Decoding Dualities at Divergent Distances

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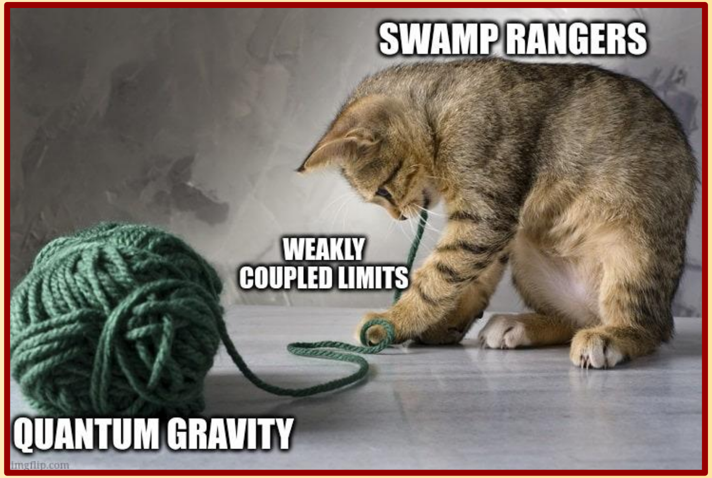
Is quantum gravity stringy?

at weak coupling, ***it looks like it…***

- *S-matrix bootstrap results*
- *black holes & holography*
- ***unexpected*** *appearances*

"emergent string conjecture"
(Lee, Lerche, Weigand, 2019)



this talk: *collect old & new insights*

Weakly coupled limits in quantum gravity

$$\mathcal{L}_{\text{eff}} = M_{\text{Pl}}^{d-2}\left(R + \boxed{\frac{\mathcal{R}^2}{\Lambda_{\text{UV}}^2} + \ldots}\right) + \boxed{\sum_k m_{\text{gap}}^{d-2k}\mathcal{R}^{2k}}$$

genuine QG effects

field theory d.o.f. to "integrate in"

(Castellano, Herráez, Ibáñez, 2023)

- *weak "quantum coupling(s)" of gravitons* $\Lambda_{\text{UV}} \ll M_{\text{Pl}}$
- *infinite-distance limits in theory space* ***factorize***

  (Stout, 2021-2022)

  - *gravity must "decouple" by equivalence principle*
  - ***tower*** *of new species (seems) required* ***(must it be light?)***

What are the endangered light species?

❖ ***assume*** *light tower (later:* ***converse*** *worldsheet argument)*

- ➢ *obvious option: KK tower* ⟶ *still EFT framework*

  T-duality & obstructions (Demulder, Lüst, Raml, 2023)

- ➢ ***string universality*** ⟶ ***only alternative*** *strings modes!*

  unique & critical *(Lee, Lerche, Weigand, 2019)*

  - ➢ *extremely* ***non-trivial*** *(dualities & obstructions)*

    (Kläwer, Lee, Weigand, Wiesner, 2020)

$$\boxed{m_{\mathrm{KK}} \ll \Lambda_{\mathrm{UV}}^{\mathrm{KK}}} \qquad \boxed{m_{\mathrm{string}} = \Lambda_{\mathrm{UV}}^{\mathrm{string}}}$$

⟶ ***in general:*** *two classes of species*

Fun with scales

- *UV cutoff: **generic** suppression in (**gravitational sector** of) EFT*
  - *"integrate in" KK modes [if present]*
  - *scale of smallest black hole* *(Dvali, 2007) (Cribiori, Lüst, Staudt, 2022) (Bedroya, Mishra, Wiesner, 2024)*
- *QG scale: strongly coupled gravity, e.g.* $\Lambda_{\mathrm{QG}}^{\mathrm{string}} \approx M_s \sqrt{\log g_s^{-2}}$

  (Gross, Mende, Ooguri, 1987-1990) (Bedroya, 2022)
  - *for KK: higher-dim. Planck scale [even w/ curvature* *(Aoufia, IB, Leone, 2024)*]
- *"**state counting**" scale: worst-case scenario* $\Lambda_{\mathrm{st}} \sim M_{\mathrm{Pl}} N_{\mathrm{sp}}(\Lambda_{\mathrm{st}})^{-\frac{1}{d-2}}$

