Generalized Global Symmetries and Global Models

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2310.12980 with F. Baume, J. J. Heckman, E. Torres, A. Turner, X. Yu

2307.13027 with M. Cvetič, J. J. Heckman, E. Torres

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Introduction

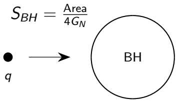
No global symmetries in quantum gravity:

[Banks, Dixon, 1988], [Banks, Seiberg, '11], [Harlow, Ooguri, '18], [McNamara, Vafa, '19], ... , [Debray, Dierigl, Heckman, Montero, '23], [Cvetič, Heckman, Hübner, Torres, '23], [Basile, Debray, Delgado, Montero, '23], [Heckman, Hübner, Murdia, '24], [Heckman, McNamara, Montero, Sharon, Vafa, Valenzuela, '24], ...

Introduction

No global symmetries in quantum gravity:

Black Hole Physics



(i)

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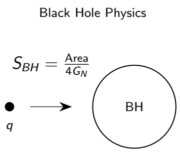

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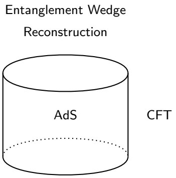

(ii)

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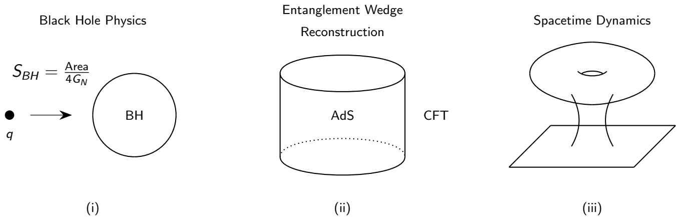

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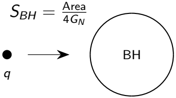
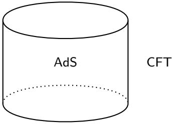


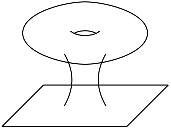
(i)

Entanglement Wedge Reconstruction



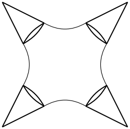
(ii)

Spacetime Dynamics



(iii)

String Compactifications



(iv)

Introduction

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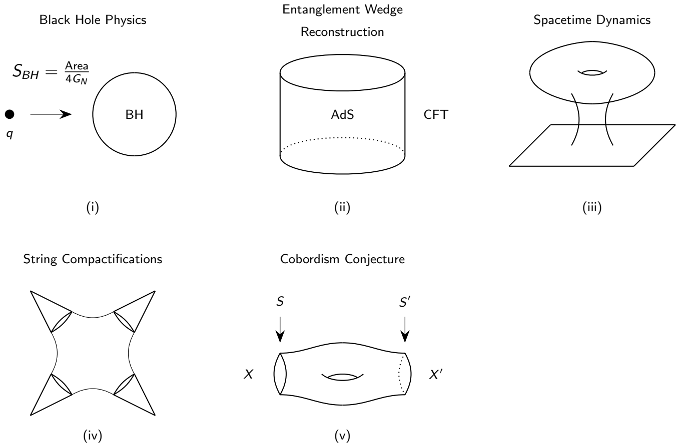

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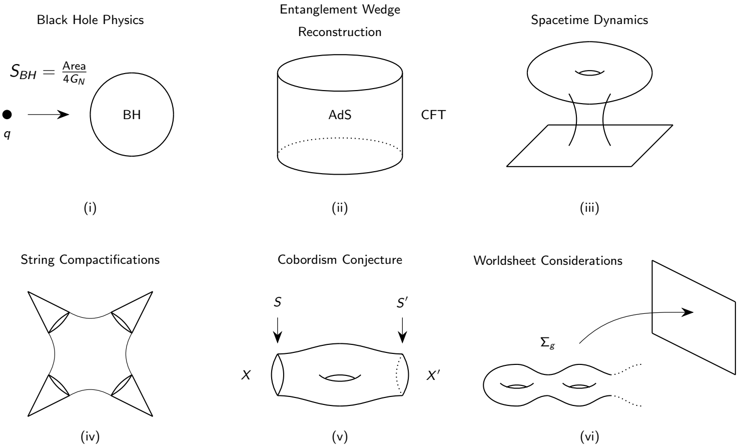

