Tensor Networks for real time: scattering in the Schwinger model

Mari-Carmen Bañuls

I. Papaestathiou (MPQ), J. Knolle (TUM)

arXiv:2402.18429

MAX PLANCK INSTITUTE
OF QUANTUM OPTICS

MCQST

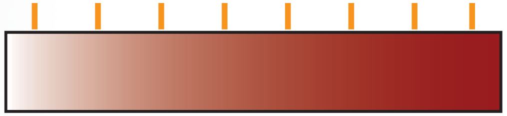
DFG FOR 5522

DFG TRR 360

QuantHEP
Munich 2.9.2024

TNS: entanglement-based ansatzes for quantum many-body states

arbitrary many-body state



d^N

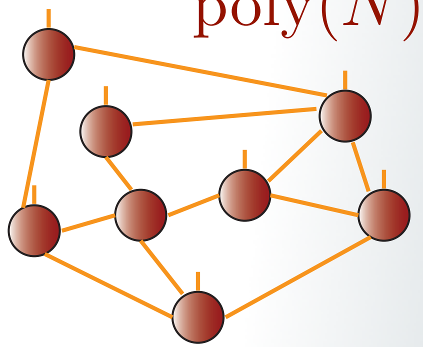
$$|\Psi\rangle = \sum_{i_j} c_{i_1 \dots i_N} |i_1 \dots i_N\rangle$$

exponential

polynomial

TNS: restricted family

$\mathrm{poly}(N)$



good ansatz for ground states and thermal equilibrium: area law

entanglement hierarchy

efficient numerics

Tensor Network States (TNS)

efficient numerical algorithms (small spatial

dim ... ing

2023 14:1;

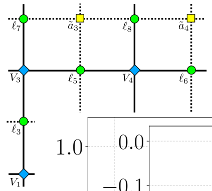
suitable for LGT

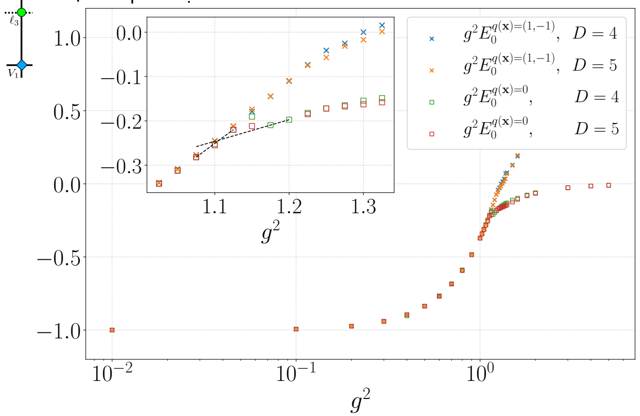
systematically explored in 1+1D

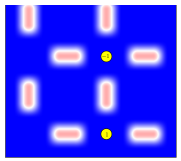
MCB, Cichy, ROPP (2020)
QTFLAG Coll. EPJD (2020)

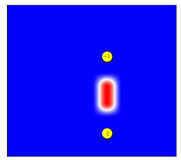
results in higher dimensions

Robaina et al PRL 2021
Magnifico et al. Nat. Com. (2021)









wor ... ium

but ... ches

al., PRL 2017

Schuch *et al.*, NJP 2008

volume law

new tools may allow us to access some of these regimes

TNS

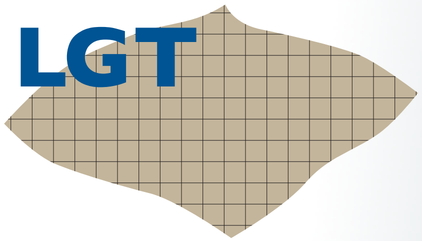
Non-perturbative for Hamiltonian systems

Extremely practical (and successful) for 1D systems (MPS)

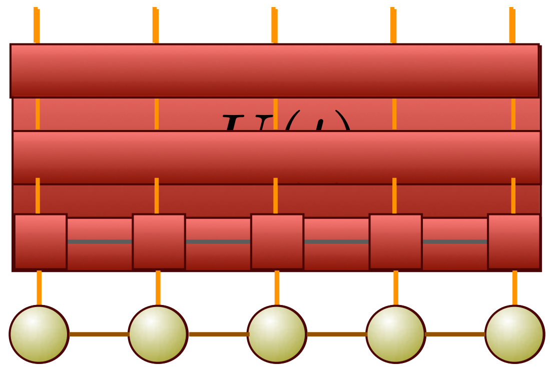
Promising improvements for higher dimensions

