Stabilizing Massless Fields in Landau-Ginzburg Models

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Based on 2406.xxxxx Becker, MR, Sengupta, Walcher, Wrase

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- Motivation
- Review of Non-Geometric Landau Ginzburg Models
- Moduli Stabilization and the Swampland
- Summary

Motivation

- Moduli Stabilization remains a major obstacle to string model building.

[Graña 05, McAllister, Quevedo '23]

- Swampland criteria provide concrete characterizations of the obstacles.
- In this work, we will not build models viable for phenomenology.
- Expanding the String Landscape is an interesting problem in its own right.

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- Non-geometric models provide novel testing grounds - *[Becker, Becker, Vafa, Walcher ’05]*

  1. AdS Distance conjecture. *[Bardzell, Gonzalo, MR, Smith, Wrase ’22]*

  2. Asymptotic Acceleration. *[Cremonini, Gonzalo, MR, Tang, Wrase’ 23]*

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Tadpole Conjecture (Type IIB) - The number of moduli stabilized by fluxes is constrained by,

$$N_{flux} > \frac{1}{3} n_{stab}$$

[Bena, Blåbäck, Graña, Lüst '20]

Becker, Bena, Blåbäck, Brodie, Coudarchet, Gonzalo, Graña, Grimm, van de Heisteeg, Herraez, Lüst, Marchesano, Monnee, Plauschinn, Prieto, Tsagkaris, Walcher, Wiesner, Wrase …

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- We need to clarify what we mean by n_{stab} -
  - $n_{stab} := \text{rank}(\partial_i \partial_j W_{flux})$
  - $n_{stab} := \text{codim}\{\partial_i W_{flux} = 0\}$

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- Conjecture has been studied extensively in the asymptotic limits of moduli space. *[Grimm, Plauschinn, van de Heisteeg ’21, Graña, Grimm, van de Heisteeg, Herraez, Plauschinn ’22]*
- Does it continue to hold in the interior? *[Becker, Gonzalo, Walcher, Wrase ’22, Lüst, Wiesner ’22]*
- Even if it continues to hold, are there models where all moduli can be stabilized?
- Fully Stabilized $\mathcal{N} = 1$ SUSY Minkowski vacua?

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- Motivated by these results in type IIA, BBVW constructed the mirror dual in type IIB. *[Becker, Becker, Vafa, Walcher '06]*

- The mirror manifold admits no geometric interpretation, but there exists a LG description. *[Vafa '89, Witten '93, Hori, Iqbal, Vafa '00]*

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- A 2d $\mathcal{N} = (2,2)$ theory as shown below admits a Landau-Ginzburg description,

$$S = \int d^2z d^4\theta \mathcal{K}\{x_i, \bar{x}_i\} + \int d^2z d^2\theta \mathcal{W}(x_i) + c\,.\,c$$

- Under RG flow, the theory flows to an IR fixed point.

- For a superpotential given by $\mathcal{W} = x^{k+2}$, the CFT at the fixed point has a central charge of,

$$c = \frac{3k}{k+2}$$

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Review of Non-Geometric LG Models

- For the 1^9 model we have 9 chiral fields with the following world sheet superpotential,

$$\mathscr{W}(\{x_i\}) = \sum_{i=1}^{9} x_i^3$$

$$g : x_i \mapsto \omega\, x_i \ , \ \ \omega = e^{\frac{2\pi \mathrm{i}}{3}}$$

- The rings formed by the chiral and anti-chiral fields correspond to the Ramond ground states by spectral flow.

$$(c, c)\ \text{ring} \qquad \mapsto \qquad \mathscr{R} = \left[\frac{\mathbb{C}[x_1, \ldots, x_9]}{\partial_{x_i}\mathscr{W}(x_1, \ldots, x_9)}\right]$$

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- The above ring is spanned by,

$$\mathbf{x}^{\mathbf{k}} = x_1^{k_1}\cdot x_2^{k_2}\cdots x_9^{k_9}$$

with $\mathbf{k} = (k_1,\ldots,k_9)$ such that $k_i \in \{0,1\}$ and $\sum_i k_i = 0 \bmod 3$.

- The monomials of the kind $x_i x_j x_k$ with $i \neq j \neq k \neq i$ form a basis of the allowed marginal deformations of the superpotential.

