Complexity of the Flux Landscape

Based on 2311.09295 with Thomas Grimm

Jeroen Monnee
Utrecht University

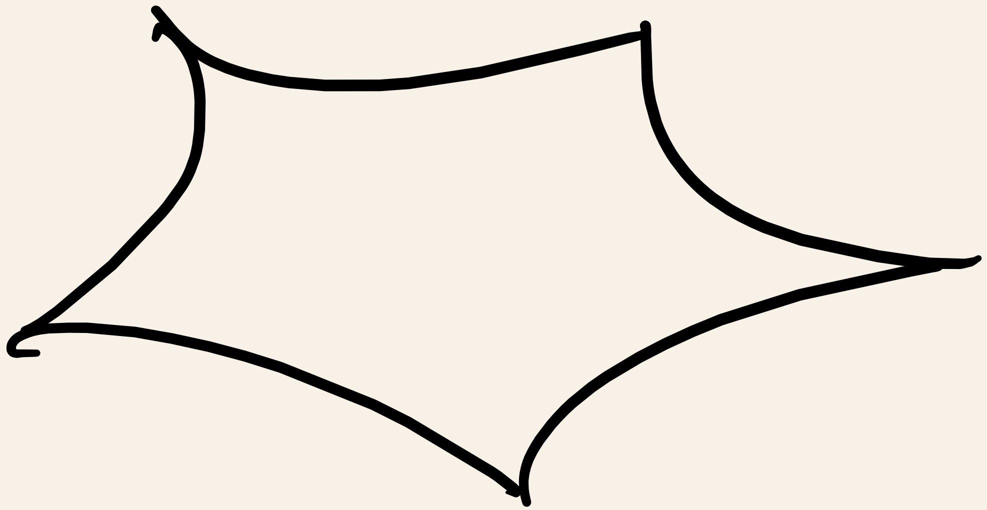
Swamplandia
May 28th 2024

Motivation

In EFT's arising from string compactifications, moduli are stabilized at the minima of a scalar potential

$$V(\text{moduli}, \text{parameters})$$

Landscape



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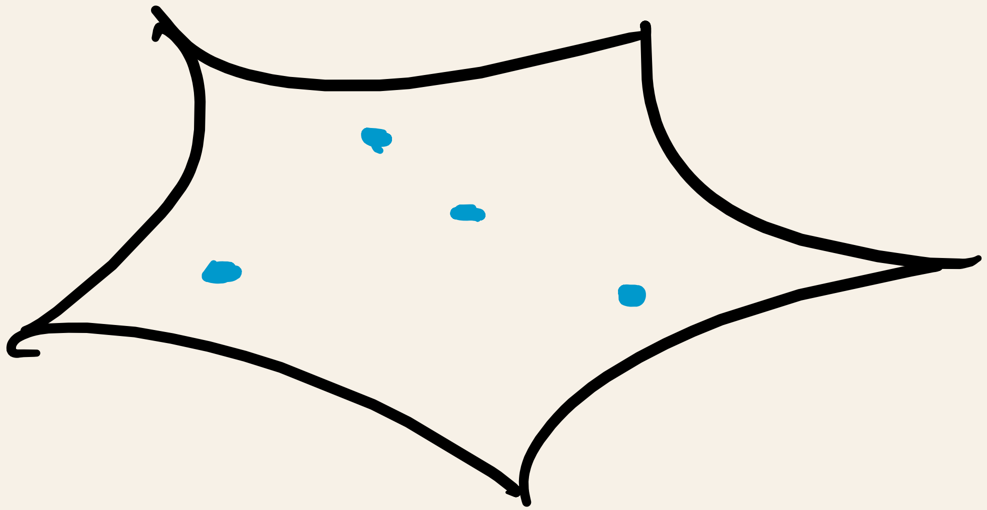

