Study of the X(3915) at Belle

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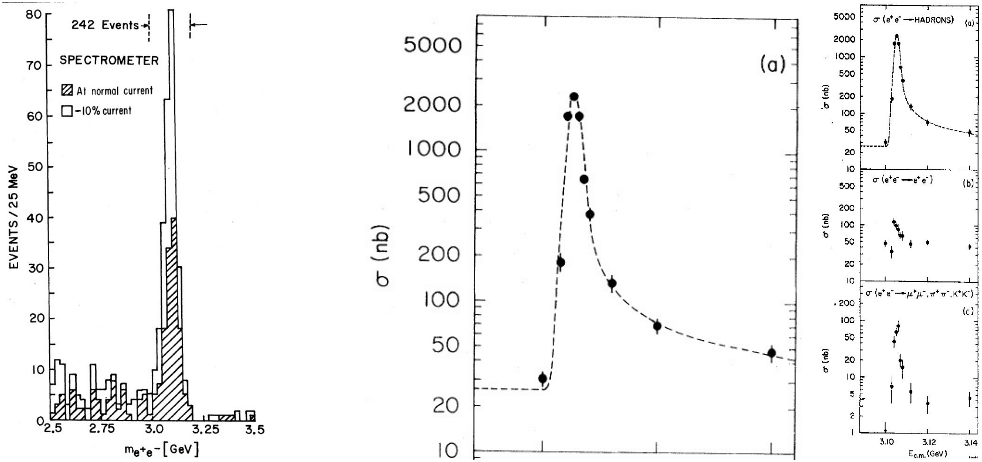
Charmonium. Origins



[1]

Discovery of the J/ψ

[1] Phys. Rev. Lett. 33, 1404 (1974), Phys. Rev. Lett. 33, 1406 (1974)

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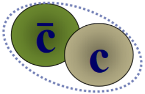
State	
$1\,^1S_0$	η_c
$1\,^3S_1$	J/ψ
$1\,^1P_1$	h_c
$1\,^3P_0$	χ_{c0}
$1\,^3P_1$	χ_{c1}
$1\,^3P_2$	χ_{c2}
$2\,^1S_0$	$\eta_c(2S)$
$2\,^3S_1$	$\psi(2S)$
$1\,^1D_2$	η_{c2}
$1\,^3D_1$	$\psi(3770)$
$1\,^3D_2$	ψ_2

[1]



[2]

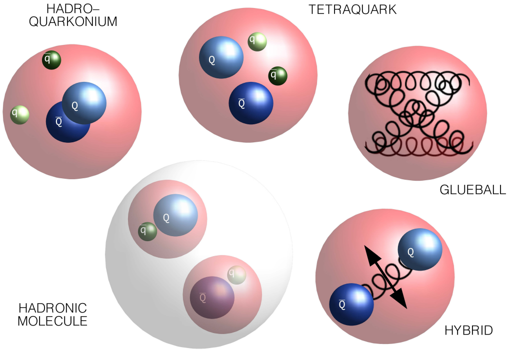
- **Charmonia - excited $c\bar{c}$ states**

Nomenclature:
n = 1, 2, 3 ...
L = 0 (S), 1 (P), 2 (D) ...
S = 0 ($\frac{1}{2} - \frac{1}{2}$) or 1 ($\frac{1}{2} + \frac{1}{2}$)
J = |L - S|, ..., |L+S|
$n^{2S+1}L_J$
Example:
χ_{c2}, $1^3P_2 \rightarrow$ L = 1, S = 1, J = 2, n = 1
$\chi_{c2}(2P)$, $2^3P_2 \rightarrow$ L = 1, S = 1, J = 2, n = 2

[1] Ed. A.J. Bevan, B. Golob, Th. Mannel, S. Prell, and B.D. Yabsley, Eur. Phys. J. C74 (2014) 3026, SLAC-PUB-15968, KEK Preprint 2014-3.
[2] (c) Galina Pakhlova, Belle

Exotic states

- New states continuously being discovered
- Interpretation of some states unclear,
  exotic nature suggested:
  tetraquark, molecular state, hybrid meson,
  glueball, ...



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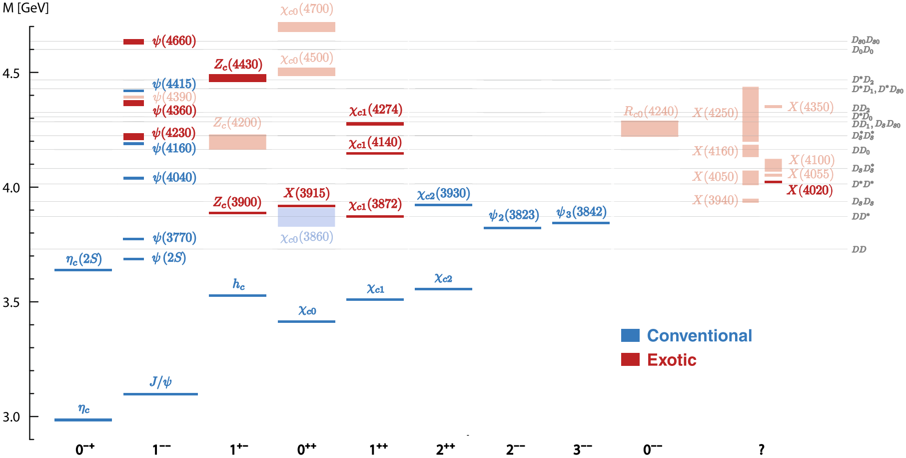
[1] https://www.fz-juelich.de/en/ias/ias-4/research/exotic-hadrons, (c) Forschungszentrum Jülich

