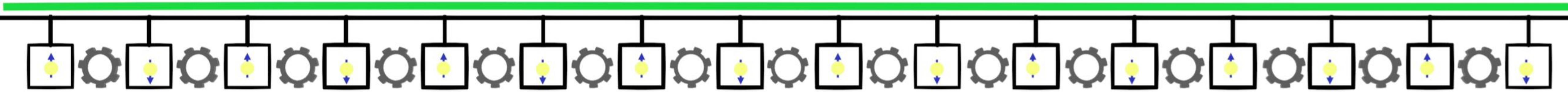
Towards preparation of scattering wave packets of hadrons on a quantum computer

A NISQ Algorithm for Lattice Gauge Theories

arXiv: 2402.00840



Saurabh Kadam[1]

with

Zohreh Davoudi[2] and Chung-Chun Hsieh[2]

Zohreh Davoudi

1. InQubator for Quantum Simulation, University of Washington

2. University of Maryland

Chung-Chun Hsieh

Gauge theories

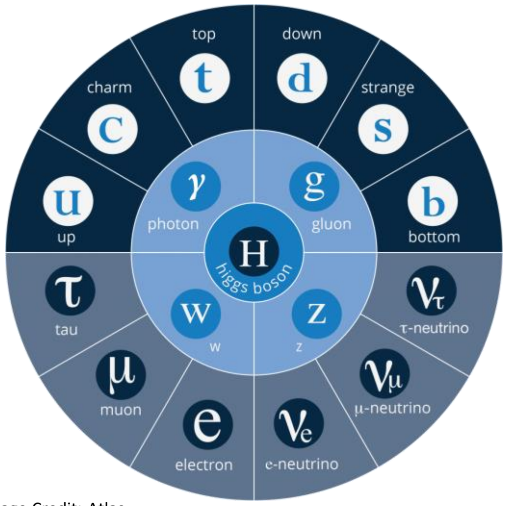
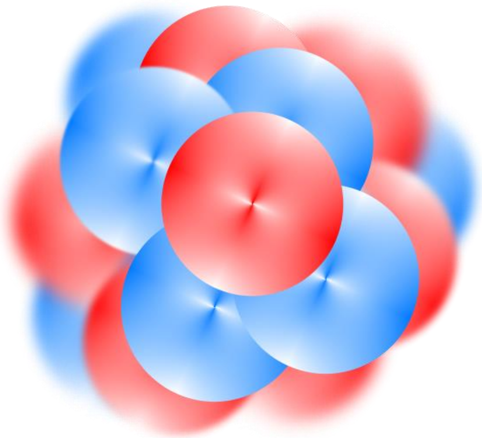


Image Credit: Atlas

Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$

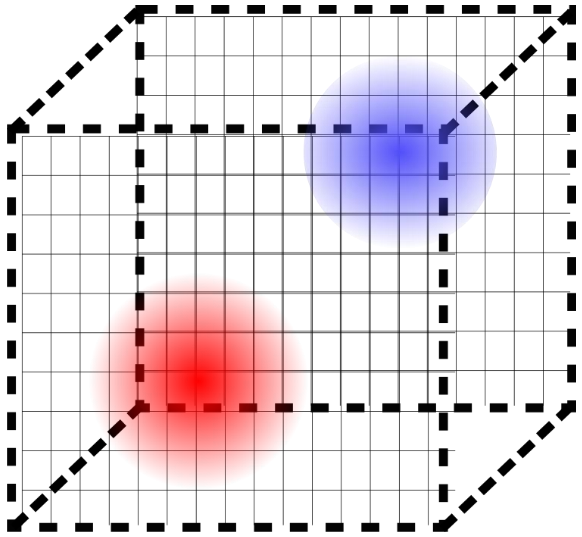
- Forces are described by gauge symmetries
- Weak and strong forces are described by non-Abelian gauge theories



Strong force: Quantum chromodynamics (QCD)

- Nuclear force: Theory of interacting quarks mediated by gluons
- Becomes strongly interacting at low energies
- Requires non-perturbative methods for calculating observables

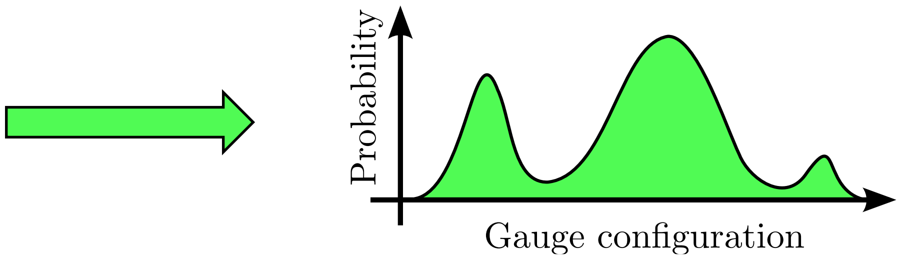
Non-perturbative method of solving QCD



Lattice QCD

- QCD Lagrangian on a discrete spacetime grid and Wick rotate to Euclidean time
- Observables are calculated using the path integral formalism
- Monte Carlo methods for probability distribution of gauge configurations

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{Z_0} \int \mathcal{D}[\bar{\psi}, \psi]\mathcal{D}[A]\ e^{-\int d^4 x_E \mathcal{L}_E} \mathcal{O}$$



Non-perturbative method of solving QCD

FLAG Review 2021
Eur. Phys. J. C 82 (2022) 10, 869

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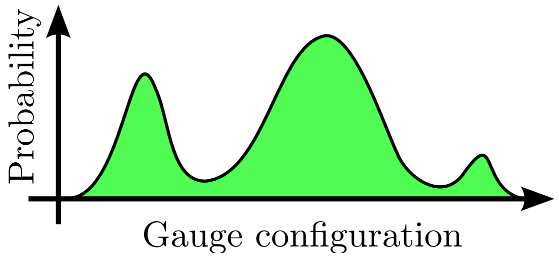


Successes

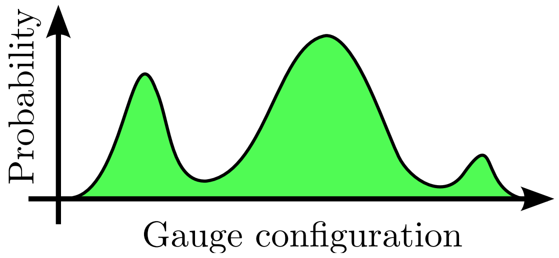
- ✓ Hadron spectrum and exotic states
- ✓ Hadron form factors
- ✓ Values of quark masses and the strong coupling constant
- ✓ Decay rates and low energy constants
- ✓ Two- and three-body scattering amplitudes

Shortcomings

- QCD phase diagram:
  Sign problem:
  Loss of probability distribution interpretation
- Euclidean time:
  Real time evolution of system
- Many-body processes are harder to obtain

Non-perturbative method of solving QCD

$$\langle\hat{\mathcal{O}}\rangle = \frac{1}{Z_0}\int\mathcal{D}[\bar{\psi},\psi]\mathcal{D}[A]\; e^{-\int d^4x_E\mathcal{L}_E}\mathcal{O}$$



Hamiltonian Formulation

$$\langle\hat{\mathcal{O}(t)}\rangle = \langle 0|e^{iHt}\hat{\mathcal{O}}(0)e^{-iHt}|0\rangle$$

1. No sign problem
2. Both real- and imaginary-time evolution
3. Many-body processes and scattering
4. Hilbert space scales exponentially with the system size

Quantum simulation

Shortcomings

- QCD phase diagram:

  Sign problem:

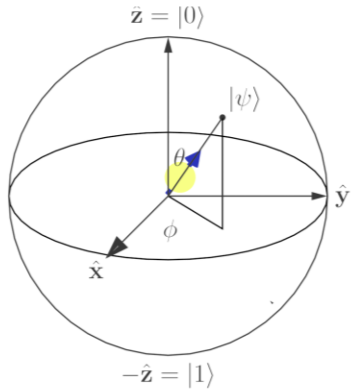
  Loss of probability distribution interpretation
- Euclidean time:

  Real time evolution of system
- Many-body processes are harder to obtain

Digital Quantum simulation

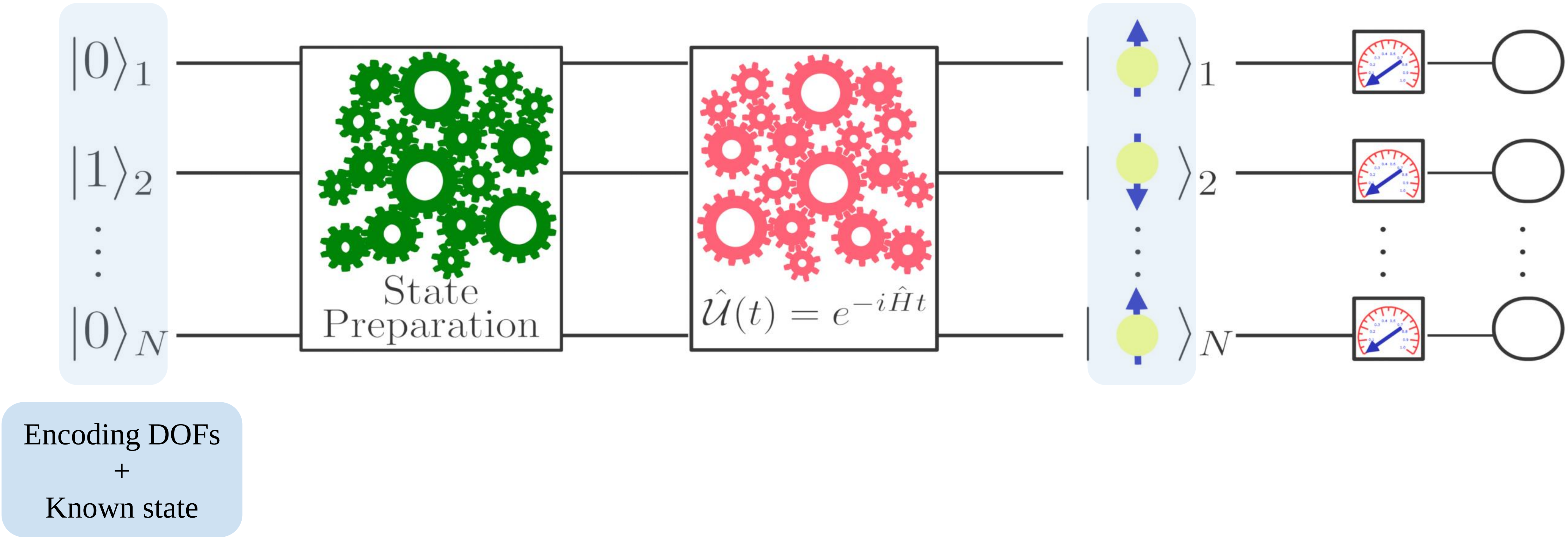
❖ Qubits:

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$$



$$|\psi\rangle_1 \otimes |\psi\rangle_2 \otimes \cdots \otimes |\psi\rangle_N \sim 2^N$$

❖ Schematic protocol for scattering



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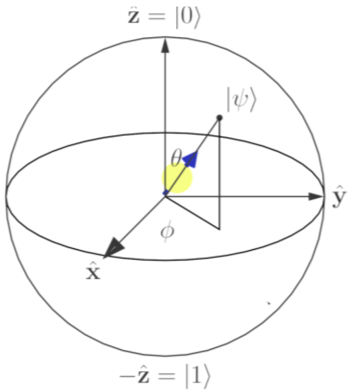

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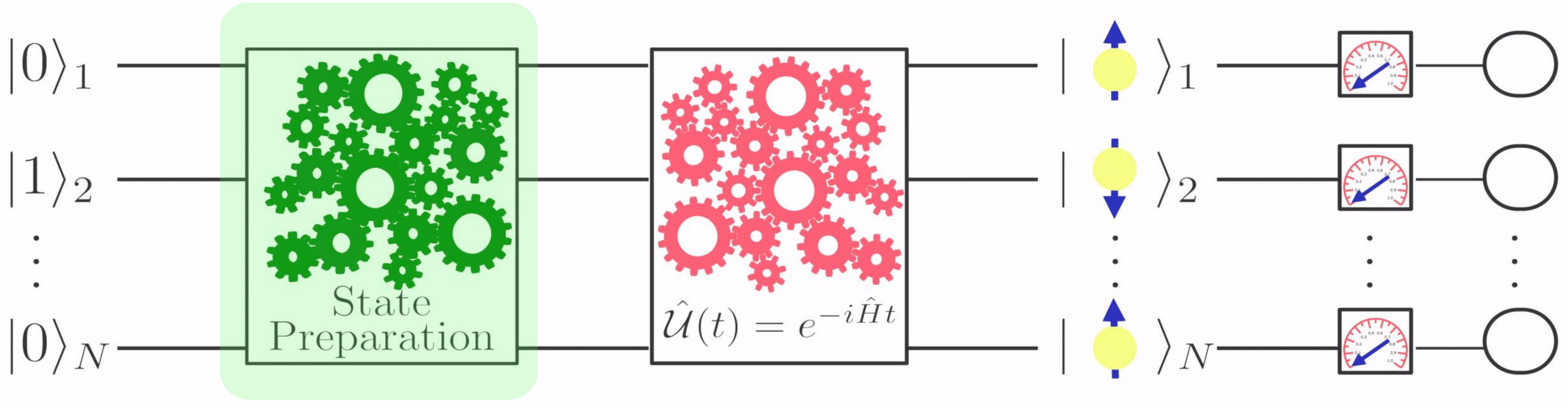

Encoding DOFs
+
Known state

Prepare a scattering state

Digital Quantum simulation

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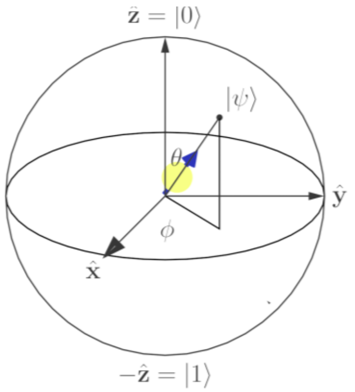

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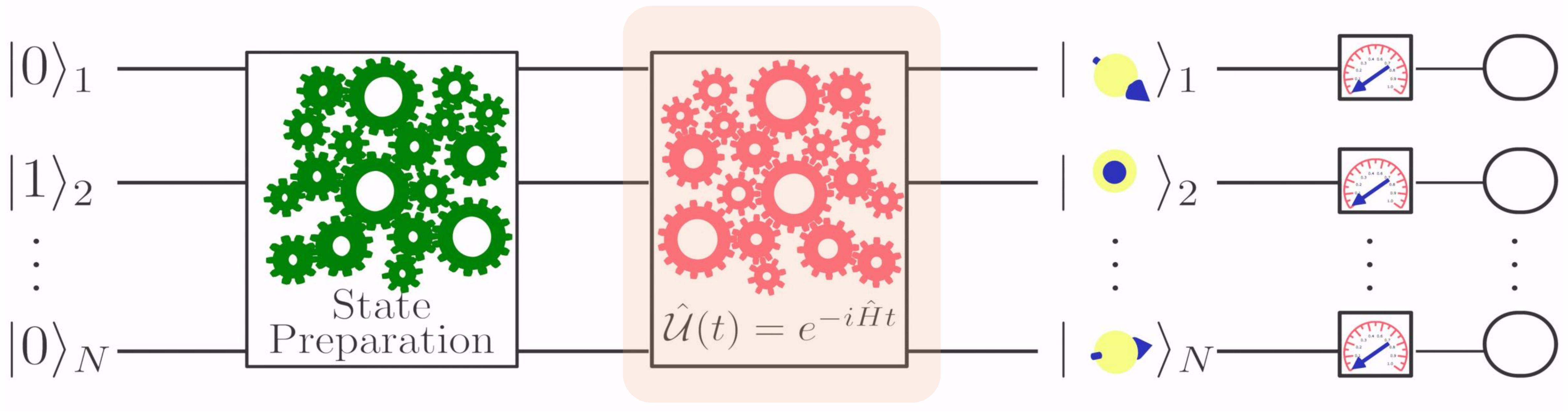

