QUANTUM AND QUANTUM-INSPIRED SIMULATION OF LATTICE GAUGE THEORIES

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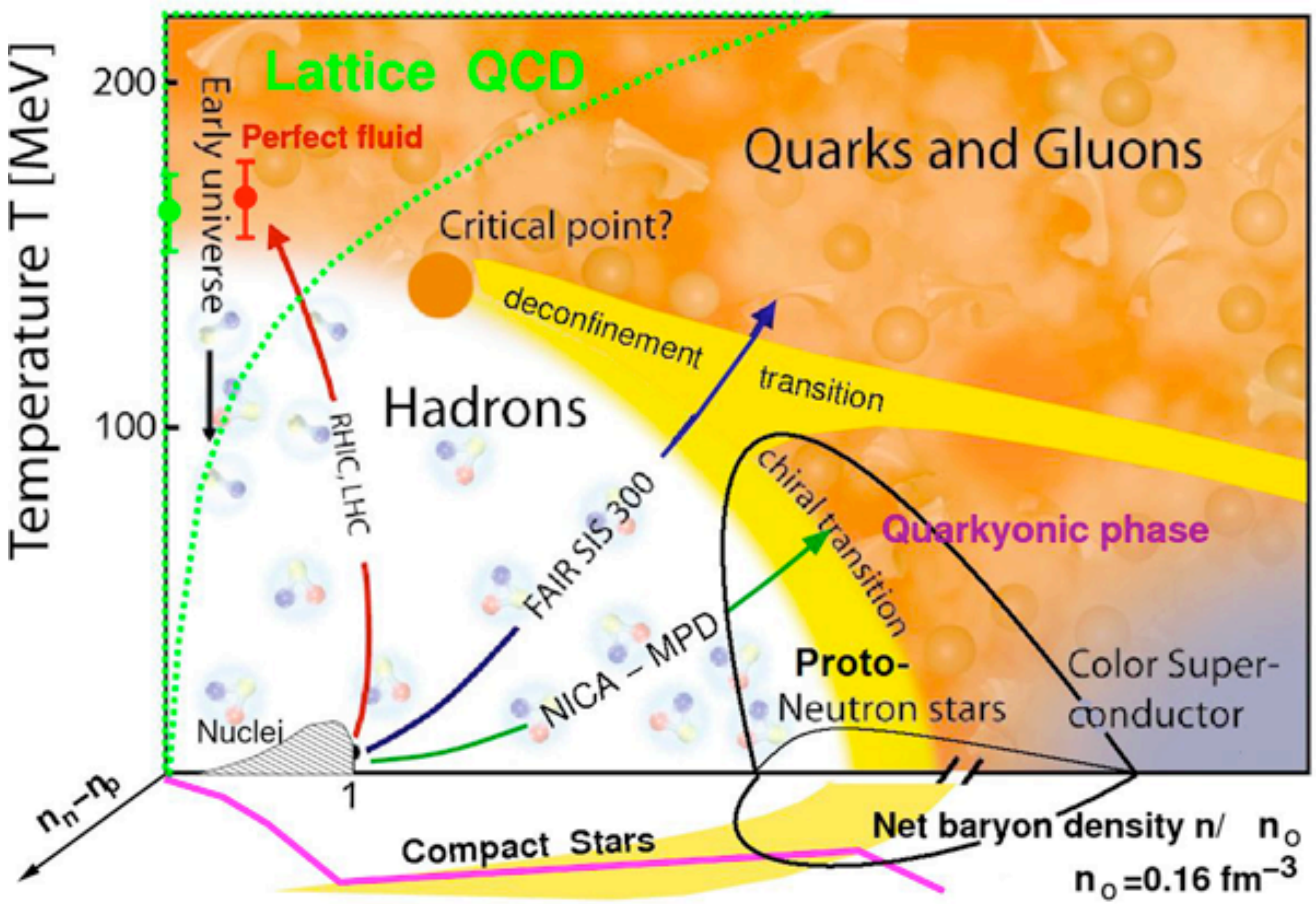
LATTICE GAUGE THEORIES

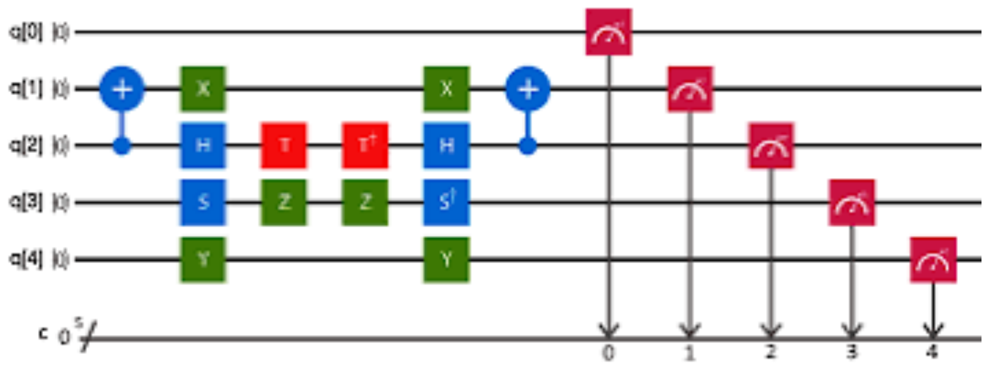


The current wisdom on the phase diagram of nuclear matter.

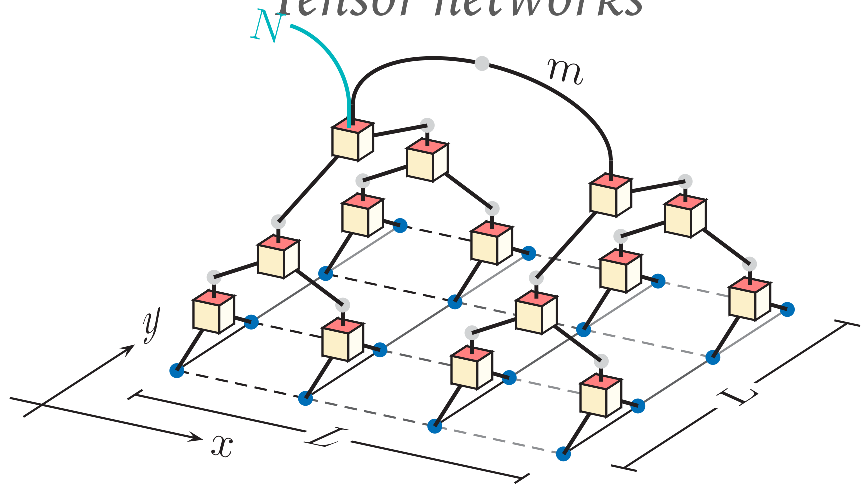
McLerran, L. Nucl.Phys.Proc.Suppl. 195 (2009) 275-280

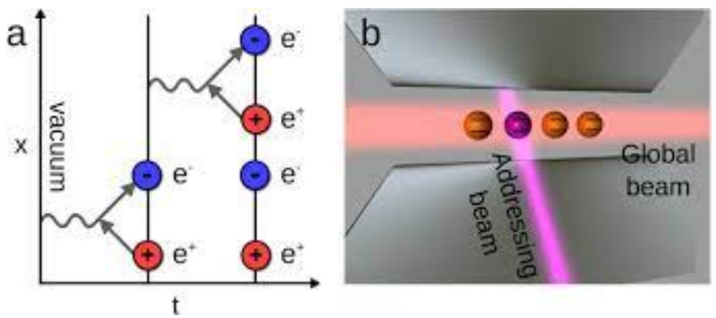
EFFICIENT WAVE FUNCTION REPRESENTATION – TODAY

Circuit quantum computation

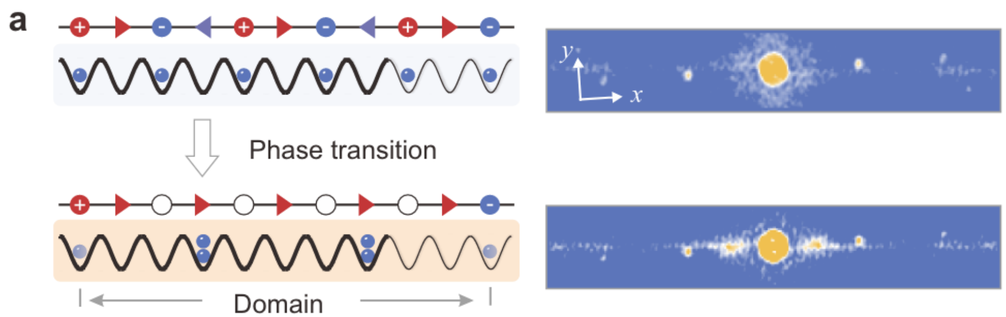


Tensor networks





IQOQI, Nature (2016, …)



Heidelberg, Nature (2020)

Quantum simulation of fundamental particles and forces

Christian W. Bauer, Zohreh Davoudi, Natalie Klco & Martin J. Savage

Nature Reviews Physics **5**, 420–432 (2023) | Cite this article

Simulating Lattice Gauge Theories within Quantum Technologies

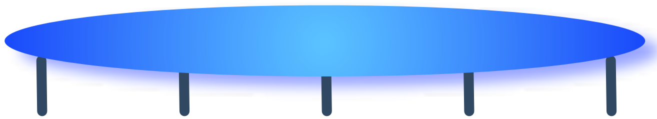
M.C. Bañuls[1,2], R. Blatt[3,4], J. Catani[5,6,7], A. Celi[3,8], J.I. Cirac[1,2], M. Dalmonte[9,10], L. Fallani[5,6,7], K. Jansen[11], M. Lewenstein[8,12,13], S. Montangero[7,14 a], C.A. Muschik[3], B. Reznik[15], E. Rico[16,17 b], L. Tagliacozzo[18], K. Van Acoleyen[19], F. Verstraete[19,20], U.-J. Wiese[21], M. Wingate[22], J. Zakrzewski[23,24], and P. Zoller[3]

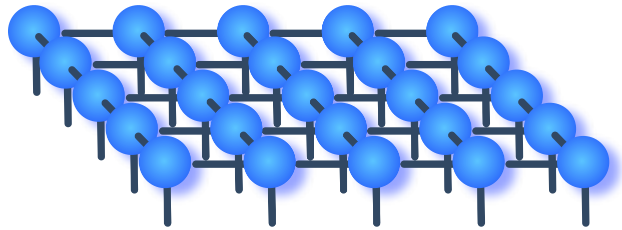
EPJD (2020)

TENSOR NETWORKS STATES

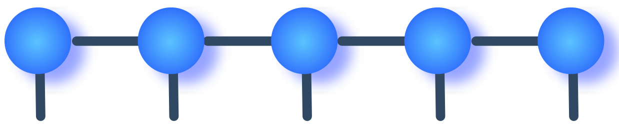
$$\psi_{\alpha_1,\alpha_2,\ldots\alpha_N}$$

$$\mathcal{O}(d^N)$$



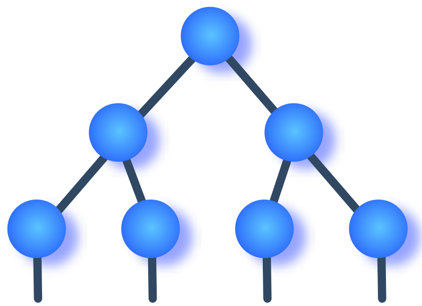


PEPS



$$\mathcal{O}(Ndm^2)$$

$$A^{\beta_1}_{\alpha_1} A^{\beta_1\beta_2}_{\alpha_2} \ldots A^{\beta_{N-1}}_{\alpha_N}$$



Tree Tensor Network

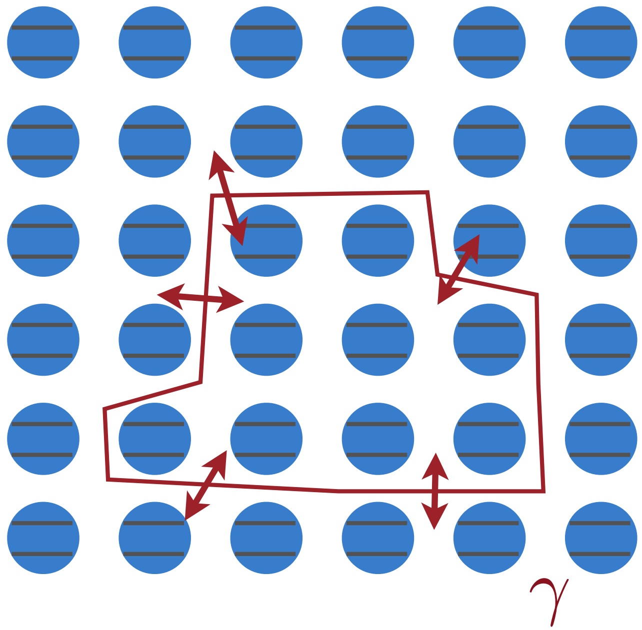
Tensor networks states are a compressed description of the system tunable between mean field and exact

ENTANGLEMENT OF PURE MANY-BODY QUANTUM SYSTEMS

For pure states:

$$\mathcal{S} = -\mathrm{Tr}\,\rho \log \rho$$

Von Neumann Entropy



Area law

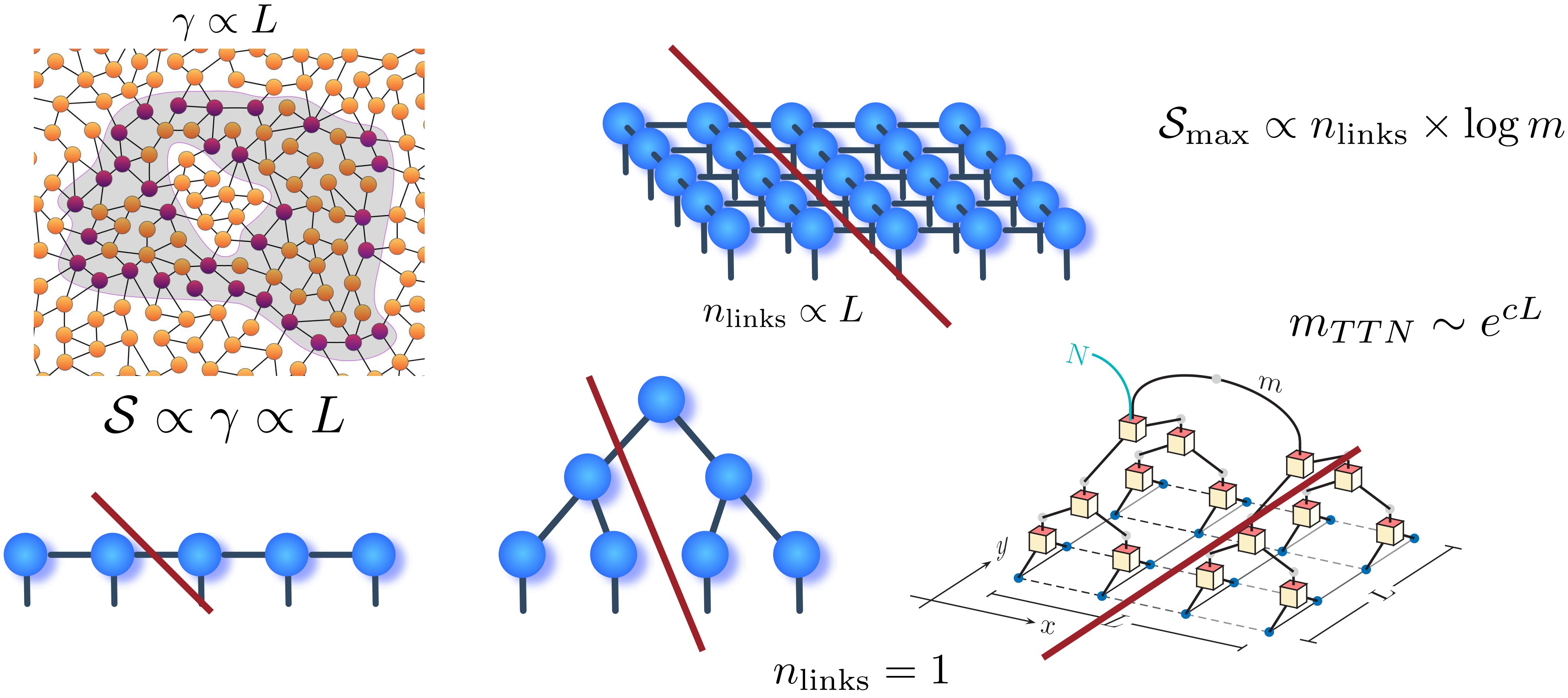
$$\mathcal{S} \propto \gamma$$

$$\mathcal{S} \propto N^{(D-1)}$$

1D critical systems:

