Università degli Studi di Padova

Dipartimento di Fisica e Astronomia Galileo Galilei

QUANTUM Information and Matter

INFN PADOVA

# Lattice QED photonic wavepackets on ladder geometries

**Mattia Morgavi**, M. Rigobello, L. Maffi, P. Silvi and S. Montangero

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PASQuanS2

QUANTUM FLAGSHIP

QUANTERA

ICSC Centro Nazionale di Ricerca in HPC, Big Data and Quantum Computing

QUANTUM COMPUTING AND SIMULATION CENTER

QuantHEP

T-NiSQ Tensor Networks in Simulation of Quantum Matter

# Introduction

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$$\hat{H} = \hat{H}_0 + \hat{H}_{\mathrm{int}}$$

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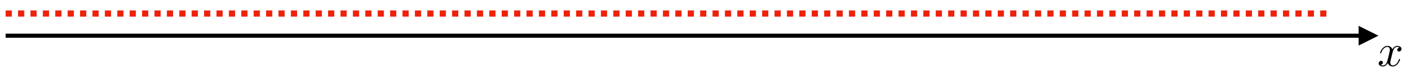


**Interacting groundstate (dressed vacuum)** $|\Omega\rangle$ $\hat{H} = \hat{H}_0 + \hat{H}_{\mathrm{int}}$

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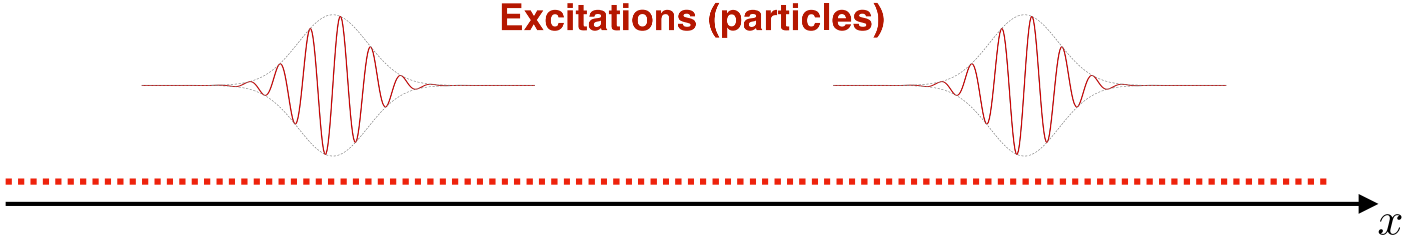


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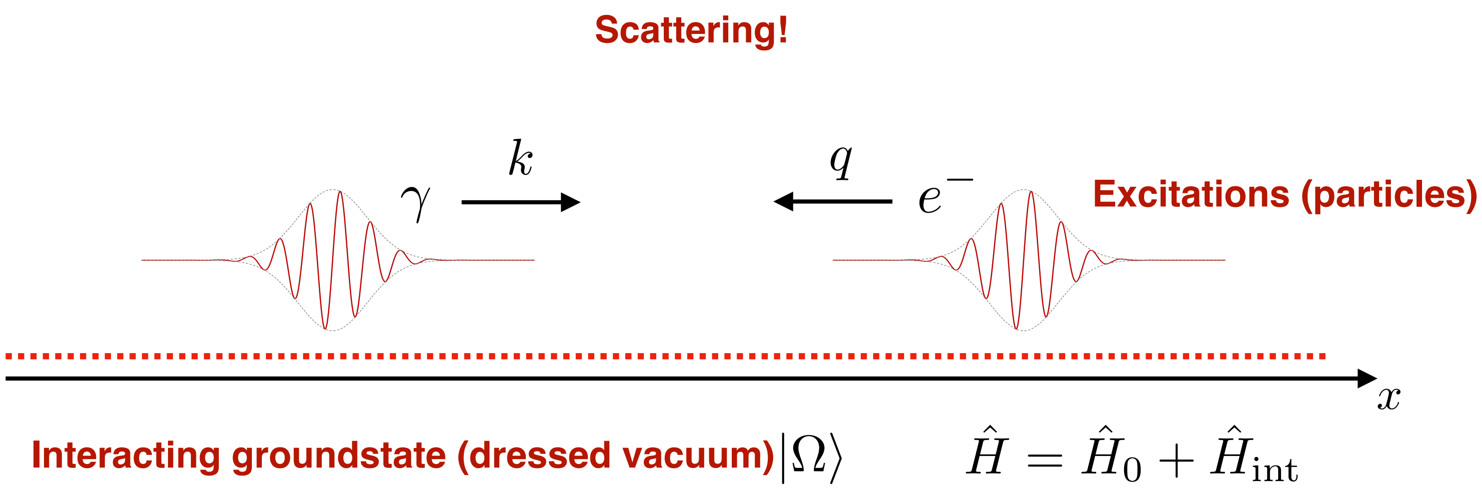

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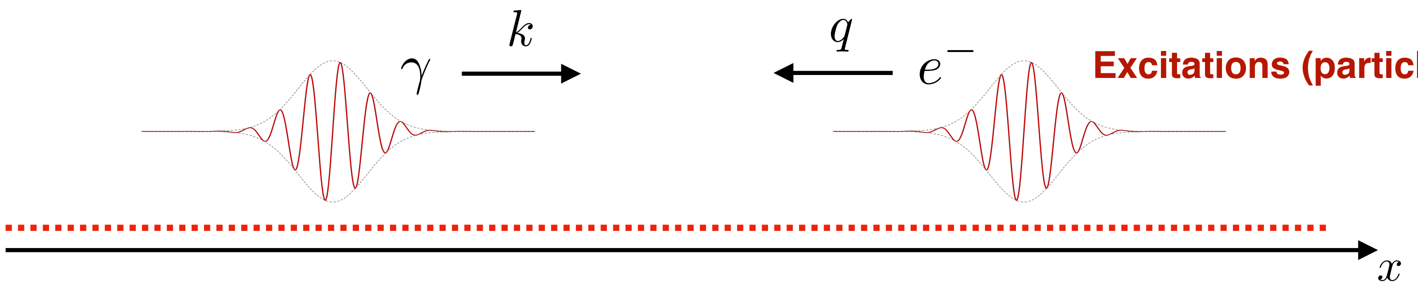



**Excitations (particles)**

**Interacting groundstate (dressed vacuum)** $|\Omega\rangle$

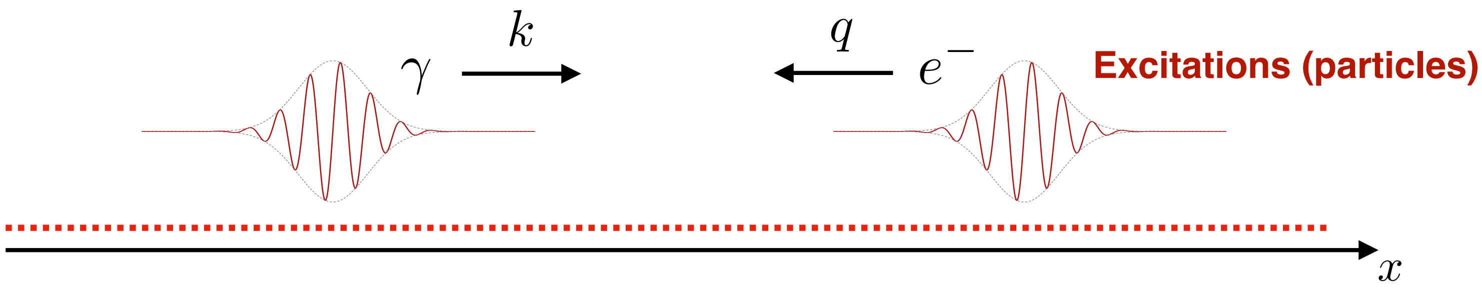
$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