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Review of Non-Geometric LG Models

$\boxed{\text{There are 84 complex structure moduli arising from the } (c, c) \text{ ring}}$

Review of Non-Geometric LG Models

There are 84 complex structure moduli arising from the (c, c) ring

There are 0 Kähler moduli arising from the (a, c) ring

Review of Non-Geometric LG Models

- The cycles wrapped by fluxes and orientifolds are represented by A-branes and B-branes respectively.

- For concreteness let us look at the single variable building block of the 1^9 model,

$$\mathscr{W} = x^3, \qquad g : x \to e^{\frac{2\pi i}{3}} x$$

- The A-branes of this model are the contours in the complex-x plane given by $Im(\mathscr{W}) = 0$

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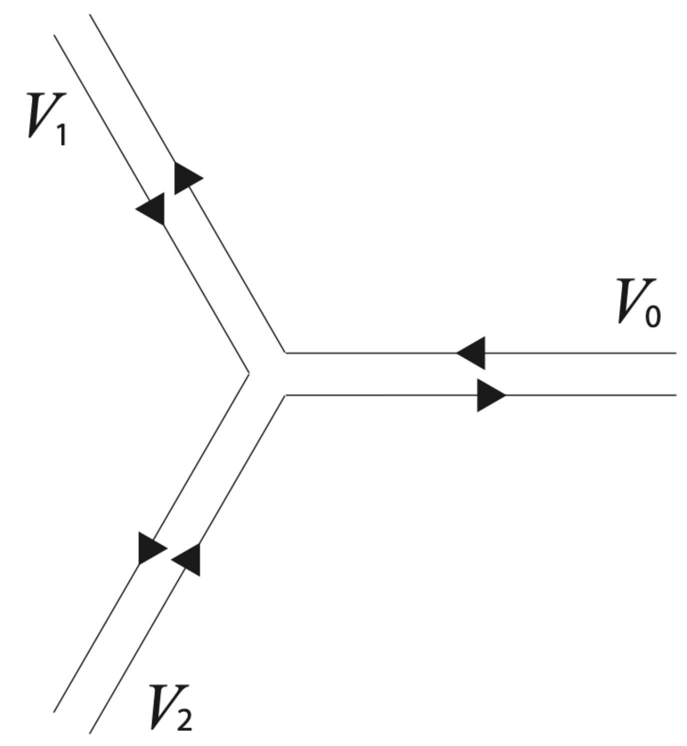

$$V_0 + V_1 + V_2 = 0$$

[Hori, Iqbal, Vafa '00]

Review of Non-Geometric LG Models

- The chiral ring of the minimal model is

$$\mathscr{R} = \mathbb{C}[x]/x^2 = \{1\,,x\}$$

$$x^{l-1} \leftrightarrow |\, l = 1,2\rangle$$

- The overlap integral between the cycles and RR ground states is then calculable,

$$\langle V_n \,|\, l\rangle = \int_{V_n} x^{l-1} e^{-x^3} dx = \frac{1}{3}\Gamma\left(\frac{l}{3}\right)(1-\omega^l)\omega^{ln}$$

with $l = 1,2$, $n = 0,1,2$ and $\omega = e^{\frac{2\pi \mathrm{i}}{3}}$

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$$\boxed{\left(\frac{\partial}{\partial t}\right)^r \bigg|_{t=0} \langle V_n | l \rangle = \int_{V_n} x^{r+l-1} e^{-x^3} dx = \frac{1}{3}\Gamma\left(\frac{r+l}{3}\right)\left(1-\omega^{r+l}\right)\omega^{(r+l)n}}$$

Review of Non-Geometric LG Models

- The RR ground states of the model are labelled by $\Omega_{\mathbf{l}}$ where $\mathbf{l} = (l_1, l_2 \ldots, l_9)$ with $l_i = 1{,}2$ -

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$\sum_i l_i$	9	12	15	18
$H^{(p,q)}$	$H^{(3,0)}$	$H^{(2,1)}$	$H^{(1,2)}$	$H^{(0,3)}$

Review of Non-Geometric LG Models

$$\boxed{G_3 = \sum_{\mathbf{I}} A^{\mathbf{I}} \Omega_{\mathbf{I}}}$$

Review of Non-Geometric LG Models

$$\boxed{G_3 = \sum_{\mathbf{l}} A^{\mathbf{l}} \Omega_{\mathbf{l}}}$$

$$\boxed{G_3 = \sum_{\mathbf{n}} \left(N^{\mathbf{n}} - \tau M^{\mathbf{n}}\right) \gamma_{\mathbf{n}}}$$

Review of Non-Geometric LG Models

- The $1^9/\mathbb{Z}_3$ model has $h^{(2,1)} = 84$ and $h^{(1,1)} = 0$.

