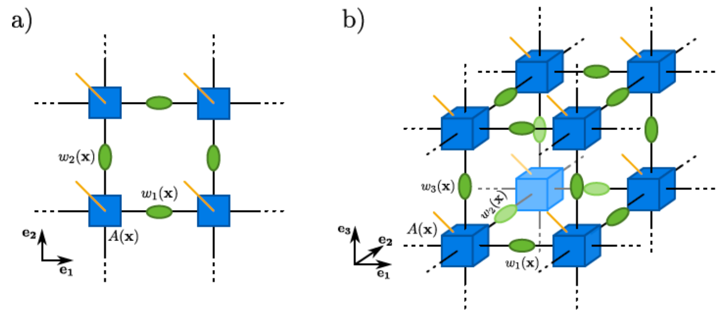
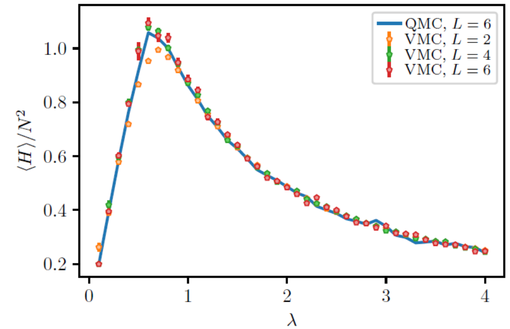
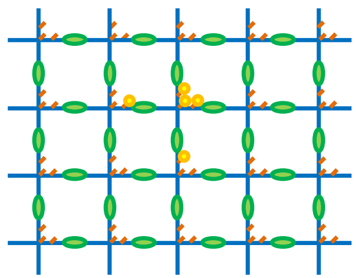
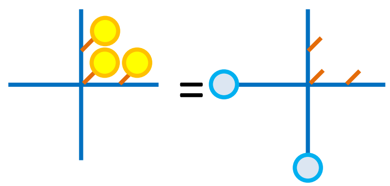
Gauged Gaussian PEPS





A tensor network approach based on sign-problem free Monte-Carlo for studying lattice gauge theories beyond 1+1d





Erez Zohar

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ERC Consolidator Grant
Oversign 2024-2029

Gauge Theories are challenging:

- Local symmetry → many constraints
- Involve non-perturbative physics
  - Confinement of quarks → hadronic spectrum
  - Exotic phases of QCD (color superconductivity, quark-gluon plasma)

→ Hard to treat experimentally (strong forces)

→ Hard to treat analytically (non perturbative)

→ Lattice Gauge Theory (Wilson, Kogut-Susskind…)

→ Lattice regularization in a gauge invariant way

Conventional LGT techniques

- Discretization of both space and time
- Monte Carlo computations on a Wick-rotated, Euclidean lattice

$$\left\langle \hat{A}\left(\hat{\Phi}\right)\right\rangle = \frac{\int \mathcal{D}\phi A(\phi) e^{iS_M}}{\int \mathcal{D}\phi e^{iS_M}}$$

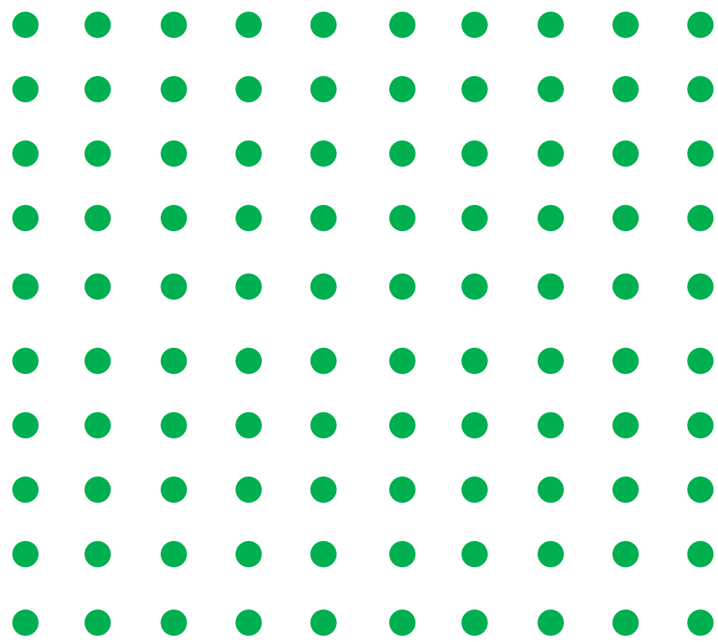
$$\underset{t\to -i\tau}{\longrightarrow} \frac{\int \mathcal{D}\phi A(\phi) e^{-S_E}}{\int \mathcal{D}\phi e^{-S_E}} \equiv \int \mathcal{D}\phi A\left(\phi\right) p\left(\phi\right)$$

- **Very (very) successful for many applications, e.g. the hadronic spectrum**
- **Problems:**
  - **Real-Time evolution:**
    - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - **Sign problem:**
    - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime

$\longrightarrow$ **New approaches: quantum simulation and computation, tensor networks.**

Many-Body Area Law, Handwaving Formulation

“The ground state of a many-body Hamiltonian with local interactions (+ a few more assumptions) **obeys an bipartite entanglement entropy area law.”**



See, e.g., Eisert, Cramer, Plenio, RMP 2010

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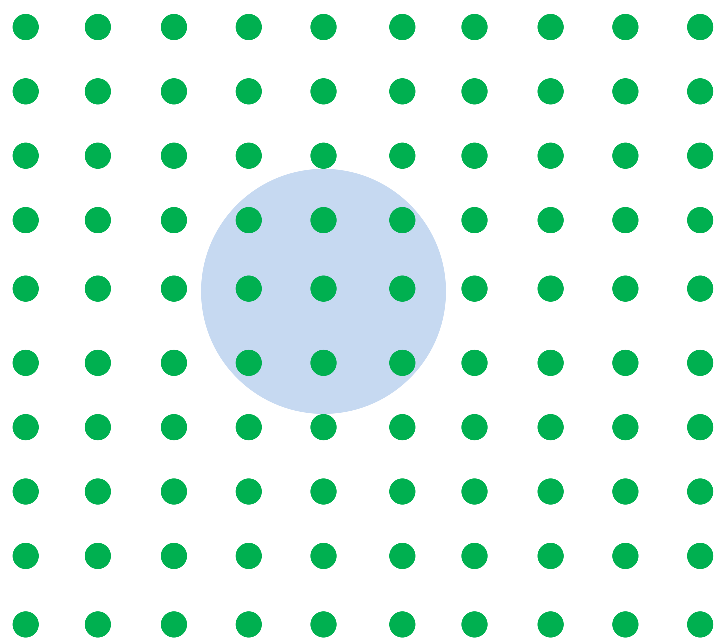

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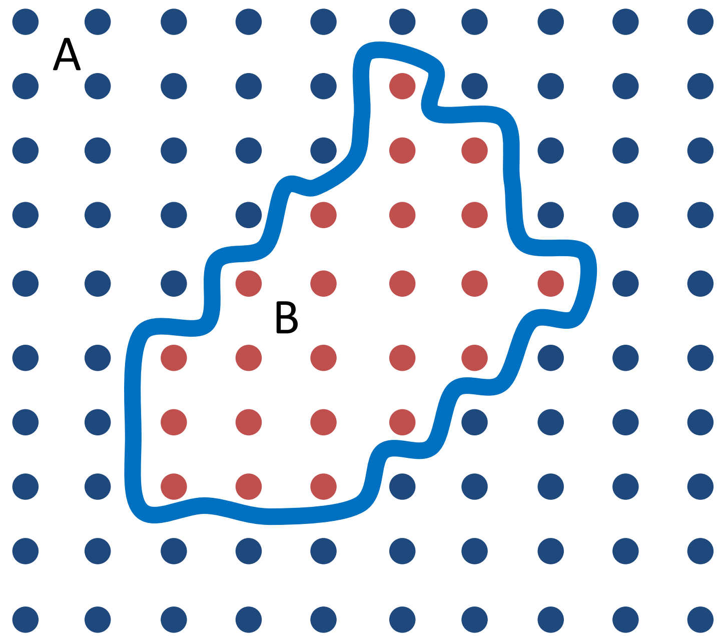

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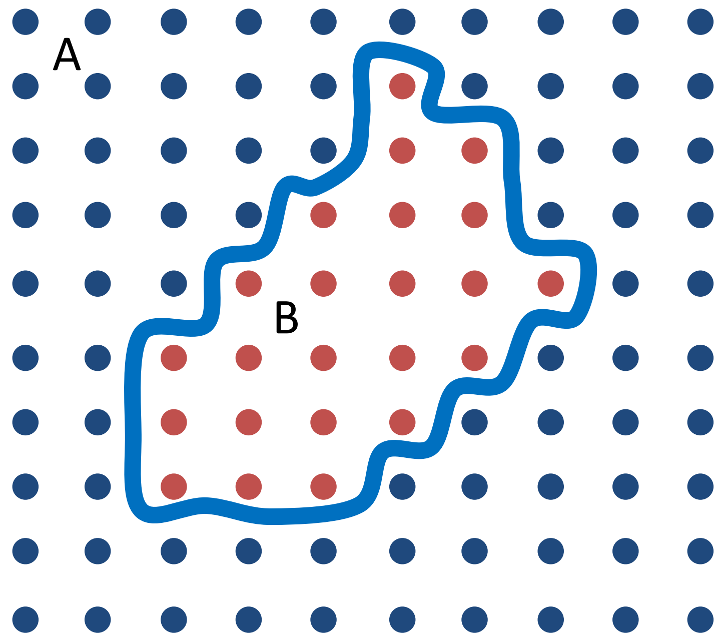
Many-Body Area Law, Handwaving Formulation

"The ground state of a many-body Hamiltonian with local interactions (+ a few more assumptions) **obeys an bipartite entanglement entropy area law."**

$$|\psi\rangle = \sum_{\mu} b_{\mu} |\mu\rangle_A |\mu\rangle_B$$

$$p_{\mu} = |b_{\mu}|^2$$

$$S = -\sum_{\mu} p_{\mu} \log p_{\mu}$$



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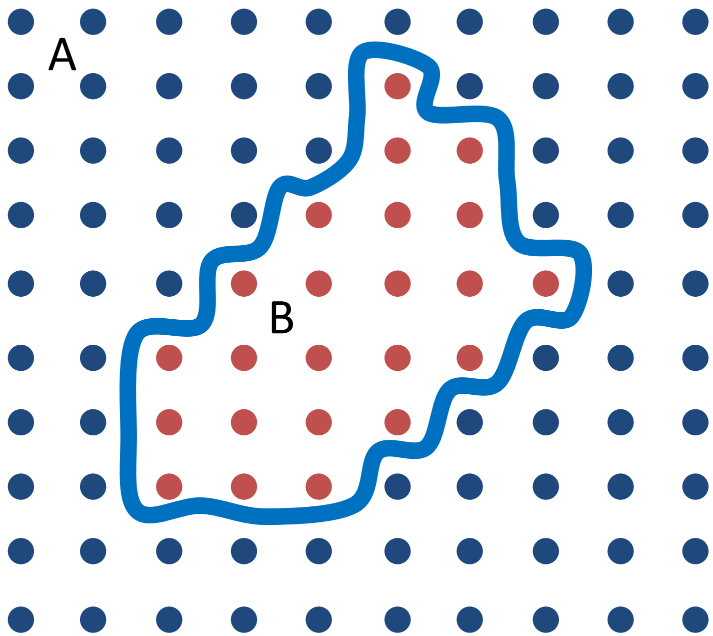
$$S = -\sum_{\mu} p_{\mu} \log p_{\mu}$$

$$S \propto \text{Area (Boundary)}$$



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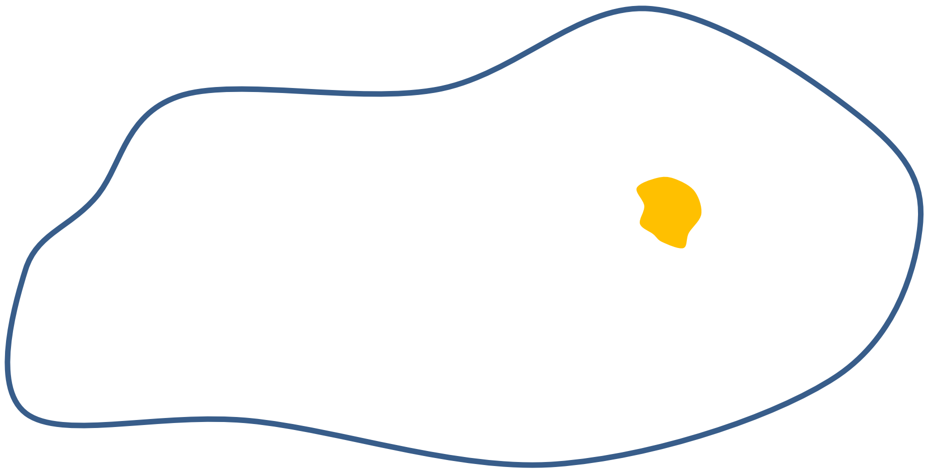
Many-Body Area Law, Handwaving Formulation

"The ground state of a many-body Hamiltonian with local interactions (+ a few more assumptions) **obeys an bipartite entanglement entropy area law."**

Physically relevant corner of the Hilbert space



See, e.g., Eisert, Cramer, Plenio, RMP 2010

Tensor Network States

- The **number of variables** needed to describe states of a many-body system **scales exponentially** with the system size. This makes it hard to simulate large systems (classically).

- **Tensor network states** are ansatz states for **many body systems**, mostly on a lattice, for either **analytical or numerical** studies, based on contractions of **local tensors that depend on few parameters**.

- In spite of their **simple description**, tensor network states describe and approximate **physically relevant states of many-body systems**.

Relevant reviews:
Schollwöck, Ann. Phys. 2011
Orus, Ann. Phys. 2014
Cirac, Perez-Garcia, Schuch, Verstraete, RMP 2021

Tensor Network Studies of LGTs

- **Real-Time evolution:**
  - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - Calculations in **quantum Hilbert spaces**, where states evolve in real time, instead of in Wick-rotated statistical mechanics analogies.

- **Sign problem:**
  - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime
  - Calculations in **quantum Hilbert spaces**: fermions are fermions, no integration over time dimension. If the problem arises, it can be the result of using a particular method, nothing general.

See the review paper by Bañuls and Cichy, Rep. Prog. Phys. 83 024401 (2020)

MPS & LGT – Numerical Approach

- **Mostly in 1+1d**, combining MPS (Matrix Product States) with **DMRG (Density Matrix Renormalization Group)**; have been widely and successfully used for various many body models, mostly from condensed matter.

- **Very successfully applied to 1+1d lattice gauge theories**, including finite chemical potential and real time evolution (string breaking) for Abelian and non-Abelian theories
  - Bañuls, Cichy, Cirac, Kühn, Jansen, Saito…
  - Dalmonte, Montangero, Pichler, Rico, Silvi, Tschirsich, Zoller…
  - Buyens, Haegeman, Hebenstreit, van Acoleyen, Verschelde, Verstraete…
  - Borla, Moroz, Grusdt, Verresen… (rather more CM-like)

- High dimensional generalizations: challenging and demanding scaling; works nicely for ladders, cylinders etc.