**continuous…**

**Model: 1D**

**first step**

# Introduction

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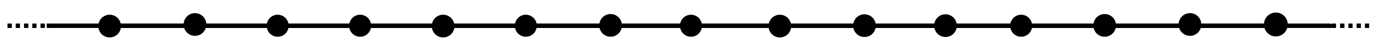
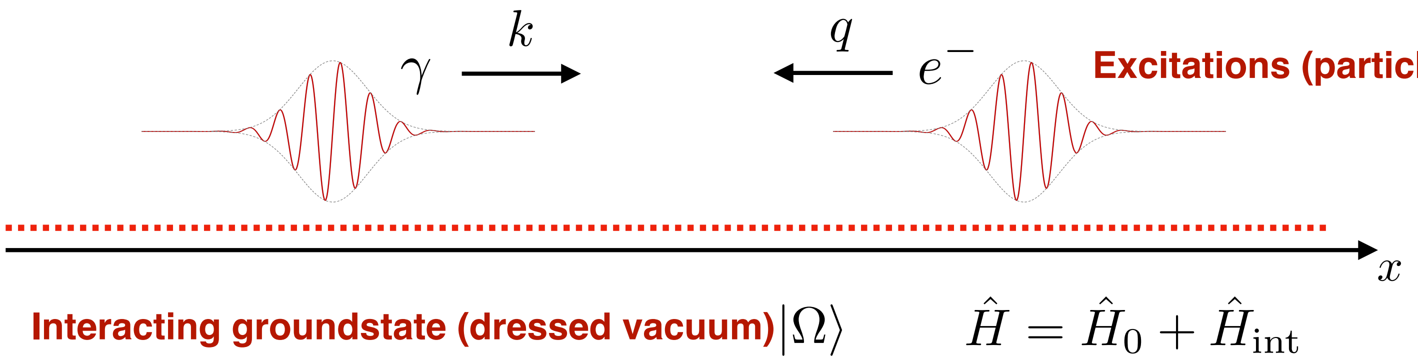


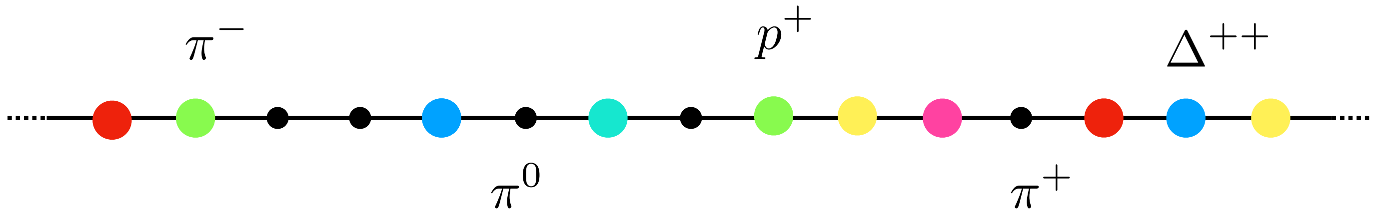
**Toy Model:** **1D** **Lattice**

**1D** — **first step**

**Lattice** — **computable**

# Introduction



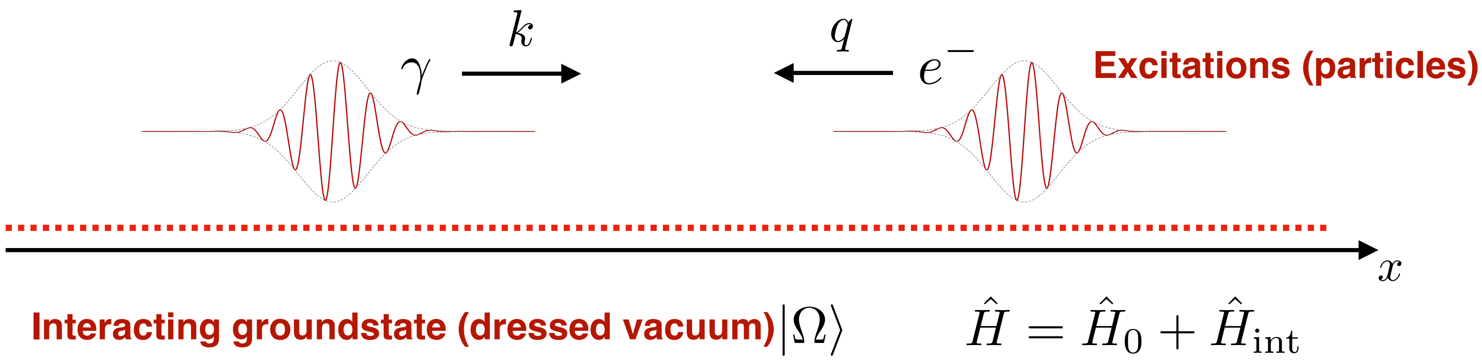



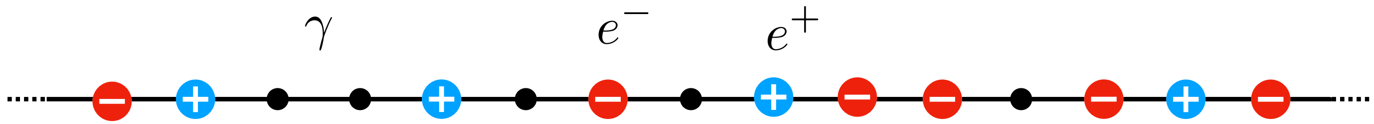

Challenging (maybe next year...)

**Toy Model:** 1D Lattice Quantum Chromo Dynamics

- 1D — first step
- Lattice — computable
- Quantum — quantum tech
- Dynamics — Scattering

# Introduction




**Toy Model:** 1D Lattice Quantum Electro Dynamics

- 1D — first step
- Lattice — computable
- Quantum — quantum tech
- Electro — first step
- Dynamics — Scattering

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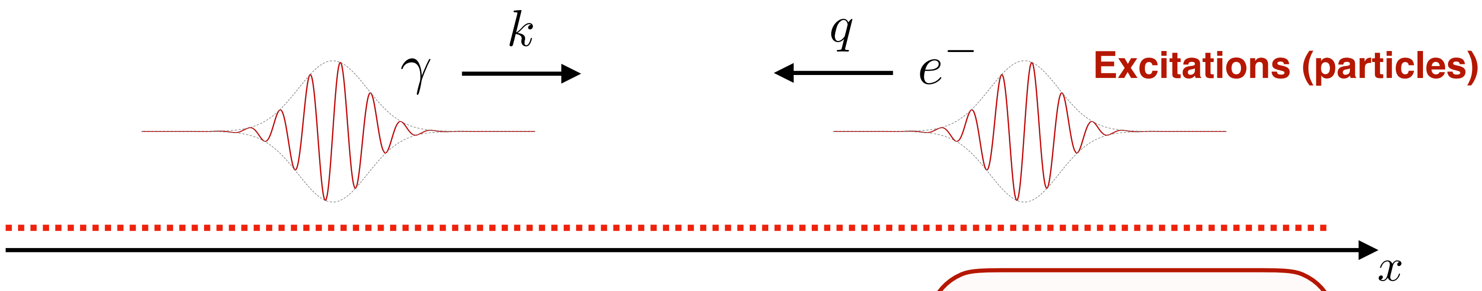


