# **QUANTUM AND QUANTUM-INSPIRED SIMULATION OF LATTICE GAUGE THEORIES**

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**HNIVERSITÀ DEGLI STUDI** DI PADOVA

### **LATTICE GAUGE THEORIES**



The current wisdom on the phase diagram of nuclear matter.

[McLerran, L.](http://inspirehep.net/author/profile/McLerran%2C%20L.?recid=823172&ln=en) Nucl.Phys.Proc.Suppl. 195 (2009) 275-280

# **EFFICIENT WAVE FUNCTION REPRESENTATION - TODAY**



#### hopping amplitudes *t*. The fluxes in the topmost row and rightmost column (faces of periodic boundary present in case of periodic boundary present in case of periodic boundary p

conditions (PBC). For PBC, additional phase twists θ*<sup>x</sup>* and θ*<sup>y</sup>* can  $\frac{1}{\sqrt{2}}$  of the definition of  $\frac{1}{\sqrt{2}}$  content between  $\frac{1}{\sqrt{2}}$ , retailer the  $\frac{1}{\sqrt{2}}$  or the first of topological

# Quantum Technologies for Lattice Gauge Theories

#### **Simulating Lattice Gauge Theories within Quantum Technologies**

M.C. Bañuls<sup>1,2</sup>, R. Blatt<sup>3,4</sup>, J. Catani<sup>5,6,7</sup>, A. Celi<sup>3,8</sup>, J.I. Cirac<sup>1,2</sup>, M. Dalmonte<sup>9,10</sup>, L. Fallani<sup>5,6,7</sup>, K. Jansen<sup>11</sup>, M. Lewenstein<sup>8,12,13</sup>, S. Montangero<sup>7,14</sup> <sup>a</sup>, C.A. Muschik<sup>3</sup>, B. Reznik<sup>15</sup>, E. Rico<sup>16,17</sup> b, L. Tagliacozzo<sup>18</sup>, K. Van Acoleyen<sup>19</sup>, F. Verstraete<sup>19,20</sup>, U.-J. Wiese<sup>21</sup>, M. Wingate<sup>22</sup>, J. Zakrzewski<sup>23,24</sup>, and P. Zoller<sup>3</sup>

*EPJD (2020)*





### **TENSOR NETWORKS STATES**



Tensor networks states are a compressed description of the system tunable between mean field and exact

#### **ENTANGLEMENT OF PURE MANY-BODY QUANTUM SYSTEMS**  $\bullet$  anisotropy  $\bullet$  and  $\bullet$  and  $\bullet$ BODY QUANIUM SYSIEMS  $\blacksquare$ Pauli matrices. The temperature *T*, defining the thermal

*For pure states:*

*Von Neumann Entropy* 



*Area law*

 $S=\frac{c}{2}$  $S = \frac{c}{3} \log N$ ID cruitul systems. *1D critical systems:*

### **AREA LAWS AND TENSOR NETWORKS**





#### AUGMENTED TREE TENSOR NETWORKS and system sizes. Finally, we compute the ground state phase diagram of two-dimensional lattice

any spatial dimension by keeping a low algorithmic com-

please to standard algorithms (see Fig. 1), algorithms (see Fig. 1), algorithms (see Fig. 1), algorithms (see <br>Experimental algorithms (see Fig. 1), algorithms (see Fig. 1), algorithms (see Fig. 1), algorithms (see Fig. 1



*T. Felser, S. Notarnicola, S. Montangero PRL (2021)* atter *Letter, S. Indianuola, S. Montanger* T. Felser, S. Notarnicola, S. Montangero PRL (2021)  $int$  $\overline{PR}$ 

### **2D RYDBERG QUANTUM SIMULATOR**



#### **RYDBERG QUANTUM SIMULATOR**

*32x32 sites*



T. Felser, S. Notarnicola, S. Montangero PRL (2021)  $F \times I = G \times I \times I \times M$  $E$ . Peiser, S. Indiamicola, S. Ind mined by performing a finite-size scale in the state of the state of the state of the state scaling and the state of the state scaling and the state scaling and the state scaling and the state scaling and the state scaling <sup>h</sup>*O*(4)*†O*(4)<sup>i</sup> against the detuning , as they satisfy the *T. Felser, S. Notarnicola, S. Montangero PRL (2021)* 

#### **FROM NOW ONEXER IS NOW ONLINE GAUGE TENSOR NETWORKS** finite dimensional. Starting from this assumption, the formulation in quantum link model language of language o i.e. a hopping term of fermions mediated by the gauge field. Also, the mass term of the fermions is a gauge invariant . . . . . . . . . . . . . *<sup>x</sup>Ux,x*+*µ<sup>x</sup> <sup>x</sup>*+*µ<sup>x</sup>* + h.c. . . . . . . *N*¯ ⇣ *<sup>N</sup>*¯  $EN5UK NEI<sup>T</sup>$ 1 r ⇣ *<sup>N</sup>*¯ 2 **. . . . .** . I ATTICE GAILGE TENSOR NETWORKS operator or parallel transporter of a *U*(1) gauge model.