See the review paper by Bañuls and Cichy, Rep. Prog. Phys. 83 024401 (2020) and refs. therein

Tensor Field Theory

A non-Hamiltonian, Euclidean (path integral) approach

See e.g. the review of Meurice, Sakai and Unmuth-Yockey (Rev. Mod. Phys. 94, 025005)

Hamiltonian LGT TNs in 2+1d and more

- **Tagliacozzo, Vidal**, **Entanglement renormalization and gauge symmetry**, PRB 2011
  - Pure gauge, Z2
- **Tagliacozzo, Celi, Lewenstein**, **Tensor Networks for Lattice Gauge Theories with Continuous Groups**, PRX 2014
  - Pure gauge, continuous groups

Gauging globally invariant ("matter") PEPS to locally invariant LGT PEPS – introducing gauge fields which lift the symmetry to a local one

- **Haegeman, Van Acoleyen, Schuch, Cirac, Verstraete, Gauging Quantum States: From Global to Local Symmetries in Many-Body Systems**, PRX 2015
  - Gauge field Hilbert space = Matter Hilbert Space (Higgs-like theories)

- **Zohar, Burrello, Building projected entangled pair states with a local gauge symmetry**, NJP 2016
  - Different Gauge Field Hilbert spaces, allowing for fermionic constructions (matching the standard model content)

Hamiltonian LGT TNs in 2+1d and more

iPEPS:

- Zapp and Orus, **Tensor network simulation of QED on infinite lattices: learning from (1+1)d, and prospects for (2+1)d**, PRD 2014
- Robaina, Bañuls, Cirac, **Simulating 2+1d Z3 lattice gauge theory with iPEPS**, PRL 2021

Tree tensor networks:

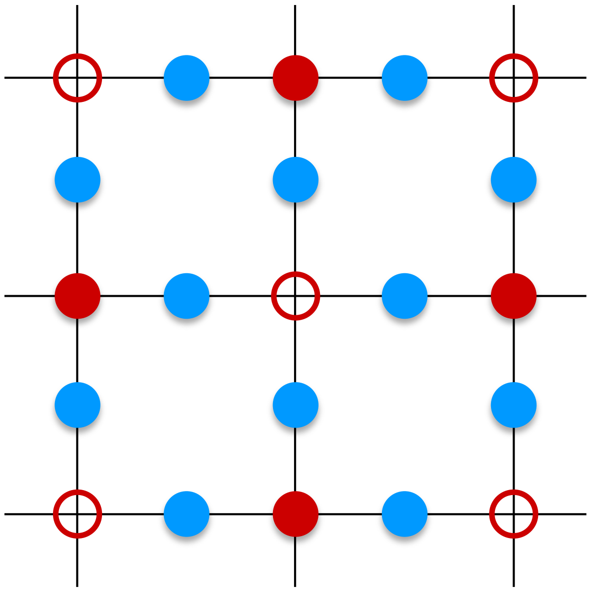
- Felser, Silvi, Collura, Montangero, **Two-dimensional quantum-link lattice Quantum Electrodynamics at finite density**, PRX 2020
- Magnifico, Felser, Silvi, Montangero, **Lattice Quantum Electrodynamics in (3+1)-dimensions at finite density with Tensor Networks**, Nat. Comm. 2021
- Montangero, Rico, Silvi, **Loop-free tensor networks for high-energy physics**, Phil. Trans. R. Soc. A, 2022
- Catataldi, Magnifico, Silvi, Montangero, **(2+1)D SU(2) Yang-Mills Lattice Gauge Theory at finite density via tensor networks,** Phys. Rev. Research 6, 033057 (2024)
- Magnifico, Catataldi, Rigobello, Majcen, Jaschke, Silvi, Montangero, **Tensor Networks for Lattice Gauge Theories beyond one dimension: a Roadmap,** arXiv:2407.03058(2024)

Gauged Gaussian Fermionic PEPS:

- Zohar, Burrello, Wahl, Cirac, **Fermionic projected entangled pair states and local U(1) gauge theories**, Ann. Phys. 2015
- Zohar, Wahl, Burrello, Cirac, **Projected Entangled Pair States with non-Abelian gauge symmetries: an SU (2) study**, Ann. Phys. 2016
- Zohar, Cirac, **Combining tensor networks with Monte Carlo methods for lattice gauge theories**, PRD 2018
- Emonts, Bañuls, Cirac, Zohar, **Variational Monte Carlo simulation with tensor networks of a pure gauge Z3 theory in 2+1d**, PRD 2020
- Emonts, Kelman, Borla, Moroz, Gazit, Zohar, **Finding the ground state of a lattice gauge theory with fermionic tensor networks: A 2+1-D Z2 demonstration**, PRD 2023
- Kelman, Borla, Elyovich, Gomelski, Roose, Emonts, Zohar, **Gauged Gaussian PEPS - A High Dimensional Tensor Network Formulation for Lattice Gauge Theories**, arXiv:2404.13123, 2024 (accepted to PRD)

The LGT Hilbert Space

- **The lattice is spatial**: time is a continuous, real coordinate.
- **Matter particles** (fermions) – on the **vertices**.
- **Gauge fields** – on the lattice's **links**
- Hamiltonian picture → Hilbert space
  → Natural way to describe constraints

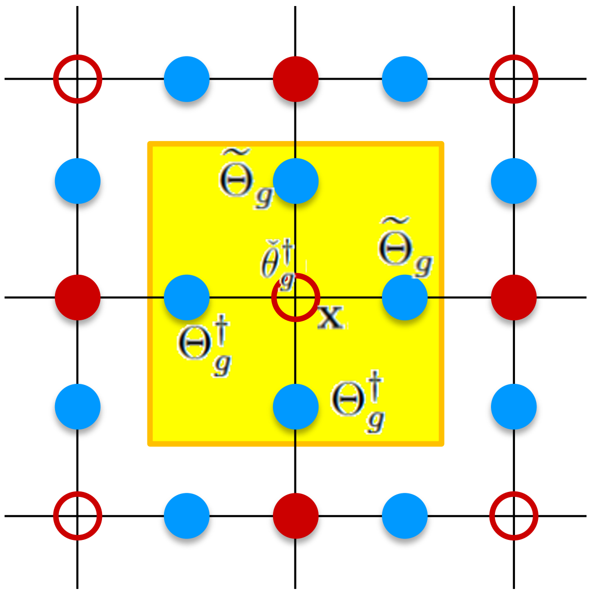


Gauge Transformations

- Act on both the matter and gauge degrees of freedom.

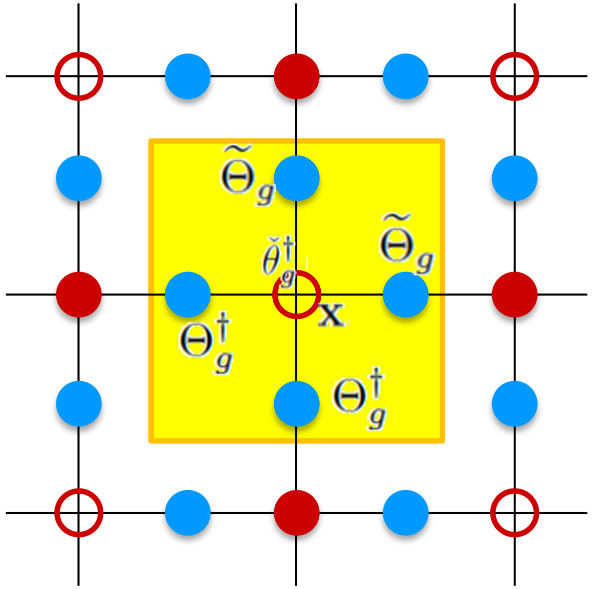
- **Local** : a unique transformation (depending on a unique element of the **gauge group**) may be chosen for each site

- The states are **invariant under each local transformation separately**.



$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d}\left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^{\dagger}\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\check{\theta}_g^{\dagger}(\mathbf{x})$$

Symmetry → Conserved Charge



– **Transformation rules on the links**

$$\{|g\rangle\}_{g\in G}$$

$$\Theta_g |h\rangle = |hg^{-1}\rangle \quad \Theta_g = e^{i\phi_a(g)R_a}$$

$$\widetilde{\Theta}_g |h\rangle = |g^{-1}h\rangle \quad \widetilde{\Theta}_g = e^{i\phi_a(g)L_a}$$

– **Gauge Transformations:**

$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d} \left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\breve{\theta}_g^\dagger(\mathbf{x})$$

$$\hat{\Theta}_g(\mathbf{x})|\Psi\rangle = |\Psi\rangle \quad \forall \mathbf{x}, g$$

– **Generators → Gauss law, left and right E fields:**

$$G_a(\mathbf{x}) = \sum_{k=1\ldots d}\left(L_a(\mathbf{x},k) - R_a\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right) - Q_a(\mathbf{x})$$

$$G_a(\mathbf{x})|\Psi\rangle = 0 \quad [G_a(\mathbf{x}), H] = 0 \quad \forall \mathbf{x}, a$$

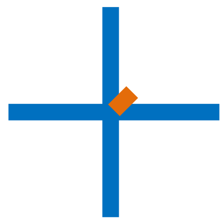
PEPS

- **Projected Entangled Pair States:** a particular tensor network construction, that
  - Allows to **encode and treat symmetries** in a very natural way.
  - Has, by construction, a **bipartite entanglement area law**, and therefore is suitable for describing “physically relevant” states.
  - Offers new approaches for the **study of phase diagrams and other properties of many body systems**.

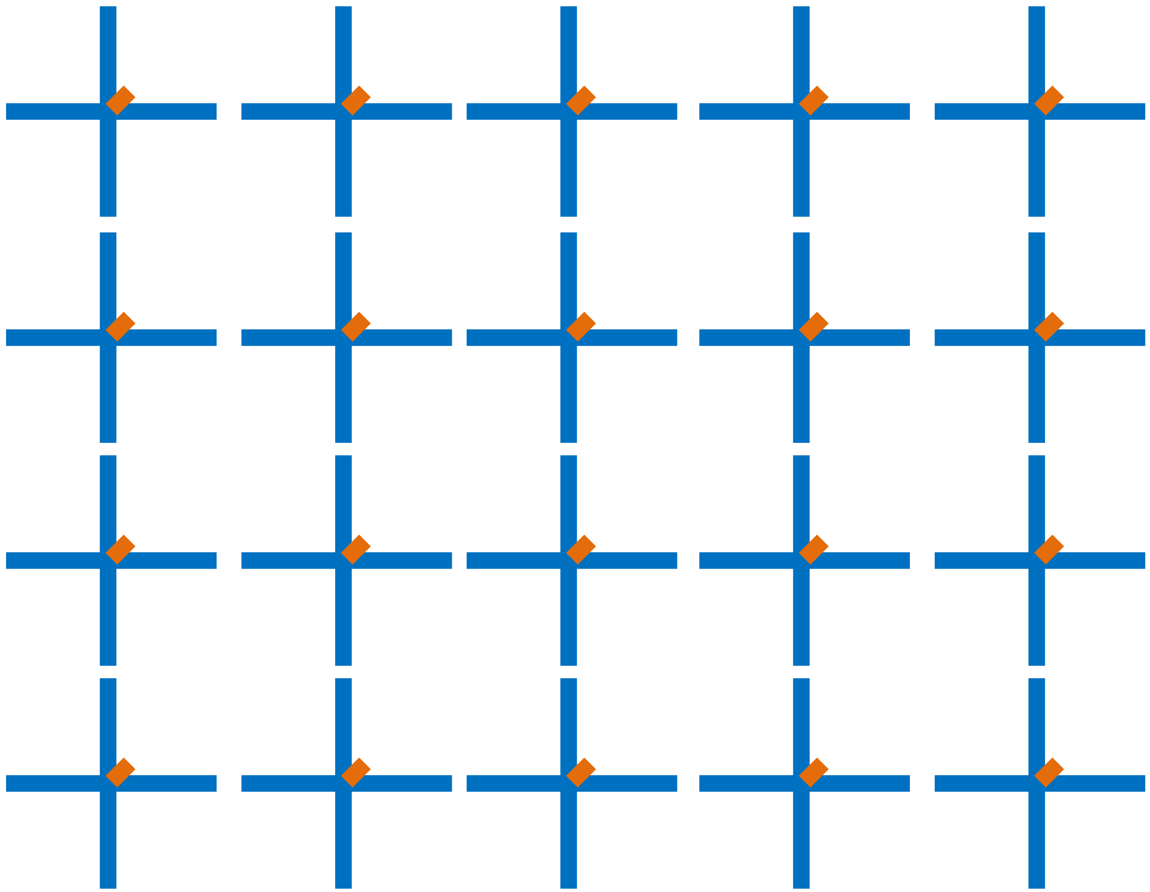
- In 1 space dimension – **MPS (Matrix Product States)**

PEPS

- Constructed out of local ingredients that include **physical** and **auxiliary** degrees of freedom.

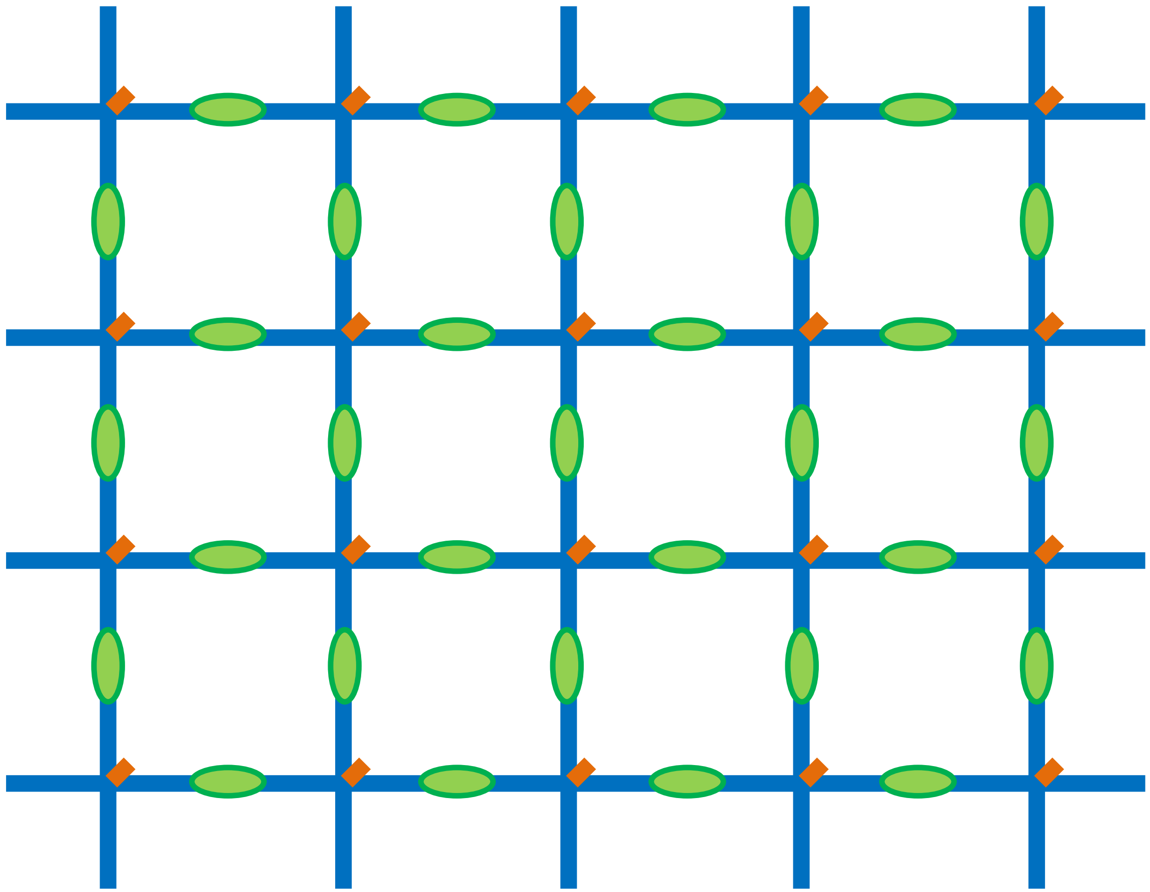


$$A(\mathbf{x})\,|\Omega\rangle$$



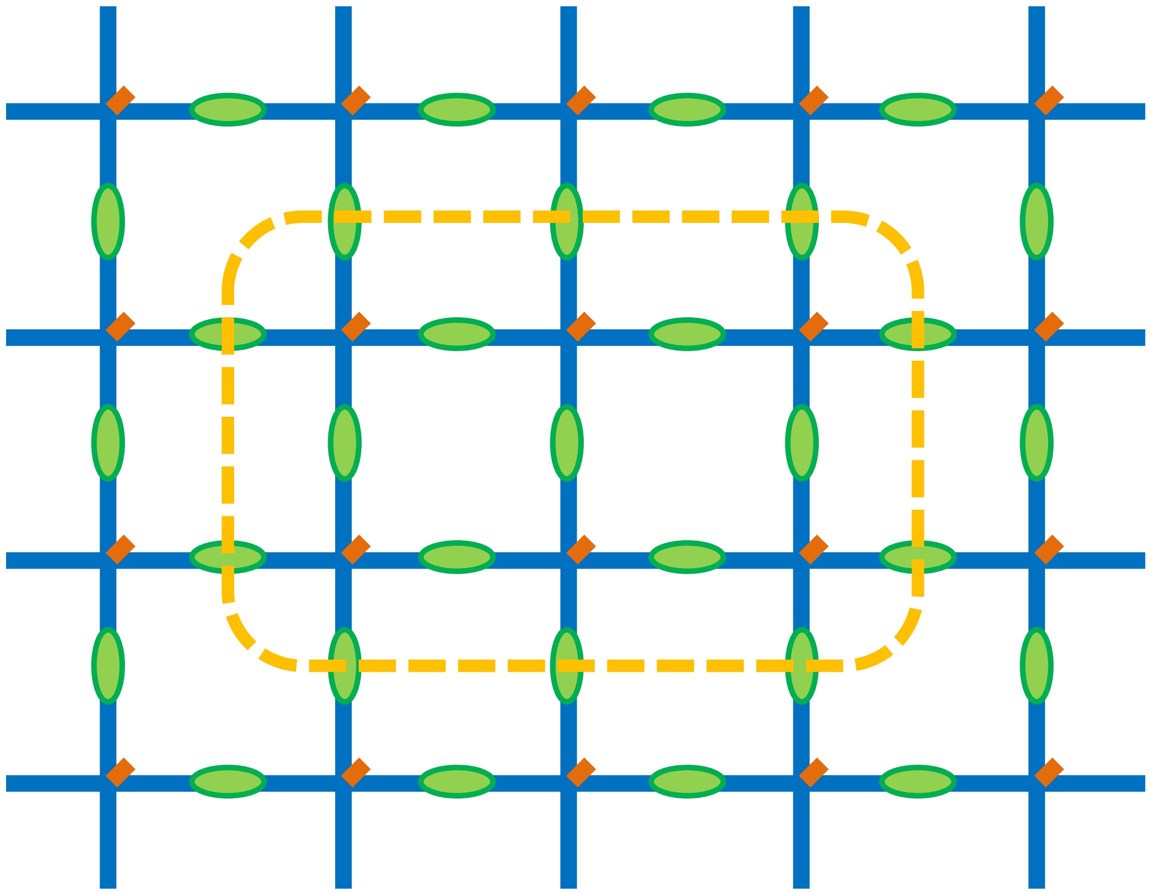
$$\prod_{\mathbf{x}} A(\mathbf{x})\,|\Omega\rangle$$