ground states
low-lying excitations
thermal states
time evolution

apply to



real time evolution with MPS



TEBD, t-DMRG

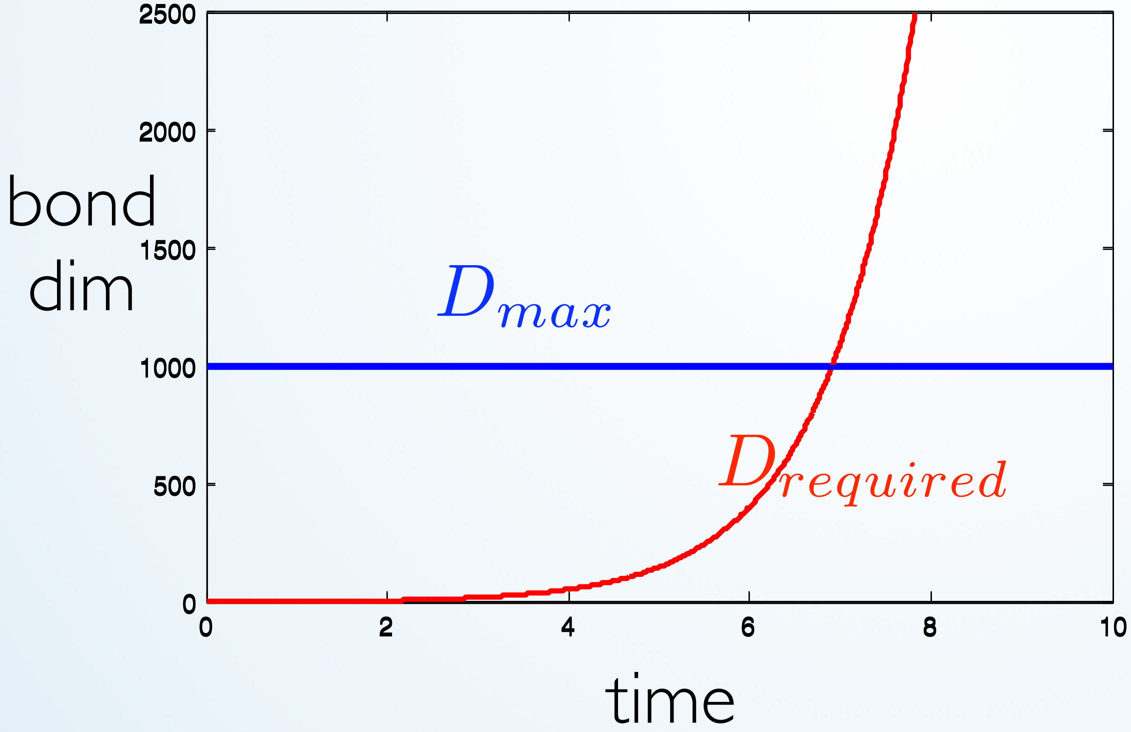
Vidal, PRL 2003, 2004

Verstraete, García-Ripoll, Cirac, PRL 2004

time evolved state approximated by MPS

but entanglement grows

Osborne, PRL 2006
Schuch et al., NJP 2008



required bond for fixed precision

$$D \sim e^{\alpha t}$$

yet many physical situations (in closed and open quantum systems) can be successfully studied!

short times, adiabatic, low energy can work well

García-Ripoll, NJP 2006
Wall, Carr NJP 2012
Paeckel et al arXiv:1901.05824

Standard evolution algorithms for LGT

Reliable for moderate times, or in some setups

Useful for quantum simulation

S. Kühn et al., Phys. Rev. A 90, 042305 (2014)
S. Kühn et al., JHEP 07 (2015) 130
Buyens et al., PRL 2014; PRX 2016
Rico et al., PRL 2014; NJP 2014; PRX 2016

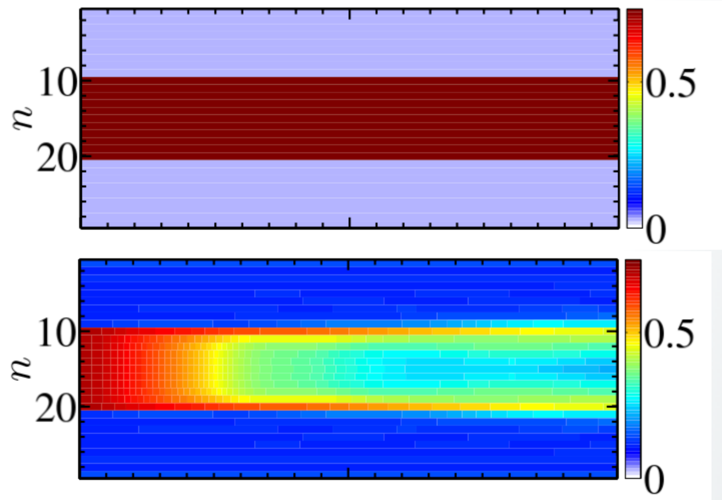
No full continuum extrapolation yet, but results near the continuum limit