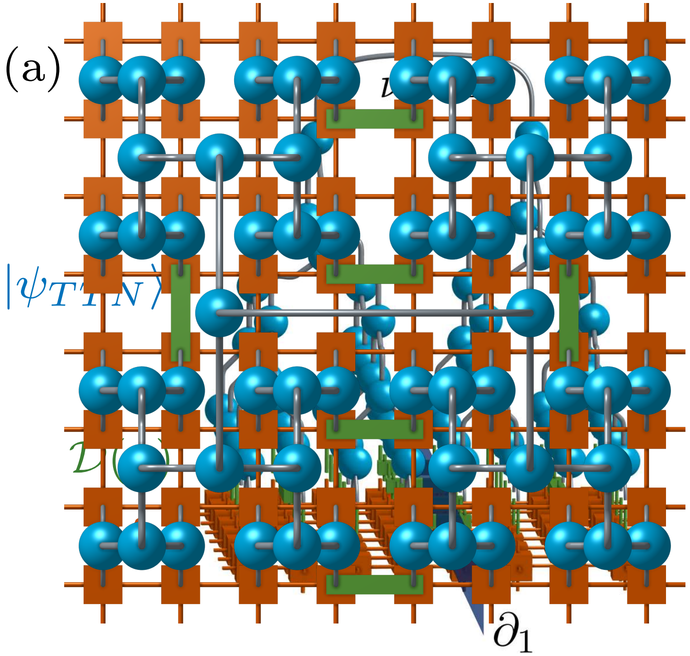
$$\mathcal{S} = \frac{c}{3} \log N$$

AREA LAWS AND TENSOR NETWORKS



Tensor Network	Complexity	Area law in 2D	Typical Bond dimensions	Exact contractable
MPS / DMRG	$\mathcal{O}\left\{\chi^3\right\}$	No (Only in 1D)	> 10.000	Yes ($\mathcal{O}\left\{\chi^3\right\}$)
TTN	$\mathcal{O}\left\{\chi^4\right\}$	No (Only in 1D)	$\approx 1.000 - 2.000$	Yes ($\mathcal{O}\left\{\chi^4\right\}$)
PEPS	$\mathcal{O}\left\{\chi^{10}\right\}$	Yes	~ 10	No ($\mathcal{O}\left\{\chi^L\right\}$)
MERA	$\mathcal{O}\left\{\chi^8\right\}$ (1D), $\mathcal{O}\left\{\chi^{16}\right\}$ (2D)	Yes	~ 10	Yes ($\mathcal{O}\left\{\chi^8\right\}$)

AUGMENTED TREE TENSOR NETWORKS



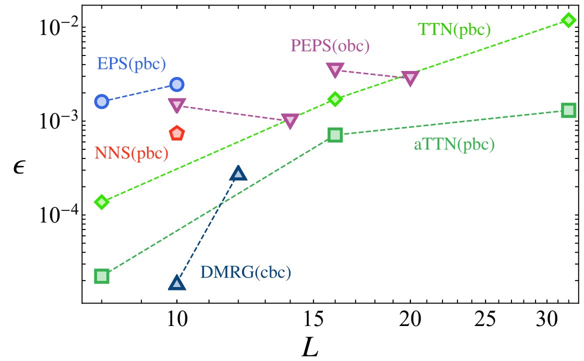
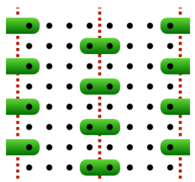
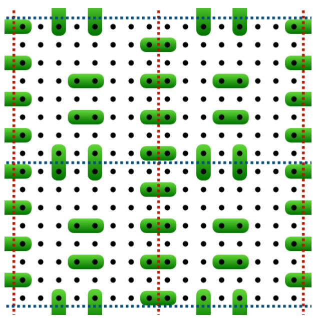
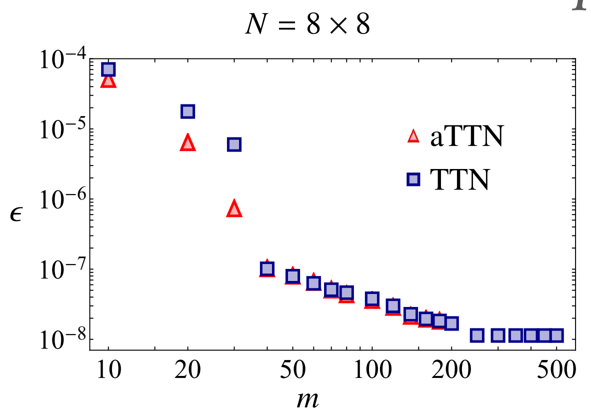


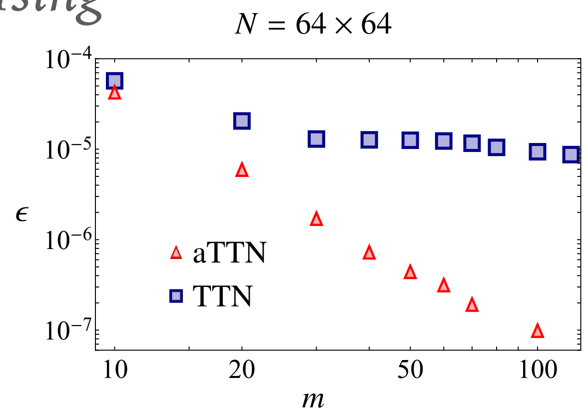
FIG. 2: Relative error ϵ of the 2D Heisenberg ground-state energy as a funciotn of the system linear size L compared with the best available estimates obtained by MC [21] for the TTN, aTTN, NNS [59], EPS [60], PEPS [61], 2D-DMRG [57]. Depending on the method open (obc), cylindrical (cbc) or periodic (pbc) boundary conditions have been chosen. For each datapoint, we compare the Monte Carlo result with the same boundary conditions.

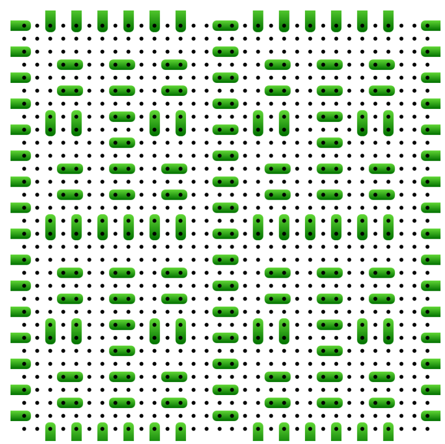
Ising





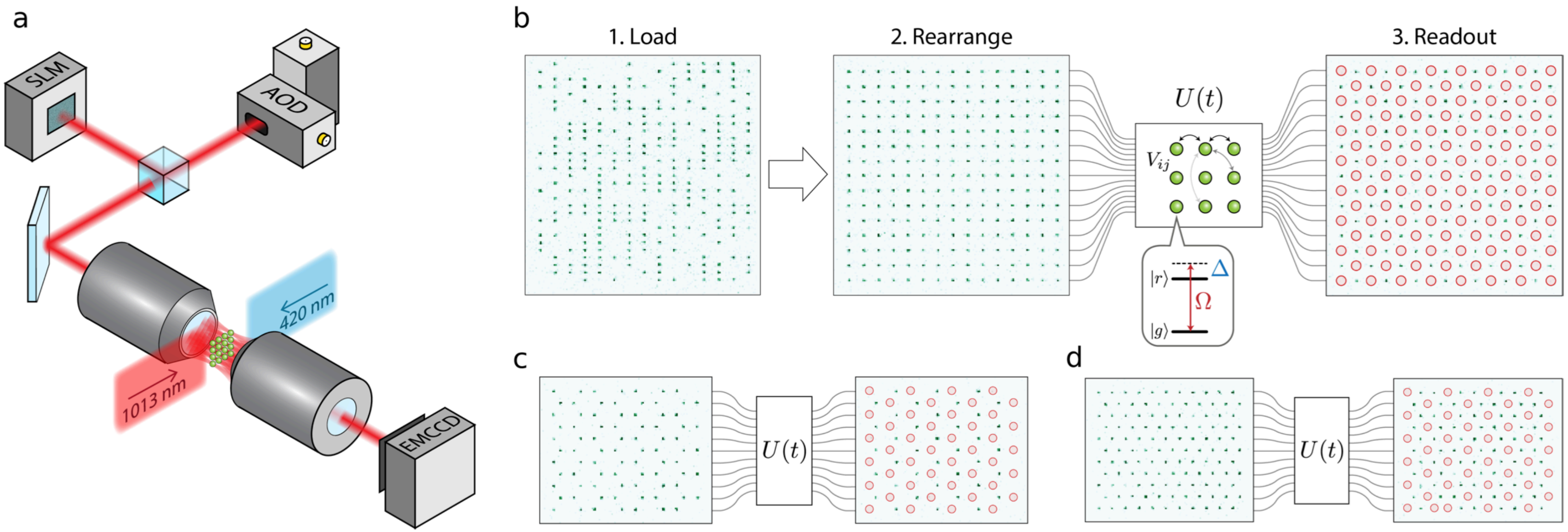


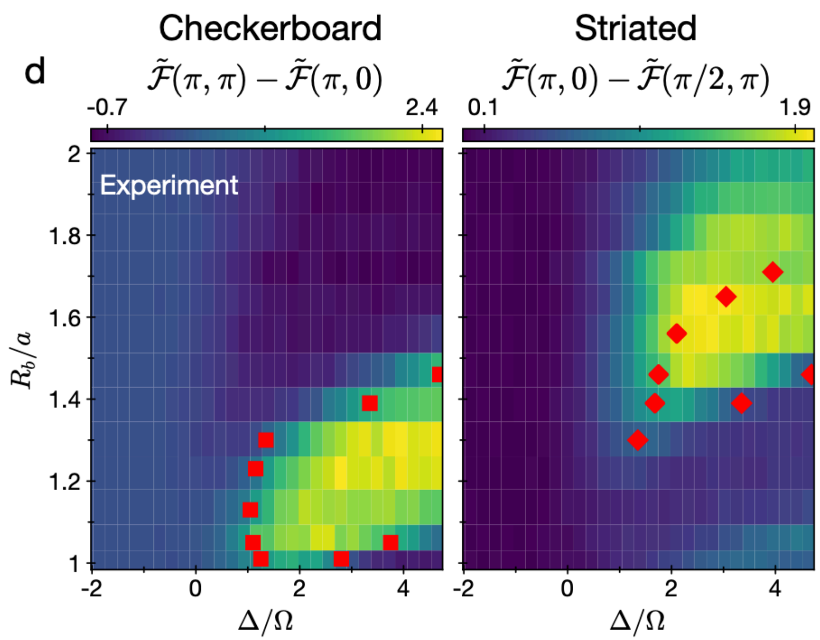




T. Felser, S. Notarnicola, S. Montangero PRL (2021)

2D RYDBERG QUANTUM SIMULATOR





Lukin's group

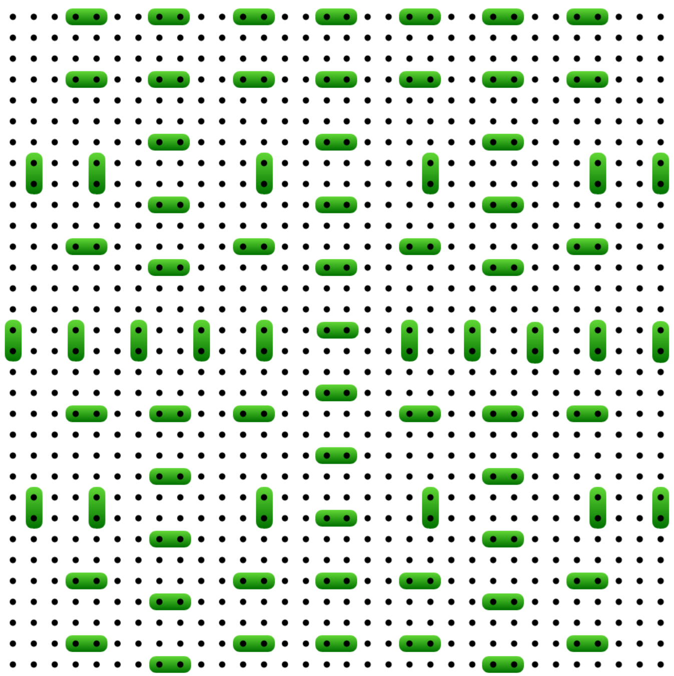
Quantum Phases of Matter

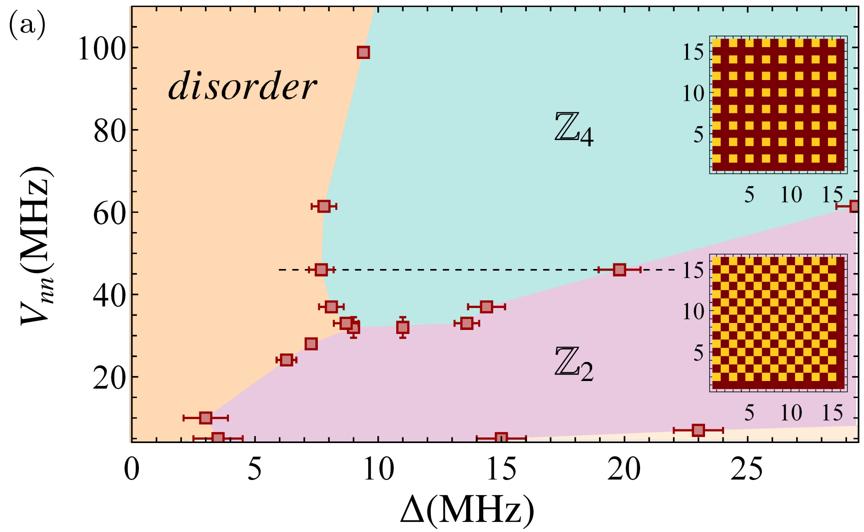
on a 256-Atom Programmable Quantum Simulator

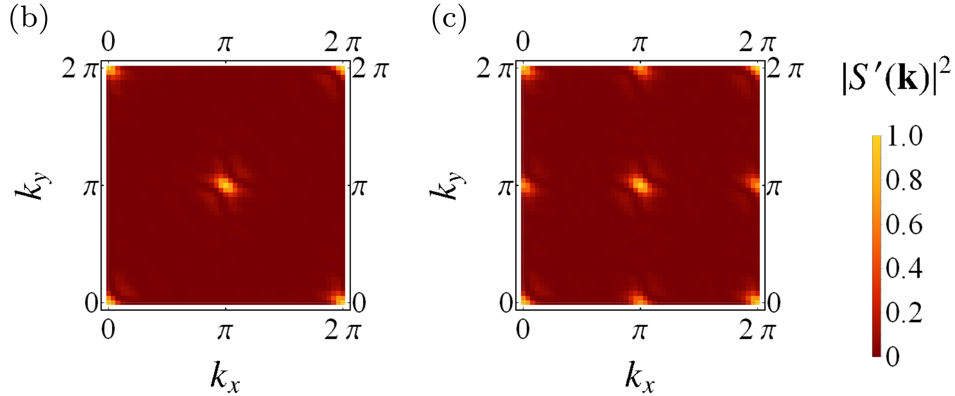
Nature (2021)

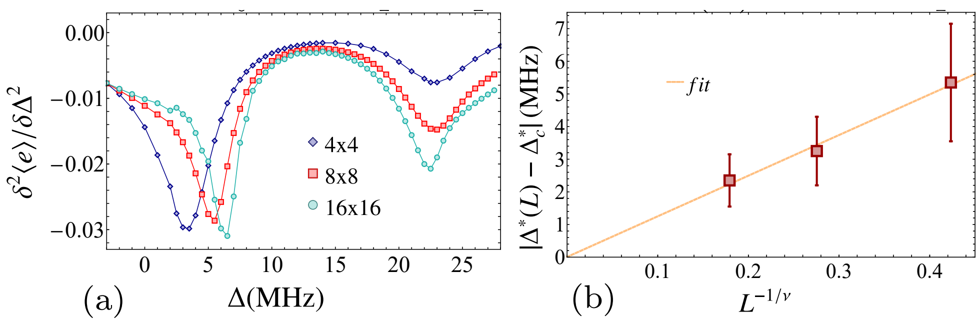
RYDBERG QUANTUM SIMULATOR

32x32 sites









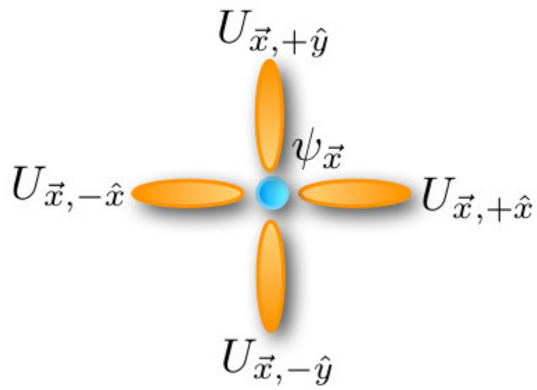
T. Felser, S. Notarnicola, S. Montangero PRL (2021)