- We would like to study orientifolds of these models. In particular, we will restrict to,

$$\sigma : (x_1, x_2 \ldots, x_9) \to -(x_2, x_1 \ldots, x_9)$$

[Becker, Becker, Vafa Walcher '06]

which has an orientifold charge of 12 that has to be cancelled by fluxes.

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$$h^{(2,1)} = 63 \qquad\qquad h^{(1,1)} = 0$$

Review of Non-Geometric LG Models

- GVW superpotential exists in these LG orbifold models as well.

$$W_{GVW} = \int_M G_3 \wedge \Omega$$

[Gukov, Vafa, Witten '99]

- The superpotential is in fact exact!

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$$\frac{1}{\tau - \bar{\tau}} \int G_3 \wedge \bar{G}_3 = \int F_3 \wedge H_3 = 12 - N_{D3}$$

Review of Non-Geometric LG Models

- How is this different from GKP? *[Giddings, Kachru, Polchinski '01]*

$$K_{GKP} = K_{CS} - 3log[-(T-\bar{T})] - log[-(\tau - \bar{\tau})]$$

- Solving the SUSY equations, $D_\tau W = D_i W = 0 \implies$ ISD fluxes

$$K_{BBVW} = K_{CS} - 4log[-(\tau - \bar{\tau})]$$

[Becker, Becker, Walcher '07]

- SUSY equations do not require ISD fluxes unlike in GKP.
- For SUSY Minkowski solutions GKP and BBVW are almost identical.

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[Becker, Becker, Walcher '07]

- SUSY equations do not require ISD fluxes unlike in GKP.

- For SUSY Minkowski solutions GKP and BBVW are almost identical.

Contents

Moduli Stabilization and the Swampland

- Finding SUSY Minkowski vacua -

1. Pick fluxes $\Omega_{l_1,l_2\ldots l_9} \in H^{(2,1)} \quad \left(\sum_i l_i = 12\right)$

2. Ensure flux quantization and tadpole cancellation

- They generically have massless directions (maximal mass matrix rank of 26).

[Becker, Gonzalo, Walcher, Wrase '22]

- A vast classification of these possible flux choices was pursed recently. [Becker, Brady, Sengupta '23]

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Moduli Stabilization and the Swampland

$$\boxed{\text{Tadpole conjecture target } = 12 \times 3 = 36 \text{ moduli}}$$

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Moduli Stabilization and the Swampland

- Consider the simple example of $W = \frac{1}{2}(\phi - \psi^2)^2$.
- This function clearly has one flat direction along $\phi = \psi^2$.
- Let us apply our algorithm for stabilizing moduli order by order to this function,
- At quadratic order in the fields, $W_2 = \frac{1}{2}\phi^2$. Solving the critical point equations gives us one non-trivial constraint ,

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$$\partial_\phi W_2 = \phi = 0$$

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- The correct thing to do would be,

$$\partial_\phi W_2 + \left(\partial_\phi W_3\right)\Big|_{\phi=\phi_{(1)}=0}$$

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Moduli Stabilization and the Swampland

- We would like to expand the superpotential around the critical points,

$$W_{expand} = \frac{1}{2!}\partial_i\partial_j W\left(t^i t^j\right) + \frac{1}{3!}\partial_i\partial_j\partial_k W\left(t^i t^j t^k\right) + \dots$$

t^i , $i = 1,2...,64$ are the deformations around the critical point.

- To determine the stabilized fields we solve the set of polynomial constraints arising from $\partial_a W = 0$ for $a = 1,2...,64$ order by order.
- The linear constraints from the quadratic terms fix the massive fields.
- The subsequent non-trivial constraints from the higher order terms can potentially fix the massless fields.