- A **physical** only state is obtained out of projecting pairs of **auxiliary** degrees of freedom, on the two sides of a link, onto maximally entangled states.

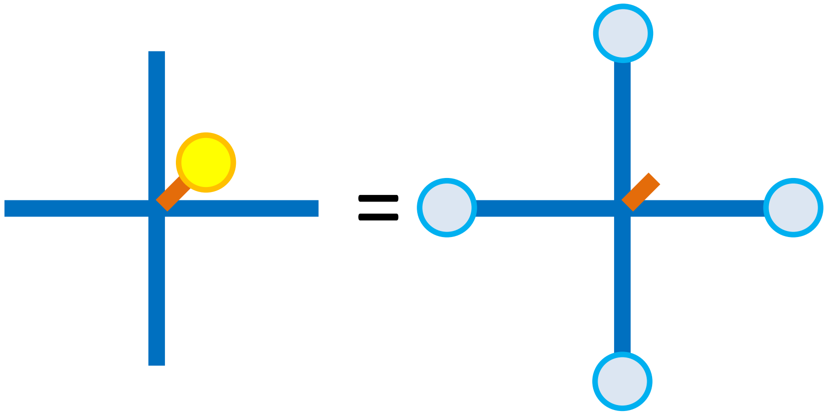


$$|\psi_0\rangle = \langle\Omega_v| \prod_{\ell} \omega_{\ell} \prod_{\mathbf{x}} A\left(\mathbf{x}\right)|\Omega\rangle$$

- An entanglement area law is satisfied by construction.

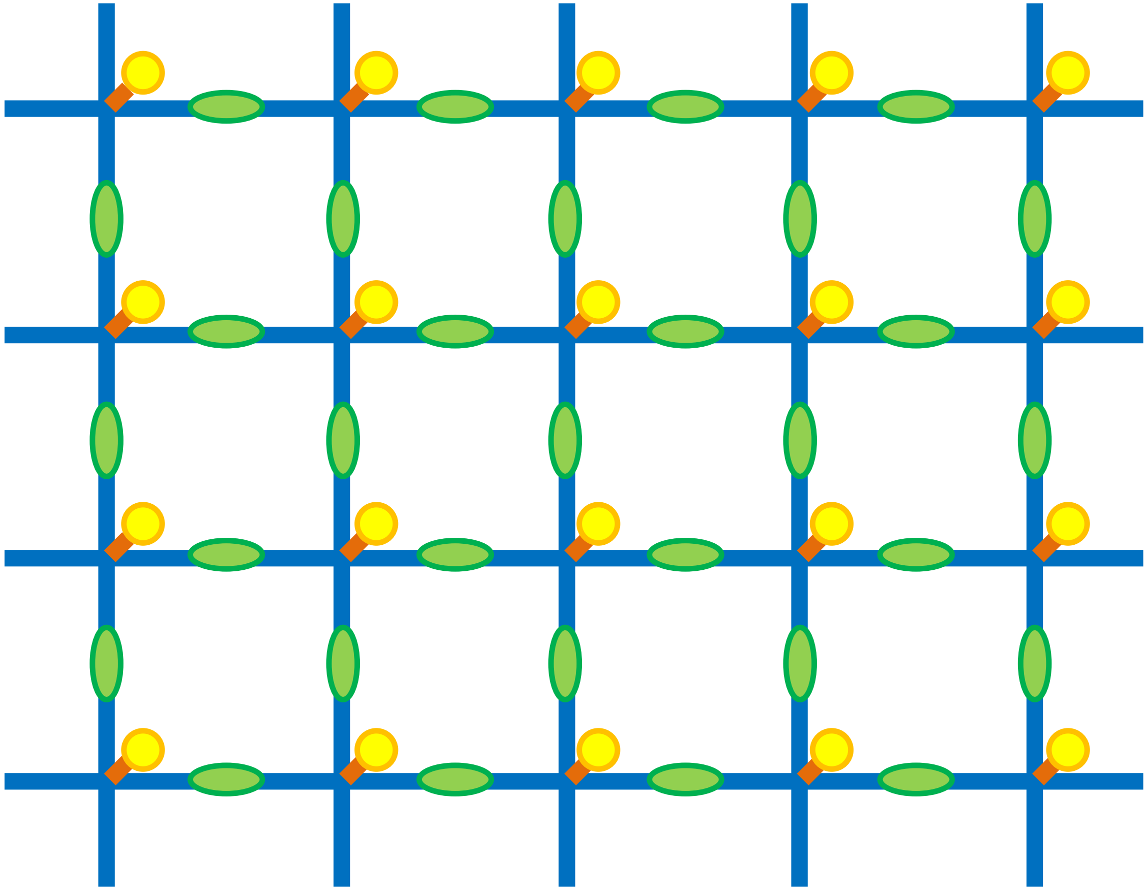


- Demanding global symmetry:
  - Acting with a group transformation on the **physical** degrees of freedom is equivalent to acting on the **auxiliary** ones.

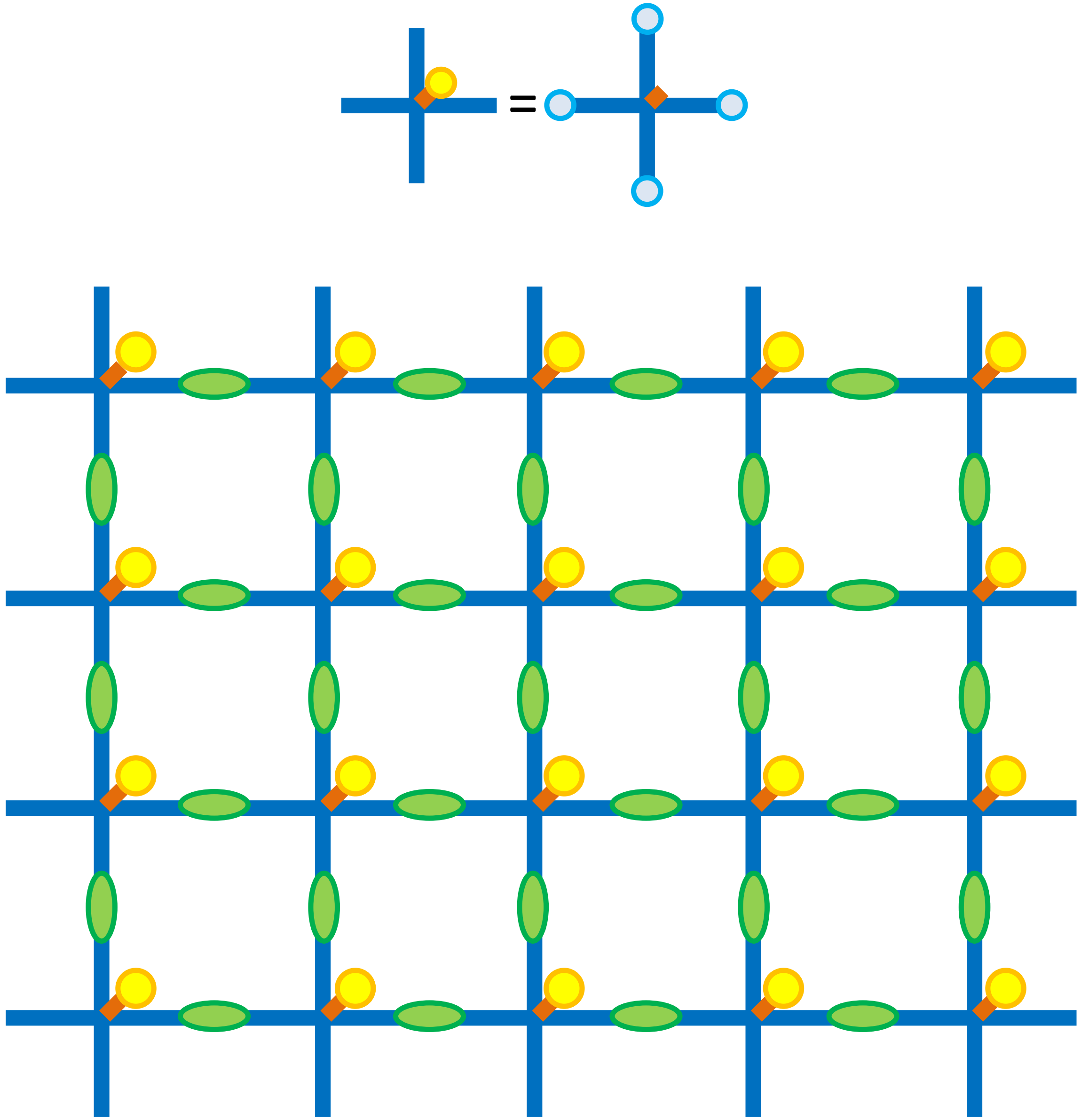


  - Projectors are invariant under group actions from both sides.

Global Transformation: $e^{i\Lambda\sum_{\mathbf{x}}Q(\mathbf{x})}|\psi_0\rangle$



$$|\psi_0\rangle = \langle\Omega_v|\prod_{\ell}\omega_\ell\prod_{\mathbf{x}}A(\mathbf{x})|\Omega\rangle$$



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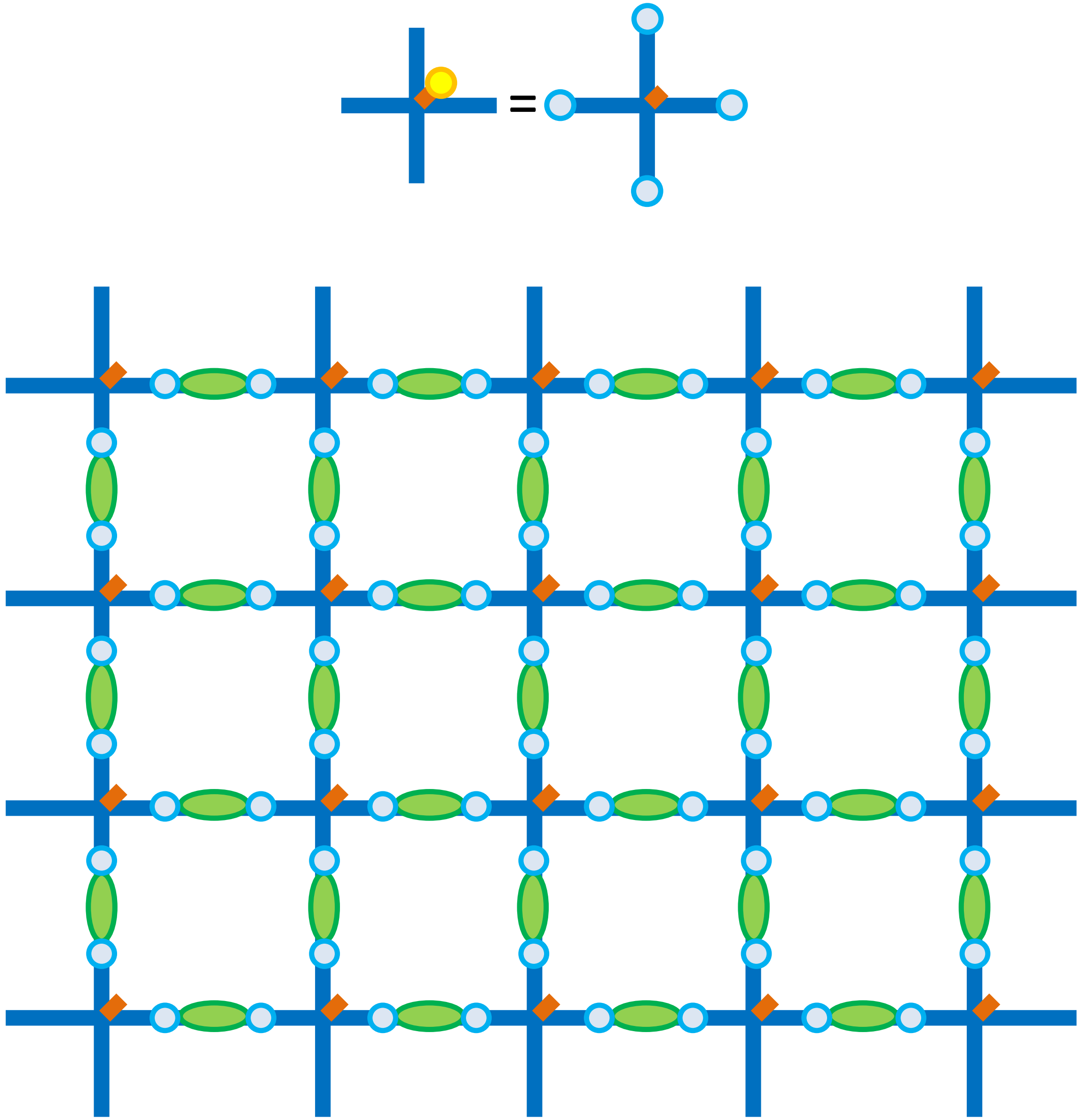

