Non-minimal Elliptic Threefolds and the Distance Conjecture

Rafael Álvarez-García
work together with Seung-Joo Lee and Timo Weigand

arXiv:2310.07761, arXiv:2312.11611 and arXiv:240X.XXXXX

29th May 2024

Swamplandia 2024 — in Bavaria — , Kloster Seeon, Germany

Universität Hamburg
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Introduction and motivation

Swampland Distance Conjecture (SDC) [Ooguri, Vafa '06]

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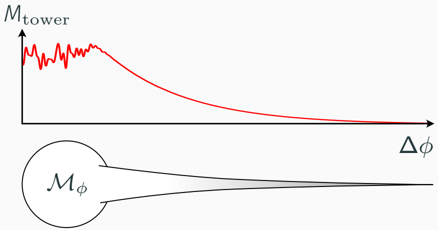
As a consequence, the effective description of the theory must break.



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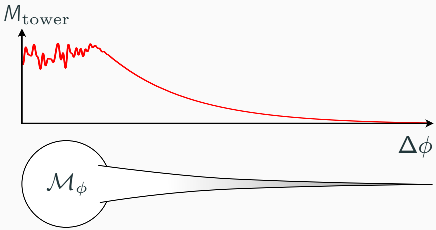
- **What theories do we encounter at infinite distance?**



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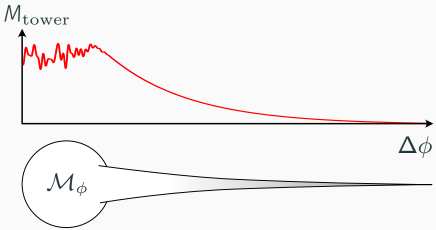

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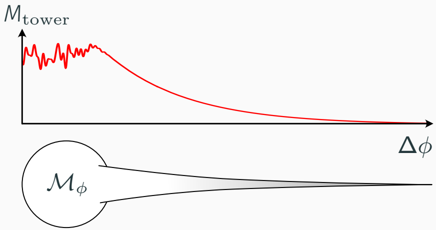

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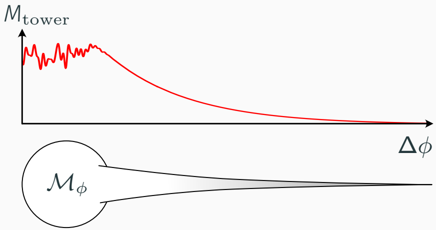

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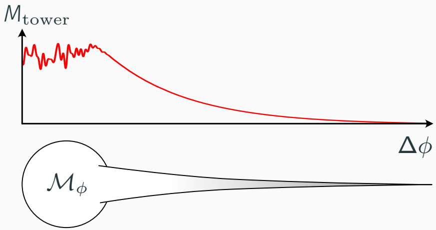

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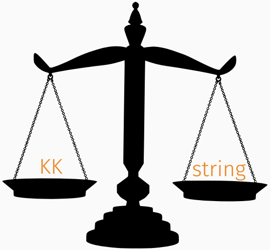
Emergent string limit: transition to a duality frame determined by a unique emergent critical weakly coupled string.



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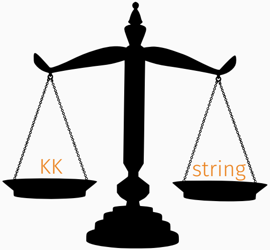
If correct, very strong constraints on the asymptotic theory:

- Bounds on exponential decay rates

  [Etheredge, Heidenreich, Kaya, Qiu, Rudelius '22]
- Behaviour of the species scale

  [van de Heisteeg, Vafa, Wiesner, (Wu) '22/'23][4], [Cribiori, Lüst, Staudt '22], [Cribiori, Lüst '23], [Cribiori, Lüst, Montella '23], [Basile, Cribiori, Lüst, Montella '24], [Marchesano, Melotti '22]
- Studies of the Emergence Proposal

  [Blumenhagen, (Cribiori), Gligovic, Paraskevopoulou '23][3]



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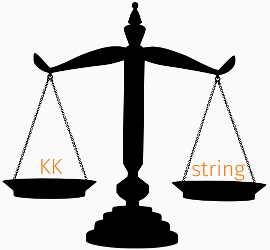

Confirmed in various non-trivial setups:

Kähler moduli F/M/IIA-theory in 6D/5D/4D	[Lee, Lerche, Weigand '18, '19, '20]
Complex structure of F-theory in 8D	[Lee, (Lerche), Weigand '21]
M-theory on G_2 manifolds	[Xu '20]
4D $\mathcal{N} = 1$ F-theory	[Lee, Lerche, Weigand '19] & [Kläwer, Lee, Weigand, Wiesner '20]
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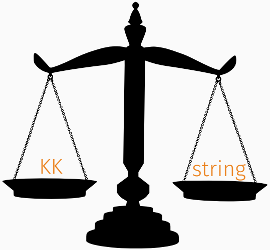

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- Elliptic fiber: τ profile.
- Base B: physical space-time.

$$y^2 = x^3 + fxz^4 + gz^6\,,$$

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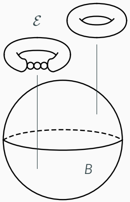

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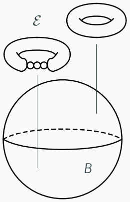

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- Singularities in codimension-one in B

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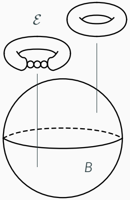

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  Gauge algebra associated with 7-branes
- Classification by Kodaira and Néron.

Algebra	Kodaira	ord(f)	ord(g)	ord(Δ)
A_n	I_{n+1}	0	0	$n+1$
D_n	I^*_{n-4}	2	3	$n+2$
E_6	IV^*	≥ 3	4	8
E_7	III^*	3	≥ 5	9
E_8	II^*	≥ 4	5	10
—	non-minimal	≥ 4	≥ 6	≥ 12

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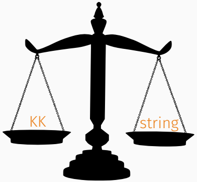

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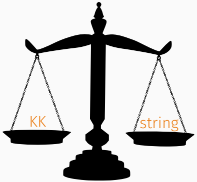

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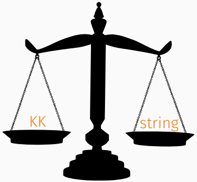

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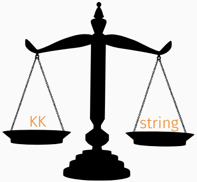

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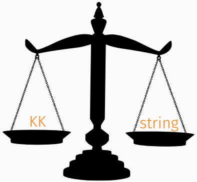

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Swampland and F-theoretic motivations

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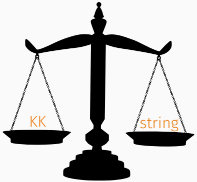

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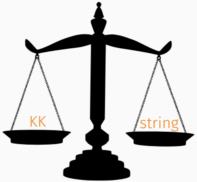

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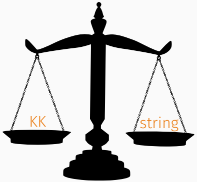

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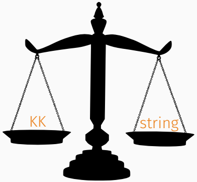

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Goal of this work

Understand the geometry and physics of the infinite-distance non-minimal singularities of CY_3.