*Jx,µ<sup>x</sup>*

*†*

Local degrees of freedom  $U_{\vec{x}+\hat{y}}$  $[\psi^a_x$  $[\psi^a_x, U^{ab}_{x,x+\mu_x}]$ Since the site of as a stead of a steady  $U_{\vec{x}, -\hat{x}}$  and  $V_{\vec{x}}$  and  $U_{\vec{x}}$  is the links of  $U_{\vec{x}}$  and  $U_{\vec{x}}$  is the lattice bonds between  $U_{\vec{x}}$  and  $U_{\vec{x}}$  is the lattice bonds between  $U_{\vec{x}}$  and  $U_{\vec{x}}$  neighboring sites, every link by a dividend by a dividend by a dividend pair of sites. Matter field Gauge field  $\overline{C}$  signs, depending on the sites, is needed for a particular type of  $\overline{C}\vec{x}, +\hat{y}$ which are the usual definition of the usual definition of the Wilson type lattice theories if we identify  $q$ operator or parallel transporter of a *U*(1) gauge model.  $\left[\psi^a_x, U^{ab}_{x,x+\mu_x}\right]$ Matter field Gauge field  $U_{\vec{x},-\hat{x}}$  on the  $U_{\vec{x},+\hat{x}}$ **Provided** the other extreme limit is *N*<sup> $II$ </sup> Local degrees or invedting  $U_{\vec{x}, +\hat{y}}$ **Matter field** Gauge field  $\sigma_{\vec{x},-\hat{x}}$  gauge  $\sigma_{\vec{x},+\hat{x}}$ 

lattice, gauge symmetry generators have a localized support, each one involving a single matter field site, and  $G_x | \varphi_{phys} \rangle = 0$ (Gauss' law) however for the same of concreteness, we now present four particular examples: the simplest  $\mathcal{L}$  $\left\langle \begin{array}{c} \alpha & \lambda \\ \end{array} \right\rangle = 0$ ور<br>م *x*<sup>*x*</sup>  $\alpha$  $G_x|\varphi_{phys}\rangle = 0$ 



*<sup>x</sup> <sup>x</sup>* (8)

<sup>2</sup> + 1⌘

*mx †*

2

 $\ddot{\phantom{0}}$ 

#### Figure 1: (Color online). a) The commutation relations [*H, G*⌫  $\mathbf{U}_x |\varphi_{phys}\rangle = 0$   $\mathbf{U}_x$  symmetry, and  $\mathbf{U}_x$  relevant models in condensed models in condensed models in condensed matter models in condensed matter models in condensed matter models in condensed matter models in *<i>x* (11) *x* (11) *x* (2) *Kogut-Susskind*

 $\mathbf{r}$  is dynamically decoupled for  $\mathbf{r}$ Gauge invariant dynamics Flamiltonian formulation of LGT  $\sigma$  The gauge invariant dynamics of  $\sigma$  *Hamiltonian formulation of LCT Gauge invariant dynamics Hamiltonian formul* toniar *controllation of LGT*<br> *commulation of LGT Hamiltonian formulation of LGT*

 $U(x)$  connected to it. c) Typical coupling Hamiltonian terms in  $\mathcal{V}(x)$  and  $\mathcal{V}($ **h**.c.) In the gauge boson is split into a pair of riskon is split into a pair of riskon is split into a pair of rishons, linked to a pair of riskon is split into a  $\mu$  $H = J \sum$ *x*  $H = J \sum (\psi_x^{\dagger} U_{x,x+1} \psi_{x+1} + \text{h.c.})$  $H = \frac{3}{x} \left( \sqrt{\varphi_x} \mathcal{O}_{x,x+1} \varphi_{x+1} + \text{h.c.} \right)$ *a,b*