  (Veneziano, 2001) (Dvali, 2007)

parametric hierarchy

$$\boxed{m_{\mathrm{tower}} \lesssim \Lambda_{\mathrm{UV}} \lesssim \Lambda_{\mathrm{QG}} \lesssim \Lambda_{\mathrm{st}} \lesssim M_{\mathrm{Pl}}}$$

Minimal black holes & species *(IB, Cribiori, Lüst, Montella, 2023-2024) (Alvaro's talk)*

❖ ***basic idea:*** *light tower* $\Leftrightarrow$ *minimal black hole* *(Dvali, Gomez, Lüst, Isermann, Stieberger, 2009-2014)*

- *spectrum w/o intermediate gaps* $m_n = m\, f(n)\ ,\ \ m_N \sim \Lambda_{\rm st}$

  max level

- *"typical" species energy* $E_{\rm sp} = \sum_{\text{level } n} d_n\, m_n$

- *thermodynamics **must** match (energy, entropy, charges...)*

$$\boxed{S_{\rm sp} \sim N_{\rm sp} + \sum_{n \leq N} d_n \log \frac{f(N)}{f(n)} - \frac{1}{2} \log\left(N_{\rm sp}\, f(N)^2\right)}$$

A familiar example

KK towers [reparametrized to "constant degeneracy"] *(Castellano, Herráez, Ibáñez, 2022)*

➢ ***if limit is an EFT**, towers gapped below cutoff **must be KK*** *(Bedroya, Mishra, Wiesner, 2024)*

$$d_n \sim 1$$

thermodynamic ↓ *consistency*

$$m_n \sim m\, n^{\frac{1}{p}}$$

$$\rightarrow \quad E_{\rm sp} = \frac{p}{p+1}\Lambda_{\rm st}^{3-d} + \mathcal{O}(\Lambda_{\rm st}) \quad ✓$$

$$\rightarrow \quad S_{\rm sp} = \frac{p+1}{p} N_{\rm sp} + \mathcal{O}(\log N_{\rm sp}) \quad ✓$$

$$\rightarrow \quad \Lambda_{\rm UV} = \Lambda_{\rm st} = \Lambda_{\rm QG} = m^{\frac{p}{p+d-2}} \quad ✓$$

***name of the game:** repeat for more general towers*

The good, the bad and the stringy

- ***punchline:*** *not any tower goes! up to reparametrizations, we found*
  - *only "**KK-like**" towers work* $d(m) \sim m^{p-1}$ *"effective" dims p ≥ 1*
  - *some **don't work*** $d(m) \sim (\log m)^{\alpha}$, $\log \log m$
  - *the rest "smells stringy", e.g.* $\Lambda_{\text{st}} \sim m \ \log m^{-1}$

may gap @ cutoff ⟶ *higher-spins* (Camanho, Edelstein, Maldacena, Zhiboedov, 2014)

Mass\Degeneracy	Log	Power	Exp	Constant
Log	*			*
Power	*			
Exp				

Living in the (S-)matrix

- *for "stringy" towers, BH argument formally works for $p \to \infty$*
  - *mild logarithmic deviations from BH matching indicate cutoff = gap << Λ_{st}*
  - *power-like DoS should degenerate into* ***exponential*** *(Bedroya, Mishra, Wiesner, 2024)*

- *highly constraining bootstrap results!*

$$\mathcal{A}_{\text{grav}} = \frac{W(s,t,u)}{stu} \prod_{n \geq 1} \frac{1 + A_n(st + tu + us) + B_n stu}{(1 - s/\lambda_n)(1 - t/\lambda_n)(1 - u/\lambda_n)}$$

string spectrum ***fixed*** ✓

no q-deformation *à la Coon* ✓

Virasoro-Shapiro *dynamics** ✓

similar ***rigidity*** *for gauge sector* ✓

(Camanho, Edelstein, Maldacena, Zhiboedov, 2014)
(Caron-Huot, Komargodski, Sever, Zhiboedov, 2016)
(Caron-Huot, Li, Parra-Martinez, Simmons-Duffin, 2022)
(Geiser, Lindwasser, 2022) (Cheung, Remmen, 2022-2023)
(Arkani-Hamed, Cheung, Figueiredo, Remmen, 2023)
(Häring, Zhiboedov, 2023) (...)