LATTICE GAUGE TENSOR NETWORKS

Local degrees of freedom

$$[\psi^a_x, U^{ab}_{x,x+\mu_x}]$$

Matter field Gauge field



Gauge symmetry generator
(Gauss' law)

$$G_x|\varphi_{phys}\rangle = 0$$

Kogut-Susskind

Hamiltonian formulation of LGT

Gauge invariant dynamics

$$H = J\sum_x \left(\psi^\dagger_x U_{x,x+1}\psi_{x+1} + \text{h.c.}\right)$$

abelian

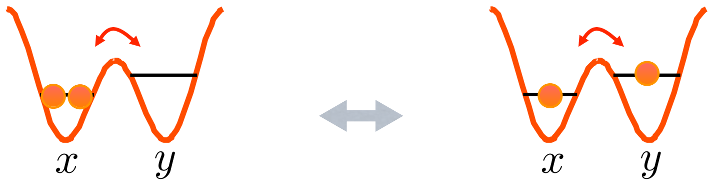
$$H = t\sum_{x,a,b}\left[\psi^{a\dagger}_x U^{ab}_{x,x+1}\psi^b_{x+1} + \text{h.c.}\right]$$

non abelian

Dynamics commutes with symmetry generator

$$\left[H^{[\text{QED}]}_{\text{int}}, G_x\right] = 0 \quad \forall x$$

QUANTUM LINK AND RISHON REPRESENTATION



Link operator $U_{x,y} \equiv S^{+}_{x,y} = c^{\dagger}_{y} c_{x}$

Electric field [U(1) generator] $E_{x,y} \equiv S^{(3)}_{x,y} = \frac{1}{2}\left[c^{\dagger}_{y} c_{y} - c^{\dagger}_{x} c_{x}\right]$

$\{c_x, c^{\dagger}_{y}\} = \delta_{x,y}$ Schwinger fermions (rishons)

$[c_x, c^{\dagger}_{y}] = \delta_{x,y}$ Schwinger bosons

Spin representation:

$N_{x,y} = c^{\dagger}_{y} c_{y} + c^{\dagger}_{x} c_{x}$ $\quad \left[\vec{S}_{x,y}\right]^2 \equiv \frac{N_{x,y}}{2}\left[\frac{N_{x,y}}{2} + 1\right]$

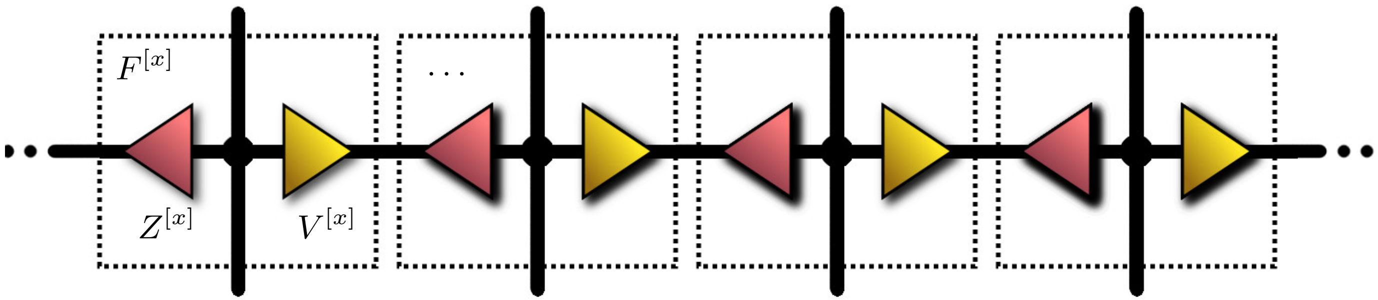
Local projection on a gauge invariant base

+

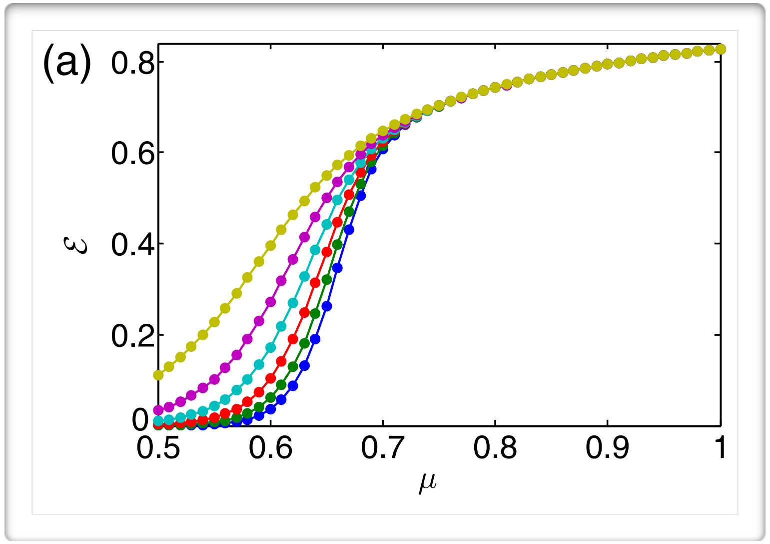
Projection on rishon number

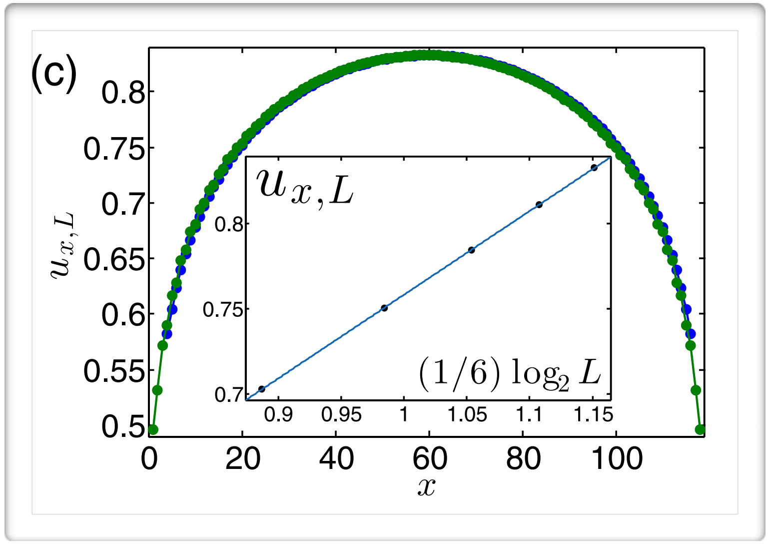
=

Matrix product operator



U(1) LATTICE GAUGE THEORY IN 1+1D



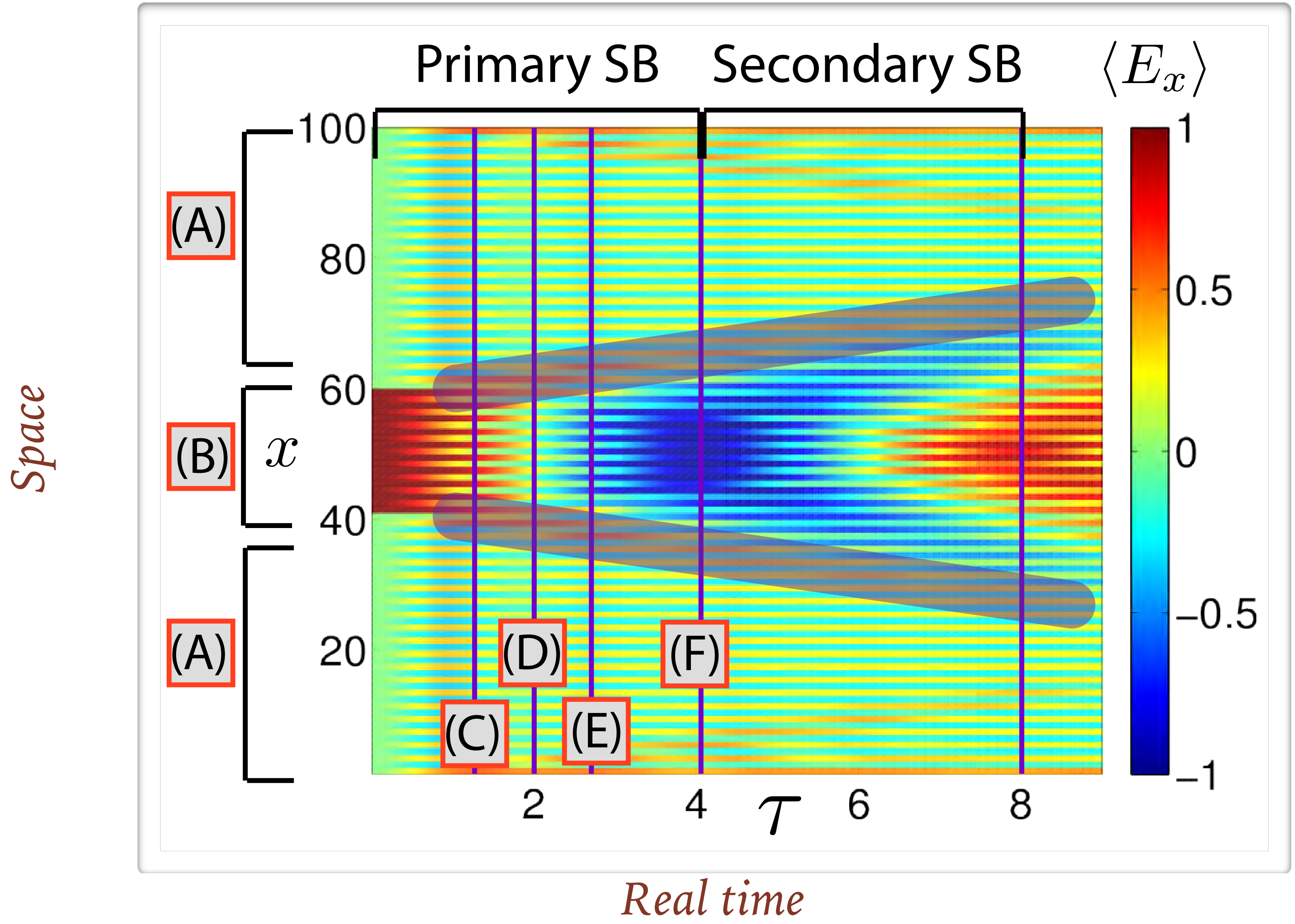


$$H = -t\sum_x \left[\psi_x^\dagger U_{x,x+1}^\dagger \psi_{x+1} + \psi_{x+1}^\dagger U_{x,x+1}\psi_x\right] + m\sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2}\sum_x E_{x,x+1}^2 .$$

$$\mathcal{E} = \sum_x \langle E_{x,x+1}\rangle / L$$

- *Quantum link and rishon representation*
- *Staggered fermions*
- *Ising universality class*
- *Central charge* $c = 0.49 \pm 0.01$
- *Confirmed by higher-link representation*

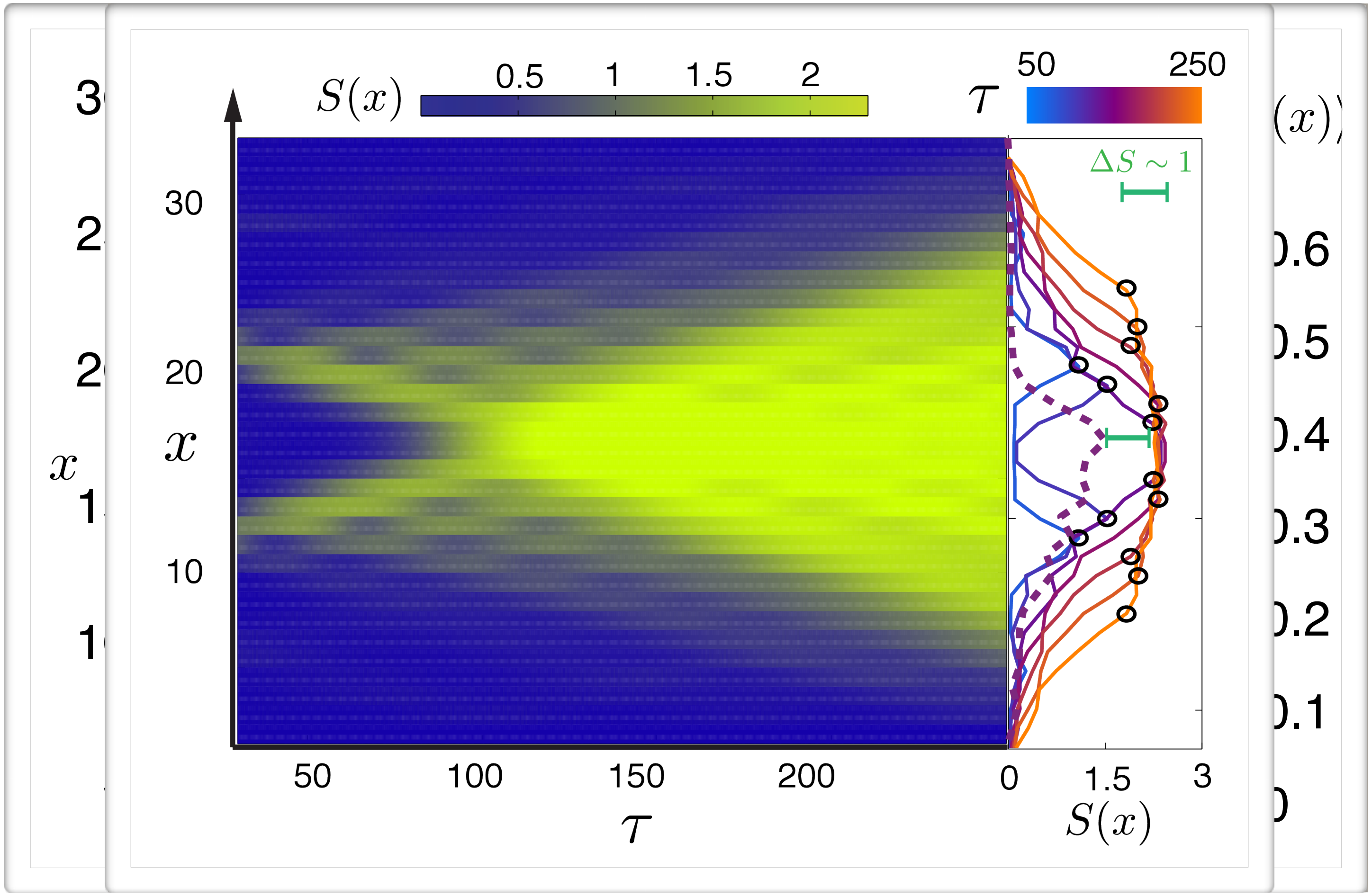
E. Rico, T. Pichler, M. Dalmonte, P. Zoller, and SM, PRL (2014)



STRING BREAKING DYNAMICS

$$H = -t\sum_x \left[\psi_x^\dagger U_{x,x+1}^\dagger \psi_{x+1} + \psi_{x+1}^\dagger U_{x,x+1}\psi_x\right] + m\sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2}\sum_x E_{x,x+1}^2.$$

