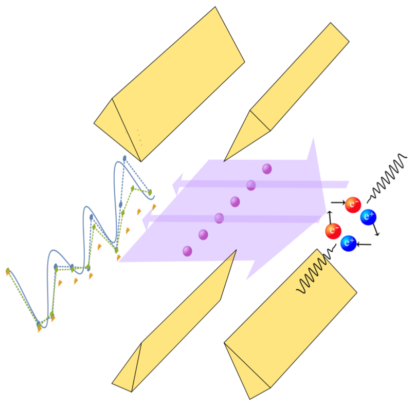
Lattice gauge theory simulations on a trapped-ion hybrid quantum simulator

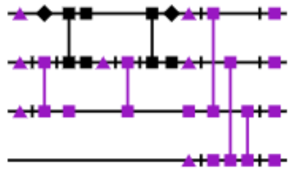
Norbert M. Linke
Duke Quantum Center, Duke University, Durham, North Carolina, USA
Joint Quantum Institute, University of Maryland, College Park, Maryland, USA

QUANTHEP, Munich, Germany
September 2, 2024



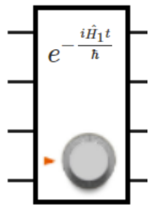
Quantum Simulation

"digital" quantum simulation



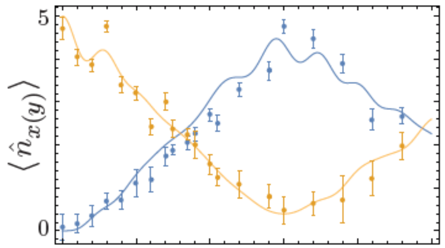
quantum computer
(universal)

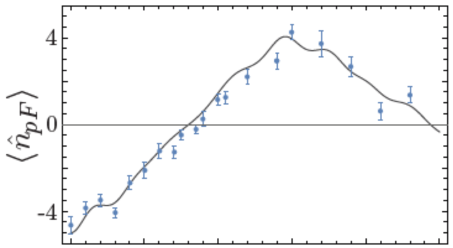
"analog" quantum simulation



tailored native evolution

approximate



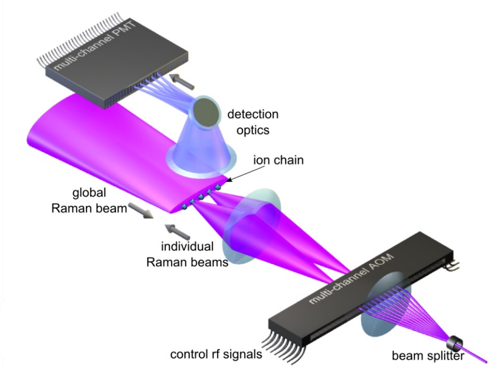


simulat**ing** system
(physical system at hand)

simulat**ed** system
(target dynamics of interest)

C. Huerta Alderete et al., arXiv:2108.05471

Overview



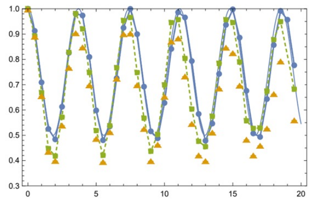
<u>Experimental system</u>

Individually-addressed $^{171}Yb^{+}$ ions
Modular operations and compiler (5-9 qubits)

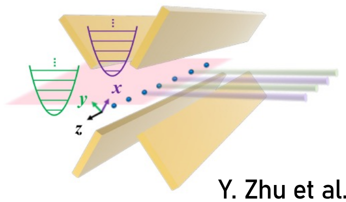
N. H. Nguyen et al., PRX Quantum 3, 020324 (2022).

<u>Applications</u>

Digital Quantum Simulation
The Schwinger model

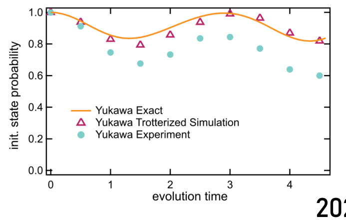


Motional mode control
Parallel entangling gates



Y. Zhu et al., Adv. Quantum Technol. 020324 (2023).

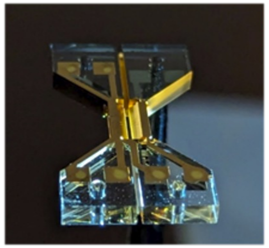
Hybrid Quantum Simulation
The Yukawa model



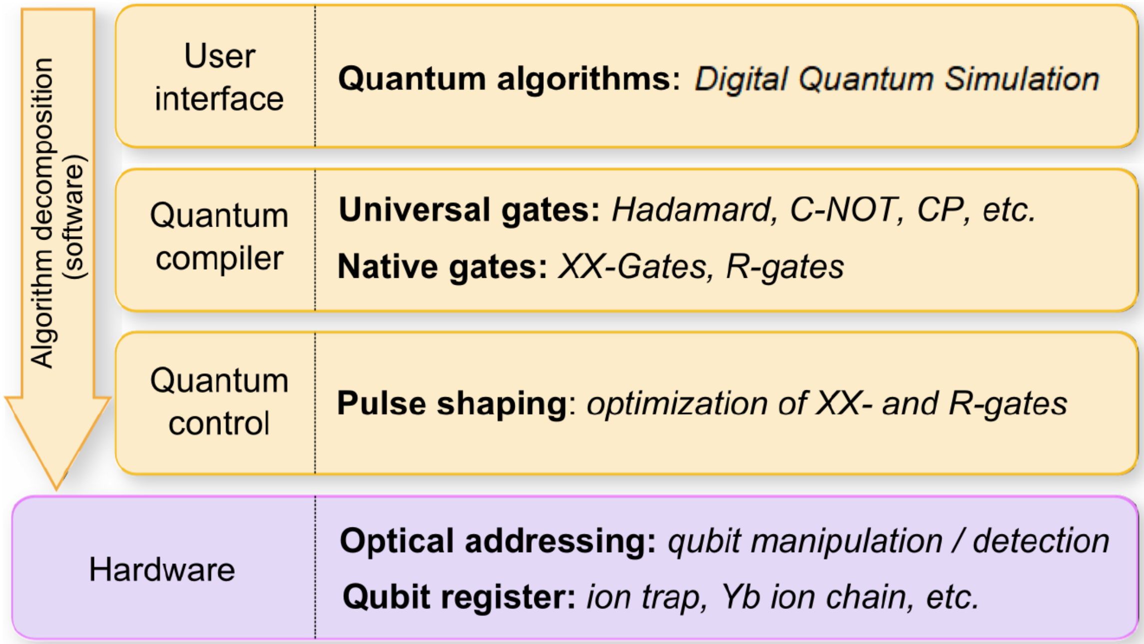
2024 based on: Z. Davoudi et al., Phys. Rev. Research 3, 043072 (2021).