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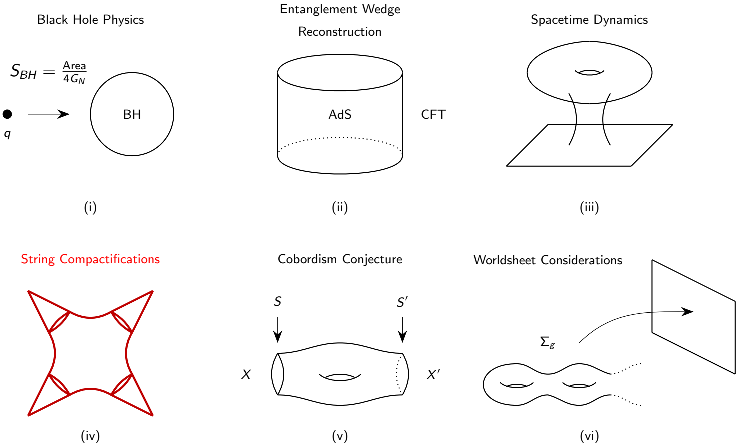
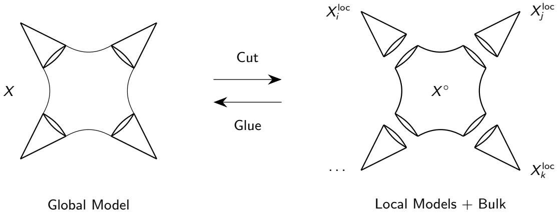


String Compactifications

Question: How are global symmetries of QFT sectors broken or gauged?



Today: Purely Geometric Backgrounds $\mathbb{R}^{1,d-1} \times X$ in M-theory

String Compactifications

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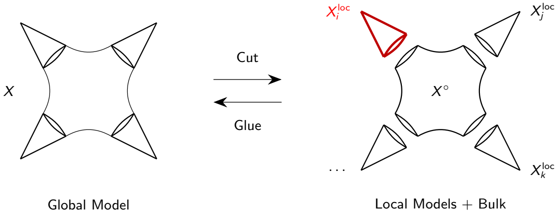
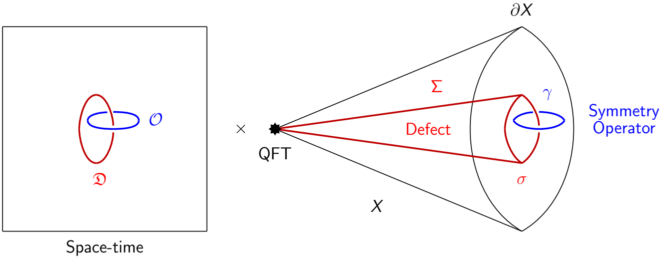


Today: Purely Geometric Backgrounds $\mathbb{R}^{1,d-1} \times X$ in M-theory

Branes and Generalized Global Symmetries

How to characterize generalized global symmetries in local string models?

[Del Zotto, Heckman, Park, Rudelius, '15], [Morrison, Schäfer-Nameki, Willet, '20], [Albertini, Del Zotto, Garcia-Etxebarria, Hosseini, '20], ..., [Garcia-Etxebarria, '22], [Apruzzi, Bah, Bonetti, Schäfer-Nameki, '22], [Heckman, Hübner, Torres, Zhang, '22], ..., [Cvetič, Heckman, Hübner, Torres, '23]

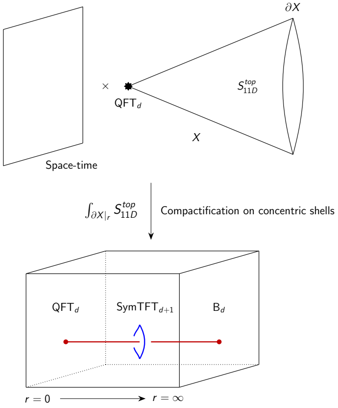


Branes for both defect and symmetry operators!

Geometry and Symmetry TFT

How to derive the Symmetry TFT from local string models?

