Towards simulate Fermionic Scattering on a QC

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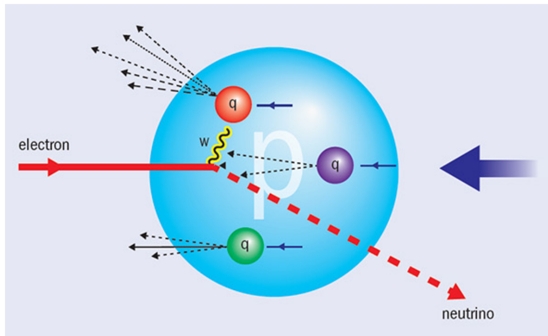
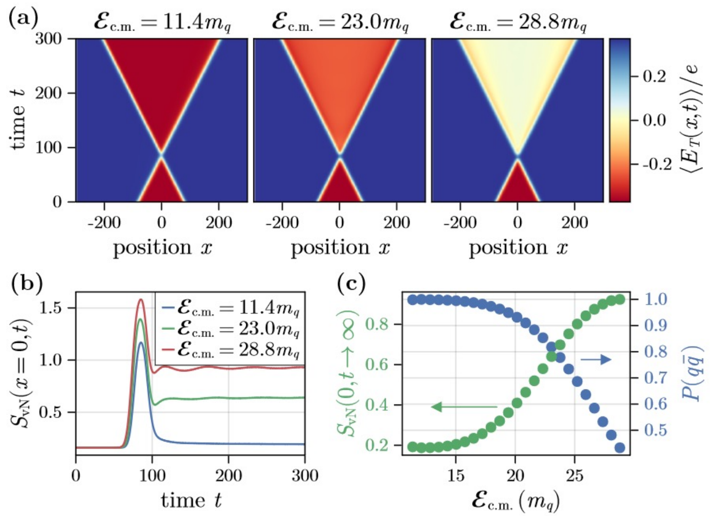
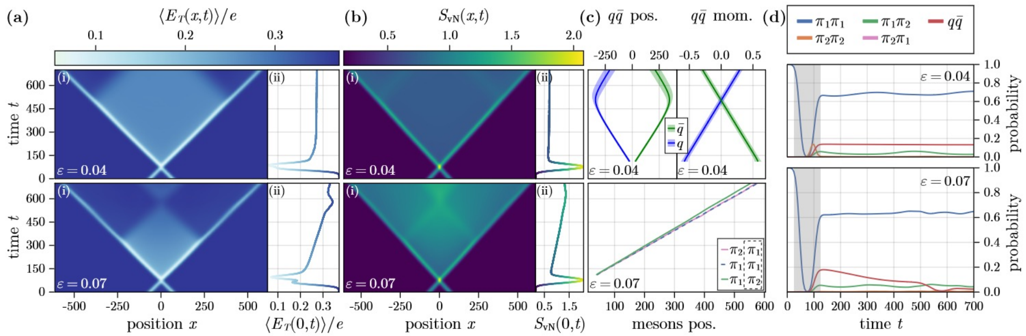
Motivation

- Real time dynamics in classical simulation
  - Monte Carlo: sign problem
  - Tensor Network : increasing entanglement with time-- increasing computation resource
- Quantum computers promise to efficiently simulate real-time dynamics