**Interacting groundstate (dressed vacuum)** $|\Omega\rangle$

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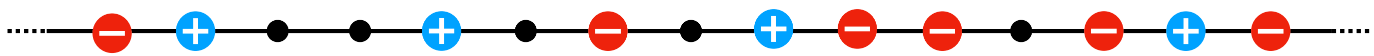

**Toy Model:** **1D Hamiltonian Lattice Quantum Electro Dynamics**

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- Hamiltonian — quantum tech
- Lattice — computable
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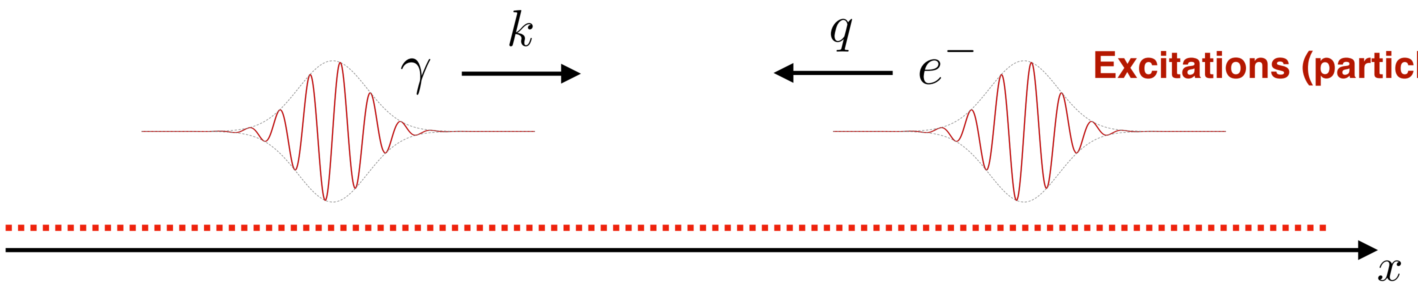
# Introduction




**Excitations (particles)**

**Interacting groundstate (dressed vacuum)** $|\Omega\rangle$

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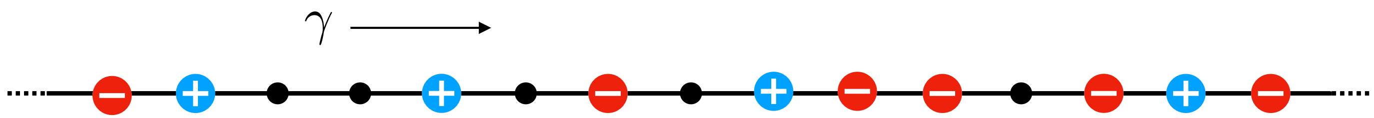
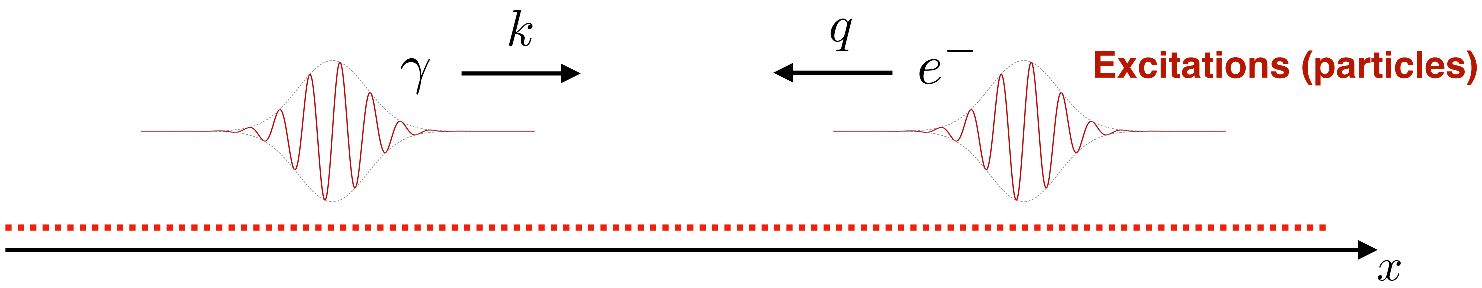



**Goal: Compton scattering**
**...we need a photon...**

**Toy Model: 1D Hamiltonian Lattice Quantum Electro Dynamics**

- **1D** — first step
- **Hamiltonian** — quantum tech
- **Lattice** — computable
- **Quantum** — quantum tech
- **Electro** — first step
- **Dynamics** — Scattering

# Introduction

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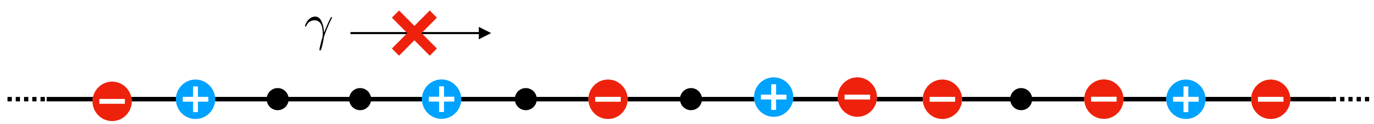


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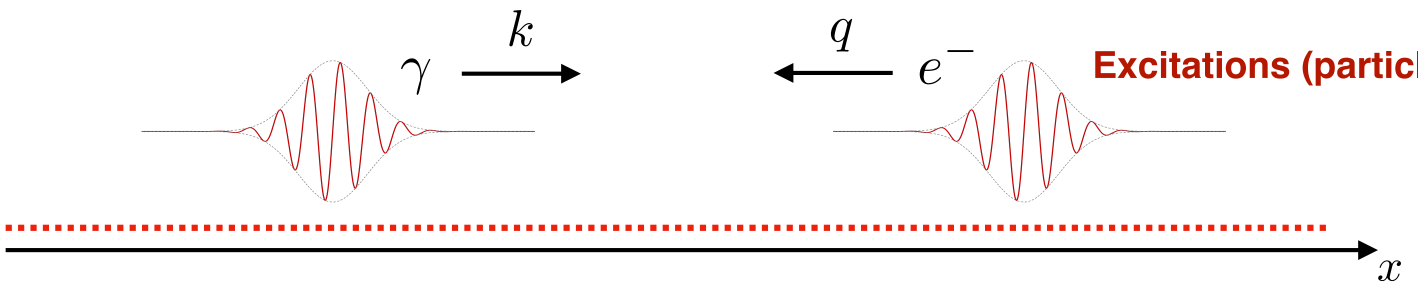