string breaking

Non-Abelian string breaking phenomena with matrix product states

JHEP07(2015)130

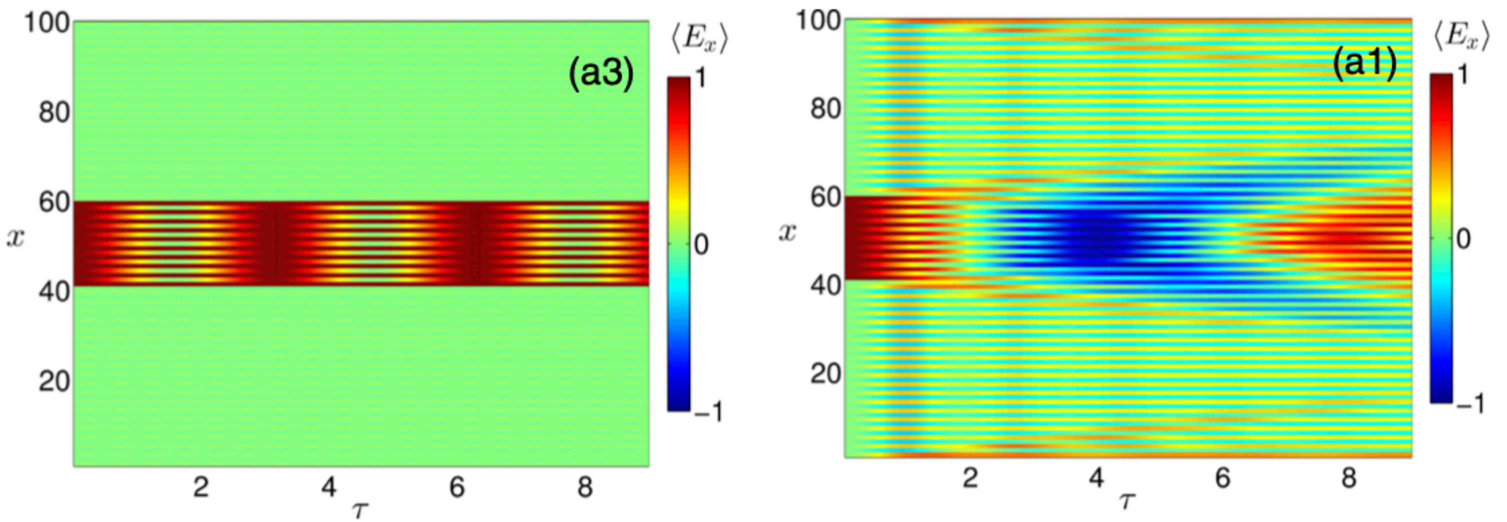
Stefan Kühn, Erez Zohar, J. Ignacio Cirac and Mari Carmen Bañuls



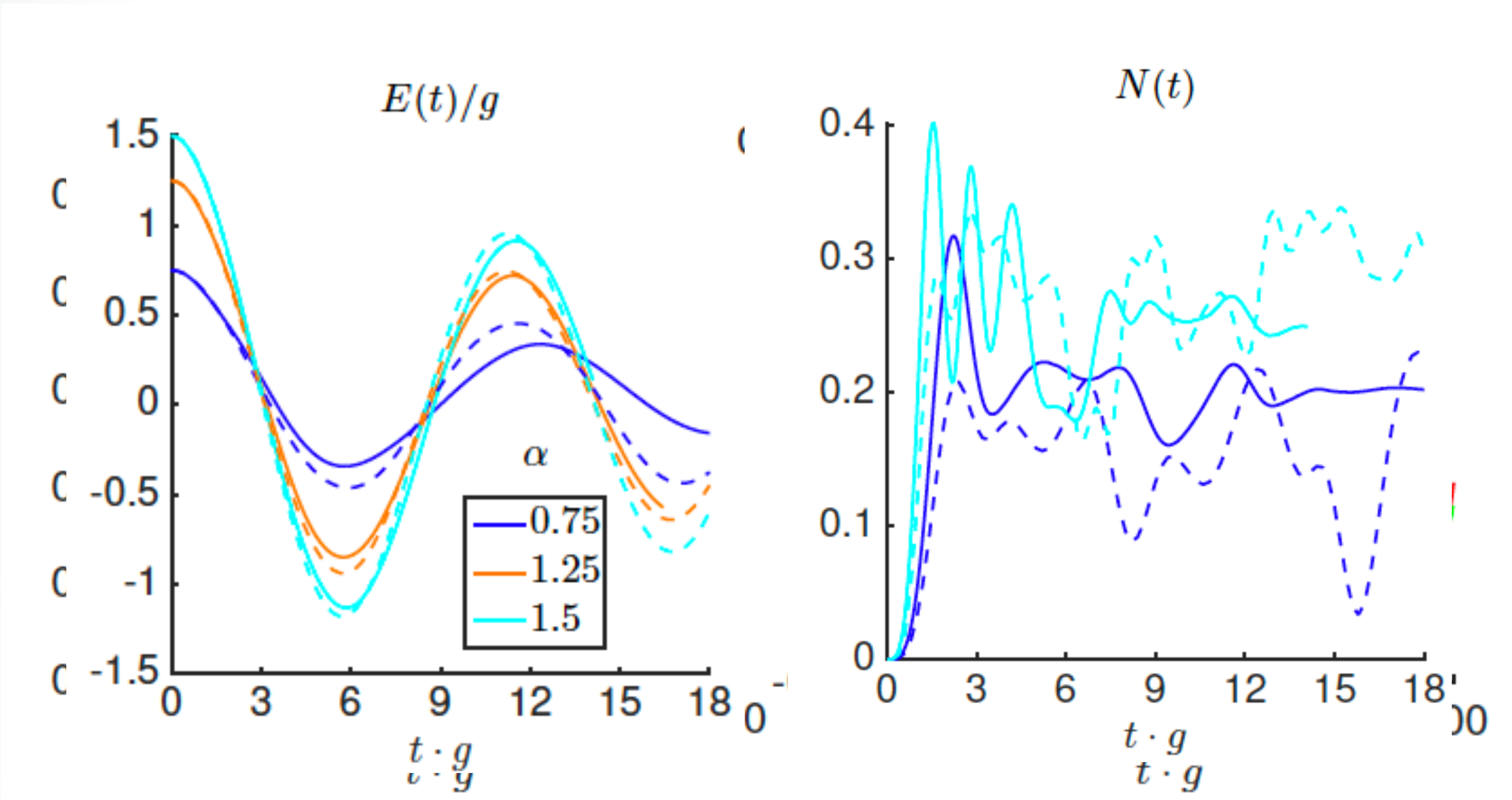
PHYSICAL REVIEW X 6, 011023 (2016)