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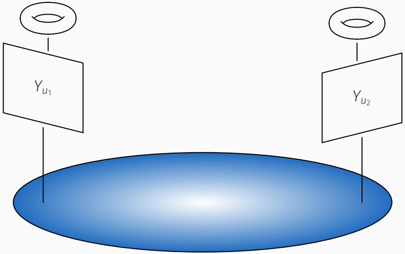

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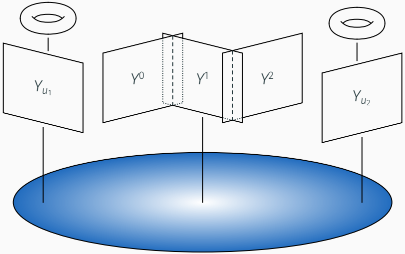

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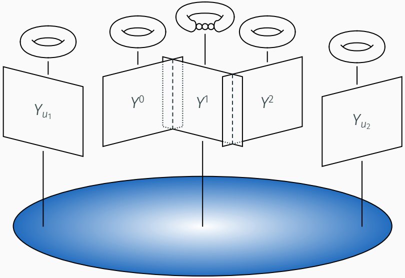

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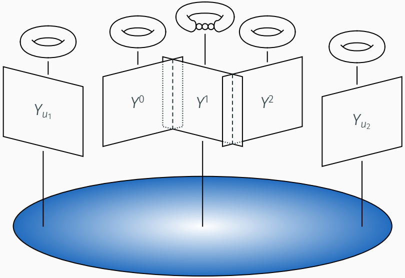

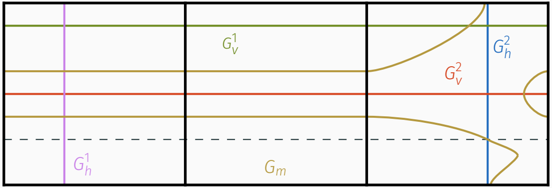

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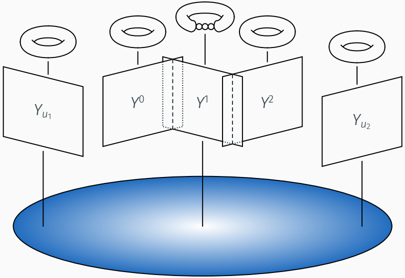

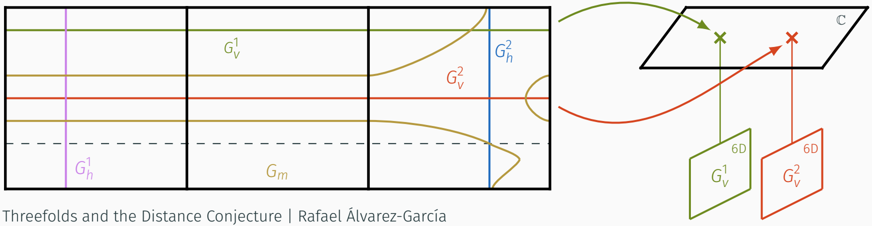

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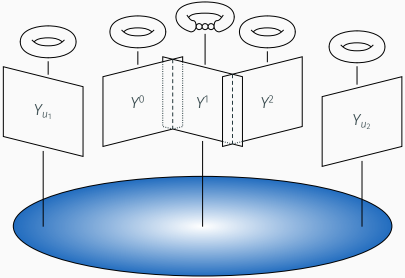
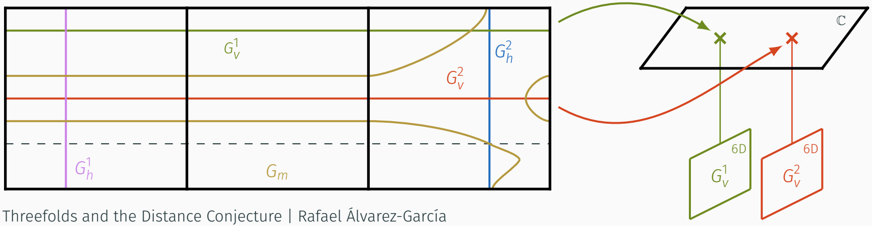


Geometric approach complementary to the asymptotic Hodge theory analysis initiated in [Grimm, Palti, Valenzuela '18].



Part I: Log Calabi-Yau Resolutions

Degenerations and modifications

Let $D := \{u \in \mathbb{C} : |u| < 1\}$ and $D^* := D \setminus \{0\}$.

Degeneration

A one-parameter family of varieties $\hat{\mathcal{Y}}$ together with a morphism $\hat{\rho} : \hat{\mathcal{Y}} \to D$ and fibers $\hat{Y}_u := \hat{\rho}^{-1}(u)$ with $u \in D$ in which we distinguish the central fiber $\hat{Y}_0$ is called a degeneration. We say that the elements $Y_{u \neq 0}$ of the family degenerate to Y_0.

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We are interested in non-minimal degenerations of elliptic threefolds, i.e.

$$\{y^2 = x^3 + f_u x z^4 + g_u z^6\}_{\mathbb{P}_{231}(\mathcal{E})}\,, \qquad u \in D$$

such that for some curve $\mathcal{C}$ we have

$$\mathrm{ord}_{\hat{\mathcal{Y}}}(f, g, \Delta)_{\mathcal{C}} \geq (4, 6, 12)\,.$$

Let $D := \{u \in \mathbb{C} : |u| < 1\}$ and $D^* := D \setminus \{0\}$.

Degeneration

A one-parameter family of varieties $\hat{\mathcal{Y}}$ together with a morphism $\hat{\rho} : \hat{\mathcal{Y}} \to D$ and fibers $\hat{Y}_u := \hat{\rho}^{-1}(u)$ with $u \in D$ in which we distinguish the central fiber $\hat{Y}_0$ is called a degeneration. We say that the elements $Y_{u\neq 0}$ of the family degenerate to Y_0.

We are interested in non-minimal degenerations of elliptic threefolds, i.e.

$$\{y^2 = x^3 + f_u x z^4 + g_u z^6\}_{\mathbb{P}_{231}(\mathcal{E})}\,, \qquad u \in D$$

such that for some curve $\mathcal{C}$ we have

$$\mathrm{ord}_{\hat{\mathcal{Y}}}(f, g, \Delta)_{\mathcal{C}} \geq (4, 6, 12)\,.$$

Endpoint of the limit

The geometrical representative Y_0 of the endpoint of the limit is not unique.

Example

Consider the Weierstrass model

$$f = s^3t^3(sv+tu)\left(suv^8+tuw^7+tv^3w^4+tv^2w^5+tvw^6\right)\,,$$

$$g = s^4t^5vw^5(sv+tu)^2\left(sw^5+tv^4+tv^3w+tv^2w^2+tvw^3\right)\,,$$

$$\Delta = s^8t^9(sv+tu)^3p_{4,24}([s:t],[v:w],u)\,.$$

$$\begin{array}{ccc} \mathcal{E} & \longrightarrow & \hat{\mathcal{Y}} \\ & & \downarrow \pi_{\text{ell}} \\ & & \hat{\mathcal{B}} \end{array}$$

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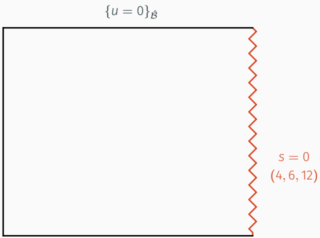
Non-minimal fibers over $\{s = u = 0\}_{\hat{\mathcal{B}}}$:

$$\mathrm{ord}_{\hat{\mathcal{Y}}} (f, g, \Delta)_{s=u=0} = (4, 6, 12) .$$



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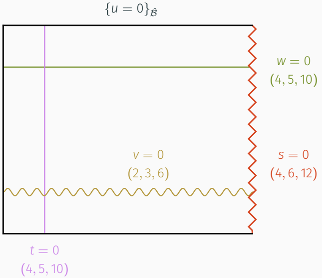
Vertical line of D_4 fibers in the central element:

$$\text{ord}_{\hat{Y}_0}(f, g, \Delta)_{v=0} = (2, 3, 6).$$



Semi-stable degenerations

We will find convenient geometrical representatives of the central fiber Y_0.