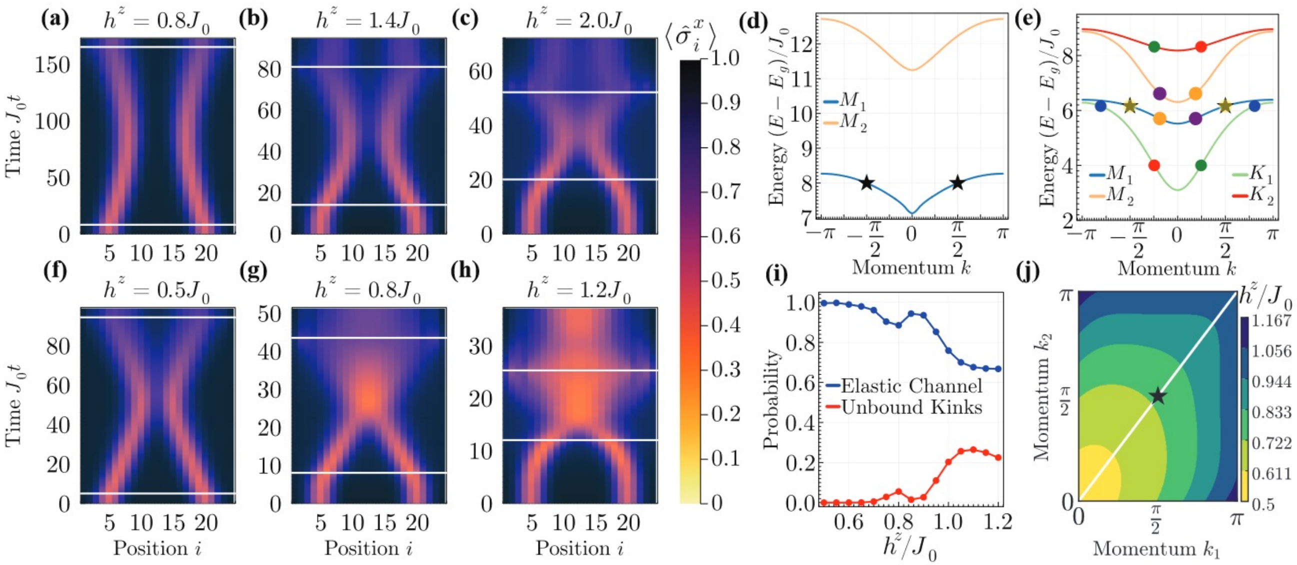
Related works





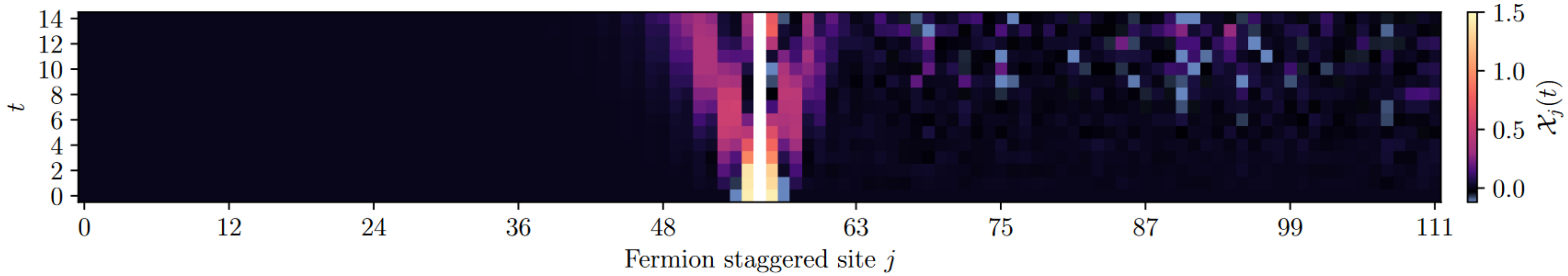
Belyansky, R., Whitsitt, S., Mueller, N., Fahimniya, A., Bennewitz, E. R., Davoudi, Z., & Gorshkov, A. V. (2024).
High-Energy Collision of Quarks and Mesons in the Schwinger Model: From Tensor Networks to Circuit QED.
Physical Review Letters, 132(9), 091903.

Related works



Bennewitz, E. R., Ware, B., Schuckert, A., Lerose, A., Surace, F. M., Belyansky, R., Morong, W., Luo, D., De, A., Collins, K. S., Katz, O., Monroe, C., Davoudi, Z., & Gorhskov, A. V. (2024).
Simulating Meson Scattering on Spin Quantum Simulators
(arXiv:2403.07061).

Related works



Farrell, R. C., Illa, M., Ciavarella, A. N., & Savage, M. J. (2024).
Quantum Simulations of Hadron Dynamics in the Schwinger Model using 112 Qubits .
PhysRevD.109.114510.

Outline

- Theoretical set up
  - The Thirring model
  - Preparation for fermion wave packet using quantum circuit
- Simulation results
  - Classical simulation for interacting case
  - Quantum simulation for noninteracting case
  - Resource estimation for interacting case

The Thirring model

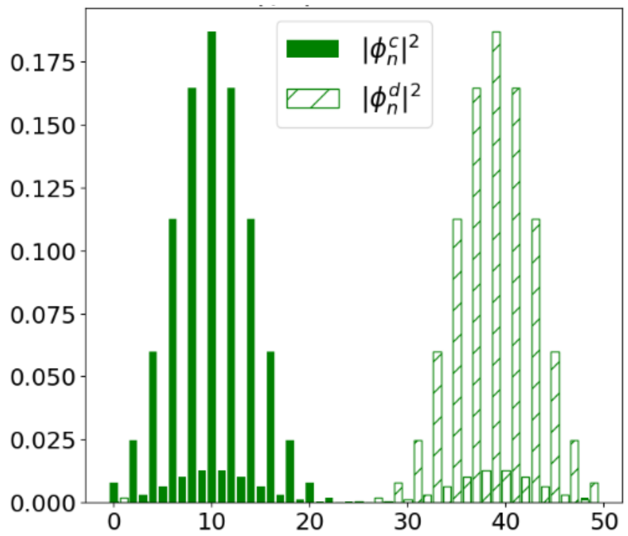
$$H = \sum_{n=0}^{N-1} \underbrace{\frac{i}{2a}\left(\xi_{n+1}^{\dagger}\xi_n - \xi_n^{\dagger}\xi_{n+1}\right)}_{\text{Kinetic term}} + m \sum_{n=0}^{N-1} \underbrace{(-1)^n \xi_n^{\dagger}\xi_n}_{\text{Mass term}} + \frac{g}{a}\sum_{n=0}^{N-1} \underbrace{\xi_n^{\dagger}\xi_n\xi_{n+1}^{\dagger}\xi_{n+1}}_{\text{Four-body interaction}}$$

- Exactly solvable fermionic model in 1+1 dimension, Kogut-Susskind formula
- $\xi_n^{\dagger}, \xi_n$: fermionic creation or annihilation operators
- a: lattice spacing, a = 1
- Periodic boundary condition: $\xi_N = \xi_0$

Scattering process

✓ Vacuum state $|\Omega\rangle$: ground state of Hamiltonian H ----- VQE…

- Initial state $|\psi(0)\rangle$: wave packets of particles



✓ Time evolution : $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$ ----- Trotterization…

Define wave packets in momentum space

- Creation operators for fermion and antifermion

$$H = \sum_k w_k \left(c_k^\dagger c_k - d_k d_k^\dagger \right)$$

$$c_k^\dagger = \frac{1}{\sqrt{N}} \sqrt{\frac{m + w_k}{w_k}} \sum_n e^{ikn} \left(\Pi_{n0} + v_k \Pi_{n1} \right) \xi_n^\dagger$$

$$d_k^\dagger = \frac{1}{\sqrt{N}} \sqrt{\frac{m + w_k}{w_k}} \sum_n e^{ikn} \left(\Pi_{n1} + v_k \Pi_{n0} \right) \xi_n$$

- Fidelity between the corresponding eigen state

N = 10, g=0.8, $k = 2\pi/N$				
m	0.01	0.1	0.5	0.8
$c_k^\dagger$	0.513	0.834	0.974	0.989
$\tilde{c}_k^\dagger$	0.693	0.90	0.989	0.995

M. Rigobello et al., Phys. Rev. D 104, 114501 (2021).

Define wave packets in momentum space

- Creation operators for wave packets

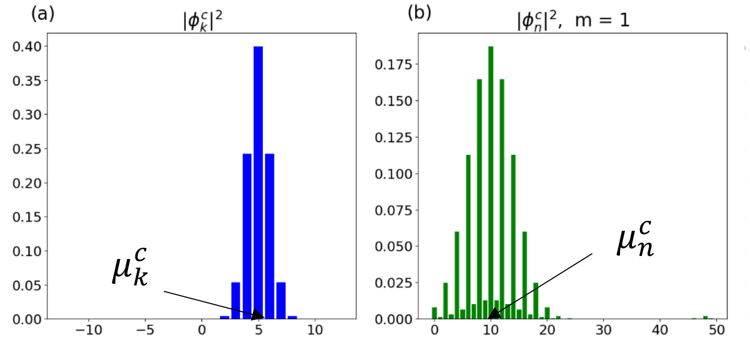
Fermion: $$C^\dagger(\phi^c) = \sum_k \phi_k^c c_k^\dagger$$

Antifermion: $$D^\dagger(\phi^d) = \sum_k \phi_k^d d_k^\dagger$$

- Amplitude of Gaussian Wave packet in momentum space

$$\phi_k^{c(d)} = \frac{1}{\mathcal{N}_k^{c(d)}} e^{-ik\mu_n^{c(d)}} e^{-(k-\mu_k^{c(d)})^2/4\sigma_k^2}$$