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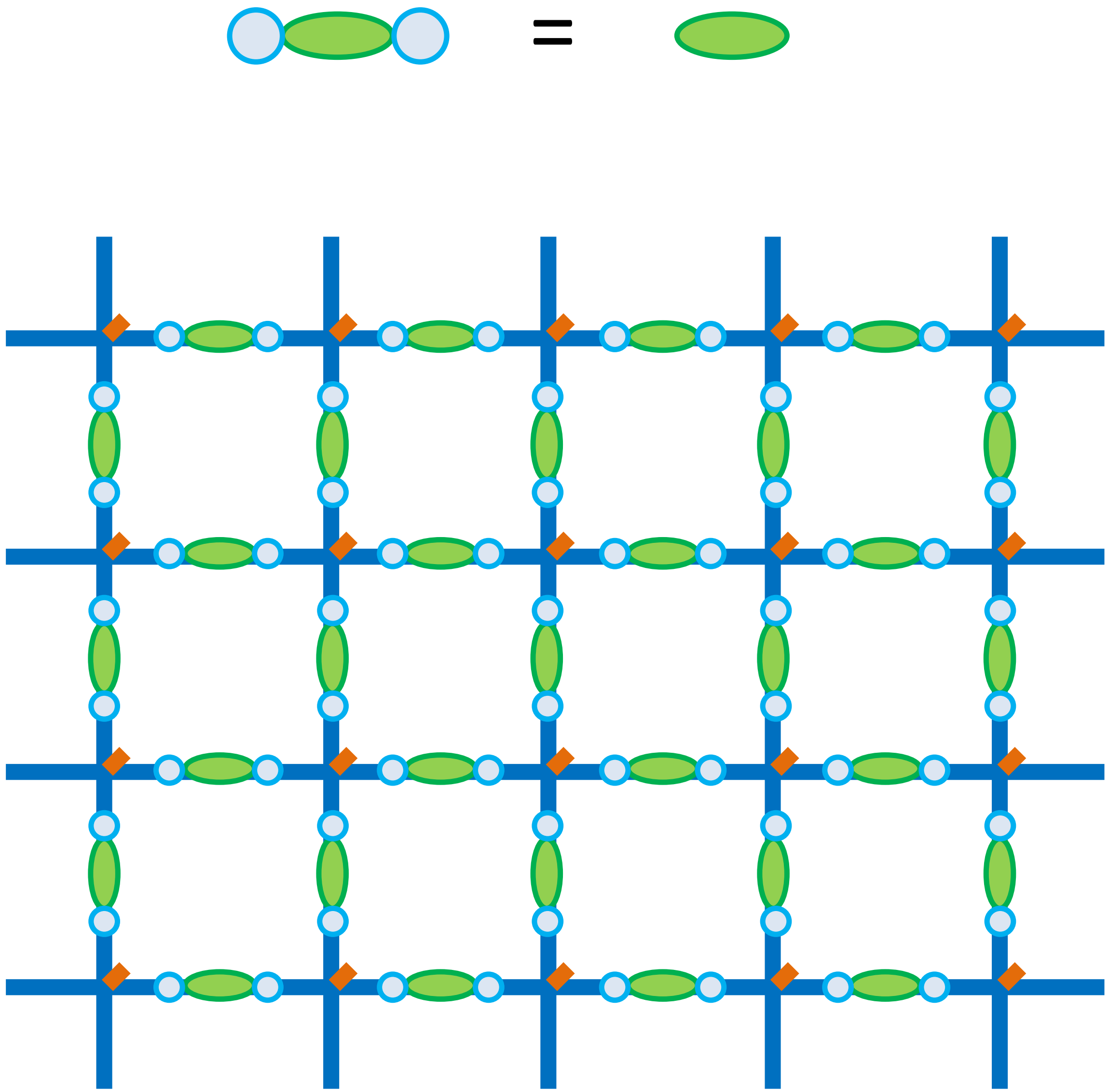

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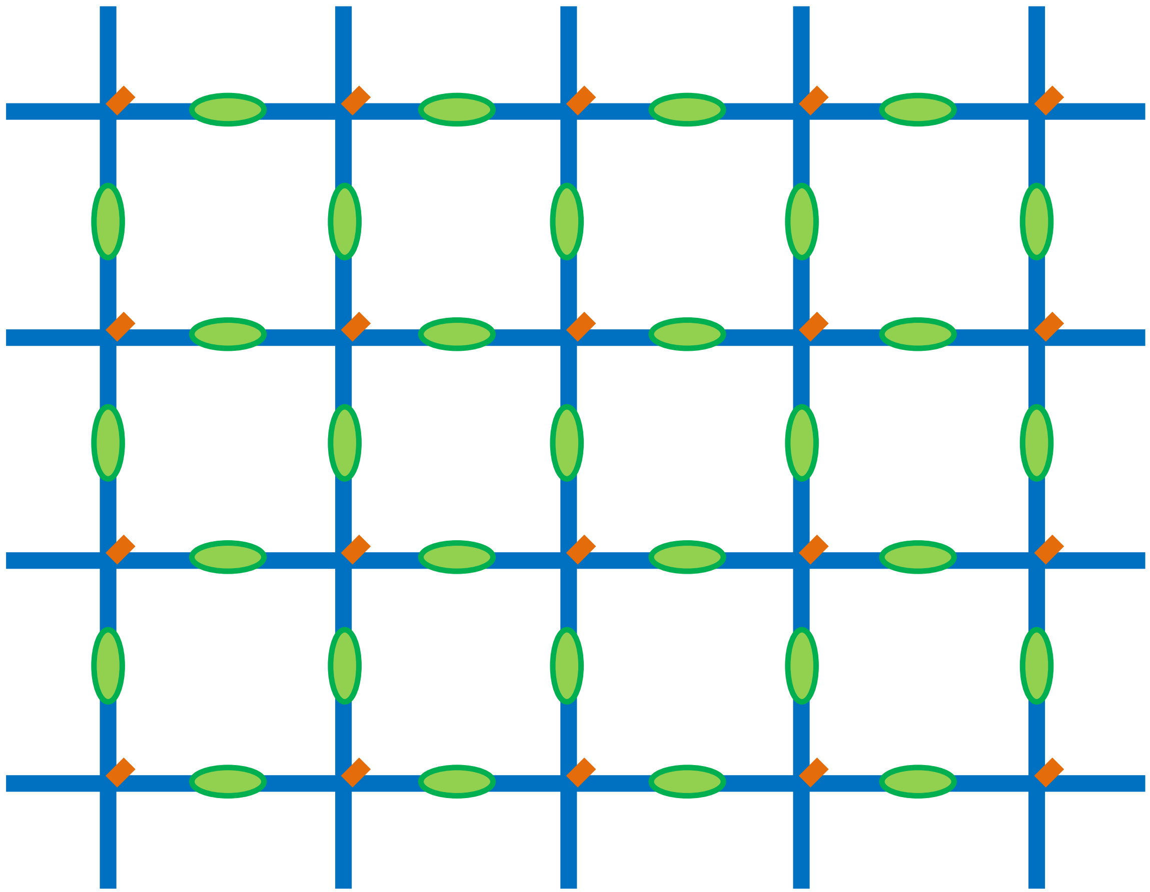
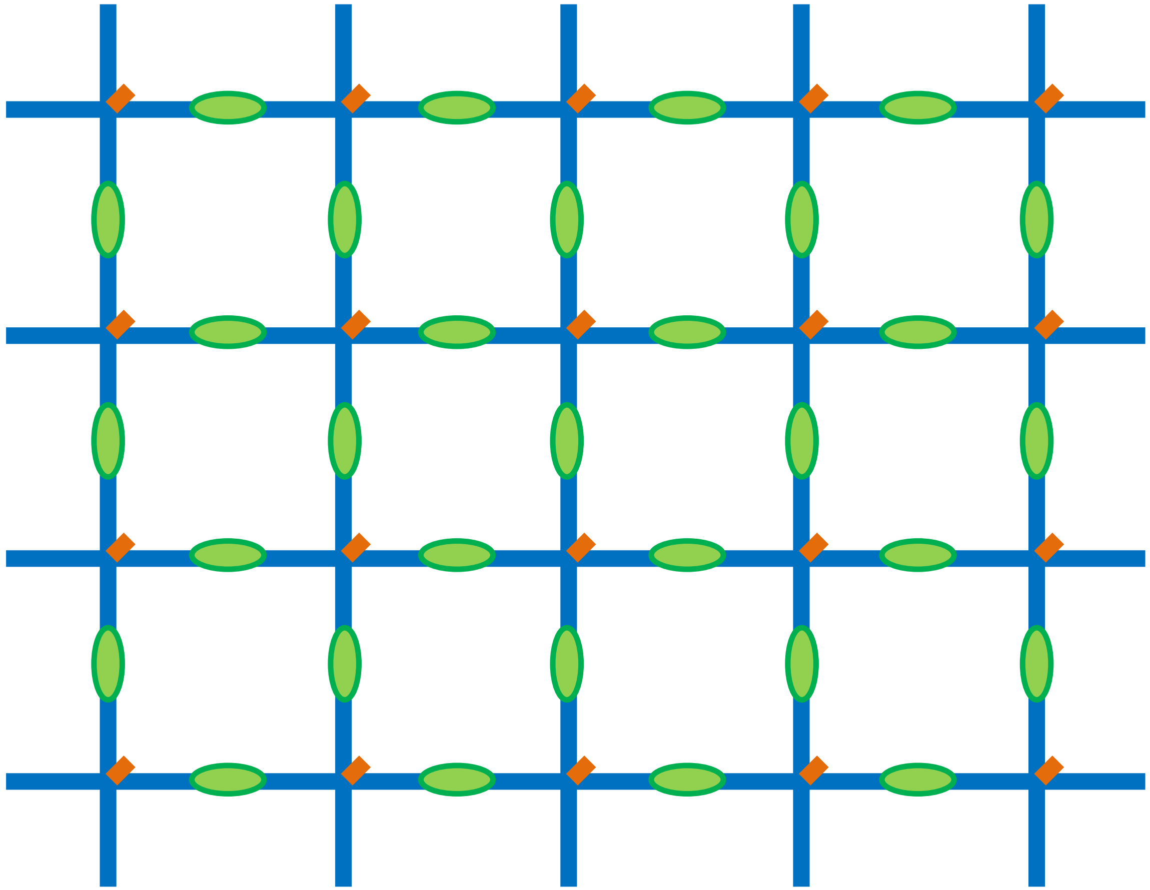


$$|\psi_0\rangle = \langle \Omega_v | \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A(\mathbf{x}) |\Omega\rangle$$

Global Symmetry: $e^{i\Lambda\sum_{\mathbf{x}}Q(\mathbf{x})}|\psi_0\rangle = |\psi_0\rangle$



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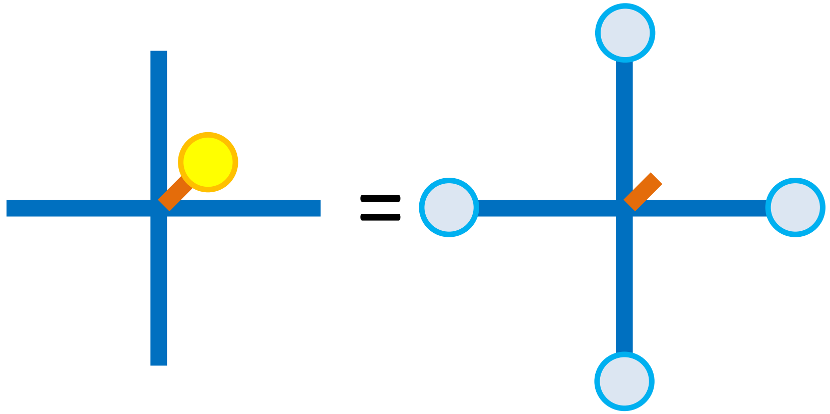
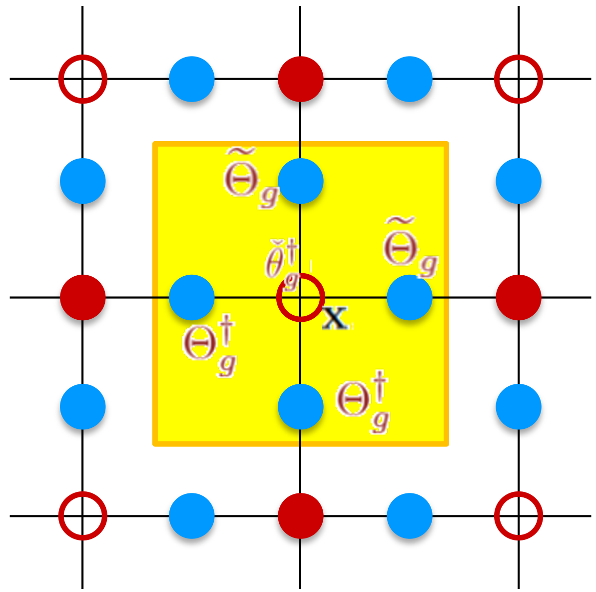
Virtual vs. Physical Gauge Invariance

Virtual - PEPS



Physical charge, but auxiliary electric fields: local symmetry exists, but it is auxiliary/virtual. The physical symmetry is global, after the bonds projection.

Physical – LGT states



$$\hat{\Theta}_g\left(\mathbf{x}\right) = \prod_{k=1\ldots d}\left(\widetilde{\Theta}_g\left(\mathbf{x},k\right)\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\check{\theta}_g^\dagger\left(\mathbf{x}\right)$$

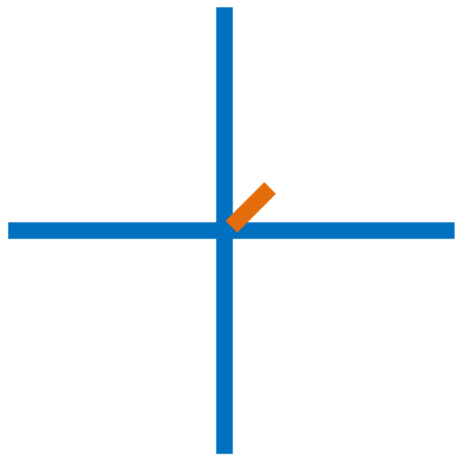
$$\hat{\Theta}_g\left(\mathbf{x}\right)\left|\Psi\right\rangle = \left|\Psi\right\rangle \quad \forall\mathbf{x},g$$

Fundamental analogy between PEPS and LGTs
- making PEPS a suitable ansatz

Gauging the PEPS: minimal coupling of a state

- Lift the virtual symmetry to be physical:
  Lift the global symmetry to a local one.

$$\left|\psi_0\right\rangle = \left\langle\Omega_v\right| \prod_{\ell} \omega_{\ell} \prod_{\mathbf{x}} A\left(\mathbf{x}\right) \left|\Omega\right\rangle$$



E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

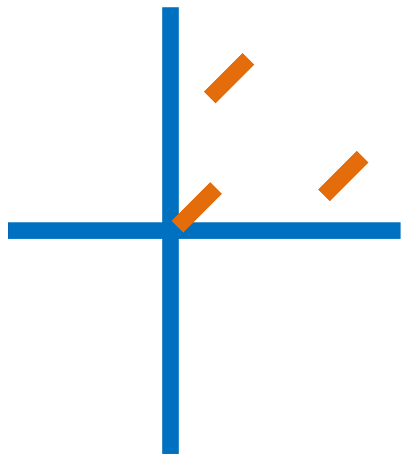
Gauging the PEPS: minimal coupling of a state

- Lift the virtual symmetry to be physical:
  The global to local.
- Step 1: Introduce gauge field Hilbert spaces on the links. Add (by a tensor product) the gauge field singlet states:

$$\left|\psi_0\right\rangle = \left\langle\Omega_v\right| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A\left(\mathbf{x}\right) \left|\Omega\right\rangle$$

$$\downarrow$$

$$\left\langle\Omega_v\right| \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} A\left(\mathbf{x}\right) \left|0\right\rangle_{\mathbf{x},1} \left|0\right\rangle_{\mathbf{x},2} \left|\Omega\right\rangle$$



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Gauging the PEPS: minimal coupling of a state

- Lift the **virtual** symmetry to be **physical**:
  The **global** to **local**.
- Step 2: Entangle the **auxiliary degrees** on the outgoing links with the **gauge fields**, by a unitary **gauging transformation** (map the auxiliary electric field information to the physical one).