T. Pichler, E. Rico, M. Dalmonte, P. Zoller, and SM, PRX (2016)



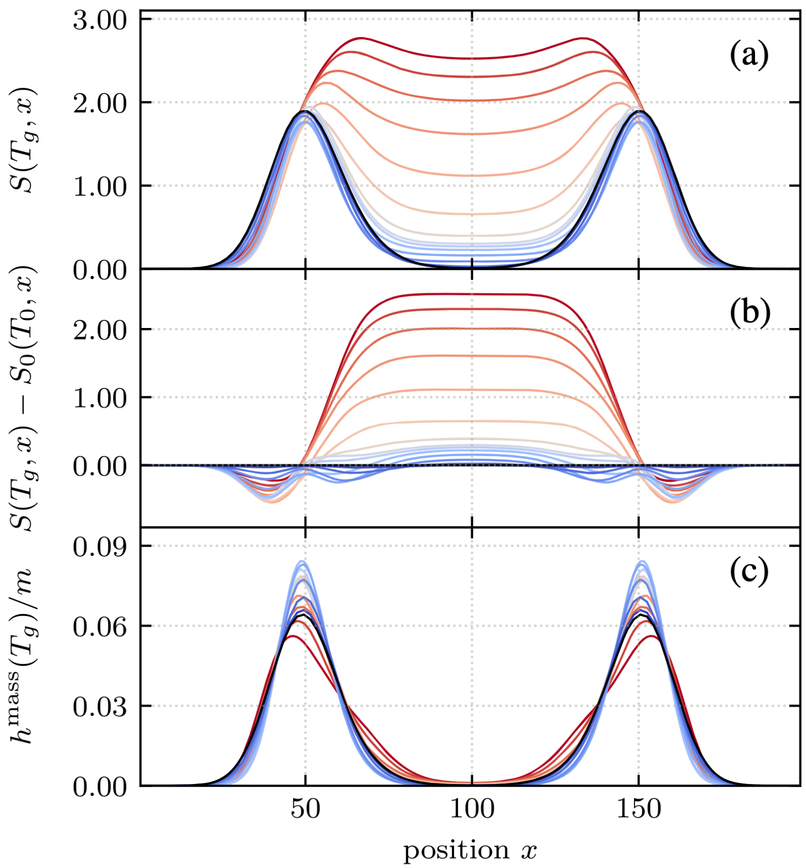
Space

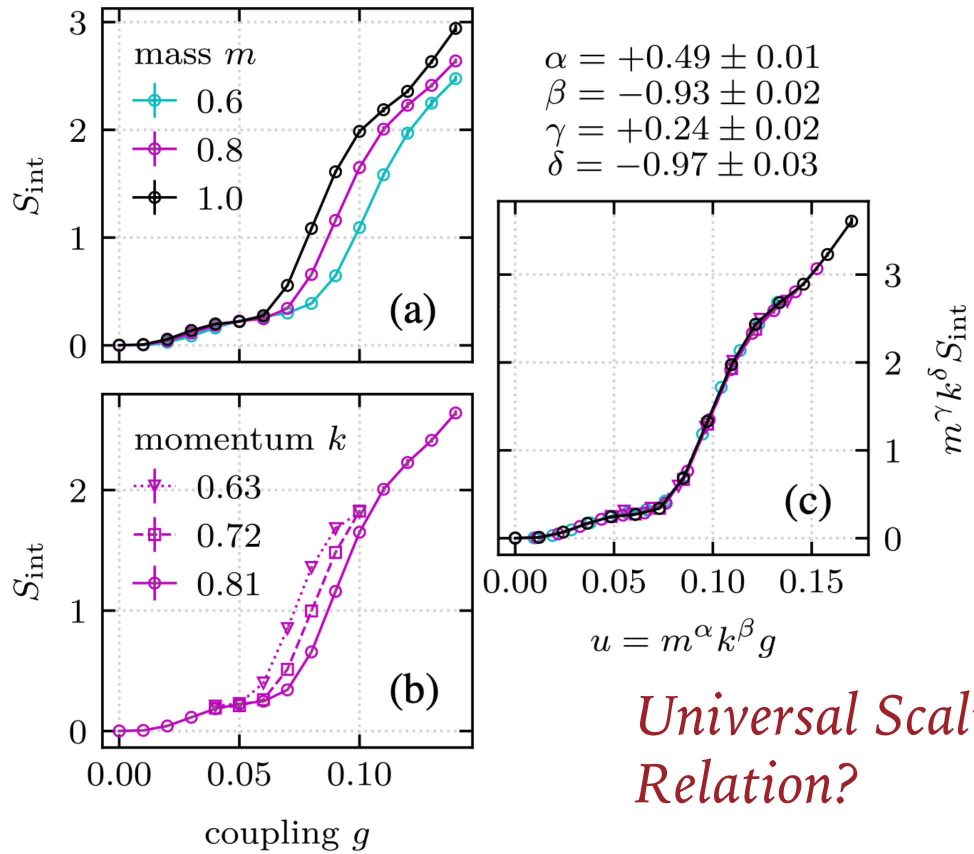
Real time

MESONS SCATTERING

T. Pichler, E. Rico, M. Dalmonte, P. Zoller, and SM, PRX (2016)

ENTANGLEMENT GENERATION IN QED SCATTERING PROCESSES

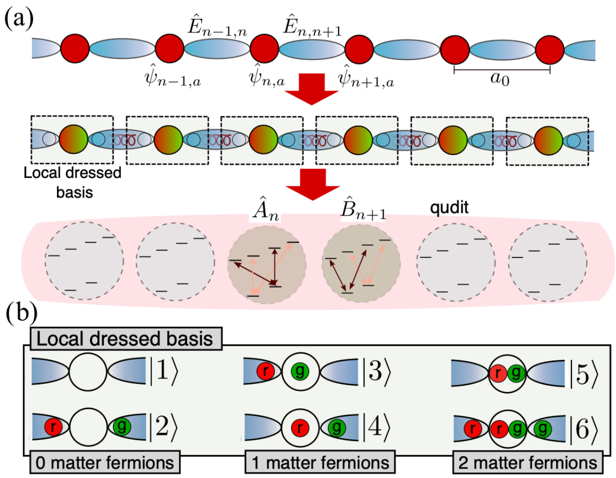


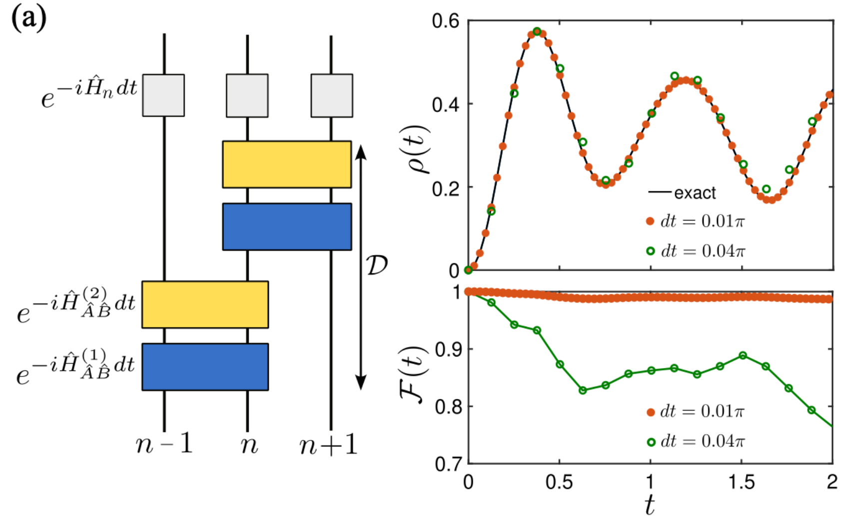


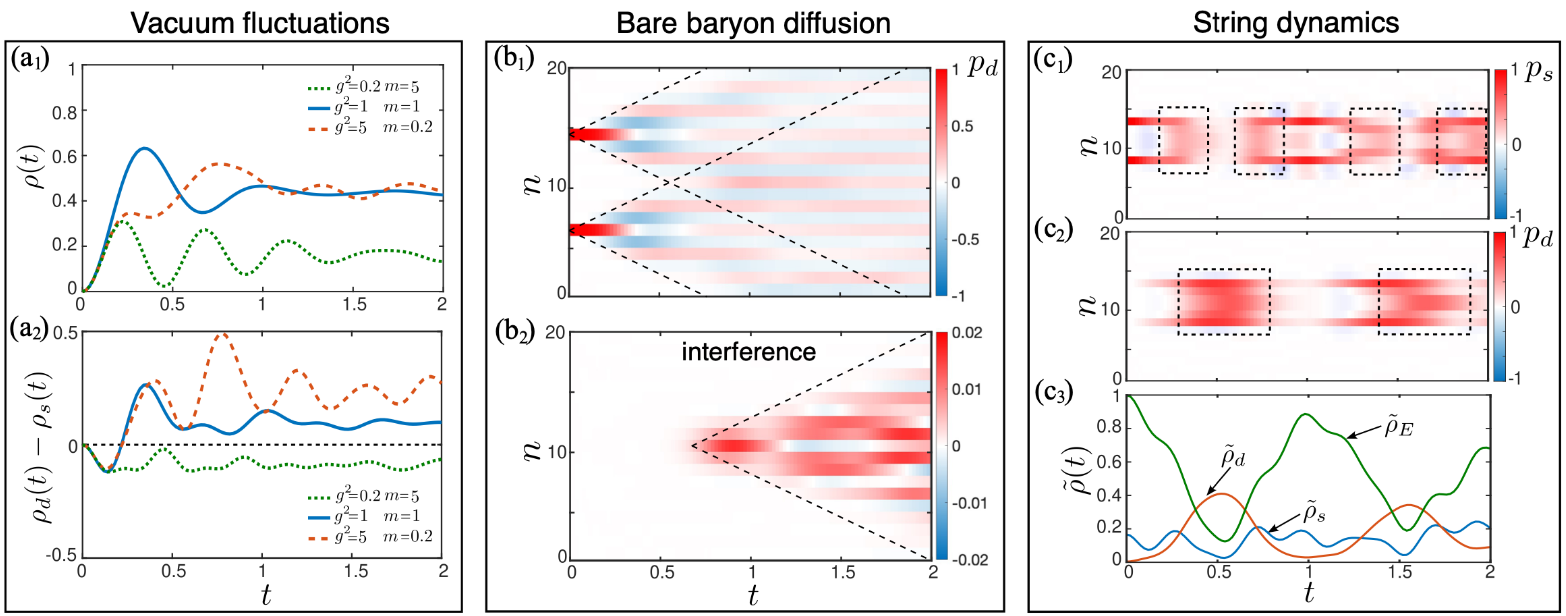
Universal Scaling Relation?

M. Rigobello, S. Notarnicola, G. Magnifico, and S. Montangero, Phys. Rev. D 104, 114501 (2021).

1+1D SU(2) LGT WITH QUDITS IN TRAPPED IONS





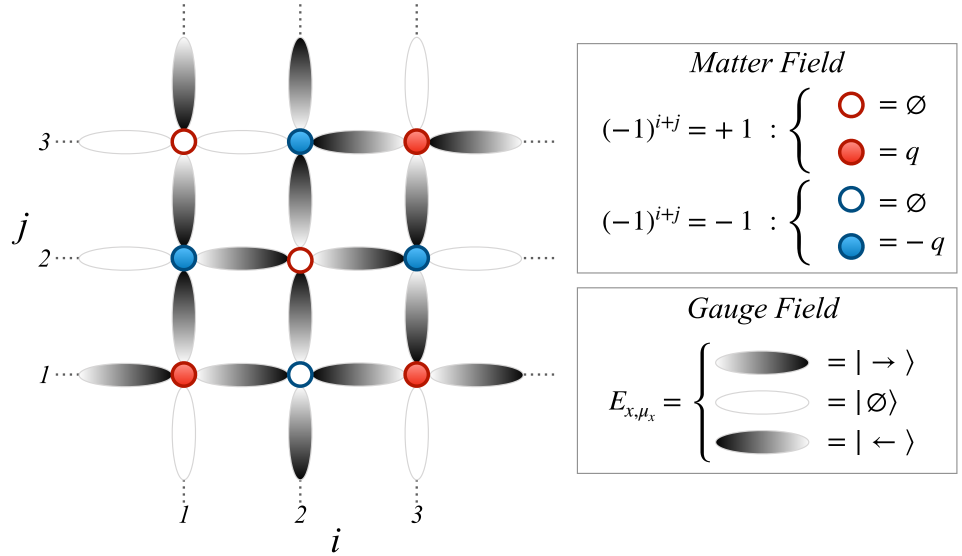


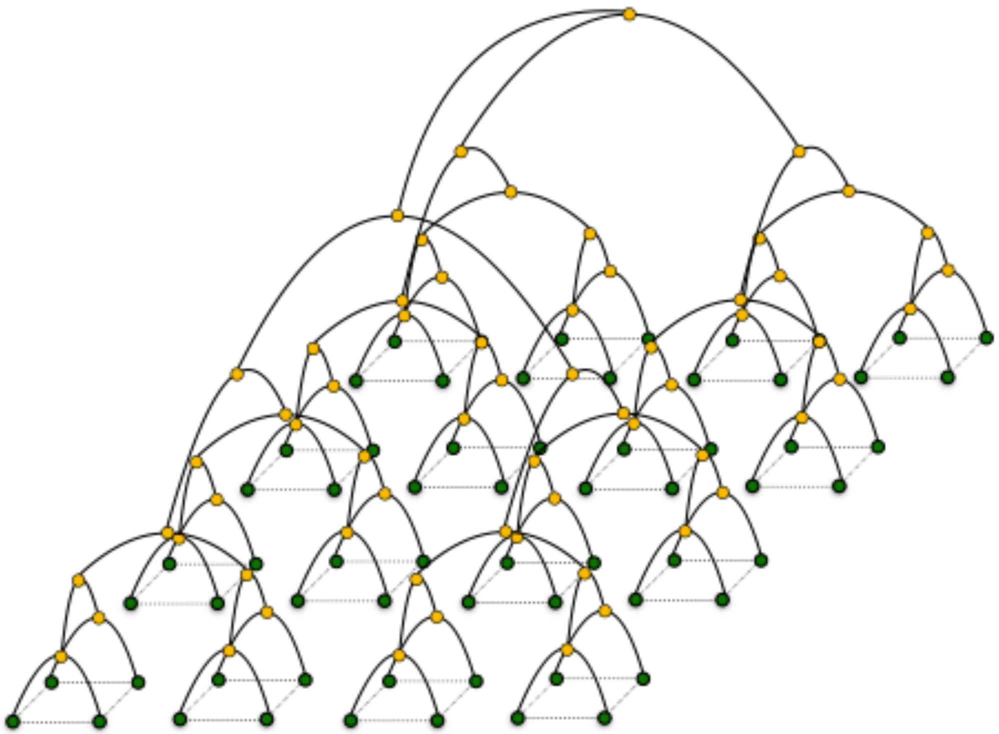
G. Calajò et al. arxiv:2402.07987

Quantum stars! arXiv:2405.13112

TWO DIMENSIONAL SIMULATION OF A LGT AT FINITE DENSITY

$$H = -t \sum_{x,\mu} \left(\psi_x^\dagger U_{x,\mu} \psi_{x+\mu} + h.c. \right) + m \sum_x (-1)^x \psi_x^\dagger U_{x,\mu} \psi_x + \frac{g_e^2}{2} \sum_{x,\mu} E_{x,\mu}^2 - \frac{g_m^2}{2} \sum_x \left(U_{x,\mu_x} U_{x+\mu_x,\mu_y} U^\dagger_{x+\mu_y,\mu_x} U^\dagger_{x,\mu_y} + h.c \right)$$





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PRX (2020)

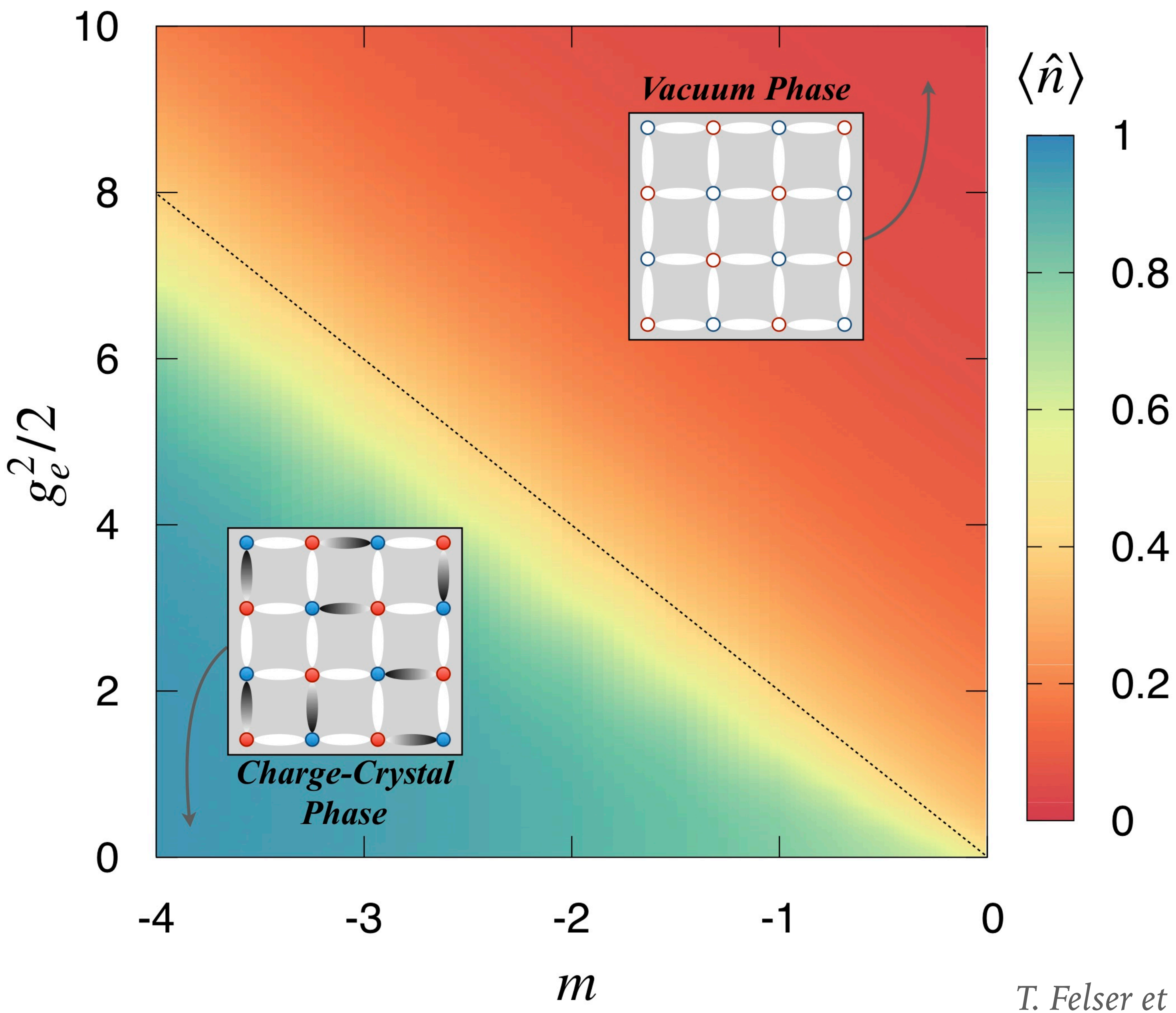
Even number of fermions =

No Jordan-Wigner strings!