<u>Outlook</u>

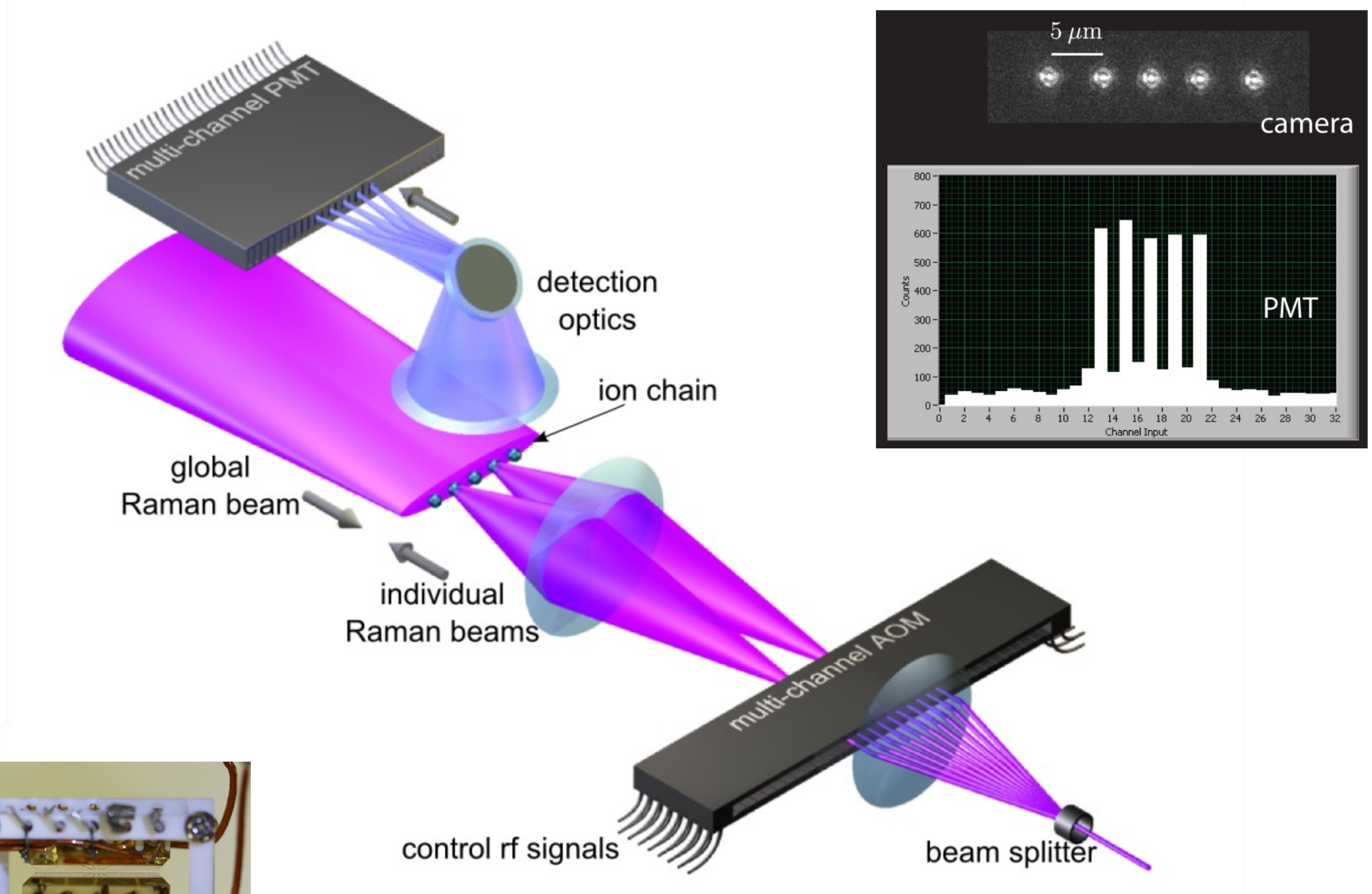
Scaling the system

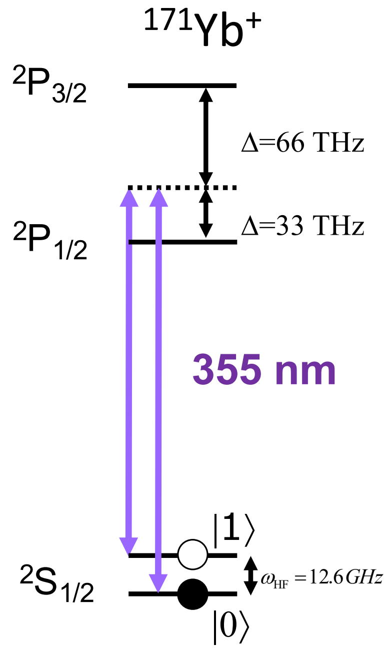


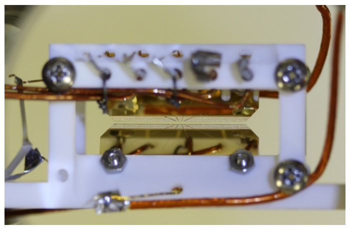
Experimental system: QC architecture



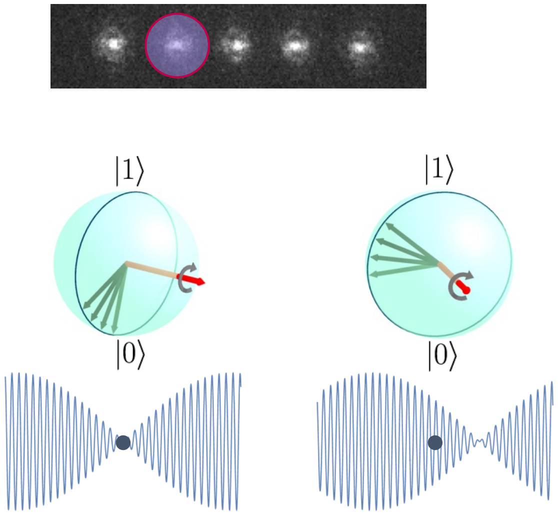
Experimental system: Hardware

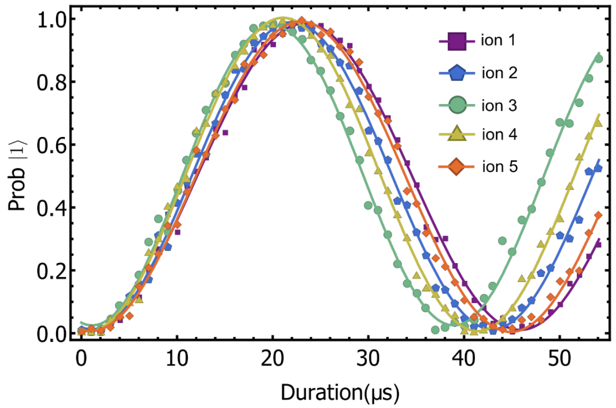




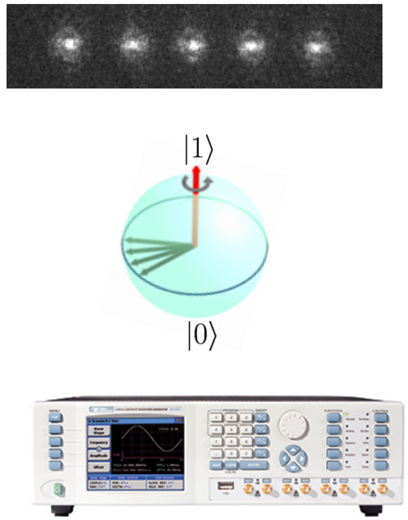


Experimental system: Single qubit gates





Raman beat note



Classical phase shifts

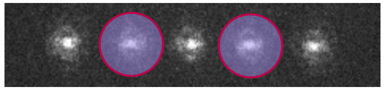
R-gate (*x*/*y* rotations)

$$R_\phi(\theta) = \begin{bmatrix} \cos(\frac{\theta}{2}) & -i\sin(\frac{\theta}{2})e^{-i\phi} \\ -i\sin(\frac{\theta}{2})e^{i\phi} & \cos(\frac{\theta}{2}) \end{bmatrix}$$

***z*-gates**

$$R_z(\theta) = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

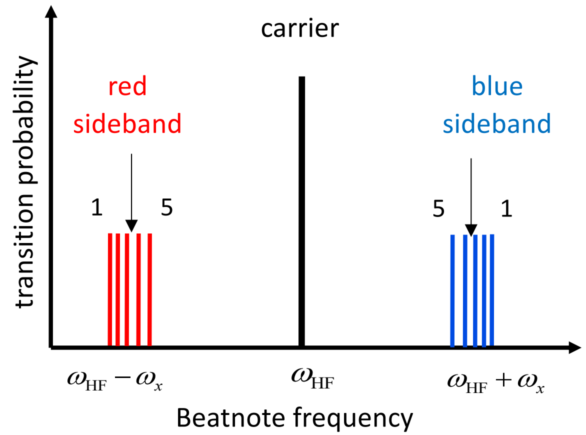
Experimental system: Two-qubit gates



$$U(t) = \exp[-i \sum_{n,k} \hat{D}(\xcancel{\alpha_n^k(t)})\sigma_x^n - i \sum_{i,j} \chi_{ij}(t)\sigma_x^i \sigma_x^j]$$

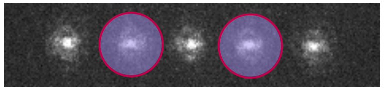
$$XX(\chi_{i,j}) = \begin{bmatrix} \cos(\chi_{i,j}) & 0 & 0 & -i\sin(\chi_{i,j}) \\ 0 & \cos(\chi_{i,j}) & -i\sin(\chi_{i,j}) & 0 \\ 0 & -i\sin(\chi_{i,j}) & \cos(\chi_{i,j}) & 0 \\ -i\sin(\chi_{i,j}) & 0 & 0 & \cos(\chi_{i,j}) \end{bmatrix}$$

$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle - i|11\rangle)$$



MS gate: PRL **82** (1999)
T. Choi et al. PRL **112**, 19502 (2014)
T. J. Green et al., PRL **114**, 120502 (2015)
P. H. Leung et al. PRL **120**, 020501 (2018)
Y. Shapira et al., PRL **121**, 180502 (2018)

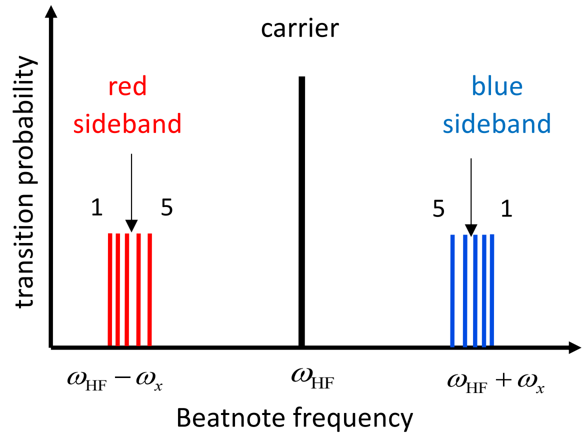
Two-qubit gates

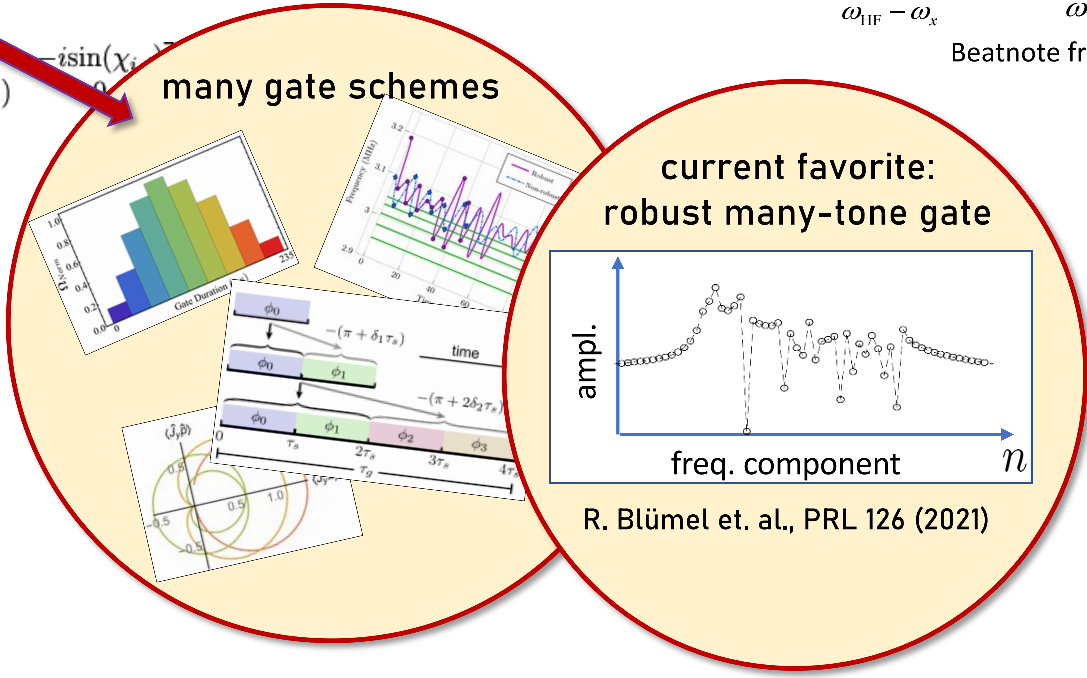


$$U(t) = \exp[-i \sum_{n,k} \hat{D}(\alpha_n^k(t))\sigma_x^n - i \sum_{i,j} \chi_{ij}(t)\sigma_x^i\sigma_x^j]$$

$$XX(\chi_{i,j}) = \begin{bmatrix} \cos(\chi_{i,j}) & 0 & 0 & -i\sin(\chi_{i,j}) \\ 0 & \cos(\chi_{i,j}) & -i\sin(\chi_{i,j}) & 0 \\ 0 & -i\sin(\chi_{i,j}) & \cos(\chi_{i,j}) & \\ -i\sin(\chi_{i,j}) & 0 & 0 & \end{bmatrix}$$

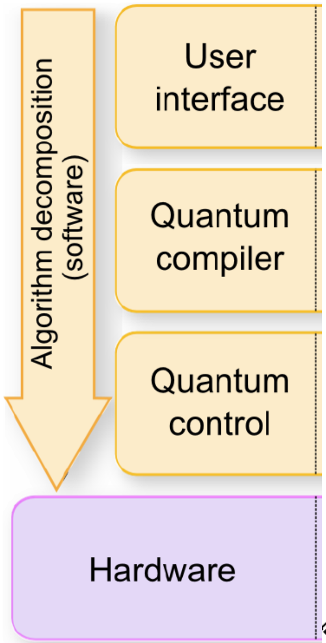
$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle - i|11\rangle)$$

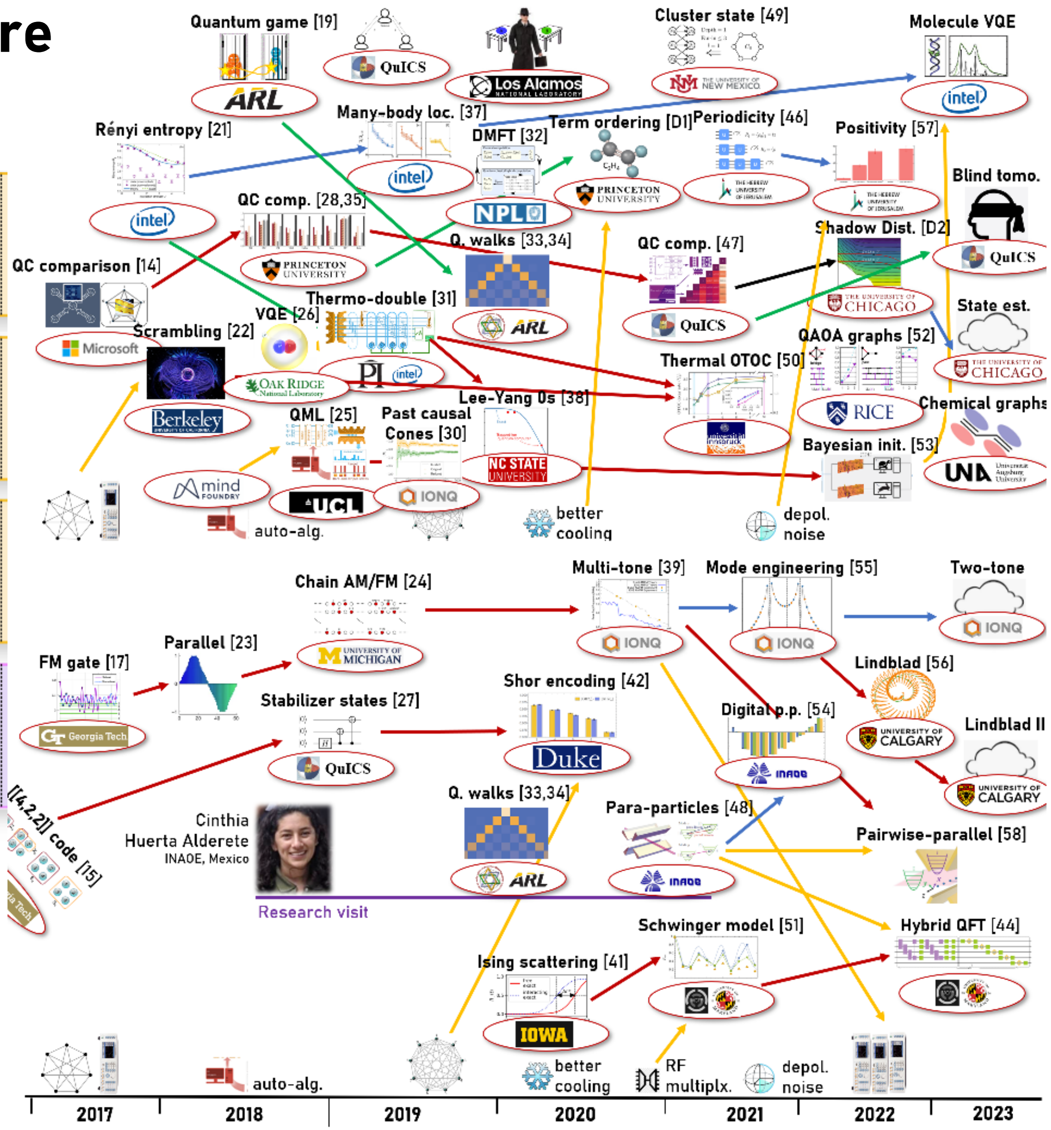




MS gate: PRL **82** (1999)
T. Choi et al. PRL **112**, 19502 (2014)
T. J. Green et al., PRL **114**, 120502 (2015)
P. H. Leung et al. PRL **120**, 020501 (2018)
Y. Shapira et al., PRL **121**, 180502 (2018)

