# Axions in the dark dimension

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Harvard University
Swamplandia, May 29, 2024

based on 2404.15414 with Cumrun Vafa

# Summary

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A localized QCD axion in the dark dimension scenario is on the precipice of being detected experimentally.

# Outline

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**Strong CP problem: why is this number so small?**

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If this is the sole contribution to the QCD axion potential, then $\langle\theta\rangle = 0$ and the strong CP problem is solved: the axion dynamically relaxes the neutron EDM.

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Experiments are sensitive to the axion-photon coupling:

$$g_{a\gamma\gamma} = \frac{\alpha_{\rm EM}}{2\pi f_a}$$

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$$g_{a\gamma\gamma} \lesssim 3.3 \times 10^{-12}\ \mathrm{GeV}^{-1}$$

i.e. $f_a \gtrsim 3.5 \times 10^{8}\ \mathrm{GeV}$

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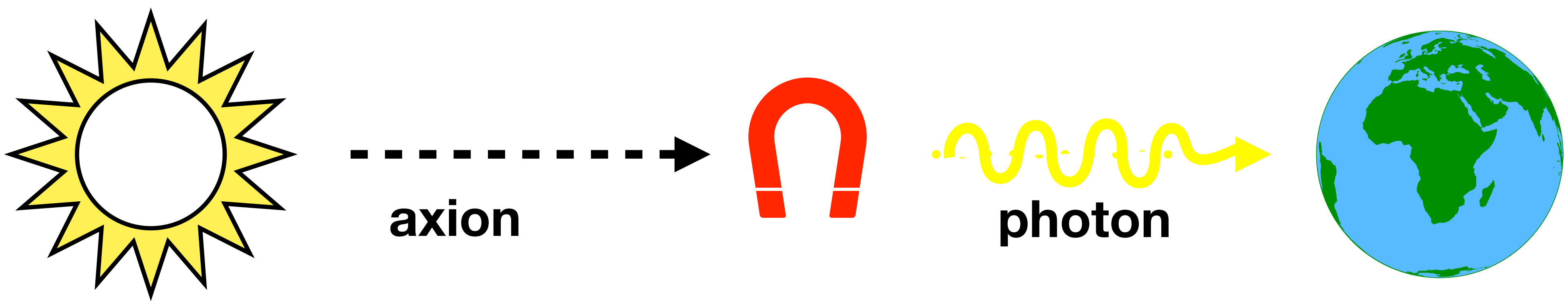

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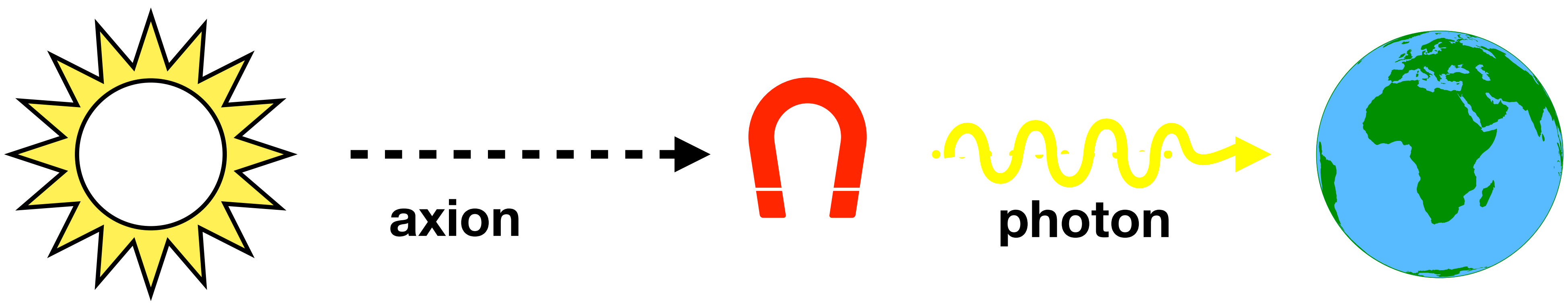


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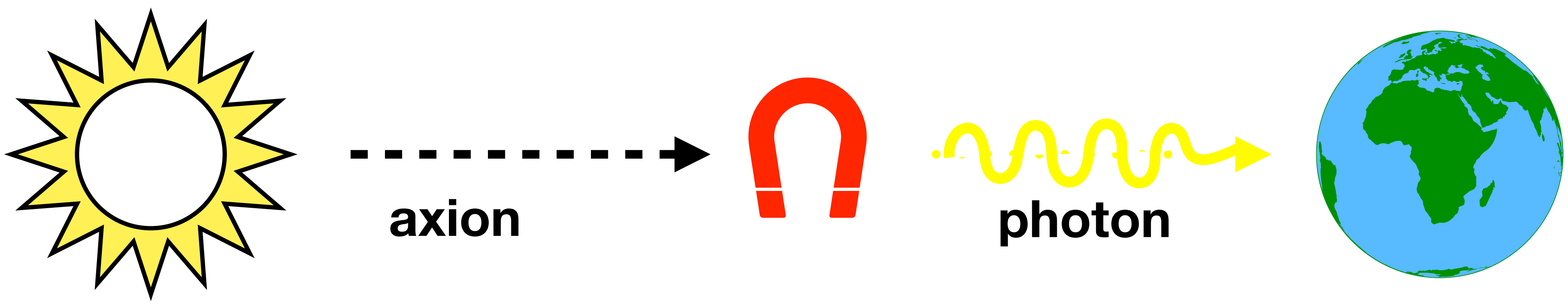


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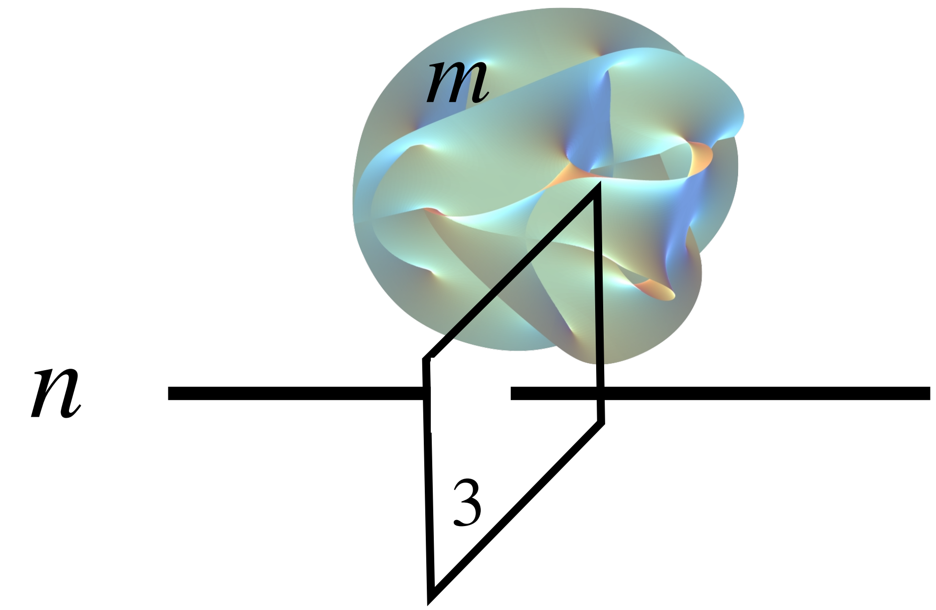