[Apruzzi, Bonetti, García Etxebarria, Hosseini, Schafer-Nameki, '21], [García Etxebarria, Hosseini, '24]



- Topological 11D supergravity terms

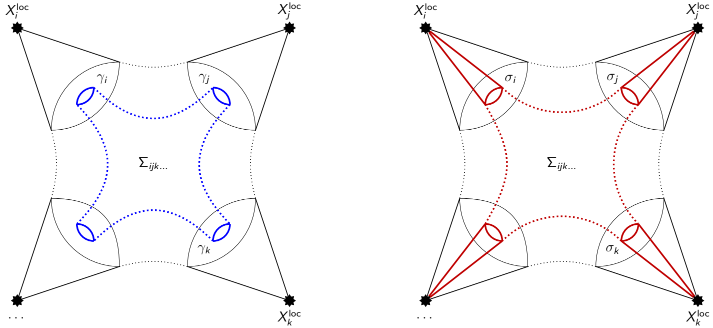
$$S^{\text{top}}_{11D} = -\frac{1}{6} C_3 \wedge G_4 \wedge G_4 - C_3 \wedge X_8$$

- Extra sandwich dimension = radius
- SUGRA boundary conditions $\rightarrow B_d$

Glued Global Models

What becomes of defect and symmetry operators of local models?

[Cvetič, Heckman, Hübner, Torres, '23]

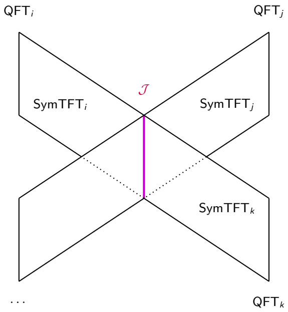


(i) : Trivialization of Symmetry Operators (ii) : Compactification of Defect Operators

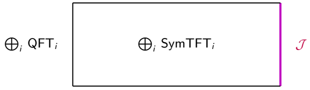
Glued Global Models

What becomes of the Symmetry TFT construction?

[Baume, Heckman, Hübner, Torres, Turner, Yu, '23]



(i): SymTree



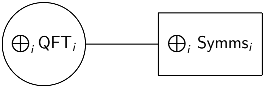
(ii): Disjoint Sandwich

The bulk $X^\circ = X \setminus X^{\text{loc}}$ maps onto the junction $\mathcal{J}$, limit $\mathrm{Vol}\, X \to \infty$

Field Theory Manipulations

How does the embedding $X^{\text{loc}} \hookrightarrow X$ manipulate the QFT sector?

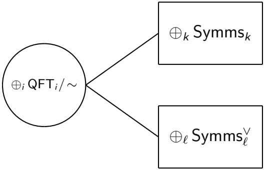
1. Direct sum of QFTs with symmetries:



Field Theory Manipulations

How does the embedding $X^{\text{loc}} \hookrightarrow X$ manipulate the QFT sector?

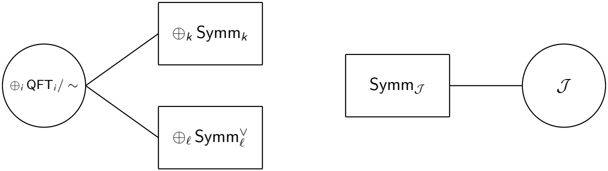
2. Gauge symmetry subset:



Field Theory Manipulations

How does the embedding $X^{\mathrm{loc}} \hookrightarrow X$ manipulate the QFT sector?

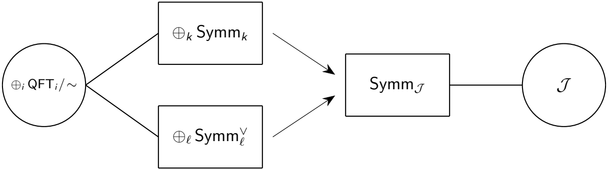
3. Add bulk / junction degrees of freedom:



Field Theory Manipulations

How does the embedding $X^{\text{loc}} \hookrightarrow X$ manipulate the QFT sector?