abelian

$$
H = t \sum_{x,a,b} \left[ \psi_x^{a\dagger} U_{x,x+1}^{ab} \psi_{x+1}^b + \text{h.c.} \right] \qquad \qquad [H_{\text{int}}^{\text{loc}} \ , G_x] = 0 \quad \forall \, x \in \mathbb{R}^n.
$$
  
non abelian

x<br>*Dynamics commutes with symmetry generator* abelian *metal potential profile for the matter field, which is a spinlet of y general and*  $\alpha$ *x*<br> *X Dynamics commutes with symmetry generator Dynamics commutes with symmetry generator*

$$
H = \sqrt[t]{\sum_{x,a,b} \left[ \psi_x^{a\dagger} U_{x,x+1}^{ab} \psi_{x+1}^b + \text{h.c.} \right]} \qquad \qquad \left[ H_{\text{int}}^{\text{[QED]}}, G_x \right] = 0 \quad \forall x
$$

### **QUANTUM LINK AND RISHON REPRESENTATION Schwinger representation**





 $U_{x,y} \equiv S_{x,y}^{+} = c_{y}^{\dagger} c_{x}$ 

**Link operator**

**Electric field [U(1) generator]**

$$
E_{x,y} \equiv S_{x,y}^{(3)} = \frac{1}{2} \left[ c_y^{\dagger} c_y - c_x^{\dagger} c_x \right]
$$

$$
\begin{aligned} \{c_x,c_y^\dagger\} &= \delta_{x,y} \quad \text{Schwinger fermions (rishons)}\\ [c_x,c_y^\dagger] &= \delta_{x,y} \qquad \qquad \text{Schwinger bosons} \end{aligned}
$$

**E=1/2 E=-1/2** *Local projection on a gauge invariant base +* 

**Spin-1:** *Projection on rishon number* 

*Matrix product operator*

*=* 

**Spin representation:**

$$
N_{x,y} = c_y^{\dagger} c_y + c_x^{\dagger} c_x \qquad \left[\vec{S}_{x,y}\right]^2 \equiv \frac{N_{x,y}}{2} \left[\frac{N_{x,y}}{2} + 1\right]
$$



#### U(1) LATTICE GAUGE THEORY IN 1+1D  $f(x) = \frac{1}{2} \int_0^x f(x) \, dx$ **THAT THROUGH THE AND ANOMALY**  $U(1)$  LATTICE GAUGE THEORY IN  $1+1D$  $s_{\rm{max}}$ With these definitions, the Hamiltonian is invariant  $\mathbf{b}$  is lacking due to the computational line  $\mathbf{c}$ ity  $i \neq i$  body problem  $i \neq j$ In the Hamiltonian formulation, its dynamics is defined and in the Hamiltonian formulation, its discussion in t<br>In the Hamiltonian formulation, it is defined as a set of the Hamiltonian formulation in the Hamiltonian formu



$$
H = -t \sum_{x} \left\{ \psi_{x}^{\dagger} U_{x,x+1}^{\dagger} \psi_{x+1}^{0.8} \psi_{x}^{\dagger} \psi_{x+1}^{0.75} \psi_{x,x+1}^{0.75} \psi_{x} \right\}
$$

$$
+ m \sum_{x} (-1)^{x} \psi_{x}^{\dagger} \psi_{x} + \frac{0.5}{20} \sum_{0.55} F_{x,x+1}^{2}
$$

$$
\mathcal{E} = \sum_{x} \langle E_{x,x+1} \rangle / L
$$

$$
0.5 \sum_{x} \langle E_{x,x+1}^{0.8} \rangle / L
$$

 $\begin{smallmatrix} \text{\large $\bullet$} \text{\large $\bullet$ 

- FIG. 2 (Color online). (a) Electric flux E as a function of uncontractom, S  $\frac{1}{6}$ antum link and rishon representation  $\frac{1}{c}$   $\frac{1}{c}$   $\frac{1}{c}$   $\frac{1}{c}$ (*x, xxtermis link, and web on redevecent sticm*)  $\sim$  Quantum time and rishon representation ➤ *Quantum link and rishon representation*
- $\bm{l}$  fermions scaling of the electric flux E shown in the critical point  $\bm{l}$ rered fermions of the different phases of the different phases are the different phases are the different phase  $\blacktriangleright$  *Staggered fermions* the electric coupling strength with *g*, where *Ex,x*+1 is the
- $~\gamma$ ersality class and blue squares, this is the system size log L: a linear fit of use log L: a linear fit results in the central fi  $\alpha$  universality class ig universumy caus ▶ *Ising universality class* 
	- harge  $c = 0.49 \pm 0.01$  $\frac{1}{2}$  $\blacktriangleright$  *Central charge*  $c = 0.49 \pm 0.01$ ➤ *Central charge*
- $\blacktriangleright$  Confi satisfying the Gauss of the Gauss o<br>*<u>i</u>* is the Gauss of the Ga  $\begin{array}{ccc} \mathbf{5} & \mathbf{5} & \mathbf{6} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} & \mathbf{15} & \mathbf{16} & \mathbf{17} & \mathbf{18} & \mathbf{18} & \mathbf{19} & \mathbf{1$