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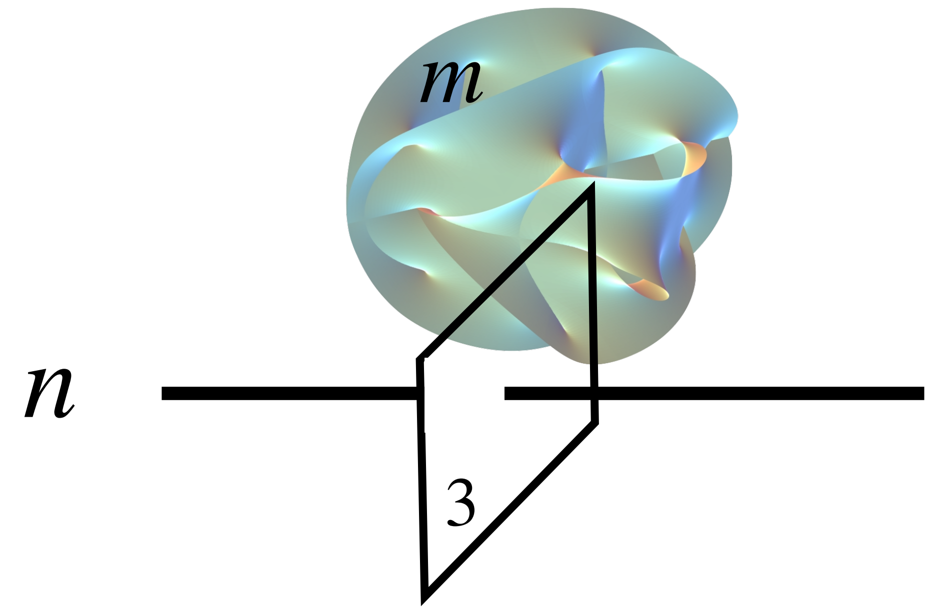


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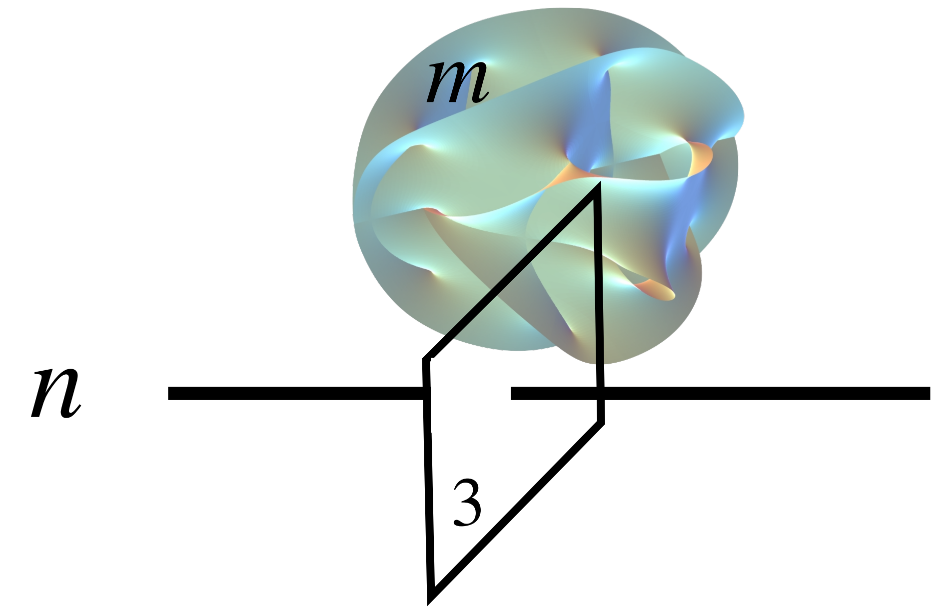


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This axion can propagate in $3 + p$ dimensions, with $p \geq 0$.

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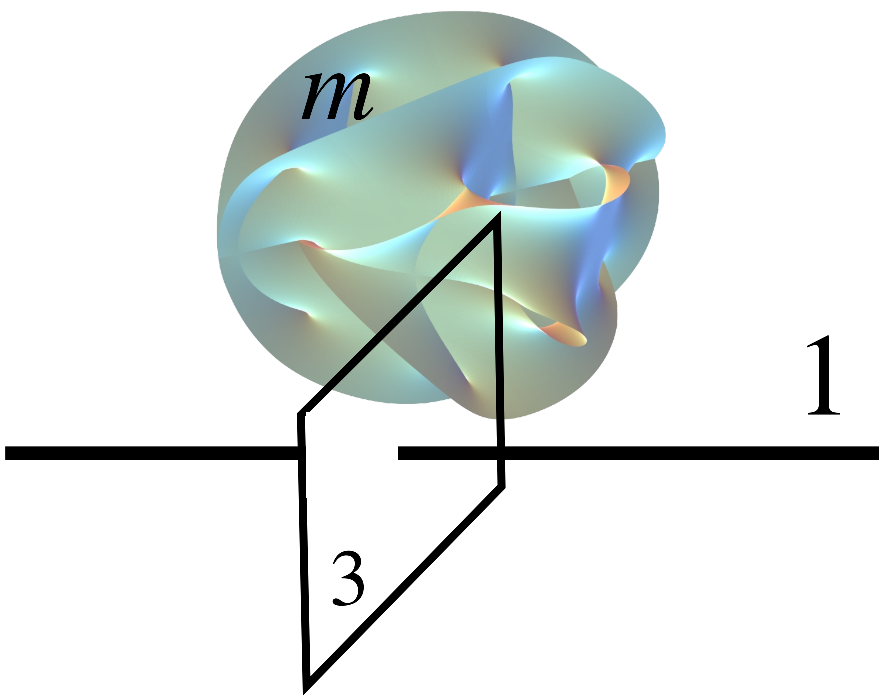


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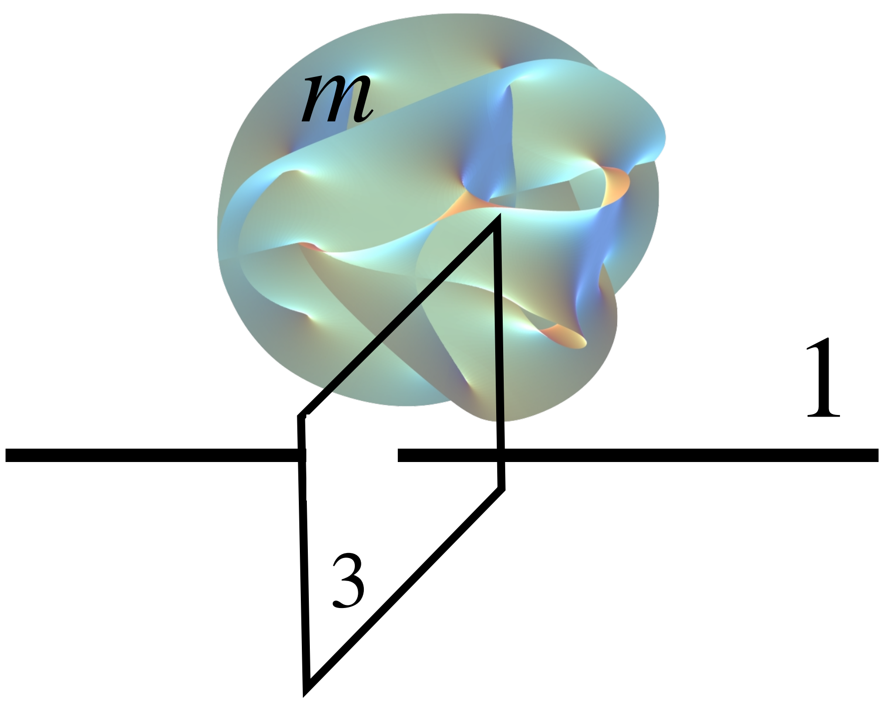