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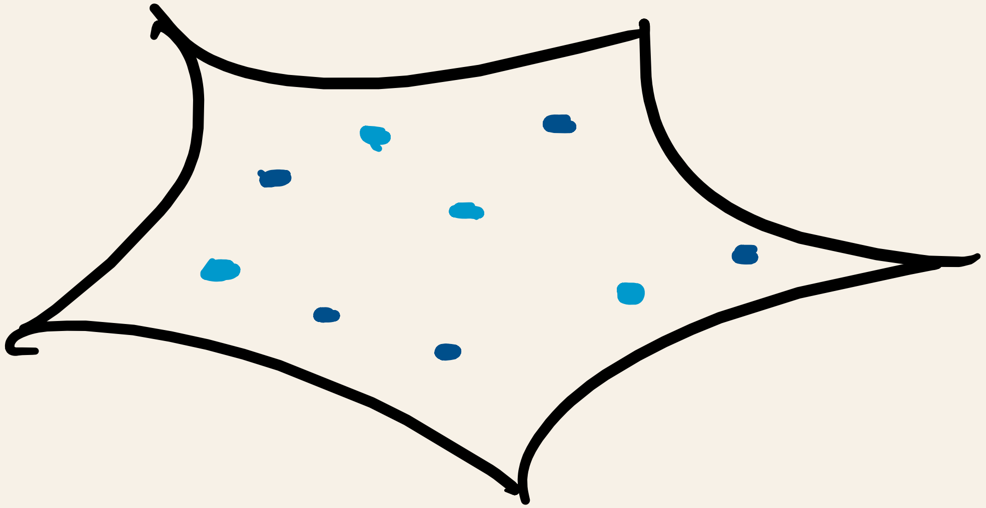

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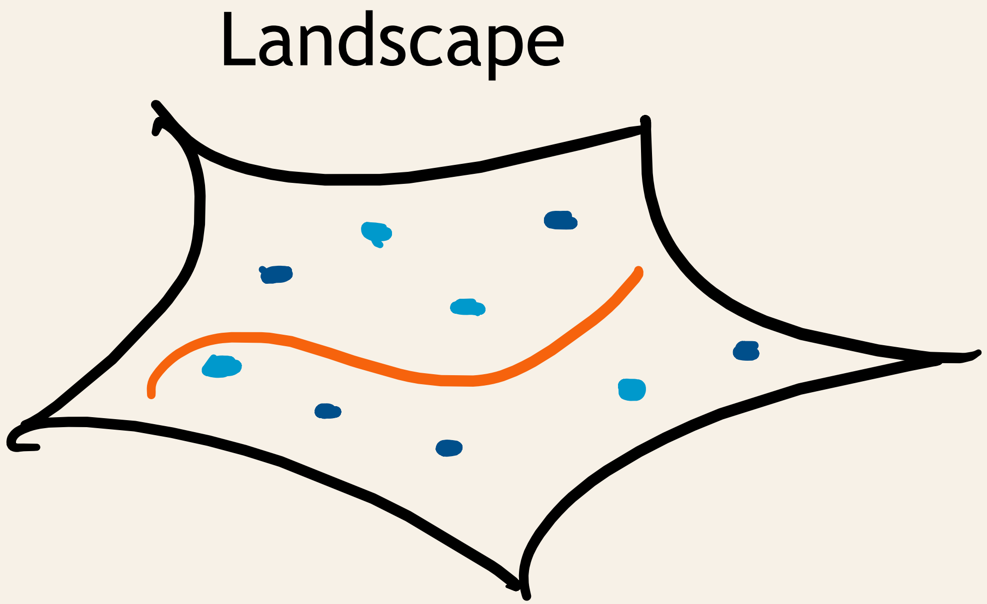

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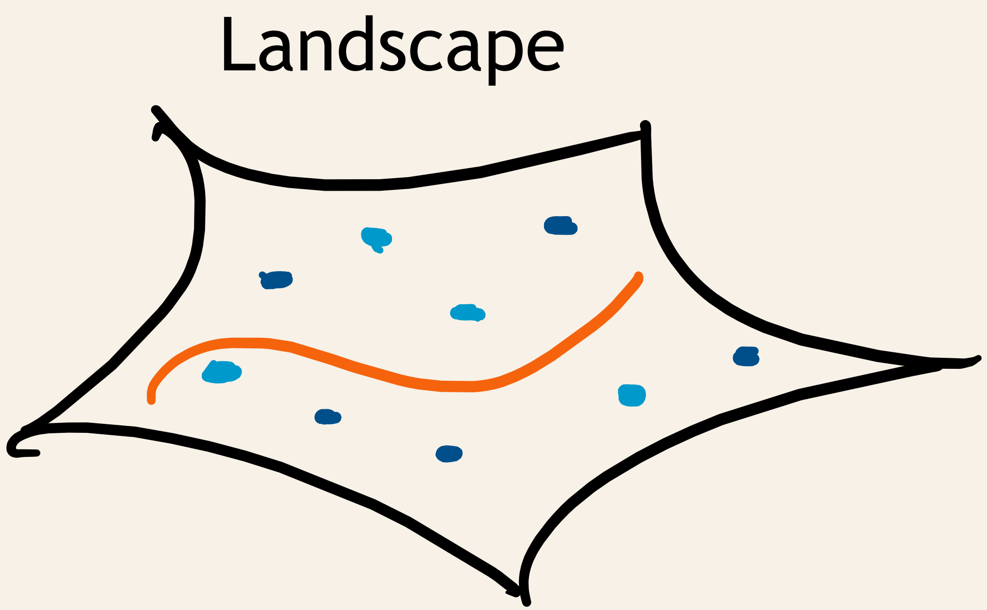