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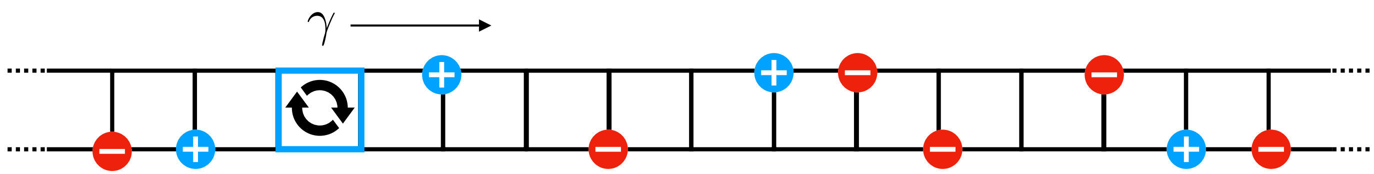
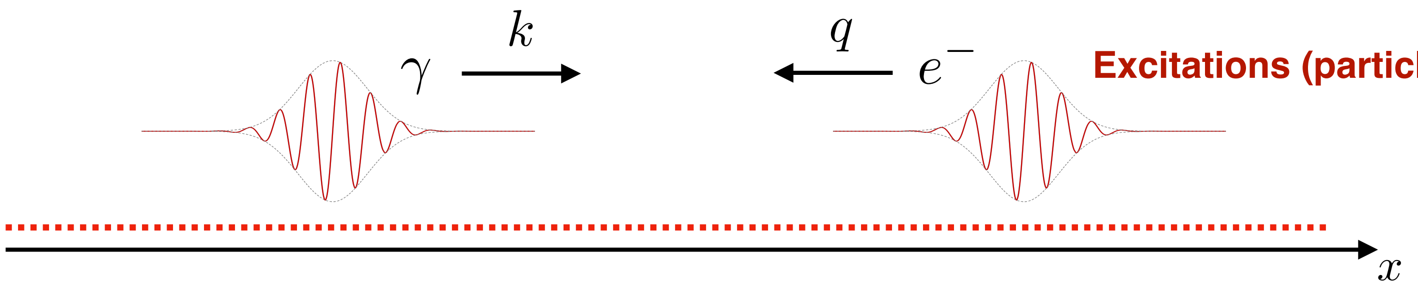



**Too difficult (for the moment)**

**Toy Model: (quasi)-1D Hamiltonian Lattice Quantum Electro Dynamics on Ladder Geometries**

- (quasi)-1D — first step
- Hamiltonian — quantum tech
- Lattice — computable
- Quantum — quantum tech
- Electro — first step
- Dynamics — Scattering
- Ladder Geometries — photon propagation

# Introduction

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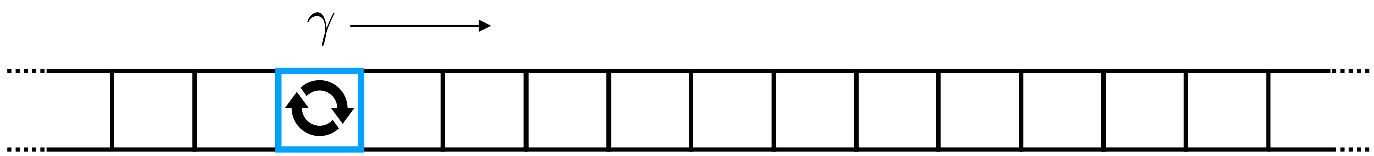
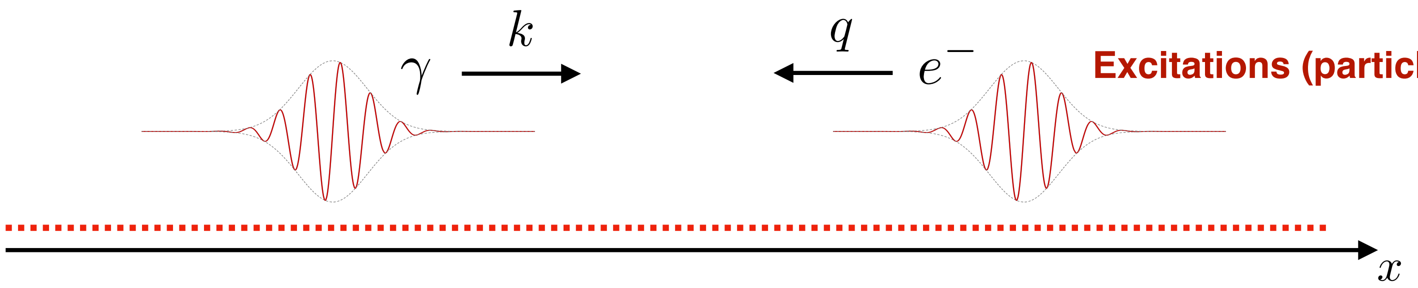



**Toy Model: (quasi)-1D Hamiltonian Lattice Pure Quantum Electro Dynamics on Ladder Geometries**

- (quasi)-1D: **first step**
- Hamiltonian: **quantum tech**
- Lattice: **computable**
- Pure: **first step (still interacting)**
- Quantum: **quantum tech**
- Electro: **first step**
- Dynamics: **Scattering**
- Ladder Geometries: **photon propagation**

# Introduction

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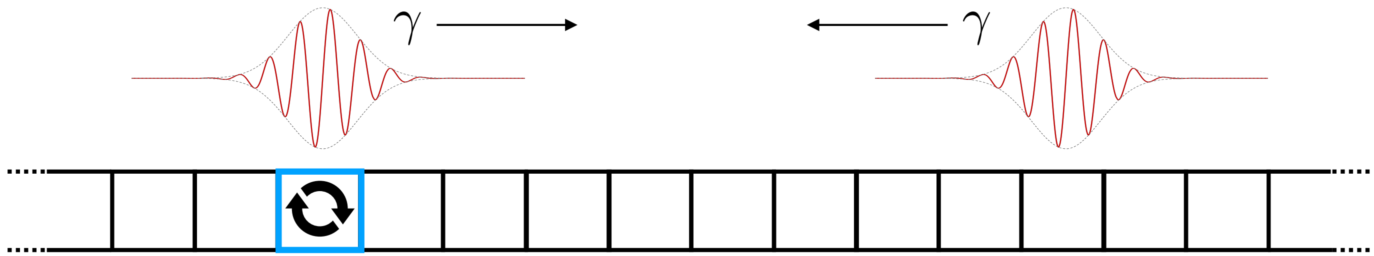


**Toy Model: (quasi)-1D Hamiltonian Lattice Pure Quantum Electro Dynamics on Ladder Geometries**

- (quasi)-1D: first step
- Hamiltonian: quantum tech
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- Dynamics: Scattering
- on Ladder Geometries: photon propagation