Semi-stable degenerations

A degeneration $\hat{\rho} : \hat{\mathcal{Y}} \to D$ is called semi-stable if $\hat{\mathcal{Y}}$ is smooth and such that the central fiber $\hat{Y}_0$ is reduced with components crossing normally.

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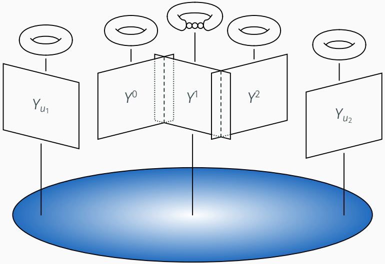

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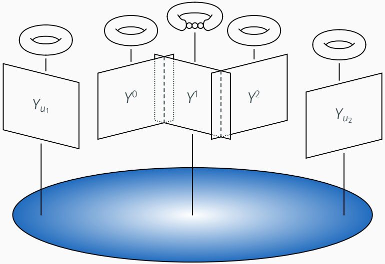
We are assured that we can always do it.

Semi-stable Reduction Theorem [Kempf, Knudsen, Mumford, Saint-Donat '73]

After a base change

$$\mu : D \longrightarrow D$$
$$u \longmapsto u^k ,$$

every degeneration admits a modification that is semi-stable.



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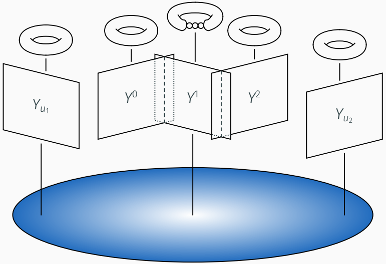
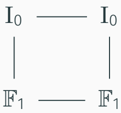


The theorem is not constructive, but we give the appropriate birational transformation for a variety of degeneration classes.

Example

“Semi-stable form” achieved for the example by:

- Base blow-up: $s \longmapsto se_1, \quad u \longmapsto e_0 e_1.$
- Line bundle shift: $(f, g, \Delta) \longmapsto (e_1^{-4} f, e_1^{-6} g, e_1^{-12} \Delta).$



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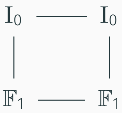


Left component $\{e_0 = 0\}_{\mathcal{B}}$:

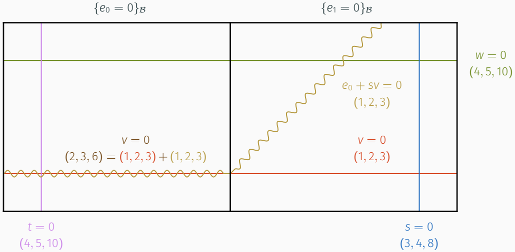
$$f_0 = t^4 v^2 w^4 \left(v^2 + vw + w^2\right),$$

$$g_0 = t^5 v^3 w^5 \left(e_1 w^5 + t v^4 + t v^3 w + t v^2 w^2 + t v w^3\right).$$

Right component $\{e_1 = 0\}_{\mathcal{B}}$:

$$f_1 = s^3 v w^4 \left(v^2 + vw + w^2\right) (e_0 + sv),$$

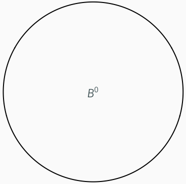
$$g_1 = s^4 v^2 w^5 (v + w) \left(v^2 + w^2\right) (e_0 + sv)^2.$$



In [RAG, Lee, Weigand '23] we are more general. Here, we focus on single infinite-distance limits.

Single infinite-distance limits

Roughly, those degenerations of elliptic threefolds with non-intersecting non-minimal curves.



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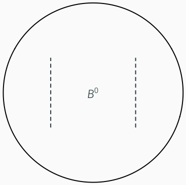

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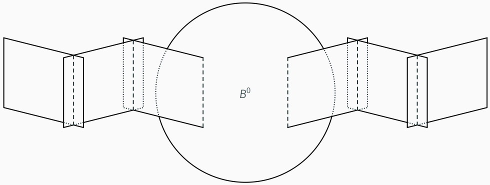

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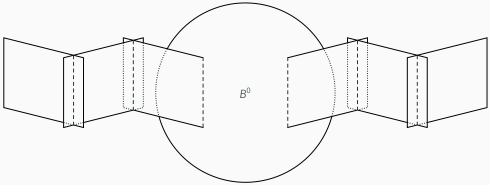

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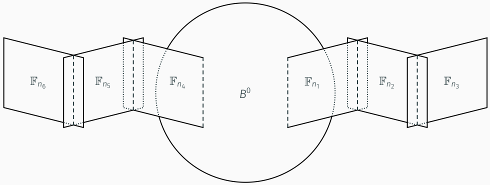
- The bases of the exceptional components are Hirzebruch surfaces $\left\{\mathbb{F}_{|C_p \cdot_{B^i} C_p|}\right\}_{1 \leq p \leq P}$.



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7-brane content

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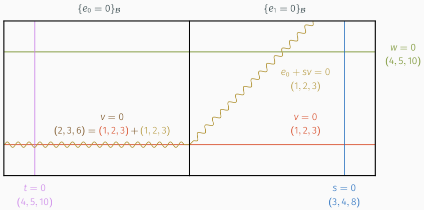

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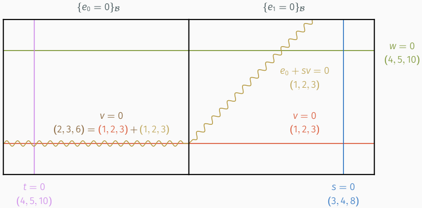
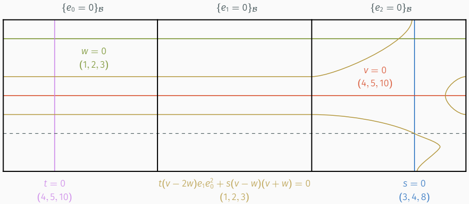


Locally reducible discriminant components:

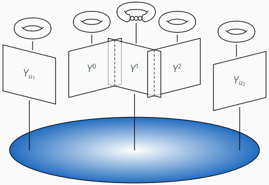


Intermediate summary

$\{y^2 = x^3 + f_u x z^4 + g_u z^6\}_{\mathbb{P}_{231}(\mathcal{E})}\,, \quad u \in D \longrightarrow$ Semi-stable form

$\downarrow$

Open-chain resolution $\longrightarrow$ Read off the physics



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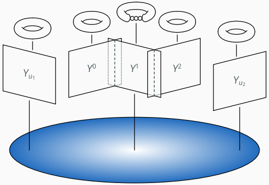

Some aspects not discussed today:

- Base change can reveal obscured infinite-distance limits.
- Heterotic K3 non-minimal singularities ⟷ codimension-one non-minimal degenerations.
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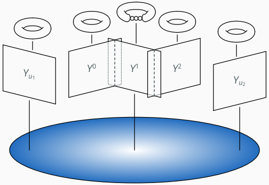

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- Some codimension-one non-minimal degenerations are finite-distance limits.

Explicit analysis of strictly non-minimal degenerations: [RAG, Lee, Weigand (to appear)].

Part II: Asymptotic Physics

Genus-zero single infinite-distance limit degenerations of Hirzebruch models can be of four types.

We analyze their asymptotic physics in Part II.