religions of the configuration of the configuration of the configuration of the continents action on the continent  $f(x) = \frac{1}{2}$ Jumont, space, we found the  $(201)$ E. Rico, T. Pichler, M. Dalmonte, P. Zoller, and SM, PRL (2014)  $v = 8.0$  and  $v = 8.0$  and  $v = 8.0$  and  $v = 8.0$  and  $v = 2.0$  i.e.,  $v = 2.0$  1.e.,  $v = 2.0$  1.e.,  $v = 2.0$  $1;$ even  $1;$   $\overline{C}M$ , ppt.  $(2,0,1,4)$ .



 $\Gamma$ . I thine, E. Kito, m. Duimonte,  $\Gamma$ . Zoner, and sin,  $\Gamma$ KX (2010) *T. Pichler, E. Rico, M. Dalmonte, P. Zoller, and SM, PRX (2016)* 



*Real time*  $F_{12}$  ,  $F_{23}$  ,  $F_{34}$  ,  $F_{44}$  ,  $F_{55}$  ,  $F_{56}$  ,  $\mathbb{R}$   $\mathbb{R}$  using a bind between sites  $\mathbb{R}$   $\mathbb{R}$  bind between sites  $\mathbb{R}$   $\mathbb{R}$  between sites  $\mathbb{R}$   $\mathbb{R}$   $\mathbb{R}$  between sites  $\mathbb{R}$   $\mathbb{R}$   $\mathbb{R}$   $\mathbb{R}$   $\mathbb{R}$   $\mathbb{R}$   $\mathbb{R}$   $\mathbb$ 

**MESONS SCATTERING** 2 cillating behavior of the entanglement entropy between zero and one. Finally, the third case with *m* = 0*.*25 and  $\overline{a}$ the system increases in the system in th nature of enhanced quantum correlations. Right panel: *S*(*x*)

> T. Pichler, E. Rico, M. Dalmonte, P. Zoller, and SM; PRX (2016) ter the collision, which enlarges as a function of time. The

#### **ENTANGLEMENT GENERATION IN QED SCATTERING PROCESSES**



*M. Rigobello, S. Notarnicola, G. Magnifico, and S. Montangero, Phys. Rev. D 104, 114501 (2021).* 

# **1+1D SU(2) LGT WITH QUDITS IN TRAPPED IONS**



*G. Calajò et al. arxiv:2402.07987 Quantum stars! arXiv:2405.13112*

# **TWO DIMENSIONAL SIMULATION OF A LGT AT FINITE DENSITY**



#### *16x16 lattice sites*

# **PHASE DIAGRAM**

*Hilbert space of ~80x80 qubits*



 $w = (9.28.8)$  $\beta$ KA (2020) *T. Felser et al. PRX (2020)*

#### **FINITE DENSITY**













 $\frac{m_c}{\sqrt{2}}$  (b)  $\frac{3}{2}$  $(g_m^2=8/g_e^2)$  $m_c \approx +0.22$  $\overline{u}$  making the transition physically relevant. electric field on links for *<sup>m</sup>* <sup>=</sup> 3*.*<sup>0</sup> (d) and *<sup>m</sup>* = 3*.*<sup>0</sup> (f) in the presence of magnetic interactions with *<sup>g</sup>*<sup>2</sup>  $g_n^2$  $\frac{2}{m} = 8/g_e^2$ 

### **SCREENING**



 $\sim$   $\sim$ 



 $g_e^2 = g^2/a, g_m^2 = 8/(g^2a)$ 

#### **FINITE DENSITY**





*Charge at distance l from boundaries* 



# **2+1D NON-ABELIAN LGT**



*(2+1)D SU(2) Yang-Mills Lattice Gauge Theory at finite density via tensor networks G. Cataldi, G. Magnifico, P. Silvi, SM Phys. Rev. Res. (2024)*