Real-Time Dynamics in U(1) Lattice Gauge Theories with Tensor Networks

T. Pichler,[1] M. Dalmonte,[2,3] E. Rico,[4,5,6] P. Zoller,[2,3] and S. Montangero[1]



quench scenario



strong field
weak field

PHYSICAL REVIEW D **96,** 114501 (2017)

Real-time simulation of the Schwinger effect with matrix product states

Boye Buyens,[1] Jutho Haegeman,[1] Florian Hebenstreit,[2] Frank Verstraete,[1,3] and Karel Van Acoleyen[1]

scattering in the Schwinger model

with I. Papaefstathiou, J. Knolle, arXiv:2402.18429

Schwinger model

Schwinger '62

simplest gauge theory with matter

QED in 1+1 dimensions
electrons & photons

shows some of the features of *full* QCD

confinement → bound states (massive bosons)

fermion condensate

testbench for lattice techniques

Schwinger model

Schwinger '62

discrete Hamiltonian (staggered) formulation

Jordan-Wigner → spin model

Kogut, Susskind '75

$$H = \frac{1}{g^2 a^2} \sum_n \left(\sigma_n^+ e^{i\theta_n} \sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n} \sigma_n^- \right) + \frac{m}{a g^2} \sum_n \left(1 + (-1)^n \sigma_n^3\right) + \sum_n L_n^2$$

Gauss Law

$$L_n - L_{n-1} = \frac{1}{2}[\sigma_n^3 + (-1)^n]$$

$$| \ldots s_e \, \ell \, s_o \, \ell \, s_e \, \ell \, s_o \ldots \rangle$$

spin-electric flux basis suitable for MPS

works by Buyens et al., PRL 2014; arXiv:1509.00246

Rico et al., PRL 2014; NJP 2014

Schwinger model

Schwinger '62

discrete Hamiltonian (staggered) formulation

Jordan-Wigner → spin model

Kogut, Susskind '75

$$\sum_n L_n^2 \rightarrow \sum_n \sum_{k<n} (N-n)\sigma_k^z \sigma_n^z$$

integrating out Gauss Law (OBC)

Hamer'82

$$L_n = \ell_0 + \frac{1}{2} \sum_{k \leq n} \sigma_k^z + \ldots$$

$$|\ell_0 \ldots s_e \; s_o \; s_e \; s_o \ldots\rangle$$

physical spin basis

e.g. MCB, Cichy, Cirac, Jansen, JHEP11(2013)158

Schwinger model

Schwinger '62

Spectrum of stable particles: bound states

distinguished by symmetries in the continuum

vector first excited state over GS

different C, P charges from GS

strong coupling: fermion-antifermion

$$|1_V\rangle = \frac{1}{\sqrt{N}} \sum_n \left(\sigma_n^+ \sigma_{n+1}^- - \sigma_{n+1}^+ \sigma_n^-\right)|0\rangle$$