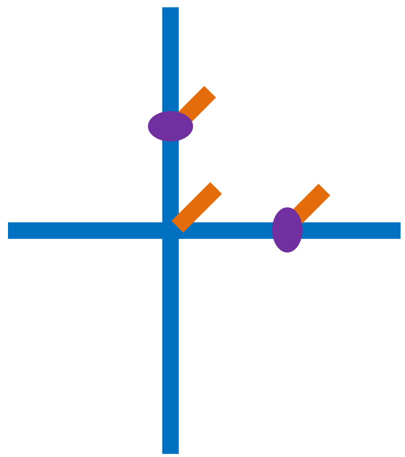
$$|\psi_0\rangle = \langle\Omega_v|\prod_{\ell}\omega_\ell\prod_{\mathbf{x}}A(\mathbf{x})|\Omega\rangle$$

$$\downarrow$$

$$\langle\Omega_v|\prod_{\ell}\omega_\ell\prod_{\mathbf{x}}A(\mathbf{x})\,|0\rangle_{\mathbf{x},1}\,|0\rangle_{\mathbf{x},2}|\Omega\rangle$$

$$\downarrow$$

$$|\psi\rangle = \langle\Omega_v|\prod_{\ell}\omega_\ell\prod_{\mathbf{x}}\mathcal{U}_G(\mathbf{x},1)\,\mathcal{U}_G(\mathbf{x},2)A(\mathbf{x})|0\rangle_{\mathbf{x},1}\,|0\rangle_{\mathbf{x},2}|\Omega\rangle$$



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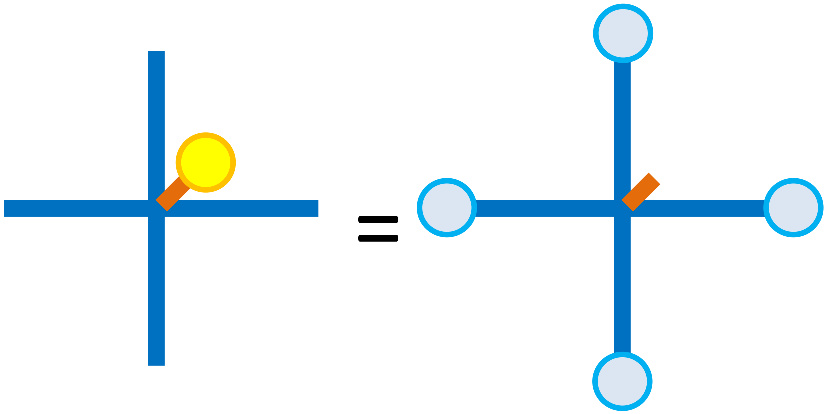
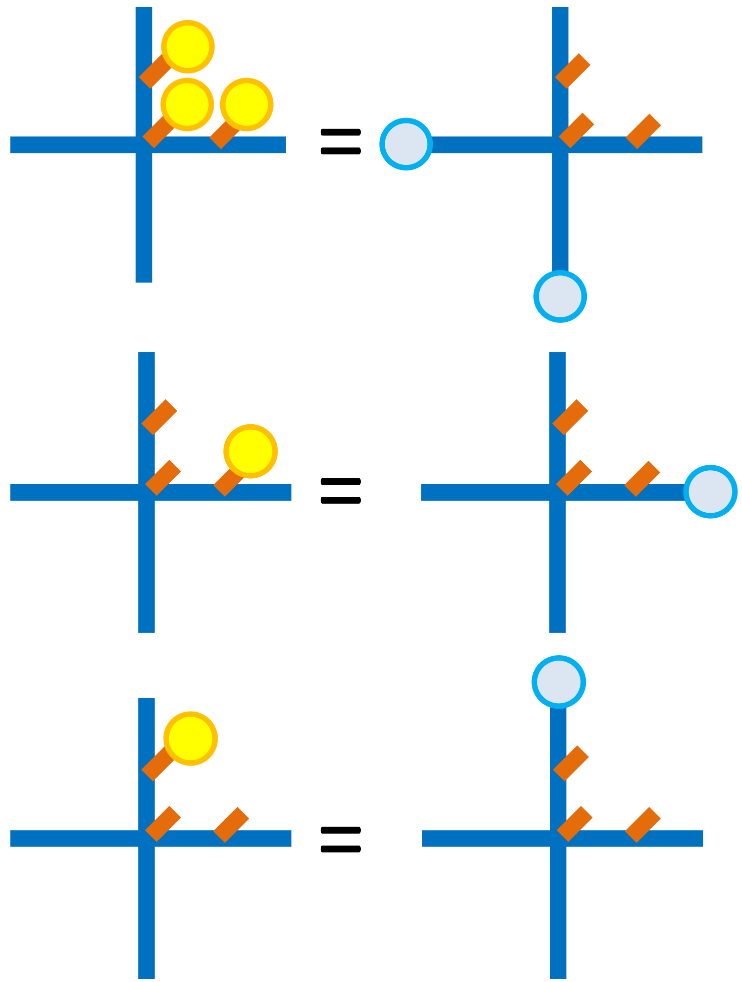
Gauging the PEPS: minimal coupling of a state

Building block of a globally invariant PEPS



Gauging Transformation

Building block of a globally invariant PEPS (gluing together the matter and gauge field tensors)



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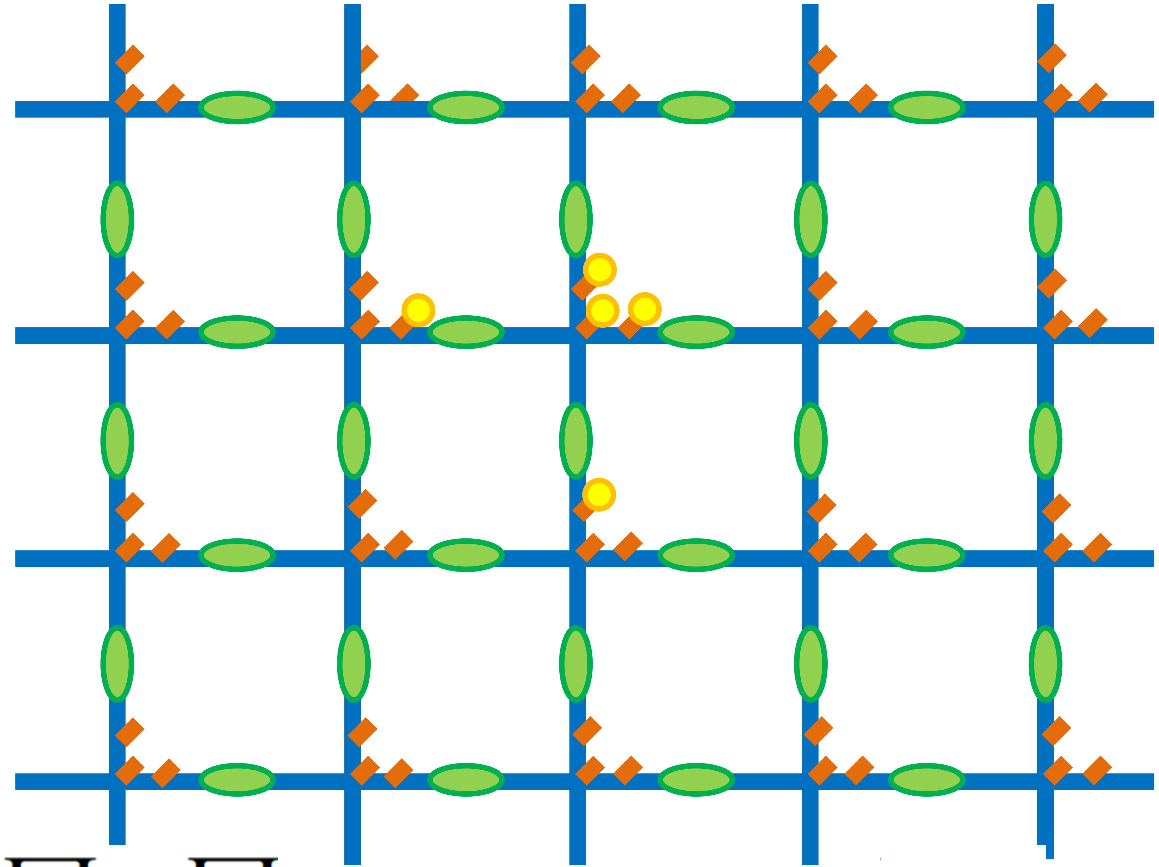
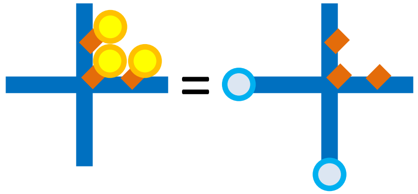
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

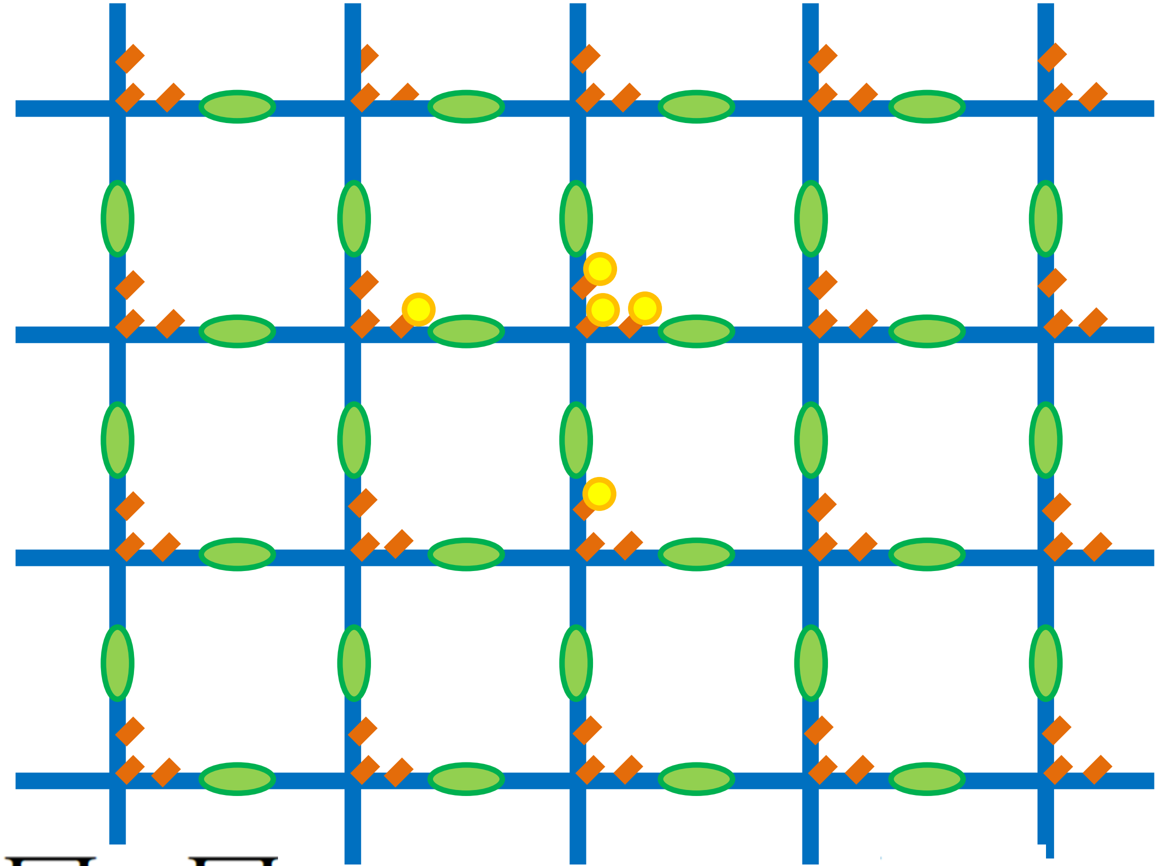
Local Transformation: $e^{i\Lambda\mathcal{G}(\mathbf{x}_0)}\left|\psi\right\rangle$



$$\left|\psi\right\rangle = \left\langle\Omega_v\right|\prod_{\ell}\omega_\ell\prod_{\mathbf{x}}\mathcal{U}_G(\mathbf{x},1)\,\mathcal{U}_G(\mathbf{x},2)A(\mathbf{x})\left|0\right\rangle_{\mathbf{x},1}\left|0\right\rangle_{\mathbf{x},2}\left|\Omega\right\rangle$$

E. Zohar and M. Burrello, New J. Phys. 18 043008 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)





$$|\psi\rangle = \langle \Omega_v | \prod_{\ell} \omega_\ell \prod_{\mathbf{x}} \mathcal{U}_G(\mathbf{x},1)\,\mathcal{U}_G(\mathbf{x},2) A(\mathbf{x}) |0\rangle_{\mathbf{x},1} |0\rangle_{\mathbf{x},2} |\Omega\rangle$$

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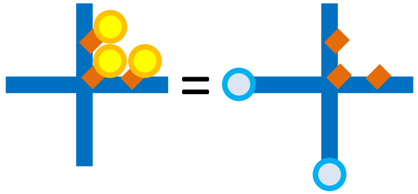

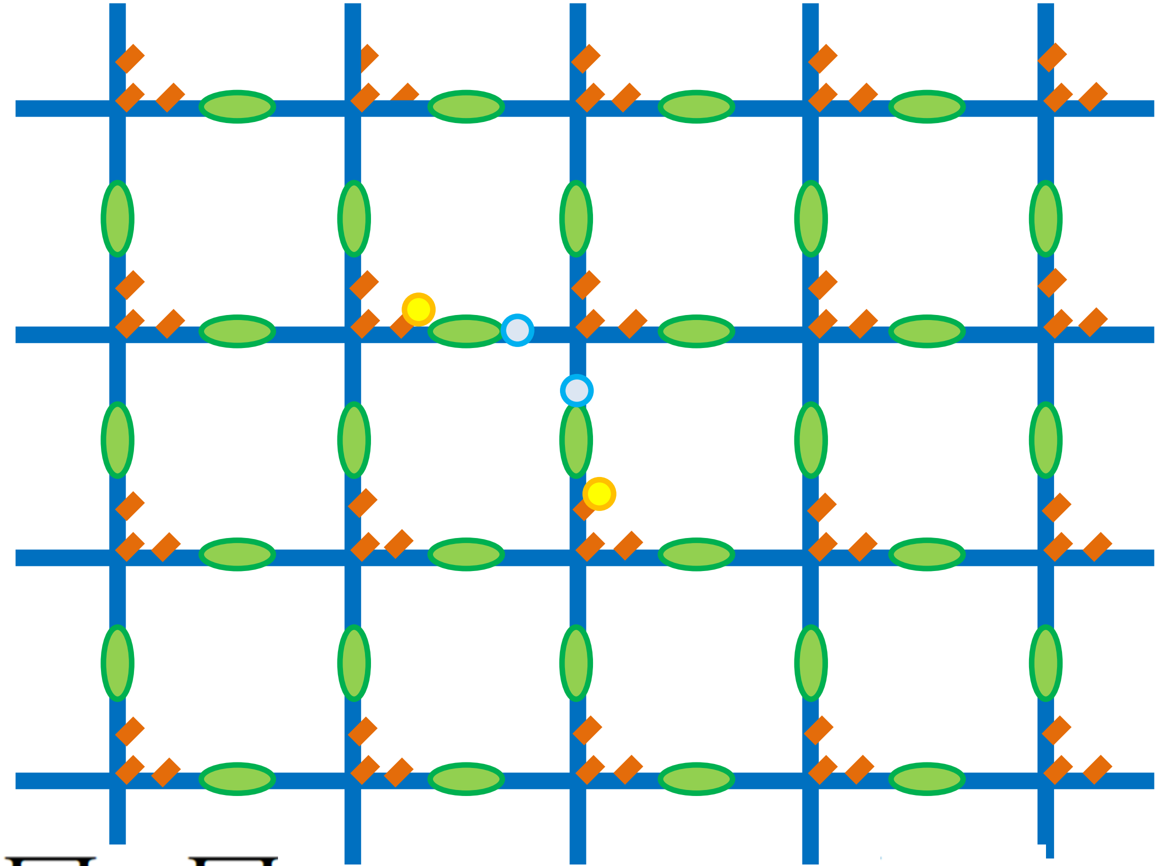

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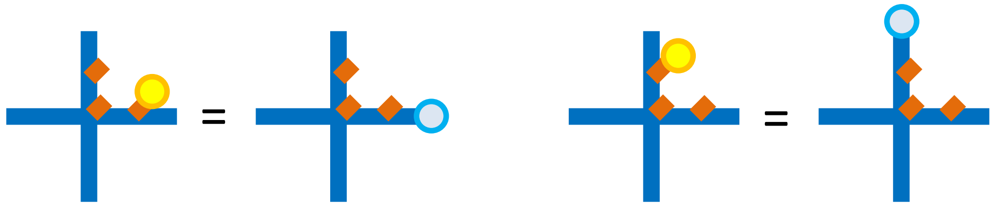