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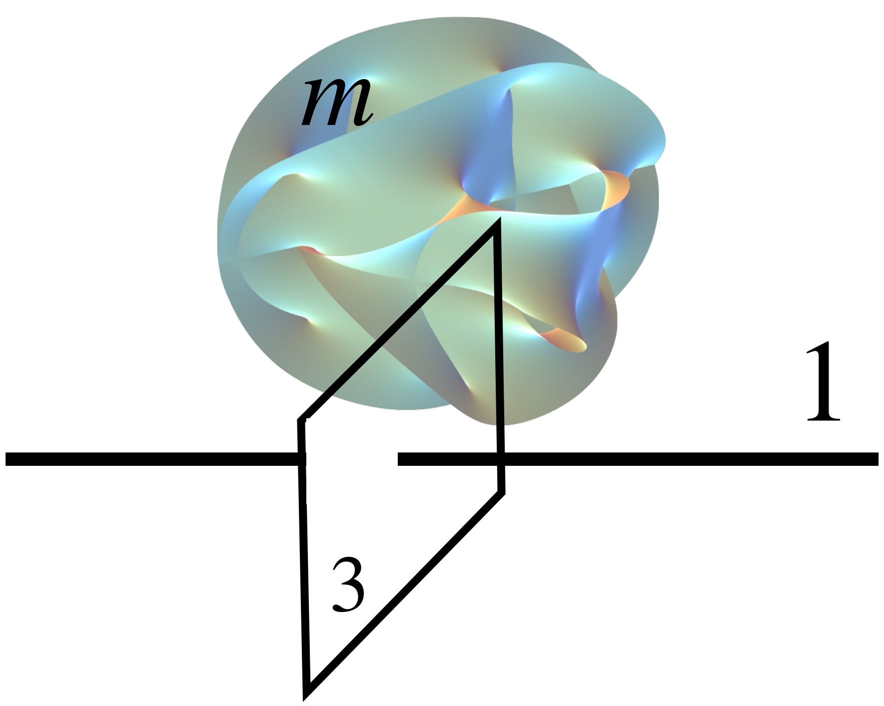


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Our question: what do axion observations say about this scenario?

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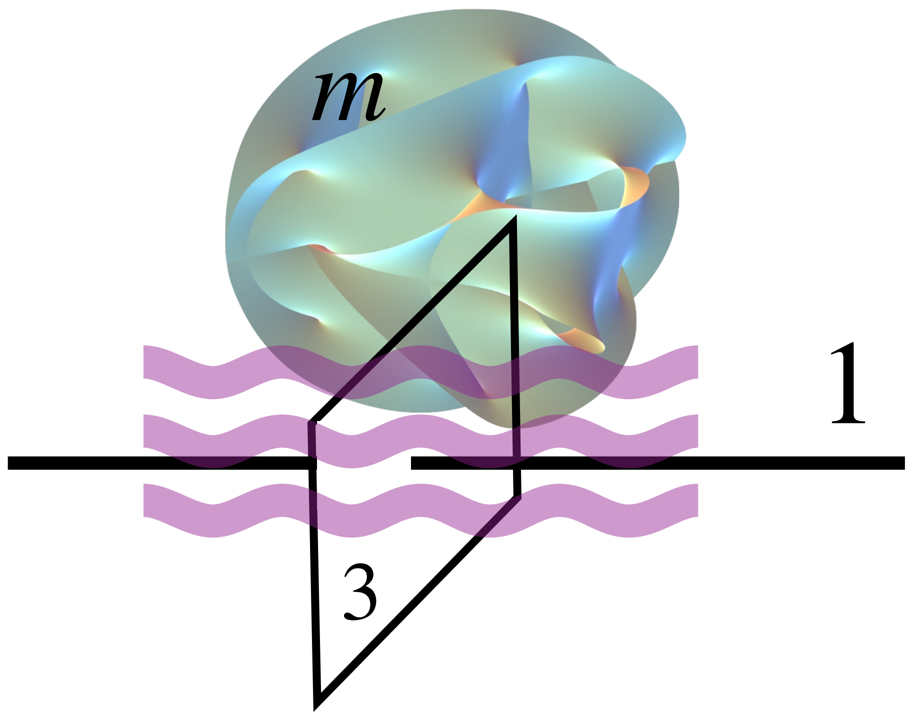

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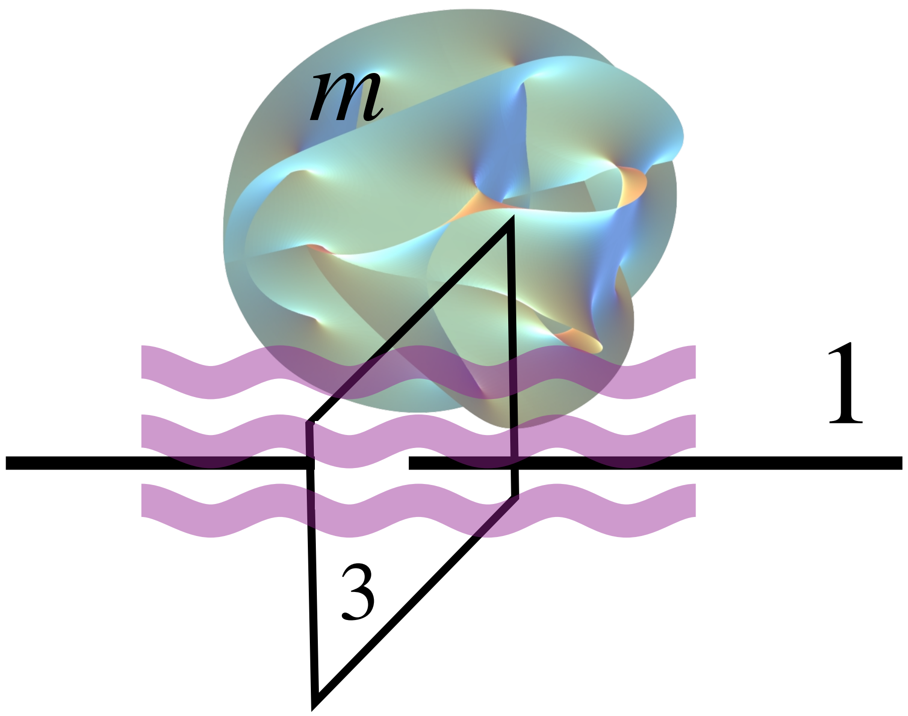


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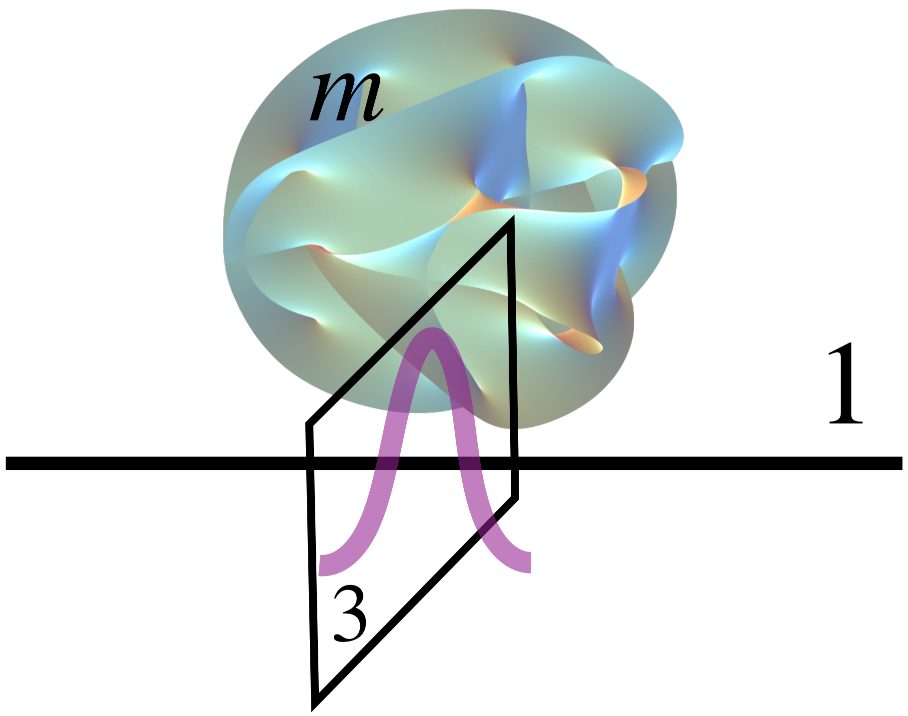