Experimental system: QC architecture



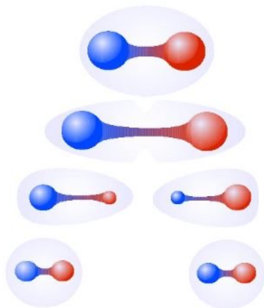


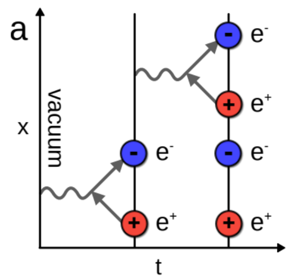
sites.duke.edu/linkelab

Digital Quantum Simulation: The Schwinger model

Digital Quantum Simulation: The Schwinger model

- Schwinger model is a Quantum Field Theory in 1+1D
- Schwinger model has Quantum Chromodynamics -like phenomena [1,2]
  - Pair creation-annihilation
  - String breaking
- Testbed for quantum simulation methods[3,4,5]
- Digital simulation with long(-ish) time dynamics



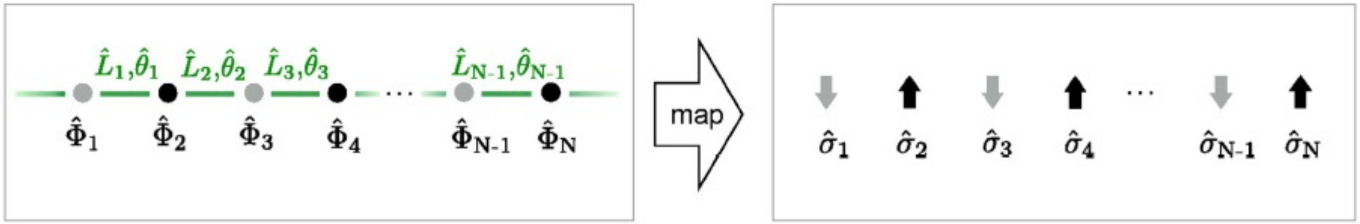


[1] Coleman Ann. Phys. 101 (1976)
[2] Hebenstreit et al PRL. 111 (2013)
[3] Martinez et al Nature 534 (2016)
[4] Surace et al PRX 10 (2020)
[5] Mil et al Science 367 (2020)

Digital Quantum Simulation: The Schwinger model

Lattice Schwinger model (spinless 1+1D QFT, discretized space, normalize)

$$\hat{H}_{\text{lat}} = -\mathrm{i}w \sum_{n=1}^{N-1} [\hat{\Phi}_n^{\dagger} \mathrm{e}^{\mathrm{i}\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{H.C.}] + m \sum_{n=1}^{N} (-1)^n \hat{\Phi}_n^{\dagger} \hat{\Phi}_n + J \sum_{n=1}^{N-1} \hat{L}_n^2,$$

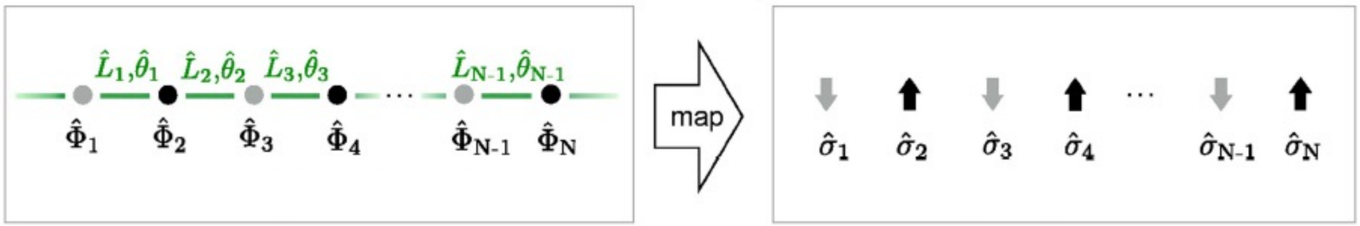


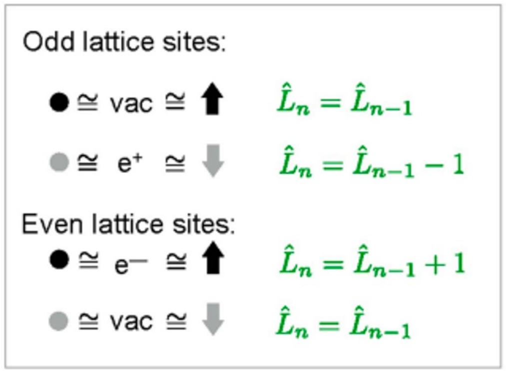
C. Muschik et al New J. Phys. 19 103020 (2018)

Digital Quantum Simulation: The Schwinger model

Lattice Schwinger model (spinless 1+1D QFT, discretized space, normalize)

$$\hat{H}_{\text{lat}} = -\mathrm{i}w\sum_{n=1}^{N-1}[\hat{\Phi}_n^{\dagger}\mathrm{e}^{\mathrm{i}\hat{\theta}_n}\hat{\Phi}_{n+1} - \text{H.C.}] + m\sum_{n=1}^{N}(-1)^n\hat{\Phi}_n^{\dagger}\hat{\Phi}_n + J\sum_{n=1}^{N-1}\hat{L}_n^2,$$





Gauss' law applies for photon link:

-> number of spin-up qubits conserved

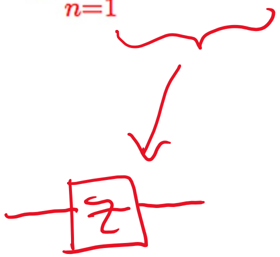
C. Muschik et al New J. Phys. 19 103020 (2018)

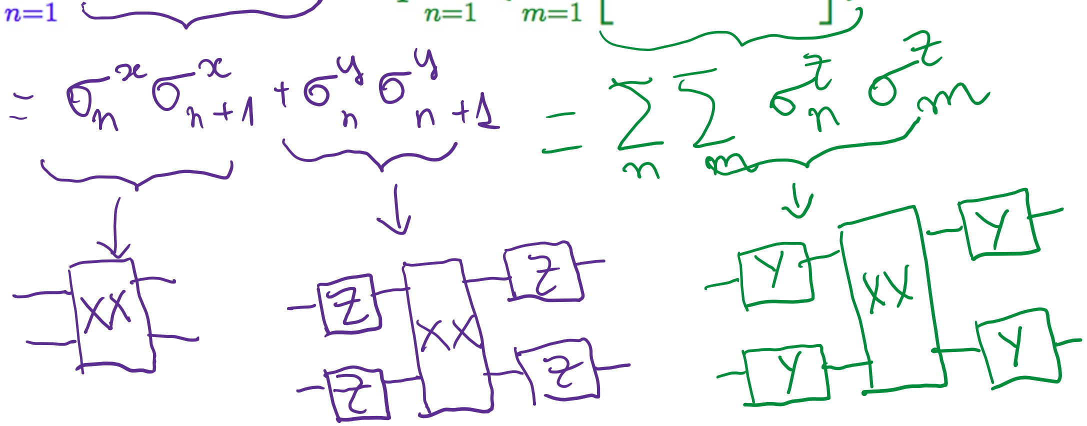
Digital Quantum Simulation: The Schwinger model

Final qubit Hamiltonian

Fermion mass, hopping on lattice, E-field interaction

$$\hat{H}_s = \frac{\mu}{2}\sum_{n=1}^{N}(-1)^n\sigma_n^z + x\sum_{n=1}^{N-1}\{\sigma_n^+\sigma_{n+1}^- + \text{h.c.}\} + \frac{1}{4}\sum_{n=1}^{N-1}\left\{\sum_{m=1}^{n}\left[\sigma_m^z + (-1)^m\right]\right\}^2$$





parameters

$x = 0.6, \quad \mu = 0.1, \quad \delta t = 0.5, \quad N_t = 20.$

Run Hamiltonian evolution as a quantum circuit: Trotterization

$$\hat{U} = e^{-i\hat{H}t/\hbar} = \lim_{n\to\infty}(\Pi_k e^{-i\hat{H}_k t/n\hbar})^n$$

First-order Trotter approximation: pick finite n

$$\delta t = t/n \quad \hat{U}_k = e^{-i\hat{H}\delta t/\hbar}$$ (steps don't commute -> term ordering)

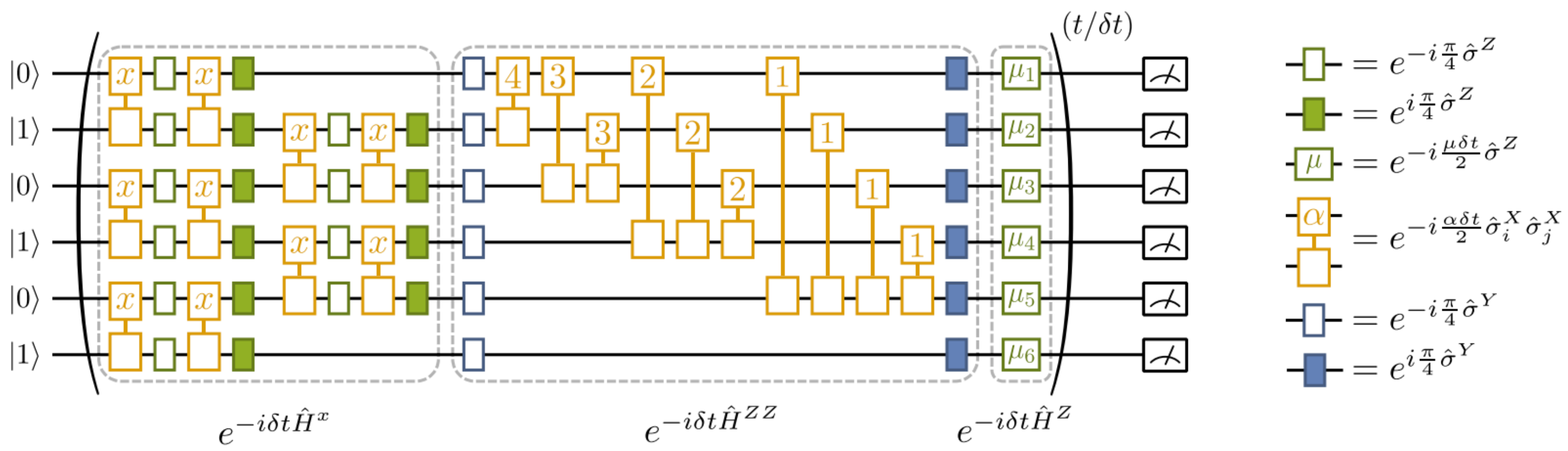
Digital Quantum Simulation: The Schwinger model