## **1+1D SU(3) 2-FLAVOURS LATTICE GAUGE THEORY**

$$
H = \sum_{x, f, c, c'} \left[ -\frac{i}{2} \psi_{x, f, c} U_{x, c; x+1, c'} \psi_{x+1, f, c'}^{\dagger} + \text{H.c.} \right]
$$
  
+ 
$$
\sum_{x, f, c} m_f (-1)^x \psi_{x, f, c}^{\dagger} \psi_{x, f, c} + \sum_x \frac{g^2}{2} E_{x; x+1}^2.
$$



*M. Rigobello, G. Magnifico, P. Silvi, SM arXiv:2308.04488*

 $=$  am  $_{phys}$ 

Ë

 $\frac{1}{6}$ 

gapless

gapped phase

 $g=1$ 

0.02

 $1/L$ 

gapless

s-pions

phase

 $g = ag_{phys}$ 

 $M = 1.01(1)$ 

(c)  $\Delta^{++}$ 

 $0.02$ 

 $1/L$ 

0.04

1.4

1.2

 $1.0$ 

 $0.00$ 

0.04

#### **CONFINEMENT IN 2D ISING MODEL**

$$
H = -J\sum_{\langle i,j\rangle} \sigma_i^z \sigma_j^z - g \sum_i \sigma_i^x \qquad \qquad \left|\downarrow \ldots \downarrow\right\rangle
$$



**FIG. 1.** Confinement in the 2D quantum Ising model. (a) Short time dependence of magnetization  $\langle \sigma^z(t) \rangle$  for a set of  $g/J$ . (b)  $\langle \sigma^z(t) \rangle$  for longer times and small  $q/J$ . (c) Time dependence of entanglement entropy  $S$  for a bipartition of two neighbouring spins and the rest of the system.  $S_{\text{max}}$  is the maximal entanglement entropy of the bipartition (in this case  $S_{\text{max}} = 2 \ln 2$ ). (d) Time dependence of a horizontal cut of the connected correlation function  $C_{ij}$  where  $i = 4$  for a set of  $g/J$ .

*L. Pavešić et al. arxiv: 2406.11979*



Spectrum of  $\sigma^z$ , given by the Fourier transform  $\mathcal{F}(\sigma^z)$ . FIG. 2. (a) Spectrum of magnetization for two cases of  $q/J$ . Dashed vertical lines correspond to  $\omega/J = 4, 8$ , and 12. (b) Heatmap of the spectrum for a range of  $q/J$ . Black dashed lines correspond to the transition energies obtained by perturbation theory.

$$
E_0 = -\frac{g^2}{8J}N^2 + \mathcal{O}(g^4),
$$
  
\n
$$
E_1 = 8J - \frac{g^2}{8J}(N^2 + 6) + \mathcal{O}(g^4),
$$
  
\n
$$
E_2 = 12J - \frac{g^2}{8J}N^2 + \mathcal{O}(g^4).
$$

#### **INTERFACE PHYSICS IN 2D ISING MODEL**



FIG. 3. Spread of correlations near an interface. (a) Sketch of the initial state, with blue representing  $\uparrow$  and red  $\downarrow$  spins. Black lines indicate the cuts shown in (d). (b) Sketch of the resonant process along the interface. (c) Heatmap of the spectral density of  $\langle \sigma^z \rangle$  (t) for a spin at the interface for a range of  $g/J$ . White dashed lines correspond to the transition energies of a freely propagating edge mode. (d) Horizontal  $(x)$  and vertical  $(y)$  cuts of the connected correlations function  $C_{ij}$  with respect to the spin at  $(4, 4)$  (white square in (a)). Blue dashed lines are  $\pm 4gt$ , showing that the interface mode carries the correlations.



*L. Pavešić et al. arxiv: 2406.11979*

#### **ENTANGLEMENT OF MIXED MANY-BODY QUANTUM SYSTEMS DODY OUANTUM CVETEME** nian energyscale (*J* = *k<sup>B</sup>* = 1). To appropriately choose a suitable number *K*<sup>0</sup> we start from *K*<sup>0</sup> = 2. We then

*For pure states:*

Von Neumann Entropy **von Neumann Entropy** 

For pure states:  
\n
$$
S = -\text{Tr } \rho \log \rho
$$
\n
$$
S = \text{Tr } \rho \log \rho
$$
\n
$$
S \propto N^{(D-1)}
$$
\n
$$
S \propto N^{(D-1)}
$$
\n
$$
D \text{ critical systems:}
$$
\n
$$
S = \frac{c}{3} \log N
$$