Observed states



Many states inconsistent with conventional $q\bar{q}$ hypothesis

[3] Eichmann, G., Fischer, C.S., Heupel, W. et al. Four-Quark States from Functional Methods. Few-Body Syst 61, 38 (2020)

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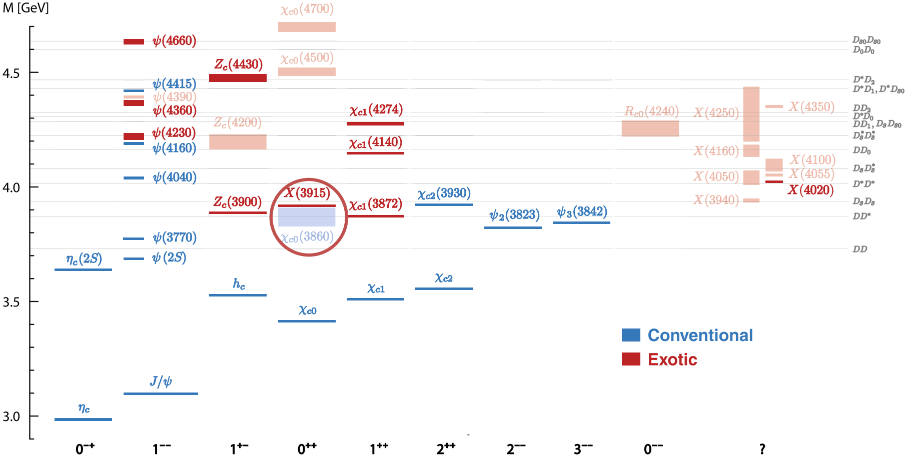

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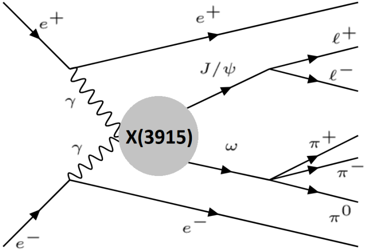
X(3915): Motivation

X(3915) - resonance, observed in e^+e^--induced process:

$$\gamma\gamma \to \boldsymbol{X(3915)} \to \boldsymbol{J/\psi\omega}$$
$$(\boldsymbol{e^+e^- \to e^+e^- J/\psi\omega})$$



- First observed at Belle and BaBar, not really consistent with predicted nearby charmonium $\chi_{c0}(2P)$
- Recently discovered X*(3860)[4] is a much better candidate for $\chi_{c0}(2P)$

$$\Downarrow$$

X(3915) is interpreted as exotic state (e.g. molecular state or hybrid meson)

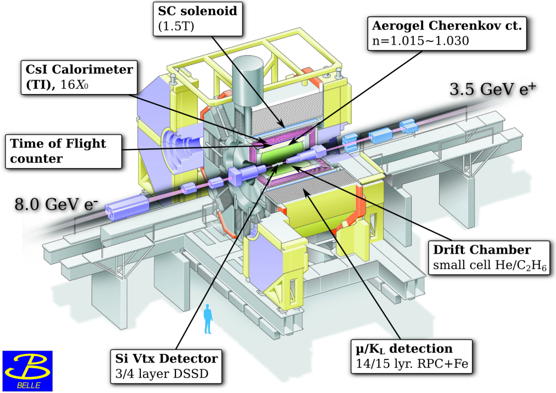
Goal: measure the quantum numbers (spin, parity) at Belle

[4] arXiv:1704.01872, Belle, PRD 95 (2017) 112003

Belle Detector

Asymmetric e^+e^- experiment mainly at the $\Upsilon(4S)$ resonance (10.58 GeV)

Our final state:
$\ell^+, \ell^-, \pi^+, \pi^-, \pi^0$
4 charged tracks and two calorimeter clusters $(\pi^0 \to \gamma\gamma)$

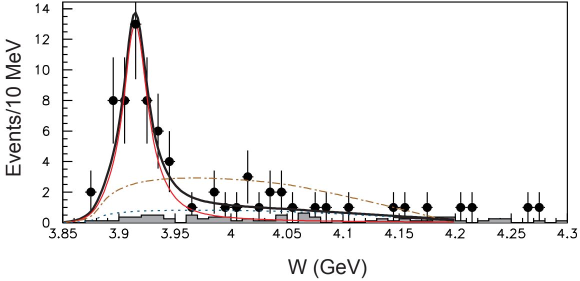


	KEKB/Belle
Operation	1999–2010
Peak luminosity	$2.11 \times 10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$
Integrated luminosity	$1\,\mathrm{ab}^{-1}$ (772 million $B\bar{B}$ pairs)