Moduli Stabilization and the Swampland

$$\boxed{\left(\frac{\partial}{\partial t}\right)^r \bigg|_{t=0} \langle V_n \,|\, l \rangle = \int_{V_n} x^{r+l-1} e^{-x^3} dx = \frac{1}{3}\Gamma\left(\frac{r+l}{3}\right)\left(1 - \omega^{r+l}\right)\omega^{(r+l)n}}$$

Moduli Stabilization and the Swampland

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$$\boxed{\frac{\partial}{\partial t^{\mathbf{k}_1}}\frac{\partial}{\partial t^{\mathbf{k}_2}}\cdots\frac{\partial}{\partial t^{\mathbf{k}_r}}\int \Omega_{\mathbf{l}} \wedge \Omega \bigg|_{t^{\mathbf{k}}=0} = \delta_{\mathbf{l}+\mathbf{L}}\,\frac{1}{3^9}\prod_{i=1}^{9}(1-\omega^{L_i})\Gamma\left(\frac{L_i}{3}\right).}$$

where, $\quad \mathbf{L} = \sum_{\alpha=1}^{r} \mathbf{k}_\alpha + \mathbf{1}$

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Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G_{(4)}^{[12,12]}$	20				

Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G^{[12,12]}_{(4)}$	20	2			

Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G_{(4)}^{[12,12]}$	20	2	0		

Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G_{(4)}^{[12,12]}$	20	2	0	4	

Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G_{(4)}^{[12,12]}$	20	2	0	4	1

Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G^{[12,4]}_{(3)}$	16				
	16				
	16				
	16				
	16				
$G^{[12,12]}_{(4)}$	20	2	0	4	1

Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G^{[12,4]}_{(3)}$	16	6			
	16	6			
	16	6			
	16	7			
	16	7			
$G^{[12,12]}_{(4)}$	20	2	0	4	1

Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G^{[12,4]}_{(3)}$	16	6	0		
	16	6	0		
	16	6	4		
	16	7	1		
	16	7	4		
$G^{[12,12]}_{(4)}$	20	2	0	4	1

Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G^{[12,4]}_{(3)}$	16	6	0	0	0
	16	6	0	0	?
	16	6	4	0	0
	16	7	1	0	0
	16	7	4	0	0
$G^{[12,12]}_{(4)}$	20	2	0	4	1

Moduli Stabilization and the Swampland

Model	massive	3rd power	4th power	5th power	6th power
$G^{[8,8]}_{(1)}$	14	0	0	0	0
$G^{[12,12]}_{(1)}$	22	0	0	0	0
$G^{[12,12]}_{(2)}$	26	0	0	0	0
$G^{[12,12]}_{(3)}$	26	0	0	0	0
$G^{[12,4]}_{(1)}$	22	0	0	0	0
$G^{[12,4]}_{(2)}$	26	0	0	0	0
$G^{[12,4]}_{(3)}$	16	6	0	0	0
	16	6	0	0	?
	16	6	4	0	0
	16	7	1	0	0
	16	7	4	0	0
$G^{[12,12]}_{(4)}$	20	2	0	4	1
	20	2	0	0	0

Moduli Stabilization and the Swampland

- The $2^6/\mathbb{Z}_4$ orientifold with tadpole charge 40 could give a way out.
- This model has 91 moduli including the axio-dilation.
- The tadpole conjecture does not imply that all 91 moduli cannot be stabilized ($40 \times 3 = 120 > 91$).
- For example, we find solutions with mass matrix rank of 84 (out of 91) moduli.

Contents

Summary

- Non-geometric LG Models are promising tools for the Swampland program.
- Moduli stabilization is possible with higher order terms in the superpotential.
- Tadpole Conjecture appears to hold in non-geometric models (for now) in the interiors of moduli space.
- Stay tuned!

Thank you!

Moduli Stabilization and the Swampland

$$G_3 = \frac{\mathrm{i}}{3\sqrt{3}} \left(\Omega_{1,1,1,1,2,1,2,1,2} - \Omega_{1,1,1,1,2,1,2,2,1} - \Omega_{1,1,1,1,2,2,1,1,2} - \Omega_{1,1,1,1,2,2,1,2,1}\right)$$

- Mass matrix rank = 16 *[Becker et al '22]*

- The already massive fields can be fixed order by order with no ambiguity. That is,

$$\partial_{\tilde{a}} W = 0$$

where $\tilde{a}$ runs over the 16 massive fields can be solved to get,

$$t_a = t_{a(1)} + t_{a(2)} + t_{a(3)} + \dots$$

Moduli Stabilization and the Swampland

- Solving the quadratic order constraints from the cubic order terms for the massless fields leads to six new stabilized directions.

$$t_{20} = t_{20(1)} + t_{20(2)} + \dots$$

- Several branches of solutions. Need to be careful to not overfix.
- An exhaustive search is cumbersome and maybe even impossible.
- Progress towards classifying the various solutions. *[Becker et al ‘23]*
- General patterns and symmetry arguments?