*Area law*

$$
\mathcal{S} \propto \gamma
$$

expressed as *E<sup>F</sup>* = log(*Nc/*<sup>3</sup>*f*(*T N<sup>z</sup>*)), or  $S=\frac{c}{2}$  $S = \frac{c}{3} \log N$ ID cruitul systems. *1D critical systems:*

this procedure to the specific case of the EoF. For mixed states:  $\rho = \sum p_j$ 

$$
\rho = \sum p_j |\psi_j\rangle\langle\psi_j|
$$

$$
E_F(\rho) = \inf_{\{p_j, \psi_j\}} \Big\{ \sum_j p_j \mathcal{S}(|\psi_j \rangle) : \rho = \sum_j p_j |\psi_j \rangle \langle \psi_j| \Big\}
$$

 $E_F(\rho,T) \stackrel{?}{\propto} \log N^{c/3}$  $L_1$  (*·* $\pi$ )  $\frac{?}{\cdot}$  legg  $N_c/3$  $E_F(\rho, T) \propto \log N^{3/3}$ 

 $Entanglement of forumation$  $\sum$ *Entanglement of formation*

C.H. Bennet et al. PRA 1996 a convex mixture of pure states *| <sup>n</sup>*i, with probabilities

#### **TREE TENSOR OPERATORS** size in the entanglement of formation in 1D critical lattice models at 1D can be finite that finite temperatur using a *K*<sup>0</sup> that scales at most *polynomially* with the system size *N*. Therefore, from a numerical viewpoint, it is meaningful represent *X* with a Tree Tensor netcomposition (SVD). In the results section, entropies are **EXPRESS OF LOGIZER OF ERATORS** unit of entanglement. For a given *K K*0, minimization in the space of the *U* is carried out via direct search methapproach introduces a novel tensor network ansatz the *Tree Tensor Operator* a positive, loopless representation for density matrices which eciently encodes information on bipartite entanglement, enabling the up-scaling of entanglement estimation. Employing this technique, we observe a finitesize scaling law for the entanglement of formation in 1D critical lattice models at finite temperature, known as the the Kraus dimension *K*0. For many-body quantum states at low temperatures, probabilities *p<sup>j</sup>* de- $\sim$  such that it is possible to approximate to approximate  $\sim$

ods, but other choices are possible. Extensive proofs of the proofs of the proofs of the proofs of the proofs o

$$
\rho = \sum p_j |\psi_j\rangle\langle\psi_j|
$$
  
\n
$$
\rho = \frac{R}{\sum_{M} K_0}
$$

$$
\hat{H}_{Ising} = J \sum_{i=1}^{N} (\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + h \hat{\sigma}_j^z) \qquad \text{Then}
$$
\n
$$
\hat{H}_{XXZ} = J \sum_{j=1}^{N} (\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + \xi \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z) \qquad \text{(Mixture of)}
$$
\n
$$
X =
$$

$$
E_F(\rho) = \min_{K \ge K_0} \inf_{\mathcal{U}} \left\{ \sum_{j=1}^K p_j \mathcal{S}(|\psi'_j\rangle) : X' = X\mathcal{U} \right\}
$$