X(3915): Previous analyses

Belle analysis[5] (2010):

- 694 fb^{-1} partial Belle dataset
- 7.7σ significance
- M $= (3915 \pm 3 \pm 2)$ MeV$/c^2$
- $\Gamma = (17 \pm 10 \pm 3)$ MeV$/c^2$



$$W = M_X - M_{\ell^+\ell^-} + M_{J/\psi}$$

$$\Gamma_{\gamma\gamma}(X(3915))\mathcal{B}(X(3915) \to \omega J/\psi) = \begin{cases} (61 \pm 17 \pm 8)\ \mathrm{eV} & \text{for } J^P = 0^+ \\ (18 \pm 5 \pm 2)\ \mathrm{eV} & \text{for } J^P = 2^+ \end{cases}$$

Confirmed by BaBar: "Data largely preferred $J^P = 0^\pm$ over 2^+; ... 0^+ over 0^-"

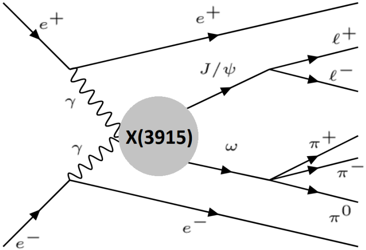
[5] Belle, PRL 104 (2010) 092001, arXiv:0912.4451

Improvements

- Uncertainty dominated by limited sample size
  $\rightarrow$ Use the full Belle dataset ($\times$1.4 larger dataset)
- None of the J^P hypotheses $0^+, 0^-, 2^+, 2^-$ is conclusively excluded
  $\rightarrow$ Use amplitude analysis formalism to construct more powerful J^P test and identify J^P preferred by data.

Event selection

- In $e^+e^- \to e^+e^- J/\psi\omega$ both recoil $e^\pm$ are not detected ("zero tag")
- Reconstruct $X(3915) \to J/\psi\omega$ with $J/\psi \to \ell^+\ell^-(\ell = e, \mu), \omega \to \pi^+\pi^-\pi^0$



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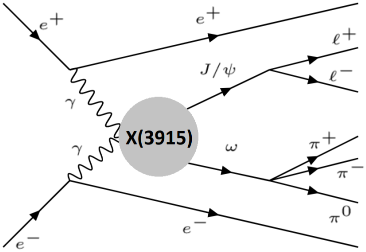

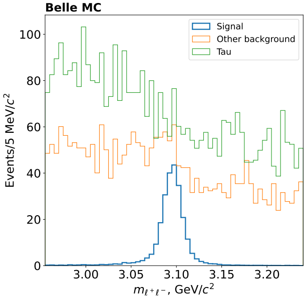

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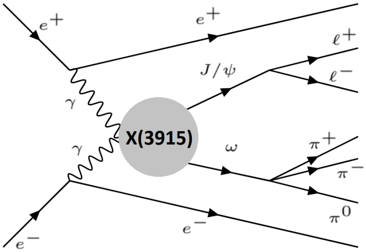

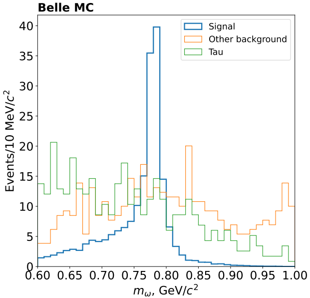

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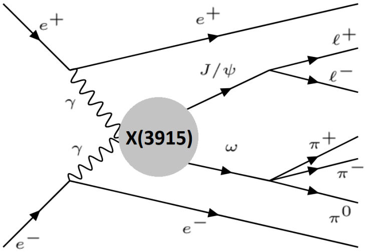

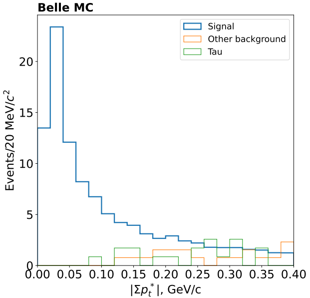

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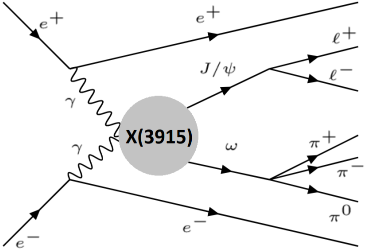