Encoding DOFs + Known state

Prepare a scattering state

Unitary time evolution

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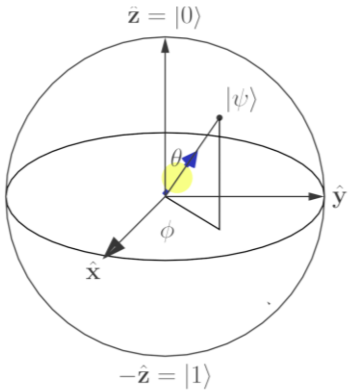

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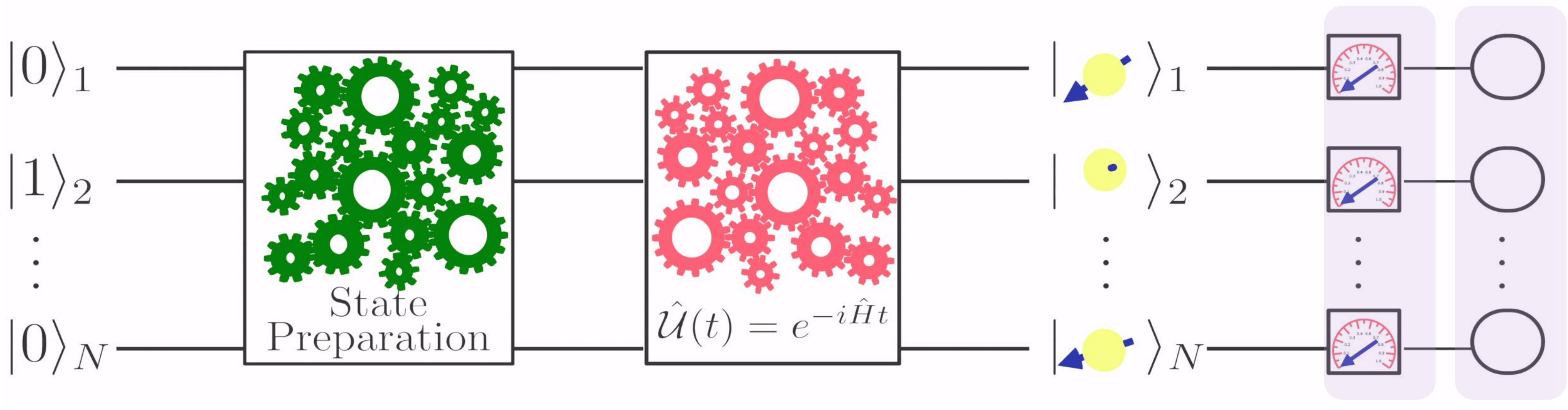


Encoding DOFs + Known state

Prepare a scattering state

Unitary time evolution

Measure

The Pioneering work

Jordan, Lee, and Preskill (JLP)

Science 336, 1130-1133 (2012)

Quantum Info. and Comp. 14, 1014-80 (2014)

Non-interacting theory

Non-interacting wave packet

$a_\psi^\dagger |\Omega\rangle_0$

Known operators

$a_\psi^\dagger = \sum_k \psi(k) a_k^\dagger$

$|\Omega\rangle_0$

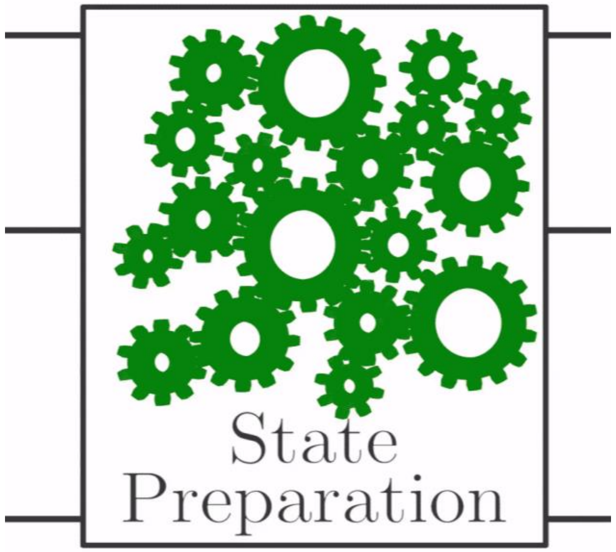
Non-interacting ground state

Interacting theory

Interacting wave packet

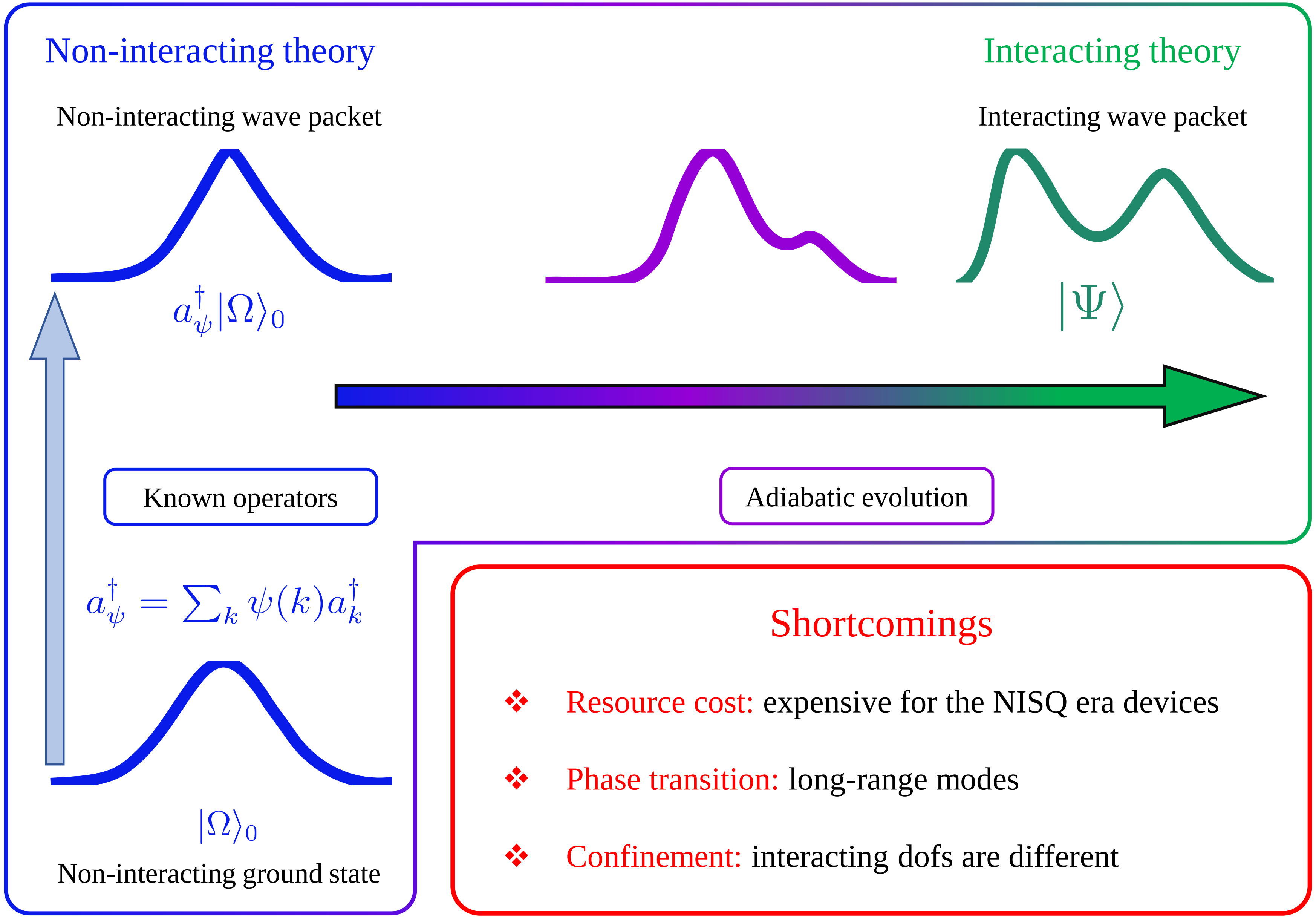
$|\Psi\rangle$

Adiabatic evolution



Shortcomings

- Resource cost: expensive for the NISQ era devices
- Phase transition: long-range modes
- Confinement: interacting dofs are different



Shortcomings

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Other methods

Jordan, Lee, and Preskill (JLP)

Science 336, 1130-1133 (2012)

Quantum Info. and Comp. 14, 1014-80 (2014)

Digital

Adiabatic

- Jordan, Lee and Preskill (JLP) arXiv:1404.7115 [hep-th] (2014)
- Barata, Mueller, Tarasov and Venugopalan Phys. Rev. A, 103, 042410 (2021)
- Farrell, Illa, Ciavarella and Savage arXiv: 2401.08044

Non-Adiabatic

- Turco,Quinta, Seixas, and Omar arXiv: 2305.07692 (2023)
- Kreshchuk, Vary, and Love arXiv: 2310.13742 (2023)
- Chai, Crippa, Jansen et al. arXiv:2312.02272 (2023)

Analog

Adiabatic

- Ciavarella, Caspar, Illa, Savage arXiv:2210.04965 (2022)

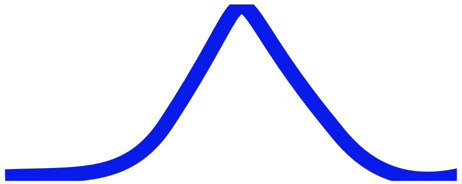
Non-Adiabatic

- Belyansky, Whitsitt, Mueller et al arXiv:2307.02522 (2023)
- Surace and Lerose New J. Phys. 23 062001 (2021)

Our approach

Non-interacting theory

Non-interacting wave packet



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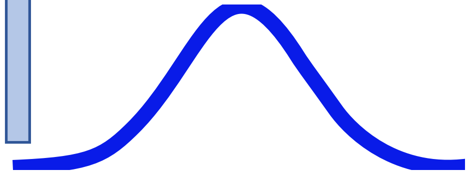


$|\Omega\rangle_0$

Non-interacting ground state

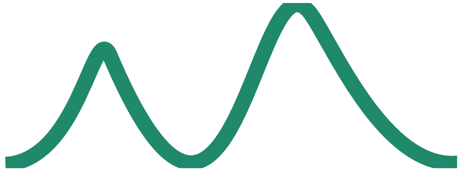
Interacting theory

Interacting wave packet

$|\Psi\rangle = b_\psi^\dagger|\Omega\rangle$

Ansatz for $b_k^\dagger$

$b_\psi^\dagger = \sum_k \psi(k) b_k^\dagger$



$|\Omega\rangle$

Interacting ground state

Rigobello, Notarnicola, Magnifico, and Montangero

Phys. Rev. D 104, 114501 (2021)

- Start with the interacting ground state
- Use an ansatz to build the interacting creation operators
- Act the wave packet creation operator on the interacting ground state

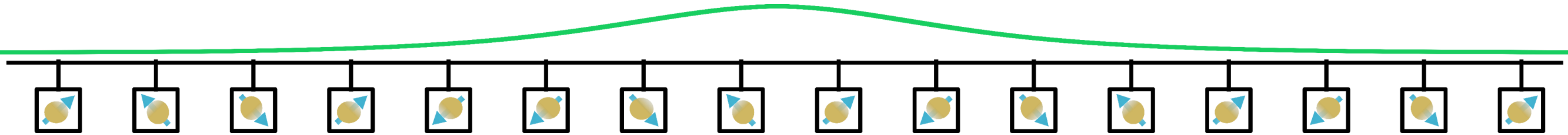
Outline

Preparation of scattering wave packets

- **Model:** *Z_2 (and U(1)) lattice gauge theory in 1+1D with matter*
- **Method:** *Construction of creation operators*
- **Mapping :** *Quantum algorithm and circuit*
- **Measurements:** *Hardware results from Quantinuum H1-1*
- **Moreover:** *Conclusions and outlook*

Model

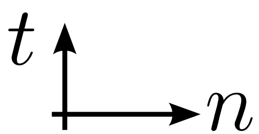
Z_2 Lattice Gauge Theory (LGT) in 1+1D with dynamical matter



Z_2 LGT in 1+1D

Motivation

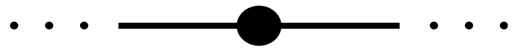
- Confined Theory
- $Z_N \xrightarrow{N\to\infty} U(1)$



Continuous time

Discretized space

Fermionic DOFs

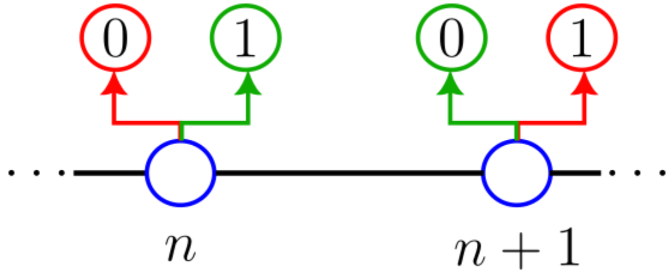


$\psi(n)$

Staggered Fermion

Dirac sea interpretation

No particle | Particle | Anti-particle | No anti-particle



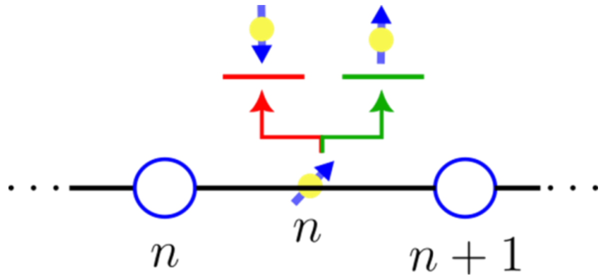
Matter → Even sites | Anti-matter → Odd sites

From staggering

$$H_M = m_f \sum_n (-1)^n \psi_n^\dagger \psi_n$$

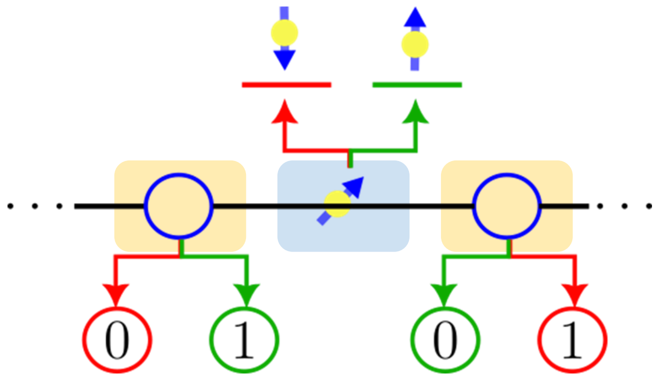
Z_2 LGT in 1+1D

Bosonic DOFs



$$H_E = \epsilon \sum_n \tilde{\sigma}_n^Z$$

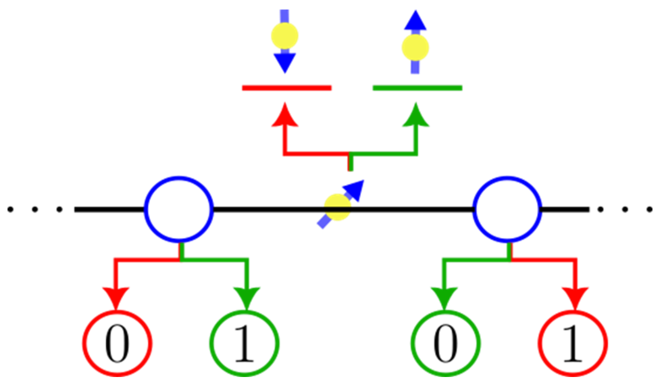
Fermionic + bosonic DOFs



$$H_I = \sum_n \psi_n^\dagger \tilde{\sigma}_n^X \psi_{n+1} + \mathrm{H.c.}$$

Z_2 LGT in 1+1D

Fermionic + bosonic DOFs



Are all 2^{2N} states physical states?

