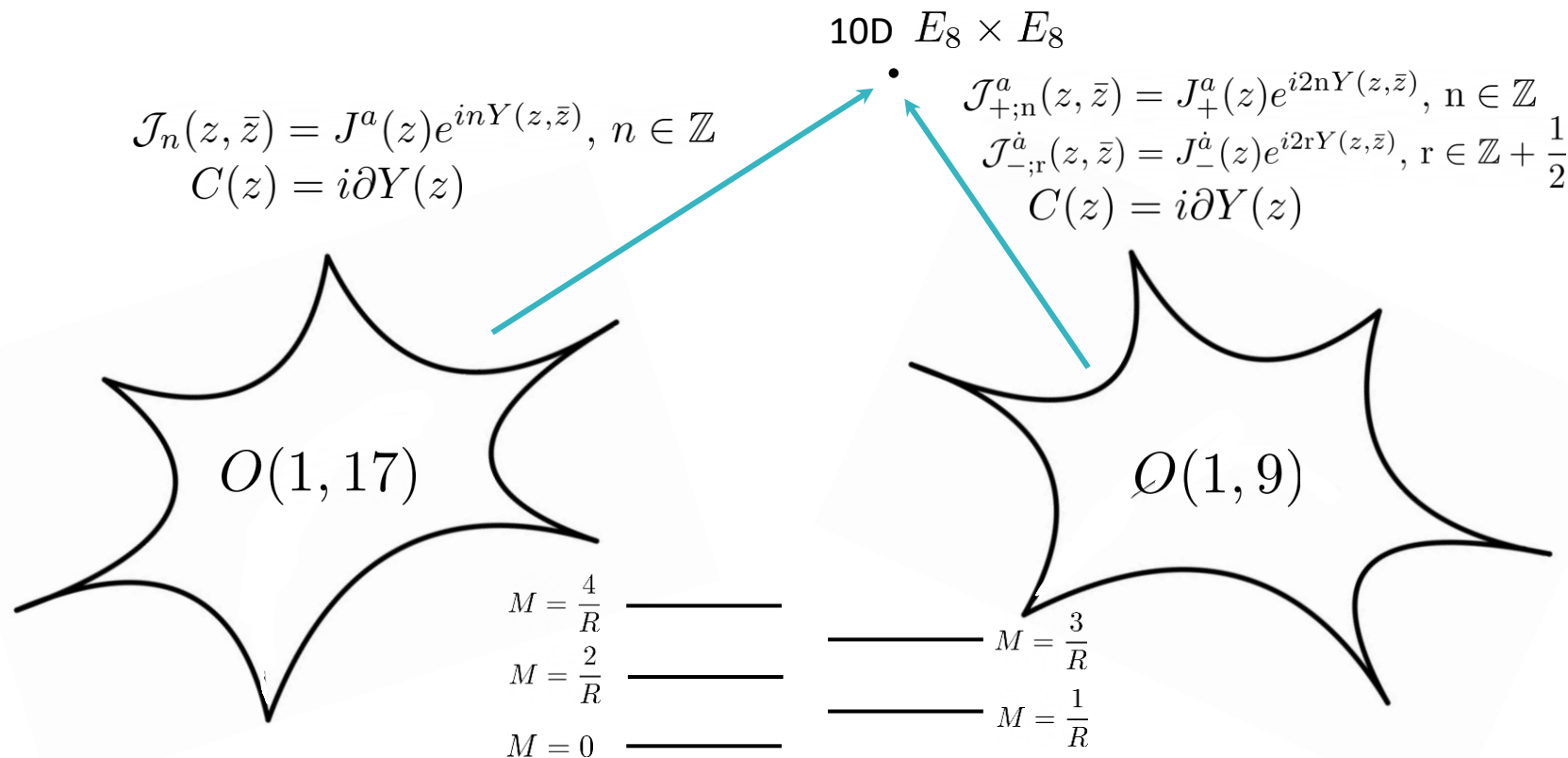


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