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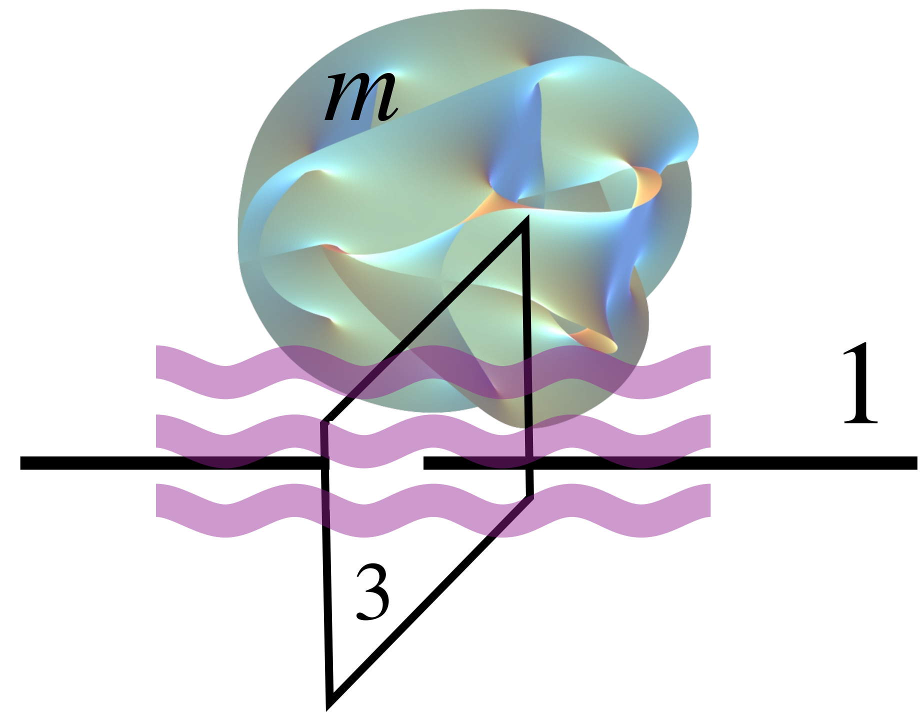


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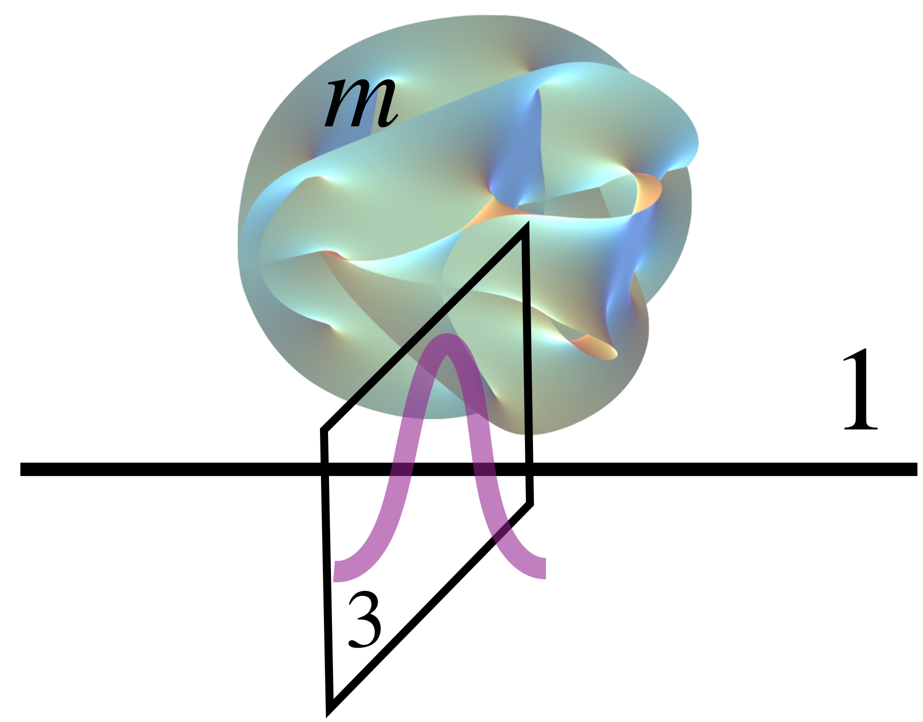


Let's study each $f_a$ in turn.

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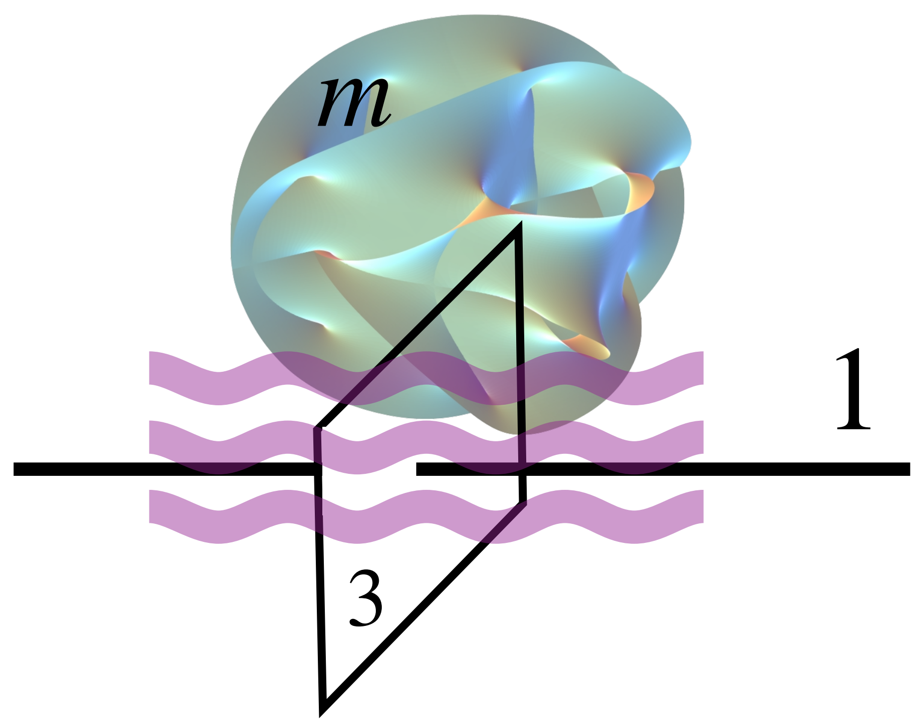