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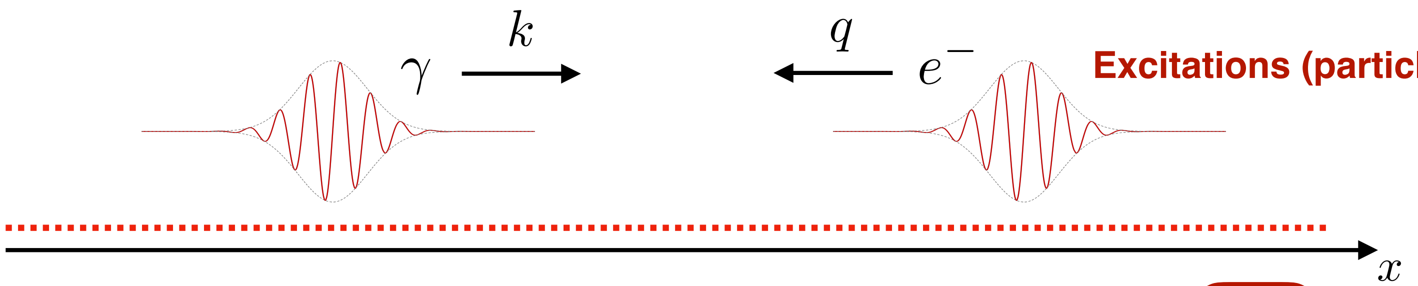
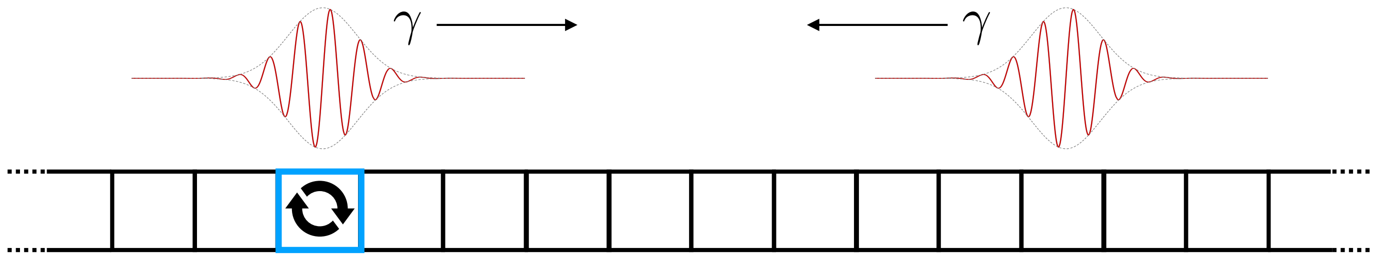



Interacting groundstate (dressed vacuum) $|\Omega\rangle$

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

due to the ladder

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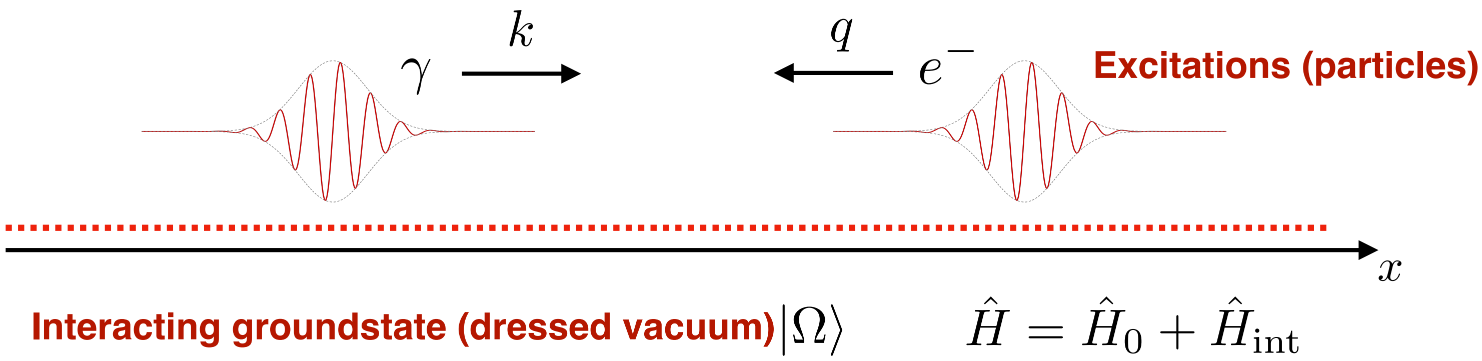


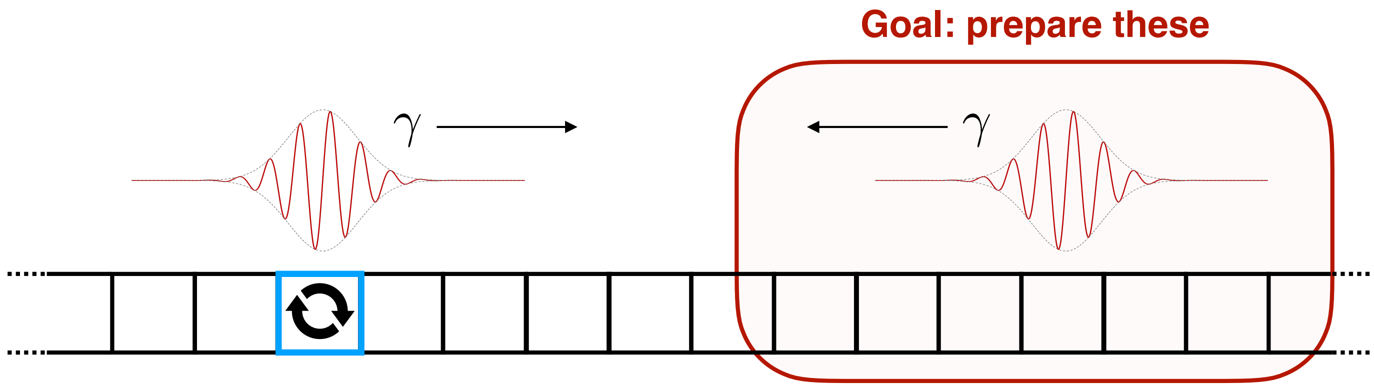


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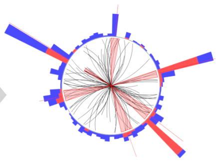
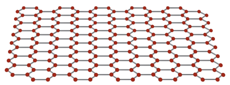


**HEP**

**Quantum Physics**



**CM**

# Motivation: problems and solutions

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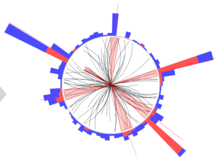
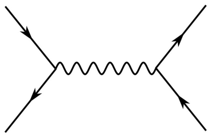
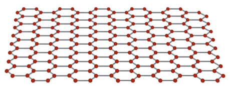


**HEP**

assumptions:



**Analytical**



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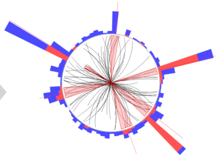

**HEP**

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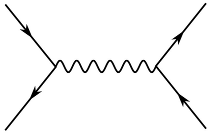
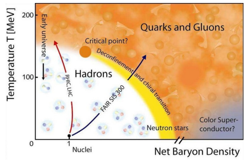
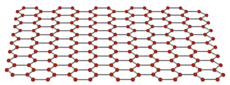


**Analytical**

QCD ?