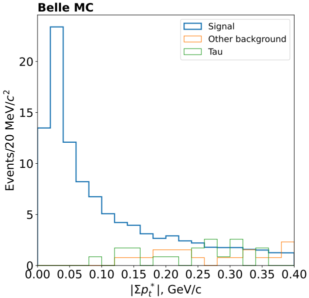

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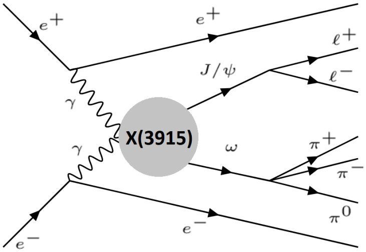
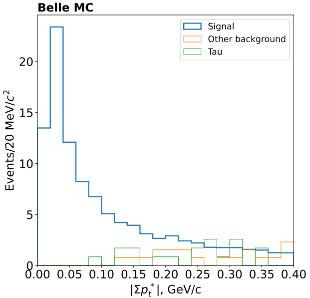
- Veto on specific background processes ($\gamma\gamma \to \pi^0\psi(2S)$, $e^+e^- \to \gamma X$)





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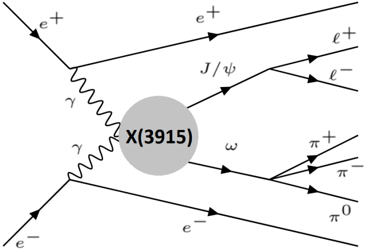
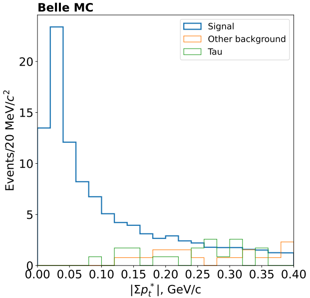
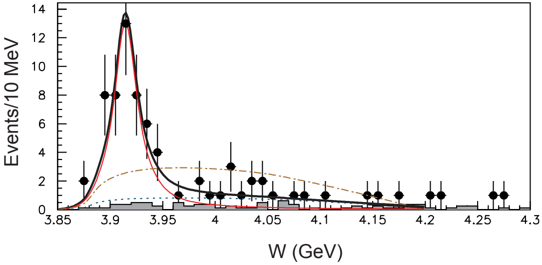
- Veto on specific background processes ($\gamma\gamma \to \pi^0\psi(2S)$, $e^+e^- \to \gamma X$)
- Other selection criteria



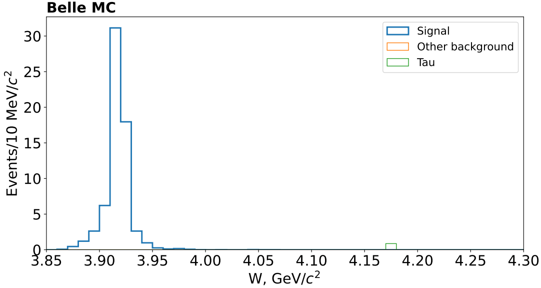


Event selection

Belle analysis (2010)



This analysis



- The background suppression is on a good level
- Event selection is approaching the quality of the previous Belle analysis

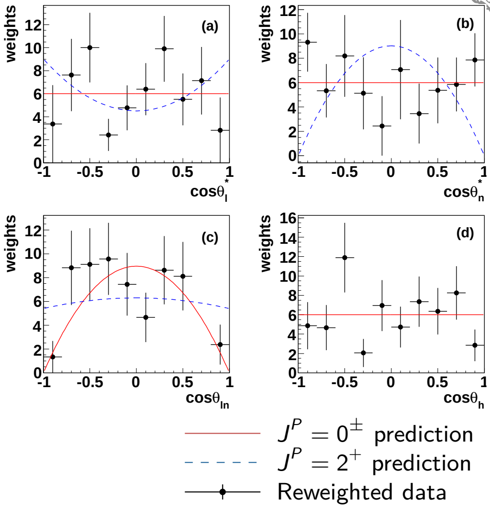
Quantum number determination

Angular analysis by BaBar[6]

Angular analysis:

- Uses 1D projections of the multi-dimensional phase-space distribution of the final-state particles
- Correlations between variables are not taken into account
- Different variable sets used for distinction between different J^P hypotheses, e.g. between 2^+ and $0^\pm$ and between 0^- and 0^+

Data prefer $J^P = 0^+$ but other hypotheses not excluded



[6] BABAR, PRD 86 (2012) 072002, arXiv:1207.2651v2

Amplitude analysis formalism

Amplitude analysis formalism:

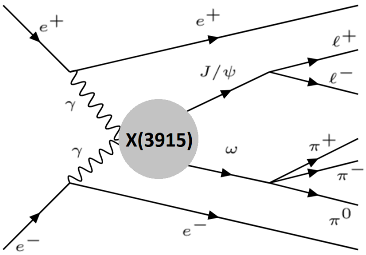
- Construct model that describes full distribution in phase space including correlations (9-dim)
- Uses complete information of measured events. Higher sensitivity expected.
- Requires a reasonable model for parametrization of signal and background

Workflow:

1. Develop models for the J^P hypotheses to be tested
2. Fit all J^P hypotheses to data
3. Choose J^P hypothesis that describes data best as a null hypothesis
4. Calculate the distances to other hypotheses (ideally larger than 5σ)

Angular analysis

- Theory model developed by Boris Grube
- Intensity depends on 9 kinematic variables:
  - θ_X, ϕ_X: decay angles for $X \to J/\psi\omega$
  - $\theta_{J/\psi}, \phi_{J/\psi}$: decay angles for $J/\psi \to \ell^+\ell^-$
  - $\alpha_\omega, \beta_\omega, m_\omega, X_\omega, Y_\omega$: decay angles and dynamic variables for $\omega \to \pi^+\pi^-\pi^0$
- J^P quantum numbers hypotheses:
  $0^+, 0^-, 2^+, 2^-, 3^+, 4^+, 4^-$
  Higher values suppressed due to low breakup momentum ($\sim$ 200 MeV)
- Maximize log-likelihood:



$$\ln \mathcal{L}(\vec{\theta}; \tau_k) = \underbrace{\sum_{k=1}^{N} \ln \mathcal{I}(\tau_k; \vec{\theta})}_{\text{Data sample}} - \underbrace{N \ln[\frac{1}{N_{MC}} \sum_{j=1}^{N_{\text{MC}}^{\text{acc}}} \mathcal{I}(\tau_j; \vec{\theta})]}_{\text{Phase space MC}},$$

where $\mathcal{I}$ - *intensity*, probability of # of produced events in the phase space

$\to$ Can be extended to account for weighted events and non-interfering background.

Partial waves of the $J/\psi\omega$ system

J^P	$P(-1)^J$	$\mathcal{A}^X_{L_X\,S_X}$	Free parameters
0^+	$+1$	$S_0,\ D_2$	2
0^-	-1	P_1	0
2^+	$+1$	$S_2,\ D_0,\ D_1,\ D_2,\ G_2$	9
2^-	-1	$P_1,\ P_2,\ F_1,\ F_2$	7
3^+	-1	$D_1,\ D_2,\ G_1,\ G_2$	7
4^+	$+1$	$D_2,\ G_0,\ G_1,\ G_2,\ I_2$	9
4^-	-1	$F_1,\ F_2,\ H_1,\ H_2$	7

Fit parameters:

- A complex coefficient for every partial-wave amplitude $\mathcal{A}^X_{L_X\,S_X}$. For selected reference amplitude it is set to 1.
- One additional real coefficient for the fraction of production of X state with helicity 2 and 0.

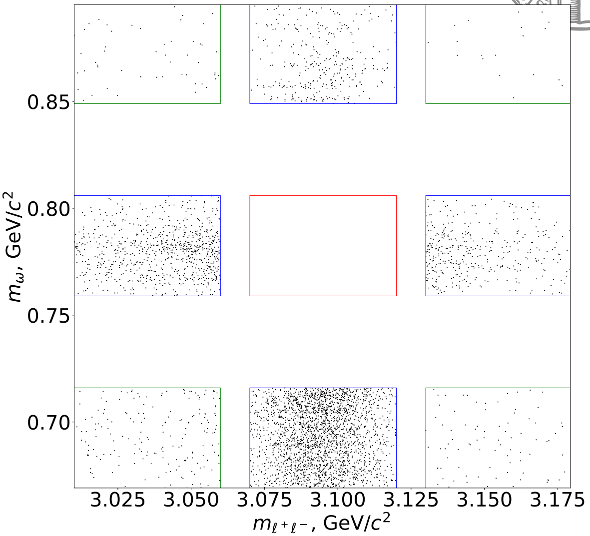
Sideband events

Subtraction scheme: S' = S - (2*B - C)/4

Technically, we put negative weights on sideband events, when summing log-Likelihoods:

$$\ln \mathcal{L}_{\text{weighted}}(\vec{\theta}; \{\tau_k, w_k\}) =$$

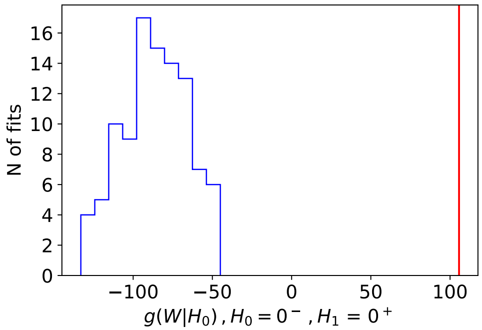
$$\sum_{k=1}^{N} w_k \ln \mathcal{I}(\tau_k; \vec{\theta}) - N \ln[\frac{1}{N_{MC}} \sum_{j=1}^{N_{\text{MC}}^{\text{acc}}} \mathcal{I}(\tau_j; \vec{\theta})]$$



Hypothesis testing: Monte Carlo study

In the simplest case we have 2 hypotheses: the null hypothesis H_0 and the alternative hypothesis H_1

