LARGE-SCALE[!] VACUUM AND HADRONIC STATE PREPARATION IN GAUGE THEORIES

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HOW TO REACH FULL QCD?

- Current quantum computers are too small and noisy to do full lattice QCD
- Simulations on noisy hardware can inform the development of techniques to reduce the effects of noise
- 3+1D calculations are limited by hardware connectivity
- Toy models lower dimensions are easier to map onto hardware
- Can be used to develop techniques that will carry over to $3+1D$
	- State preparation
	- Constructing physical observables
- Some non-trivial physics can be studied
	- Jet Fragmentation
	- **Hadronization**

STATE PREPARATION

• Simulating physics requires preparing physically relevant states

Adiabatic

- Theoretical guarantees
- Potentially deep circuit depths
- Mostly restricted to theoretical studies
- Variational
- Heuristic method, depends on circuit ansatz
- Requires optimization of circuits
- Lower circuit depth
- Too many works studying these methods to list

PREPARING THE SCHWINGER M O D E L VAC U U M PRX Quantum 5 (2), 020315

- QED in 1+1D
- Gapped and translationally invariant
- Confining, like QCD in 3+1D
- We looked at preparing the vacuum state as a step towards studying QCD

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Quantum Simulations of Hadron Dynamics in the Schwinger Model using 112 Qubits

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THE LATTICE SCHWINGER MODEL

$$
H = \frac{1}{2} \sum_{n=0}^{2L-2} (\psi_n^{\dagger} U_n \psi_{n+1} + \text{h.c.}) + m \sum_{n=0}^{2L-1} (-1)^n \psi_n^{\dagger} \psi_n + \frac{g^2}{2} \sum_{n=0}^{2L-2} |E_n|^2
$$

$$
\frac{\psi_0}{e^-} = \frac{U_0, E_0}{e^+} = \frac{U_1, E_1}{e^-} = \frac{U_2, E_2}{e^+} = \frac{U_2, E_2}{e^+} = \frac{1}{e^-} = \frac{1}{e^+} = \frac{1}{e^-} = \frac
$$

$$
\hat{H} = \hat{H}_m + \hat{H}_{kin} + \hat{H}_{el} = \frac{m}{2} \sum_{j=0}^{2L-1} \left[(-1)^j \hat{Z}_j + \hat{I} \right] + \frac{1}{2} \sum_{j=0}^{2L-2} \left(\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \text{h.c.} \right) + \frac{g^2}{2} \sum_{j=0}^{2L-2} \left(\sum_{k \le j} \hat{Q}_k \right)^2
$$

$$
\hat{Q}_k = -\frac{1}{2} \left[\hat{Z}_k + (-1)^k \hat{I} \right].
$$

VQE

Quantum Prepare some state $|\psi(\theta)\rangle$ Measure the energy of the state, (and possibly the gradient as well)

- Classical Use a classical optimization algorithm to update θ
- Hybrid algorithm that can be used to prepare ground states.
- Previously has been applied to studies of the Schwinger model (PhysRevA.98.032331, Nature 569 355–360 (2019), Phys. Rev. Lett. 126, 220501), SU(2) hadrons (Nature Communications 12, 6499 (2021)), …
- Use of VQE at scale will require an appropriate scalable ansatz circuit and optimization algorithm

SCALABLE CIRCUITS

- Construct an operator pool that respects translation invariance and other symmetries of the Hamiltonian
- Perform ADAPT-VQE on several small lattices to optimize a state prep circuit
- Provided the parameters were computed on a lattice larger than multiple correlation lengths, the convergence will be exponentially fast
- Extrapolate the parameters in lattice size to use on a larger lattice

RUNNING ON HARDWARE

Quantum computers are noisy and to perform reliable calculations, this noise needs to be corrected

Two types of noise on quantum computers

- Incoherent noise: relaxation and dephasing
- Coherent noise: unitary rotations caused by miscalibration or cross-talk

We can mitigate incoherent noise, but not coherent noise.

However, we can convert coherent noise into incoherent noise

Pauli Twirling (or randomized compiling)

HARDWARE RESULTS

$$
\chi = \frac{1}{L} \sum_{i} \langle \overline{\psi}_{i} \psi_{i} \rangle = \frac{1}{2L} \sum_{i} [(-1)^{i} Z_{i} + I] \equiv \frac{1}{2L} \sum_{i} \chi_{i}
$$

OPERATOR DECOHERENCE RENORMALIZATION

- Remaining errors in the simulation are incoherent
- Measured observable is proportional to noiseless one (under mild assumptions)

$$
\langle \hat{O} \rangle_{\text{meas}} = (1 - \eta_O) \langle \hat{O} \rangle_{\text{pred}}
$$

• Measure the noise parameter by running the same circuit with single qubit rotation angles set to 0

IMPLEMENTATION ON UP TO 100 QUBITS

- All circuits were optimized classically with up to $L=14$ sites (28 qubits)
- Errors were mitigated using operator decoherence renormalization

$(SC)^2$ -ADAPT-VQE

- ADAPT-VQE can choose different sequences of operators on different lattice sizes.
- This problem can be avoided by doing ADAPT-VQE on one lattice size and optimizing the same operator sequence on different lattice sizes.
- Optimization doesn't have to minimize energy. One can instead maximize the overlap with a surrogate for a given state, ex. MPS representation of the vacuum.

HADRONIC STATES

- Scattering simulations will likely require the ability to prepare hadronic wavepackets.
- Wavepackets are not eigenstates, so VQE can't be directly applied.
- Adiabatic state preparation can be used in principle, but wavepackets propagate and spread during adiabatic switching.

PREPARING HADRON STATES

- To study scattering or other dynamics, one needs to be able to prepare a state with hadrons.
- Previous studies of quantum simulations of scalar field theories proposed using adiabatic switching with forward and backwards evolution.
- In the Schwinger model, adiabatic switching can be performed from the strong coupling vacuum.
- Not necessary to use a large lattice or do adiabatics on the quantum computer. The same variational techniques can be used on a small lattice and extrapolated. $\langle \psi_{\rm WP} | E_i \rangle |^2$

TIME EVOLUTION

- Time evolution on a quantum computer is done by Trotterization, i.e. one breaks up the Hamiltonian into individual pieces that can be implemented in a sequence.
- QQ terms in the Hamiltonian give rise to long range interactions, due to confinement QQ interactions are exponentially suppressed at long distances can be neglected beyond a certain distance.
- Propagation of hadrons was tracked by measuring the disturbance of the chiral condensate from its value in vacuum

112 Qubits on ibm_torino CNOT Depth 370 13,858 CNOT gates $10⁷$ shots per time step

Q UA N T U M V S C L A S S I C A L S I M U L AT I O N

- Running all of the quantum circuits and processing the results took ~30 minutes per time slice
- The classical simulation took roughly 30 minutes total
- Simulating multiple hadrons will take the same amount of time to run a quantum computer (Necessary for scattering or simulating dense systems)
- The time to simulate multiple hadrons on a classical computer will grow exponentially

SUMMARY

- Variational calculations can be extrapolated to larger system sizes.
- This has enabled preparation of vacuum and single hadron states on quantum computers.
- This approach is capable of reaching the continuum limit of the Schwinger model.
- These techniques should scale to higher dimensions.

REAL TIME EVOLUTION