The bottom-up story — recap

- *information-theoretic **factorization** links distances and towers* (Stout, 2021-2022)
- *test w/ "**minimal black hole** ⇔ **light species**" correspondence*
- ***upshot:** if tower is light,*
  - *gapped below cutoff* ⟶ *KK-like +* $p \to \infty$ *limit*
  - *gapped at cutoff* ⟶ *exponential DoS + higher-spins + rigid S-matrix*

(Bedroya, Mishra, Wiesner, 2024) (Alvaro's talk)

leftover questions:

- *must **light** towers exist?*
- *are these **actually** KK?*

⟶ ***worldsheet approach***

(Aoufia, IB, Leone, 2024)

Strings and (non-)geometry

❖ *string perturbation theory can probe* ***both questions***

- ➢ *tool:* ***modular invariance*** *(here: RNS-RNS)*
- ➢ *quantity: 1-loop* **R^4** ***Wilson coefficient*** *from 2-to-2 graviton scattering*

(Green, Schwarz, Brink, 1982) (Kiritsis, Pioline, 1997) (Green, Gutperle, Vanhove, 1997) (Obers, Pioline, 1999) (Green, Vanhove, 1999) (Green, Russo, Vanhove, 2008) (Green, Russo, Vanhove, 2010) (Angelantonj, Florakis, Pioline, 2012) (Blumenhagen, Cribiori, Gligovic, Paraskevopoulou, 2024) (Bedroya, van de Heisteeg, Vafa, Wiesner, Wu, 2022-2024)

"reduced"/primary partition function
(Afkhami-Jeddi, Cohn, Hartman, Tajdini, 2021)

$$\boxed{\alpha_{R^4} = \left(\frac{\mathcal{Z}_{S^2}}{g_s^2}\right)^{\frac{8-d}{d-2}} \Bigg(\overbrace{2\zeta(3)\,\frac{\mathcal{Z}_{S^2}}{g_s^2}}^{\text{tree- level}} + 2\pi \overbrace{\int_{\mathcal{F}} d\mu\, \mathcal{Z}^{\text{reg}}_{T^2}}^{\text{1-loop}}\Bigg)}$$

emergent string limit

$$\alpha_{R^4} \overset{g_s \ll 1}{\sim} \left(\frac{M_{\text{Pl}}}{M_s}\right)^6$$

decompactification

$$\int_{\mathcal{F}} d\mu\, \mathcal{Z}^{\text{reg}}_{T^2} \sim \left(\frac{m_{\text{gap}}}{M_s}\right)^{-p}$$

Modular invariance to the rescue

- ***can prove*** *(Aoufia, IB, Leone, 2024)*
  - *Wilson coefficient **diverges*** $\Leftrightarrow$ $\exists$ ***light tower***
  - *scaling w/ spectral gap is **geometric** ("emergent geometry")*

exp. decay
*distance **diverges***
(Ooguri, Wang, 2024)

$$\boxed{\int_{\mathcal{F}} d\mu \, Z^{\text{reg}}_{T^2}(t) \overset{t \gg 1}{\sim} \Delta_{\text{gap}}(t)^{-\frac{c_{\text{int}}}{2}}}$$

***bonus:** limiting CFT contains $\mathbb{R}^N$ sigma model* ✓ *(Ooguri, Wang, 2024)*

One more recap for the road

- *information-theoretic **factorization** links distances and towers*
- ***if light:***
  - *gapped below cutoff* ⟶ *KK-like +* $p \to \infty$ *limit*
  - *gapped at cutoff* ⟶ *exponential DoS + higher-spin + rigid S-matrix*

- *worldsheet allows **converse:***
  - *small cutoff* ⇔ *(exponentially)* ***light tower*** + ***emergent geometry***

Outlook

- ❖ *"bootstrapping" strings from many directions*
- ❖ *coherent emerging picture* ***from the bottom up***
  - ➢ *light species & information theory*
  - ➢ *(minimal) black holes & thermodynamics*
- ❖ ***emergence of geometry*** *& clarifications from top-down*

Geometric decompactifications — EFT estimates

❖ *compactify on* n*-dim. manifold* X

➢ *heat kernel* $K_X(t)$ *determines one-loop contribution*

$$S_{\text{1-loop}} \sim -\frac{1}{2(4\pi)^{\frac{d}{2}}} \int_{\Lambda_{\text{st}}^{-2}}^{\infty} \frac{dt}{t^{1+\frac{d}{2}}} K_X(t) \sum_{k\geq 0} a_{2k}(\mathcal{R})\, t^k$$

curvature ops.