FIG. 2. Scaling of computational times *versus N*, for thermal

(a semi-unitary matrix satisfying *UU†* = **1**) of dimension



t her mat equition tum state (Mixture of  $K_0$  Boltzmann factors) ( $\left( +1 \right)$  **E**  $\left( 2T + 1 \right)$  is the energy of  $\left( 2T + 1 \right)$  is the energy of  $\left( 2T + 1 \right)$  is the energy of  $\left( 2T + 1 \right)$  $X = \frac{1}{\sqrt{Z}} \sum_{j}^{K_0} e^{-E_j/2T} |\psi_j\rangle\langle j|$  $Thermal equilibrium state$ resentation, with *Mermal equilibrium state* a function of the size *N*, for both Ising and XXZ models  $X = \frac{1}{\sqrt{Z}} \sum_{j}^{K_0} e^{-E_j/2T} |\psi_j\rangle\langle j|$ as shown in the inset.  $\hat{\sigma}^y_{j+1} + \xi \, \hat{\sigma}^z_j \hat{\sigma}^z_{j+1}$ the *M* needed to achieve 99% of the exact EoF value as a fuermal equilibrium situe  $\overline{a}$ estimators are often accessible in numerical simulations of  $\mathcal{L}_{\mathcal{A}}$ many-body quantum systems, and especially in loopless  $t_{\rm max}$  ansatz states, where the calculation complexity scales politically with the system size  $\alpha$  in  $\alpha$ Conversely, for *mixed* global quantum states, the problem  $\tau \zeta \sigma_i \sigma_{i+1}$ is more involved, both conceptually and technically and technically and technically and technically.  $\sqrt{Z}$ equilibrium state monotones pro  $\mathbf{y}$ <sup>armer</sup> samt samt mea- $\mathbb{R}^3$ . At a technical level, the core problem is to core problem in the core problem is to core proble  $\alpha$  Boltzmann factors. Even by  $\sigma$  that can be evaluated by linear algebra operations,  $\sigma$  $\overline{V}$  and  $\overline{V}$  and  $\overline{V}$  $\sum_{i=1}^{N} e^{-E_j/2T} |\psi_i\rangle \langle i|$  $\mathcal{L}$ j many important monotones with a clear physical ph  $\tilde{j}$ ) Thermal equilibrium state  $\overline{M}_{\text{inter}}$ *(Mixture of K<sub>0</sub> Boltzmann factors )*  $\mathcal{I}$  system, and the system, and the system, and thus are e $\mathbf{v}$  are e $\mathbf{v}$  are e $\mathbf{v}$  $X = \sqrt{Z} \sum_j C_j$ . If  $\ket{\psi_j}$  $\alpha$ <sup>1</sup>  $\alpha$ <sup>1</sup>  $\alpha$ <sup>1</sup>  $\alpha$ <sup>2</sup>  $\alpha$ <sup>2</sup> tum lattice models *H* via TTO. We first obtain  $X=\frac{1}{\sqrt{2}}$ *Z*  $\sum_j^{K_0} e^{-E_j/2T} |\psi_j\rangle\langle j|$ 

L. Arceci, P. Silvi, and S. Montangero PRL (2022) *R*, after having compressed the state with some maximal bond *<sup>j</sup>*+1 + ⇠ ˆ*<sup>z</sup> <sup>j</sup>* ˆ*<sup>z</sup> <sup>j</sup>*+1 (3) ntangero  $PRL(2022)$ L. Arceci, P. Silvi, and S. Montangero PRL (2022) boundary conditions (PBC) and  $\mathcal{P}$  and  $\mathcal{P}$  and  $\mathcal{P}$ Arceci, P. Silvi, and S. Montangero PRL (.  $\mathcal{O}$  mann entropy  $\mathcal{O}$  $f(2)$ tensor *R*, containing all the information about entanglement L. Arceci, P. Silvi, and S. Montangero PRL (2022)

*U*, precisely

## **CONFORMAL SCALING OF ENTANGLEMENT OF FORMATION** <sup>4</sup>

$$
E_F = \log(N^{c/3} f(TN^z))
$$

0  $5$  10  $15$  20

 $k_B T = 0.1, 0.2, 0.3 J$ 

 $N = 16$ 

*<sup>E</sup><sup>F</sup>* (*T,N*) = *<sup>c</sup>*

1*.*2

0*.*25

0*.*50

*E*

 $\left( \bigodot_{\mathbf{k}_1} 0.75 \right)$ 

 $1.00 \cdot$ model in transverse field (*N* = 16) at di↵erent temperatures  $\frac{1}{k}$  0.70 kept in the the the theorem in the the theorem in the th  $\overline{E}$  are  $\overline{E}$ Circles correspond to data for *K* = *K*0, while crosses are for  $0.25$ already enough to achieve a very good estimate for the  $\mathbb{R}$ 

 $\sum_{\tau} 0.75$ 

 $\frac{0.25 + 1.28 + 2.88}{0.25 + 2.88}$ 

ment by looking at the Von-Neumann entropy of the den-

 $\sigma_{12}$  for the correction model in the 16 million of *versus* the number of states *K*<sup>0</sup> kept in the thermal ensemble. Temperatures in the key are expressed in units of *J/kB*.  $1.00 \frac{1}{3}$ 

0*.*75

1*.*00

*S*()