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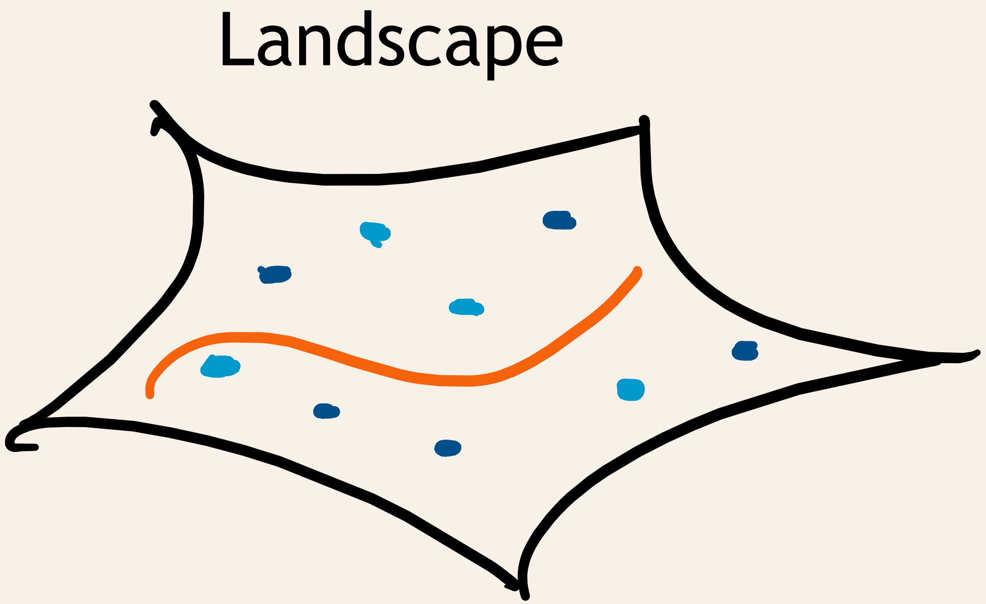

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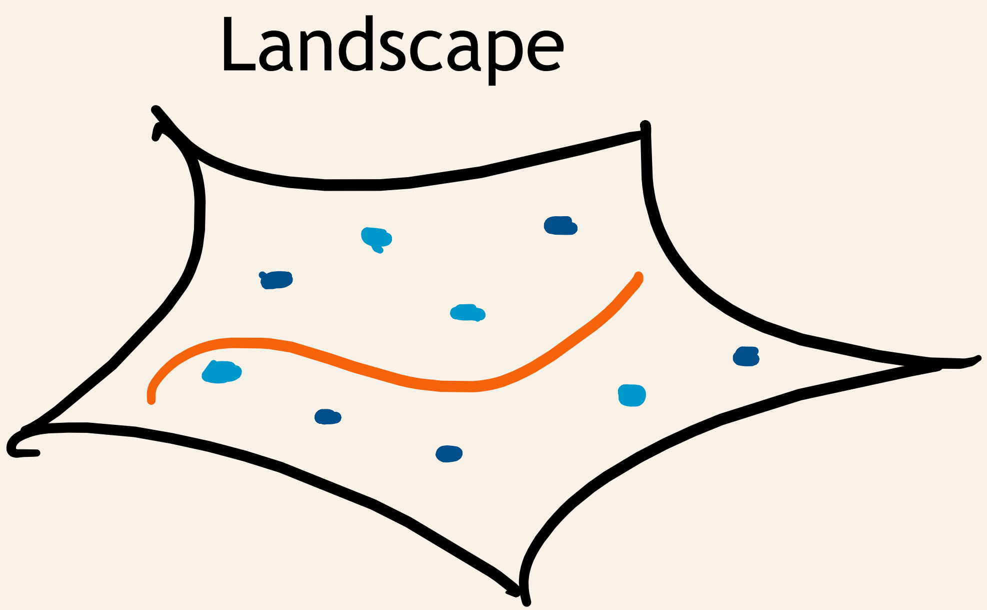