Position ($\mu_n^{c(d)}$) Momentum ($\mu_k^{c(d)}$) Width (σ_k^2)



M. Rigobello et al., Phys. Rev. D 104, 114501 (2021).

Define wave packets in position space

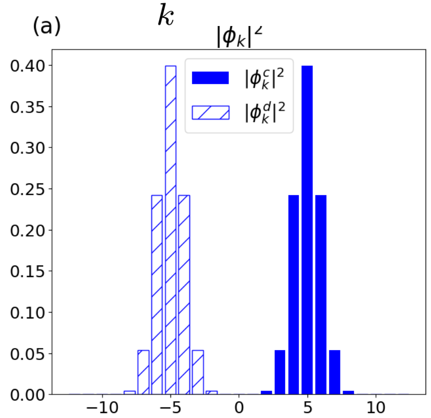
$C^\dagger(\phi^c)$

$D^\dagger(\phi^d)$

- Momentum space

$$\sum_k \phi_k^c c_k^\dagger$$

$$\sum_k \phi_k^d d_k^\dagger$$



$$\phi_k^{c(d)} = \frac{1}{\mathcal{N}_k^{c(d)}} e^{-ik\mu_n^{c(d)}} e^{-(k-\mu_k^{c(d)})^2/4\sigma_k^2}$$

Fourier transformation:

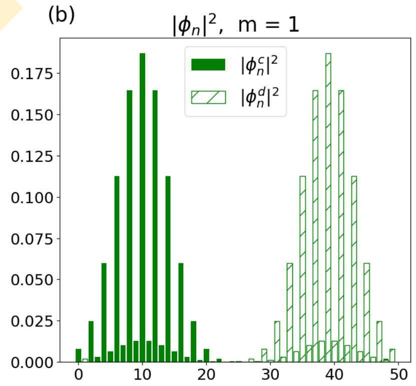
$c_k^\dagger \to \xi_n^\dagger$

$d_k^\dagger \to \xi_n$

- Position space

$$\sum_n \phi_n^c \xi_n^\dagger$$

$$\sum_n \phi_n^d \xi_n$$



$$\phi_n^c = \frac{1}{\sqrt{N}} \sum_k \phi_k^c \sqrt{\frac{m+w_k}{w_k}} e^{ikn} (\Pi_{n0} + v_k \Pi_{n1})$$

$$\phi_n^d = \frac{1}{\sqrt{N}} \sum_k \phi_k^d \sqrt{\frac{m+w_k}{w_k}} e^{ikn} (\Pi_{n1} + v_k \Pi_{n0})$$

Summary for operators

- Initial state: $|\psi(0)\rangle = D^{\dagger}(\phi^{d})\, C^{\dagger}(\phi^{c})|\Omega\rangle$

$$C^{\dagger}(\phi^{c}) = \sum_{n} \phi_{n}^{c} \xi_{n}^{\dagger}, \qquad D^{\dagger}(\phi^{d}) = \sum_{n} \phi_{n}^{d} \xi_{n}$$

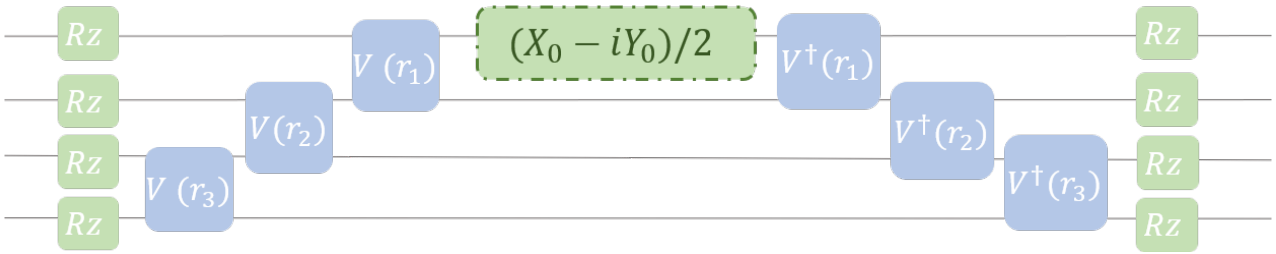
$$\phi_{n}^{c}(\mu_{k}^{c}, \mu_{n}^{c}, \sigma_{k}^{c}), \quad \phi_{n}^{d}(\mu_{k}^{d}, \mu_{n}^{d}, \sigma_{k}^{d})$$

- **How to implement the linear combination of operators in a quantum computing?**

Circuit for wave packet preparation

$$C^\dagger(\phi^c) = \sum_{n=0}^{N-1} \xi_n^\dagger \phi_n^c = V(\phi_n^c)\sigma_0^- V^\dagger(\phi_n^c)$$

- Circuit decomposed by Givens rotation



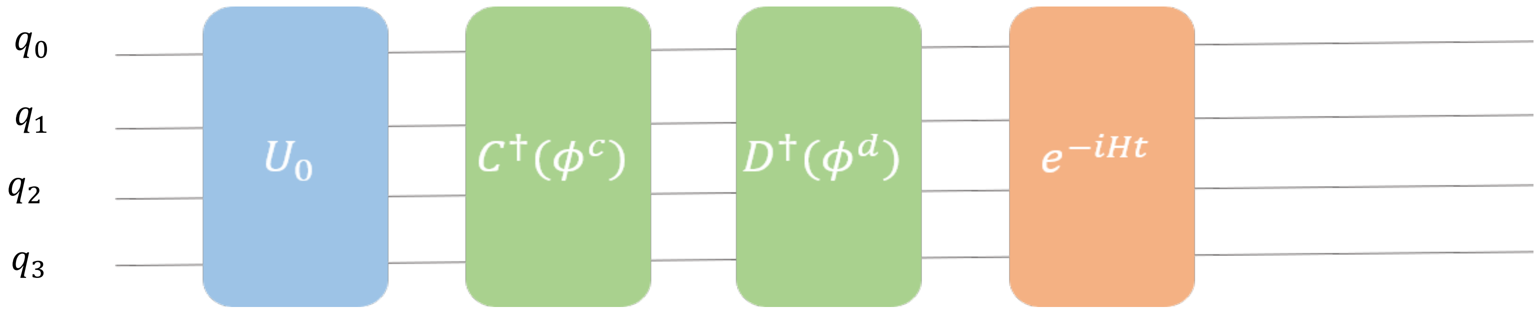
$$V^\dagger(r_n) = \exp\left(i\frac{\theta_n}{2}\left[\sigma_{n-1}^x\sigma_n^y - \sigma_{n-1}^y\sigma_n^x\right]\right)$$