Final qubit Hamiltonian

Fermion mass, hopping on lattice, E-field interaction

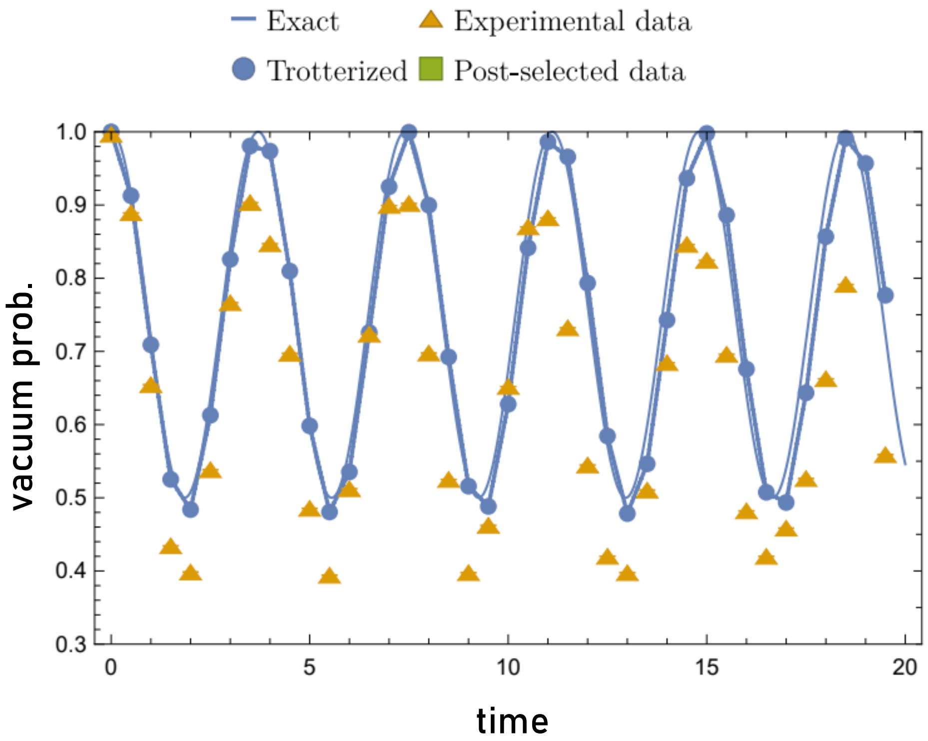
$$\hat{H}_s = \frac{\mu}{2}\sum_{n=1}^{N}(-1)^n\sigma_n^z + x\sum_{n=1}^{N-1}\{\sigma_n^+\sigma_{n+1}^- + \text{h.c.}\} + \frac{1}{4}\sum_{n=1}^{N-1}\left\{\sum_{m=1}^{n}\left[\sigma_m^z + (-1)^m\right]\right\}^2$$

- Start at ground state of x=0
- Evolve with x≠0 to time t (Trotterized) $|\text{vac}\rangle = |0101...01\rangle$
- Measure vacuum survival prob. / particle number density / E-field density



Digital Quantum Simulation: The Schwinger model

Results (1-site model, 2 qubits)

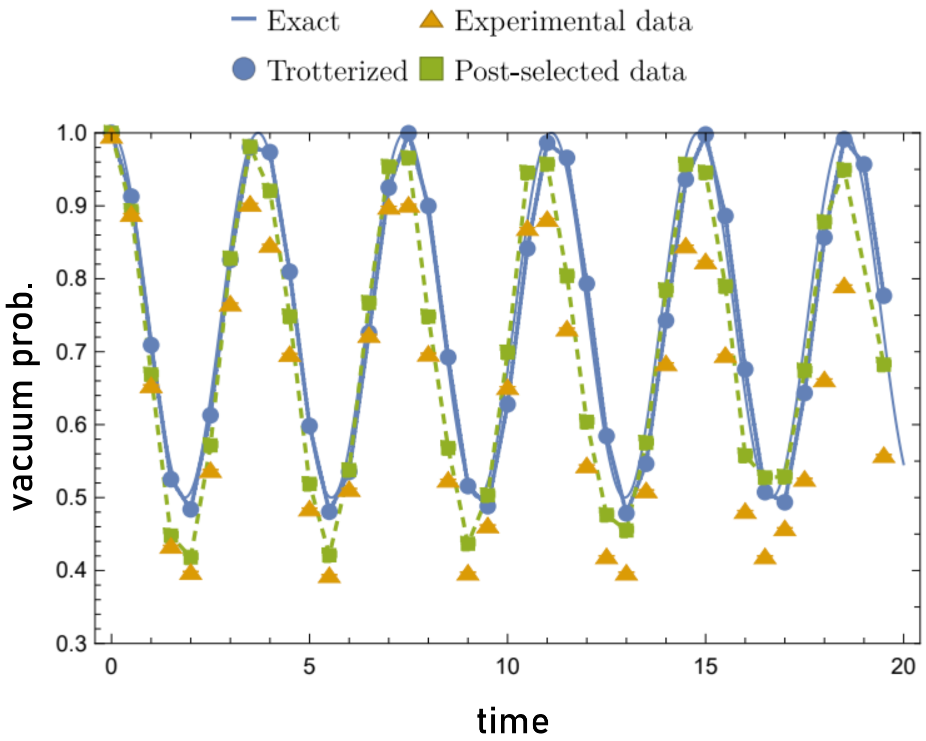


N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

see also Innsbruck group: E. Martinez, Nature 516, 534 (2016)

Digital Quantum Simulation: The Schwinger model

Results (1-site model, 2 qubits)



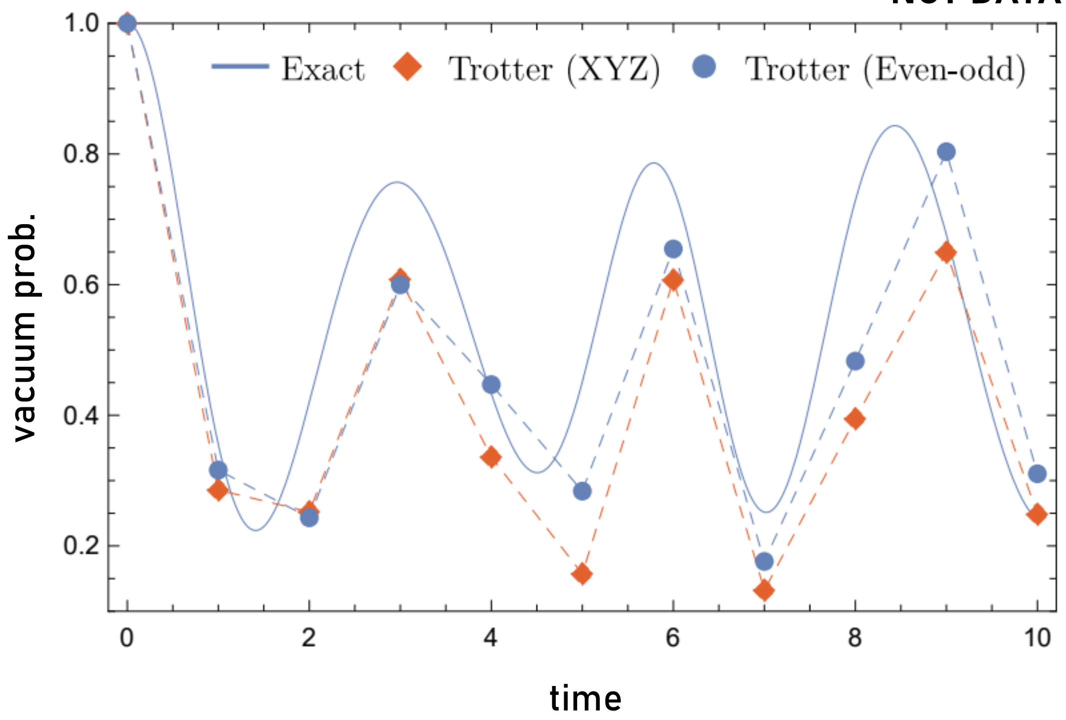
N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

see also Innsbruck group: E. Martinez, Nature 516, 534 (2016)

Digital Quantum Simulation: The Schwinger model

Different term ordering choices

NOT DATA

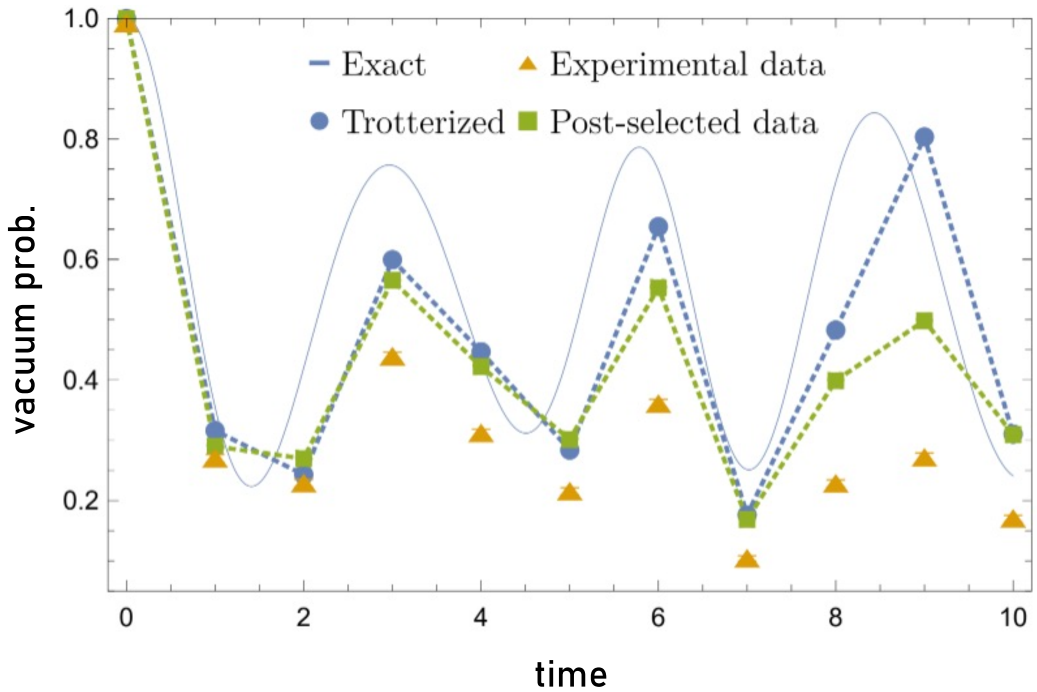


N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

see A. Childs et al., Phys. Rev. Lett. 123, 050503 (2019)

Digital Quantum Simulation: The Schwinger model

Results (2-site model, 4 qubits)

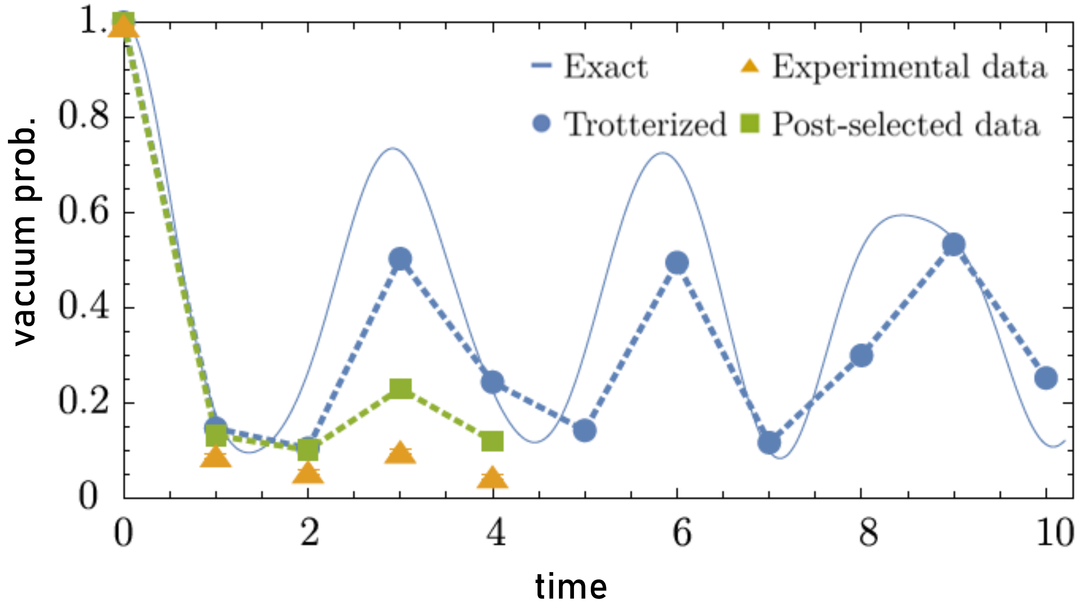


N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

note term ordering (Trotter error): A. Childs et al., Phys. Rev. Lett. 123, 050503 (2019)

Digital Quantum Simulation: The Schwinger model

Results (3-site model, 6 qubits)