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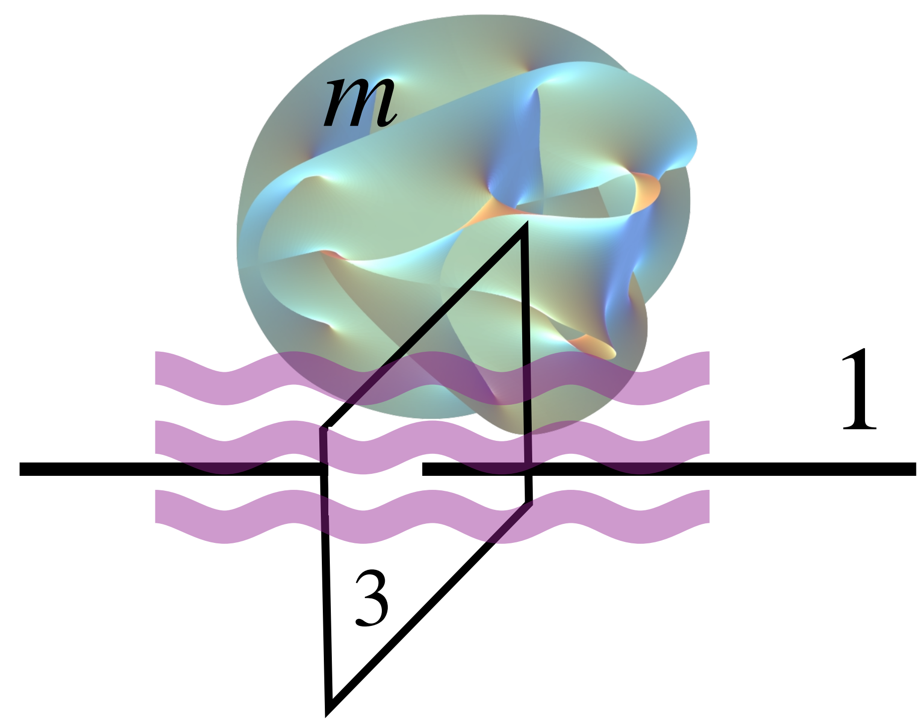

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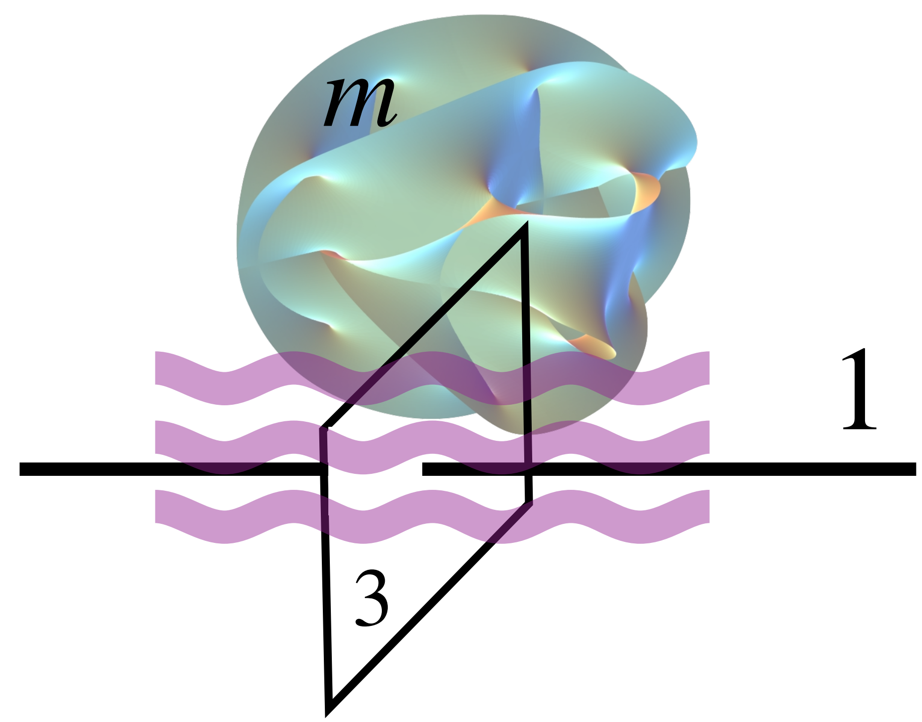

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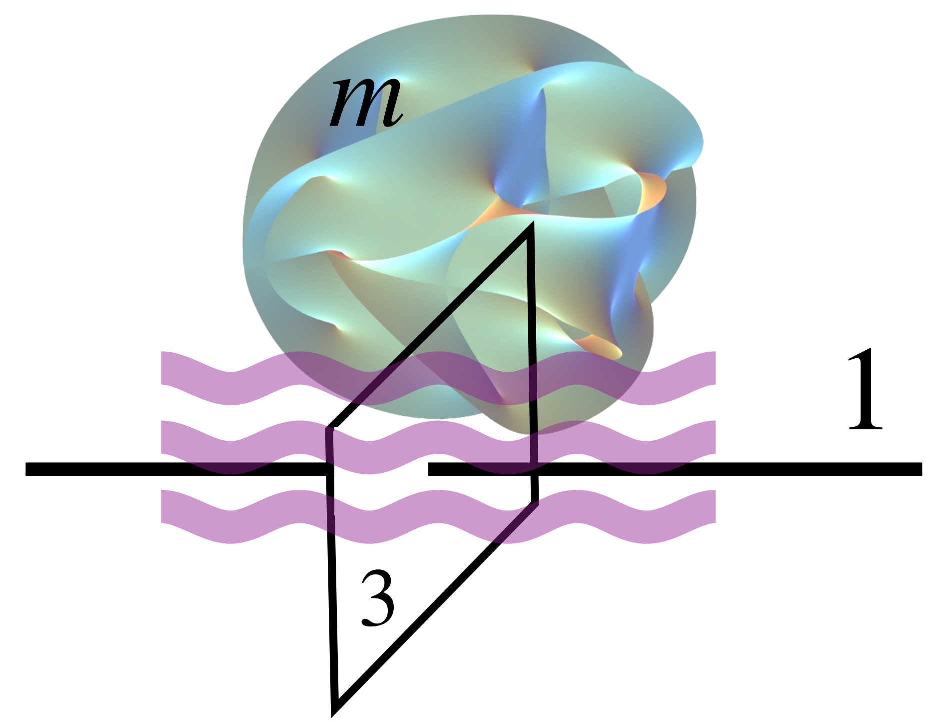

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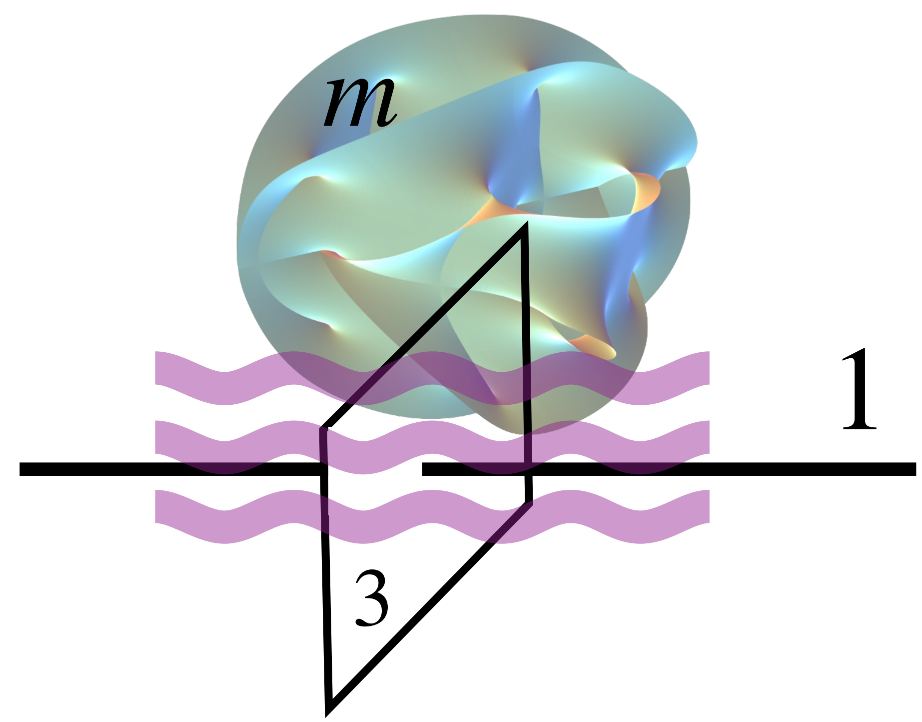