No!!

Z_2 LGT Hamiltonian

$$H_E = \epsilon \sum_n \tilde{\sigma}_n^Z$$

$$H_I = \sum_n \psi_n^\dagger \tilde{\sigma}_n^X \psi_{n+1} + \text{H.c.}$$

$$H_M = m_f \sum_n (-1)^n \psi_n^\dagger \psi_n$$

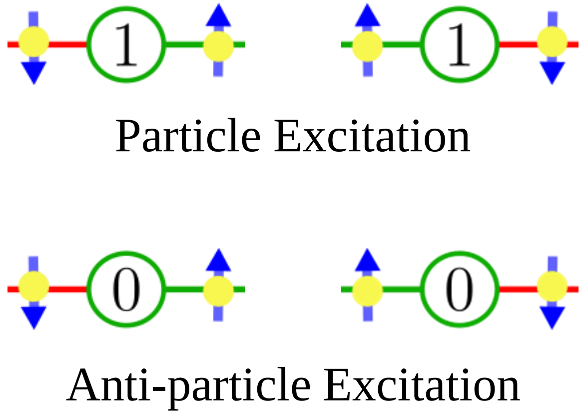
$$H_{Z_2} = H_E + H_I + H_M$$

Physical Hilbert space

Gauge invariance → Gauss's law

$$G_n |\psi\rangle_{\text{Phys}} = |\psi\rangle_{\text{Phys}} \quad \forall n$$

$$G_n = \tilde{\sigma}_{n-1}^Z \tilde{\sigma}_n^Z e^{i\pi\left(\psi_n^\dagger \psi_n - \frac{1-(-1)^n}{2}\right)}$$



Z_2 LGT in 1+1D

Periodic boundary condition with charge 0 sector

Z_2 LGT Hamiltonian

$$H_E = \epsilon \sum_n \tilde{\sigma}_n^Z$$

$$H_I = \sum_n \psi_n^\dagger \tilde{\sigma}_n^X \psi_{n+1} + \text{H.c.}$$

$$H_M = m_f \sum_n (-1)^n \psi_n^\dagger \psi_n$$

$$H_{Z_2} = H_E + H_I + H_M$$

Consequences

- Physical Hilbert space is a tiny fraction of the full Hilbert space

	Hilbert Space	
# Sites	Possible	Physical
4	256	12
6	4096	40

- Only mesonic excitations

Strong Coupling Vacuum (SCV)

No fermionic excitation

Low-energy boson configuration

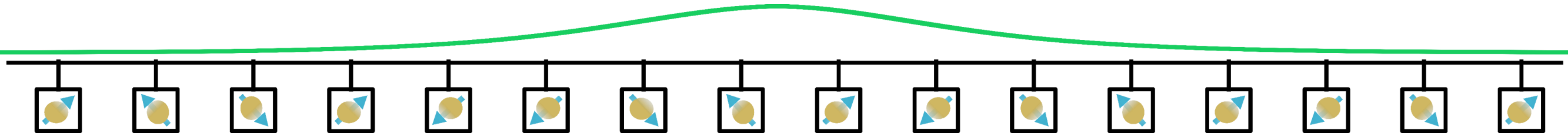
Example of a length-3 meson

Starts at particle site

Ends at anti-particle site

Method

Building creation operators in the interacting theory



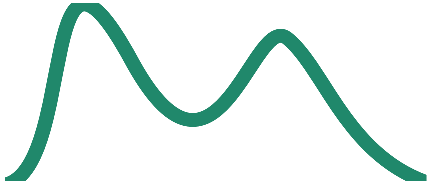
Building creation operators in interacting theory

Rigobello, Notarnicola, Magnifico, and Montangero

Phys. Rev. D 104, 114501 (2021)

Interacting theory

Interacting wave packet



$|\Psi\rangle = b_\psi^\dagger |\Omega\rangle$

Ansatz for $b_k^\dagger$

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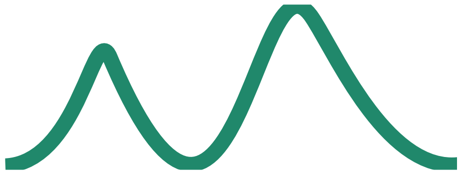


$|\Omega\rangle$

Interacting ground state

Build momentum creation operator from mesonic excitations

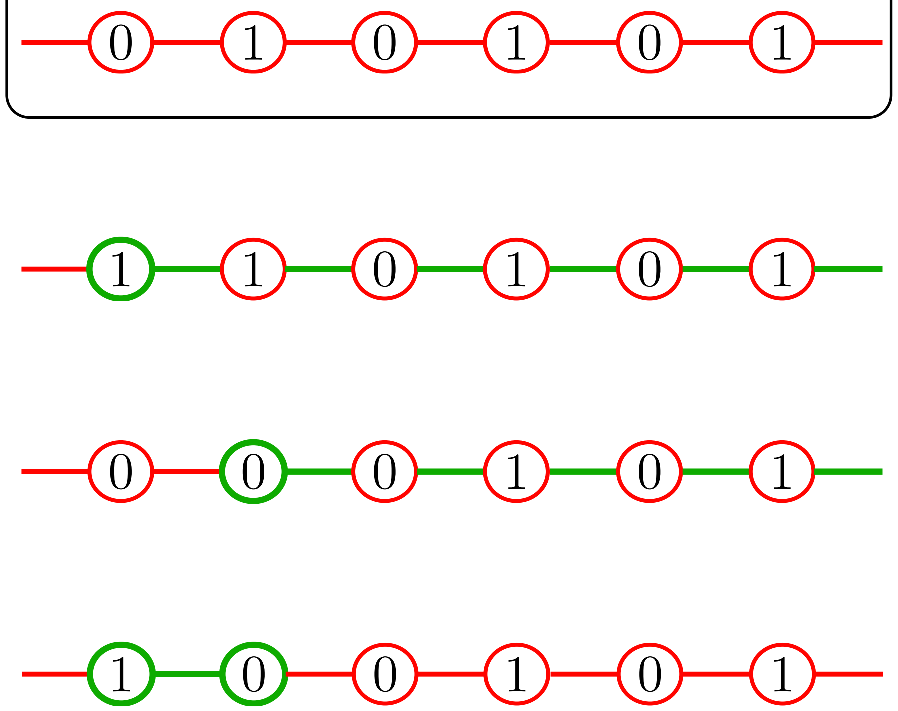
$$b_k^\dagger = \frac{2\pi}{aN} \sum_{p,q} \delta_{p+q}^k \, \eta_{pq} \, \mathcal{B}(p,q)$$

$$a\sqrt{\frac{m+\omega_p}{2\pi\omega_p}} \sum_n \left[\Pi_{n0} + \Pi_{n1} v_p\right] e^{ipna} \quad \psi_n^\dagger \prod_{i\geq n} \tilde{\sigma}_i^X$$

$$a\sqrt{\frac{m+\omega_q}{2\pi\omega_q}} \sum_m \left[\Pi_{n1} - \Pi_{n0} v_q\right] e^{iqma} \quad \psi_m \prod_{i\geq m} \tilde{\sigma}_i^X$$

$$\# \sum_{m,n} \left[\text{Projections}\right] e^{i(pn+qm)a} \quad \psi_n^\dagger \psi_m \prod_{n\leq i\leq m} \tilde{\sigma}_i^X$$

Action on the SCV



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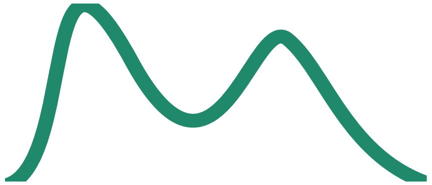

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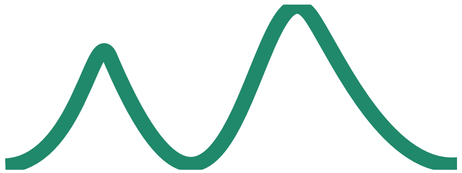

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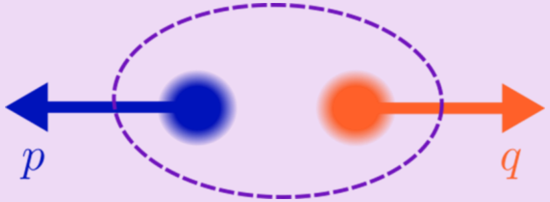
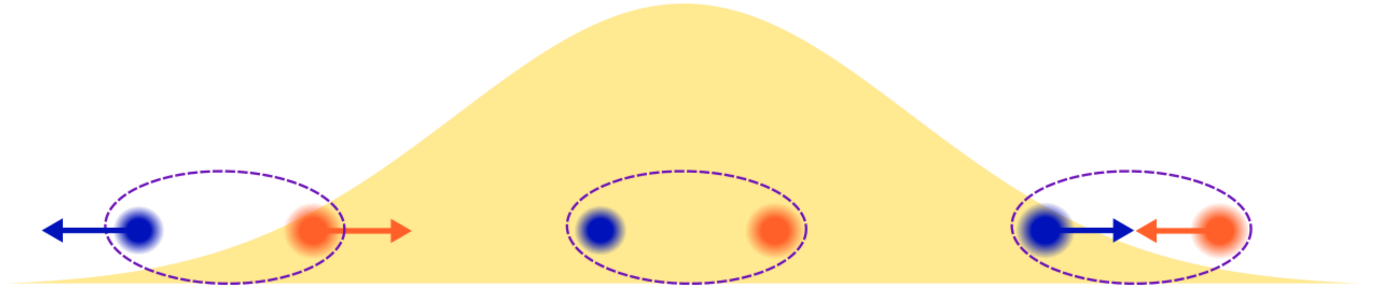


$$\eta_{pq} = N_\eta \exp\left(\frac{i(p-q)\mu_k^A}{2}\right) \exp\left(\frac{-(p-q)^2}{4\sigma_k^{A2}}\right)$$

$k = p + q$



Example for $k = 0$

Building creation operators in interacting theory

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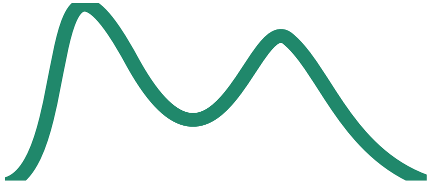
Interacting theory

Interacting wave packet



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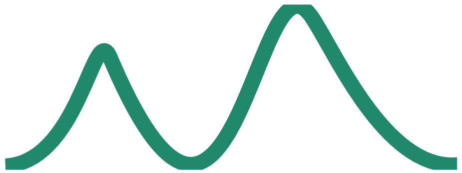

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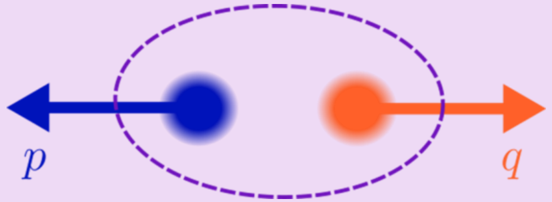


$$\eta_{pq} = N_\eta \exp\left(\frac{i(p-q)\boldsymbol{\mu}^{\boldsymbol{A}}_{\boldsymbol{k}}}{2}\right) \exp\left(\frac{-(p-q)^2}{4\boldsymbol{\sigma}^{\boldsymbol{A}\,2}_{\boldsymbol{k}}}\right)$$

$$k = p + q$$

Optimize first excited energy eigenstates for each k

$$|k\rangle = b^\dagger_k(\boldsymbol{\mu}^{\boldsymbol{A}}_{\boldsymbol{k}}, \boldsymbol{\sigma}^{\boldsymbol{A}}_{\boldsymbol{k}})\, |\Omega\rangle \qquad E^{(1)}_k = \langle k | H_{Z_2} | k \rangle$$

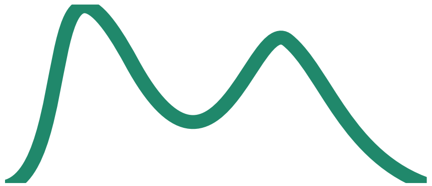
- Classical/Quantum: We did it classically, but we checked that VQE works
- Works for U(1) LGT in 1+1D as well: We were limited by computational resources

Wave packet constructions

Rigobello, Notarnicola, Magnifico, and Montangero
Phys. Rev. D 104, 114501 (2021)

Interacting theory

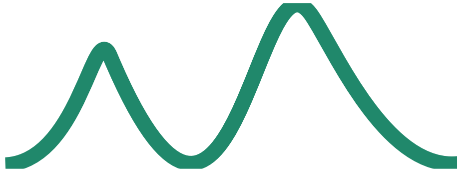
Interacting wave packet



$|\Psi\rangle = b^\dagger_\psi|\Omega\rangle$

Ansatz for $b_k^\dagger$

$b^\dagger_\psi = \sum_k \psi(k) b_k^\dagger$



$|\Omega\rangle$

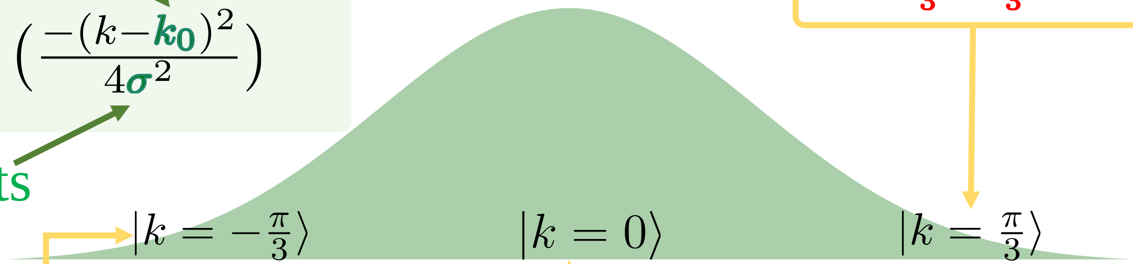
Interacting ground state

$$|\Psi\rangle = b^\dagger_\psi|\Omega\rangle = \sum_k \boldsymbol{\psi(k)} b_k^\dagger(\boldsymbol{\mu_k^A}, \boldsymbol{\sigma_k^A})|\Omega\rangle$$

$$\boldsymbol{\psi(k)} = \exp(-ik\boldsymbol{\mu}) \exp\left(\frac{-(k-\boldsymbol{k_0})^2}{4\boldsymbol{\sigma}^2}\right)$$