1*.*4

 $N = 16, 32, 64, 128$  $\frac{1.6}{\sqrt{2}}$  $E_F(N,T)-\frac{1}{6}\log_2 N$ 0.4  $0.3\,$  $F(X \cup Y \cup Y \cup Y \cup Y) = 1.4 + \frac{1}{2}$  $\mathcal{L}$ <sup>1.4</sup>  $\sim$   $\mathcal{L}$   $\sim$   $\mathcal{L}$  $0.2\,$ (top) and of the EoF *E<sup>F</sup>* (⇢) (bottom) in the number *K*<sup>0</sup>  $\blacksquare$  states considered in the theorem and the theorem  $\blacksquare$ that connects lengthscales to entanglement, while *z* is  $0.1$  $\frac{15}{\sqrt{2}}$   $\frac{20}{\sqrt{2}}$   $\frac{1}{2}$   $\frac{1}{2}$  $0.00$ 0.25  $0.50\,$ 0.75  $k_B T = 0.1, 0.2, 0.3 J$  $E_{B1} = 0.1, 0.2, 0.3J$  $T N$  $\frac{1}{\sqrt{2}}$  and  $\frac{1}{\sqrt{2}}$  for  $\frac{1}{\sqrt{2}}$  estimation respectively. (top) and of the EoF *E<sup>F</sup>* (⇢) (bottom) in the number *K*<sup>0</sup>  $\begin{bmatrix} 0.75 \\ 4 \end{bmatrix}$  and  $N = 16$   $\begin{bmatrix} 4 & 1.0 \\ 0.50 & 0.50 \end{bmatrix}$  $\Box$   $\Box$   $1.0$ of states considered in the theorem in th  $\begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix}$  **from are non-universal and depend on the microscopic on the m**  $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 &$ st<del>ubre that in the contract of the contract of the contract of the contract of the con</del><br>The contract of the contract of 0  $5 \t\t \t\t \frac{10}{K_0}$   $15 \t\t 20$   $3.5$   $-2$  $\frac{1}{0.00}$   $\frac{1}{0.01}$ 

 $0.02$ 

 $\overline{T}$ 

0.03

0.04

 $0.05$ 

Numerical complexity depends on entropy, not entanglement!  $N$ umorical complexity depends  $\emph{Numerical complexity depends}$ *on entropy, not entanglement!* or very close to its corresponding cross. while crosses correspond to *K* = *K*<sup>0</sup> + 2. We observe that there is no need to enlarge the parameter space for  $\mathbf{I}$ 

0  $5$  10  $15$  20  $K_0$ 

 $\frac{5}{L}$  and  $\frac{10}{L}$  is  $\frac{15}{L}$ 

*L. Arceci, P. Silvi, and S. Montangero PRL (2022)* 

### **RYDBERG ARRAYS AT FINITE TEMPERATURE**



*N. Reinic et al. arxiv: 2405.18477*

## **ROADMAP FOR LGT QUANTUM SIMULATION**

 $\ell^*$ 







TABLE I. Dressed site Hilber creasing number  $\ell$  of allowed in some 2- and 3-dimensional namical matter and gauge grou

TABLE II. *Estimated simulation time*. We derive the baseline from a single-tensor optimization of a  $64 \times 64$  quantum Ising simulation with  $\mathbb{Z}_2$  symmetry taking 7192s on a A100 GPU. Further, we assume that single-tensor update, one tensor and one GPU per MPI thread, and 50 sweeps for the baseline. To extrapolate to larger systems, we assume a scaling with  $\mathcal{O}(\chi^4 N^{D-1})$  as well as seven (thirty-one) tensors per MPI thread for  $256 \times 256$  (64  $\times$  64  $\times$  64) systems. The empirical a QED plaquette for a grid scalings are approximately a factor of 2.3 for doubling the system size and 13 for doubling the bond dimension, which we obtain from smaller simulations with  $\chi = 225$  and for  $32 \times 32$  qubits. The times are valid for any  $d < \chi$ .



 $^{-2}$ , 10<sup>1</sup>] and  $g^2 \in [10^{-1}, 10^1]$ .  $\ell^*$  required to reach a preciic energy  $\langle \text{Re}\,\hat{U}_{\square} \rangle$ . (c) Cor- $V S$  associated with a symte.

*G. Magnifico et al. arxiv:2407.03058*

### **TAKE HOME MESSAGES**

- ➤ Tensor network algorithms will benchmark, verify, support and guide quantum simulations/computations development
- ➤ High-dimensional tensor network simulations are becoming more and more available (PEPS, aTTN,…)
- ➤ Entanglement of mixed many-body states can be quantified
- ➤ Scalability to full HPC will be necessary to produce quantitative results
- ➤ Interaction with HEP is becoming more and more relevant
- ➤ Interesting developments also in other directions (classical optimisers/annealers, machine learning)