- CNOT number: 4(N-1)

Z. Jiang et al., Phys. Rev. Appl. 9, 044036 (2018)

Circuit for scattering process

- Acting time evolution on initial state: $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$



Observables

- Focus on the sector of $\sum_n \xi_n^\dagger \xi_n = N/2$
- Monitor the excess fermion density with respect to the ground state $|\Omega\rangle$ over time

$$\Delta\langle \xi_n^\dagger \xi_n \rangle_t = \langle \psi(t)|\xi_n^\dagger \xi_n|\psi(t)\rangle - \langle \Omega|\xi_n^\dagger \xi_n|\Omega\rangle$$

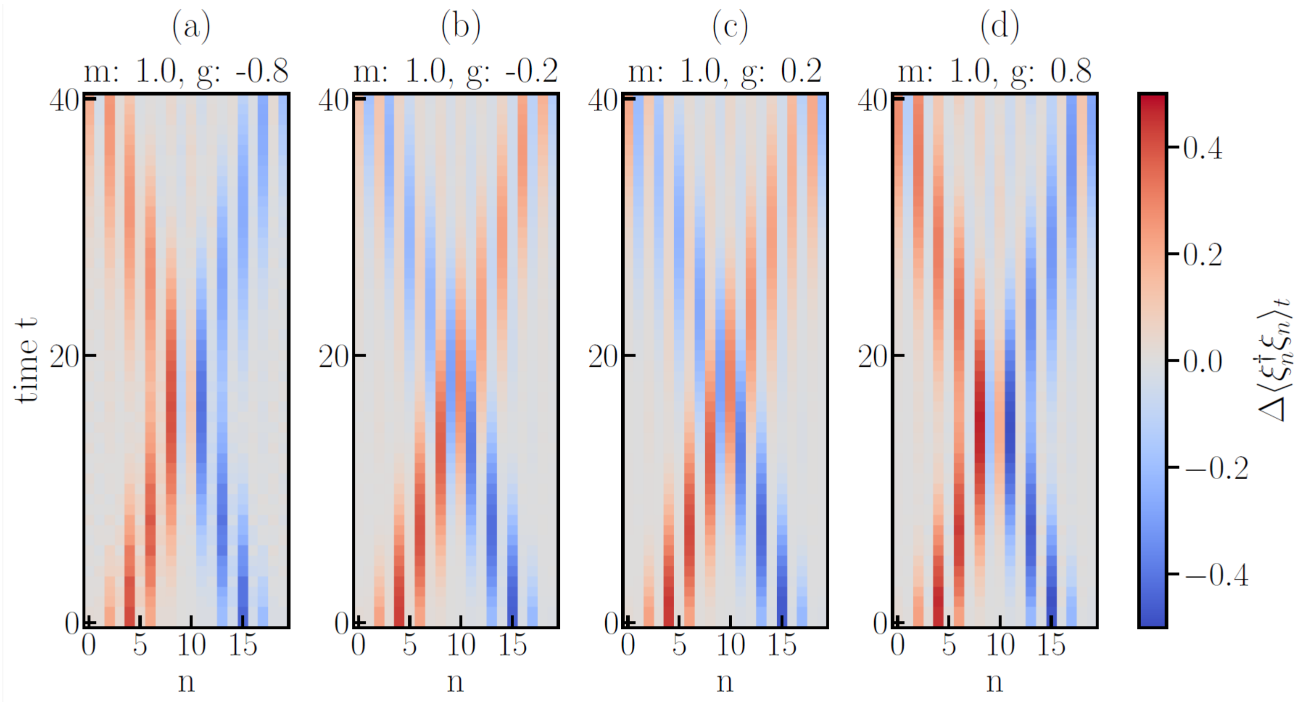
- Monitor the entropy production over time compared to two wave packets moving individually,

$$\Delta S_2(n,t) = \Delta S_1(n,t) - \left(\Delta S_{1,C}(n,t) + \Delta S_{1,D}(n,t)\right)$$

where $\Delta S_1(n,t) = S(n,t) - S_\Omega(n)$ is the difference with respect to the ground state

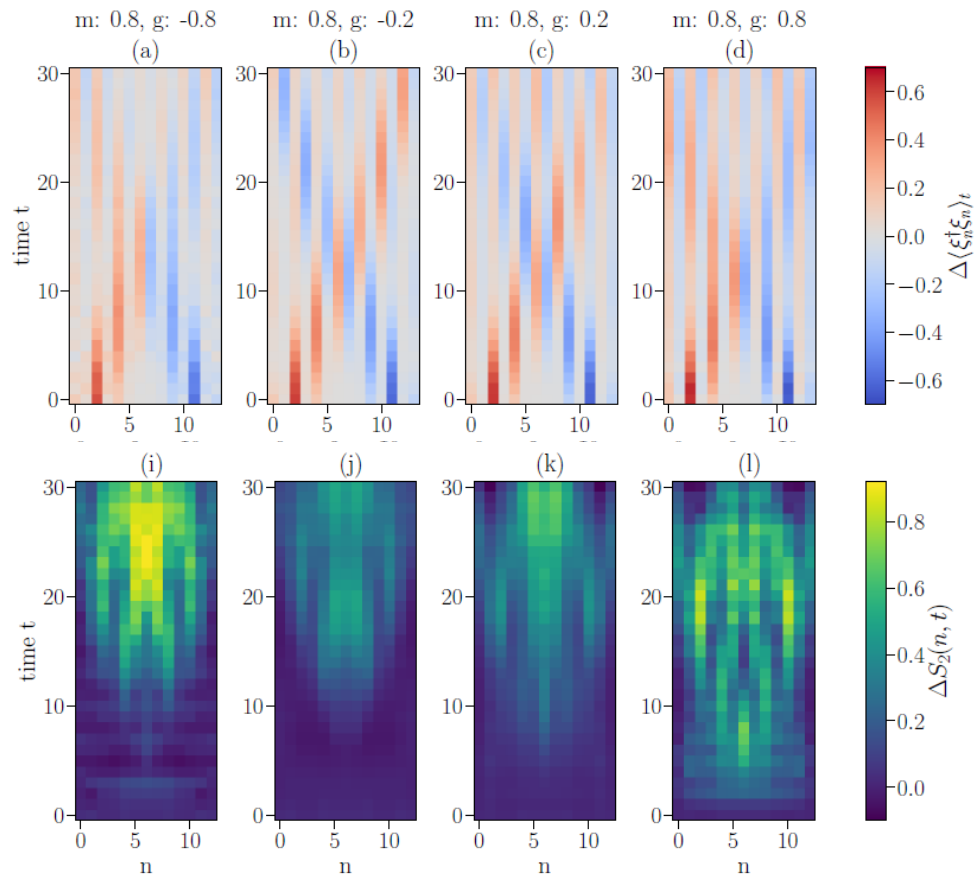
Classical simulation for the interacting case