➢ *"relevant" vs. "irrelevant" ops.*

(IB, Lüst, Montella, 2023) (Aoufia, IB, Leone, 2024)

$$m_{\text{gap}}^{d-2k} \int_{\frac{m_{\text{gap}}^2}{\Lambda_{\text{st}}^2}} \frac{ds}{s}\, s^{k-\frac{d+n}{2}} \sim \begin{cases} \boxed{\Lambda_{\text{st}}^{2-2k}} & 2k<d+n \\ \boxed{m_{\text{gap}}^{d-2k}} & 2k>d+n \end{cases}$$

Planck scale appears (QG effect)

no Planck scale (field theory effect)

(Castellano, Herráez, Ibáñez, 2023)

(Bedroya, Mishra, Wiesner, 2024)

Some gory details — existence of light tower

- *assume* N *states go below some threshold weight* Δ_{th}
  - *bound modular integral with strip integral* [$\tau = x+iy$]

$$|\widetilde{\mathcal{Z}_{T^2}}| \leq y^{\frac{c}{2}} \sum_{j,\Delta>0} e^{-2\pi\Delta y} + |E_{\frac{c}{2}} - y^{\frac{c}{2}}|$$

  - *split sum into* $Z_{\text{below}} + Z_{\text{above}}$ *with* $Z_{\text{below}} \leq N$ $\longrightarrow$ *modular invariance*

$$Z_{\text{above}} \overset{y>1}{\leq} \frac{e^{2\pi\left(\frac{1}{y}-y\right)\Delta_{\text{th}}}}{1-e^{2\pi\left(\frac{1}{y}-y\right)\Delta_{\text{th}}}} Z_{\text{below}} \qquad Z_{\text{above}} \overset{y<1}{\leq} \frac{1}{1-e^{2\pi\left(y-\frac{1}{y}\right)\Delta_{\text{th}}}} Z_{\text{below}}$$

$\longrightarrow$ $\boxed{\textit{finite } N \Rightarrow \textit{finite Wilson coeff.}}$

More gory details — modular differential equation

- *assume weights are* ***light*** $\Delta = \Delta_0 f(t) \sim \Delta_0/t$ *or* ***heavy*** $\Delta >> 1$
  - ***asymptotic*** *differential equation (akin to Narain lattice sum)*

$$\left(-t^2\partial_t^2 - (2-c)t\partial_t\right)\mathcal{Z}_{T^2} \overset{t\gg 1}{\sim} \left(\Delta_\tau - \frac{c}{2}\left(1-\frac{c}{2}\right)\right)\mathcal{Z}_{T^2}$$

- *regulate & integrate over fundamental domain*

(Rankin, 1939) (Selberg, 1940) (Zagier, 1982) (Angelantonj, Florakis, Pioline, 2011) (Angelantonj, Cardella, Elitzur, Rabinovici, 2011)

  - ***geometric scaling*** *of integral*

$$\boxed{I(t) \overset{t\gg 1}{\sim} t^{\frac{c}{2}} \sim \Delta_{\text{gap}}^{-\frac{c}{2}}}$$

Last one, I promise — factorized CFT limits

- *relax spectrum to factorization* Z(t) = A(t)B *("partial decompactification")*
  - ***harmonic decomposition*** *w.r.t. fundamental domain*

    (Benjamin, Collier, Fitzpatrick, Maloney, Perlmutter, 2021)

$$\widetilde{A} = \underbrace{\frac{3}{\pi} I_A(t)}_{\textit{preceding result}} + \sum_{n>0} a_n(t)\, \underbrace{\nu_n}_{\textit{Maass cusp forms}} + \int_{\mathrm{Re}(s)=\frac{1}{2}} ds\, \alpha_s\, \underbrace{E_s}_{\textit{real analytic Eisenstein series}}$$

$$\boxed{I_{AB}(t) \overset{t\gg 1}{\sim} \underbrace{a\, t^{\frac{c_A}{2}}}_{\textit{QG geometric scaling}} + \underbrace{b\, t^{\frac{c_A+c_B-2}{2}}}_{\textit{field theory gap contribution}}}$$

bonus: *log threshold terms [when expected]* ✔