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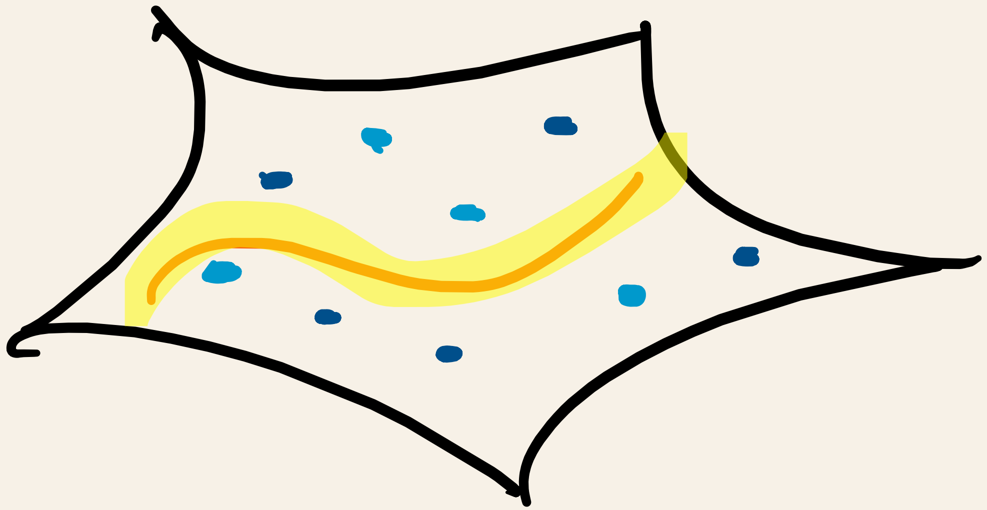

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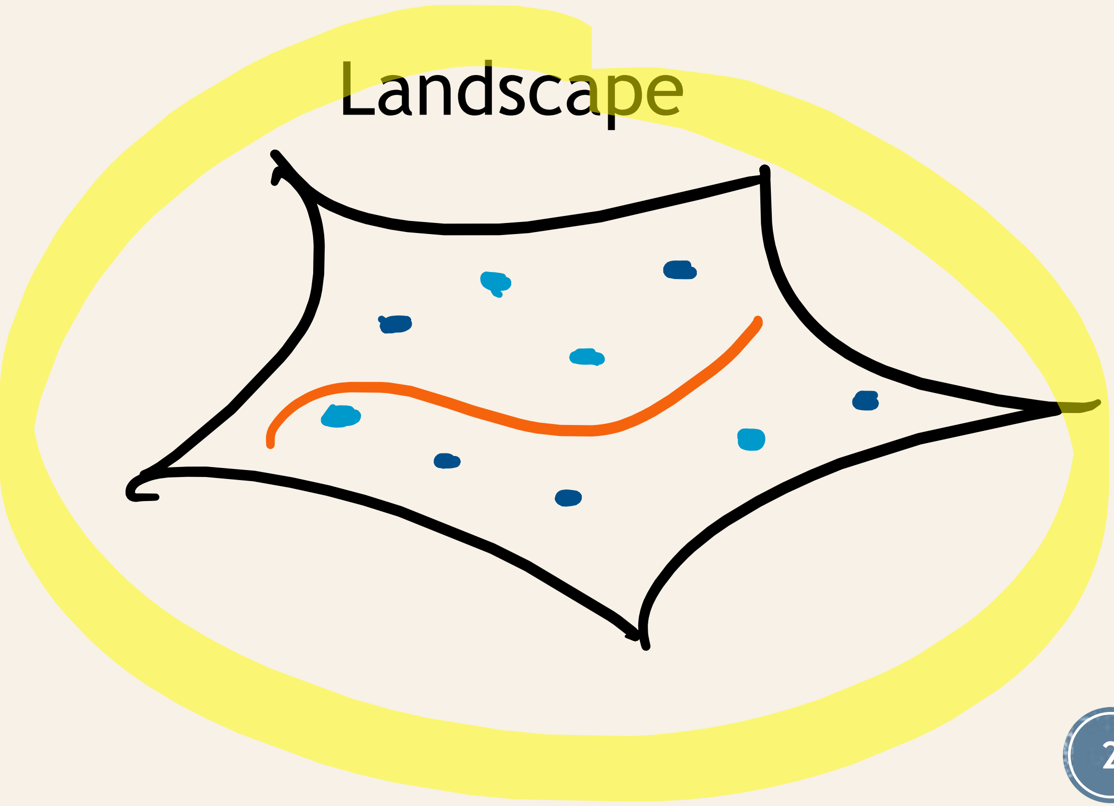
- Complexity?