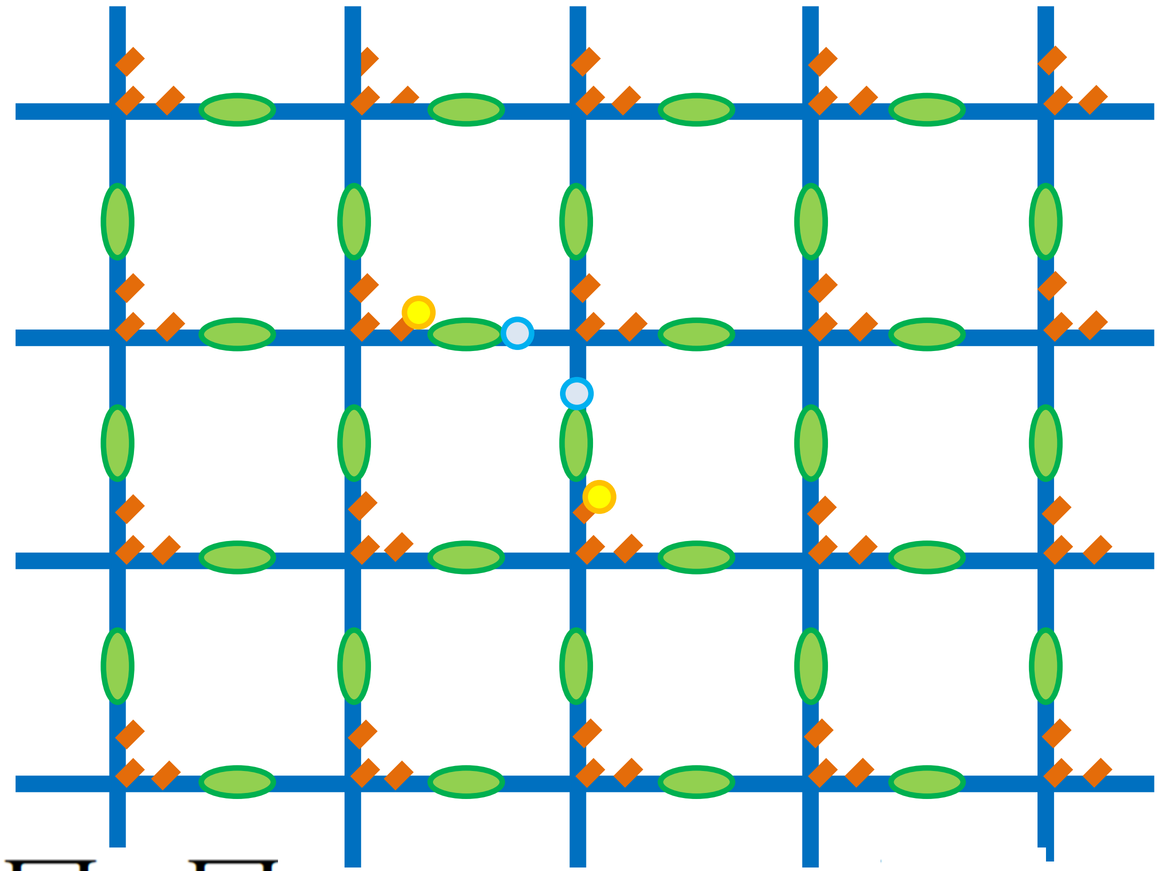

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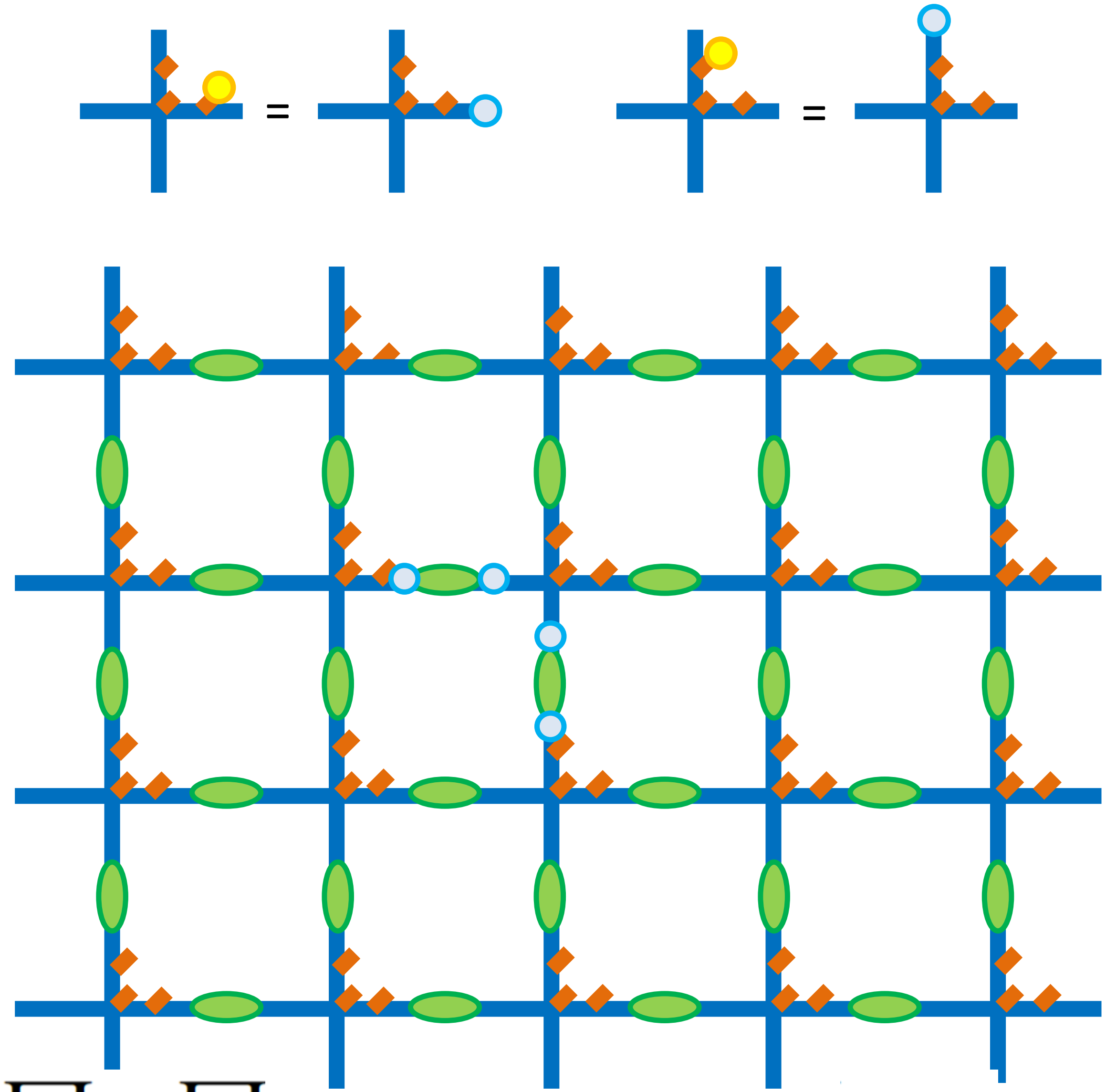

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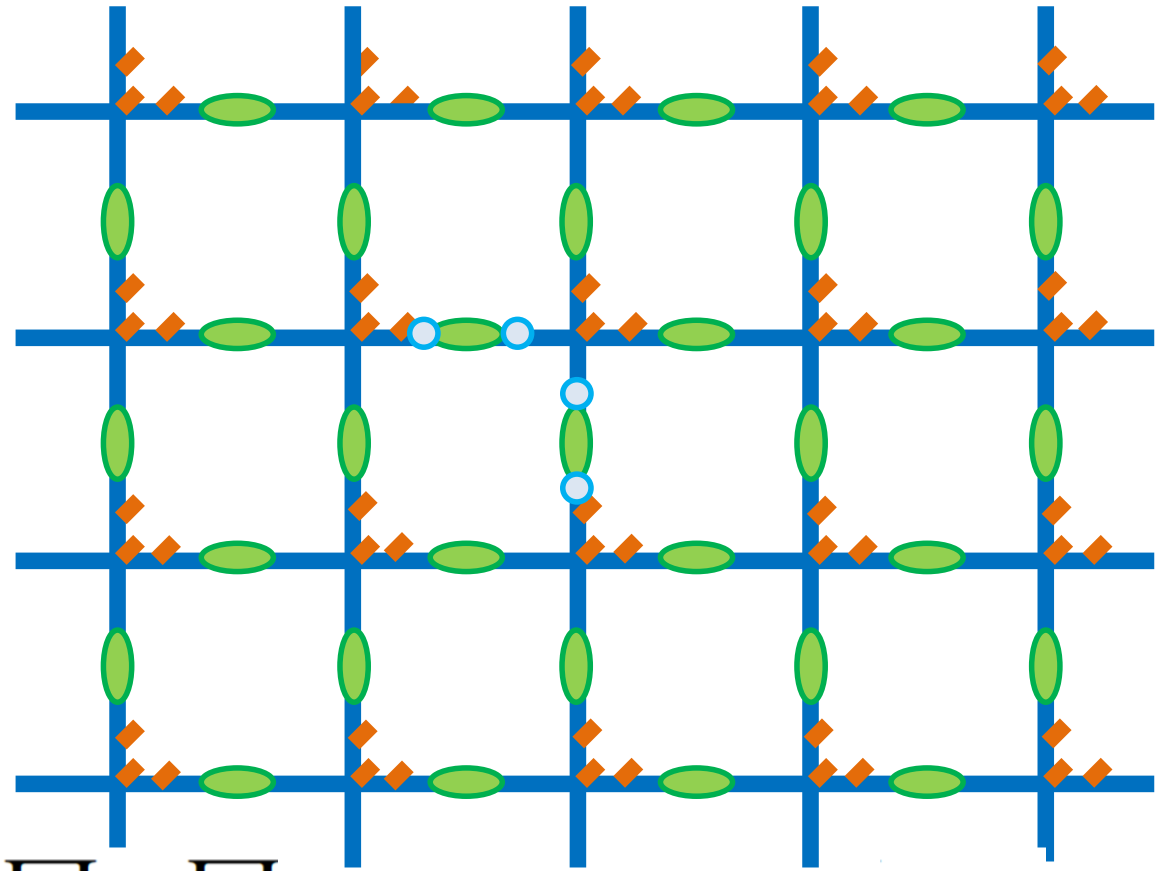

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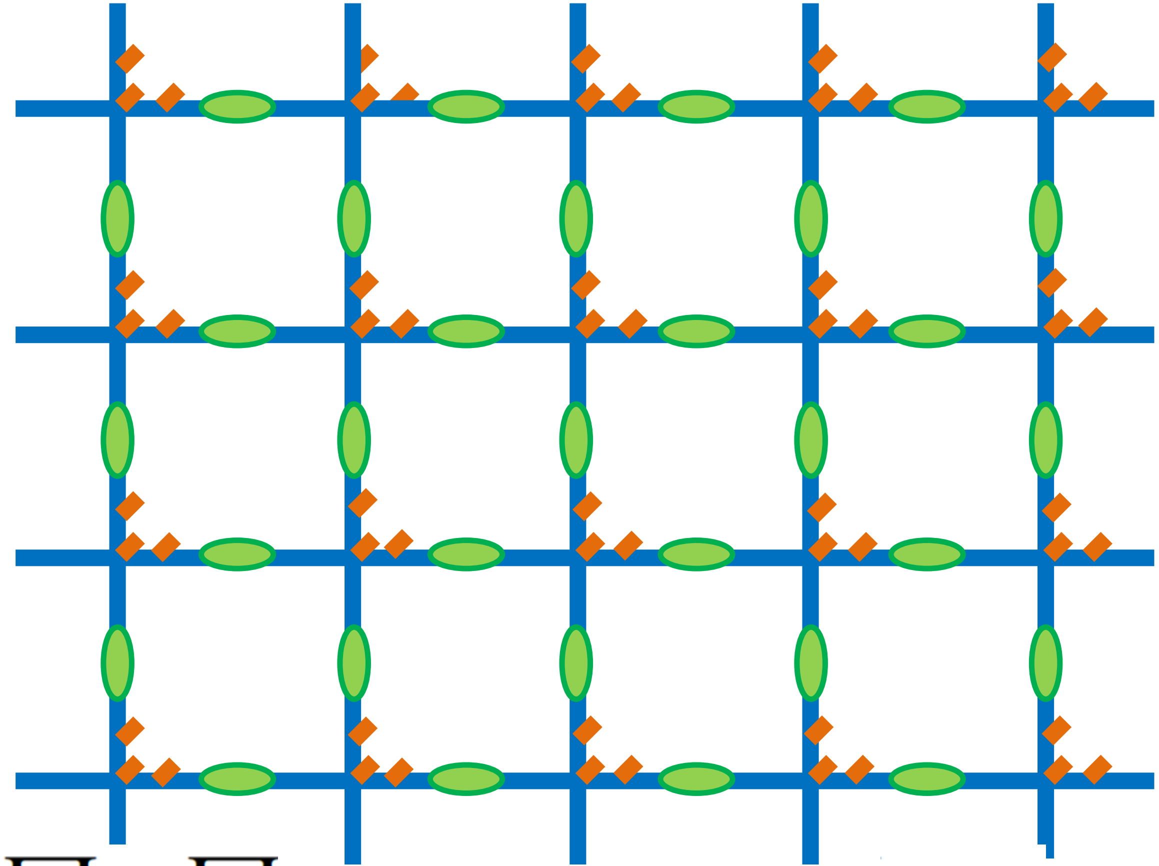
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Local Symmetry: $e^{i\Lambda\mathcal{G}(\mathbf{x}_0)}\left|\psi\right\rangle = \left|\psi\right\rangle$



$$\left|\psi\right\rangle = \left\langle\Omega_v\right|\prod_{\ell}\omega_{\ell}\prod_{\mathbf{x}}\mathcal{U}_G\left(\mathbf{x},1\right)\mathcal{U}_G\left(\mathbf{x},2\right)A\left(\mathbf{x}\right)\left|0\right\rangle_{\mathbf{x},1}\left|0\right\rangle_{\mathbf{x},2}\left|\Omega\right\rangle$$

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E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Locally gauge invariant fermionic PEPS

- We We wish to describe PEPS of fermionic matter coupled to dynamical gauge fields.

- Starting point – **Gaussian fermionic PEPS** with a global symmetry.
  - **Gaussian states** – ground states of quadratic Hamiltonians, completely described by their covariance matrix. Very easy to handle analytically with the use of the Gaussian formalism.
  - **Fermionic PEPS** – defined with fermionic creation operators acting on the Fock vacuum. Easy to parameterize if they are Gaussian (Kraus, Schuch, Verstraete, Cirac, PRA 2011)

E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)
E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)
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- **Start with these, then make the symmetry local and add the gauge field.**

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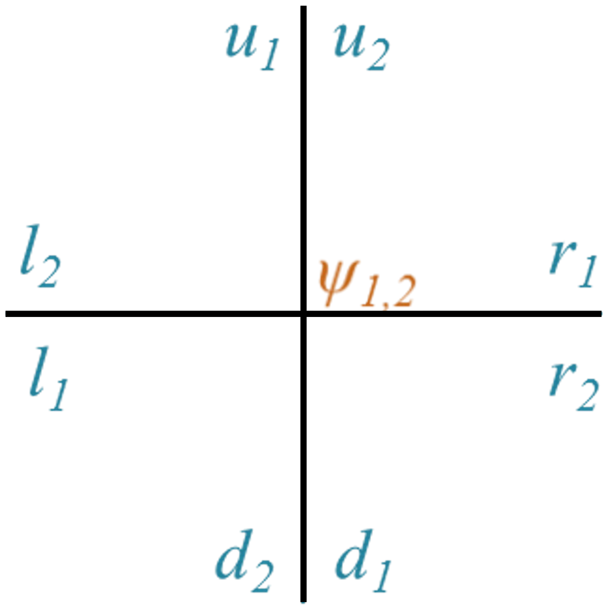
- **Start with these, then make the symmetry local and add the gauge field.** Similar to **minimal coupling**: **Gauge a free matter state → obtain an interacting matter-gauge field state** without introducing further parameters.