scalar C, P sector of GS

~bound state: pair of vectors

Numerically explored with standard MPS methods

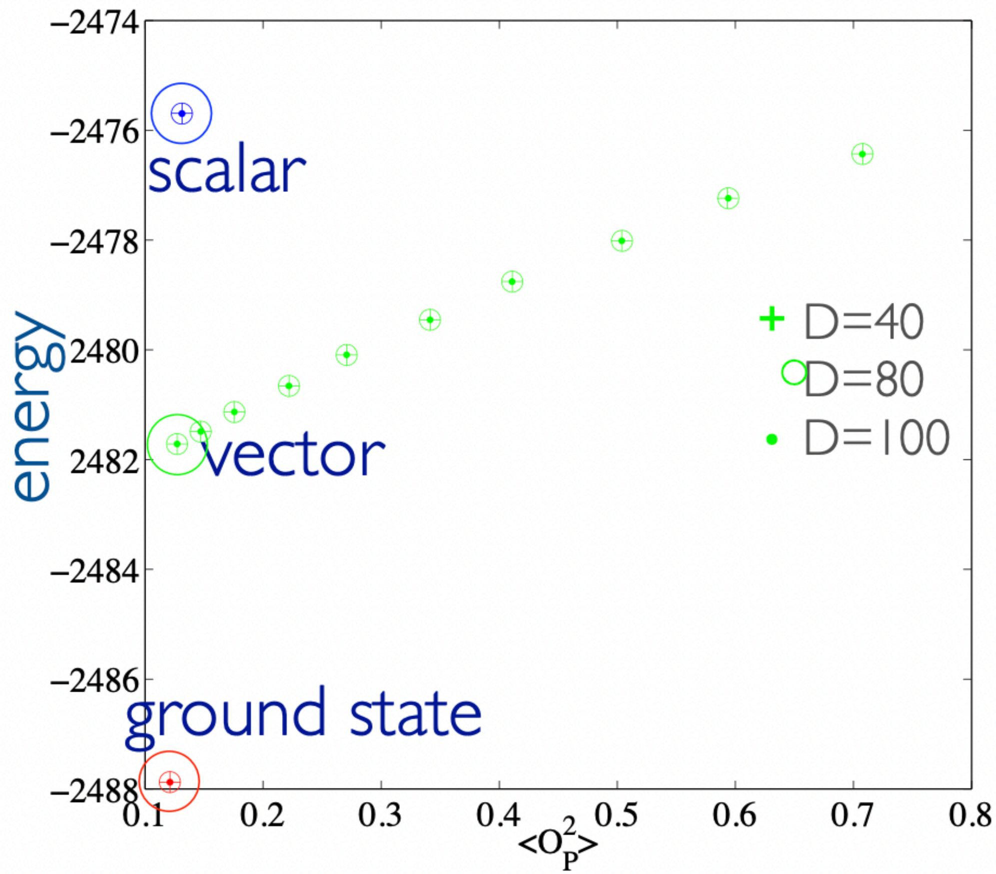
precise continuum limit

review 1910.00257

Schwinger model

Schwinger '62

Spectrum of stable particles: bound states



Numerically explored with standard MPS methods

precise continuum limit

review 1910.00257

we are interested in simulation of
(inelastic) scattering

scattering with MPS

uMPS formalism provides ansatz for quasiparticles

PHYSICAL REVIEW B **92**, 125136 (2015)

Scattering particles in quantum spin chains

Laurens Vanderstraeten,[1] Frank Verstraete,[1,2] and Jutho Haegeman[1]

PHYSICAL REVIEW RESEARCH **3**, 013078 (2021)

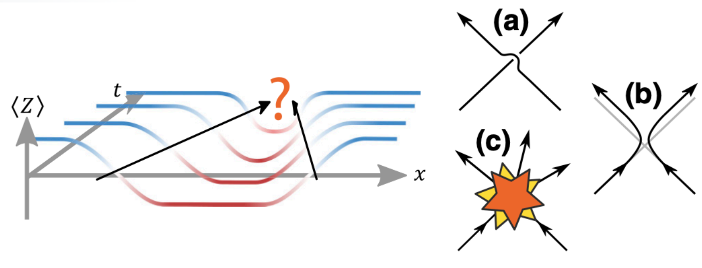
Real-time scattering of interacting quasiparticles in quantum spin chains

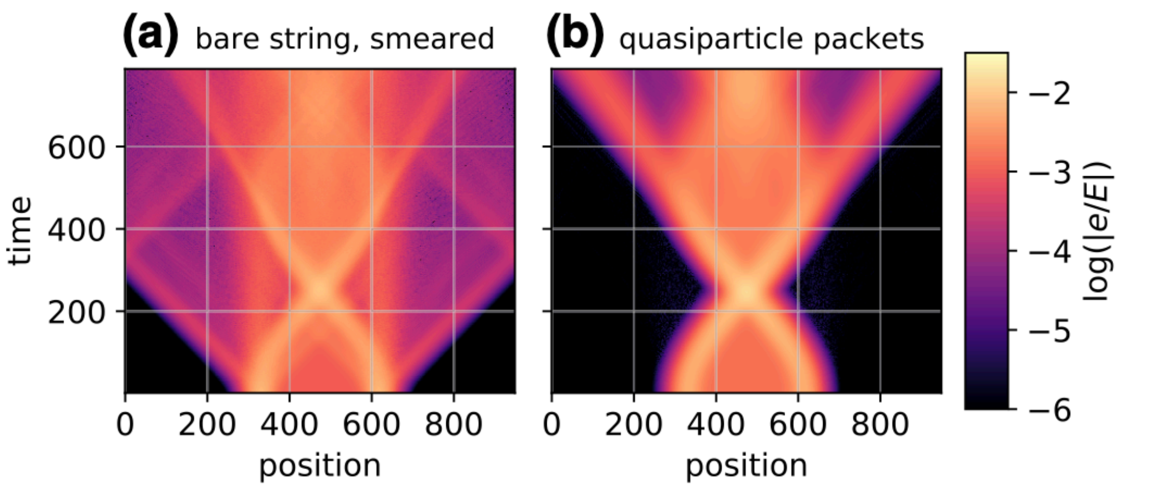
Maarten Van Damme,[*] Laurens Vanderstraeten, Jacopo De Nardis, Jutho Haegeman, and Frank Verstraete

PRX QUANTUM **3,** 020316 (2022)

Collisions of False-Vacuum Bubble Walls in a Quantum Spin Chain

Ashley Milsted,[1,2,3,4,*,‡] Junyu Liu,[1,2,†] John Preskill,[1,2,4,‡] and Guifre Vidal[3,5]





scattering in LGT

PHYSICAL REVIEW X **6,** 011023 (2016)