Review of Non-Geometric LG Models

- Similarly we can indentify the cohomology and homology bases starting from the building block of the $2^6/\mathbb{Z}_4$ model, $W_{ws} = x^4$.

- A cohomology basis is given by the RR ground states of the minimal model $|\,l\rangle$with $l = 1,2,3$. A homology basis is given by V_0, V_1, V_2, V_3 with $V_0 + V_1 + V_2 + V_3 = 0$.

- The overlap integral between the cycles and RR ground states is then calculable,

$$\langle V_n|\,l\rangle = \int_{V_n} x^{l-1}e^{-x^4}dx = \frac{1}{4}\Gamma\left(\frac{l}{4}\right)(1-\omega^l)\omega^{ln}$$

[Hori et al '00]

with $l = 1,2,3$, $n = 0,1,2,3$ and $\omega = e^{\frac{2\pi i}{4}}$

Review of Non-Geometric LG Models

- The $2^6/\mathbb{Z}_4$ model has $h^{(2,1)} = 90$ and $h^{(1,1)} = 0$. $\left(W_{2^6} = \sum_{i=1}^{6} x_i^4, g : x_i \to e^{\frac{2\pi \mathrm{i}}{4}} x_i \right)$
- The RR ground states of the model are labelled by $\Omega_{\mathbf{l}}$ where $\mathbf{l} = (l_1, l_2 \ldots, l_6)$ with $l_i = 1,2,3$ -

  1. For $\Omega_{l_1, l_2, \ldots, l_6} \in H^{(2,1)}$, $\sum_i l_i = 10.$
  2. For $\Omega_{l_1, l_2, \ldots, l_6} \in H^{(3,0)}$, $\sum_i l_i = 6$

- The orientifold involution we will work with is,

$$\sigma : (x_1, x_2 \ldots, x_6) \to e^{\frac{2\pi \mathrm{i}}{4}} (x_1, x_2 \ldots, x_6)$$

which has an orientifold charge of 40 that has to be canceled by fluxes. *[Becker et al '06]*

Moduli Stabilization and the Swampland

- A flux choice that gives 84 massive fields,

$$\begin{aligned} G_3 = &-\frac{1}{2}\Omega_{1,1,3,3,1,1} + \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{1,2,1,1,3,2} - \left(\frac{1}{4}-\frac{i}{4}\right)\Omega_{1,2,2,3,1,1} - \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{1,2,3,1,1,2} \\ &+ \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{1,3,1,1,2,2} + \frac{1}{2}i\Omega_{1,2,1,1,3,2} - \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{1,3,2,1,1,2} + \left(\frac{1}{4}-\frac{i}{4}\right)\Omega_{1,3,2,2,1,1} \\ &+ \left(\frac{1}{2}-\frac{i}{2}\right)\Omega_{1,3,3,1,1,1} + \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{2,1,1,1,3,2} - \frac{1}{2}\Omega_{2,1,2,3,1,1} - \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{2,1,3,1,1,2} \\ &- \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{2,1,3,2,1,1} + \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{2,2,1,1,2,2} + \frac{1}{2}i\Omega_{2,2,1,1,3,1} - \left(\frac{1}{4}-\frac{i}{4}\right)\Omega_{2,2,1,3,1,1} \\ &- \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{2,2,2,1,1,2} + \left(\frac{1}{4}-\frac{i}{4}\right)\Omega_{2,2,3,1,1,1} + \frac{1}{2}i\Omega_{2,3,1,1,2,1} + \left(\frac{1}{4}-\frac{i}{4}\right)\Omega_{2,3,1,2,1,1} \\ &+ \left(\frac{1}{2}-\frac{i}{2}\right)\Omega_{2,3,2,1,1,1} + \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{3,1,1,1,2,2} + \frac{1}{2}i\Omega_{3,1,1,1,3,1} - \frac{1}{2}\Omega_{3,1,1,3,1,1} \\ &- \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{3,1,2,1,1,2} - \left(\frac{1}{4}+\frac{i}{4}\right)\Omega_{3,1,2,2,1,1} - \frac{1}{2}i\Omega_{3,1,3,1,1,1} + \frac{1}{2}i\Omega_{3,2,1,1,2,1} \\ &+ \left(\frac{1}{4}-\frac{i}{4}\right)\Omega_{3,2,2,1,1,1} + \frac{1}{2}\Omega_{3,3,1,1,1,1} \end{aligned}$$