Inputs

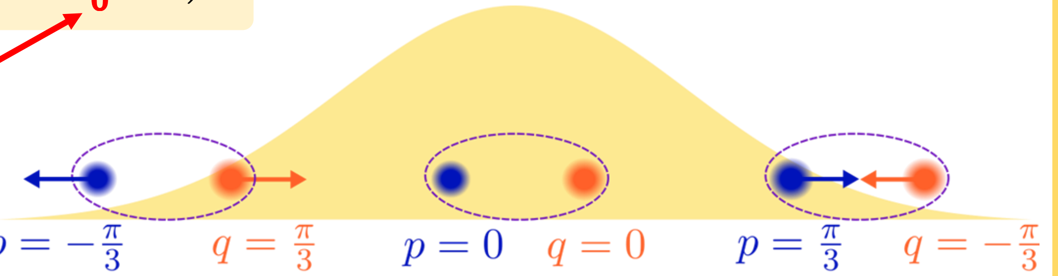
$b_k^\dagger(\boldsymbol{\mu^A_{\frac{\pi}{3}}}, \boldsymbol{\sigma^A_{\frac{\pi}{3}}})\,|\Omega\rangle$



$b_k^\dagger(\boldsymbol{\mu^A_{-\frac{\pi}{3}}}, \boldsymbol{\sigma^A_{-\frac{\pi}{3}}})\,|\Omega\rangle$

$$\eta_{pq} = N_\eta \exp\left(\frac{i(p-q)\boldsymbol{\mu_0^A}}{2}\right) \exp\left(\frac{-(p-q)^2}{4\boldsymbol{\sigma_0^A}^2}\right)$$

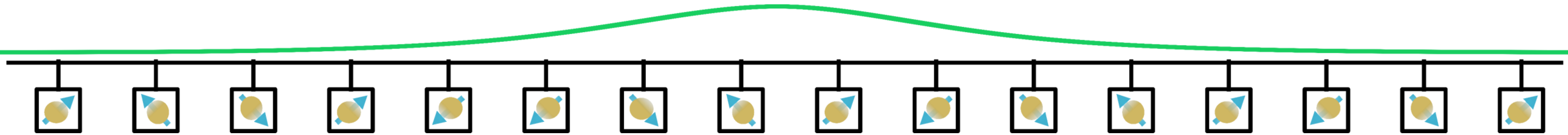
Optimized



Example for $k=0$

Mapping

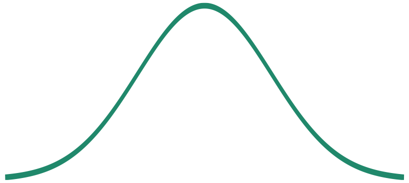
Algorithm and circuit



Overview

Interacting theory

Interacting wave packet



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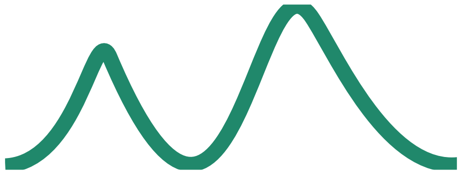
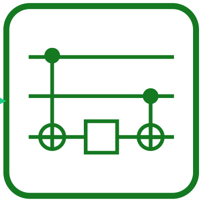


$|\Omega\rangle$

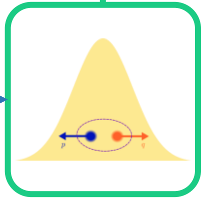
Interacting ground state

4



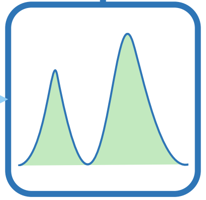
$|\Psi\rangle$ Circuit for the wave packet

3



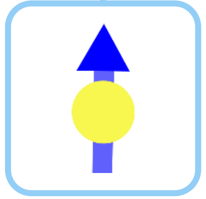
$b_k^\dagger$ Optimize the momentum creation operators

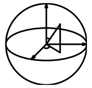
2



$|\Omega\rangle$ Prepare the interacting ground state

1

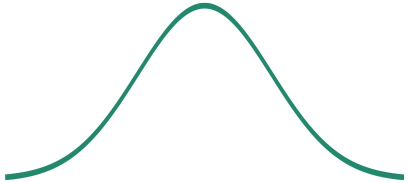


 Map the fermionic + bosonic DOFs to qubits

Overview

Interacting theory

Interacting wave packet



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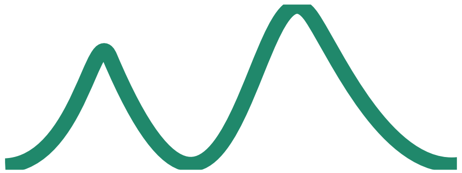
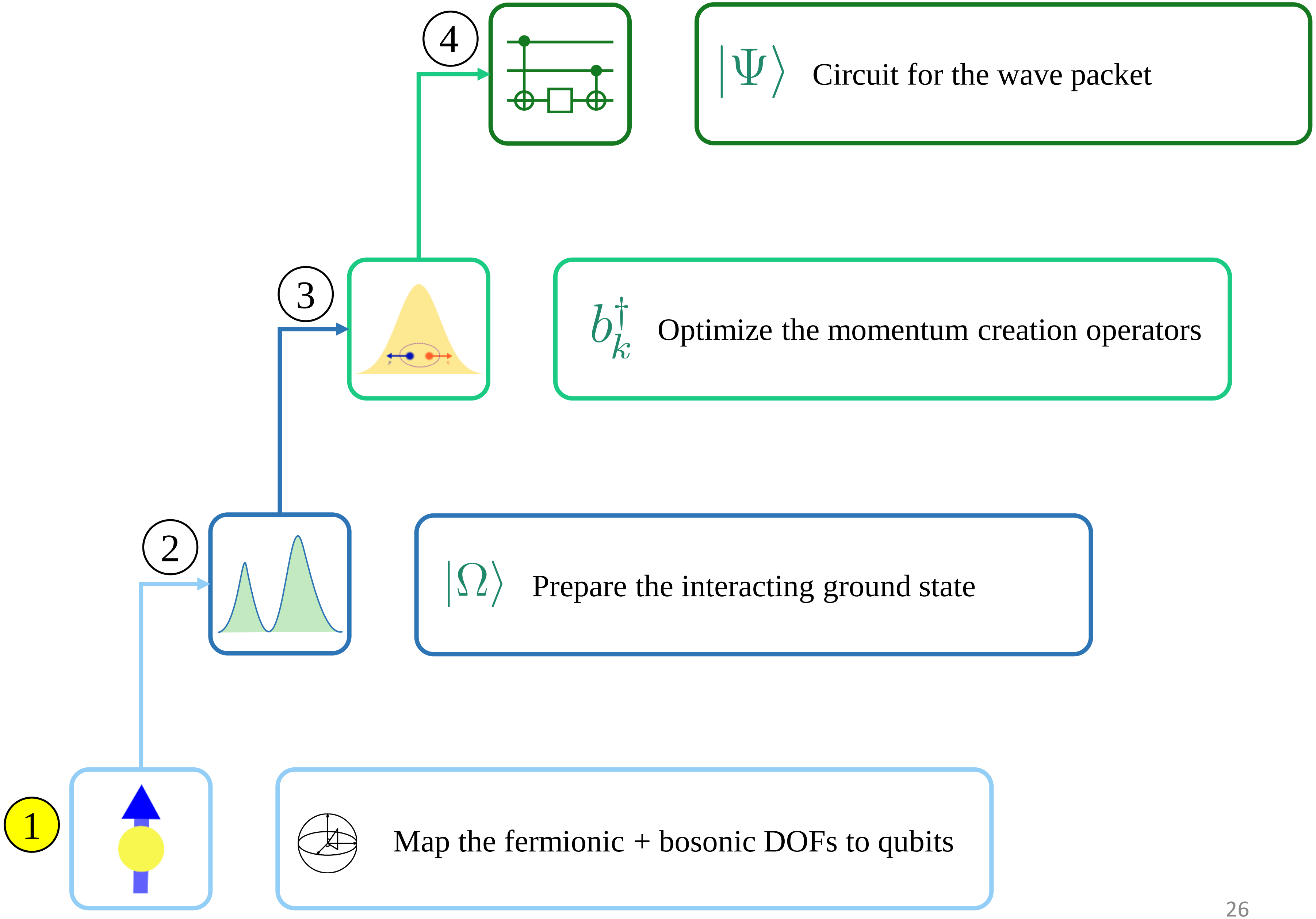
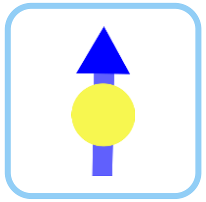


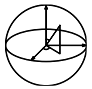
$|\Omega\rangle$

Interacting ground state



1



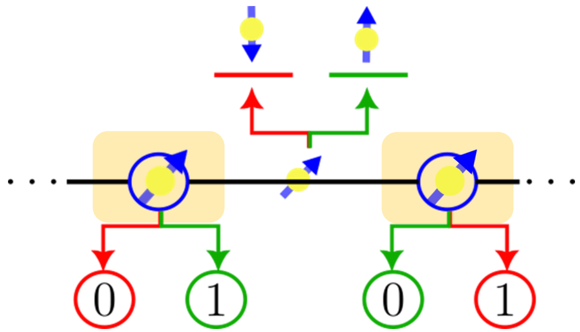


Map the fermionic + bosonic DOFs to qubits

- Bosonic links: as it is
- Fermions: Jordan-Wigner transformation

$$\psi_n^\dagger = \prod_{i<n} \sigma_i^Z \sigma_n^-$$

$$\psi_n = \prod_{i<n} \sigma_i^Z \sigma_n^+$$



$$|\Psi\rangle = b_\psi^\dagger |\Omega\rangle = \sum_k \psi(k) b_k^\dagger |\Omega\rangle$$

Inputs

$$\frac{2\pi}{aN} \sum_{p,q} \delta_{p+q}^k \, \eta_{pq} \, \mathcal{B}(p,q)$$

Optimized

$$\sum_{m,n} \left[\text{Projections}\right] e^{i(pn+qm)a} \; \psi_n^\dagger \psi_m \prod_{n\le i\le m} \tilde{\sigma}_i^X$$

Jordan-Wigner

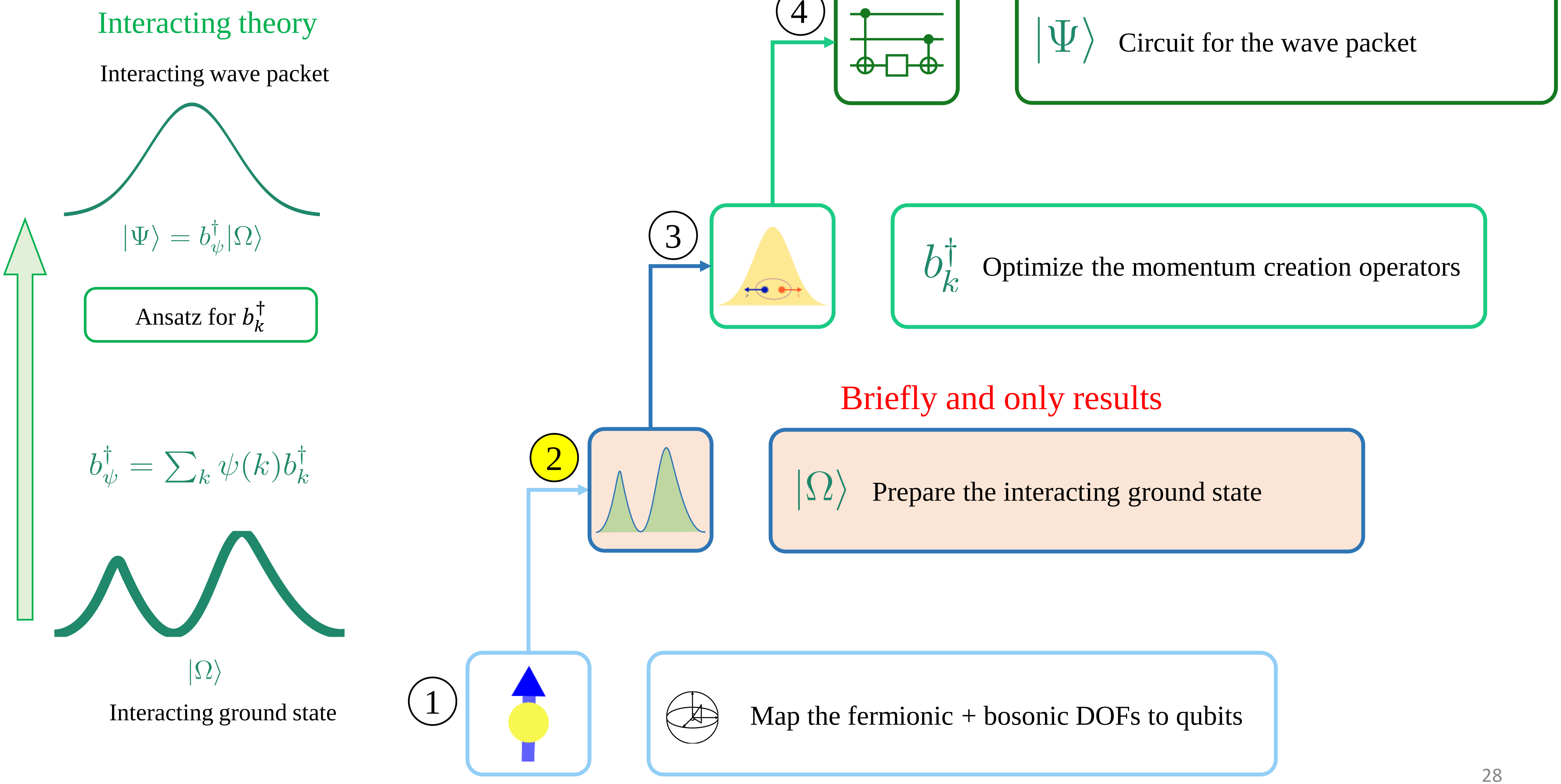
$$b_\psi^\dagger = \sum_{m,n} C_{m,n} \; \sigma_n^- \prod_{n<j<m} \sigma_j^Z \prod_{n\le i\le m} \tilde{\sigma}_i^X \; \sigma_m^+$$

Inputs Optimized

From here onwards:

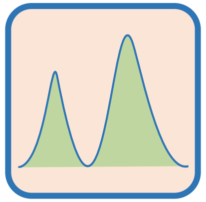
- ✓ 6 staggered sites
- ✓ 3 momenta: $k = \frac{\pi}{3}, 0, -\frac{\pi}{3}$
- ✓ 12 qubits = 4096 states
- ✓ 40 Physical states