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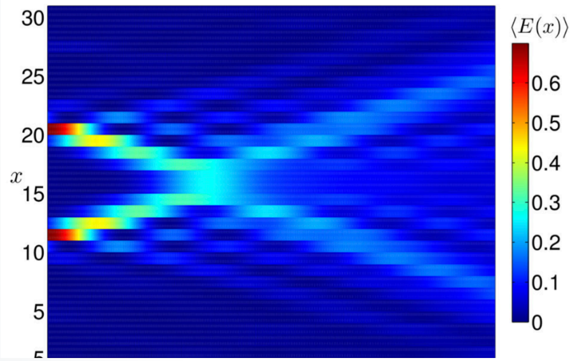
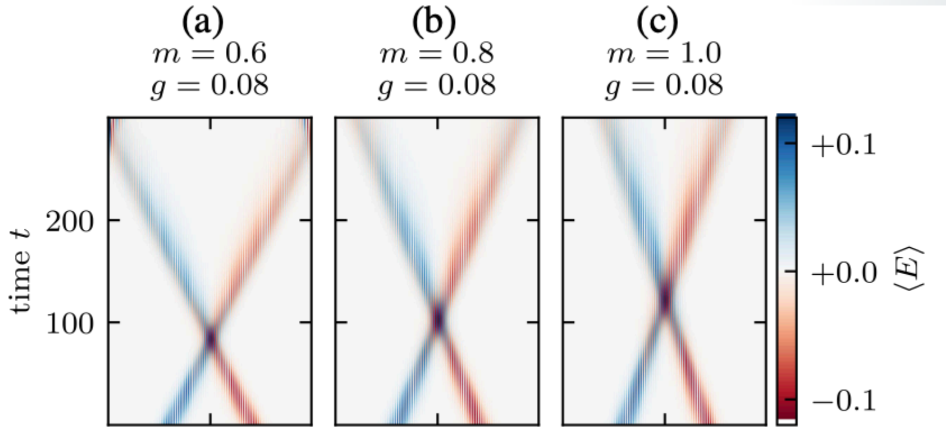


PHYSICAL REVIEW D **104,** 114501 (2021)

Entanglement generation in (1+1)D QED scattering processes

Marco Rigobello,* Simone Notarnicola, Giuseppe Magnifico, and Simone Montangero

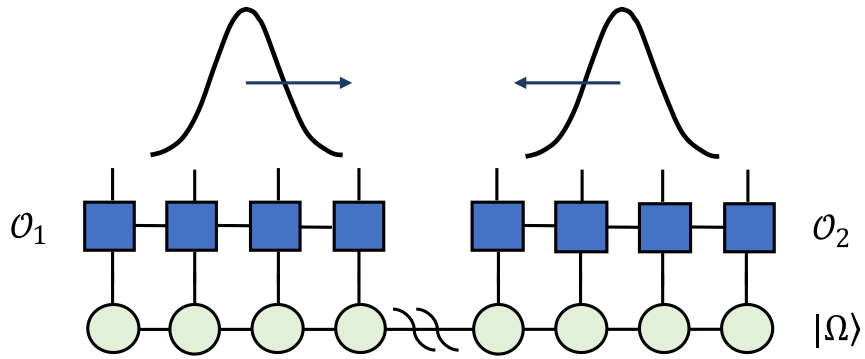


notice also:
Surace, Lerose, New J. Phys. 23 (2021) 062001
Vovrosh et al. PRX Quantum 3, 040309 (2022)
Su, Osborne, Halimeh arXiv:2401.05489

inelastic scattering in the Schwinger model

collision of two vector mesons can produce two scalars

initial setup



$$\sum_n e^{-(n-n_0)^2/(2\sigma^2)} e^{-ikn} \left(\sigma_n^+ \sigma_{n+1}^- - \sigma_{n+1}^+ \sigma_n^-\right) |\Omega\rangle$$

Gaussian wavepacket with momentum k

probe the inelastic threshold

strong coupling regime

Papaefstathiou, Knolle, MCB, arXiv:2402.18429

inelastic scattering in the Schwinger model

collision of two vector mesons can produce two scalars

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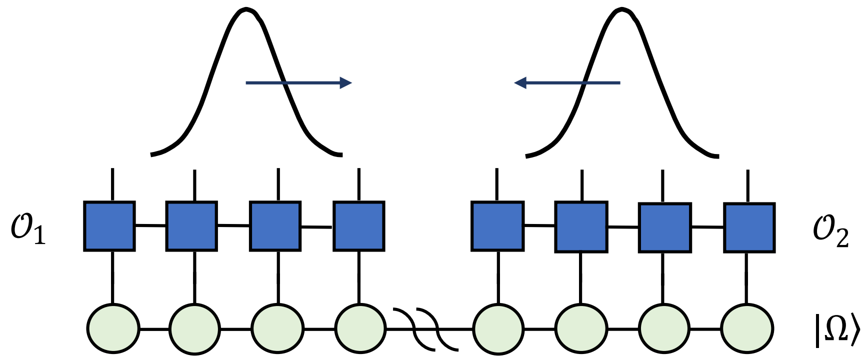