**Some problems remain intractable...**



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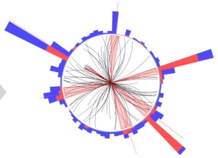
Quantum Physics



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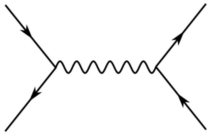

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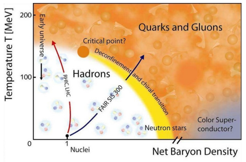


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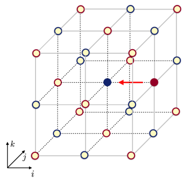
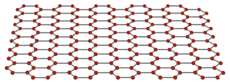
discretization:



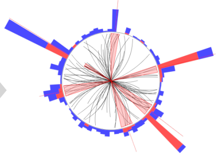
**LGT**



**CM**

# Motivation: problems and solutions

Quantum Physics



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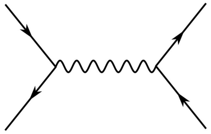

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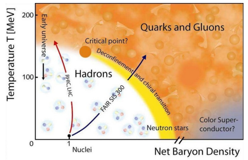


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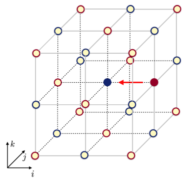

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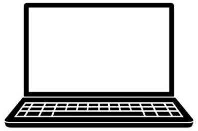
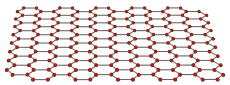


**Numerical**



**CM**

# Motivation: problems and solutions

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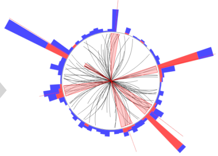

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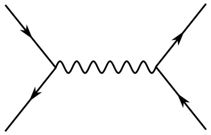

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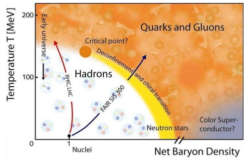


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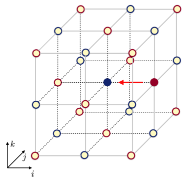
discretization:



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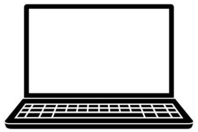

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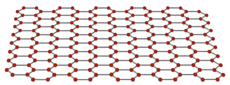

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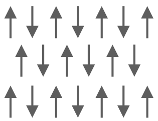

**Models**

# Motivation: problems and solutions

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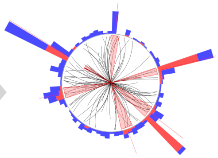
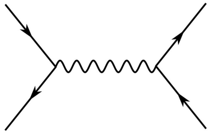


**HEP**

assumptions:



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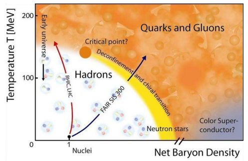


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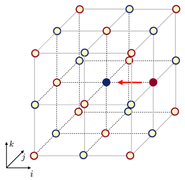

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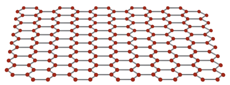

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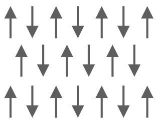
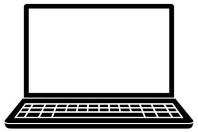


**Models**



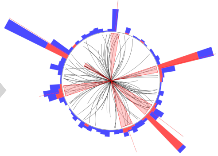
**Numerical**

**problem:**

$$\dim \mathcal{H}$$
$$||$$
$$\mathcal{O}(d^L)$$

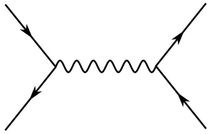
# Motivation: problems and solutions

Quantum Physics



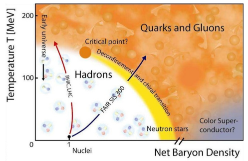
**HEP**

assumptions:



**Analytical**

QCD ?

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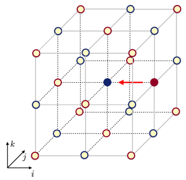
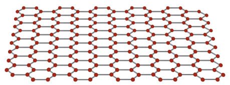
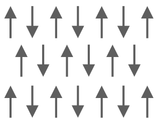
discretization:



**LGT**



**CM**



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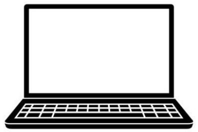

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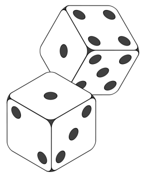

**Monte Carlo**

# Motivation: problems and solutions

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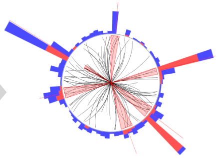

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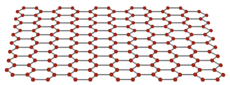

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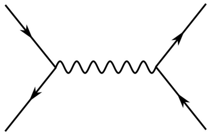

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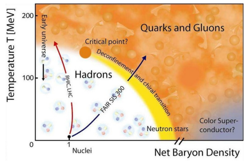


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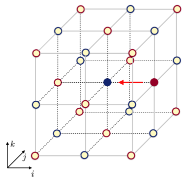

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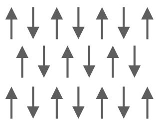

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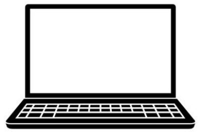

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**problem:**

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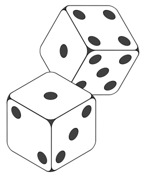


**Monte Carlo**

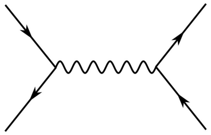
**sign problem...**

finite density (fermions)

**out-of-equilibrium** (scattering…)

# Motivation: problems and solutions

assumptions:



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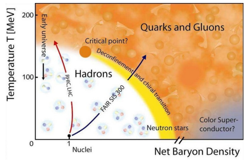


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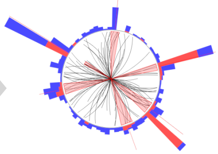
**Quantum Physics**



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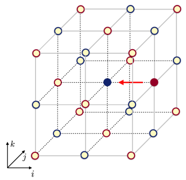

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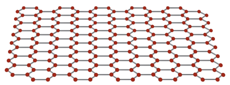

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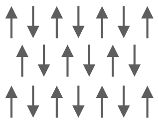
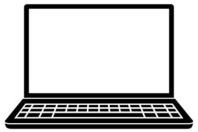


**Models**



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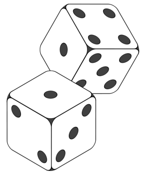
$$\dim \mathcal{H} \; || \; \mathcal{O}(d^L)$$