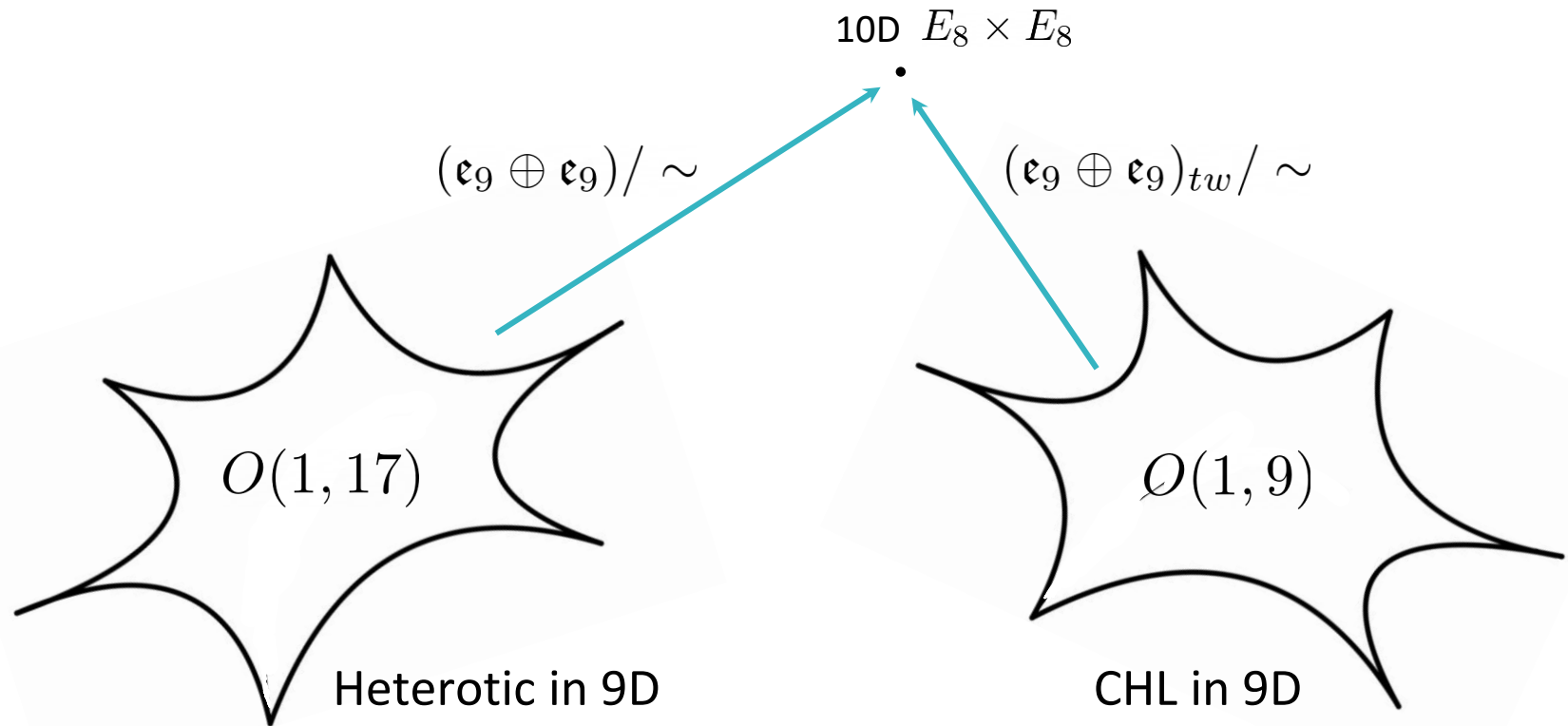