inelastic threshold

continuum $2\sqrt{p^2 + M_V^2} = 2\sqrt{M_S^2}$

lattice $k = p/(g\sqrt{x})$

$$x = 1;\ N = 100;\ \mu = 2 \cdot 10^{-5} \Rightarrow k_{\mathrm{thr}} = 1.12$$

Papaefstathiou, Knolle, MCB, arXiv:2402.18429

inelastic scattering in the Schwinger model

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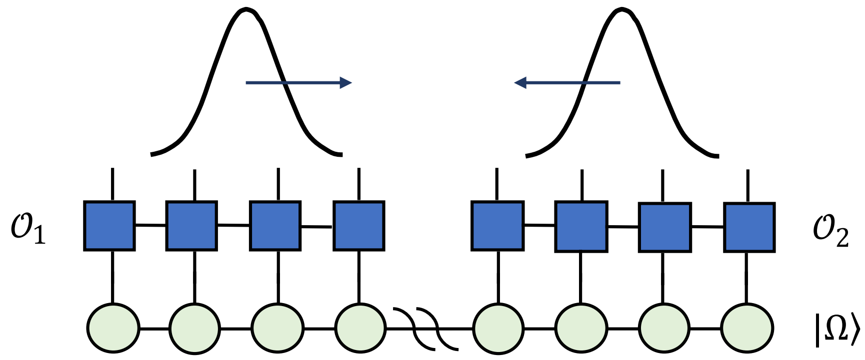
initial setup



observables (mostly local)

entropy of two sites

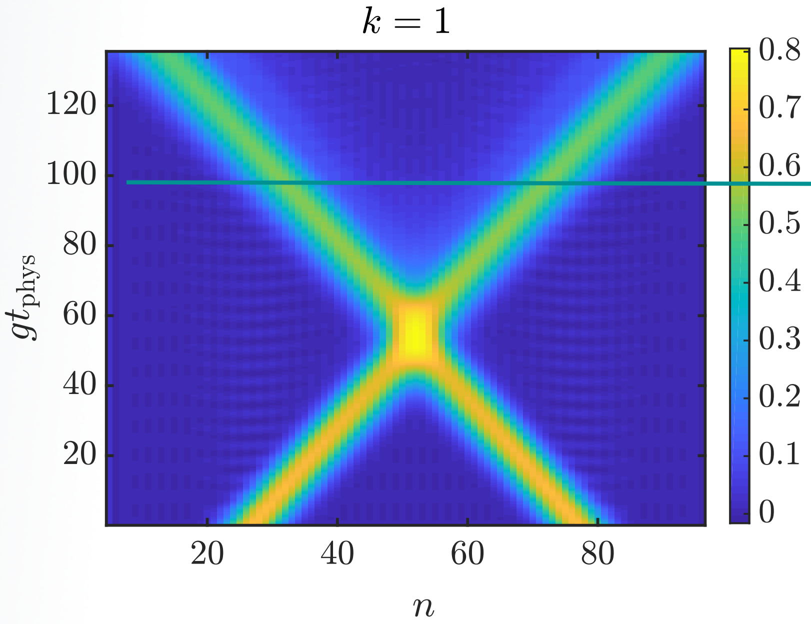
4-fermion projector (strong coupling)

electric flux correlator (not local)

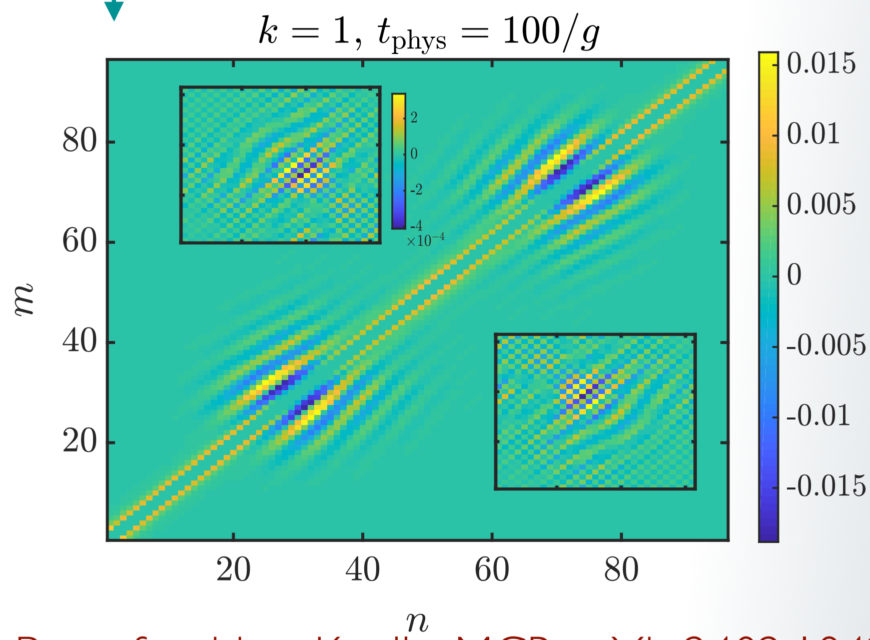
Papaefstathiou, Knolle, MCB, arXiv:2402.18429