E. Zohar, M. Burrello, T.B. Wahl, and J.I. Cirac, Ann. Phys. 363, 385-439 (2015)
E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)
E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

The PEPS ingredients

- At each site –
  - Physical fermions
    - For example,
      - 1 For *U*(1)
      - 2 For *SU*(2), in the fundamental rep.
  - Virtual fermions on the legs (e.g. 2 per leg, 8 in total)
- The local **Gaussian state** involving both physical and virtual modes is created from the Fock vacuum with

$$A = \exp\left(\sum_{ij} T_{ij} \alpha_i^\dagger \alpha_j^\dagger\right)$$



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E. Zohar, T.B. Wahl, M. Burrello, and J.I. Cirac, Ann. Phys. 374, 84-137 (2016)

Magnetic Basis

- The physical Hilbert space: $\mathcal{H}_{\text{phys}} = \mathcal{H}_{\text{phys}}\left(\{q(\mathbf{x})\}\right) \subset \mathcal{H}_{\text{gauge}} \times \mathcal{H}_{\text{matter}}$

- Gauge field configuration states:

$$|\mathcal{G}\rangle = \bigotimes_{\mathbf{x},k} |g(\mathbf{x},k)\rangle$$

$$\mathcal{D}\mathcal{G} = \bigotimes_{\mathbf{x},k} dg(\mathbf{x},k)$$

- **General gauge invariant state**:

$$|\psi\rangle = \int \mathcal{D}\mathcal{G} |\mathcal{G}\rangle_{\text{Gauge}} |\psi(\mathcal{G})\rangle_{\text{Matter}}$$

Where $|\psi(\mathcal{G})\rangle$ represents matter coupled to an external (classical) gauge field $\mathcal{G}$.

- E.g. for *U*(1): $|\Phi\rangle = \bigotimes_{\mathbf{x},k} |\phi(\mathbf{x},k)\rangle$

$$\mathcal{D}\Phi = \bigotimes_{\mathbf{x},k} d\phi(\mathbf{x},k)$$

$$|\psi\rangle = \int \mathcal{D}\Phi |\Phi\rangle_{\text{Gauge}} |\psi(\Phi)\rangle_{\text{Matter}}$$

Monte Carlo with gauged Gaussian fPEPS

- Expressing our states in the **magnetic basis** that allows us to perform **efficient Monte-Carlo calculations**

$$\left|\Psi\right\rangle = \int \mathcal{D}\mathcal{G} \left|\mathcal{G}\right\rangle \left|\psi\left(\mathcal{G}\right)\right\rangle$$

- $\left|\mathcal{G}\right\rangle$ is a **fixed configuration state of the gauge field on the links**.

$$\left|\mathcal{G}\right\rangle \equiv \bigotimes_{\mathbf{x},k} \left|g\left(\mathbf{x},k\right)\right\rangle \qquad \mathcal{D}\mathcal{G} = \prod_{\mathbf{x},k} dg\left(\mathbf{x},k\right)$$

$$\left\langle\mathcal{G}'|\mathcal{G}\right\rangle = \delta\left(\mathcal{G}',\mathcal{G}\right)$$

- $\left|\psi\left(\mathcal{G}\right)\right\rangle$ is a **fermionic Gaussian state**, representing **fermions coupled to a static, background gauge field** $\mathcal{G}$.

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Monte Carlo with gauged Gaussian fPEPS

- Expressing our states in the **magnetic basis** that allows us to perform **efficient Monte-Carlo calculations**

$$|\psi\rangle = \int \mathcal{D}\Phi |\Phi\rangle_{\text{Gauge}} |\psi(\Phi)\rangle_{\text{Matter}}$$

- $|\Phi\rangle$ is a **fixed configuration state of the gauge field on the links**.

$$|\Phi\rangle = \bigotimes_{\mathbf{x},k} |\phi(\mathbf{x},k)\rangle \qquad \mathcal{D}\Phi = \bigotimes_{\mathbf{x},k} d\phi(\mathbf{x},k)$$

$$\langle\Phi'|\Phi\rangle = \prod_{\mathbf{x},k} \delta(\phi(\mathbf{x},k) - \phi'(\mathbf{x},k))$$

- $|\psi(\Phi)\rangle$ is a **fermionic Gaussian state**, representing **fermions coupled to a static, background gauge field** Φ.

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Monte Carlo with gauged Gaussian fPEPS

- Expressing our states in the **magnetic basis** that allows us to perform **efficient Monte-Carlo calculations**

$$|\psi\rangle = \int \mathcal{D}\Phi |\Phi\rangle_{\text{Gauge}} |\psi(\Phi)\rangle_{\text{Matter}}$$

- Gauge field configuration states are eigenstates of functions of group element operators:

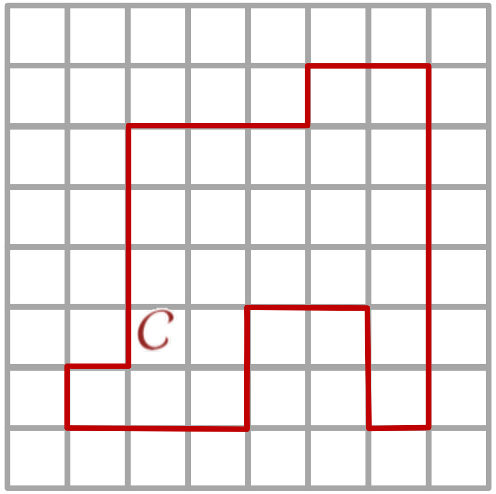
$$U|\phi\rangle = e^{i\phi}|\phi\rangle \qquad |\Phi\rangle = \bigotimes_{\mathbf{x},k} |\phi(\mathbf{x},k)\rangle$$

$$F(\{U(\mathbf{x},k)\})|\Phi\rangle = F\left(\left\{e^{i\phi(\mathbf{x},k)}\right\}\right)|\Phi\rangle$$

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Monte Carlo with gauged Gaussian fPEPS

- Wilson Loops: $$W(C) = \prod_{\ell \in C} U(\ell)$$



$$W(C)|\Phi\rangle = e^{i \sum_{\ell \in C} \phi(\ell)} |\Phi\rangle$$

Detecting confinement of static charges (Wilson, 1974)

- Expectation value for $|\psi\rangle = \int \mathcal{D}\Phi |\Phi\rangle_{\text{Gauge}} |\psi(\Phi)\rangle_{\text{Matter}}$:

$$\langle W(C)\rangle = \langle\psi| W(C) |\psi\rangle / Z = \int \mathcal{D}\Phi\, p(\Phi)\, e^{i \sum_{\ell \in C} \phi(\ell)}$$

$$\langle\psi|\psi\rangle = \int \mathcal{D}\Phi \langle\psi(\Phi)|\psi(\Phi)\rangle \equiv Z$$

$$p(\Phi) = \langle\psi(\Phi)|\psi(\Phi)\rangle / Z$$

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Monte Carlo with gauged Gaussian fPEPS

- Wilson Loops: $$W(C) = \mathrm{Tr}\left(\prod_{\{\mathbf{x},k\}\in C} U(\mathbf{x},k)\right)$$

  - exp. value for $|\Psi\rangle = \int \mathcal{DG}\,|\mathcal{G}\rangle\,|\psi(\mathcal{G})\rangle$:

$$\langle W\rangle = \frac{\int \mathcal{DG}\,\mathrm{Tr}\left(\prod_{\{\boldsymbol{x},k\}\in C} D(g(\boldsymbol{x},k))\right)\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}{\int \mathcal{DG}\,\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}$$

- The function

$$p(\mathcal{G}) = \frac{\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}{\int \mathcal{DG}'\,\langle\psi(\mathcal{G}')|\psi(\mathcal{G}')\rangle}$$

is a probability density.

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Monte Carlo with gauged Gaussian fPEPS

- Wilson Loops: $$W(C) = \mathrm{Tr}\left(\prod_{\{\mathbf{x},k\}\in C} U(\mathbf{x},k)\right)$$

  - exp. value for $|\Psi\rangle = \int \mathcal{D}\mathcal{G}\,|\mathcal{G}\rangle\,|\psi(\mathcal{G})\rangle$:

$$\langle W\rangle = \frac{\int \mathcal{D}\mathcal{G}\,\mathrm{Tr}\left(\prod_{\{\boldsymbol{x},k\}\in C} D\left(g\left(\boldsymbol{x},k\right)\right)\right)\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}{\int \mathcal{D}\mathcal{G}\,\langle\psi(\mathcal{G})|\psi(\mathcal{G})\rangle}$$

- **The fermionic calculation is easy, through the gaussian formalism: very efficient**
- **No sign problem: the probability density is obtained from a norm of a state, and thus is real and positive.**

→ **Monte Carlo integration!**

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

Monte Carlo with gauged Gaussian fPEPS

- The method is extendable to further physical observables, always involving the **probability density function**

$$p\left(\mathcal{G}\right)=\frac{\left\langle \psi\left(\mathcal{G}\right)|\psi\left(\mathcal{G}\right)\right\rangle}{\int\mathcal{D}\mathcal{G}'\left\langle \psi\left(\mathcal{G}'\right)|\psi\left(\mathcal{G}'\right)\right\rangle}$$

and possibly elements of the **covariance matrix of the Gaussian state** $\left|\psi\left(\mathcal{G}\right)\right\rangle$, which could be calculated very efficiently.

 – For example, mesonic operators $M\left(\mathbf{x},\mathbf{y},C\right)=\psi^{\dagger}\left(\mathbf{x}\right)\prod_{\ell\in C}U\left(\ell\right)\psi\left(\mathbf{y}\right)$

$$\left\langle M\left(\mathbf{x},\mathbf{y},C\right)\right\rangle=\int\mathcal{D}\Phi\, p\left(\Phi\right)e^{i\sum_{\ell\in C}\phi(\ell)}\frac{\left\langle \psi\left(\Phi\right)\right|\psi^{\dagger}\left(\mathbf{x}\right)\psi\left(\mathbf{y}\right)\left|\psi\left(\Phi\right)\right\rangle}{\left\langle \psi\left(\Phi\right)|\psi\left(\Phi\right)\right\rangle}$$

(given for U(1) for simplicity).

- **It is possible to contract gauged Gaussian fPEPS beyond 1+1d, and without the sign problem of conventional LGT methods (it is not a Euclidean path integral).**

E. Zohar, J.I. Cirac, Phys. Rev. D 97, 034510 (2018)

First benchmark: pure gauge Z_3 PEPS in 2+1d

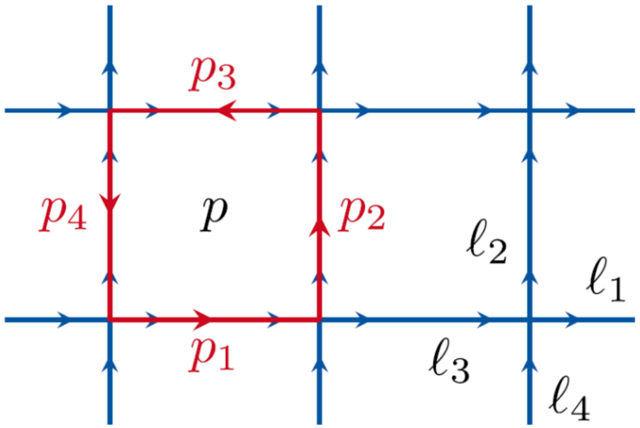
$$P_\ell^N = Q_\ell^N = 1 \qquad P_\ell^\dagger P_\ell = Q_\ell^\dagger Q_\ell = 1$$

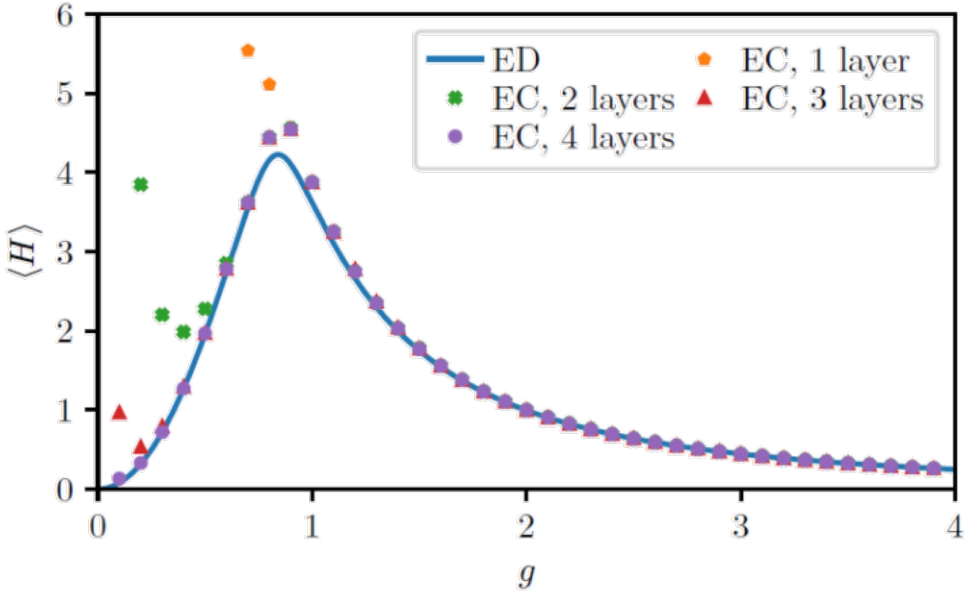
$$P_\ell^\dagger Q_\ell P_\ell = e^{i\delta Q_\ell} \qquad \delta = \frac{2\pi}{N}$$

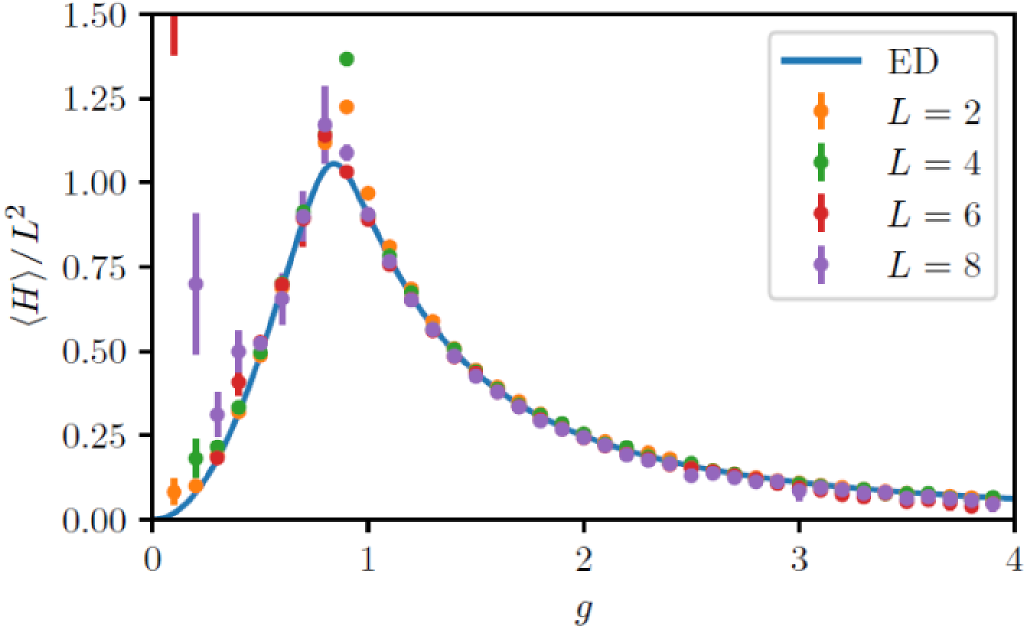
$$H = H_E + H_B$$
$$= \frac{g^2}{2}\sum_\ell [2 - (P_\ell + P_\ell^\dagger)]$$
$$+ \frac{1}{2g^2}\sum_p [2 - (Q_{p_1}^\dagger Q_{p_2}^\dagger Q_{p_3} Q_{p_4} + \text{H.c.})]$$

$$\Theta(\mathbf{x}) = P_{\mathbf{x},r} P_{\mathbf{x},u} P^\dagger_{\mathbf{x}-\hat{\mathbf{e}}_1,r} P^\dagger_{\mathbf{x}-\hat{\mathbf{e}}_2,u}$$