	Non-minimal curves	Central component structure	Component line bundles and discriminants
Case A (horizontal)	$\mathscr{C}_1 = \{h\}$ $\mathscr{C}_1 = \{h + nf\}$ $\mathscr{C}_2 = \{h, h + nf\}$	$\mathrm{I}_{n_0} - \cdots - \mathrm{I}_{n_p} - \cdots - \mathrm{I}_{n_P}$ $\mathbb{F}_n - \cdots - \mathbb{F}_n - \cdots - \mathbb{F}_n$	$\mathcal{L}_0 = S_0 + (2+n)V_0$ $\mathcal{L}_p = 2V_p$ $\mathcal{L}_P = S_P + 2V_P$ $\Delta'_0 = (12 + n_0 - n_1)S_0 + (24 + 12n)V_0$ $\Delta'_p = (2n_p - n_{p-1} - n_{p+1})S_p + (24 + n(n_p - n_{p-1}))V_p$ $\Delta'_P = (12 + n_P - n_{P-1})S_P + (24 + n(n_P - n_{P-1}))V_P$
Case B (vertical)	$\mathscr{C}_1 = \{f\}$	$\mathrm{I}_{n_0} - \cdots - \mathrm{I}_{n_p} - \cdots - \mathrm{I}_{n_P}$ $\mathbb{F}_n - \cdots - \mathbb{F}_0 - \cdots - \mathbb{F}_0$	$\mathcal{L}_0 = 2S_0 + (1+n)W_0$ $\mathcal{L}_p = 2S_p$ $\mathcal{L}_P = 2S_P + W_P$ $\Delta'_0 = 24S_0 + (12 + 12n + n_0 - n_1)W_0$ $\Delta'_p = 24S_p + (2n_p - n_{p-1} - n_{p+1})W_p$ $\Delta'_P = 24S_P + (12 + n_P - n_{P-1})W_P$
Case C	$\mathscr{C}_1 = \{h + (n+\alpha)f\}$ $\alpha = 1$ with $n \leq 6$ $\alpha = 2$ with $n = 0$	$\mathrm{I}_{n_0} - \cdots - \mathrm{I}_{n_p} - \cdots - \mathrm{I}_{n_P}$ $\mathbb{F}_{n+2\alpha} - \cdots - \mathbb{F}_{n+2\alpha} - \cdots - \mathbb{F}_n$	$\mathcal{L}_0 = S_0 + (2 + (n+2\alpha))V_0$ $\mathcal{L}_p = 2V_p$ $\mathcal{L}_P = S_P + (2-\alpha)V_P$ $\Delta'_0 = (12 + n_0 - n_1)S_0 + (24 + 12(n+2\alpha))V_0$ $\Delta'_p = (2n_p - n_{p-1} - n_{p+1})S_p + (24 + (n+2\alpha)(n_p - n_{p-1}))V_p$ $\Delta'_P = (12 + n_P - n_{P-1})S_P + ((24 - 12\alpha) + (n+\alpha)(n_P - n_{P-1}))V_P$
Case D	$C = 2h + bf$ $(n,b) = (0,1)$ $(n,b) = (1,2)$	$\mathrm{I}_{n_0} - \cdots - \mathrm{I}_{n_p} - \cdots - \mathrm{I}_{n_P}$ $\mathbb{F}_4 - \cdots - \mathbb{F}_4 - \cdots - \mathbb{F}_n$	$\mathcal{L}_0 = S_0 + (2 + (n+4))V_0$ $\mathcal{L}_p = 2V_p$ $\mathcal{L}_P = V_P$ $\Delta'_0 = (12 + n_0 - n_1)S_0 + (24 + 12 \cdot 4)V_0$ $\Delta'_p = (2n_p - n_{p-1} - n_{p+1})S_p + (24 + 4(n_p - n_{p-1}))V_p$ $\Delta'_P = 2(n_P - n_{P-1})S_P + (12 + (n+1)(n_P - n_{P-1}))V_P$

Degenerations of Hirzebruch models

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Degenerations of Hirzebruch models

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We analyze their asymptotic physics in Part II.

Here we focus on Case A (horizontal models):

- They are relative versions of the 8D models.

	Non-minimal curves	Central component structure	Component line bundles and discriminants
Case A (horizontal)	$\mathscr{C}_1 = \{h\}$ $\mathscr{C}_1 = \{h + nf\}$ $\mathscr{C}_2 = \{h, h + nf\}$	$\begin{array}{ccccccccc} \mathrm{I}_{n_0} & — & \cdots & — & \mathrm{I}_{n_p} & — & \cdots & — & \mathrm{I}_{n_P} \\ \vert & & & & \vert & & & & \vert \\ \mathbb{F}_n & — & \cdots & — & \mathbb{F}_n & — & \cdots & — & \mathbb{F}_n \end{array}$	$\mathcal{L}_0 = S_0 + (2+n)V_0$ $\mathcal{L}_p = 2V_p$ $\mathcal{L}_P = S_P + 2V_P$ $\Delta'_0 = (12 + n_0 - n_1)S_0 + (24 + 12n)V_0$ $\Delta'_p = (2n_p - n_{p-1} - n_{p+1})S_p + (24 + n(n_p - n_{p-1}))V_p$ $\Delta'_P = (12 + n_P - n_{P-1})S_P + (24 + n(n_P - n_{P-1}))V_P$
Case B (vertical)	$\mathscr{C}_1 = \{f\}$	$\begin{array}{ccccccccc} \mathrm{I}_{n_0} & — & \cdots & — & \mathrm{I}_{n_p} & — & \cdots & — & \mathrm{I}_{n_P} \\ \vert & & & & \vert & & & & \vert \\ \mathbb{F}_n & — & \cdots & — & \mathbb{F}_0 & — & \cdots & — & \mathbb{F}_0 \end{array}$	$\mathcal{L}_0 = 2S_0 + (1+n)W_0$ $\mathcal{L}_p = 2S_p$ $\mathcal{L}_P = 2S_P + W_P$ $\Delta'_0 = 24S_0 + (12 + 12n + n_0 - n_1)W_0$ $\Delta'_p = 24S_p + (2n_p - n_{p-1} - n_{p+1})W_p$ $\Delta'_P = 24S_P + (12 + n_P - n_{P-1})W_P$
Case C	$\mathscr{C}_1 = \{h + (n+\alpha)f\}$ $\alpha = 1 \quad \text{with} \quad n \leq 6$ $\alpha = 2 \quad \text{with} \quad n = 0$	$\begin{array}{ccccccccc} \mathrm{I}_{n_0} & — & \cdots & — & \mathrm{I}_{n_p} & — & \cdots & — & \mathrm{I}_{n_P} \\ \vert & & & & \vert & & & & \vert \\ \mathbb{F}_{n+2\alpha} & — & \cdots & — & \mathbb{F}_{n+2\alpha} & — & \cdots & — & \mathbb{F}_n \end{array}$	$\mathcal{L}_0 = S_0 + (2 + (n+2\alpha))V_0$ $\mathcal{L}_p = 2V_p$ $\mathcal{L}_P = S_P + (2-\alpha)V_P$ $\Delta'_0 = (12 + n_0 - n_1)S_0 + (24 + 12(n+2\alpha))V_0$ $\Delta'_p = (2n_p - n_{p-1} - n_{p+1})S_p + (24 + (n+2\alpha)(n_p - n_{p-1}))V_p$ $\Delta'_P = (12 + n_P - n_{P-1})S_P + ((24 - 12\alpha) + (n+\alpha)(n_P - n_{P-1}))V_P$
Case D	$C = 2h + bf$ $(n, b) = (0, 1)$ $(n, b) = (1, 2)$	$\begin{array}{ccccccccc} \mathrm{I}_{n_0} & — & \cdots & — & \mathrm{I}_{n_p} & — & \cdots & — & \mathrm{I}_{n_P} \\ \vert & & & & \vert & & & & \vert \\ \mathbb{F}_4 & — & \cdots & — & \mathbb{F}_4 & — & \cdots & — & \mathbb{F}_n \end{array}$	$\mathcal{L}_0 = S_0 + (2 + (n+4))V_0$ $\mathcal{L}_p = 2V_p$ $\mathcal{L}_P = V_P$ $\Delta'_0 = (12 + n_0 - n_1)S_0 + (24 + 12 \cdot 4)V_0$ $\Delta'_p = (2n_p - n_{p-1} - n_{p+1})S_p + (24 + 4(n_p - n_{p-1}))V_p$ $\Delta'_P = 2(n_P - n_{P-1})S_P + (12 + (n+1)(n_P - n_{P-1}))V_P$

Degenerations of Hirzebruch models

Genus-zero single infinite-distance limit degenerations of Hirzebruch models can be of four types.