PHASE DIAGRAM

16x16 lattice sites

Hilbert space of ~80x80 qubits

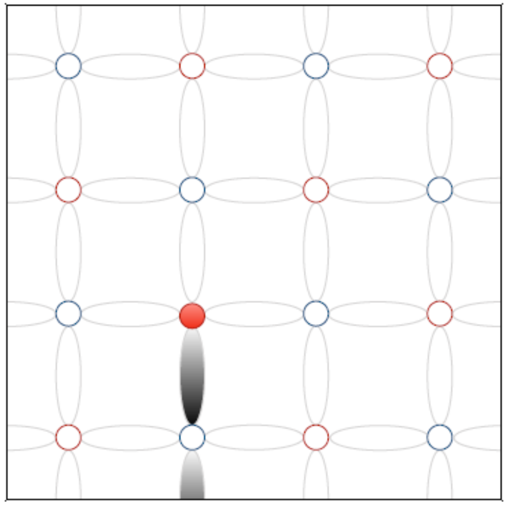


T. Felser et al. PRX (2020)

FINITE DENSITY

$m > \frac{g_e^2}{2}$

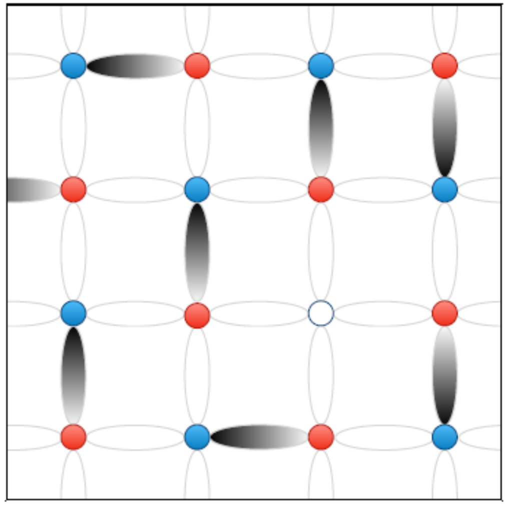
Vacuum phase



$\langle \hat{H} \rangle = m + 3g_e^2/4$

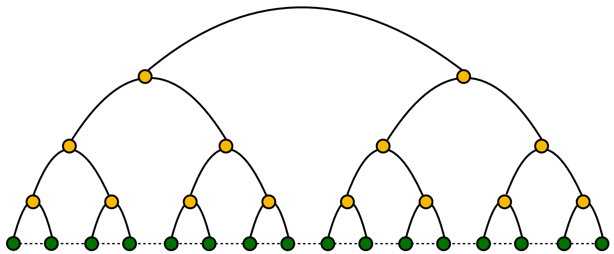
$m < \frac{g_e^2}{2}$

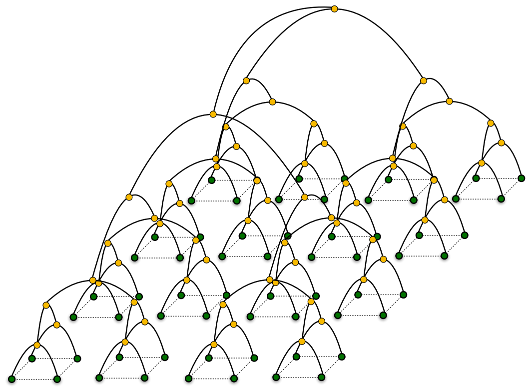
Charge-crystal phase



$\langle \hat{H} \rangle = (L^2/2 - 1/2)(2m + g_e^2/2)$

3D TREE TENSOR NETWORK





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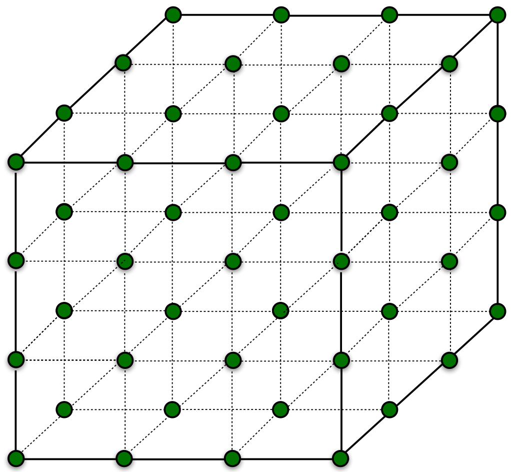
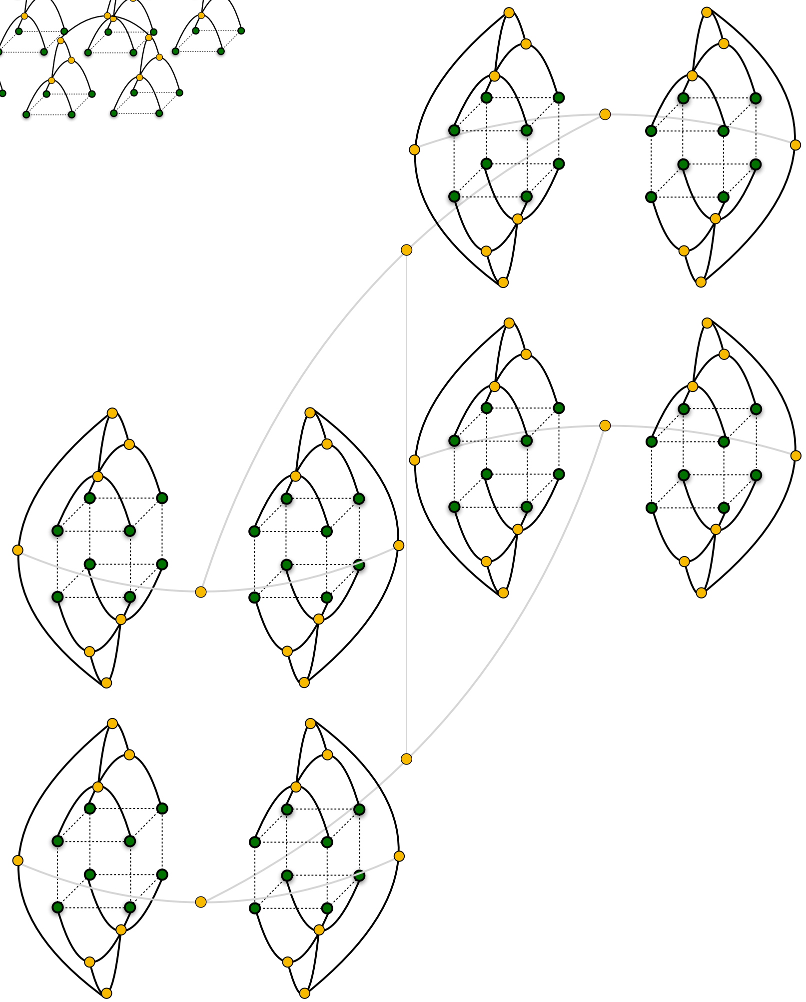
S. Montangero
PRX (2020)

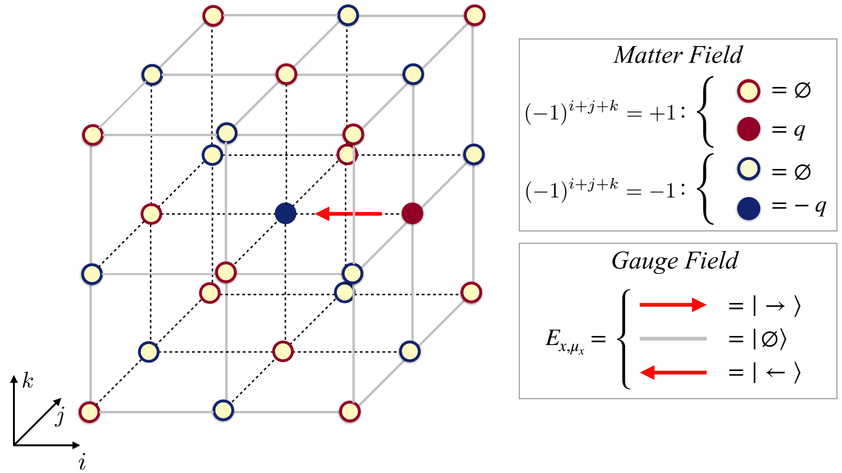




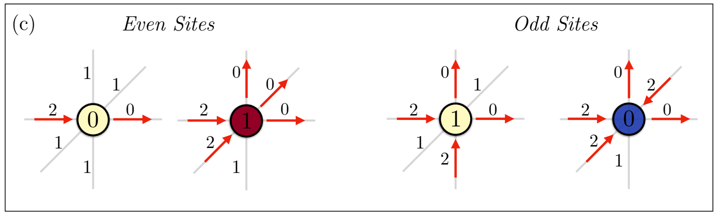
G. Magnifico, T. Felser, P. Silvi, and S. Montangero
Nat. Comm. (2021)

3D QUANTUM-LINK FORMULATION OF QED

$$\hat{H} = -t\sum_{x,\mu}\left(\hat{\psi}^{\dagger}_{x}\,\hat{U}_{x,\mu}\,\hat{\psi}_{x+\mu} + \mathrm{H.c.}\right)$$
$$+\, m\sum_{x}(-1)^{x}\hat{\psi}^{\dagger}_{x}\hat{\psi}_{x} + \frac{g_e^2}{2}\sum_{x,\mu}\hat{E}^{2}_{x,\mu}$$
$$-\,\frac{g_m^2}{2}\sum_{x}\left(\Box_{\mu_x,\mu_y} + \Box_{\mu_x,\mu_z} + \Box_{\mu_y,\mu_z} + \mathrm{H.c.}\right)$$



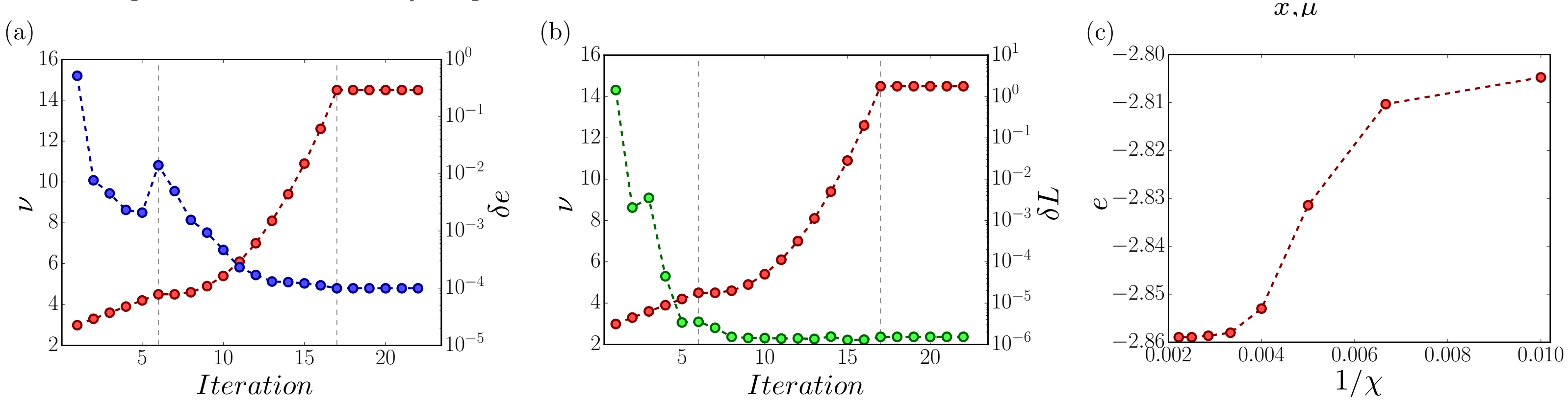
$$\hat{G}_x = \hat{\psi}^{\dagger}_{x}\hat{\psi}_{x} - \frac{1-(-1)^{x}}{2} - \sum_{\mu}\hat{E}_{x,\mu}$$



Local dimension 267, up to 12288 Hamiltonian operators

$$H_{pen} = \nu\sum_{x,\mu}\left(1-\delta_{2,\hat{L}_{x,\mu}}\right)$$

Up to 5 weeks x 64 cores of computational time

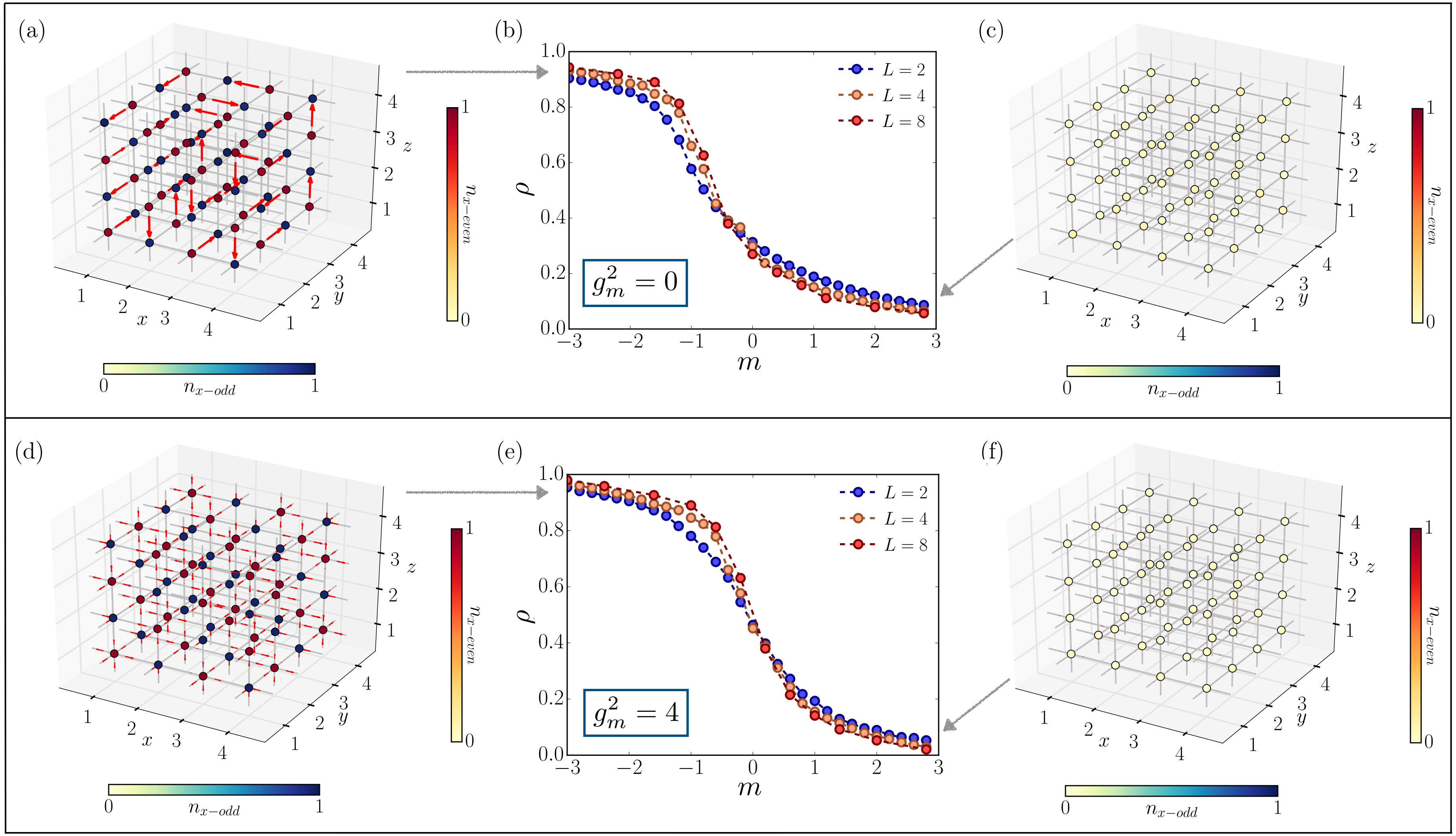


QUANTUM PHASES

Hilbert space of

~16x16x16 qubits!