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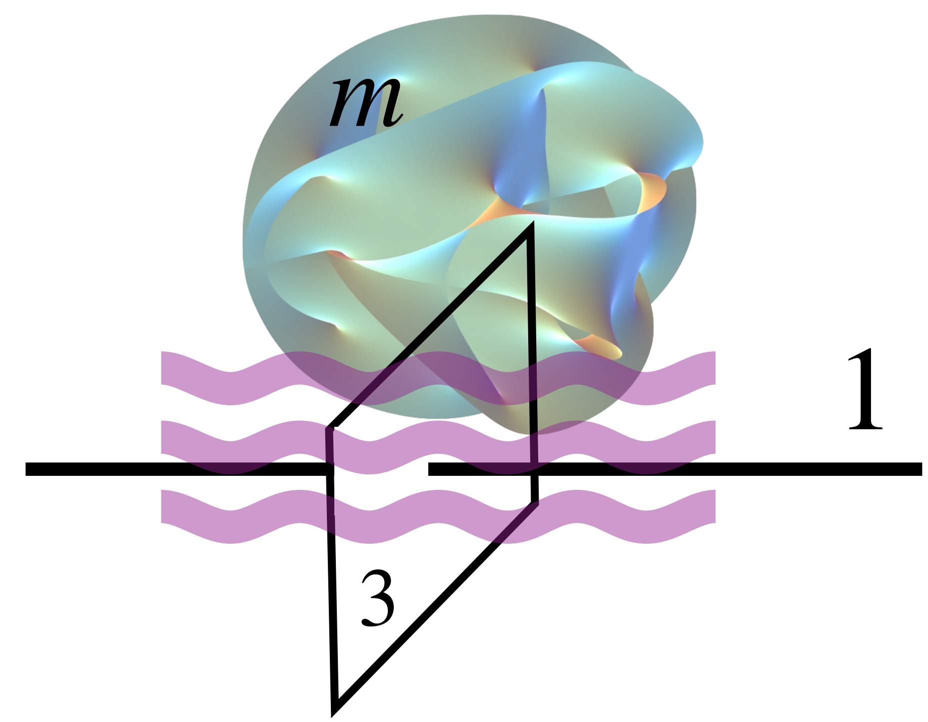


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Upshot: Bulk QCD axion in the dark dimension is not constrained.

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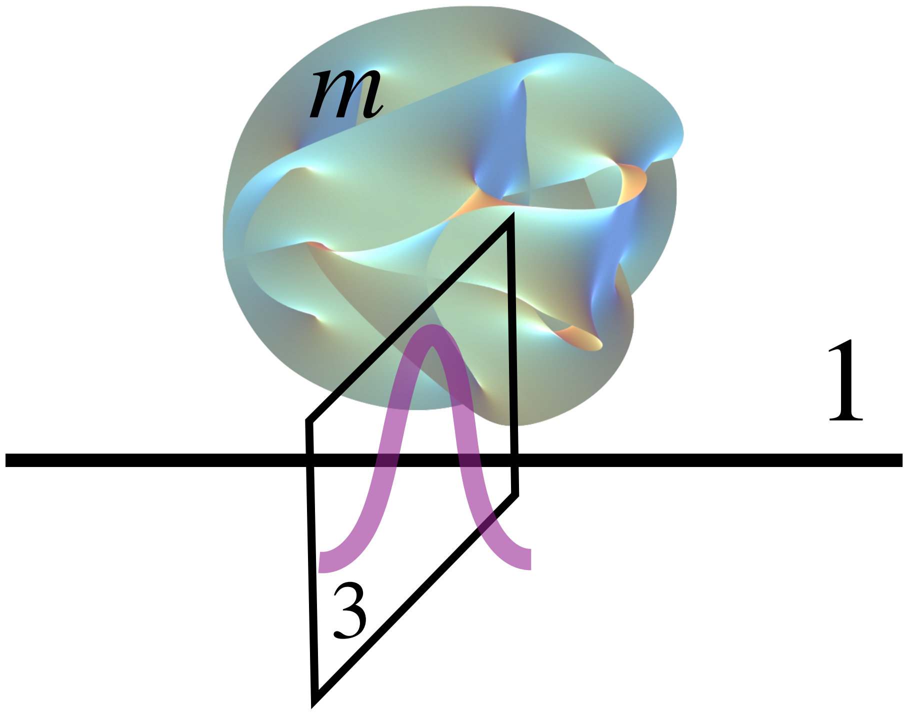

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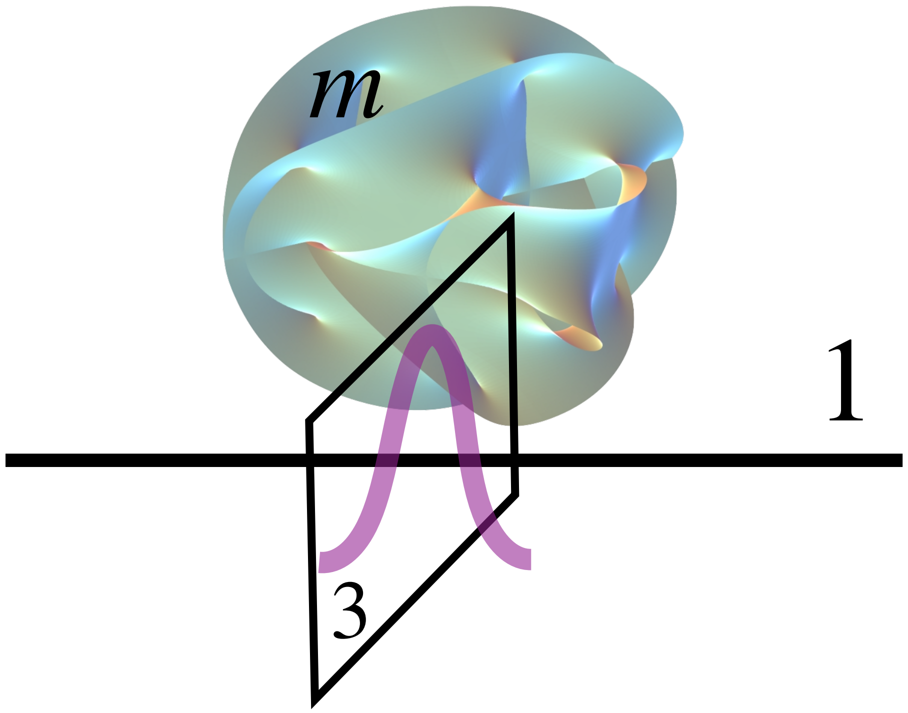