Overview

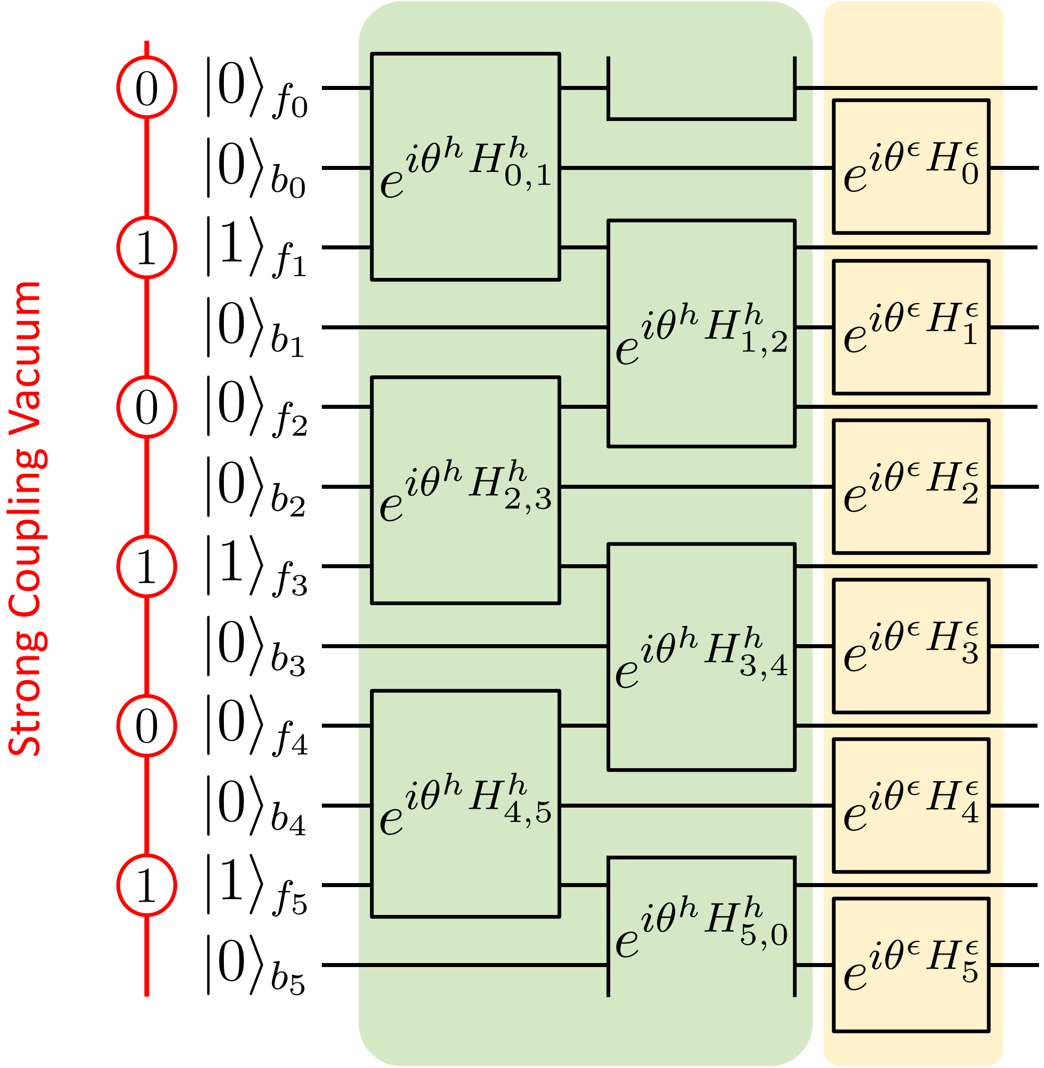


Interacting theory
Interacting wave packet
$|\Psi\rangle = b_\psi^\dagger |\Omega\rangle$
Ansatz for $b_k^\dagger$
$b_\psi^\dagger = \sum_k \psi(k) b_k^\dagger$
$|\Omega\rangle$
Interacting ground state
4
$|\Psi\rangle$ Circuit for the wave packet
3
$b_k^\dagger$ Optimize the momentum creation operators
Briefly and only results
2
$|\Omega\rangle$ Prepare the interacting ground state
1
Map the fermionic + bosonic DOFs to qubits

2



$|\Omega\rangle$ Prepare the interacting ground state

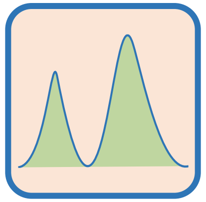


Lumia, Torta, Mbeng, Santoro, Ercolessi, Burrello and Wauters
Phys. Rev. X Quantum 3, 020320 (2022)

Variational Quantum Eigensolver (VQE) for the GS preparation:

- Parameterized circuit with 2 parameters
  - ✓ Inspired from the Hamiltonian
  - ✓ Gauge invariant by construction
- Calculate energy with the Quantum circuit
- Optimize the parameters classically

2



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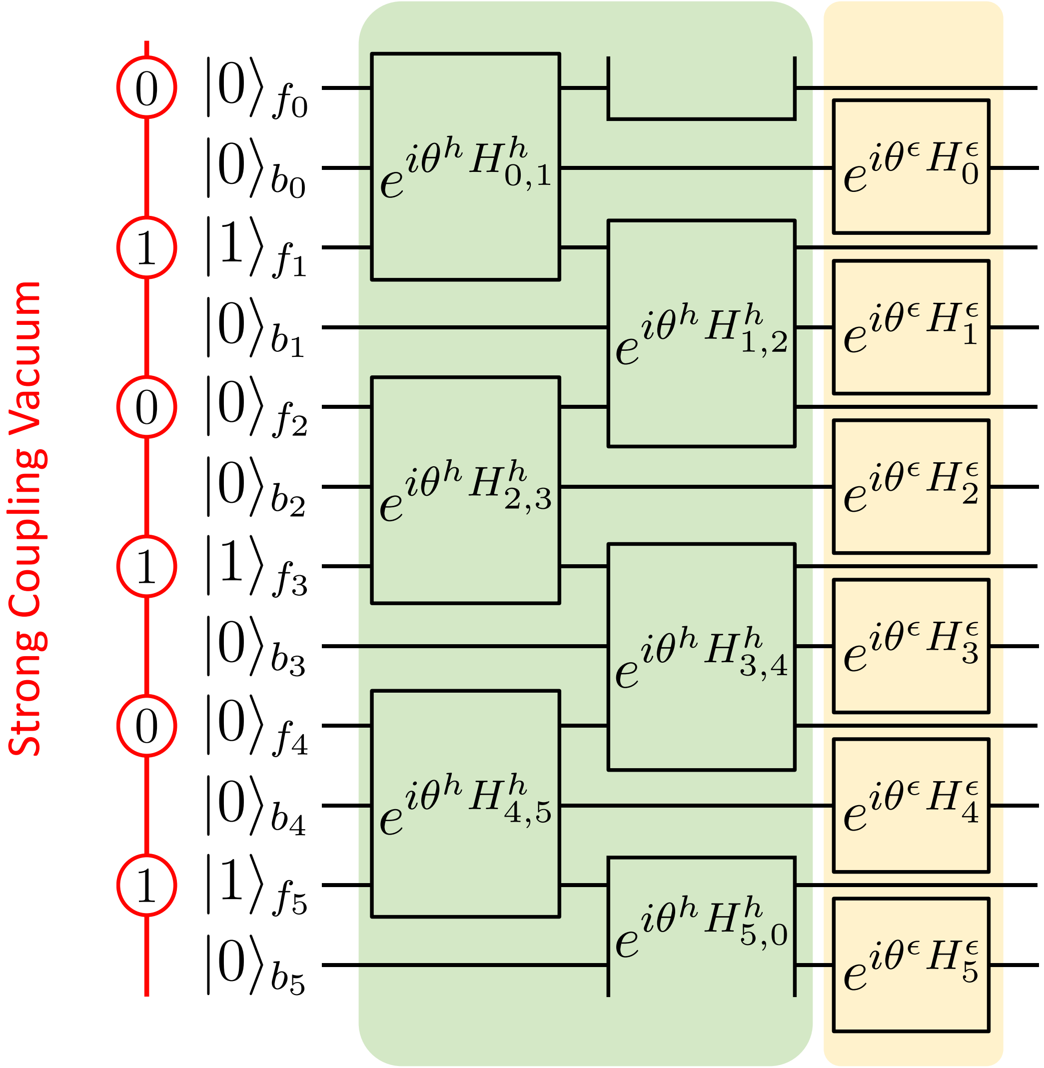
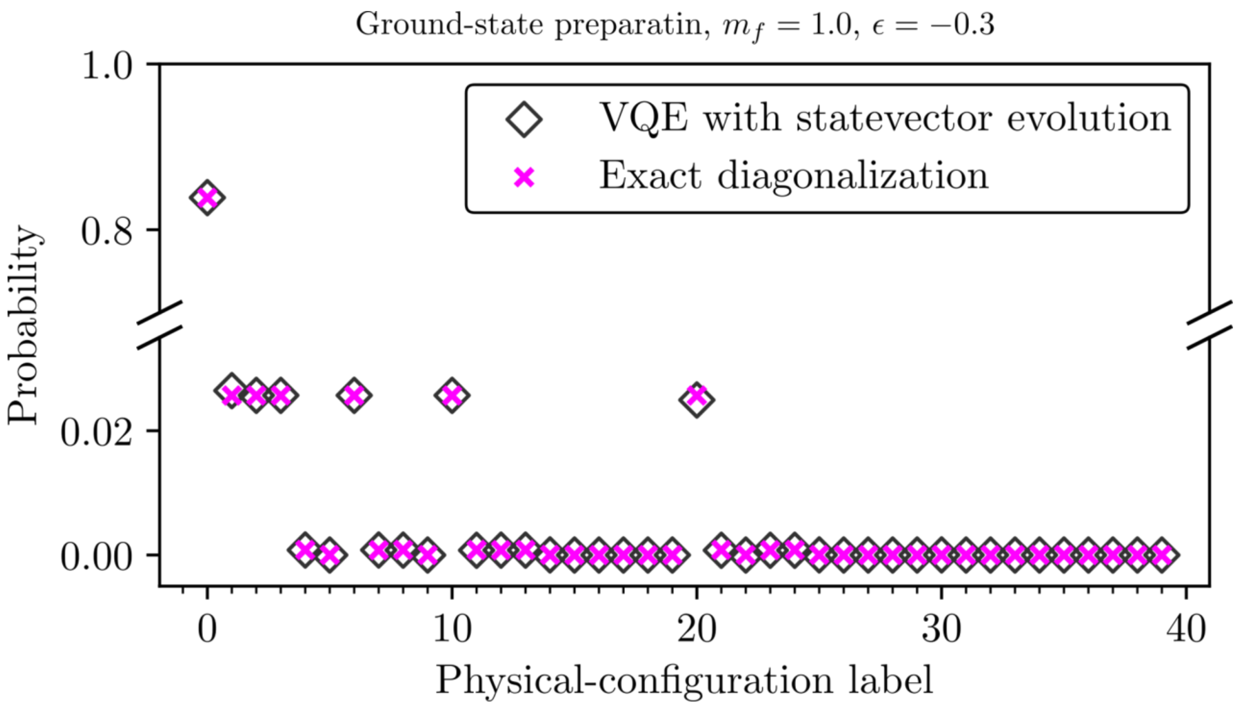




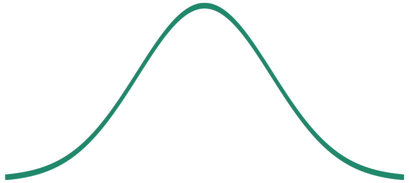
$$E_\Omega^{\text{Exact}} = -5.32483 \qquad \mathcal{F} = |\langle \Omega_{\text{Exact}} | \Omega_{\text{VQE}} \rangle|^2$$

$$E_\Omega^{\text{VQE}} = -5.32452 \qquad = 0.99992$$

Overview

Interacting theory

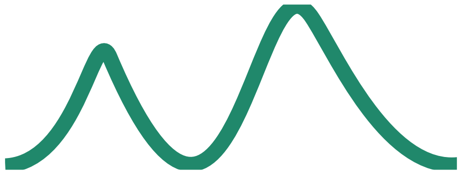
Interacting wave packet



$|\Psi\rangle = b^{\dagger}_{\psi}|\Omega\rangle$

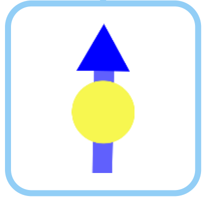
Ansatz for $b^{\dagger}_{k}$

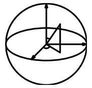
$b^{\dagger}_{\psi} = \sum_k \psi(k) b^{\dagger}_{k}$

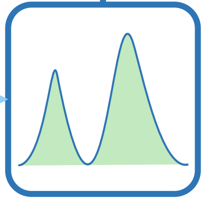


$|\Omega\rangle$

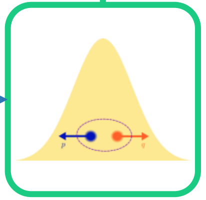
Interacting ground state

1 

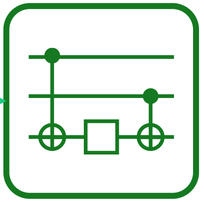
 Map the fermionic + bosonic DOFs to qubits

2 

$|\Omega\rangle$ Prepare the interacting ground state

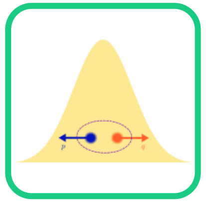
3 

$b^{\dagger}_{k}$ Optimize the momentum creation operators

4 

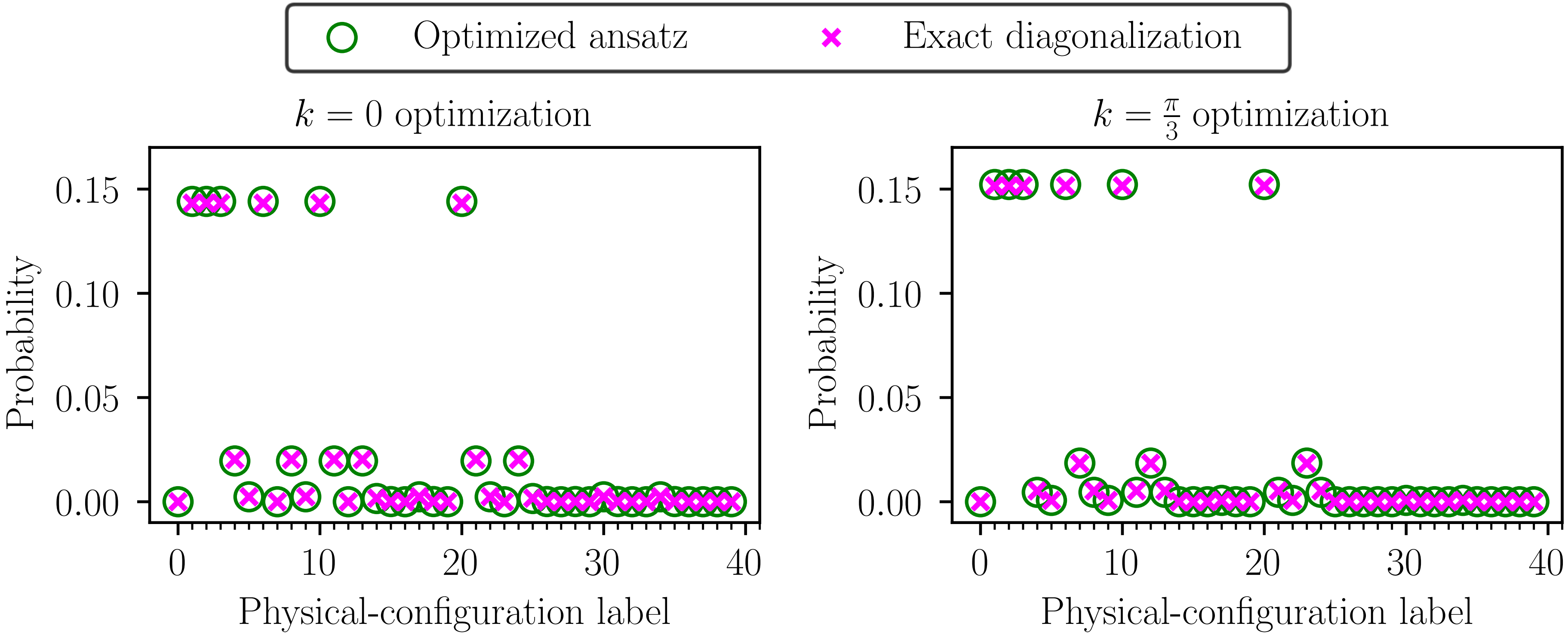
$|\Psi\rangle$ Circuit for the wave packet

3



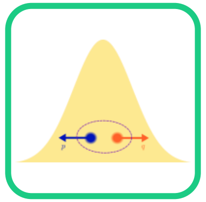
$b_k^\dagger$ Optimize the momentum creation operators

$$\eta_{pq} = N_\eta \exp\left(\frac{i(p-q)\mu_k^A}{2}\right) \exp\left(\frac{-(p-q)^2}{4\sigma_k^{A^2}}\right)$$