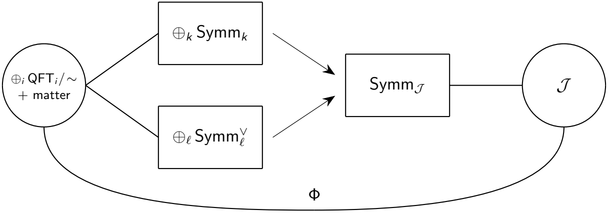
4. Identify some symmetries:



Field Theory Manipulations

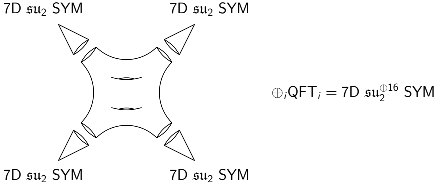
How does the embedding $X^{\text{loc}} \hookrightarrow X$ manipulate the QFT sector?

5. Add matter, break symmetries:



Example: M-theory on K3

M-theory on $X = T^4/\mathbb{Z}_2$: 7D SUGRA theory with 7D SYM sector



Focus: Invertible 1-form (electric $\leftrightarrow$ M2) and 4-form (electric $\leftrightarrow$ M5)

Aka center symmetries $\Rightarrow$ consider gauge group G_X + matter representations

$$G_X = \frac{SU(2)^{16}/\mathbb{Z}_2^5 \times U(1)^6}{\mathbb{Z}_2^6}$$

Matter breaks all symmetries $\Rightarrow$ no global 1-form / 4-form symmetries

Example: M-theory on K3

Formalize via Mayer-Vietoris sequence for the covering:

$$X = \mathrm{X}^{\mathrm{loc}} \cup X^\circ$$

Mayer-Vietoris sequence contains the exact subsequence, for $T^4/\mathbb{Z}_2$:

$$0 \to H_2(X^\circ) \to H_2(X) \to H_1(\partial X^{\mathrm{loc}}) \to H_1(X^\circ) \to 0$$

$$0 \to \mathbb{Z}^6 \to \mathbb{Z}^6 \oplus \mathbb{Z}_2^5 \to \mathbb{Z}_2^{16} \to \mathbb{Z}_2^5 \to 0$$

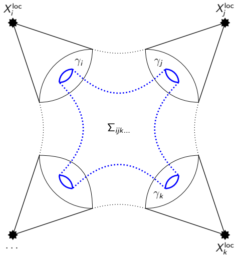
Exactness $\Rightarrow$ no global 1-form / 4-form symmetries

Emergent 1-form Symmetry $\cong H_1(X^\circ)$

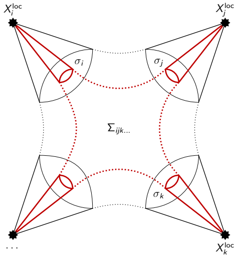
Example: M-theory on K3

Bulk Mapping: $\iota : H_1(\partial X^{\text{loc}}) \to H_1(X^\circ)$

Boundary Mapping: $\partial : H_2(X) \to H_1(\partial X^{\text{loc}})$

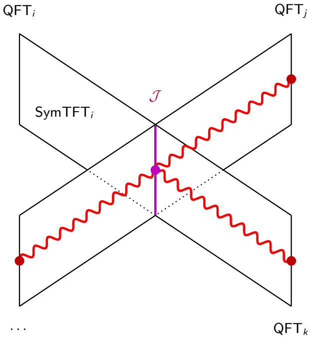


(i) : Trivialization of Symmetry Operators



(ii) : Compactification of Defect Operators

Example: K3 SymTree



Relative QFT$_i$: 7D $\mathfrak{su}_2$ SYM theory

Junction degrees of freedom $\mathcal{T}_{\mathcal{J}}$: 7D $\mathfrak{u}_1^{\oplus 6}$ vector multiplet

Junction $\mathcal{J}$: theory $\mathcal{T}_{\mathcal{J}}$ and topological boundary conditions