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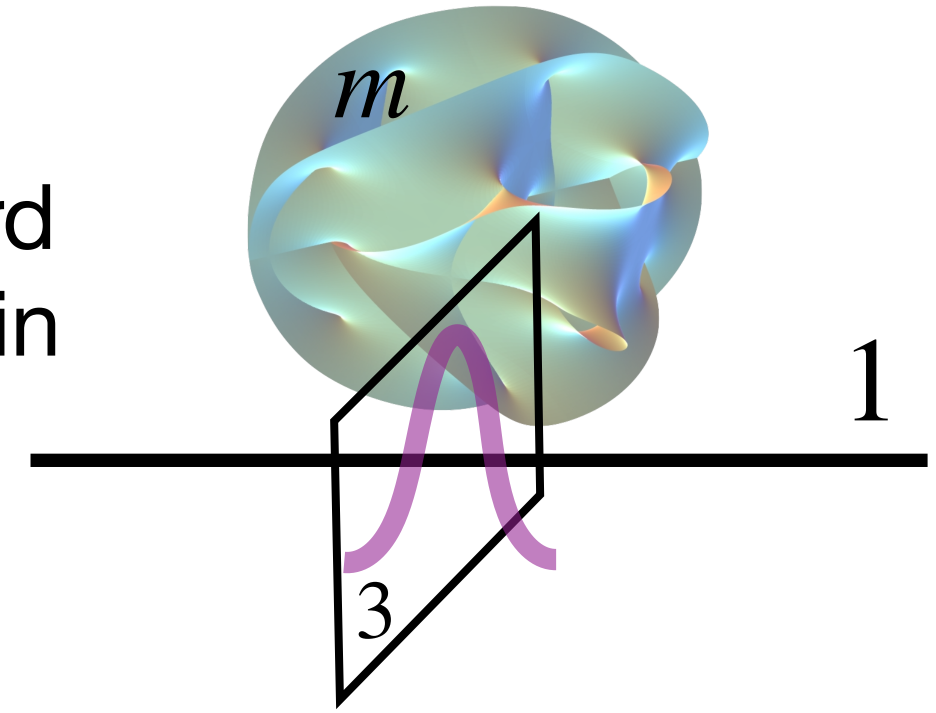

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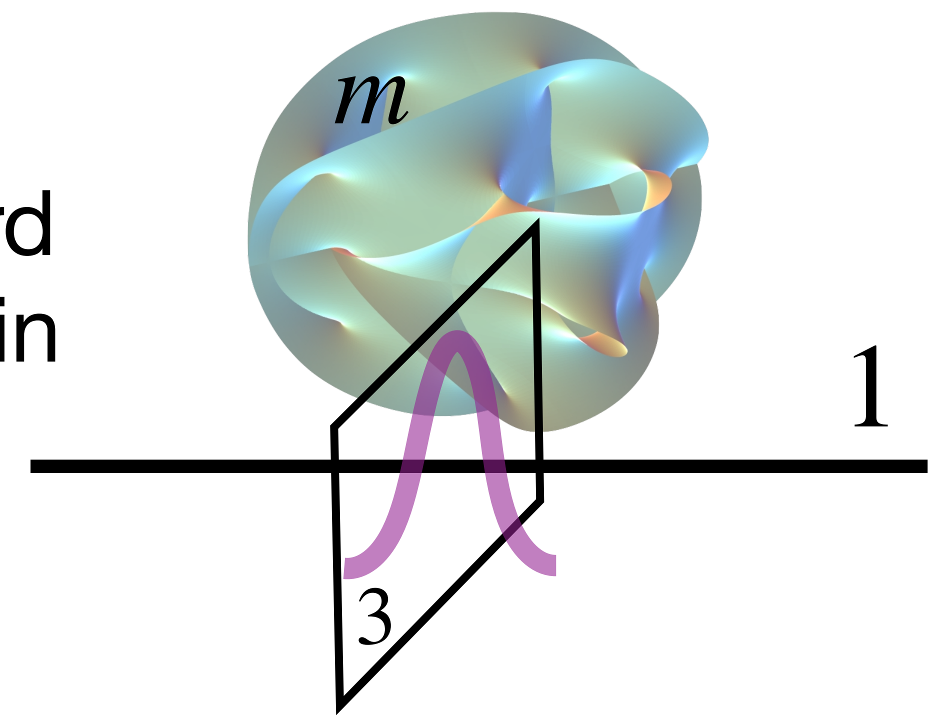


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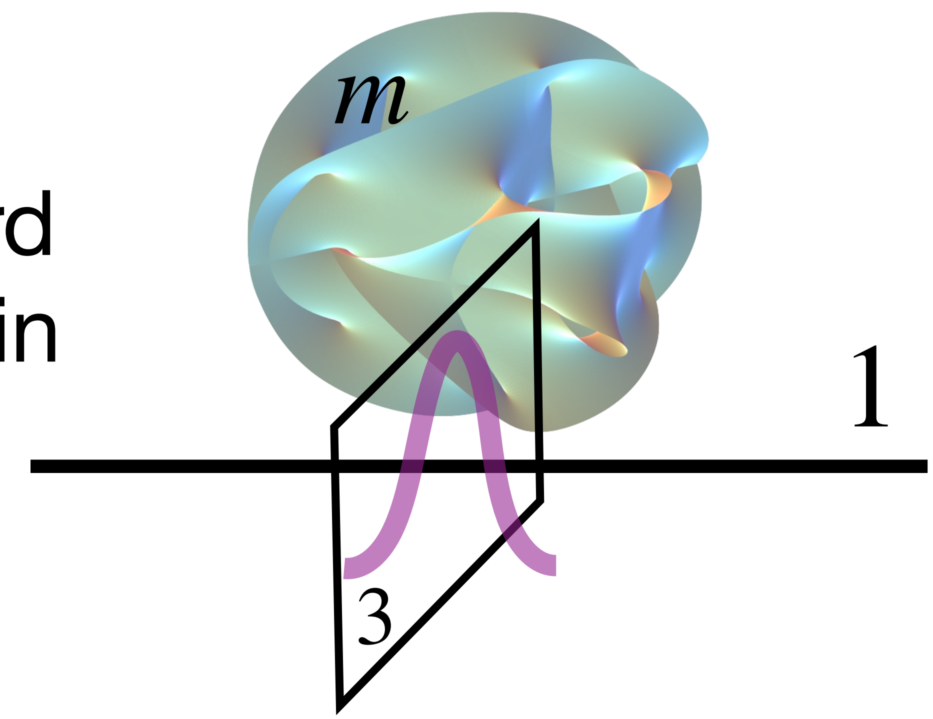


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What is the 4-dimensional axion decay constant?

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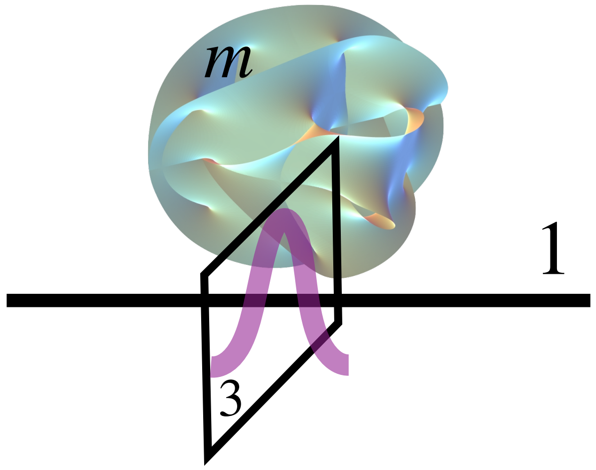