Outline

1. F-theory landscape
2. Finiteness of the flux landscape
3. Refining the flux landscape
4. Conclusion

F-THEORY LANDSCAPE

4

D=4, N=1 from F-Theory

Consider F-theory compactified on an elliptically fibered Calabi-Yau fourfold with fluxes (see [Grana: 2005], [Douglas, Kachru: 2006], [Denef: 2008] for reviews)

$$G_4 \in H^4\left(Y_4, \mathbb{Z}\right) \ , \qquad \frac{1}{2}\int_{Y_4} G_4 \wedge G_4 \leq \frac{\chi(Y_4)}{24}$$

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This yields a D=4, N=1 supergravity theory:

$$S = \int \frac{1}{2} R \star 1 - g_{i\bar{\jmath}}\, \mathrm{d}z^i \wedge \star \mathrm{d}\bar{z}^{\bar{\jmath}} - V(z^i, G_4) \star 1 + \cdots$$

complex structure moduli

F-Theory Flux Vacua

- Scalar potential:

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- A special subset of vacua are

$$G_4 \in H^4(Y_4, \mathbb{Z}) \cap H^{2,2}(Y_4, \mathbb{C}) \qquad \Longleftrightarrow \qquad D_i W = W = 0$$

Hodge class

FINITENESS OF THE FLUX LANDSCAPE

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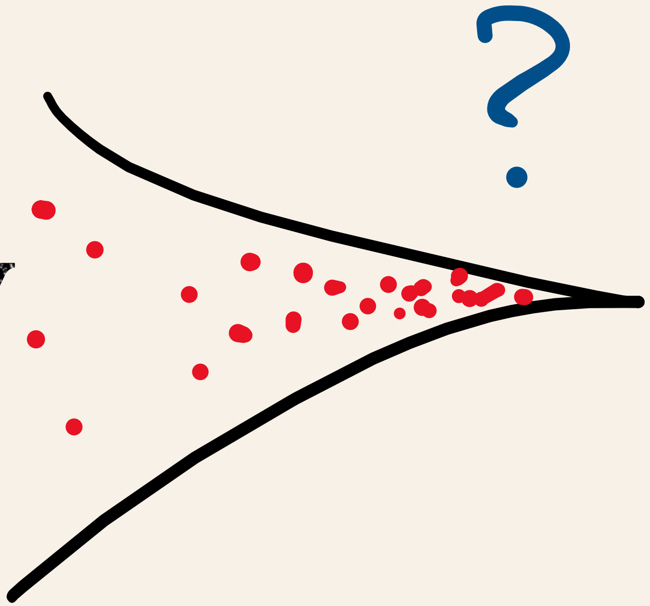

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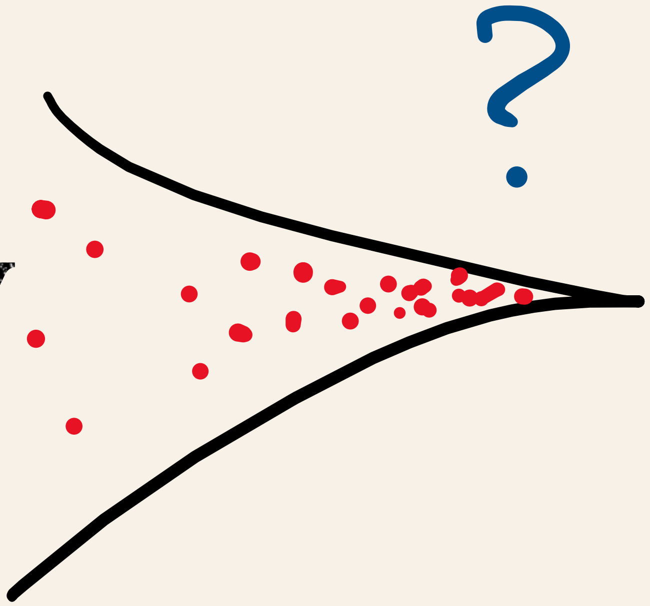

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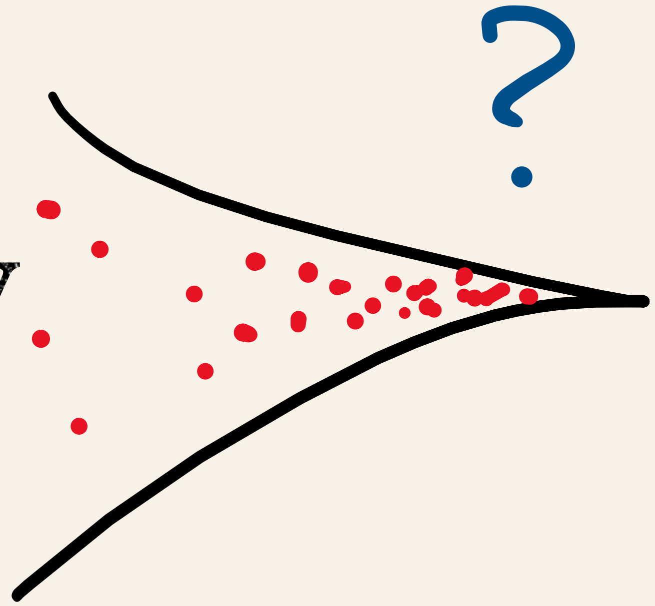