1. Generate N H_0 datasets, fit with both H_1 and H_0



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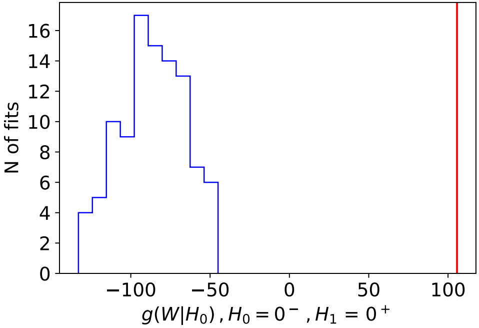

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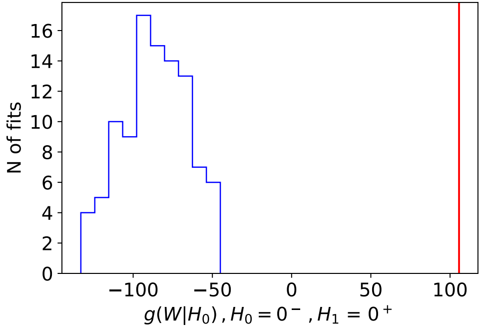

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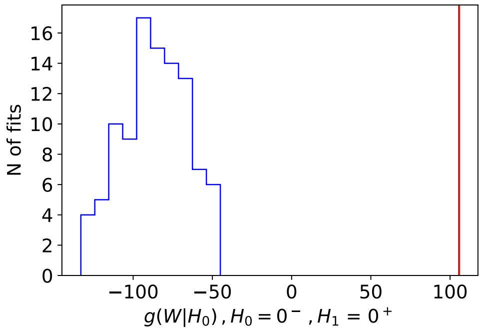

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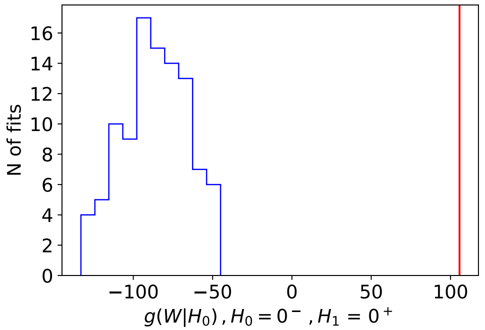

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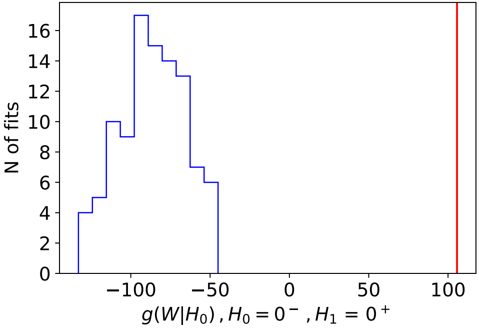

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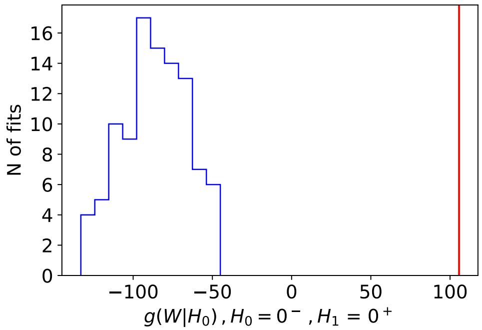


For multiple hypotheses pick the fit with highest log-likelihood as null hypothesis and test other hypothesis against it.

Summary

- The resonance X(3915) is observed in $\gamma\gamma$ processes at e^+e^- colliders
- The state is being interpreted as an exotic one, rather than charmonium
- Previous analyses do not exclude any of the J^P hypotheses $(0^+, 0^-, 2^+, 2^-)$.
  This is important for determining the nature of X(3915)
- Uncertainties are dominated by statistics. More data is needed for a more precise determination

⋆ Current analysis operates with more statistics, by working with full Belle dataset

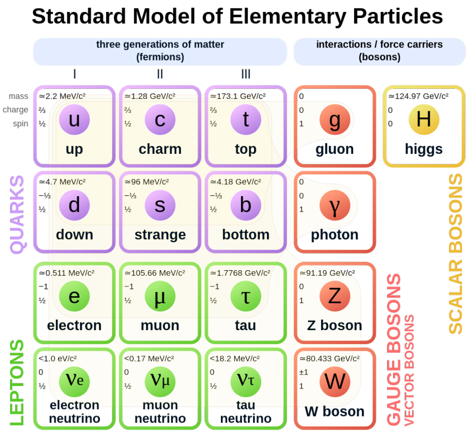
⋆ Amplitude analysis will be used to extract quantum numbers in a more efficient manner

⋆ Currently the fit is finalised.

Thank you for your attention!