	$\mathcal{F} = \lvert\langle k_{\text{Exact}} \vert k_{\text{Optimized}}\rangle\rvert^2$	$E_k^{\text{Optimized}}$	E_k^{Exact}
$k = 0$	0.98756	-2.45688	-2.46734
$k = \pm\frac{\pi}{3}$	0.99977	-2.57561	-2.57613

3



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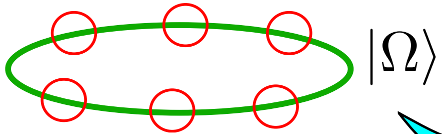
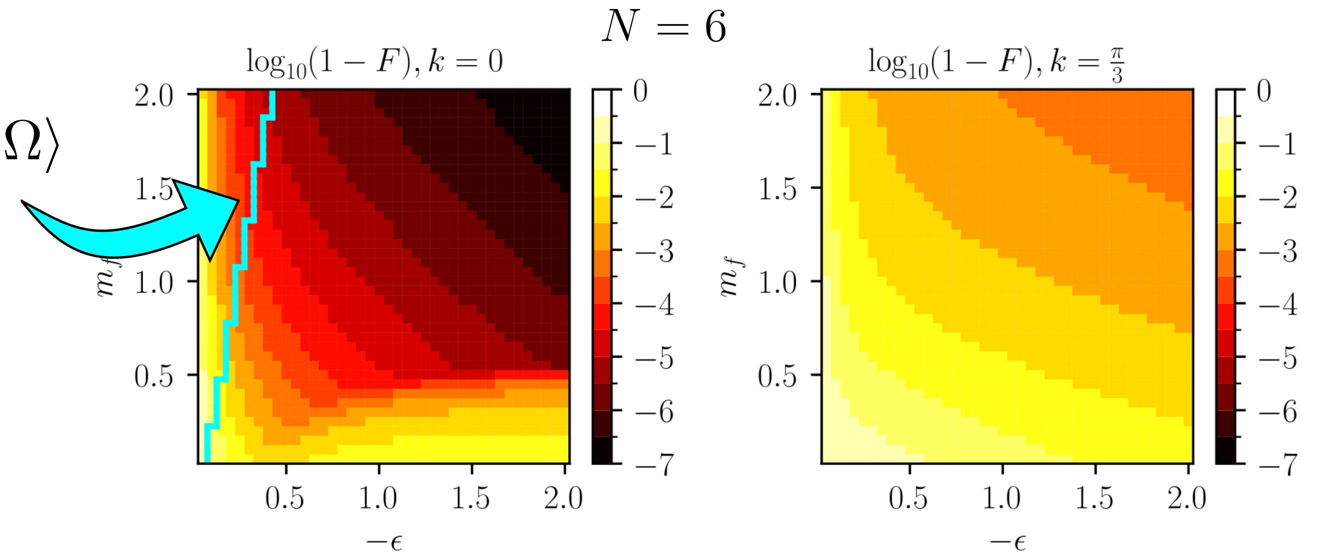
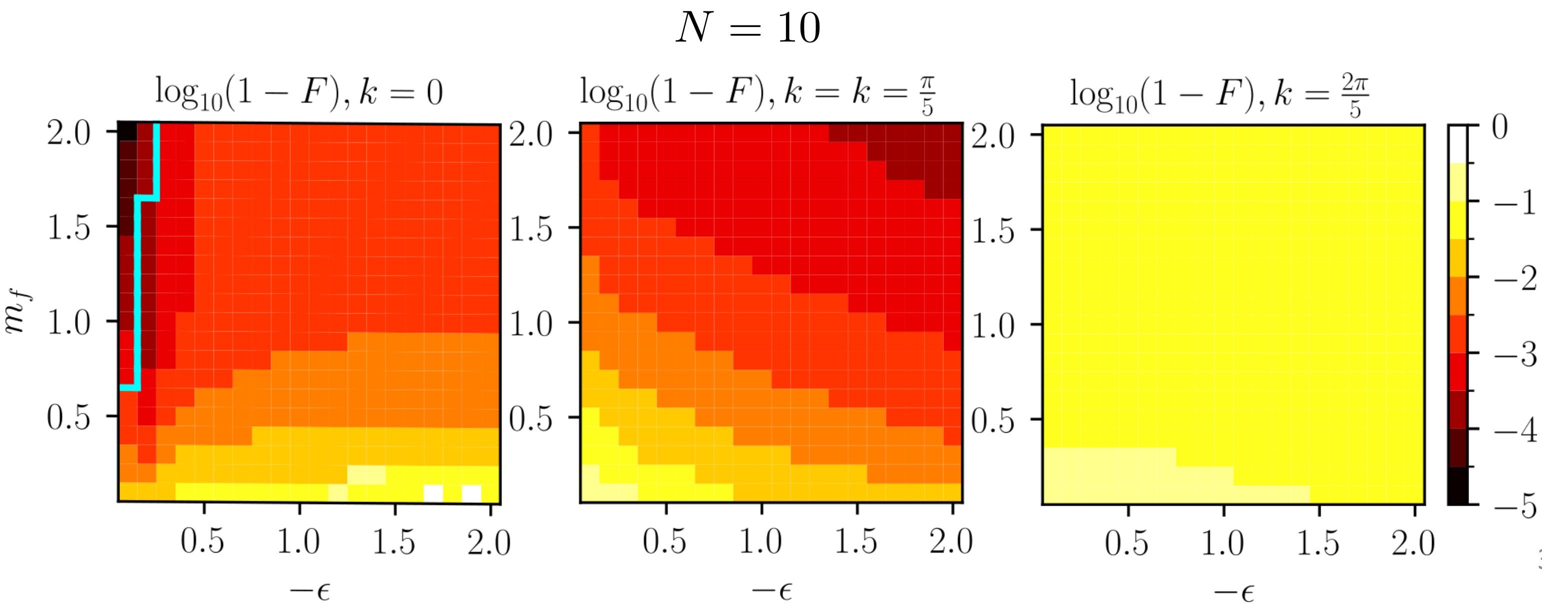




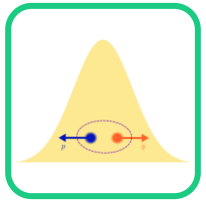
Z_2 LGT

How well does the ansatz work for different Hamiltonian parameters?

$$\mathcal{F} = |\langle k_{\text{Exact}} | k_{\text{Optimized}} \rangle|^2$$



3



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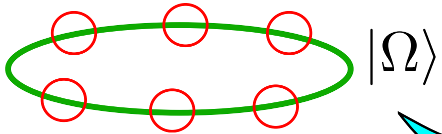
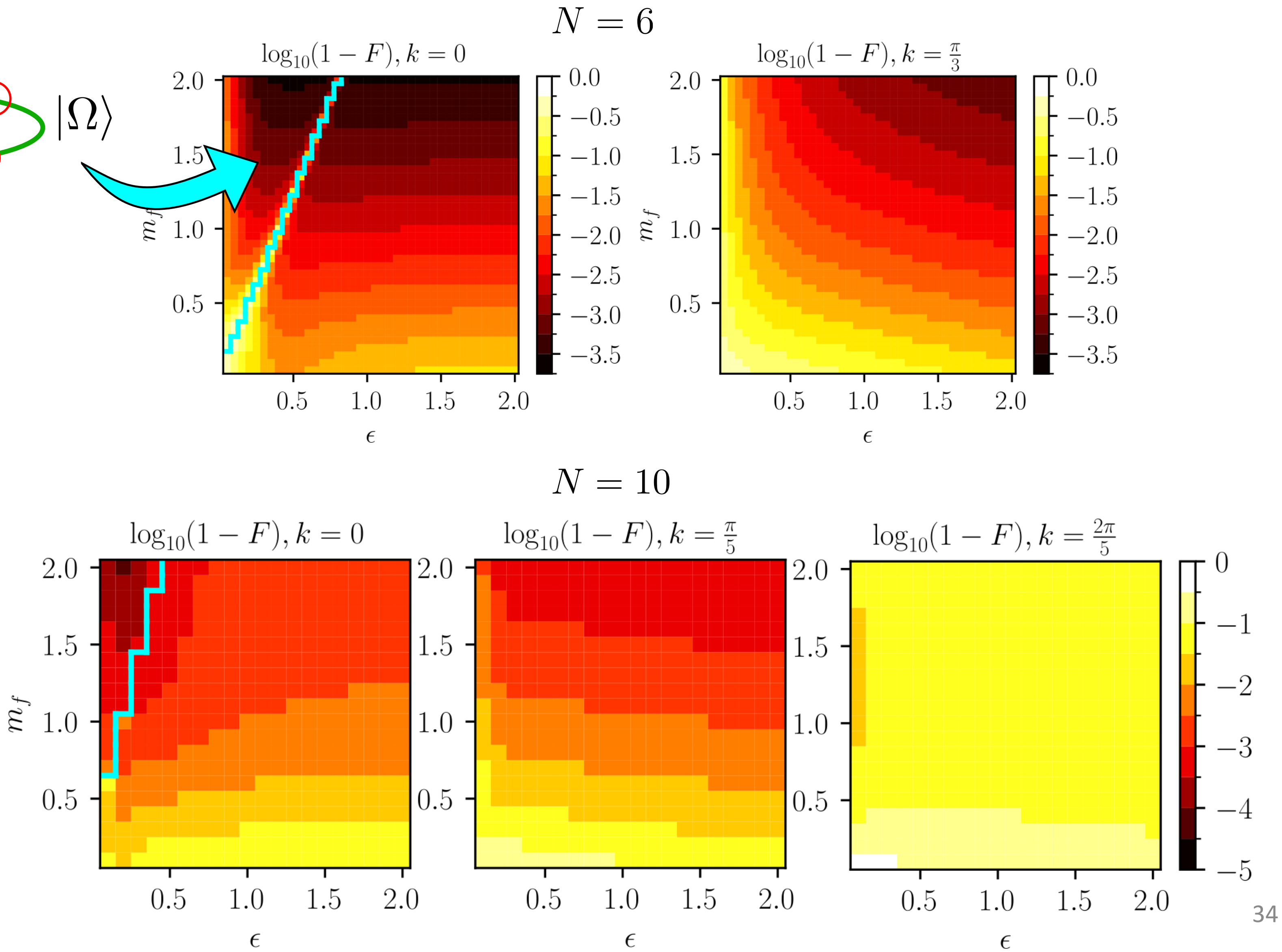


U(1) LGT

How well does the ansatz work for different Hamiltonian parameters?

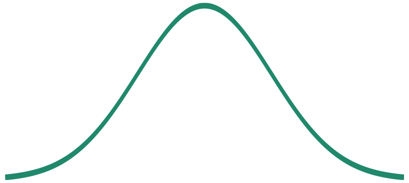
$$\mathcal{F} = |\langle k_{\text{Exact}}|k_{\text{Optimized}}\rangle|^2$$



Overview

Interacting theory

Interacting wave packet



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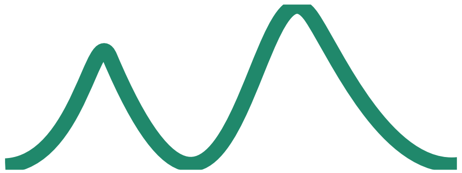
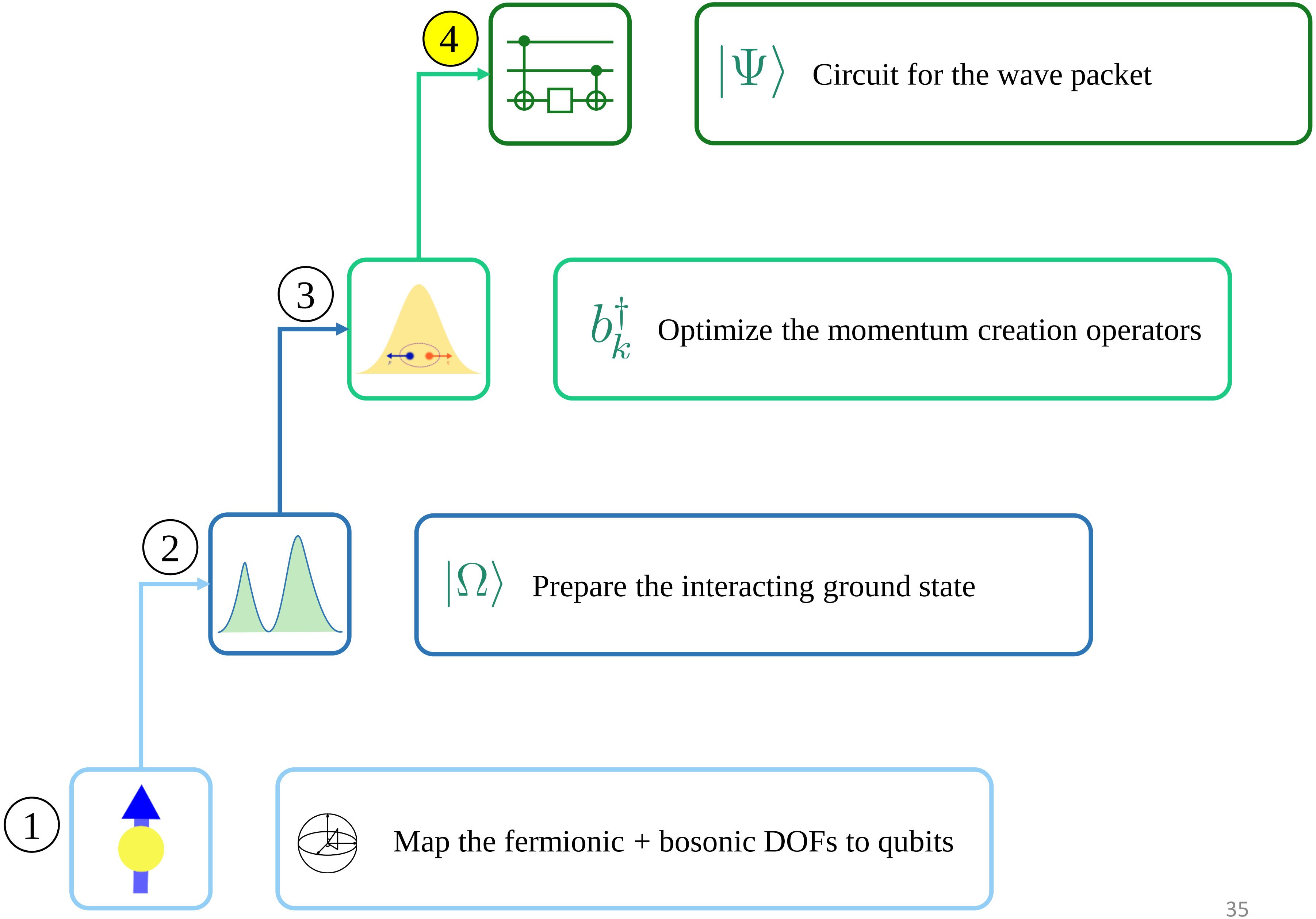


$|\Omega\rangle$

Interacting ground state



4

$|\Psi\rangle$ Circuit for the wave packet

$$b_\psi^\dagger = \sum_{m,n} C_{m,n}\ \sigma_n^- \prod_{n<j<m} \sigma_j^Z \prod_{n\leq i\leq m} \tilde{\sigma}_i^X\ \sigma_m^+$$