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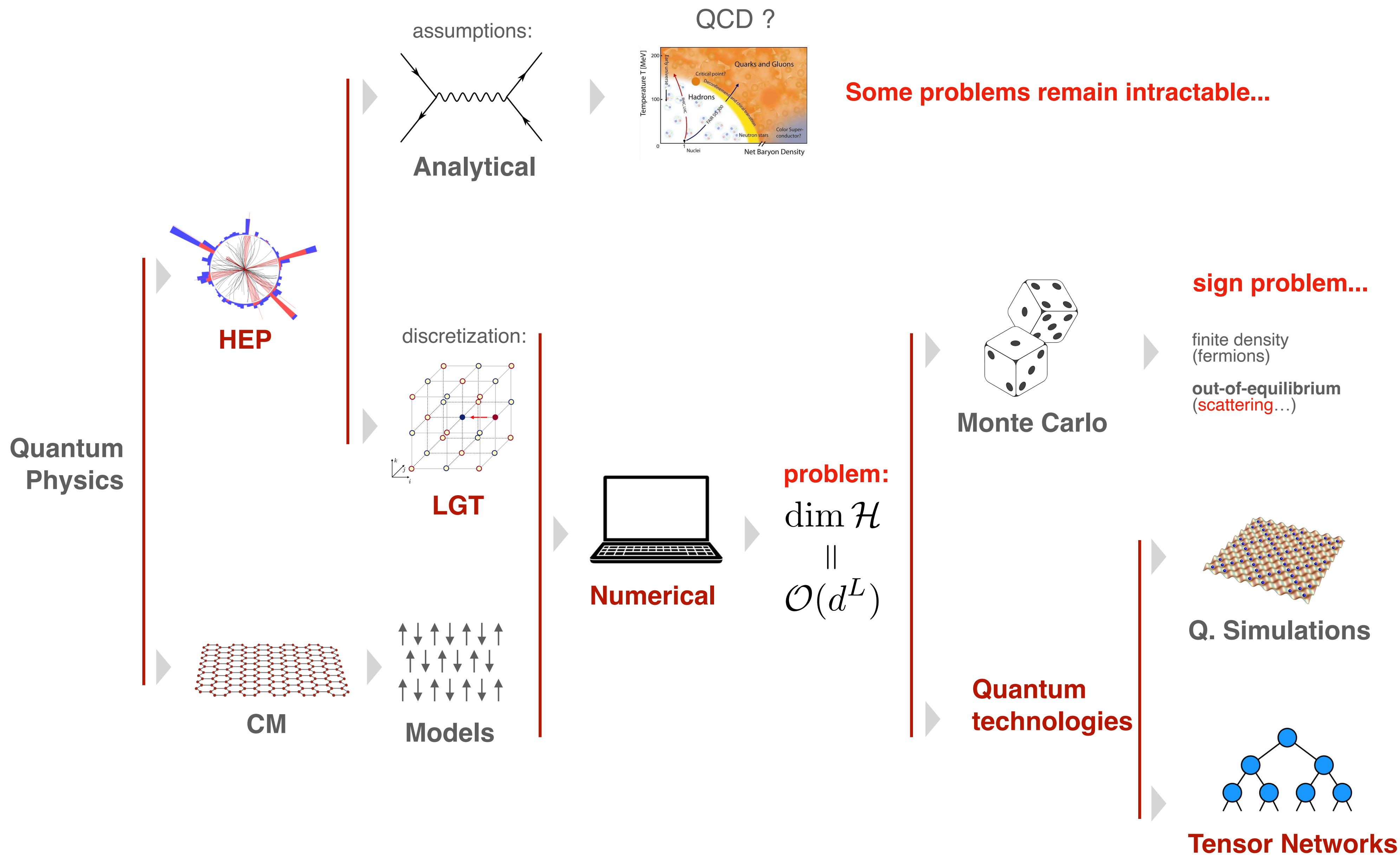
**Quantum technologies**

# Motivation: problems and solutions



# 1. Theoretical background

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- **(2+1)D pure EM Hamiltonian** $$H = \frac{1}{2}\int d^2x\,\left(E^2 + B^2\right)$$

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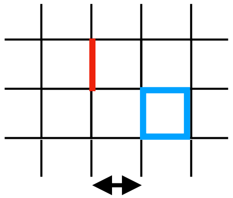
**(2+1)D pure EM Hamiltonian**

$$H = \frac{1}{2} \int d^2x \left(E^2 + B^2\right)$$

(Quantum d.o.f survive!)

1 **Space Discretization**



Spacing $a$

# 1. Theoretical background

**(2+1)D pure EM Hamiltonian**

$$H = \frac{1}{2}\int d^2x\,\left(E^2 + B^2\right)$$

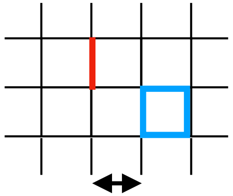
Divergence of E → Vertex flux

Gauge invariance → Gauss' Law

1

(Quantum d.o.f survive!)

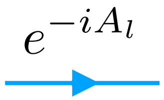
**Space Discretization**



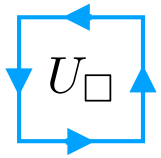
Spacing $a$

**Electric**

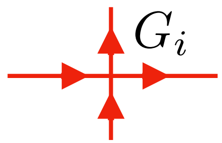

**Wilson line**




**Wilson loop**




**Vertex flux**




**Gauss' Law**

$$G_i = 0$$

# 1. Theoretical background

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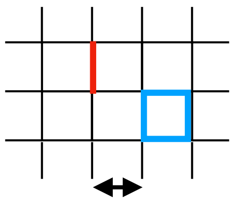
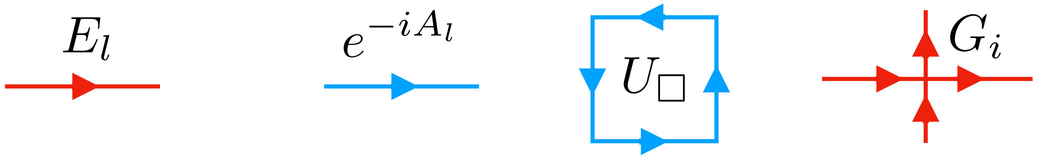
Space **Discretization**



Spacing $a$

Electric | Wilson line | Wilson loop | Vertex flux | Gauss' Law




$$G_i = 0$$

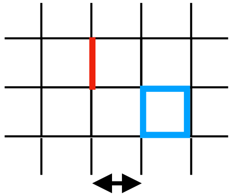
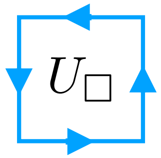
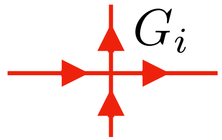
**2** **Canonical quantization**

$$[\hat{E}_l, \hat{U}_m] = \delta_{lm}\hat{U}_l$$

$\mathcal{H}$ Hilbert space

# 1. Theoretical background

**(2+1)D pure EM Hamiltonian** $$H = \frac{1}{2}\int d^2x\,(E^2 + B^2)$$

| | | | | Divergence of E | Gauge invariance |
|---|---|---|---|---|---|
| **1** (Quantum d.o.f survive!) Space **Discretization** | Electric | Wilson line | Wilson loop | Vertex flux | Gauss' Law |
|  Spacing $a$ |  $E_l$ |  $e^{-iA_l}$ |  $U_\square$ |  $G_i$ | $G_i = 0$ |
| **2** **Canonical quantization** | $[\hat{E}_l, \hat{U}_m] = \delta_{lm}\hat{U}_l$ | | $\hat{U}_\square$ | $\hat{G}_i$ | $\hat{G}_i\lvert\psi\rangle = 0$ |
| $\mathcal{H}$ Hilbert space | Electric op. | Comparator | Plaquette operator | Gauge generator | Physical states |

# 1. Theoretical background

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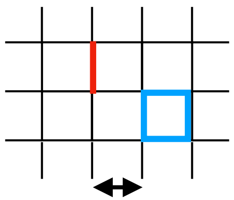
Space **Discretization**



**Spacing** $a$

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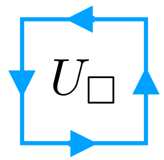