Backup

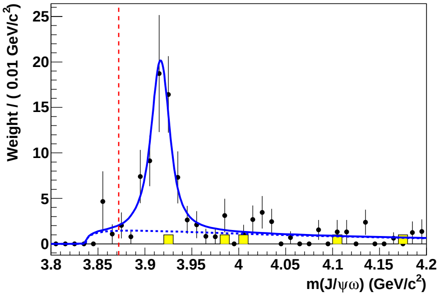
Standard Model



Previous analyses

BaBar analysis[7] (2012):

- 519.2 fb^{-1} data
- 7.6σ significance
- M = (3919.4 ±2.2 ±1.6) MeV
- Γ = (13 ±6 ±3) MeV



$$\Gamma_{\gamma\gamma}(X(3915))\mathcal{B}(X(3915)\to\omega J/\psi) = \begin{cases} (52\pm10\pm3)\ \mathrm{eV} & \text{for } J^P = 0^+ \\ (10.5\pm1.9\pm0.6)\ \mathrm{eV} & \text{for } J^P = 2^+ \end{cases}$$

"Data largely preferred $J^P = 0^{\pm}$ over 2^+; ... 0^+ over 0^-"

All results consistent with those of Belle!!

[7] BABAR, PRD 86 (2012) 072002, arXiv:1207.2651v2

Selection criteria

Track selection

- $|d_0| < 6cm$
- $|z_0| < 6cm$
- $actPIDBelle(3,2) < 0.8$ $(= P(K|\pi) < 0.8)$
- $N_{cleanedtracks} = 4$

LeptonID pair selection

1. If either of the lepton tracks has $elID > 0.6$ accept as $J/\psi \to e^+e^-$
2. Else, if either of the lepton tracks has $muID > 0.1$ accept as $J/\psi \to \mu^+\mu^-$
3. Otherwise discard

π^0 selection

- $\chi^2 < 4$

1. If 1 candidate at $p_t > 0.1 GeV/c$ - take it
2. If >1 candidate at $p_t > 0.1 GeV/c$ - discard event
3. If 0 candidates at $p_t > 0.1 GeV/c$ - preserve all and do best candidate selection by χ^2

Mass windows

- $3{,}07 GeV/c^2 < M_{J/\psi} < 3.12 GeV/c^2$
- $0.813 GeV/c^2 < M_\omega < 0.753 GeV/c^2$
- $M_X < 4.3 GeV/c^2$

Selection criteria (cont.)

ISR and $\psi(2S)$ rejection

- $P_z > (M_5^2 - 49\,GeV^2/c^4)/14\,GeV/c^3 + 0.6\,GeV/c$
- $|M_{\ell^+\ell^-\pi^+\pi^-} - M_{\ell^+\ell^-} - 0.589\,GeV/c^2| > 0.01\,GeV/c^2$

Transverse momentum balance

- $|\Sigma \mathbf{p}_t^*| < 0.1\,GeV/c$

Fitting

$$\ln \mathcal{L}(\vec{\theta}; \tau_k) = \sum_{k=1}^{N} \ln \mathcal{I}(\tau_k; \vec{\theta}) - N \ln[\frac{1}{N_{MC}} \sum_{j=1}^{N_{\mathrm{MC}}^{\mathrm{acc}}} \mathcal{I}(\tau_j; \vec{\theta})], \quad (1)$$

1. Maximize log likelihood
   - τ_i - measured angles in an event
   - $\vec{\theta}$ - amplitude values (fitted parametres)

Fitting

$$\ln \mathcal{L}(\vec{\theta}; \tau_k) = \sum_{k=1}^{N} \ln \mathcal{I}(\tau_k; \vec{\theta}) - N \ln[\frac{1}{N_{MC}} \sum_{j=1}^{N_{\rm MC}^{\rm acc}} \mathcal{I}(\tau_j; \vec{\theta})], \quad (1)$$

1. Maximize log likelihood
   - τ_i - measured angles in an event
   - $\vec{\theta}$ - amplitude values (fitted parametres)
2. Intensity for each amplitude is theoretically derived

Fitting

$$\ln \mathcal{L}(\vec{\theta}; \tau_k) = \sum_{k=1}^{N} \ln \mathcal{I}(\tau_k; \vec{\theta}) - N \ln[\frac{1}{N_{MC}} \sum_{j=1}^{N_{\rm MC}^{\rm acc}} \mathcal{I}(\tau_j; \vec{\theta})], \quad (1)$$