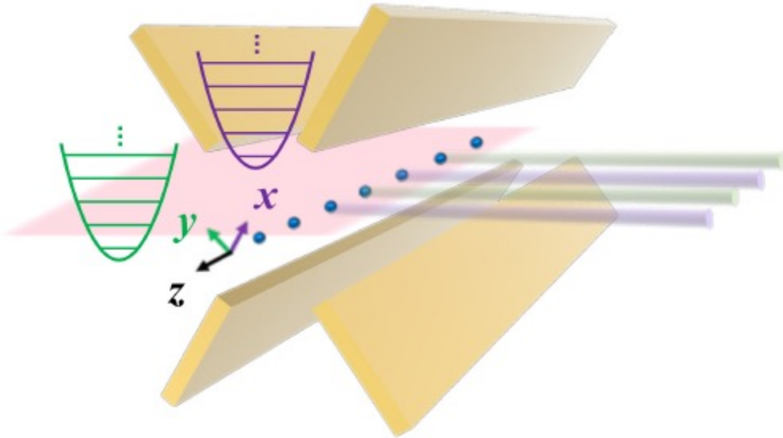


N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

Pairwise parallel gates

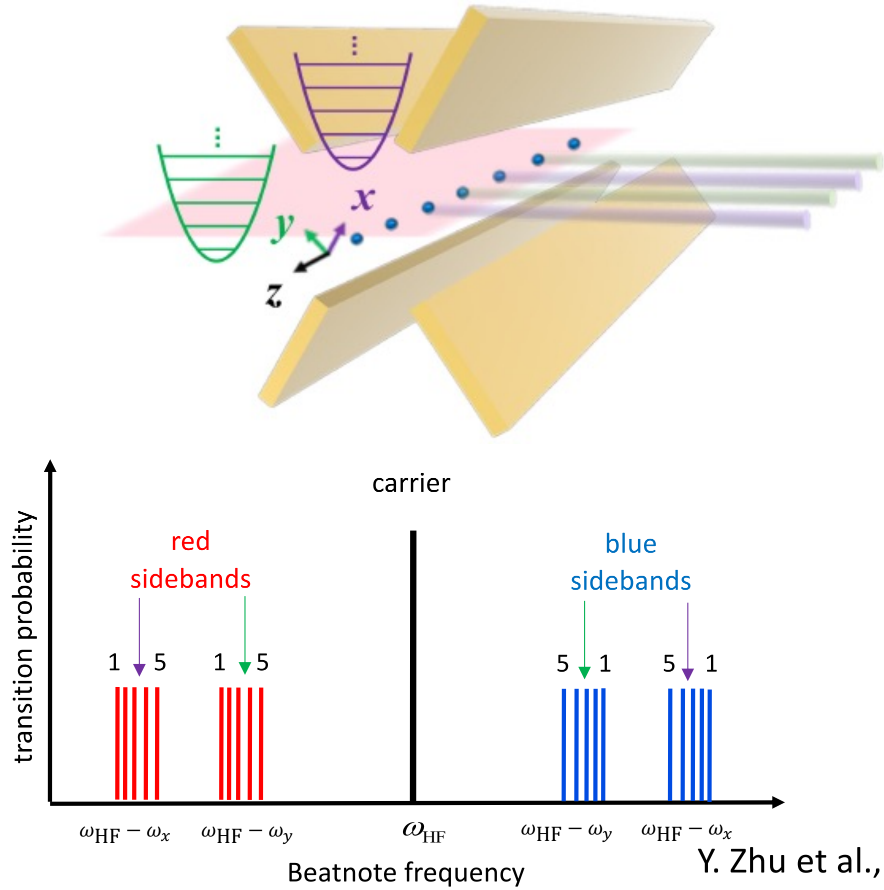
Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates



Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

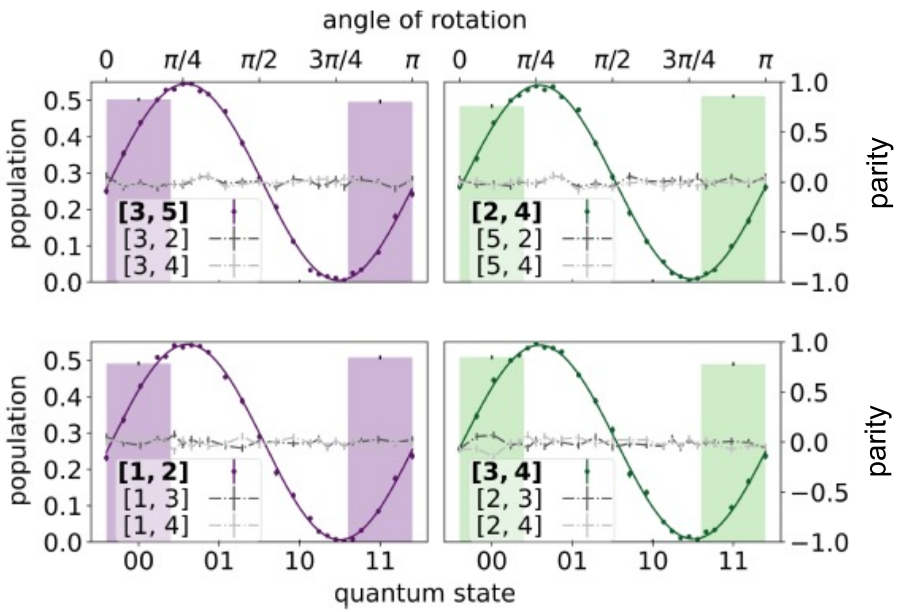
Pairwise parallel gates



Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates

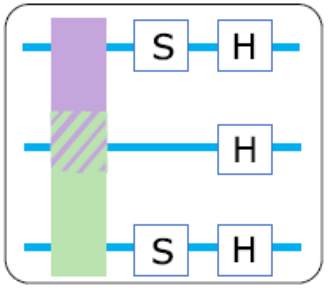
$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle - i|11\rangle)$$



Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates

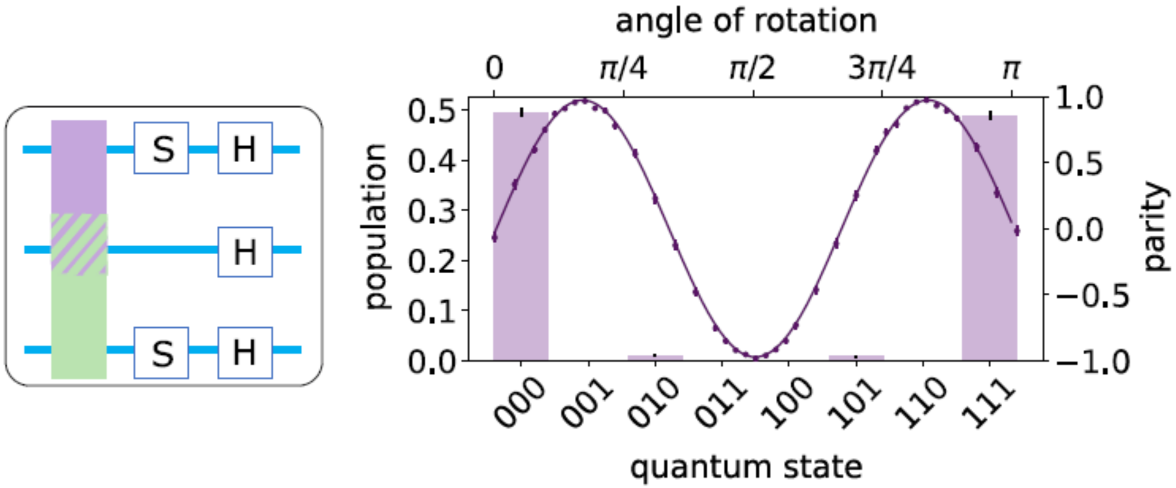
Parallel gates with overlapping ion: GHZ state



Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates

Parallel gates with overlapping ion: GHZ state

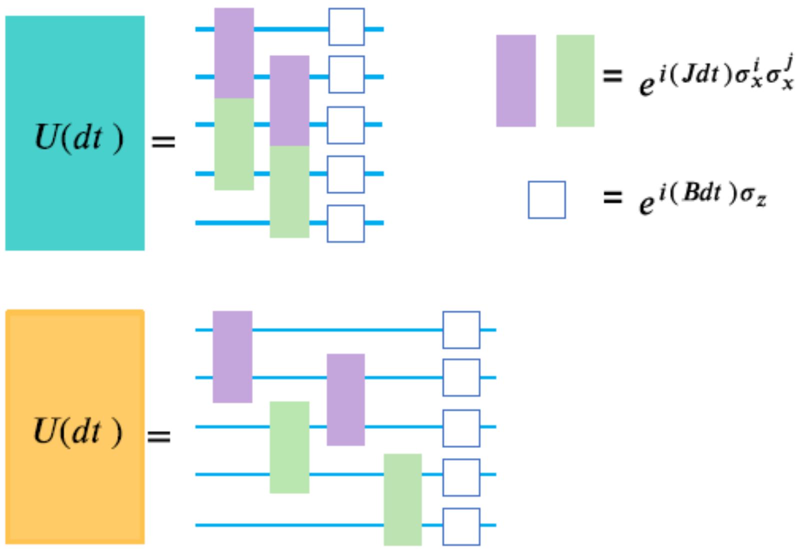


Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates

Transverse-field Ising model:

$$H = -J \sum_{i=1}^{4} \sigma_x^i \sigma_x^{i+1} - B \sum_{i=1}^{5} \sigma_z^i$$

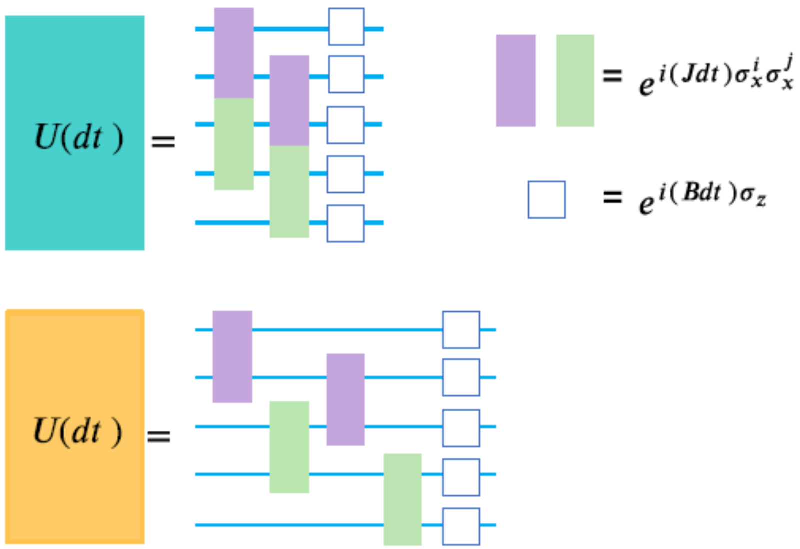


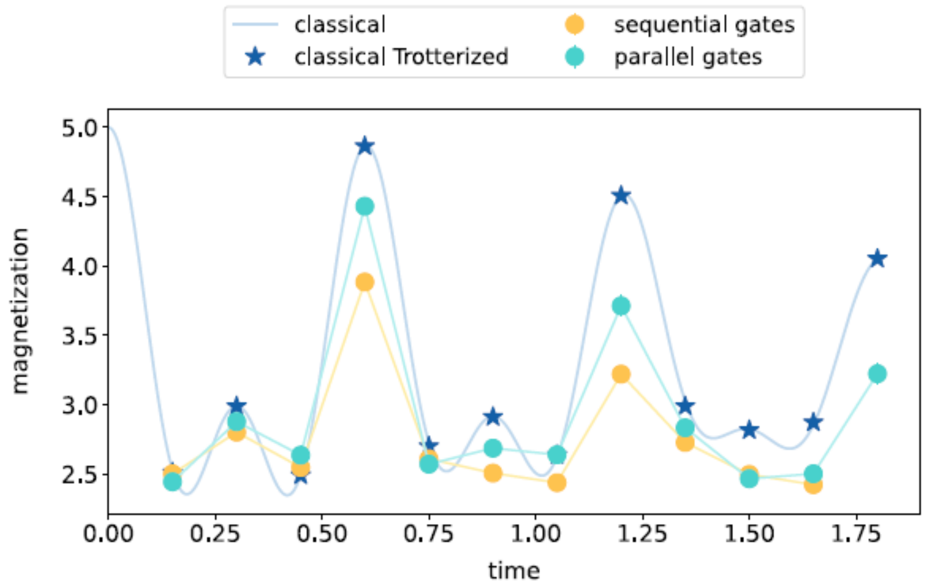
Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates

Transverse-field Ising model:

$$H = -J\sum_{i=1}^{4}\sigma_x^i\sigma_x^{i+1} - B\sum_{i=1}^{5}\sigma_z^i$$

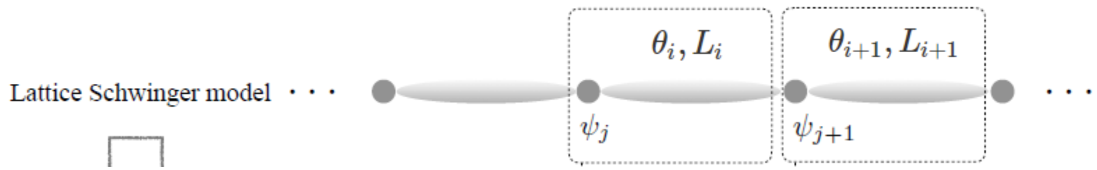




Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Difficult Quantum Simulation… such as the Schwinger model

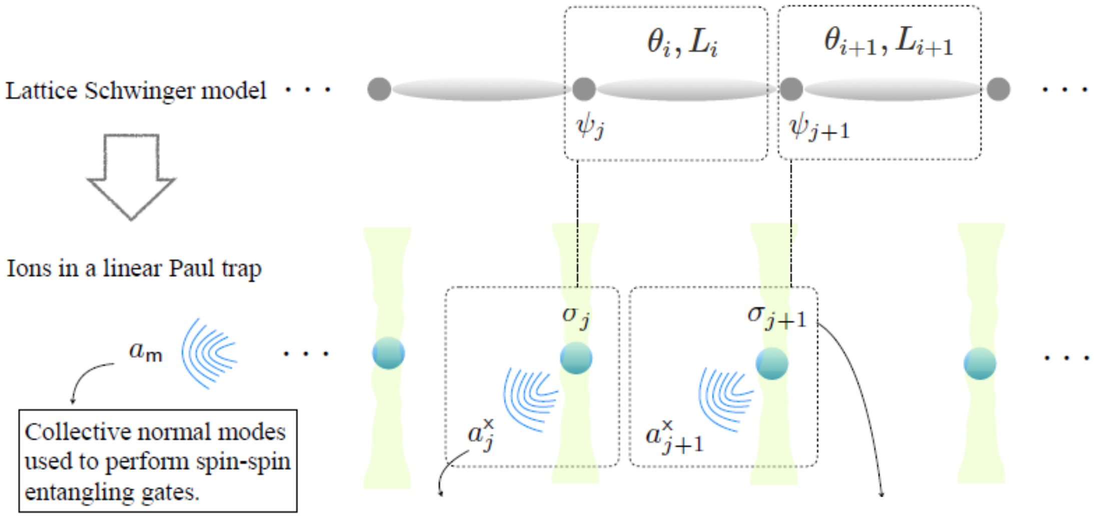
Analog Quantum Simulation Alternative ?



Zohreh Davoudi, NML, G. Pagano, Phys. Rev. Research 3, 043072 (2021)

Hybrid Quantum Simulation

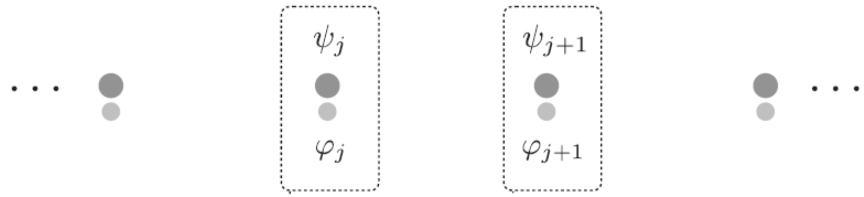
The Schwinger model



Zohreh Davoudi, NML, G. Pagano, Phys. Rev. Research 3, 043072 (2021)

Hybrid Quantum Simulation: The Yukawa model

$$V(r) = -\frac{g^2}{4\pi}\,\frac{1}{r}\,e^{-\mu r}$$



- Scalar field theory: scalar bosons (field) interact with fermions (matter)
- Describes the Higgs field interacting with the leptons and quarks
- Explains mass generation

$$H_{\text{Yukawa}} = H^{(I)}_{\text{Yukawa}} + H^{(II)}_{\text{Yukawa}} + H^{(III)}_{\text{Yukawa}}$$

$$H^{(III)}_{\text{Yukawa}} = gb \sum_{j=1}^{N} \psi_j^{\dagger} \varphi_j \psi_j$$

$$\varphi_j = \frac{1}{\sqrt{Nb}} \sum_{k=-N/2}^{N/2-1} \frac{1}{\sqrt{2\varepsilon_k}} (d_k^{\dagger} e^{-i2\pi kj/N} + d_k e^{i2\pi kj/N})$$

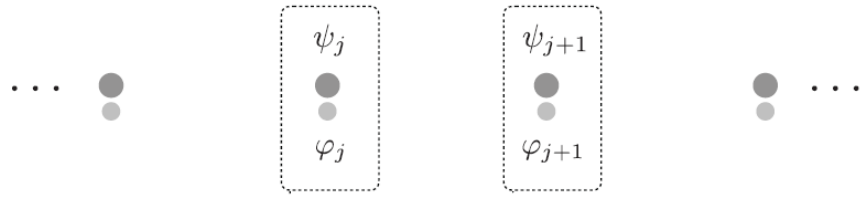
$$H^{(II)}_{\text{Yukawa}} = \sum_{k=-N/2}^{N/2-1} \varepsilon_k \left(d_k^{\dagger} d_k + \frac{1}{2} \right)$$

$$\varepsilon_k = \sqrt{(\tfrac{2\pi k}{Nb})^2 + m_{\varphi}^2}$$

$$H^{(I)}_{\text{Yukawa}} = \sum_{j=1}^{N} \left[\frac{i}{2b} (\psi_j^{\dagger} \psi_{j+1} - \psi_{j+1}^{\dagger} \psi_j) + m_{\psi} (-1)^j \psi_j^{\dagger} \psi_j \right]$$

Hybrid Quantum Simulation: The Yukawa model

$$V(r) = -\frac{g^2}{4\pi}\,\frac{1}{r}\,e^{-\mu r}$$



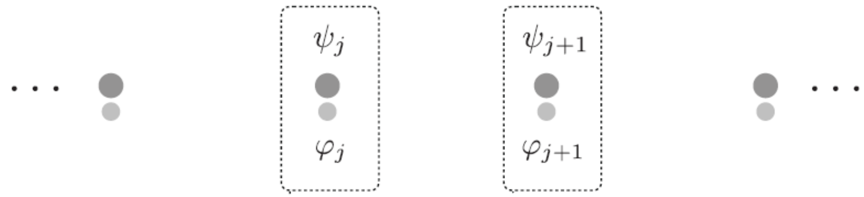
- Scalar field theory: scalar bosons (field) interact with fermions (matter)
- Describes the Higgs field interacting with the leptons and quarks
- Explains mass generation

$$H_{\text{Yukawa}} = H_{\text{Yukawa}}^{(I)} + H_{\text{Yukawa}}^{(II)} + H_{\text{Yukawa}}^{(III)}$$

$$H_{\text{Yukawa}}^{(III)} = gb \sum_{j=1}^{N} \psi_j^\dagger \varphi_j \psi_j$$

Hybrid Quantum Simulation: The Yukawa model

$$V(r) = -\frac{g^2}{4\pi}\,\frac{1}{r}\,e^{-\mu r}$$



- Scalar field theory: scalar bosons (field) interact with fermions (matter)
- Describes the Higgs field interacting with the leptons and quarks
- Explains mass generation

$$H_{\text{Yukawa}} = H^{(I)}_{\text{Yukawa}} + H^{(II)}_{\text{Yukawa}} + H^{(III)}_{\text{Yukawa}}$$

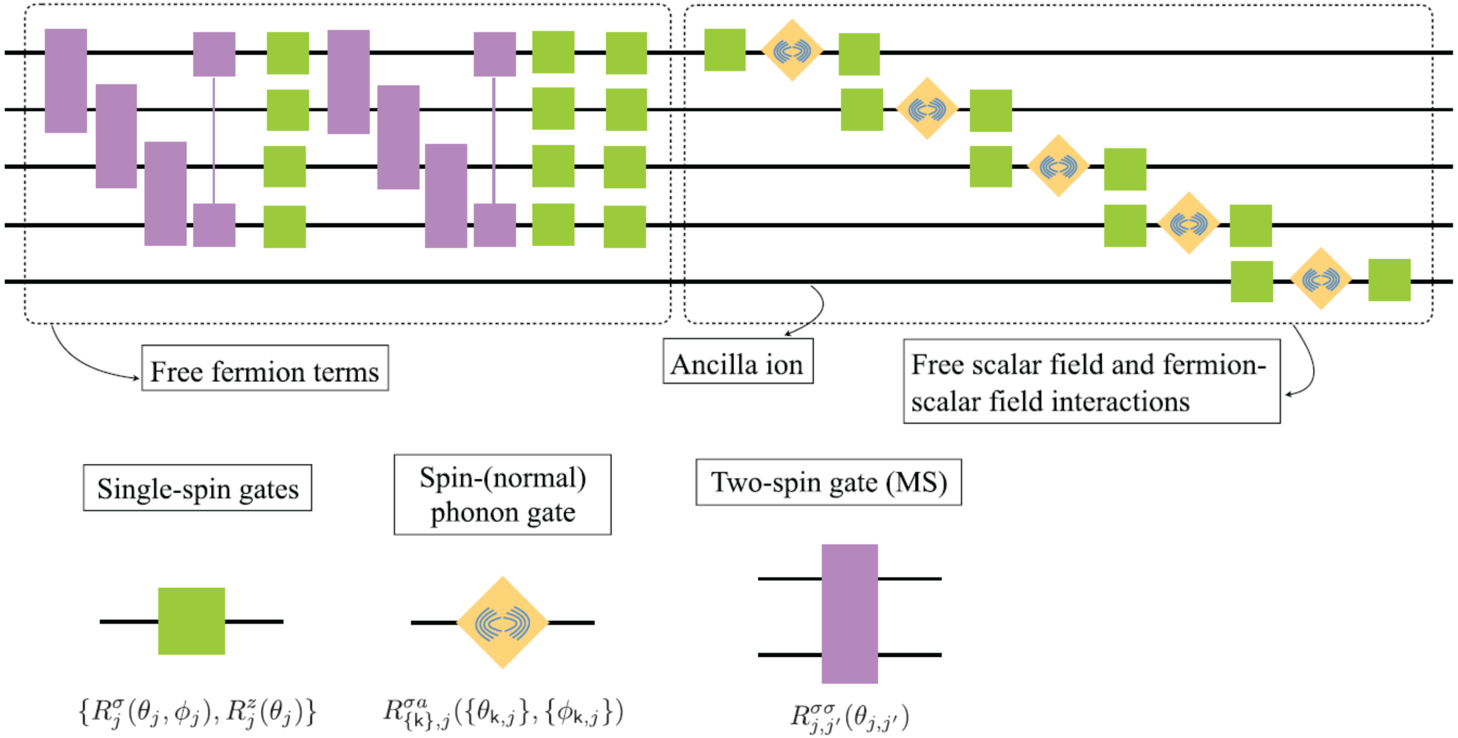
$$H^{(III)}_{\text{Yukawa}} = \sqrt{\frac{g^2 b}{8N}} \sum_{j=1}^{N} (\mathbb{I}_j + \sigma_j^z) \sum_{\mathrm{m}=1}^{N} \frac{1}{\sqrt{\varepsilon_{\mathrm{m}}}} \left(a^\dagger_{\mathrm{m}} e^{-i\frac{2\pi j}{N}(\mathrm{m}-\frac{N}{2}-1)} + a_{\mathrm{m}} e^{i\frac{2\pi j}{N}(\mathrm{m}-\frac{N}{2}-1)}\right) + \sum_{\mathrm{m}=1}^{N} \varepsilon_{\mathrm{m}} \left(a^\dagger_{\mathrm{m}} a_{\mathrm{m}} + \frac{1}{2}\right)$$

ancilla ion

red & blue sideband (rwa, detuned)