Inputs

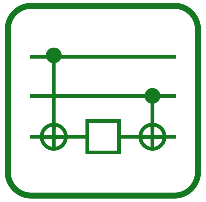
Optimized

$$\psi(k) = \exp(-ik\mu)\exp\left(\frac{-(k-k_0)^2}{4\sigma^2}\right)$$

Issues

- ❖ Non-Unitary operator
- ❖ Needs efficient circuit design

4



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❖ Non-Unitary operator

❖ Needs efficient circuit design

- Ancilla encoding:

Jordan, Lee, and Preskill (JLP)
Quantum Info. and Comp. 14, 1014-80

Embed the state into a larger Hilbert space using an ancilla qubit

If $b_\psi|\Omega\rangle = 0 \qquad [b_\psi, b_\psi^\dagger] = \hat{1}$

Then

$$\Theta = b_\psi^\dagger \otimes |1\rangle\langle 0|_a + b_\psi \otimes |0\rangle\langle 1|_a$$

$$e^{-i\frac{\pi}{2}\Theta}|\Omega\rangle \otimes |0\rangle_a = -i\, b_\psi^\dagger|\Omega\rangle \otimes |1\rangle_a$$

- Applicable for $\Theta = \sum_{\{m,n\}} \Theta_{m,n}$ upon Trotterization

- Singular Value Decomposition (SVD)

Find a basis that diagonalizes $\Theta_{m,n}$

Davoudi, Shaw and Stryker
Quantum 7, 1213 (2023)

If $b_{m,n}^{\dagger\,2} = b_{m,n}^2 = 0 \quad \& \quad b_{m,n} = VSW^\dagger$

Then

$$e^{-i\frac{\pi}{2}\Theta_{m,n}} = \mathcal{U}_{m,n}^\dagger e^{-i\frac{\pi}{2}\mathcal{D}_{m,n}}\mathcal{U}_{m,n}$$

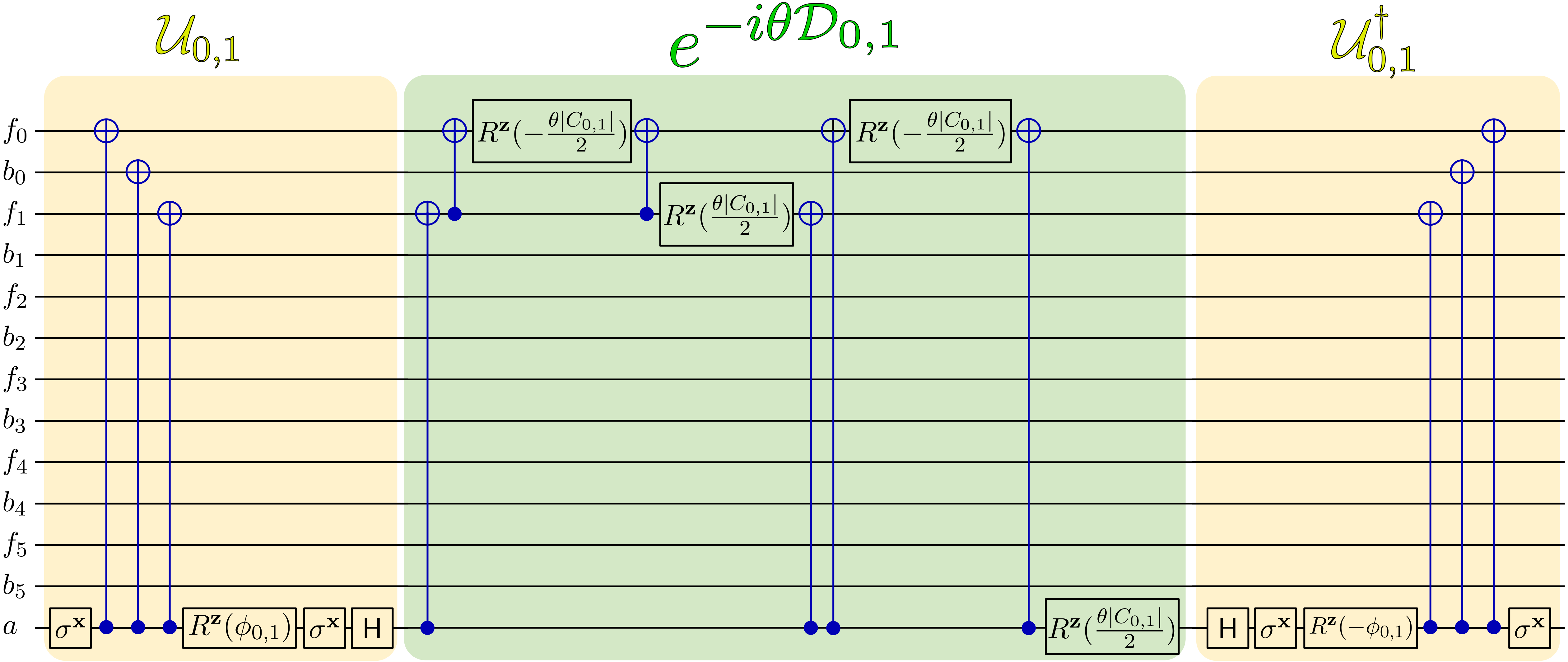
$$\mathcal{U}_{m,n} = \mathrm{Had}_a\left(V^\dagger \otimes |0\rangle\langle 0|_a + W^\dagger \otimes |1\rangle\langle 1|_a\right)$$

$$\mathcal{D}_{m,n} = S_{m,n} \otimes Z_a$$

4

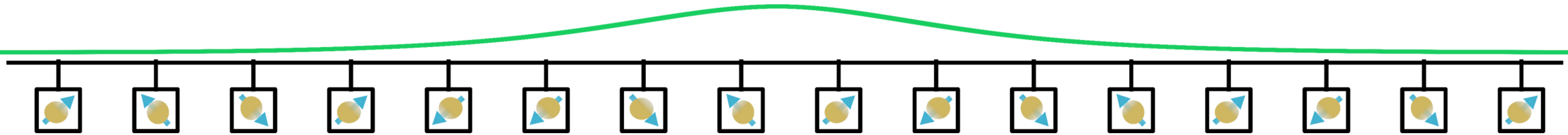
$|\Psi\rangle$ Circuit for the wave packet

$$b_{0,1}^{\dagger} = e^{i\phi_{0,1}} |C_{0,1}| \, \sigma_0^- \tilde{\sigma}_0^X \sigma_1^+$$



Measurements

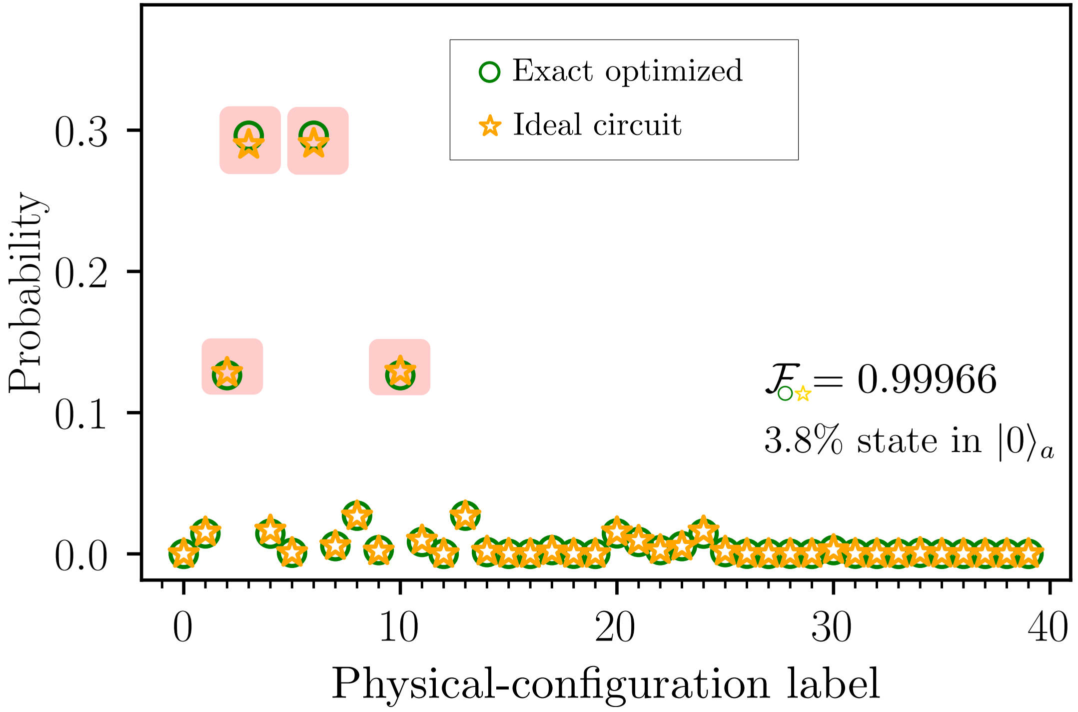
Hardware results



Results

The ideal quantum circuit

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$



☆ Ideal quantum circuit

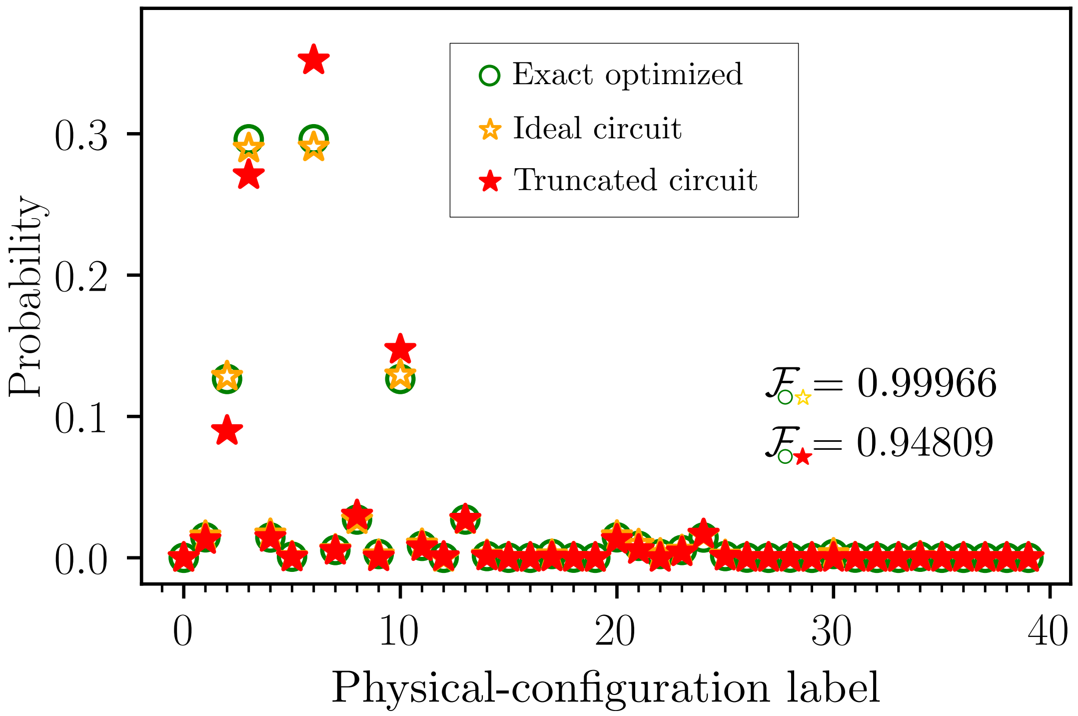
- Circuit statevector method & many Trotter steps
- Systematic error from:

$$b_\psi |\Omega\rangle \simeq 0 \qquad [b_\psi, b_\psi^\dagger] \simeq \hat{1}$$

Results

Truncated quantum circuit

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$



☆ Ideal quantum circuit

- Circuit statevector method & many Trotter steps
- Systematic error from:

$$b_\psi|\Omega\rangle \simeq 0 \qquad [b_\psi, b_\psi^\dagger] \simeq \hat{1}$$

★ Truncated quantum circuit

- Resource limitation:

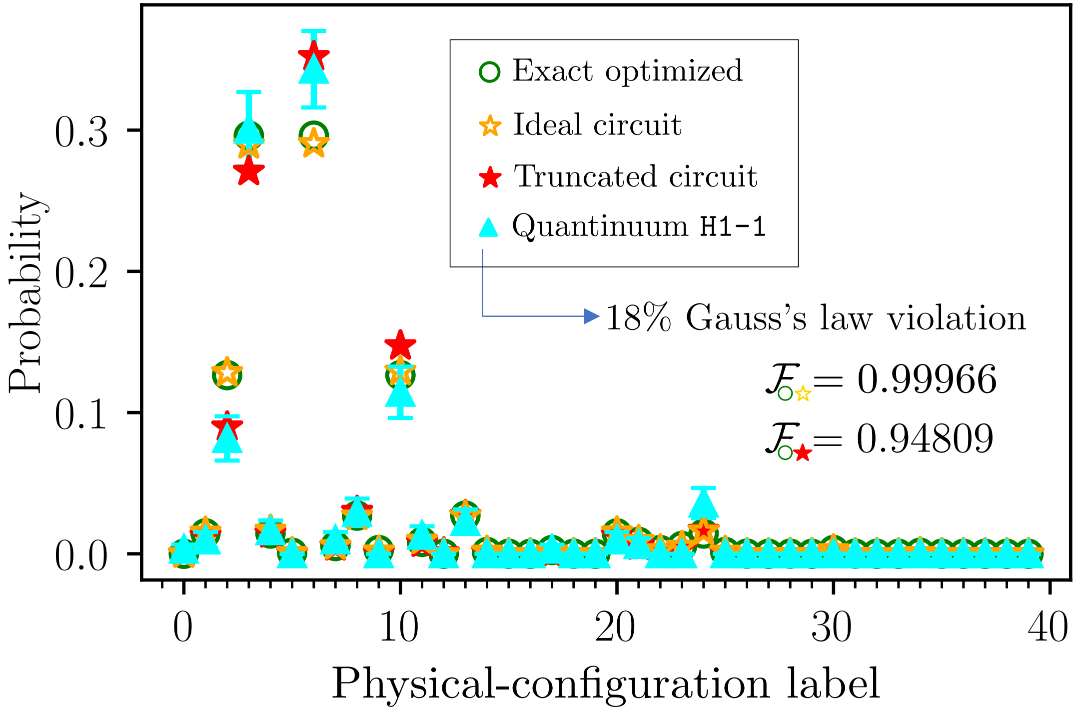
  Only $|C_{m,n}| \geq 0.1$ terms were implemented

- 2nd order Trotter with 1 Trotter step

Results

Quantinuum Hardware results

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$



QUANTINUUM

H1-1

- Trapped ion with 20 qubits
- all-to-all connectivity
- $\sim 10^{-5}$ single-qubit gate infidelity
- $\sim 10^{-3}$ two-qubit gate infidelity

☆ Ideal quantum circuit

- Circuit statevector method & many Trotter steps
- Systematic error from:

$$b_\psi |\Omega\rangle \simeq 0 \qquad [b_\psi, b_\psi^\dagger] \simeq \hat{1}$$

★ Truncated quantum circuit

- Resource limitation:

  Only $|C_{m,n}| \geq 0.1$ terms were implemented

- 2^{nd} order Trotter with 1 Trotter step

▲ Quantinuum H1-1

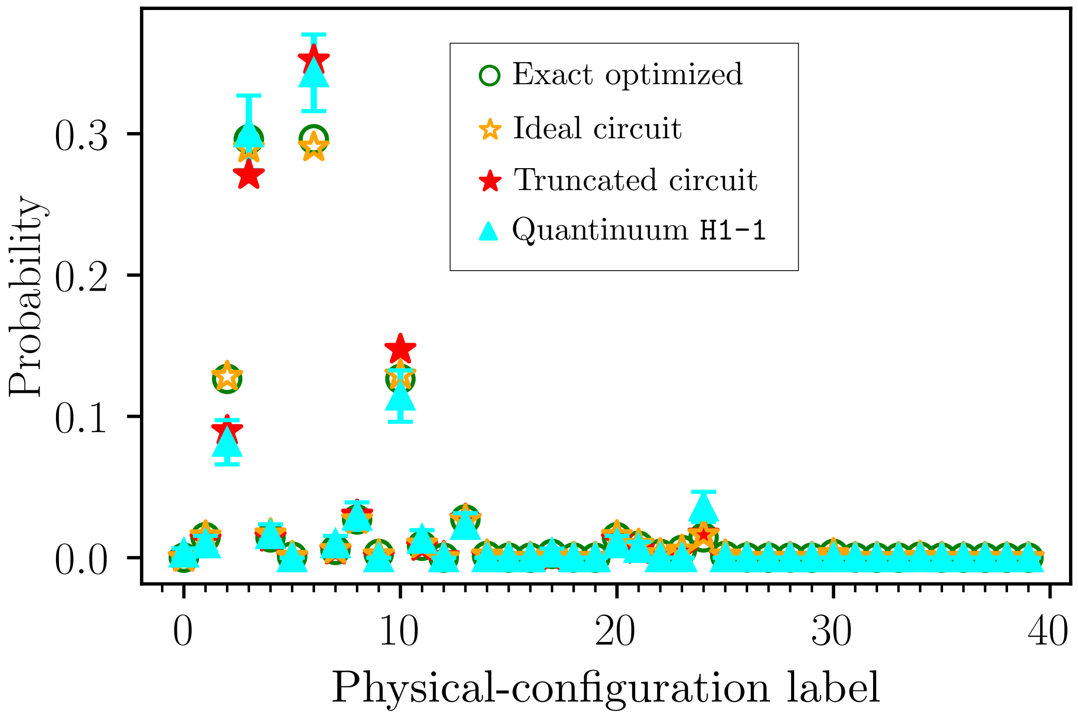
- ~300 (350) two- (single-) qubit gates with 500 shots
- Error mitigation using the gauge invariant nature of our method

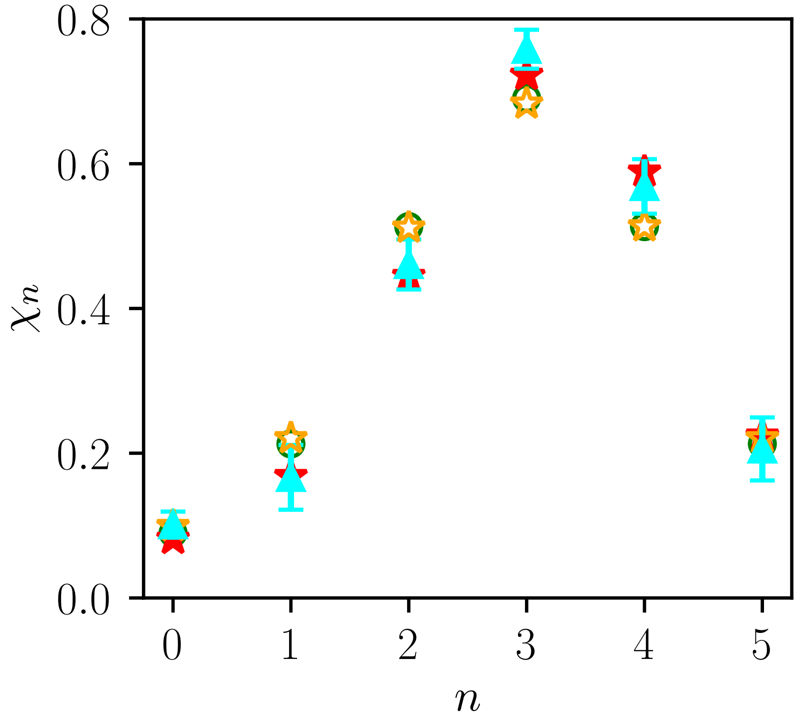
Results

Staggered number density

$$\chi(n) = \begin{cases} \langle \psi^\dagger(n)\psi(n) \rangle & n \text{ even} \\ 1 - \langle \psi^\dagger(n)\psi(n) \rangle & n \text{ odd} \end{cases}$$

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$

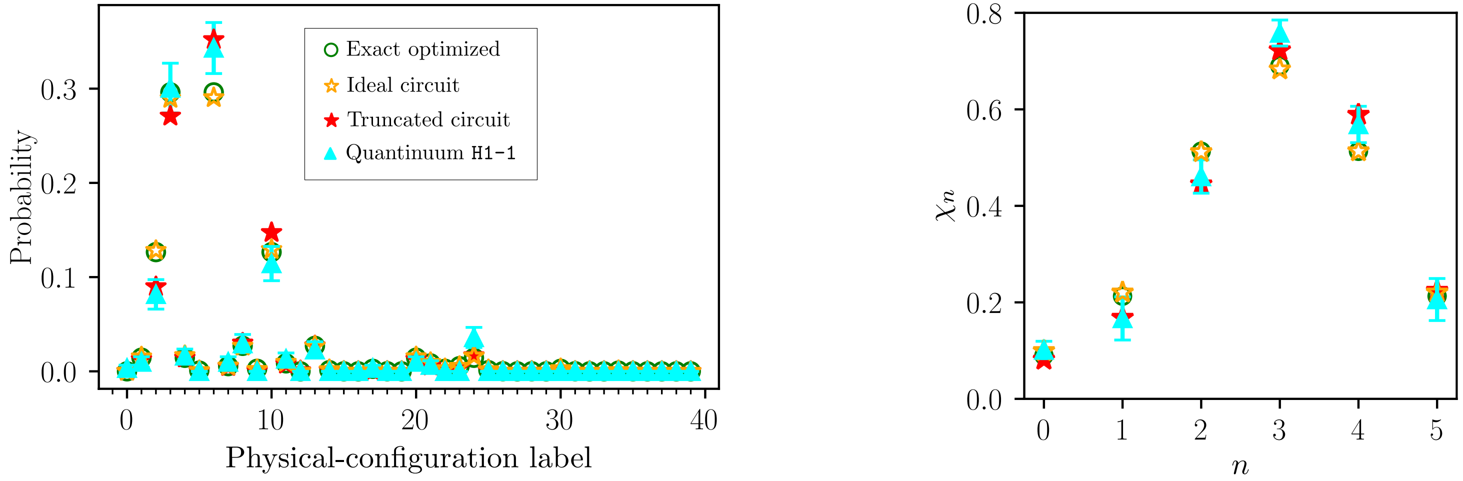




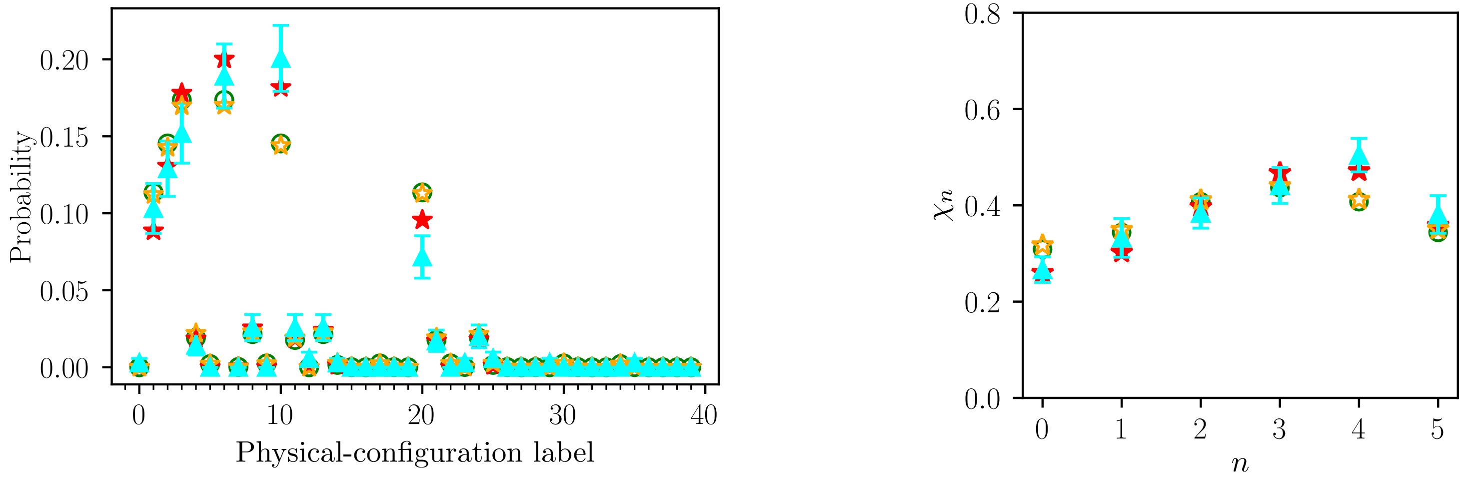
Results

Different WP widths

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$

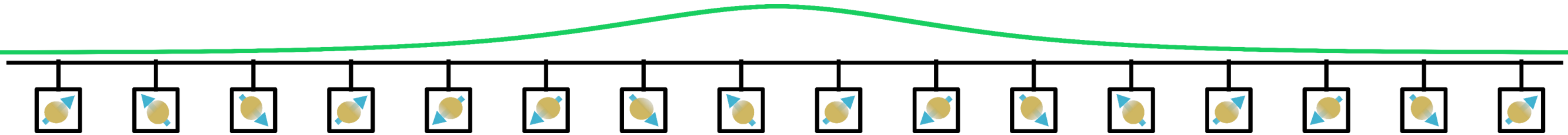


6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{10}$, $\mu = 3$, and $k_0 = 0$



Moreover

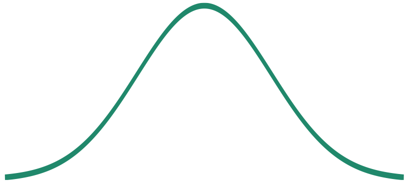
Summary and outlook



Summary

Interacting theory

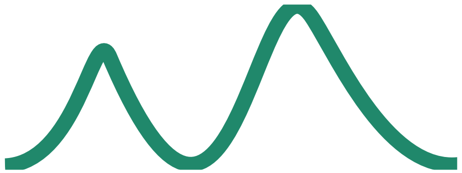
Interacting wave packet



$|\Psi\rangle = b_\psi^\dagger |\Omega\rangle$

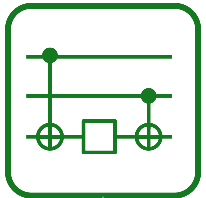
Ansatz for $b_k^\dagger$

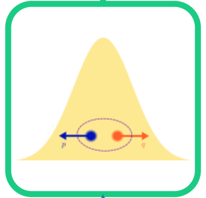
$b_\psi^\dagger = \sum_k \psi(k) b_k^\dagger$

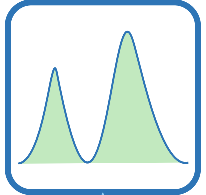


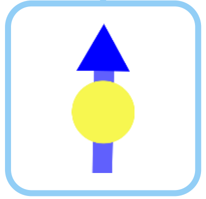
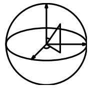
$|\Omega\rangle$

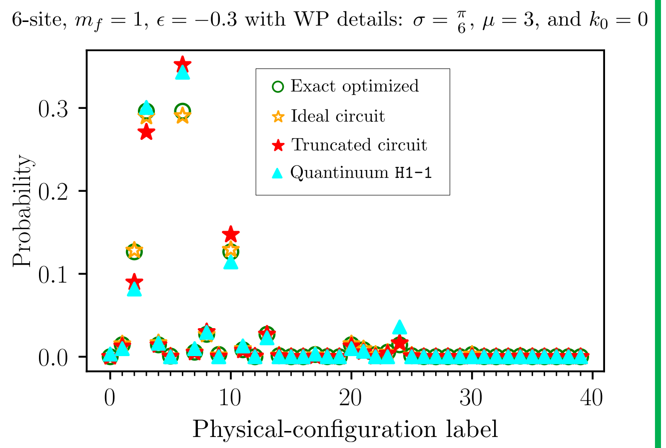
Interacting ground state

4  $|\Psi\rangle$ Circuit for the wave packet

3  $b_k^\dagger$ Optimize the momentum creation operators

2  $|\Omega\rangle$ Prepare the interacting ground state

1   Map the fermionic + bosonic DOFs to qubits

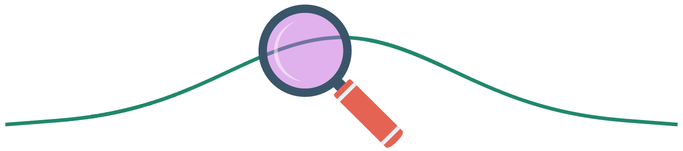


- ✓ Gauge invariant construction
- ✓ VQE based method
- ✓ Systematic error has been identified
- ✓ Works for the U(1) LGT in 1+1D
- ✓ Can be implemented on NISQ devices

Outlook

Thank You !

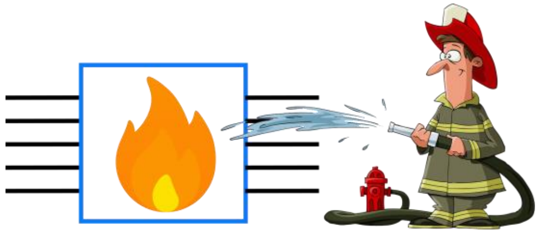
- What's more?

  - More observable for measuring the wave packet fidelity

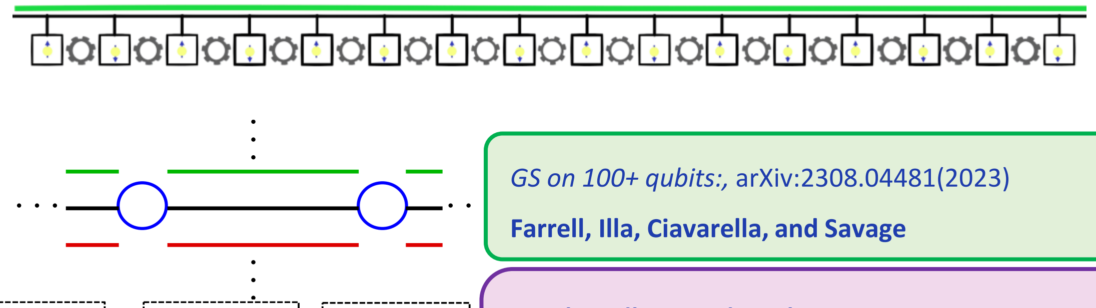
  

  - Analytical bounds on systematic errors

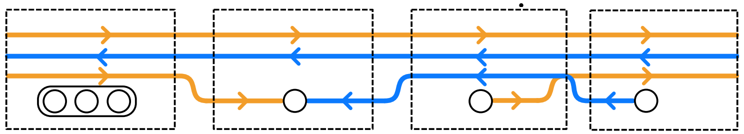
  - Advanced noise mitigation techniques

- What's next?

  - Prepare two wave packets and perform scattering
  - Wave packet in the U(1) LGT on larger devices

  GS on 100+ qubits:, arXiv:2308.04481(2023)
  Farrell, Illa, Ciavarella, and Savage

  - Ansatz for non-Abelian theories

  Raychowdhury and Stryker
  Phys. Rev. D 101, 114502 (2020)
  Kadam, Raychowdhury and Stryker
  Phys. Rev. D 107, 094513 (2023)