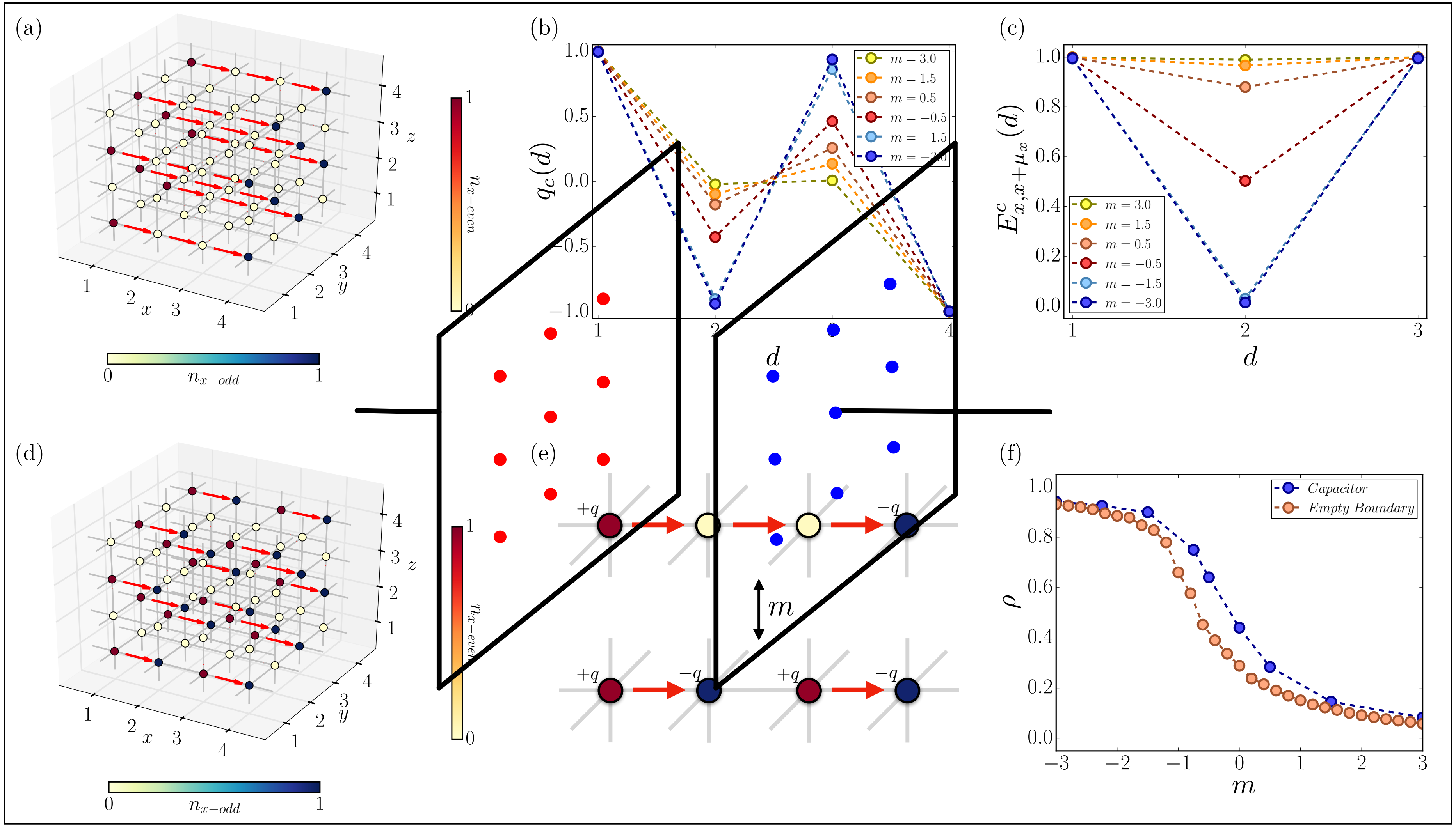
4Kb QRAM



$$m_c \approx +0.22$$

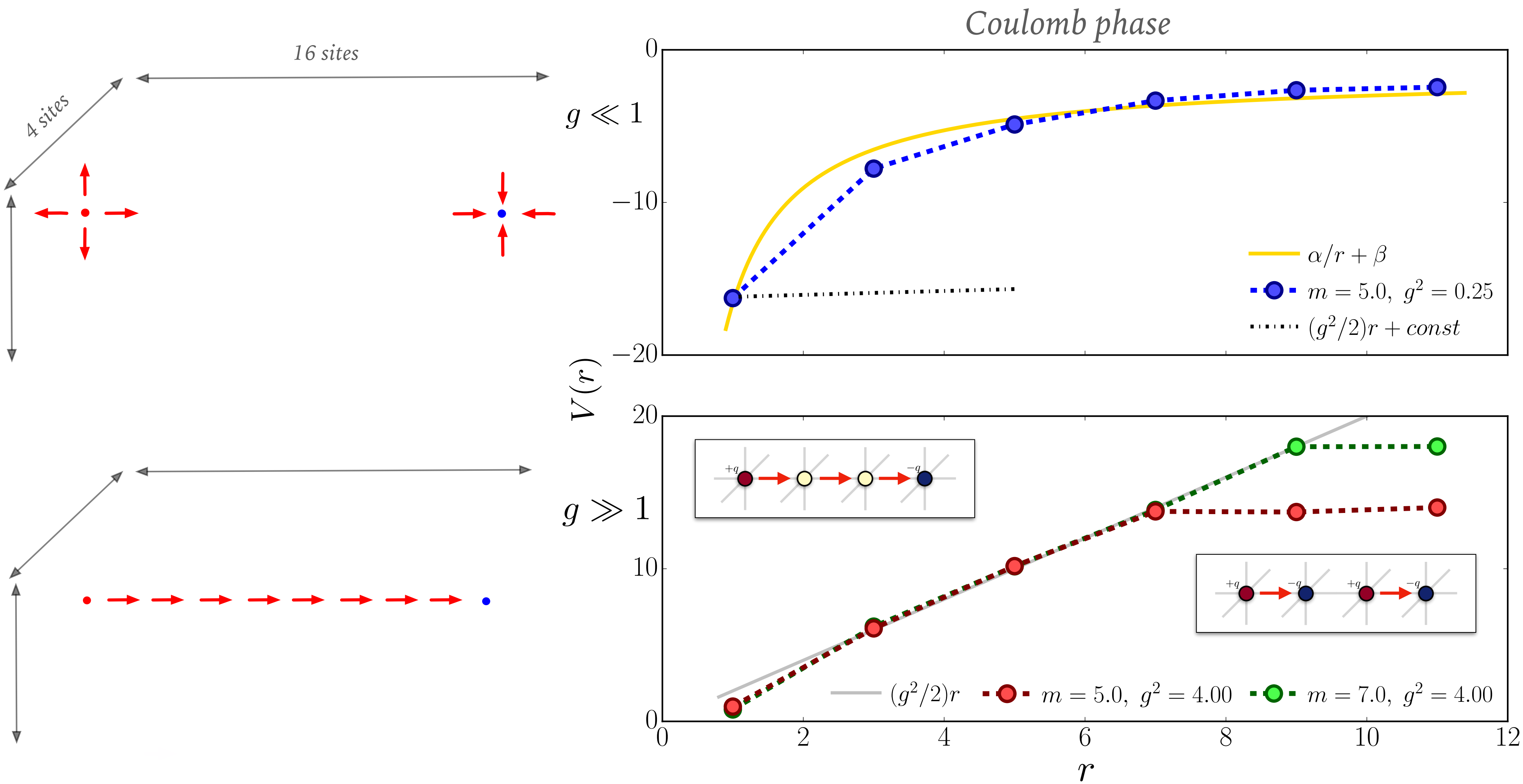
$$g_m^2 = 8/g_e^2$$

SCREENING



(a)
(b)
(c)
(d)
(e)
(f)
n_{x-odd}
n_{x-even}
$q_c(d)$
$E^c_{x,x+\mu_x}(d)$
d
$m = 3.0$
$m = 1.5$
$m = 0.5$
$m = -0.5$
$m = -1.5$
$m = -3.0$
$+q$
$-q$
m
ρ
Capacitor
Empty Boundary

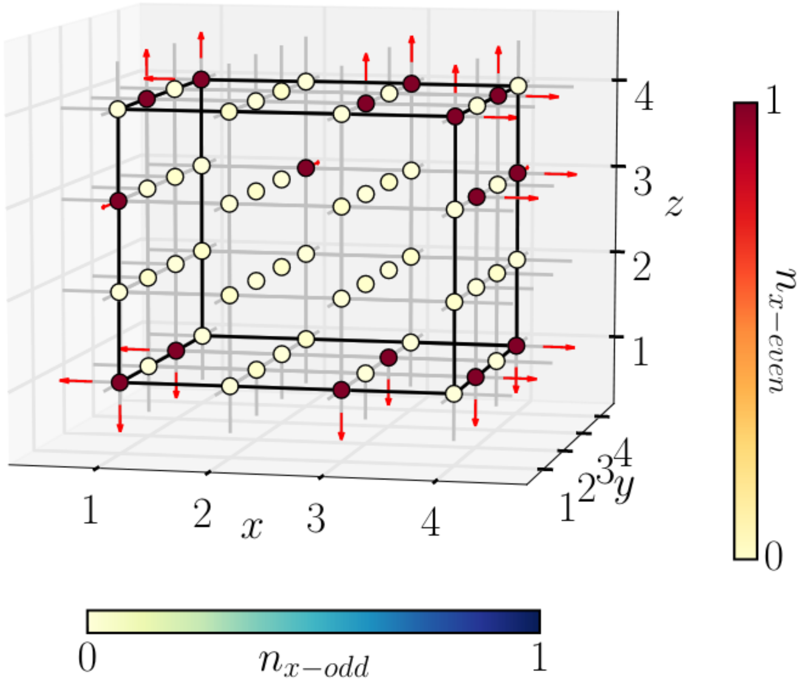
CONFINEMENT

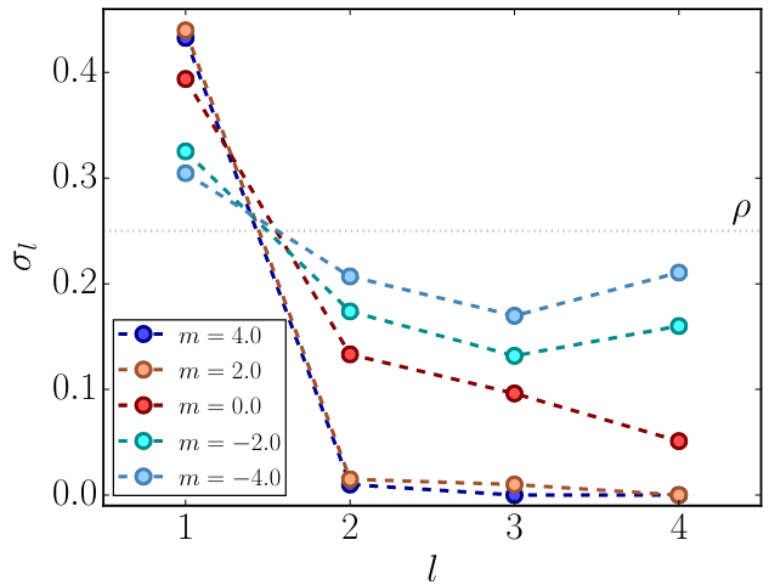


$$g_e^2 = g^2/a,\ g_m^2 = 8/(g^2 a)$$

FINITE DENSITY

m = 4.0



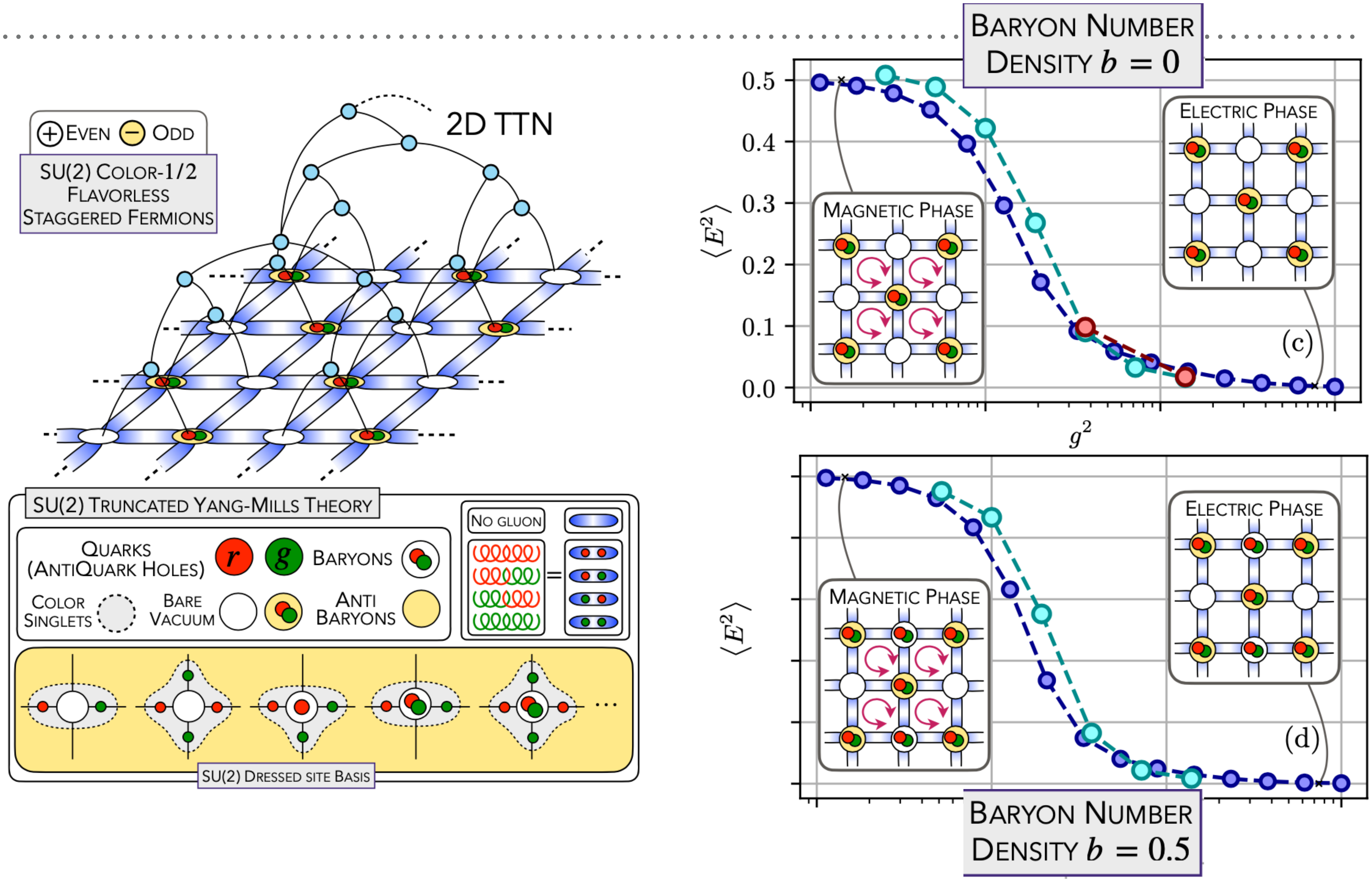


$$\sigma(l) = \frac{1}{A(l)} \sum_{x \in A(l)} \langle \psi_x^\dagger \psi_x \rangle$$

Finite charge density $\rho = Q/L = 1/4$

Charge at distance l from boundaries

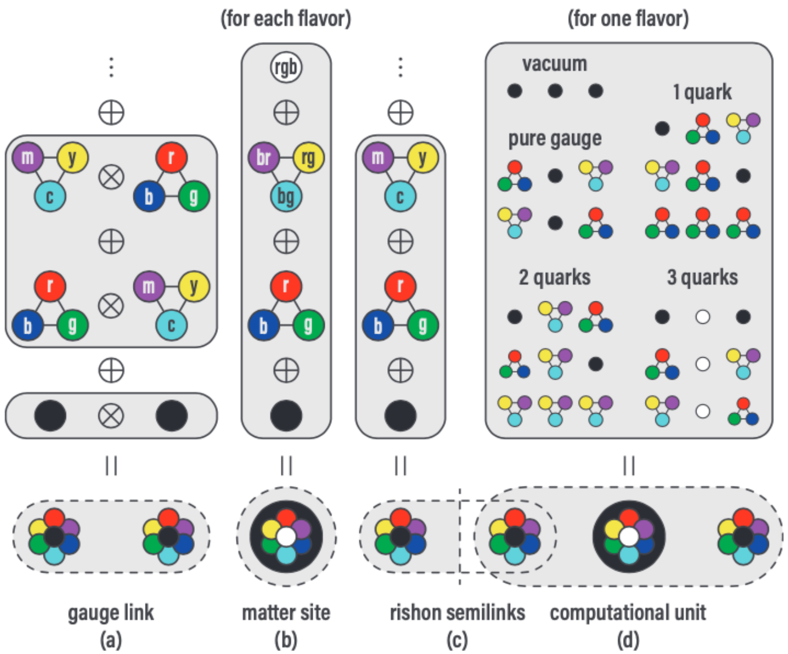
2+1D NON-ABELIAN LGT

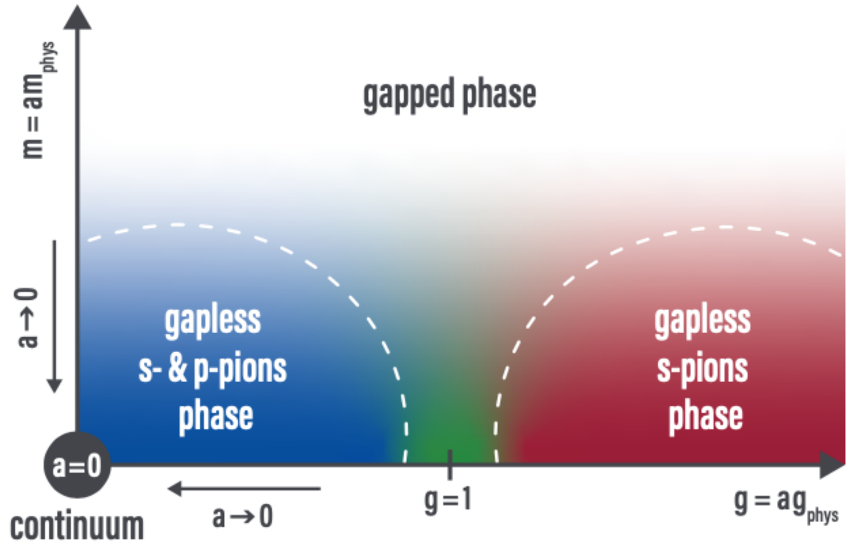


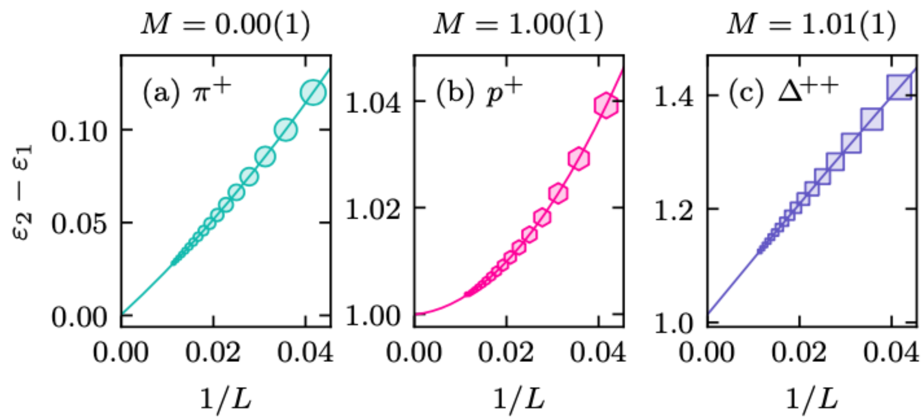
(2+1)D SU(2) Yang-Mills Lattice Gauge Theory at finite density via tensor networks
G. Cataldi, G. Magnifico, P. Silvi, SM Phys. Rev. Res. (2024)

