Exploring the AdS conjecture: a positive metric over DGKT vacua

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- **AdS Distance conjecture** (in Planck units) [Lüst, Palti, Vafa, '19]

$$m_\infty \sim |\Lambda|^\gamma \qquad \text{with} \qquad \gamma \sim O(1)\,.$$

- Ideas underlying the proposal:
  - The exists a notion of distance between different vacua of string theory.
  - As infinite geodesic distance a tower of states appears $m_\infty \sim e^{-\gamma\Delta}$.
  - Two AdS vacua gets a contribution of the tpye $\Delta \sim -\log\left|\frac{\Lambda_2}{\Lambda_1}\right|$.
- First steps with Freund-Rubin vacua in [Li, Palti, NP, '23]. What about scale separation? DGKT vacua.
- **Metric positivity conjecture**: families of solutions in quantum gravity always have a positive metric on them.

- In Minkowski vacua this is well-understood: distance on moduli space. [Ooguri, Vafa, '06].
- In AdS we need a universal prescription to extract a metric:
  - Take a family of AdS solutions of string theory parametrized by a constant parameter σ.
  - Gives this parameter an infinitesimal spatial dependence over AdS.
  - Extract the two-derivatives from the action: $S[\sigma(x)] \subset \int -K_{\sigma\sigma}(\partial\sigma)^2$.
- Various terms in the action can contribute. These terms are those ones ultimately related to variations of the vacuum energy.

In AdS vacua one can always take variations of the vacuum energy

$$ds_d^2 = e^{2\sigma}\, d\hat{s}^2_{\mathrm{AdS}_d} \qquad \text{with} \qquad \frac{\Lambda}{\hat{\Lambda}} = e^{-2\sigma}\,.$$

We compute the "off-shell" action for $e^{2\sigma}$ rescalings:

$$R_d = e^{-2\,\sigma}\left(\hat{R}_d - (d-1)(d-2)\hat{g}_d^{mn}\partial_m\,\sigma\partial_n\,\sigma - 2(d-1)\,\hat{\nabla}_d^2\,\sigma\right)$$

$$S_d \supset \frac{1}{2}\int d^dx\sqrt{-g_d}\,\left(\tilde{R}_d - K_{\sigma\sigma}(\partial\,\sigma)^2\right)$$

$$K_{\sigma\sigma} = -(d-1)(d-2) \longleftarrow \textbf{negative definite metric!}$$

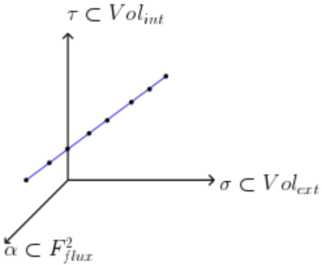
- This is the manifestation of the Conformal Factor Problem.

  [De Witt, '67] [Gibbons, Hawking, Perry, '78]

- In string theory variations of the vacuum energy are the result of various contributions: internal volume variations, fluxes, etc. We must include all these contributions [Lüst, Palti, Vafa, '19].

- AdS vacua are identified by a set of **on-shell conditions**: the variations contributing to the vacuum energy are related each other.

- One-parameter families of AdS solutions:



$$ds^2_{d+k} = e^{2\sigma} ds^2_{\mathrm{AdS}_d} + e^{2\tau} ds^2_{Y_k}$$

$$F_d = e^{\alpha}\,\mathrm{vol}_d$$

$$R_{ext} + R_{int} + F^2_{flux} \to K_{\mathrm{tot}}(\partial\sigma)^2$$

- Contribution from internal volume variations:

$$S_d \supset \frac{1}{2} \int \left[(d-1)(d-2)(\partial\sigma)^2 - k^2 \left(\frac{d-1}{d-2} - \frac{k-1}{k} \right) (\partial\tau)^2 \right] .$$

- On-shell condition: $\tau = a\sigma$.

- Total metric over metric variations:

$$K_{metric} = K_{\sigma\sigma} + K_{\tau\tau} = (d-1)(d-2) - a^2 k^2 \left(\frac{d-1}{d-2} - \frac{k-1}{k} \right)$$

- Strong scale separation leads to a negative metric.

- Example: $K_{\mathsf{DGKT}} = -\frac{10}{3}$. We need to include flux and dilaton variations.

- DGKT vacua: type IIA CY orientifold compactifications to $\mathcal{N}=1$ AdS$_4$. [DeWolfe, Giryavets, Kachru, Taylor. 05.]
- Internal fluxes: $H_3 = q^\lambda \alpha_\lambda - p_k \beta^k \,, \qquad F_0 = m \,, \qquad F_4 = e_a \tilde{\omega}^a$.
- The F_4 flux is unrestricted by tadpoles and can be varied parametrically: $|e_a| \sim n$.
- Internal volume variations from 2-cycles: $v^a = e^{2\tau}\, \hat{v}^a \to ds_6^2 = e^{2\tau} d\hat{s}_6^2$. The 10D dilaton Φ contributes. The axions do not contribute.
- Vacuum energy: $V_{\text{min}} = \Lambda = -\frac{6m^2}{25}\, e^{4\Phi - 6\tau}$.