$$[\Theta(\mathbf{x}), H] = 0 \quad \forall \;\; \mathbf{x}$$





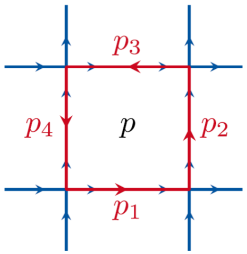


P. Emonts, M.C. Bañuls, J. I. Cirac, **E. Zohar**, Phys. Rev. D 102, 074501 (2020)

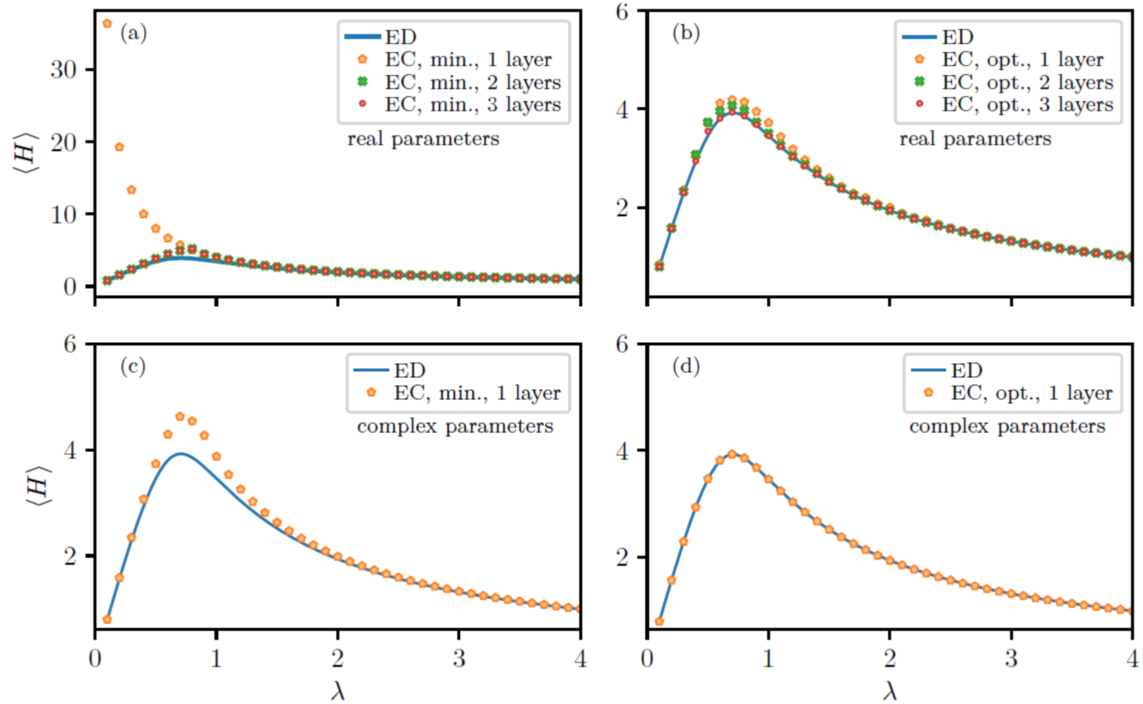
Improved Algorithm + Benchmark: pure gauge Z_2 PEPS in 2+1d

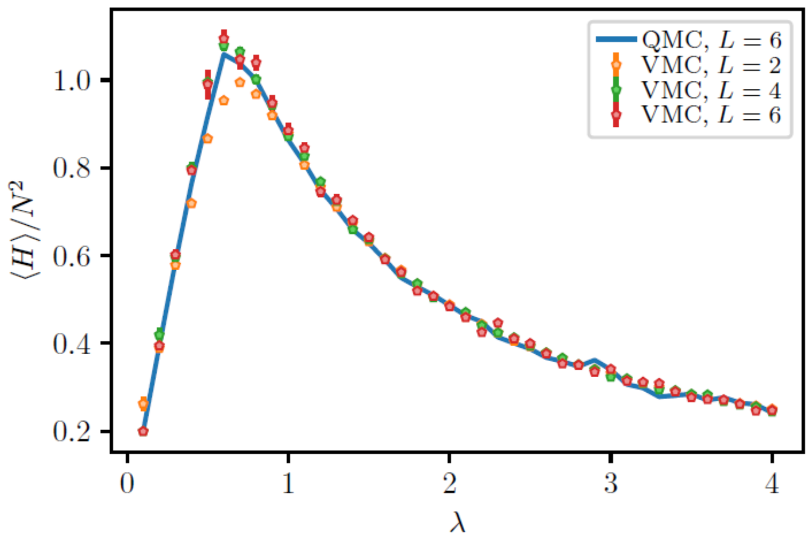
- Solved a numerical bottleneck (Pfaffians)
- Improved the ansatz states analytically

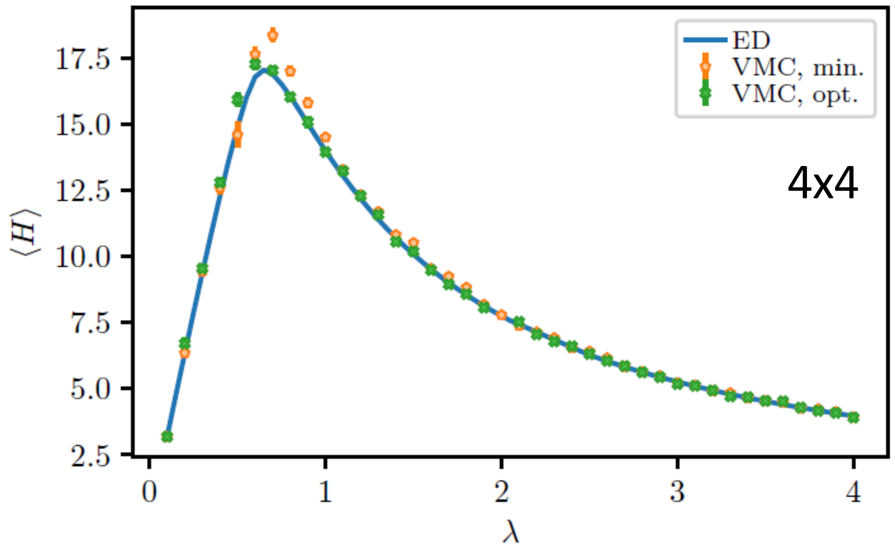
$$H = \lambda \sum_{\ell} [1 - P_\ell] + \frac{1}{\lambda} \sum_{p} [1 - Q_{p_1} Q_{p_2} Q_{p_3} Q_{p_4}]$$



$$P = P^\dagger = \sigma_z$$
$$Q = Q^\dagger = \sigma_x$$



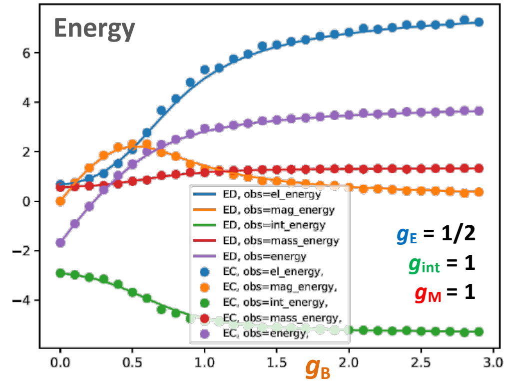




P. Emonts, A. Kelman, U. Borla, S. Moroz, S. Gazit, E. Zohar, Phys. Rev. D 107, 014505 (2023)

Non-published Preliminary Results – Numerics

- Z_2 in 2+1-D with a single fermionic flavor
- Early results, exact contraction, 2x2

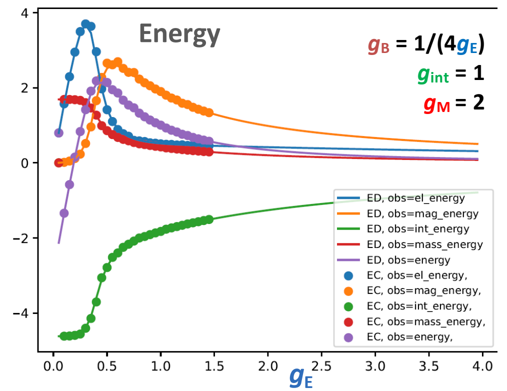


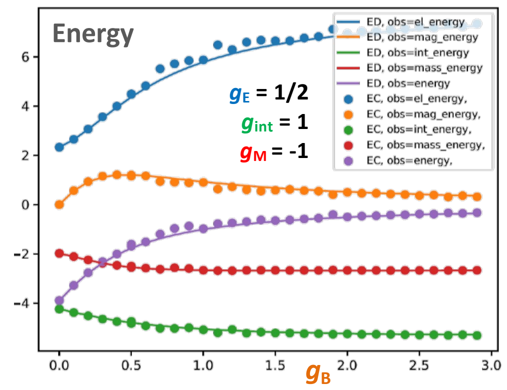
Next steps: larger systems with Monte-Carlo; two flavors

First expected breakthrough: introduction of a chemical potential, into a sign-problem regime

$$H = g_E H_E + g_B H_B + g_{int} H_{int} + g_M H_M$$

Electric
Magnetic
Interaction
Mass



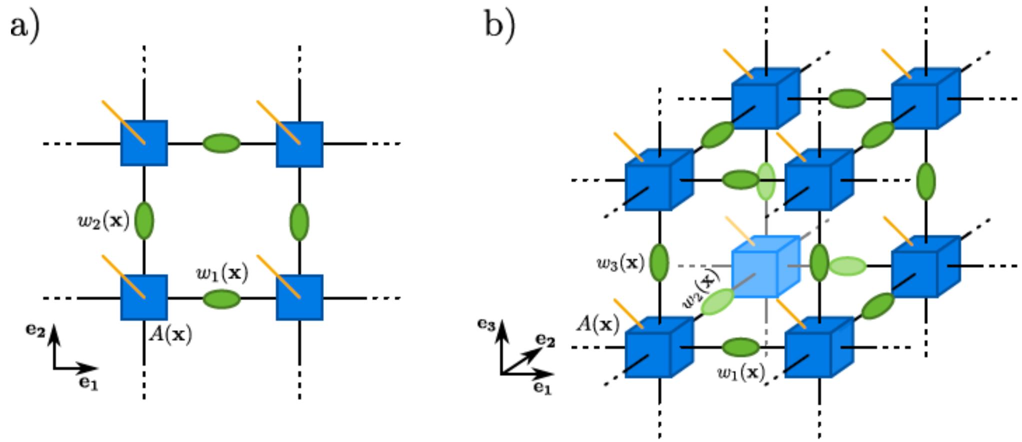


Soon:

- Kelman, Borla, Emonts, Zohar – Z_2 with fermions

Analytics 1: 3+1-d: fermions ready for gauging

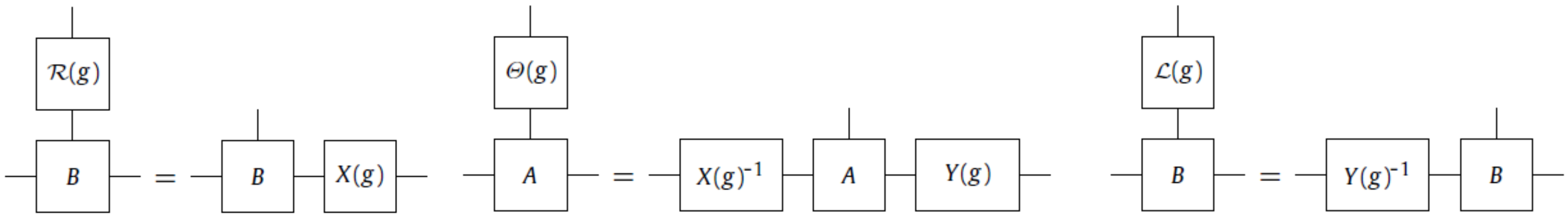
- The Gaussian fermionic PEPS formalism can be extended in a straight forward way to three space dimensions.
- As expected, "spin" emerges from demanding lattice rotation invariance; assuming very naively that the fermions are "spinless" leads to Kogut's staggering.
- It is possible to formally express the ground states of lattice Dirac Hamiltonians exactly as such states.
- Gauging can be done exactly the same, as well as Monte-Carlo sampling; the dependence on the dimension is indirect, only through the number of links to be integrated.
- To be continued



P. Emonts, E. Zohar, Phys. Rev. D 108, 014514 (2023)

Analytics 2: Is this the way to gauge?

- We have shown that the current gauging mechanism produces LGT state with proper gauge invariance. But does this cover all the options for gauge invariant PEPS?
- YES!
  - It was rigorously proven for MPS (PEPS in one space dimension) that once simple physical and mathematical properties (injectivity etc) are satisfied, gauge invariance implies the described PEPS structure.
    - Kull, Molnar, Zohar, Cirac, Ann. Phys 386, 199-241 (2017)



  - Recently completed a proof of the theorem for higher dimensions and arbitrary geometries.
    - Blanik, Garre-Rubio, Molnar, Zohar – soon!

Funded by the
European Union

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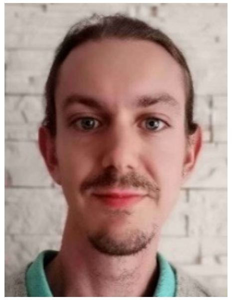
The Gauged Gaussian Fermionic PEPS team:

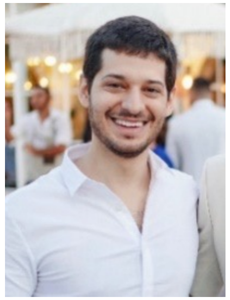
At the Hebrew University of Jerusalem:


Erez Zohar
PI

Gertian Roose
Postdoc


Umberto Borla
Postdoc


Ariel Kelman
PhD Student


Jonathan Elyovich
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The Racah Institute
לפיסיקה
of Physics

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM
الجامعة العبرية في اورشليم القدس

Soon:

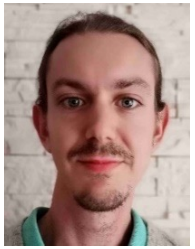
- Roose, Zohar – analytical study of the ansatz suitability
- Kelman, Borla, Emonts, Zohar – Z_2 with fermions
- Blanik, Garre-Rubio, Molnar, Zohar – soon!

Quantum Information & Many Body Physics Group

Racah Institute of Physics, Hebrew University of Jerusalem, Israel

Erez Zohar

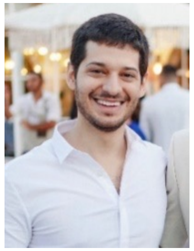
Gertian Roose (Postdoc)



Umberto Borla (Postdoc)

Guy Pardo (Phd)

Ariel Kelman (Phd)



Jonathan Elyovich (PhD)

Itay Gomelski (PhD)

Uri Friedman (Master)

- Locally and unitarily mapping fermions to bosons in the presence of lattice gauge fields
  - **Pardo**, **Greenberg**, **Fortinsky**, Katz, **Zohar**, PRR 2023
  - Popov, Meth, Lewenstein, Hauke, **Zohar**, Kasper, PRR 2024
- Photon-Mediated Quantum Simulation of LGT plaquette interactions.
  - **Armon**, **Ashkenazi**, Garcia-Moreno, Gonzalez-Tudela, **Zohar**, PRL 2021
- Physically blocked U(1) LGTS with qubits
  - **Shir**, **Zohar**, PRD 2024
- Building non-Abelian LGT quantum simulators using dynamical decoupling ("non-Abelian rotating wave approximation").
  - Kasper, Zache, Jendrzejewski, Lewenstein, **Zohar**, PRD 2023
- Sign-problem free tensor network construction for studying LGTs with fermionic matter
  - Emonts, **Kelman**, **Borla**, Moroz, Gazit, **Zohar**, PRD 2023
  - Emonts, **Zohar**, PRD 2023
  - **Kelman, Borla, Gomelski, Elyovich,** Emonts, **Zohar**, PRD 2024
- Analytical and entanglement properties of LGT PEPS
  - **Zohar**, PRR 2021
  - **Knaute**, **Feuerstein**, **Zohar**, JHEP 2024
  - Feldman, **Knaute**, **Zohar**, Goldstein, JHEP 2024
  - **Roose**, **Zohar** – soon
  - Blanik, Garre-Rubio, Molnar, **Zohar** – soon
- Physically implementing Duality Transformations using Local Unitaries and Measurements.
  - **Ashkenazi**, **Zohar**, PRA 2022
- Continuous tensor networks for relativistic QFTs
  - Shachar, **Zohar**, PRD 2023
  - **Rigobello**, Shachar, **Zohar** – soon!

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المؤسسة الإسرائيلية للعلوم
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