We analyze their asymptotic physics in Part II.

Here we focus on Case A (horizontal models):

- They are relative versions of the 8D models.
- Some of them have controlled heteretic duals.

	Non-minimal curves	Central component structure	Component line bundles and discriminants
Case A (horizontal)	$\mathscr{C}_1 = \{h\}$ $\mathscr{C}_1 = \{h + nf\}$ $\mathscr{C}_2 = \{h, h + nf\}$	$\mathrm{I}_{n_0} - \cdots - \mathrm{I}_{n_p} - \cdots - \mathrm{I}_{n_P}$ $\mid \quad \mid \quad \mid$ $\mathbb{F}_n - \cdots - \mathbb{F}_n - \cdots - \mathbb{F}_n$	$\mathcal{L}_0 = S_0 + (2+n)V_0$ $\mathcal{L}_p = 2V_p$ $\mathcal{L}_P = S_P + 2V_P$ $\Delta'_0 = (12 + n_0 - n_1)S_0 + (24 + 12n)V_0$ $\Delta'_p = (2n_p - n_{p-1} - n_{p+1})S_p + (24 + n(n_p - n_{p-1}))V_p$ $\Delta'_P = (12 + n_P - n_{P-1})S_P + (24 + n(n_P - n_{P-1}))V_P$
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Kulikov Type II.a models

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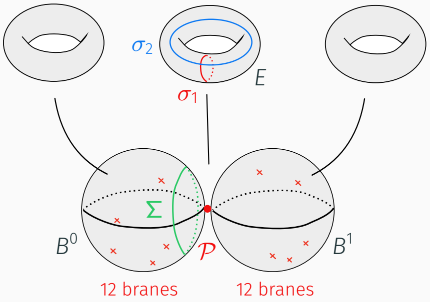

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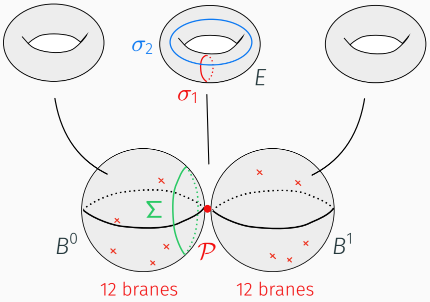
- All components have codimension-zero I_0 type fibers; no local weak coupling.



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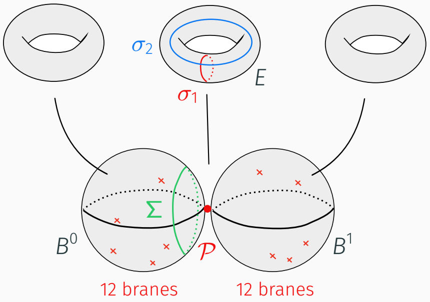

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- All components have codimension-zero I_0 type fibers; no local weak coupling.
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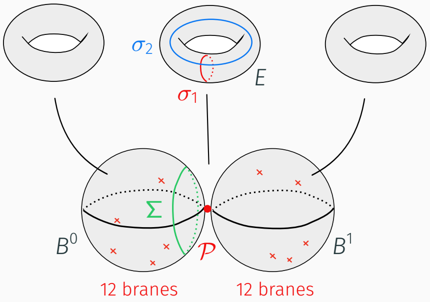

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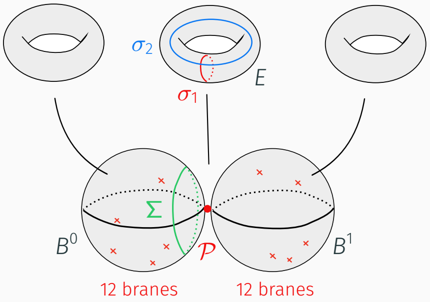

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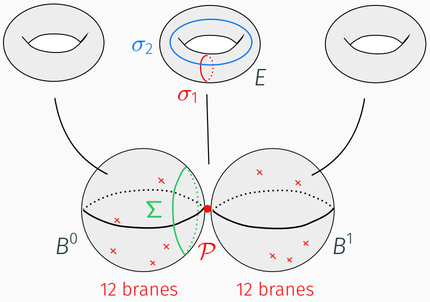

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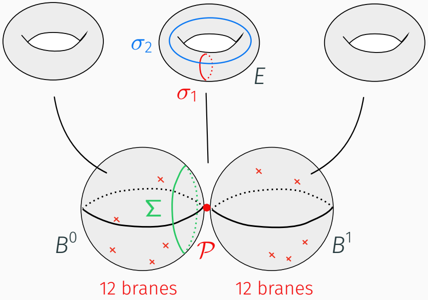

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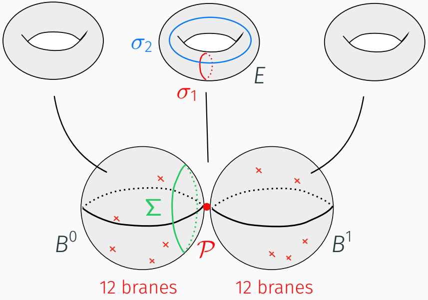

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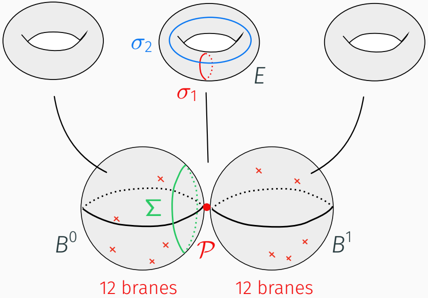

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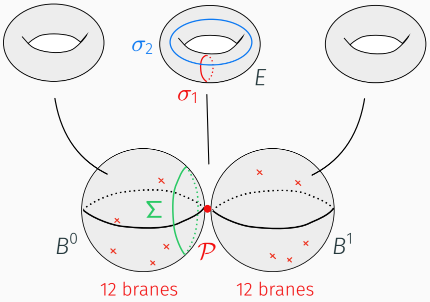

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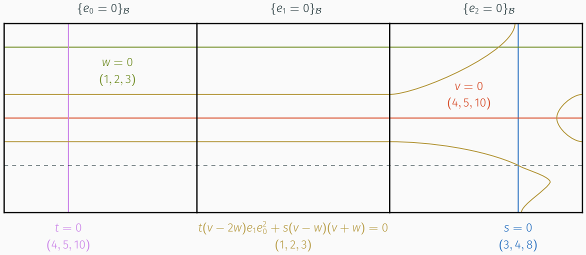
- Double loop enhancement $G_\infty = \left(\hat{E}_9 \oplus \hat{E}_9\right) / \sim$ $\Rightarrow$ After decompactification: $G_{10D} = E_8 \oplus E_8$.