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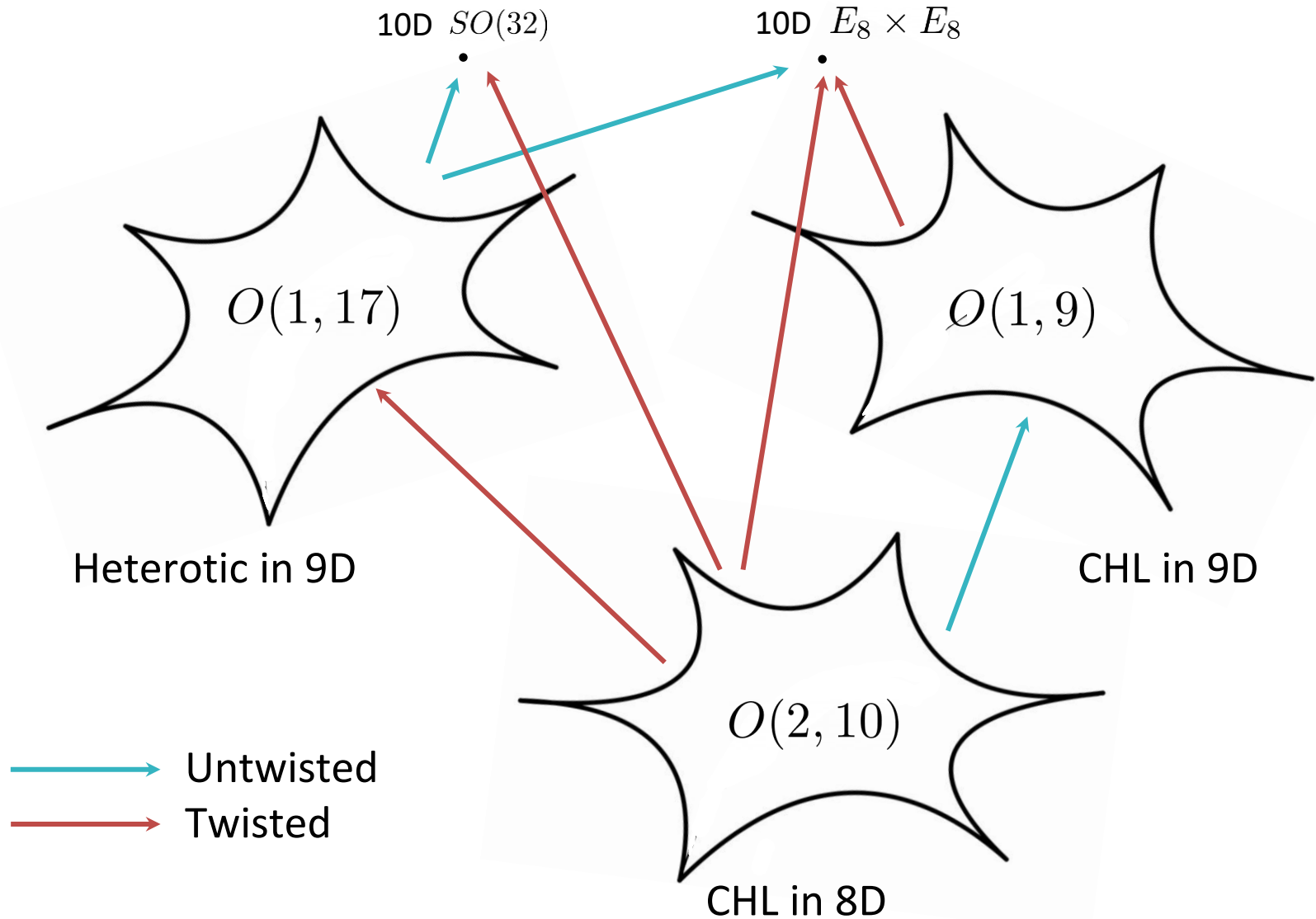
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How do you see these limits from the lower dimensional perspective?

→ compute the current algebras

CHL string: decompactification limit



Conclusions

- The boundaries of the symmetric moduli spaces can be explicitly parametrized from the algebra.
- Assuming a charge lattice, the string spectrum in this limit is consistent with the SDC. [\[Cecotti '15\]](#)
- The physics of decompactification limits can be inferred from the current algebras, which come in their affine version from the point of view of the lower dimensional theory. [\[Lee, Lerche, Weigand, '21\]](#)
[\[Cvetic, Dierigl, Lin, Zhang '22\]](#)
- Going from the CHL to the heterotic components of the moduli spaces leads to twisted affine algebras.

Thank you!