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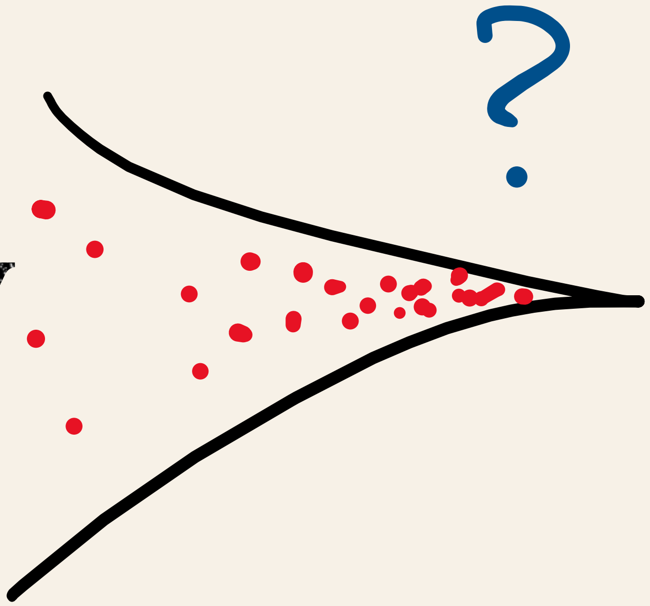

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- Major technical advances in computation of Hodge norms [Grimm, JM: 2023]

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- **Crucial** to go beyond leading order ‘Sl(2)-orbit’ approximation
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- Can use these methods to refine previous analyses
  - Classification of F-theory scalar potentials [Grimm, Li, Valenzuela: 2020]
  - WGC for D3-particles [Bastian, Grimm, van de Heisteeg: 2021] [Gendler, Valenzuela: 2021]
  - Asymptotic accelerated expansion [Calderón-Infante, Ruiz, Valenzuela: 2023]
  - …

REFINING THE FLUX LANDSCAPE

Beyond finiteness & asymptotics

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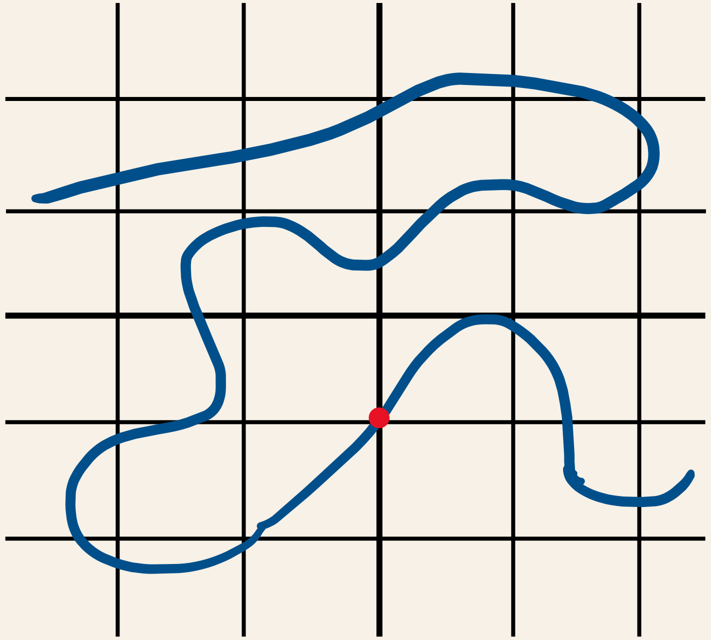