Fermionic scattering: $g \neq 0$



- Elastic scattering between the fermion and antifermion for large values of $|g|$

Fermionic scattering: $\boldsymbol{g \neq 0}$



- We again observe excess entropy with respect to the vacuum

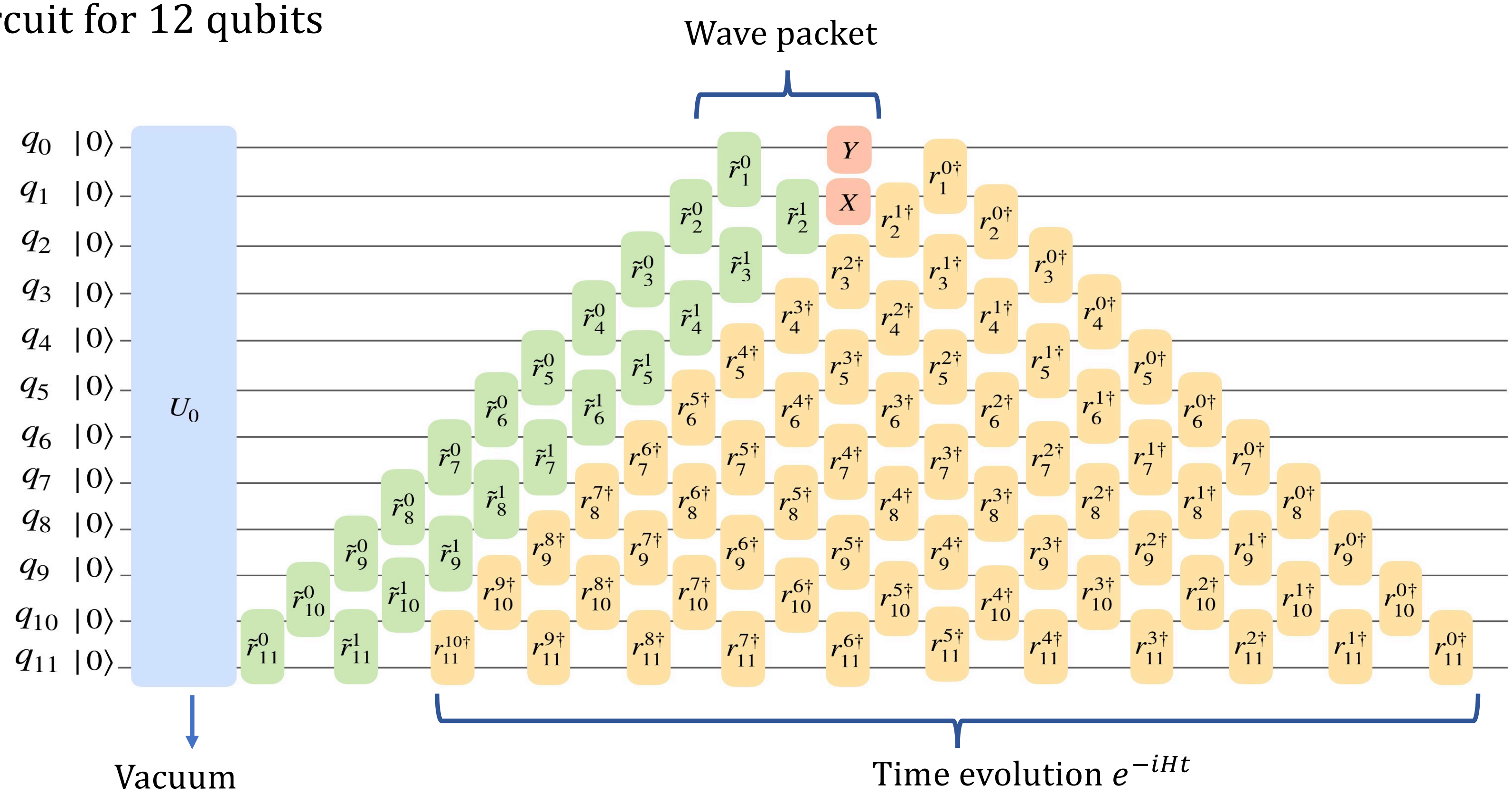
- This time $\Delta S_2(n,t)$after the Collision:
  - Effect of the interaction
  - Entropy production is larger for larger $|g|$

Quantum simulation for the noninteracting case

$$H = \sum_{n=0}^{N-1} \frac{i}{2a}\left(\xi_{n+1}^{\dagger}\xi_n - \xi_n^{\dagger}\xi_{n+1}\right) + m\sum_{n=0}^{N-1}(-1)^n \xi_n^{\dagger}\xi_n + \frac{g}{a}\sum_{n=0}^{N-1}\xi_n^{\dagger}\xi_n\xi_{n+1}^{\dagger}\xi_{n+1}$$

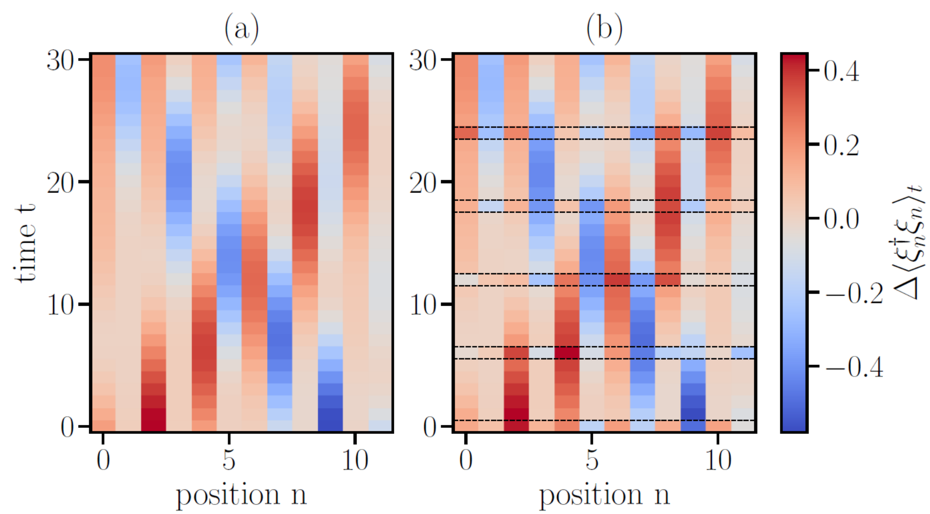
Quantum simulation: $g = 0$

- Circuit for 12 qubits



Quantum simulation: $\boldsymbol{g = 0}$

- Results from ibmq_peekskill after applying Pauli twirling, dynamical decoupling and zero noise extrapolation