SymTFT$_i$: 8D TFT with action:

$$S_i = \frac{2}{2\pi}\int_{8\mathrm{D}} B_2^{(i)} \cup dC_5^{(i)} + \frac{\alpha}{2\pi}\int_{8\mathrm{D}} B_2^{(i)} \cup B_2^{(i)} \cup J_4^{(i)}$$

$$\alpha = \eta_{\not{D}_L} - \eta_{\not{D}} = \frac{1}{N}\sum_{k=1}^{N-1} \frac{(-1)^k(\omega^k - 1)}{(\omega^{pk/2} - \omega^{-pk/2})(\omega^{qk/2} - \omega^{-qk/2})}$$

with $N = 2$ and $p = 1$ and $q = N - 1 = 1$

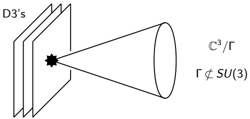
General Comments / Summary

- p-form symmetries: Exactness of Mayer-Vietoris → no global symmetries

$$\dots \rightarrow H_n(X) \rightarrow H_{n-1}(\partial X^{\text{loc}}) \rightarrow H_{n-1}(X^\circ) \rightarrow \dots$$

  (General orbifolds: Mayer-Vietoris spectral sequence)

- Purely topological → applies to non-supersymmetric setups
  [Braeger, Chakrabhavi, Heckman, Hübner, '24]



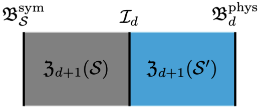
- Fate of 2-group symmetries also analyzed via cutting and gluing
  [Lee, Ohmori, Tachikawa, '21], [Cvetič, Heckman, Hübner, Torres, '22]

$$0 \rightarrow \mathcal{A} \rightarrow \widetilde{\mathcal{A}} \rightarrow \widetilde{Z}_G \rightarrow Z_G \rightarrow 0$$

General Comments / Summary

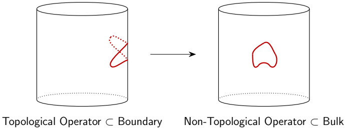
- SymTree analysis: non-geometric formulation applying more broadly

  [Bhardwaj, Bottini, Pajer, Schäfer-Nameki, '24]



- Symmetries from branes VS branes from symmetries

  [Heckman, Hübner, Murdia, '24]



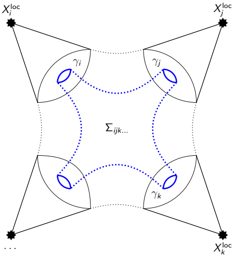
Outlook: Non-invertible Symmemtries

Homology considerations are very 'group-like'.

What about non-invertible symmetries?

Outlook: Non-invertible Symmemtries

Crucial question was: can symmetry operators fuse / deform to the identity operator in the bulk X°?

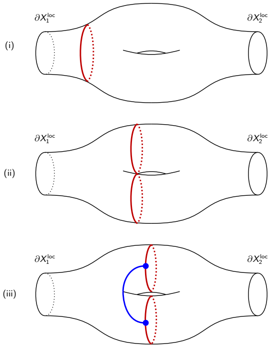


Trivialization of Symmetry Operators

Invertible symmetries: Homotopy problem = Homology problem

Outlook: Non-invertible Symmemtries

Non-invertible symmetries: Homotopy problem $\neq$ Homology problem



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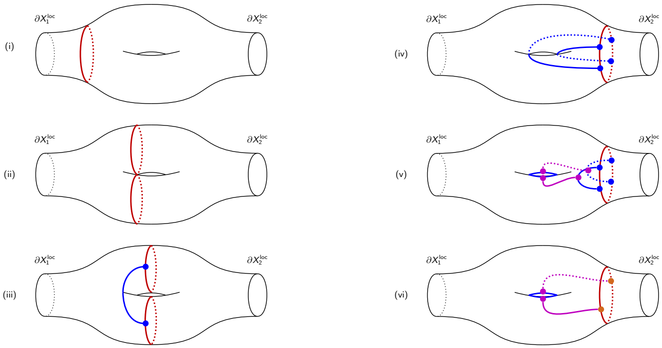
Non-invertible symmetries: Homotopy problem $\neq$ Homology problem



Lower-dimensional symmetry operators remain after ‘annihilation’

$\Rightarrow$ corresponding symmetries need to be broken

Thank you for your attention!