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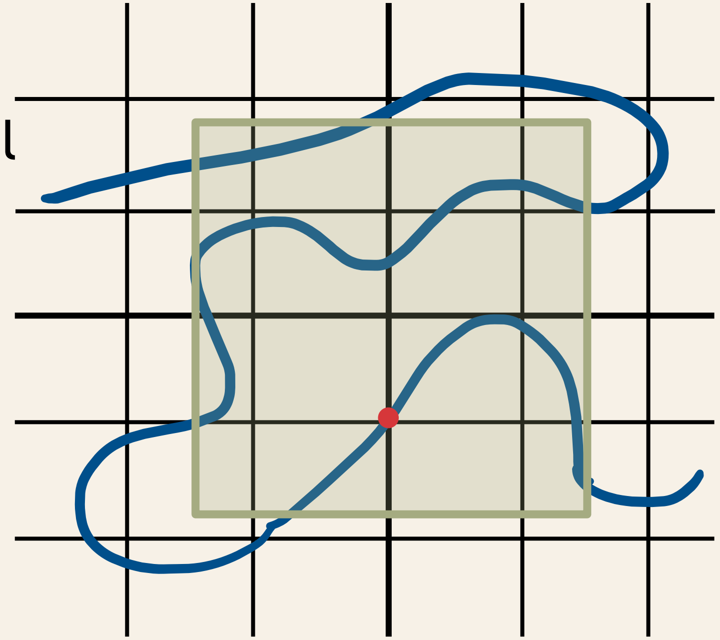
- Pila-Wilkie counting theorem: [Pila, Wilkie: 2006]

  "The number of lattice points in the transcendental part of a tame set grows sub-polynomially in the height"

$$\#\text{contained lattice points} < C(\epsilon)L^{\epsilon}$$



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$\ell_{\mathcal{M}} = 0, 1, 2$

elliptic curve, K3

$\mathrm{K3} \times \mathrm{K3}$, $\mathrm{CY}_3 \times T^2$

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CY_3, CY_4, ...

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- **Important:** the level can locally reduce along special “symmetry” loci, on which W becomes algebraic, due to appearance of additional Hodge tensors [Grimm, van de Heisteeg: 2024]

Conjecture 1: Counting W=0 vacua (simplified)

If $\ell_{\mathcal{M}} \geq 3$ the number of isolated* points in the set

$$(z^i, G_4): \quad G_4 \in H^4(Y_4, \mathbb{Z}) \cap H^{2,2}(Y_4, \mathbb{C}) \ , \quad \int_{Y_4} G_4 \wedge G_4 = L$$

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- For positive-dimensional vacuum loci, there are even stronger **finiteness** theorems, independent of L! [Baldi, Klingler, Ullmo: 2021]

The Tadpole Conjecture

$$\alpha h^{3,1} < \frac{1}{2}\int_{Y_4} G_4 \wedge G_4\Big|_{\text{all moduli stabilized}} \leq \frac{\chi(Y_4)}{24} \lesssim \frac{1}{4}h^{3,1}$$

- If α is too large -> cannot stabilize all moduli within bound! [Bena et al: 2021]

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- Restrict to Hodge classes (W=0) -> more tools for possible proof

Conjecture 3: Generalized tadpole conjecture for the Hodge locus

Consider a variation of polarized Hodge structure of weight D. Fix a positive integer L and write

$$E_{\text{Hodge}}(L) = \{(z^i, v) : v \in H^{k,k} \cap H_{\mathbb{Z}}\,, (v,v) \leq L\}\,, \qquad D = 2k\,, \tag{1}$$

We conjecture that for certain positive constants C_1, C_2, which are independent of L and $\dim \mathcal{M}$, the following holds: if

$$\dim \mathcal{M} > C_1 \qquad \text{and} \qquad \dim \mathcal{M} > C_2 \cdot L\,, \tag{2}$$

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- For Calabi-Yau fourfold case: expect $C_2 \sim \mathcal{O}(1)$
- Gives a criterion for when the Hodge locus does not contain points
  - Interesting for mathematicians
  - Deep finiteness results for Hodge tensors [Baldi, Klingler, Ullmo: 2021]
  - Hodge conjecture + reduction theorems as possible testing ground

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- Sharp o-minimality [Binyamini, Novikov: 2022]
- Assigns a "sharp complexity" (F,D) to a definable set, reflecting its geometric complexity

$$\#\ \text{connected components} < \text{poly}_{\mathrm{F}}(D)$$

- E.g. for polynomials: D = degree, F = number of variables

(see also [Grimm, Schlechter, van Vliet: 2023] for application in physics)

- $\mathbb{R}_{\mathrm{an,exp}}$ is **not** sharply o-minimal
- It is conjectured that periods are definable in a sharply o-minimal structure [Binyamini, Novikov: 2022]