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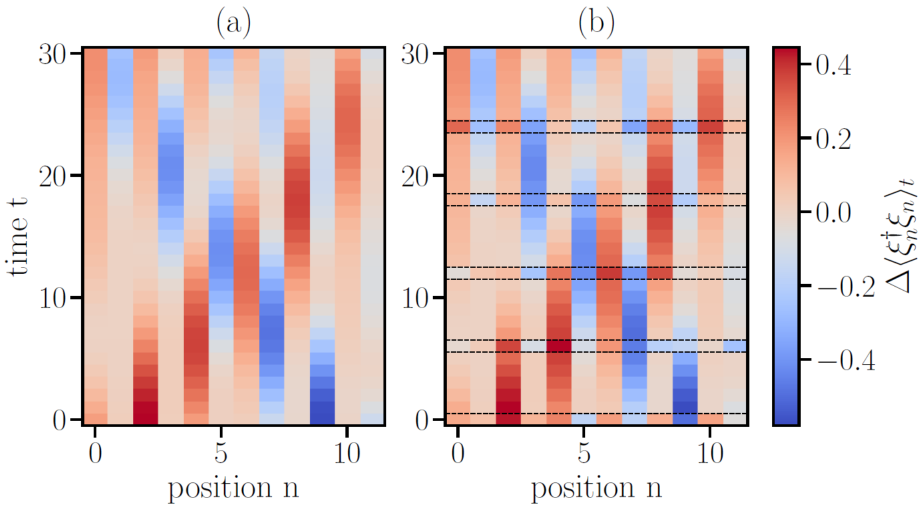

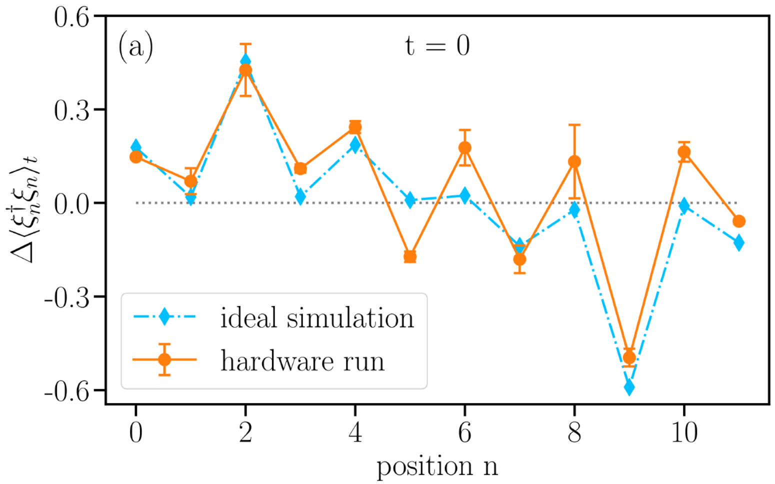

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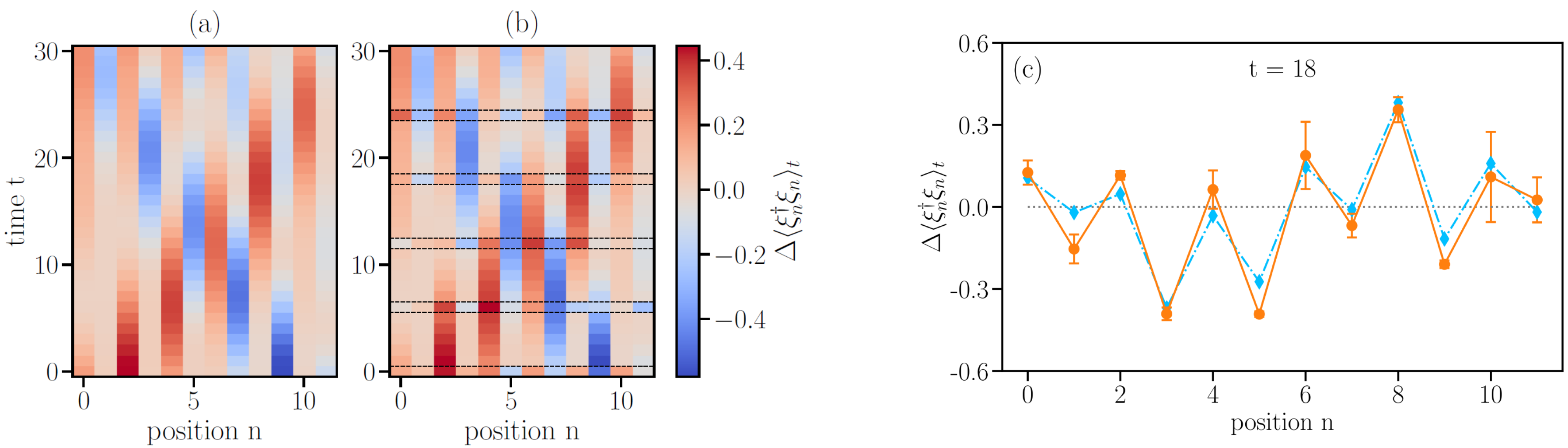


Resource estimation for the interacting case

$$H = \sum_{n=0}^{N-1} \frac{i}{2a}\left(\xi_{n+1}^{\dagger}\xi_n - \xi_n^{\dagger}\xi_{n+1}\right) + m\sum_{n=0}^{N-1}(-1)^n \xi_n^{\dagger}\xi_n + \frac{g}{a}\sum_{n=0}^{N-1} \xi_n^{\dagger}\xi_n\xi_{n+1}^{\dagger}\xi_{n+1}$$