1+1D SU(3) 2-FLAVOURS LATTICE GAUGE THEORY

$$H = \sum_{x,f,c,c'} \left[-\frac{i}{2}\psi_{x,f,c} U_{x,c;x+1,c'} \psi^{\dagger}_{x+1,f,c'} + \text{H.c.} \right] + \sum_{x,f,c} m_f (-1)^x \psi^{\dagger}_{x,f,c}\psi_{x,f,c} + \sum_{x} \frac{g^2}{2} E^2_{x;x+1} \,.$$







M. Rigobello, G. Magnifico, P. Silvi, SM arXiv:2308.04488

CONFINEMENT IN 2D ISING MODEL

$$H = -J\sum_{\langle i,j\rangle}\sigma_i^z\sigma_j^z - g\sum_i \sigma_i^x \qquad |\downarrow\ldots\downarrow\rangle$$

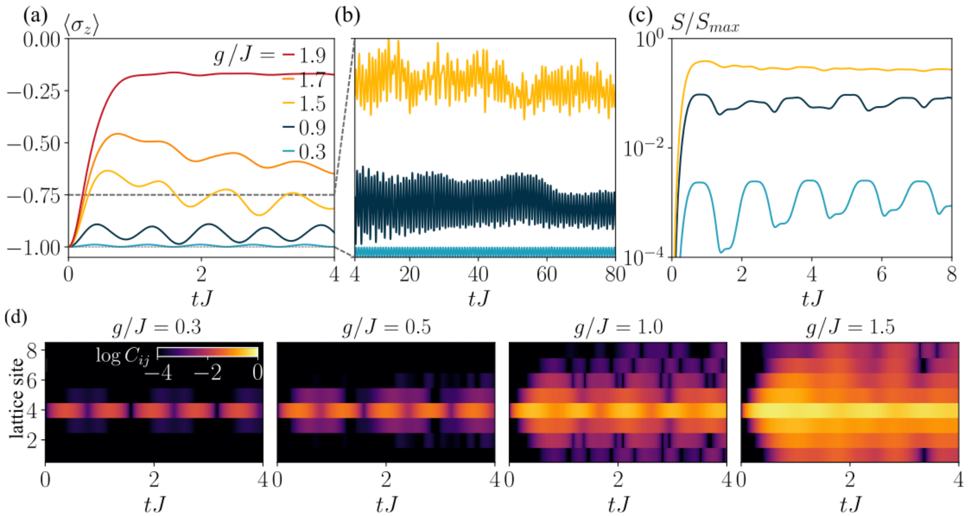


FIG. 1. Confinement in the 2D quantum Ising model. (a) Short time dependence of magnetization $\langle\sigma^z(t)\rangle$ for a set of g/J. (b) $\langle\sigma^z(t)\rangle$ for longer times and small g/J. (c) Time dependence of entanglement entropy S for a bipartition of two neighbouring spins and the rest of the system. $S_{\max}$ is the maximal entanglement entropy of the bipartition (in this case $S_{\max} = 2\ln 2$). (d) Time dependence of a horizontal cut of the connected correlation function C_{ij} where $i = 4$ for a set of g/J.

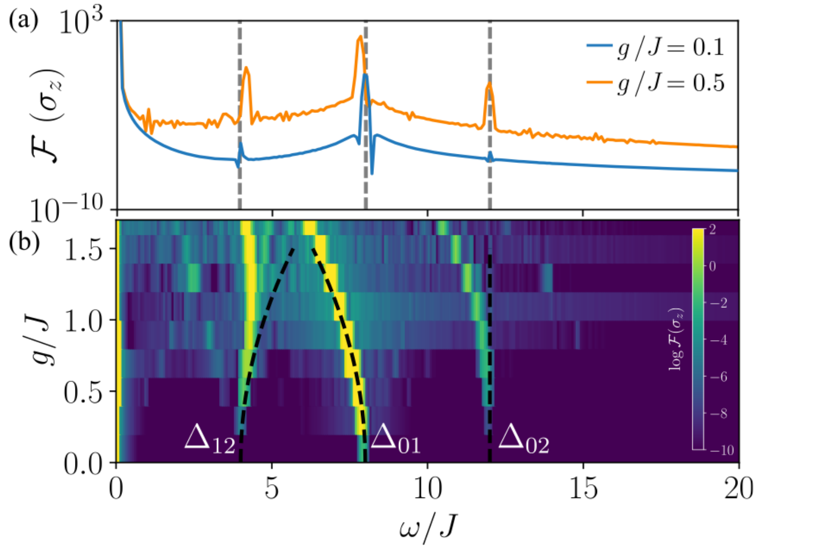


FIG. 2. Spectrum of σ^z, given by the Fourier transform $\mathcal{F}(\sigma^z)$. (a) Spectrum of magnetization for two cases of g/J. Dashed vertical lines correspond to $\omega/J = 4$, 8, and 12. (b) Heatmap of the spectrum for a range of g/J. Black dashed lines correspond to the transition energies obtained by perturbation theory.

$$E_0 = -\frac{g^2}{8J}N^2 + \mathcal{O}(g^4),$$

$$E_1 = 8J - \frac{g^2}{8J}\left(N^2+6\right) + \mathcal{O}(g^4),$$

$$E_2 = 12J - \frac{g^2}{8J}N^2 + \mathcal{O}(g^4).$$

L. Pavešić et al. arxiv: 2406.11979

INTERFACE PHYSICS IN 2D ISING MODEL

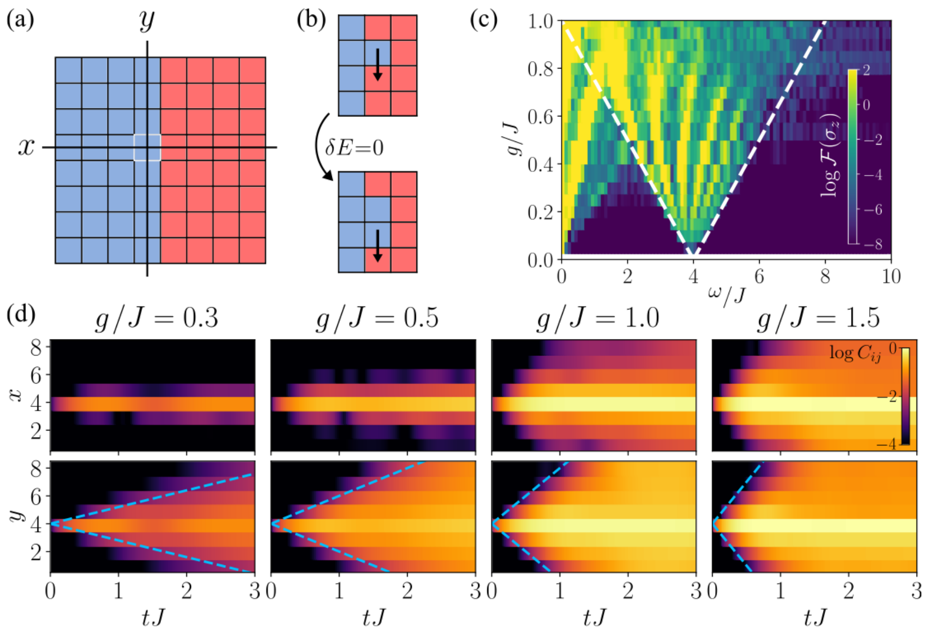
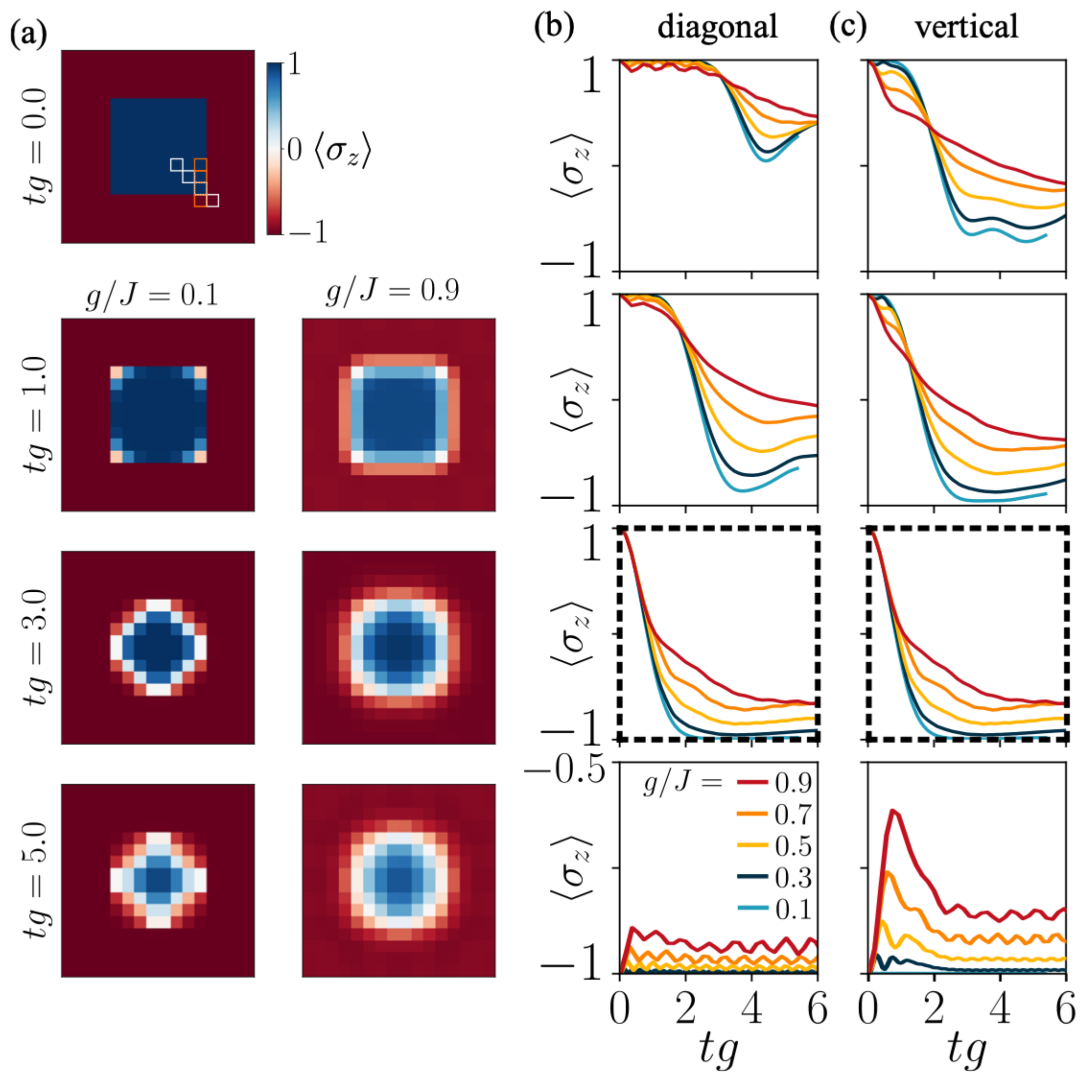


FIG. 3. Spread of correlations near an interface. (a) Sketch of the initial state, with blue representing ↑ and red ↓ spins. Black lines indicate the cuts shown in (d). (b) Sketch of the resonant process along the interface. (c) Heatmap of the spectral density of $\langle\sigma^z\rangle(t)$ for a spin at the interface for a range of g/J. White dashed lines correspond to the transition energies of a freely propagating edge mode. (d) Horizontal (x) and vertical (y) cuts of the connected correlations function C_{ij} with respect to the spin at $(4, 4)$ (white square in (a)). Blue dashed lines are $\pm 4gt$, showing that the interface mode carries the correlations.



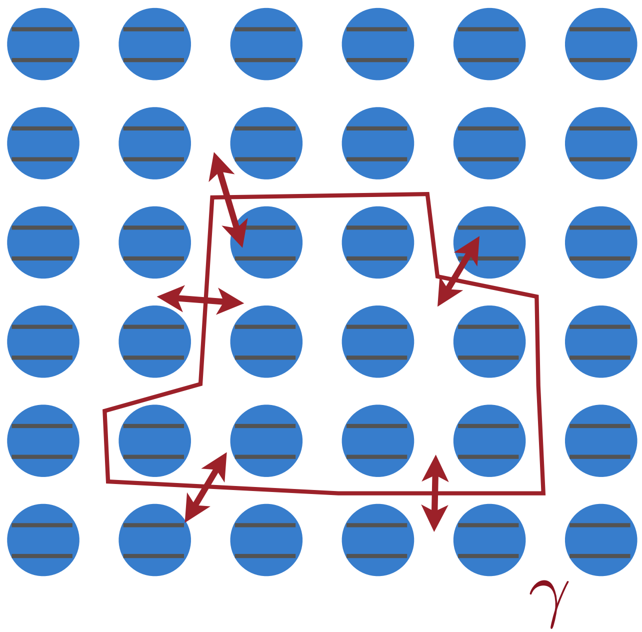
L. Pavešić et al. arxiv: 2406.11979

ENTANGLEMENT OF MIXED MANY-BODY QUANTUM SYSTEMS

For pure states:

$$\mathcal{S} = -\mathrm{Tr}\,\rho\log\rho$$

Von Neumann Entropy



Area law

$$\mathcal{S} \propto \gamma$$

$$\mathcal{S} \propto N^{(D-1)}$$

1D critical systems:

$$\mathcal{S} = \frac{c}{3}\log N$$

For mixed states:

$$\rho = \sum p_j |\psi_j\rangle\langle\psi_j|$$

$$E_F(\rho) = \inf_{\{p_j,\psi_j\}}\left\{\sum_j p_j \mathcal{S}(|\psi_j\rangle) : \rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|\right\}$$