1. Maximize log likelihood
   - τ_i - measured angles in an event
   - $\vec{\theta}$ - amplitude values (fitted parametres)
2. Intensity for each amplitude is theoretically derived Looks like this

$$\mathcal{I}_X^{J\,P}(\tau_X, m^2_{\pi^-\pi^0}, m^2_{\pi^0\pi^+}) = \overline{\mathcal{N}}\, |\mathcal{D}_\omega(m_\omega)|^2\, |\mathcal{D}_\omega(m^2_{\pi^-\pi^0}, m^2_{\pi^0\pi^+}; m_{\omega,0})|^2 \times \left[|\overline{\Psi}_X^{J\,P\,(\lambda=0)}(\tau_X)|^2 + |\overline{\Psi}_X^{J\,P\,(\lambda=+2)}(\tau_X)|^2 \right], \quad (2)$$

$$\overline{\Psi}_X^{J\,P\,\lambda\,\uparrow\downarrow}(\tau_X) = \sum_{L_X, S_X} \mathcal{D}_X(m_X)\, F_{L_X}(m_X)$$

$$\sum_{\lambda_{J/\psi}=-1}^{+1} \sum_{\lambda_\omega=-1}^{+1} \overline{\mathcal{A}}_{L_X\,S_X}^{X\,\lambda}\, (1\,\lambda_{J/\psi}, 1\ -\lambda_\omega | S_X\,\lambda_X)\, (L_X\,0, S_X\,\lambda_X | J\,\lambda_X)$$

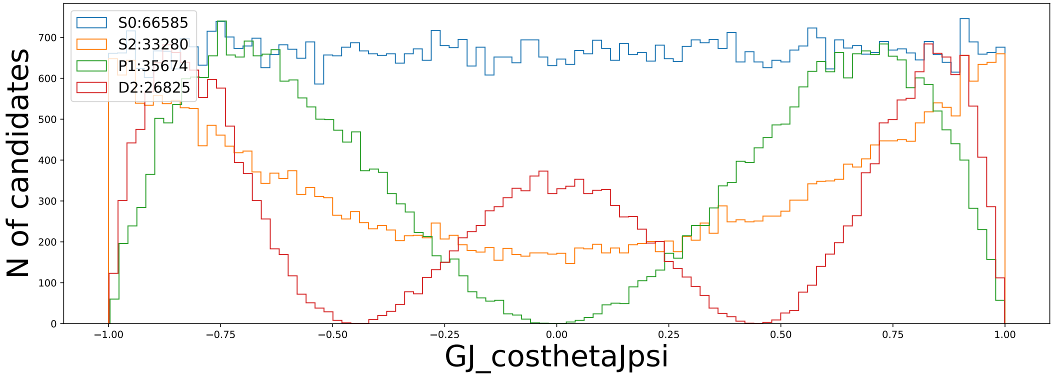
$$\times\, D^{J*}_{\lambda\,(\lambda_{J/\psi}-\lambda_\omega)}(\phi_X^{\rm GJ}, \vartheta_X^{\rm GJ}, 0)\, D^{1*}_{\lambda_{J/\psi}\ 1}(\phi^{\rm HF}_{J/\psi}, \vartheta^{\rm HF}_{J/\psi}, 0)\, D^{1*}_{\lambda_\omega\ 0}(\alpha_\omega, \beta_\omega, 0). \quad (3)$$

Fitting

1. Maximize log likelihood
   - τ_i - measured angles in an event
   - $\vec{\theta}$ - amplitude values (fitted parametres)
2. Intensity for each amplitude is theoretically derived
3. One quantum number hypothesis fitted is several amplitudes summed incoherently

J^P	$P(-1)^J$	$\mathcal{H}^X_{\lambda_{J/\psi}\,\lambda_\omega}$	$\mathcal{A}^X_{L_X\,S_X}$
0^+	$+1$	$\mathcal{H}^X_{0\,0},\ \mathcal{H}^X_{1\,-1}$	$S_0,\ D_2$
0^-	-1	$\mathcal{H}^X_{1\,-1}$	P_1
2^+	$+1$	$\mathcal{H}^X_{0\,0},\ \mathcal{H}^X_{1\,-1},\ \mathcal{H}^X_{1\,0},\ \mathcal{H}^X_{0\,1},\ \mathcal{H}^X_{1\,1}$	$S_2,\ D_0,\ D_1,\ D_2,\ G_2$
2^-	-1	$\mathcal{H}^X_{1\,-1},\ \mathcal{H}^X_{1\,0},\ \mathcal{H}^X_{0\,1},\ \mathcal{H}^X_{1\,1}$	$P_1,\ P_2,\ F_1,\ F_2$
3^+	-1	$\mathcal{H}^X_{1\,-1},\ \mathcal{H}^X_{1\,0},\ \mathcal{H}^X_{0\,1},\ \mathcal{H}^X_{1\,1}$	$D_1,\ D_2,\ G_1,\ G_2$
4^+	$+1$	$\mathcal{H}^X_{0\,0},\ \mathcal{H}^X_{1\,-1},\ \mathcal{H}^X_{1\,0},\ \mathcal{H}^X_{0\,1},\ \mathcal{H}^X_{1\,1}$	$D_2,\ G_0,\ G_1,\ G_2,\ I_2$
4^-	-1	$\mathcal{H}^X_{1\,-1},\ \mathcal{H}^X_{1\,0},\ \mathcal{H}^X_{0\,1},\ \mathcal{H}^X_{1\,1}$	$F_1,\ F_2,\ H_1,\ H_2$

Phase space MC. Reweighting



There are in total 9 angles fitted. For a given amplitude they are all correlated.