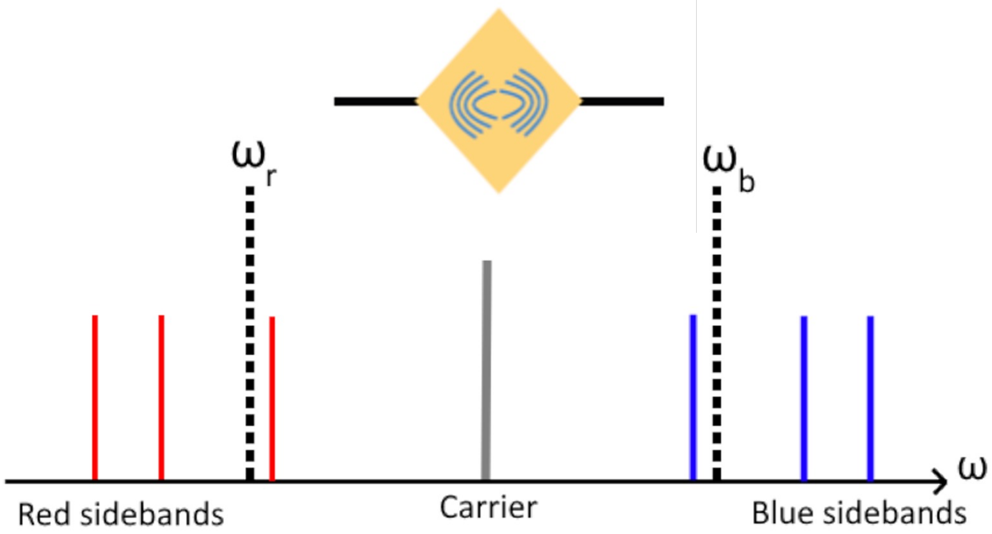
Hybrid Quantum Simulation: The Yukawa model

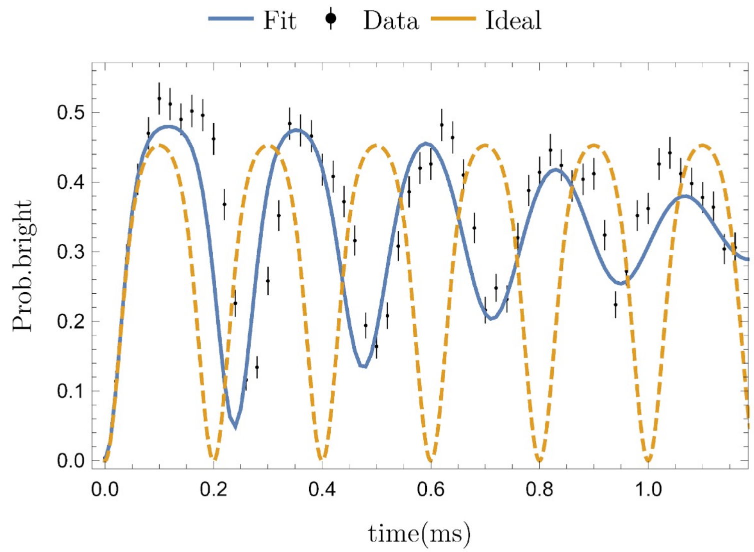
Map fermions to spins and bosons to phonons



Z. Davoudi, NML, G. Pagano, Phys. Rev. Research 3, 043072 (2021)

Hybrid Quantum Simulation: The Yukawa model





apply ω_r and ω_b to one ion:

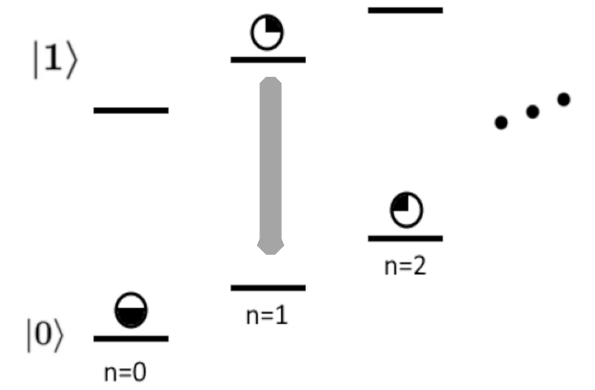
$$\hat{H}_i^{\mathrm{rb}} \approx \frac{\eta_{ik}\Omega_i}{2}\left(\mathrm{e}^{-i\delta_k t - i\phi_i^m}\hat{a}_k + \mathrm{e}^{i\delta_k t + i\phi_i^m}\hat{a}_k^\dagger\right)\hat{\sigma}_i^{\phi_i^s - \pi/2}$$

probability of bright state:

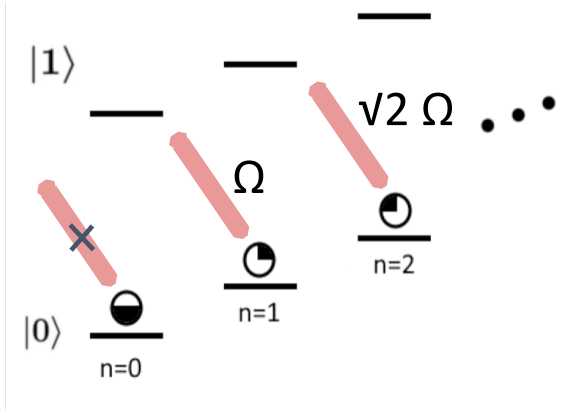
$$\frac{1}{2}\left\{1 - \exp\left[-\left(\frac{2\eta_{ik}\Omega_i}{\delta_k}\right)^2 \sin^2\left(\frac{\delta_k \tau}{2}\right)\right]\right\}$$

Hybrid Quantum Simulation: The Yukawa model

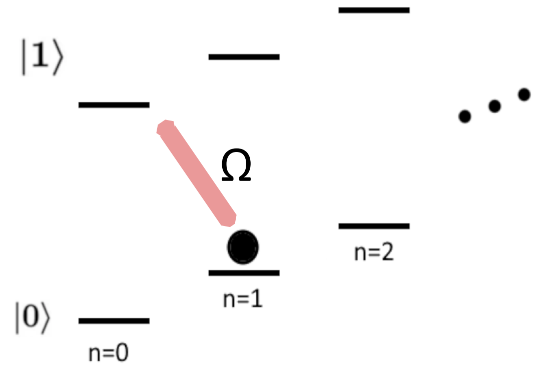
1. Pump to $|0\rangle$

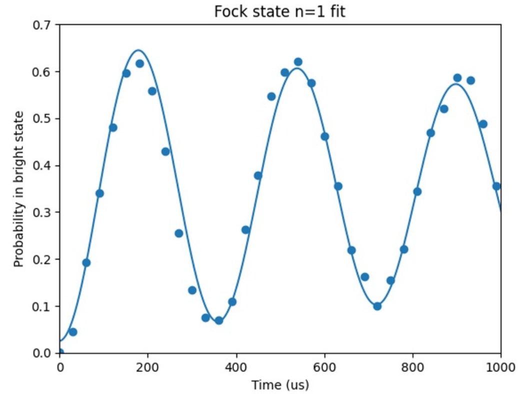


2. Apply red sideband and measure



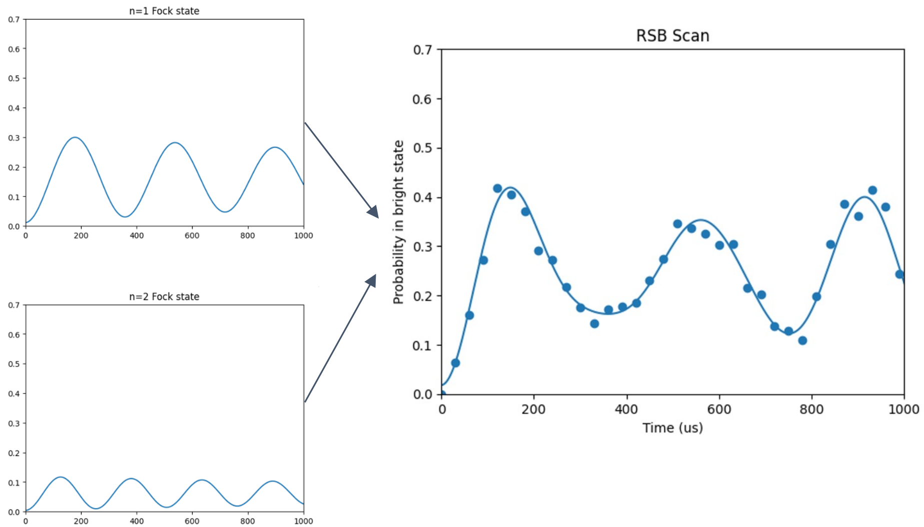
3. Fit red sideband flop for pure Fock state





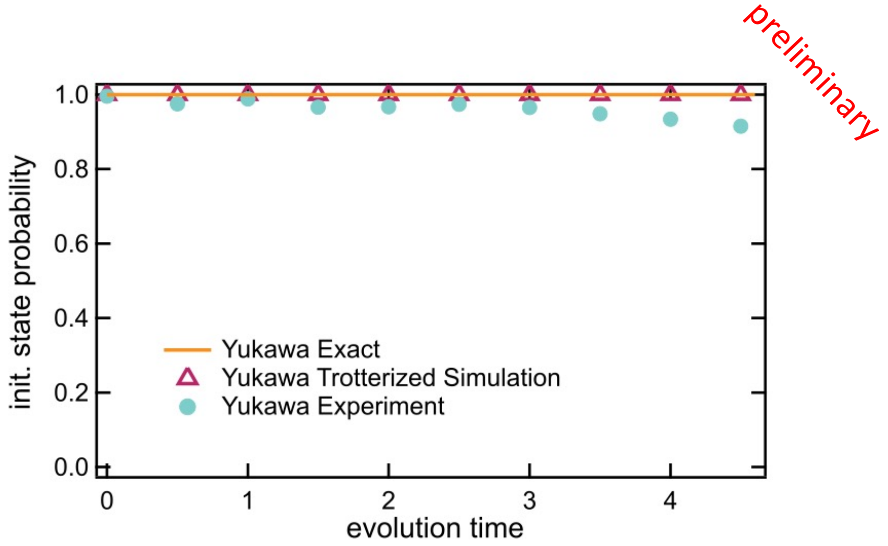
Hybrid Quantum Simulation: The Yukawa model

4. Fit superposition of Fock states



Hybrid Quantum Simulation: The Yukawa model

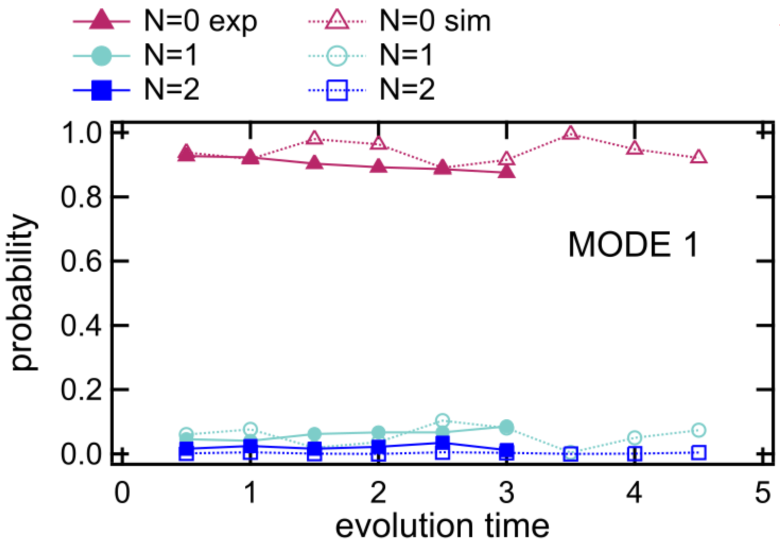
First implementation of Yukawa simulation with closed boundary conditions: spin evolution