Trotterization of time evolution

- Hamiltonian after Jordan-Wigner transformation

$$H = H_{even}^{g} + H_{odd}^{g} + H_{z}^{g}$$

$$H_{even}^{g} = \frac{1}{4} \sum_{n \in \{0,2,\cdots,N-2\}} (\sigma_{n+1}^{y}\sigma_{n}^{x} - \sigma_{n+1}^{x}\sigma_{n}^{y} + g \cdot \sigma_{n+1}^{z}\sigma_{n}^{z}),$$

$$H_{odd}^{g} = \frac{1}{4} \sum_{n \in \{1,3,\cdots,N-3\}} (\sigma_{n+1}^{y}\sigma_{n}^{x} - \sigma_{n+1}^{x}\sigma_{n}^{y} + g \cdot \sigma_{n+1}^{z}\sigma_{n}^{z}))$$

$$+ \frac{(-1)^{\frac{N}{2}-1}}{4} \left(\sigma_{0}^{y}\sigma_{N-1}^{x} - \sigma_{0}^{x}\sigma_{N-1}^{y}\right) + \frac{g}{4} \cdot \sigma_{0}^{z}\sigma_{N-1}^{z}),$$

$$H_{z}^{g} = \sum_{n=0}^{N-1} \left(\frac{-m}{2} \cdot (-1)^{n} - \frac{g}{2} \right) \sigma_{n}^{z}$$

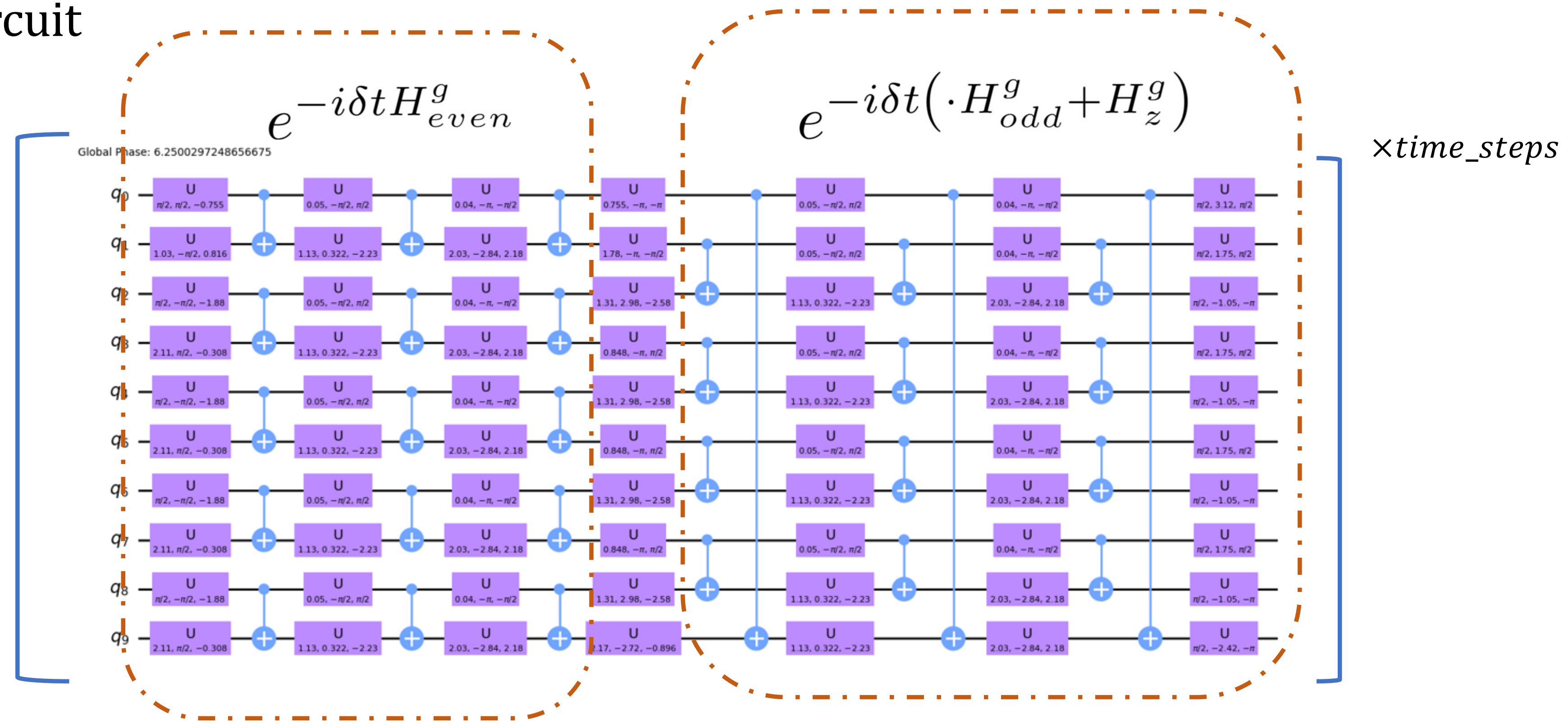
Trotterization of time evolution

- Second order Trotterization

$$
\begin{aligned}
U(t) &= \left(e^{-i\frac{\delta t}{2}\cdot H^g_{even}} \cdot e^{-i\delta t\left(\cdot H^g_{odd}+H^g_z\right)} \cdot e^{-i\frac{\delta t}{2}\cdot H^g_{even}}\right)^k, \\
&= e^{-i\frac{\delta t}{2}\cdot H^g_{even}} \cdot e^{-i\delta t\left(\cdot H^g_{odd}+H^g_z\right)} \cdot e^{-i\delta t/2\cdot H^g_{even}} \\
&\quad \cdot e^{-i\delta t/2\cdot H^g_{even}} \cdot e^{-i\delta t\left(\cdot H^g_{odd}+H^g_z\right)} \cdot e^{-i\delta t/2 H^g_{even}} \cdots, \\
&= e^{-i\frac{\delta t}{2}\cdot H^g_{even}} \cdot e^{-i\delta t\left(\cdot H^g_{odd}+H^g_z\right)} \cdot e^{-i\delta t\cdot H^g_{even}} \\
&\quad \cdot e^{-i\delta t\left(\cdot H^g_{odd}+H^g_z\right)} \cdot e^{-i\delta t H^g_{even}} \cdots \\
&\quad \cdot e^{-i\delta t\left(\cdot H^g_{odd}+H^g_z\right)} \cdot e^{-i\frac{\delta t}{2}\cdot H^g_{even}}.
\end{aligned}
$$