Horizontal Type II.a models: 7-brane types



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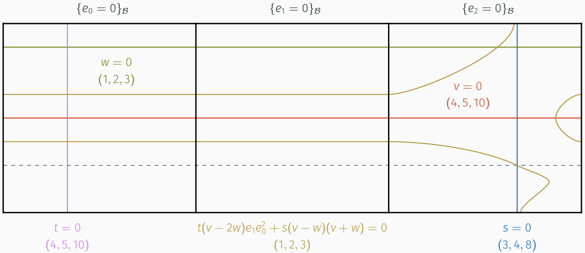

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  Localized in one of the base components

Analogue of the 8D 7-branes

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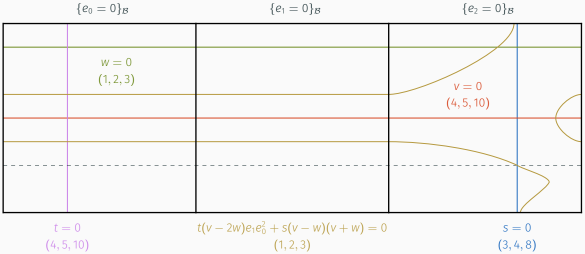

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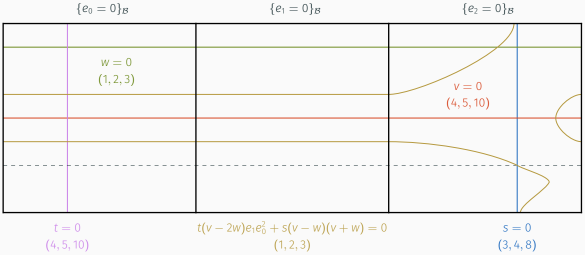

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  Recombination of the previous two types

Horizontal Type II.a models: generic vertical slices

Horizontal Type II.a limits

Horizontal limits with $\mathrm{I}_0 - \cdots - \mathrm{I}_0$ are the relative version of 8D Type II.a limits.

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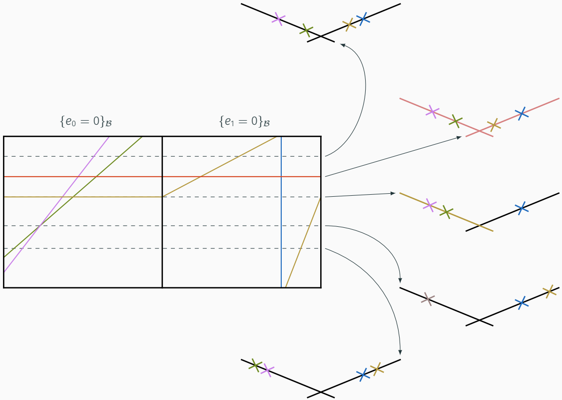

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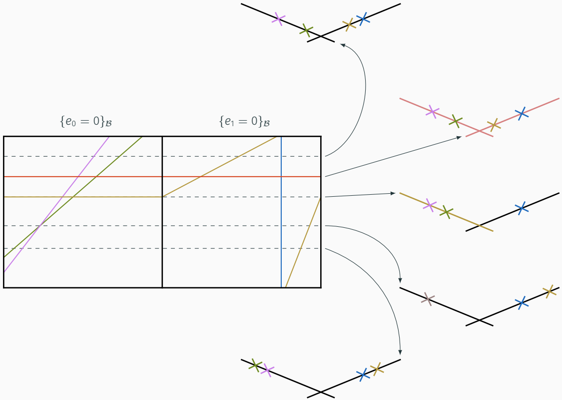

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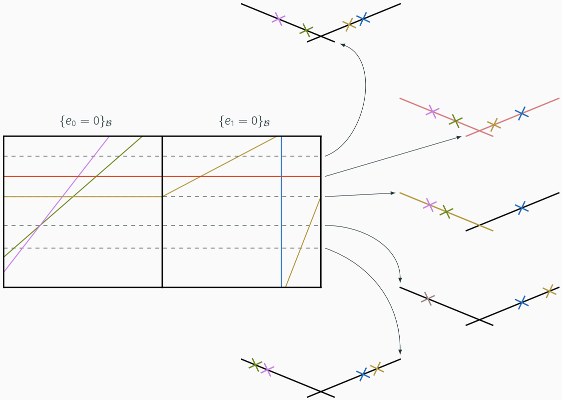

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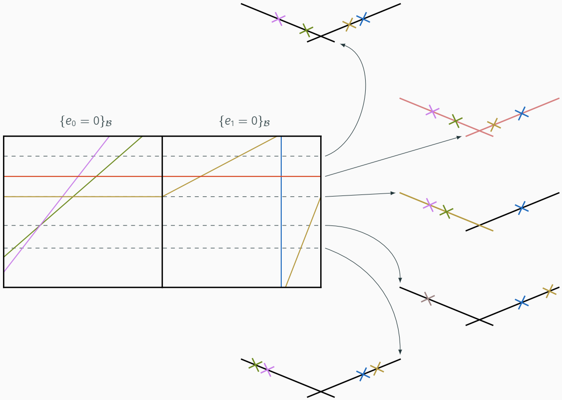
- The picture fails at 24 points

$$\Delta_0' \cdot S_0 = \Delta_1' \cdot T_1 = 24\,,$$

(the singular fibers of the het. K3).



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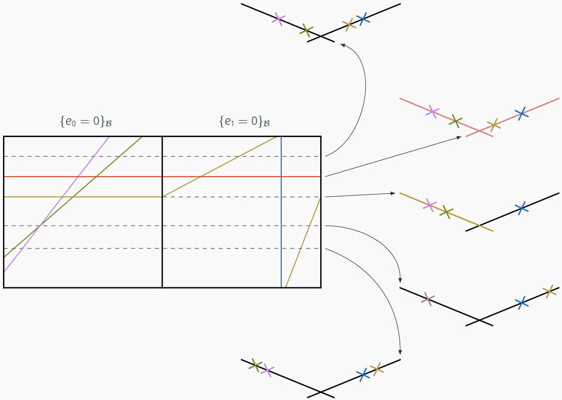

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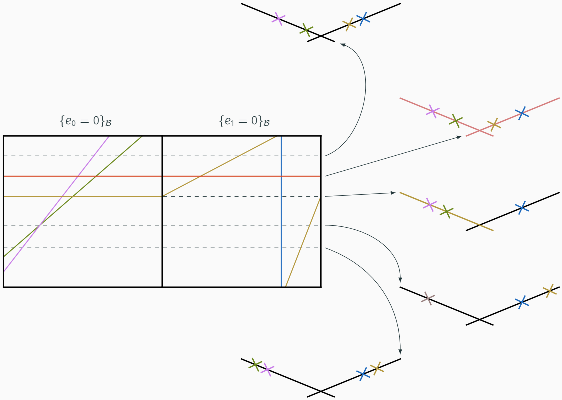

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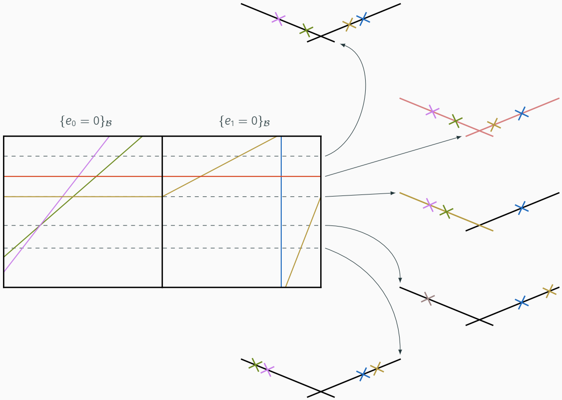