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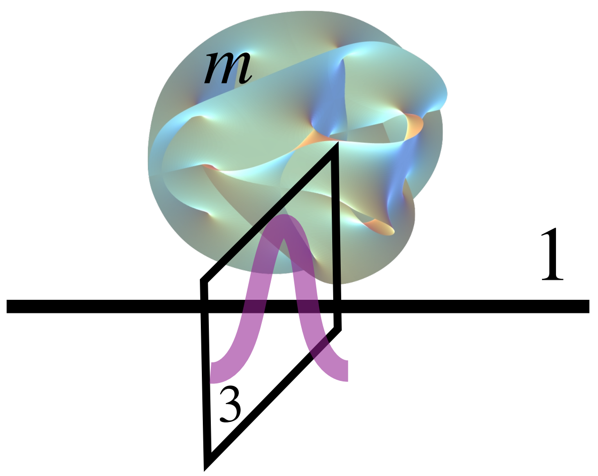

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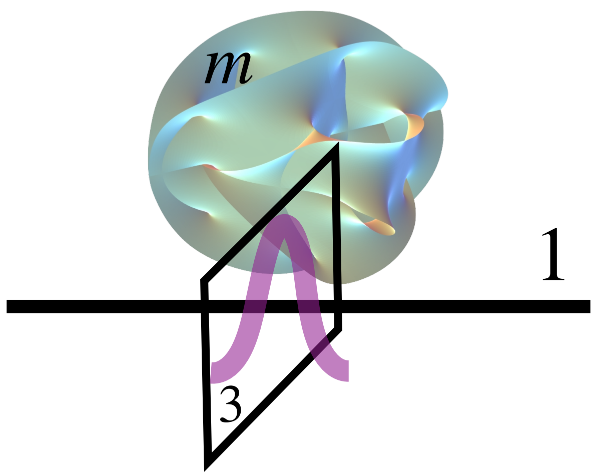

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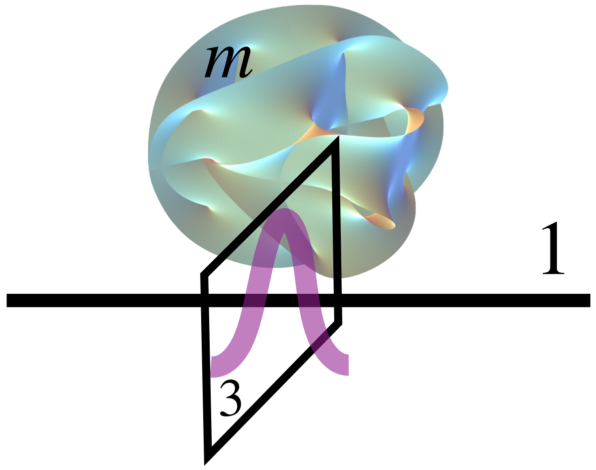

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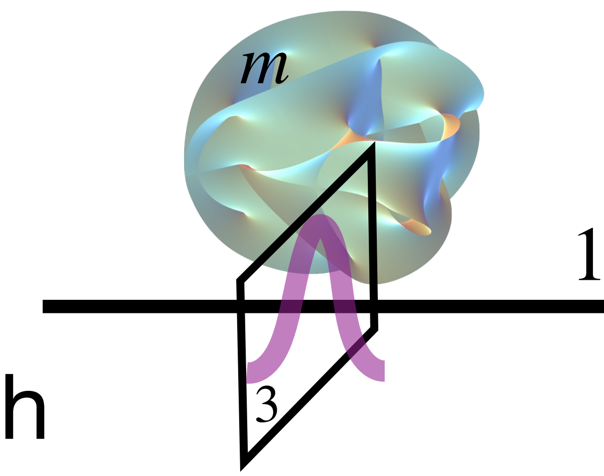


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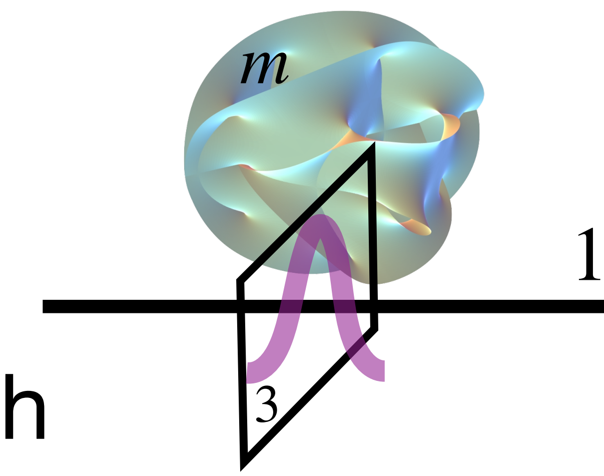


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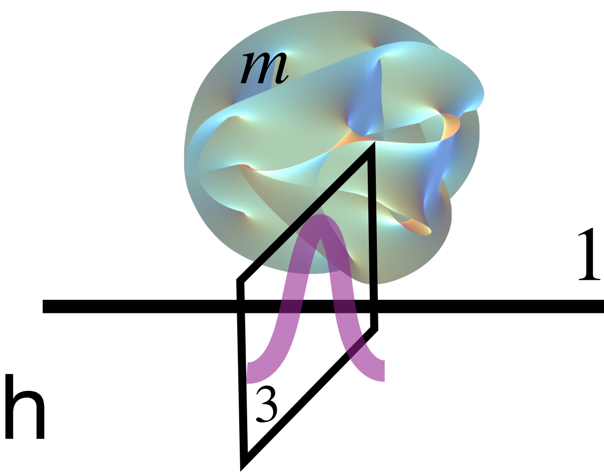


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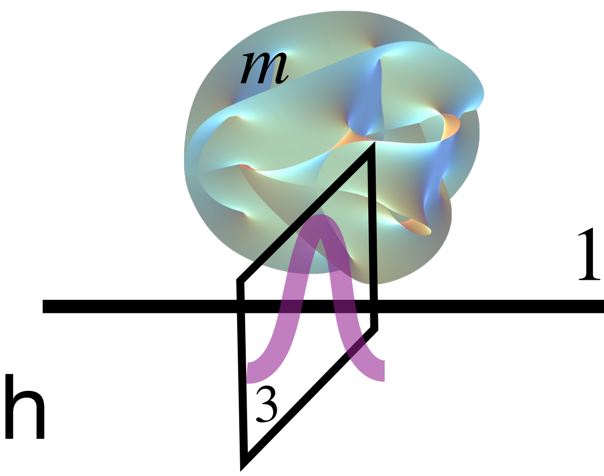
Then we can integrate the axion field over the fifth dimension, with compact support on a region of size $1/M_5$.

# Case 2: Localized QCD axion

Again, a reasonable expectation is that $f_5 \lesssim M_5$.

But what is $f_a$ in the four-dimensional theory?

We argue that for a *localized* axion, $f_a \lesssim M_5$.




Easy way to see this: start with a bulk axion, with

$$S_5 \supset \frac{1}{2}\int d^5x\ f_5^3(\nabla\theta)^2$$

Now suppose there exists a mechanism for localizing the axion.

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$$f_a^2 \lesssim \frac{f_5^3}{M_5} \lesssim M_5^2\,.$$

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**masses of localized particles are bounded by the higher dimensional Planck mass.**

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Gives a narrow window for the QCD axion:

$$m_a \sim (1 - 10)\ \mathrm{meV}.$$

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- This parameter regime will be probed by the next generation of solar axion experiments (IAXO).

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- Misalignment dark matter production from this axion would be negligible—$\mathcal{O}(0.1 - 1\%)$ of the total dark matter abundance.

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- Case 1: the bulk QCD axion lives in the same parameter space as the field theoretic axion.
- Case 2: the localized QCD axion has a decay constant bounded by $M_5$, which is a very strong constraint from the experimental perspective.
- Upshot: a link between the **size of a mesoscopic extra dimension**, and the **axion-photon coupling**.