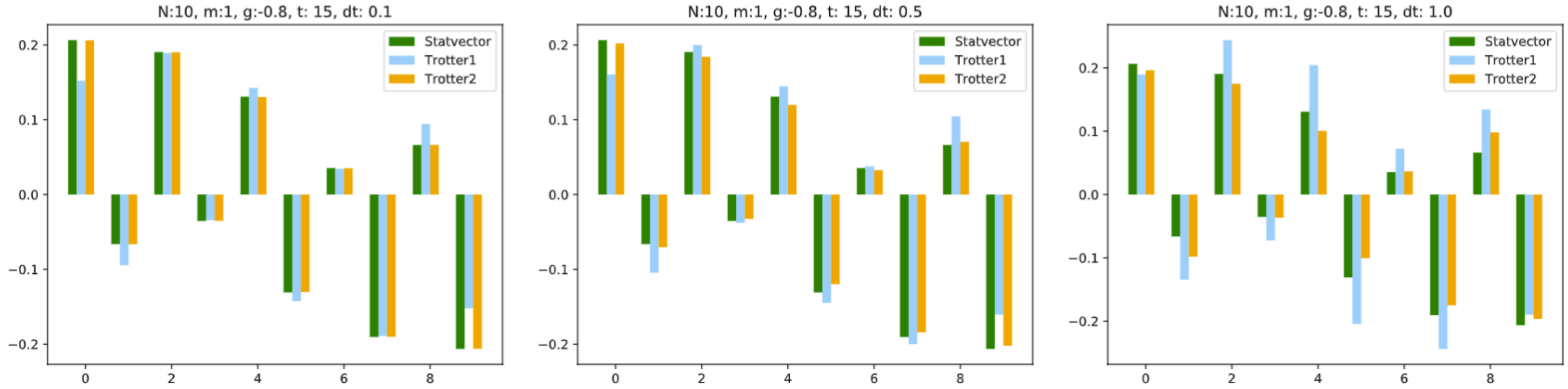
Trotterization of time evolution

- Circuit



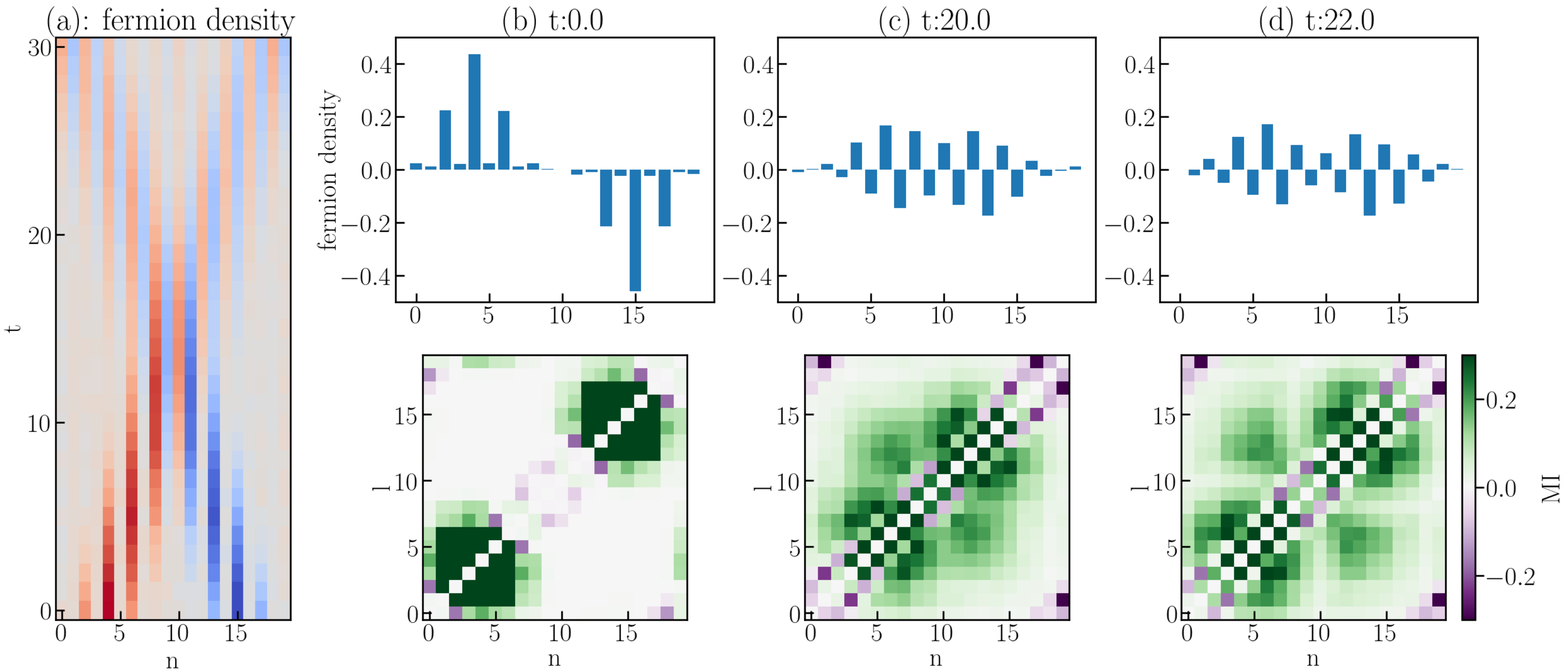
Trotterization of time evolution

- Comparison of first and second order Trotterization



Mutual Information

$$MI(n,l) = S(n,n+1) + S(l,l+1) - S(n,n+1,l,l+1)$$



m=0.8, g=0.4, dt=1

Resource estimation

- Simulate the scattering process for interacting Thirring model on quantum hardware

N = 40, interacting, T = 22, dt = 1, vacuum reps=3				
Steps	vacuum	wave packet	time evolution	in total
CNOT layer	12	95	135	242
CNOT number	240	318	2700	3258

Summary and outlooks

- Propose the framework to simulate fermionic scattering on a digital quantum computing approach.
  - Simulated the elastic scattering process in the Thirring model classically
  - Successful implementation for the noninteracting case on quantum hardware

Summary and outlooks

- Outlook:
  - Study the interacting Thirring model on quantum hardware
  - Apply the method to other fermionic models
  - Extension to gauge models
  - Approximate physical fermion in small mass value

Thank you!

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arXiv:2312.02272