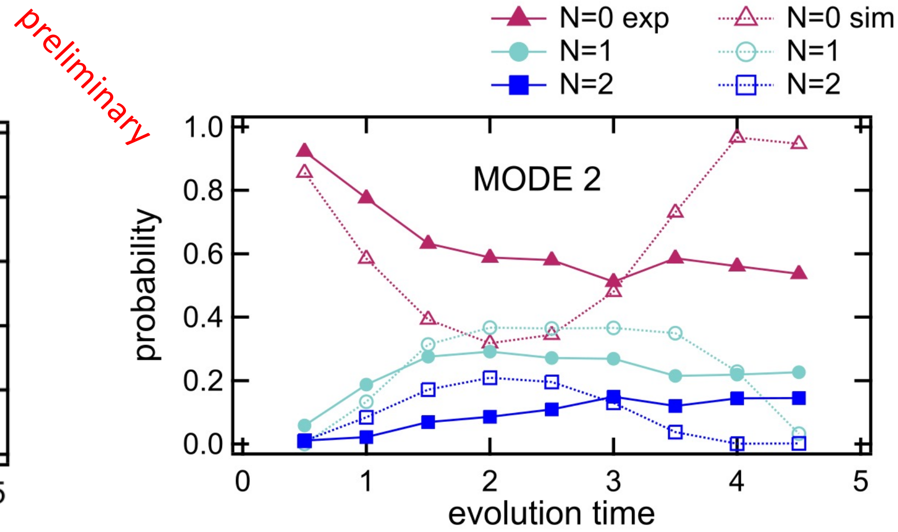


parameters: g=2, b=1, m_ψ=1, m_ϕ=1.5

Hybrid Quantum Simulation: The Yukawa model

First implementation of Yukawa simulation with closed boundary conditions: motional modes

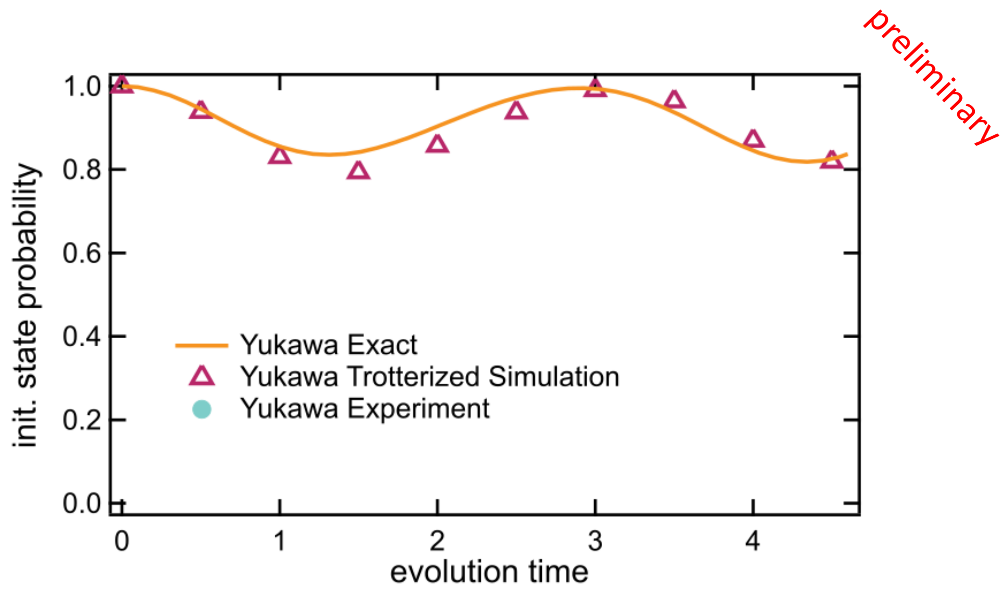




parameters: $g=2$, $b=1$, $m_{\psi}=1$, $m_{\phi}=1.5$

Hybrid Quantum Simulation: The Yukawa model

First implementation of Yukawa simulation with **open** boundary conditions: spin evolution

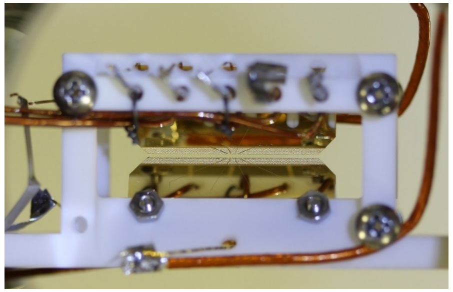


parameters: $g=2$, $b=1$, $m_{\psi}=1$, $m_{\phi}=1.5$

Outlook: A new ion trap platform

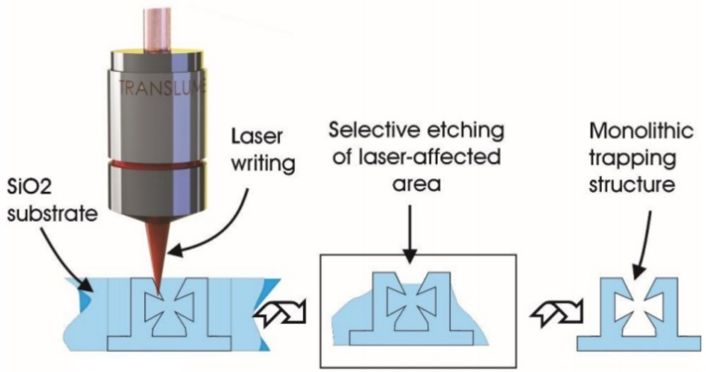
collaboration with G. Pagano (Rice)

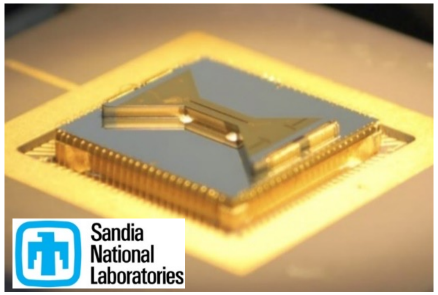
Outlook: A new ion trap platform



3D blade trap (with problems)

Monolithic 3D trap made of Fused Silica by Translume Inc.

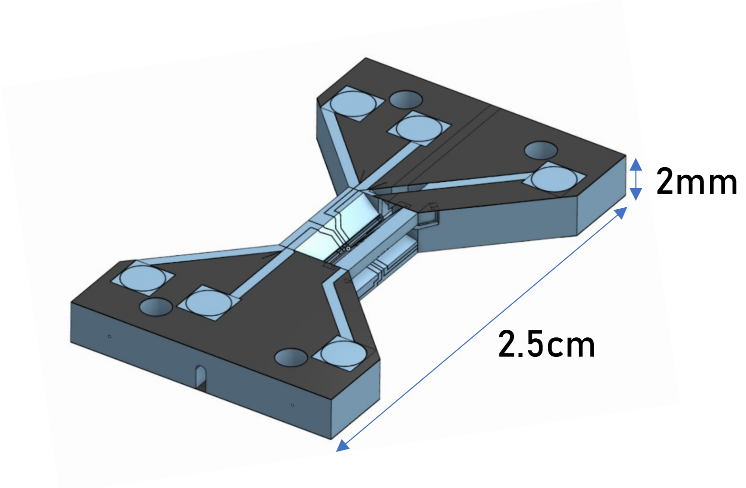


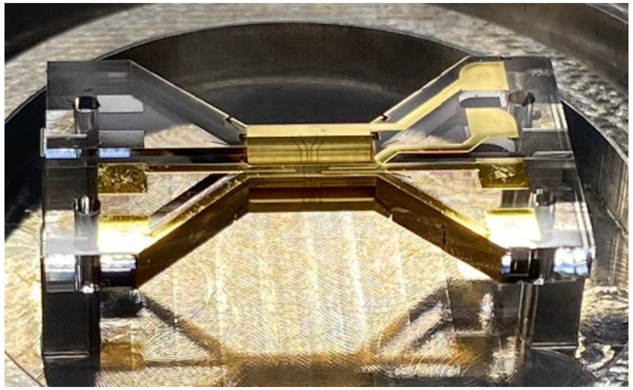


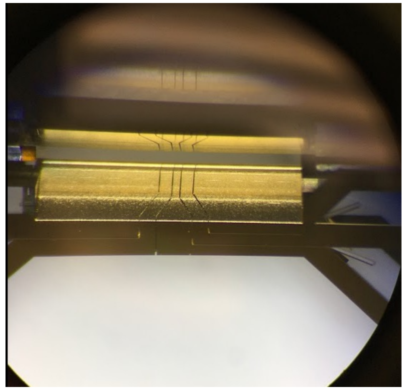
2D surface traps (with fewer/different problems):
DQC groups, PTB, NICT, Honeywell, NIST etc.

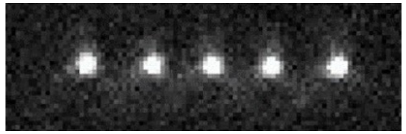
collaboration with G. Pagano (Rice)

Outlook: A new ion trap platform



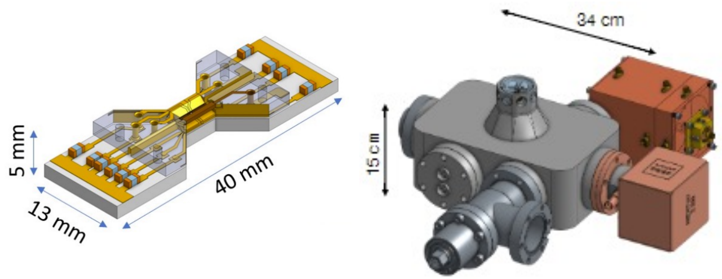






TAMOS est. Oct. 2023

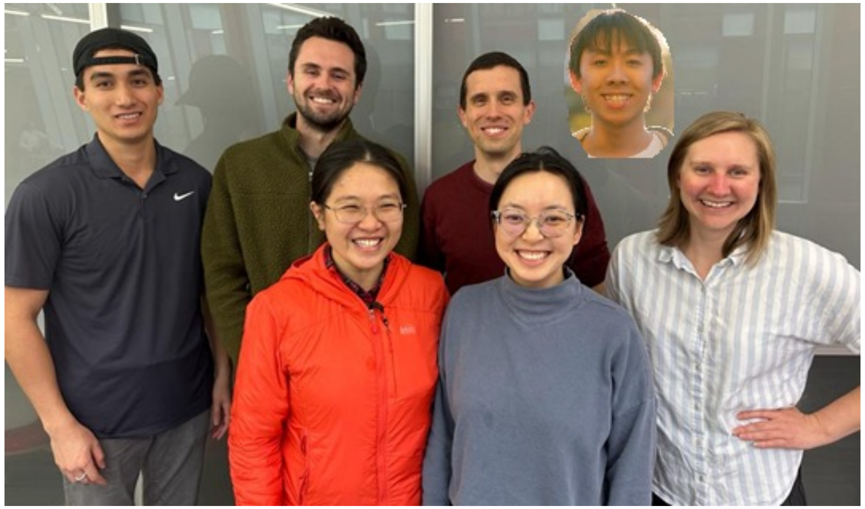
R. Zhuravel G. Pagano NML



UNIVERSITY OF MARYLAND

Duke Quantum Center

Elijah Mossman (u) | Liam Jeanette | NML | **Anton Than** | **Alaina Green**



Nhung Nguyen -> Quantinuum | **Yingyue Zhu**

Michael Straus | Xinyi Dai | Thomas Kim (u) | Devon Valdez | Denton Wu

NML | Yuanheng Xie | Ecem Duman (u) | Ana Ferrari | Mika Chmielewski

Zohreh Davoudi (UMD)

Saurabh Kadam (UMD)

Guido Pagano (Rice)

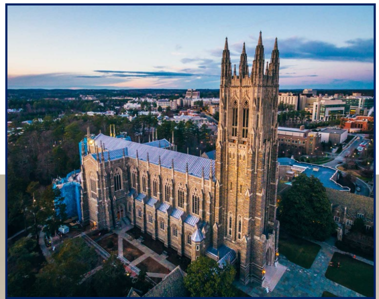
QUANTUM SYSTEMS ACCELERATOR

Catalyzing the Quantum Ecosystem

NSF Convergence Accelerator

Institute for Robust Quantum Simulation

Duke Quantum Center



Brown

Calderbank

Cetina

Kim

Klco

Kozhanov

Linke

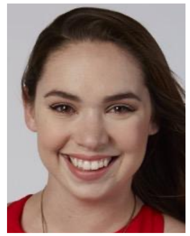
Loh

Marvian

Monroe

Nicholson

Barthel



Noel