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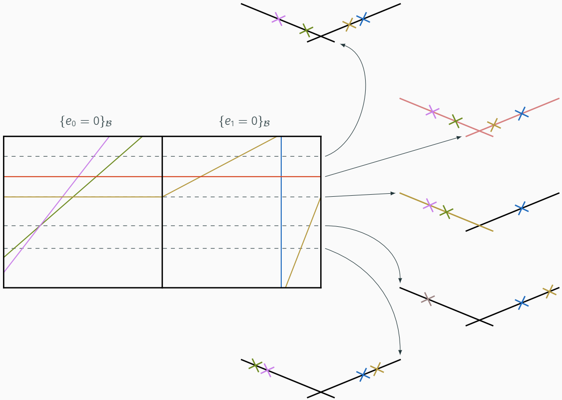

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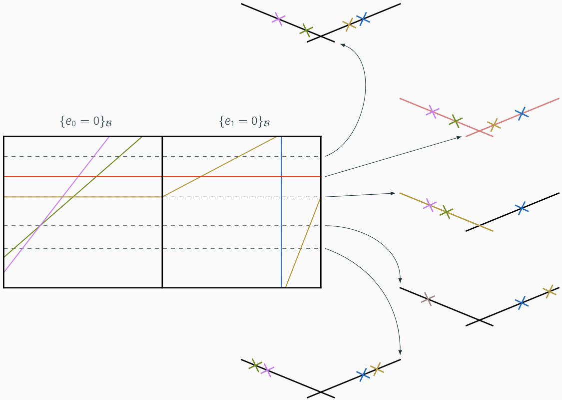

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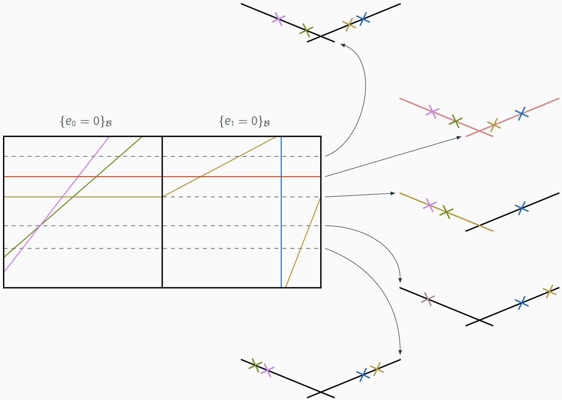

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We would like to determine when global weak coupling limits possible:

$$\textbf{Global weak coupling limit} \quad \Leftrightarrow \quad \mathrm{I}_{n_0} - \cdots - \mathrm{I}_{n_P} \quad \text{with} \quad n_p > 0, \forall p.$$

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Horizontal models admit global weak coupling iff they are constructed over $\mathbb{F}_n$ with $0 \leq n \leq 4$.

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This can be argued for in several ways:

- a physical argument,
- the geometry of the central fiber,
- and the Sen-limits of Tate models.

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Horizontal models admit global weak coupling iff they are constructed over $\mathbb{F}_n$ with $0 \leq n \leq 4$.

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Local weak coupling requires an accidental cancellation structure

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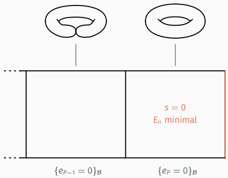

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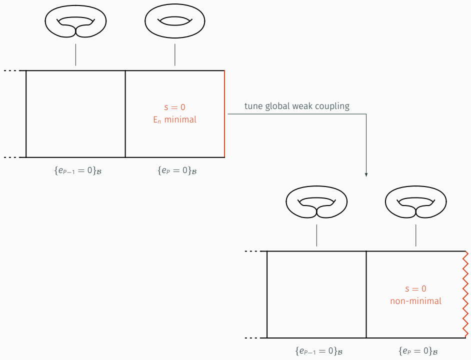

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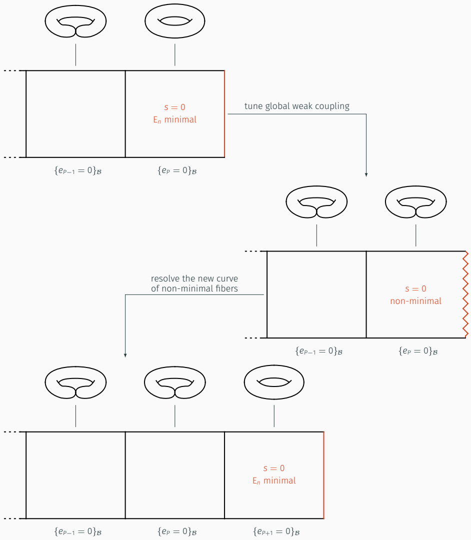

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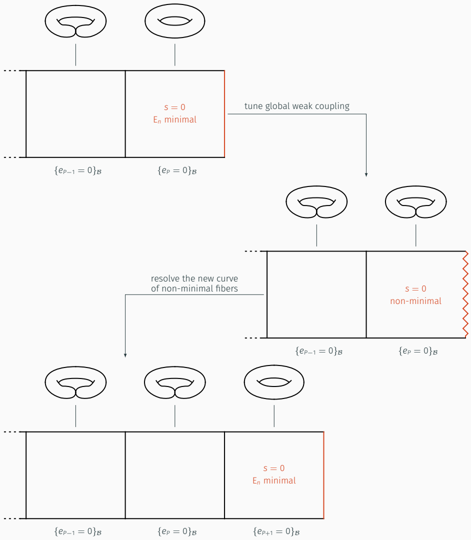
Cases B & C: $n \leq 2$, **Case D:** $n \leq 1$.



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  $\updownarrow$

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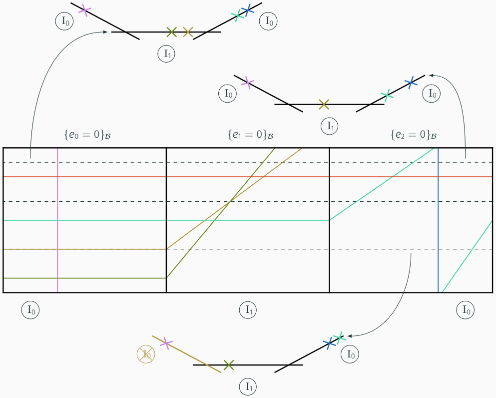

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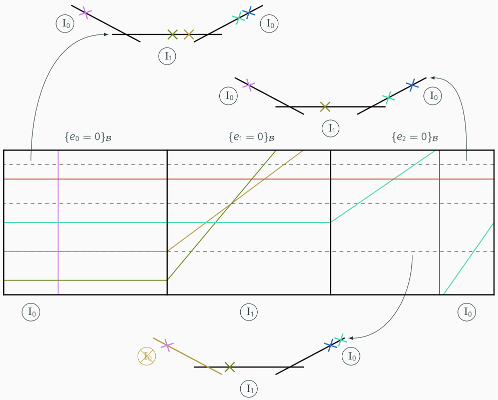

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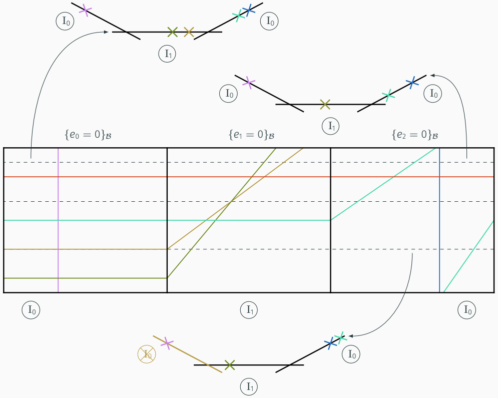