- On-shell flux: $F_6 = -3\, e^{\alpha} e^{a} \mathrm{vol}_{\mathsf{AdS}_4} \wedge w_a$ with $F_4 = \star F_6$.
- We introduce the Poincaré patch: $ds^2_{\mathsf{AdS}_4} = \frac{1}{z^2}\left(ds^2_{M_3} + dz^2\right)$.
- We "gauge" the flux parameter $\alpha = \alpha(z)$:

$$C_5 = -e^{\alpha}\, e^{a}\, z^{-3} \mathrm{vol}_{M_3} \wedge w_a \to F_6 = dC_5 = -e^{\alpha} e^{a}\, (3 - z\partial_z \alpha)\, \mathrm{vol}_{\mathsf{AdS}_4} \wedge w_a$$

- Compute the contribution to the action $\int |F_4|^2 = -\int |F_6|^2$:

$$S_4 \supset \int d^4x \sqrt{-g_E}\, e^{4\Phi - 6\tau} \left(-\frac{27m^2}{25} - \frac{3m^2}{25} z^2 (\partial_z \alpha)^2 - \frac{18m^2}{25} z \partial_z \alpha \right)$$

- The holographic coordinate z is picked as special.

How can we handle with the linear term?

$$ds_4^2 = \frac{e^{2\sigma}}{z^2}\left(ds_{M_3}^2 + dz^2\right) \qquad ds_4^2 = \frac{e^{2\sigma}}{z^2}\left(ds_{M_3}^2 + e^{2\sigma_1}dz^2\right)$$

From the type IIA action we get

$$S_4 \supset \frac{1}{2}\int d^4x\sqrt{-g_E}\Big(\tilde{R}_E + 6\,(\partial_z\,\sigma)^2 - 24\,(\partial_z\tau)^2 + 12(\partial_z\Phi)(\partial_z\tau)$$

$$-2\,(\partial_z\Phi)^2 - 6e^{-2(\sigma+\sigma_1)+2\Phi-6\tau}z\,\partial_z\sigma_1\Big)\,.$$

$$S_4 \supset \int d^4x\sqrt{-g_E}\left(-\frac{3}{2}(\partial_z\alpha)^2 + 9e^{-2(\sigma+\sigma_1)+2\Phi-6\tau}z\partial_z\alpha\right)$$

- We have the freedom to choose σ_1! We choose it to precisely cancel the linear term in flux variations

$$\sigma_1 = \frac{3}{2}\,\alpha$$

On-shell relations for DGKT:

$$\tau = \frac{1}{3}(\sigma + \sigma_1) + \frac{1}{6}\log\left(\frac{q^2}{32}\right)\,, \qquad \Phi = -(\sigma + \sigma_1) + \frac{1}{2}\log\left(\frac{25}{2m^2}\right)\,,$$

$$\alpha = 3\sigma + 2\tau - \Phi + \log\left(\frac{5}{3\sqrt{2}m}\right)\,, \qquad \sigma_1 = \frac{3}{2}\,\alpha,$$

[DeWolfe, Giryavets, Kachru, Taylor. 05.]

We obtain a unique solution:

$$\tau = -\frac{11}{9}\sigma + \frac{1}{18}\log\left(\frac{3^6 q^2}{2^5}\right)\,, \qquad \Phi = \frac{11}{3}\sigma + \log\left(\frac{5\,q^{2/3}}{2^{1/6}\,12m}\right)\,,$$

$$\sigma_1 = -\frac{14}{3}\sigma + \frac{1}{3}\log\left(\frac{2^5\,3^3}{q^2}\right)\,, \qquad \alpha = -\frac{28}{9}\sigma + \frac{2}{9}\log\left(\frac{2^5\,3^3}{q^2}\right)\,.$$

$$\downarrow$$

$$S_4 \supset \frac{1}{2}\int d^4x\sqrt{-g_E}\,\left[-K_{\mathsf{DGKT}}(\partial_z\sigma)^2\right] \qquad \text{with} \qquad K_{\mathsf{DGKT}} = \frac{3376}{27}\,.$$

- We can write the result in term of the traditional exponential dependence on the distance of the KK modes

$$m_\infty \sim e^{-\gamma\Delta}\,, \qquad\qquad \gamma = \frac{7}{9\sqrt{K_{\mathsf{DGKT}}}}\,.$$

  with $\Delta = -\frac{1}{2}\int\sqrt{K}d\log\Lambda$. [Lüst, Palti, Vafa, '19].
- This gives the explicit value $\gamma \sim 0.06 < \gamma_{moduli} = 0.7$. [Etheredge, Heidenreich, Kaya, Qiu, Rudelius, 22].
- Our procedure is very involved, but is rigid.
- Test our **metric positivity conjecture** in various setups.
- Big deal: holographic interpretation, distance between CFTs.

Thank you!