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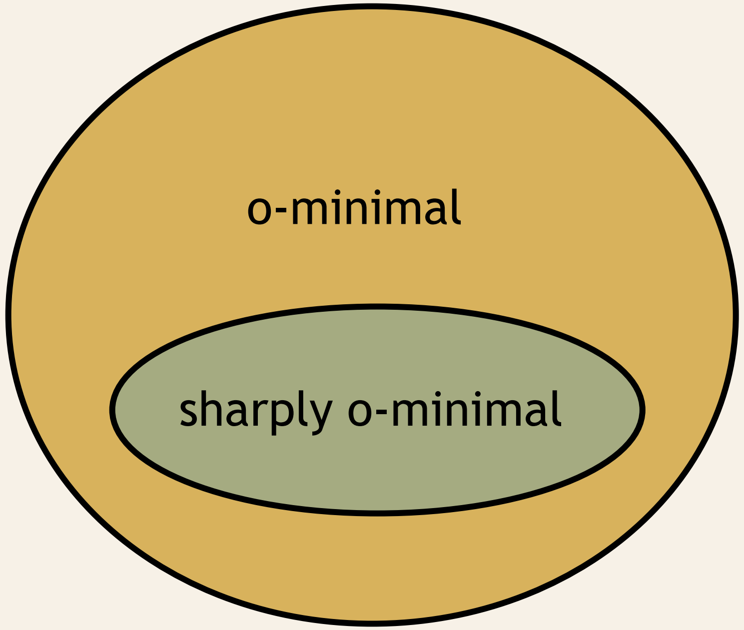

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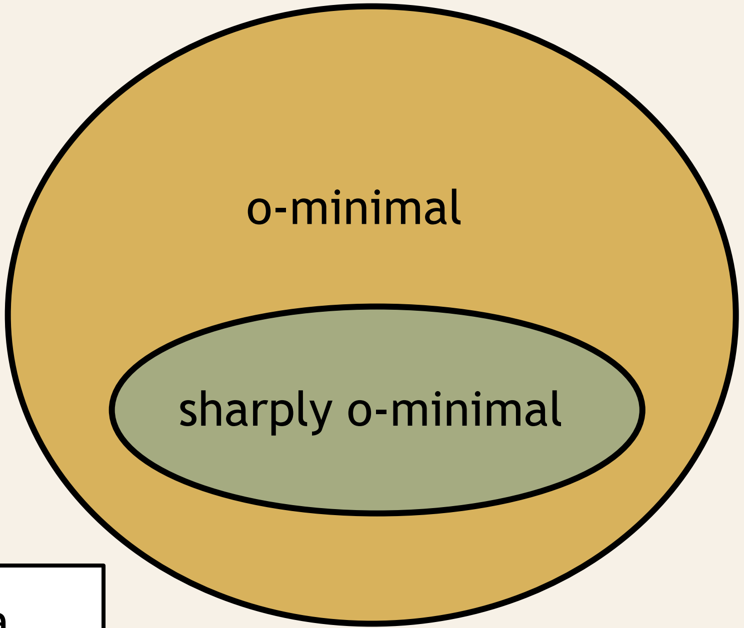

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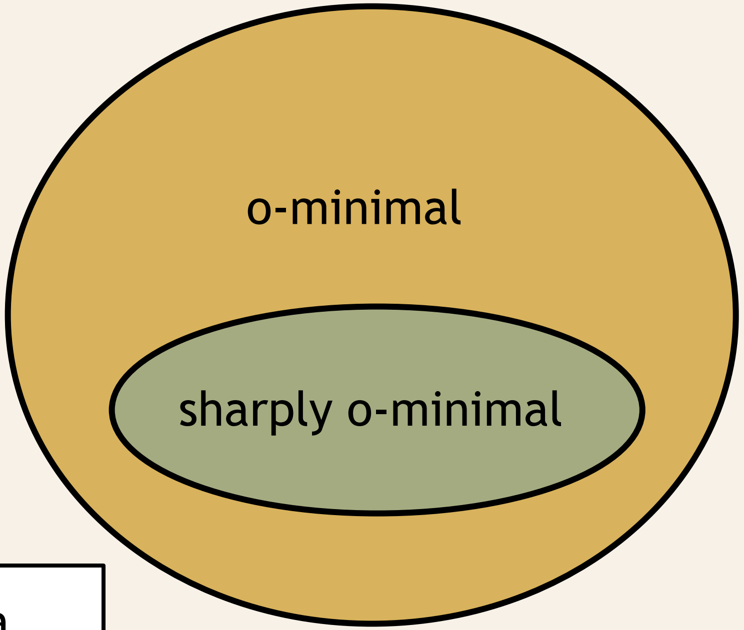

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- Results in sharp o-minimality + (F,D) -> new bounds on counting of vacua
- Formalizes the counting result of Ashok-Denef-Douglas

Conclusions

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