$$E_F(\rho, T) \overset{?}{\propto} \log N^{c/3}$$

Entanglement of formation

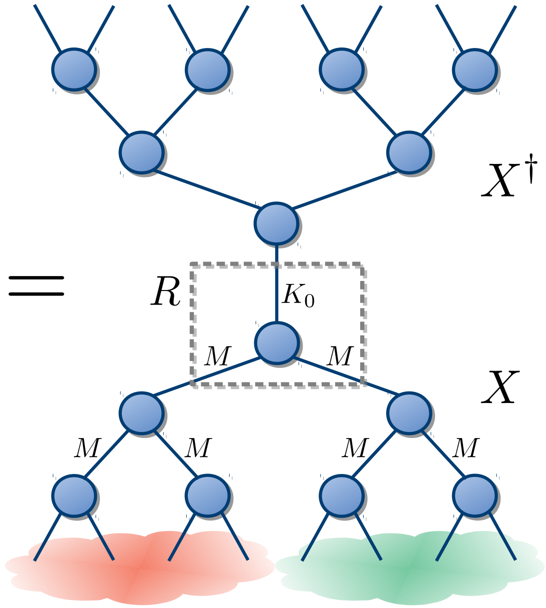
C.H. Bennet et al. PRA 1996

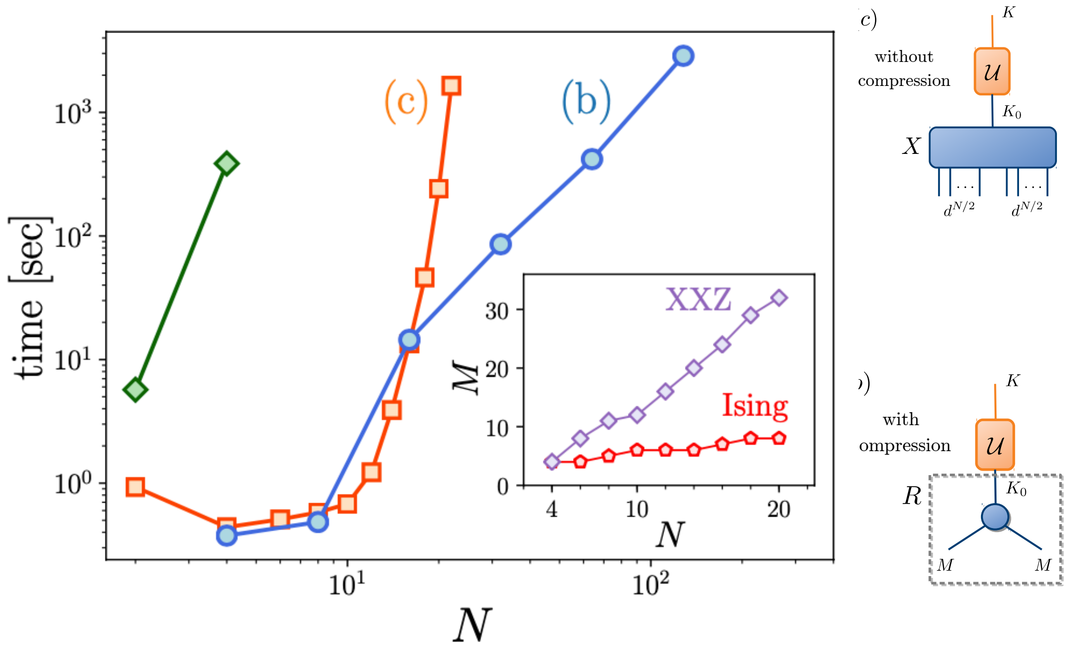
TREE TENSOR OPERATORS

$$\rho = \sum p_j |\psi_j\rangle\langle\psi_j|$$

$$E_F(\rho) = \min_{K \geq K_0} \inf_{\mathcal{U}} \left\{ \sum_{j=1}^{K} p_j \mathcal{S}(|\psi'_j\rangle) : X' = X\mathcal{U} \right\}$$

$\rho =$





$$\hat{H}_{Ising} = J \sum_{i=1}^{N} \left(\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + h \hat{\sigma}_j^z \right)$$

$$\hat{H}_{XXZ} = J \sum_{j=1}^{N} \left(\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + \xi \, \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z \right)$$

Thermal equilibrium state

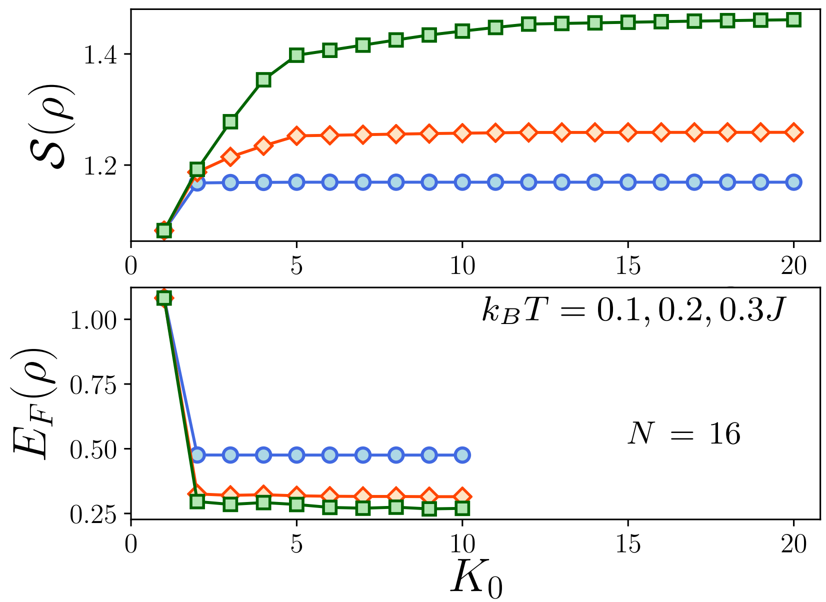
(Mixture of K_0 Boltzmann factors)

$$X = \frac{1}{\sqrt{Z}} \sum_j^{K_0} e^{-E_j/2T} |\psi_j\rangle\langle j|$$

L. Arceci, P. Silvi, and S. Montangero PRL (2022)

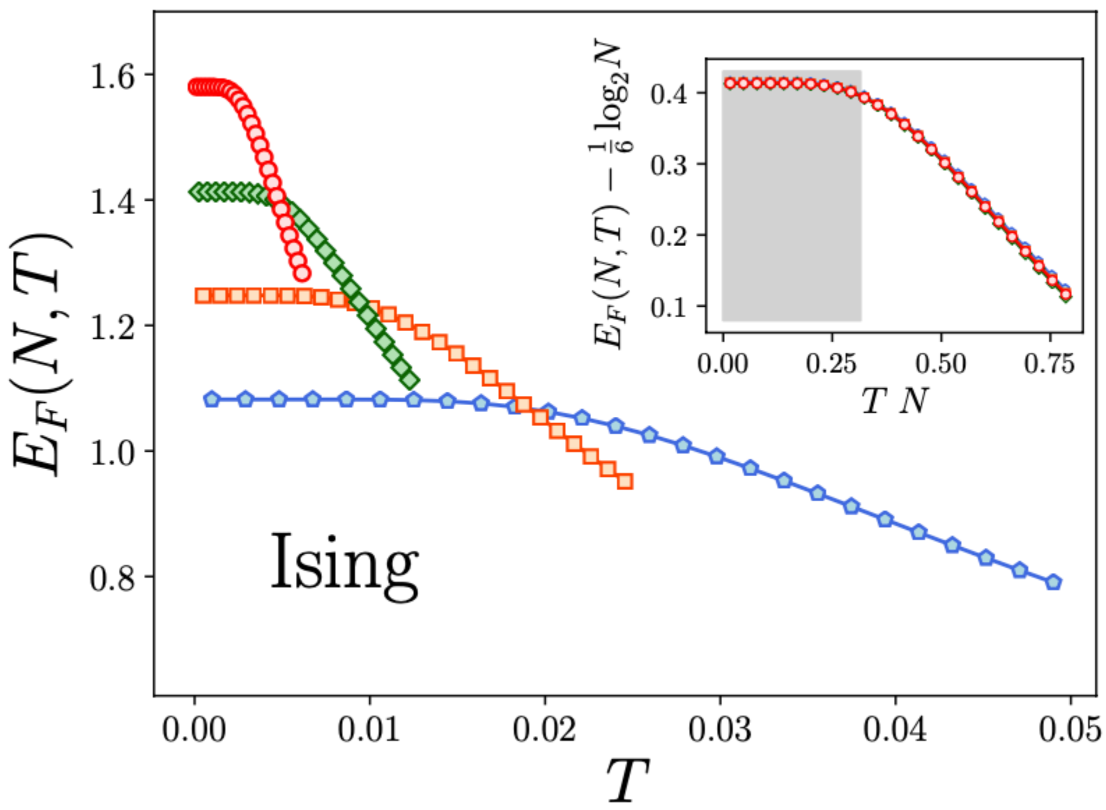
CONFORMAL SCALING OF ENTANGLEMENT OF FORMATION

$$E_F = \log(N^{c/3} f(T N^z))$$



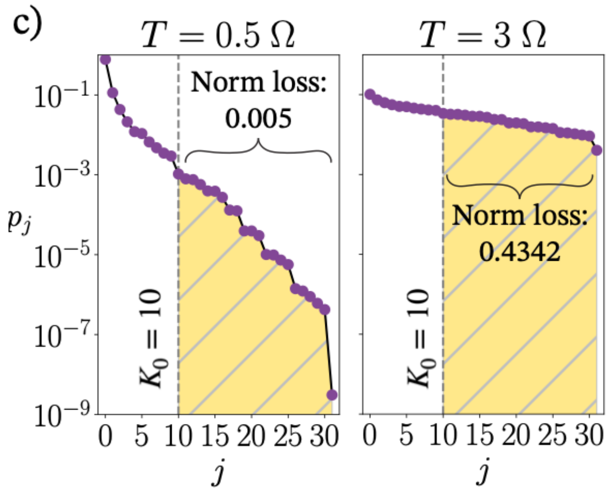
Numerical complexity depends on entropy, not entanglement!

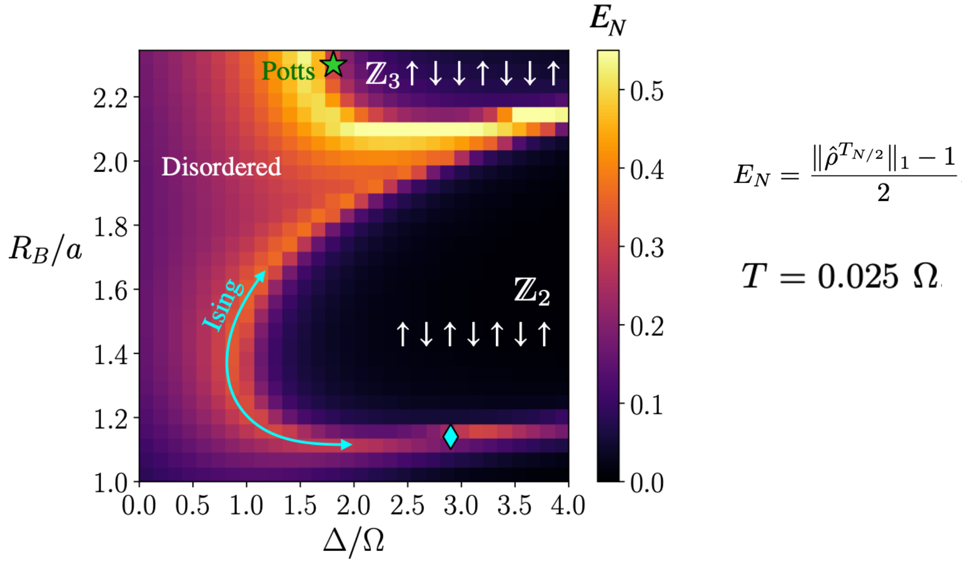
$$N = 16, 32, 64, 128$$

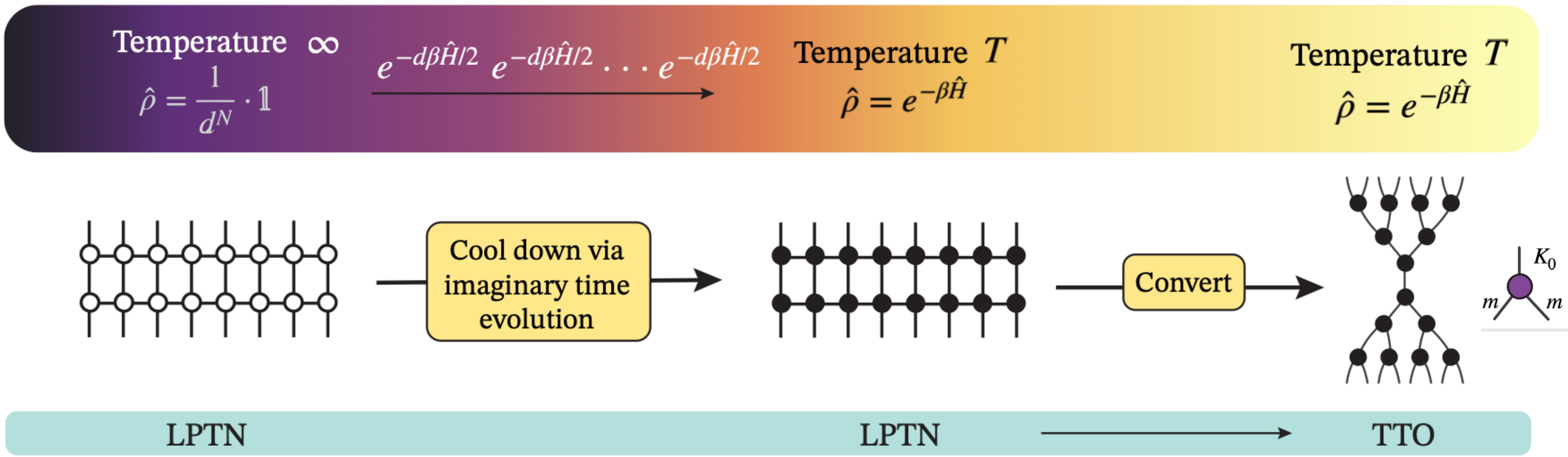


L. Arceci, P. Silvi, and S. Montangero PRL (2022)

RYDBERG ARRAYS AT FINITE TEMPERATURE







N. Reinic et al. arxiv: 2405.18477

ROADMAP FOR LGT QUANTUM SIMULATION

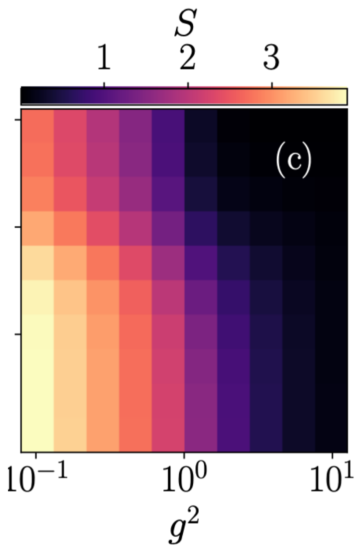
ℓ	(2 + 1)-dimensions U(1)	SU(2)	SU(3)	U
1	35	30	164	
2	165	168	752	3
3	455	600	3738	18
4	969	1650	19878	64
5	1771	3822	43698	177
6	2925	7840	82128	408
7	4495	14688	212496	835
8	6545	25650	333538	1561

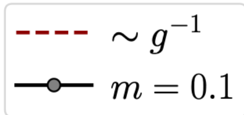
TABLE I. Dressed site Hilber creasing number ℓ of allowed in some 2- and 3-dimensional namical matter and gauge grou

ℓ^*

System size	χ	Factor	Estimated walltime
64×64	450	T_{base}	4.16 days
64×64	900	$16 \cdot T_{\text{base}}$	66.6 days
256×256	450	$28 \cdot T_{\text{base}}$	116.5 days
256×256	900	$448 \cdot T_{\text{base}}$	5.1 years
$16 \times 16 \times 16$	450	$4 \cdot T_{\text{base}}$	16.6 days
$16 \times 16 \times 16$	900	$64 \cdot T_{\text{base}}$	266 days
$64 \times 64 \times 64$	450	$1984 \cdot T_{\text{base}}$	23 years
$64 \times 64 \times 64$	900	$31744 \cdot T_{\text{base}}$	362 years

TABLE II. *Estimated simulation time.* We derive the baseline from a single-tensor optimization of a 64×64 quantum Ising simulation with $\mathbb{Z}_2$ symmetry taking 7192s on a A100 GPU. Further, we assume that single-tensor update, one tensor and one GPU per MPI thread, and 50 sweeps for the baseline. To extrapolate to larger systems, we assume a scaling with $\mathcal{O}(\chi^4 N^{D-1})$ as well as seven (thirty-one) tensors per MPI thread for 256×256 ($64 \times 64 \times 64$) systems. The empirical scalings are approximately a factor of 2.3 for doubling the system size and 13 for doubling the bond dimension, which we obtain from smaller simulations with $\chi = 225$ and for 32×32 qubits. The times are valid for any $d < \chi$.





a QED plaquette for a grid $^{-2}, 10^1]$ and $g^2 \in [10^{-1}, 10^1]$. ℓ^* required to reach a preci- ic energy $\langle \mathrm{Re}\, \hat{U}_{\square} \rangle$. (c) Cor- y S associated with a sym- te.

G. Magnifico et al. arxiv:2407.03058

TAKE HOME MESSAGES

- Tensor network algorithms will benchmark, verify, support and guide quantum simulations/computations development
- High-dimensional tensor network simulations are becoming more and more available (PEPS, aTTN,…)
- Entanglement of mixed many-body states can be quantified
- Scalability to full HPC will be necessary to produce quantitative results
- Interaction with HEP is becoming more and more relevant
- Interesting developments also in other directions (classical optimisers/annealers, machine learning)

Thank you for your attention!

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Giuseppe Magnifico Simone Notarnicola
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Marco Ballarin Mattia Morgavi
Peter Majcen Alice Pagano
Nora Reinić Marco Tesoro
Asmita Datta Rocco Barač

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