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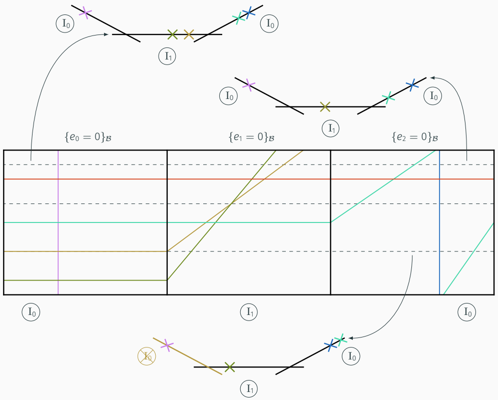

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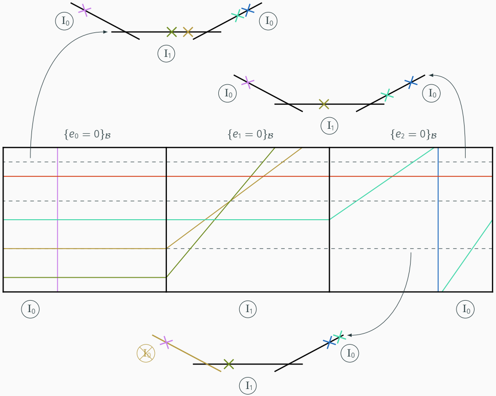

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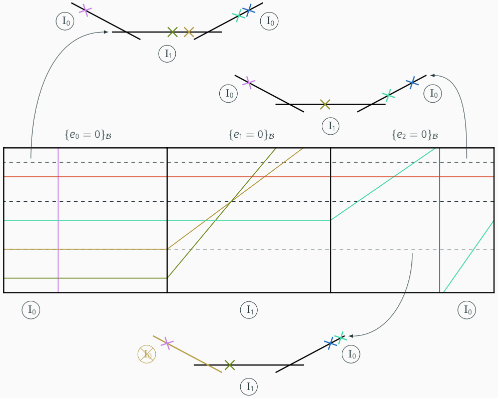

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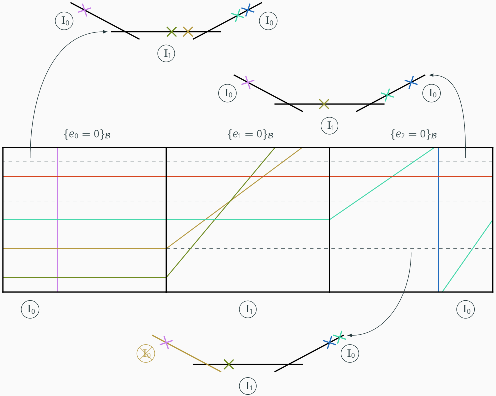

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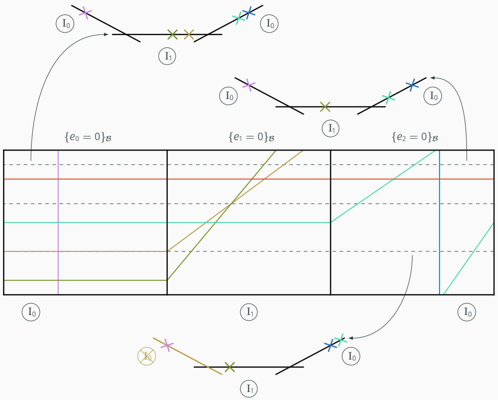

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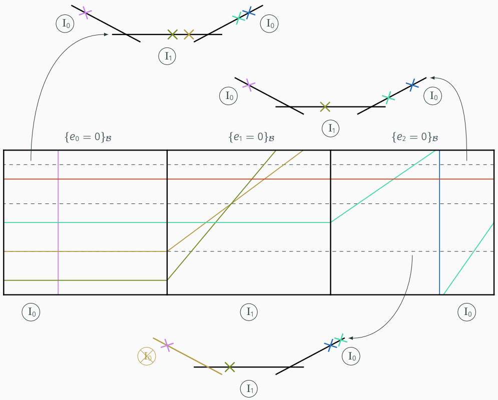
- Constructive proof of the existence of the birational transformations for degenerations of Hirzebruch models [RAG, Lee, Weigand (to appear)].



Thank you!

The example presents a discrepancy between the family and component vanishing orders:

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The role of the base change transformation

$$\begin{aligned} \delta_k : D &\longrightarrow D \\ u &\longmapsto u^k\,, \end{aligned} \quad \sim \quad \text{“taking the limit at a faster rate”}$$

is to reveal these infinite-distance limits.

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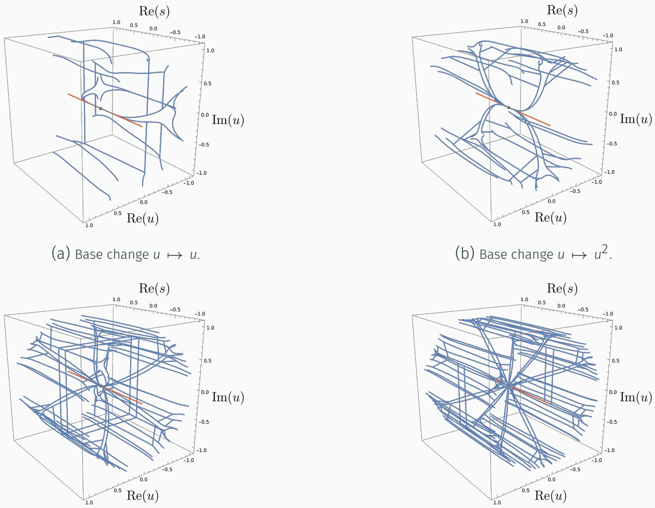

(a) Base change $u \mapsto u$.

(b) Base change $u \mapsto u^2$.

(c) Base change $u \mapsto u^4$.

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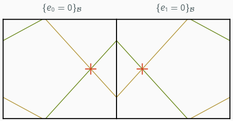

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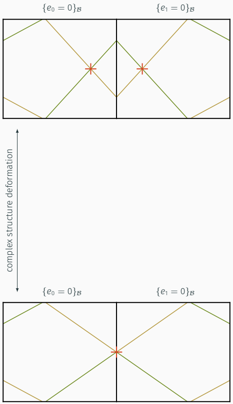

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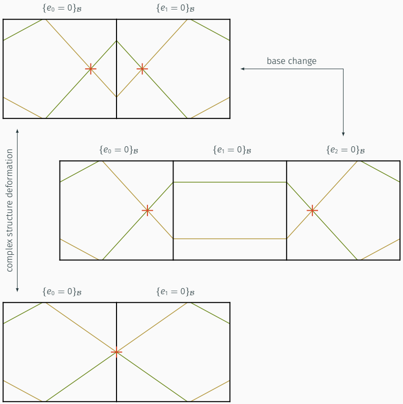

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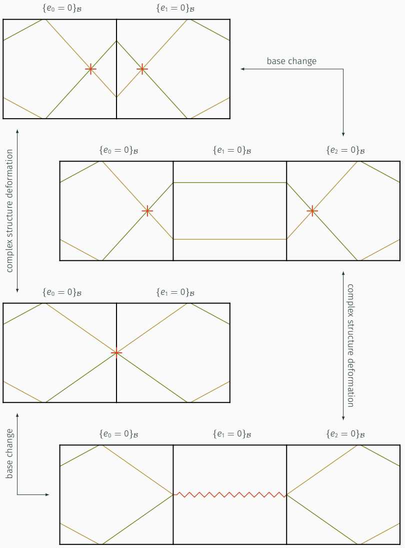

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In [RAG, Lee, Weigand (to appear)] we explicitly study the composition of base change, blow-up and blow-down transformations necessary to achieve this and improve on the K3 results of [Lee, Weigand '21].

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