below momentum threshold

entropy of two sites



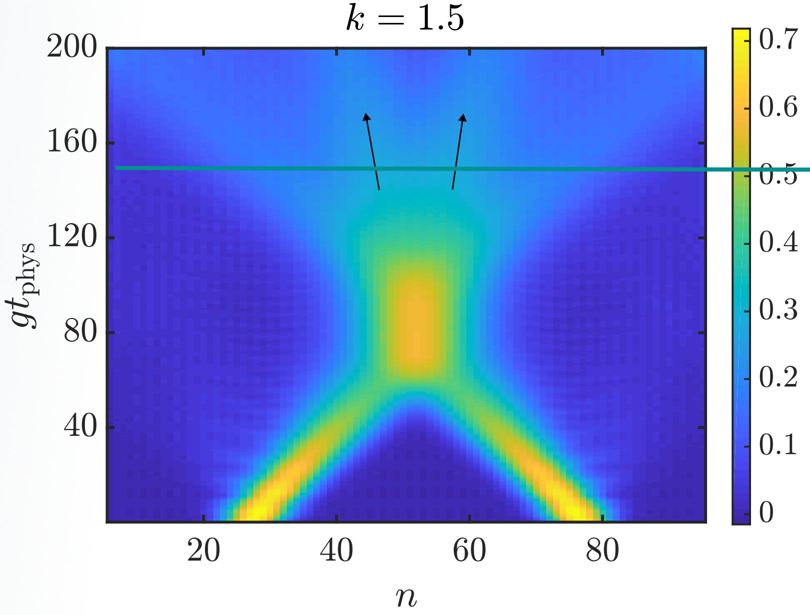
electric flux correlator

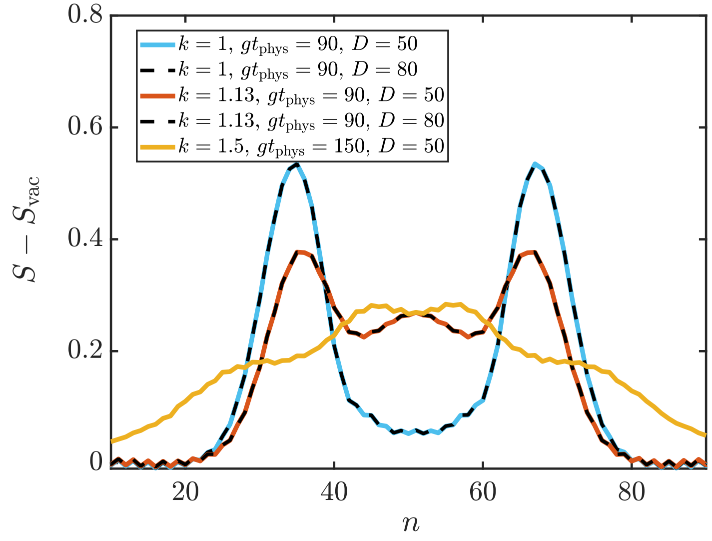


Papaefstathiou, Knolle, MCB, arXiv:2402.18429

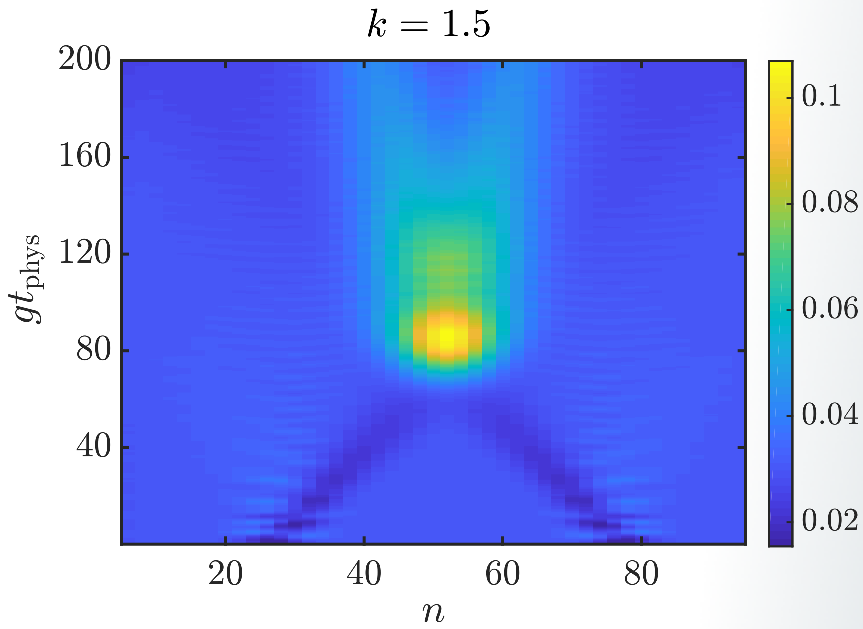
above momentum threshold

entropy of two sites





4-fermion projector



Papaefstathiou, Knolle, MCB, arXiv:2402.18429

Thanks for your attention!

TNS can be a suitable ansatz also for LGT/QFT

real time is more challenging than equilibrium

standard methods limited to short times

current results: inelastic scattering

Papaefstathiou, arXiv:2402.18429

- momentum threshold observed
- production detected through local observables

further directions

- challenge: continuum limit