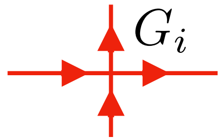

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**Coupling** $g^2 = e^2 a$

J. Kogut, L. Susskind, Phys. Rev. D (1975) 395-408

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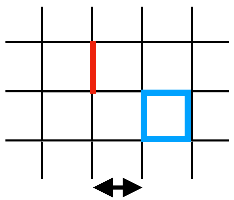

**Spacing** $a$

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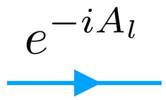
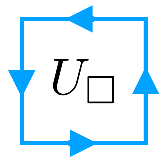
Wilson line




Wilson loop

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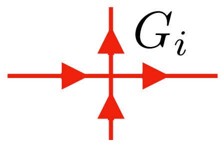


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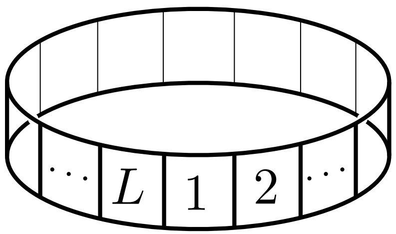
Choose **geometry**, **topology**

**Size** $L$ (# plaquettes)



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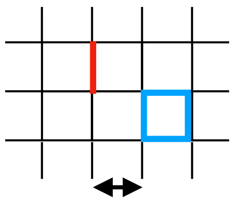

**Spacing** $a$

| Electric | Wilson line | Wilson loop | Vertex flux | Gauss' Law |
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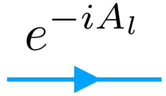


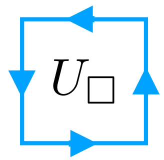


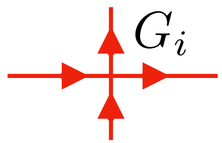


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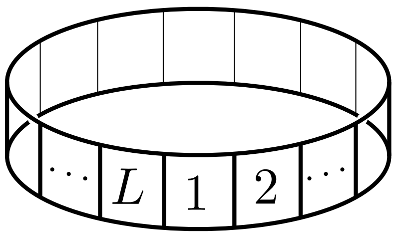
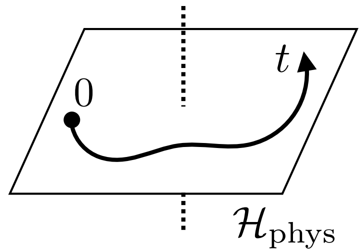



Physical Hilbert space



# 1. Theoretical background

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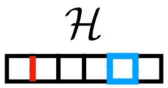
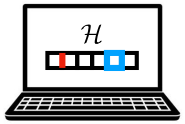
To simulate 


We want $\mathcal{H}$ **finite**-dimensional!

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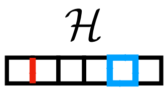
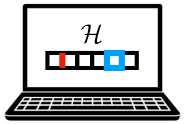
To simulate 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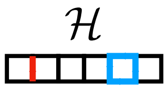
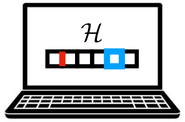
**U(1) Lattice QED** → **U(1) Quantum Link Model (QLM)**

U. J. Wiese, Nuc. Phys. B 492 (1997) 455-471

# 1. Theoretical background

To simulate 




We want $\mathcal{H}$ **finite**-dimensional!

**U(1) Lattice QED** → **U(1) Quantum Link Model (QLM)**

$\forall$ link Hilbert space $\mathcal{H}_{\rm link}$ → $\mathrm{SU}(2)$ irreducible representations $s \in \mathbb{N}/2$

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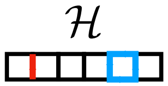
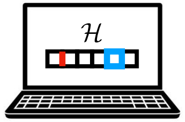
To simulate 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**Operators**

| | |
|---|---|
| $\hat{E}$ | $\hat{S}^z$ |
| $\hat{U}$ | $\hat{S}^+$ |
| $\hat{U}^\dagger$ | $\hat{S}^-$ |

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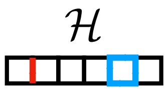
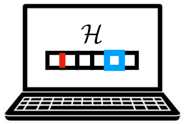
To simulate 

→ 

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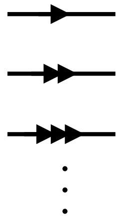
Arrow notation



$s_z = 1/2$

$s_z = 1$

$s_z = 3/2$

$\vdots$

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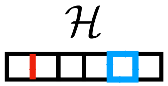
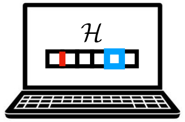
To simulate  ▶  We want $\mathcal{H}$ **finite**-dimensional!

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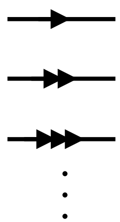
Arrow notation 

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$\sigma(\hat{E})$ ▶ $\sigma(\hat{S}^z) = \{-s, \ldots, s\}$ **finite!** ✓

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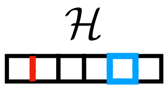
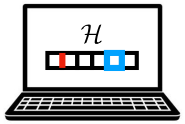
To simulate  ▶  We want $\mathcal{H}$ **finite**-dimensional!

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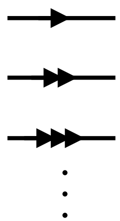

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**CCR** $[\hat{E}, \hat{U}] = \hat{U}$ ▶ $[\hat{S}^z, \hat{S}^+] = \hat{S}^+$ ✓

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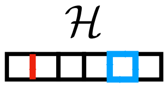
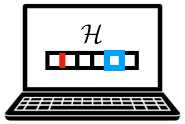
To simulate  ▶  We want $\mathcal{H}$ **finite**-dimensional!

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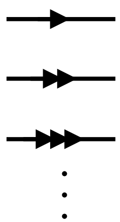

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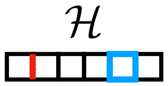
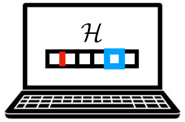
**Unitarity** $\hat{U}\hat{U}^\dagger = \mathbb{I}$ ▶ $\hat{S}^+\hat{S}^- \neq \mathbb{I}$ ✗

# 1. Theoretical background

To simulate  ▶  We want $\mathcal{H}$ **finite**-dimensional!

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| **Operators** $\hat{E}$ | ▶ | $\hat{S}^z$ |
| $\hat{U}$ | ▶ | $\hat{S}^+ / s$ |
| $\hat{U}^\dagger$ | ▶ | $\hat{S}^- / s$ |
| $\sigma(\hat{E})$ | ▶ | $\sigma(\hat{S}^z) = \{-s, \ldots, s\}$ **finite!** ✓ |
| **CCR** $[\hat{E}, \hat{U}] = \hat{U}$ | ▶ | $[\hat{S}^z, \hat{S}^+] = \hat{S}^+$ ✓ |
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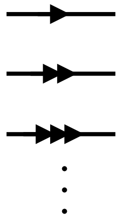
Arrow notation 

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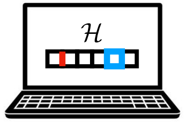
To simulate $\mathcal{H}$  ▸ 

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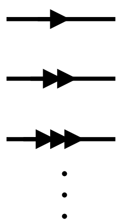
Arrow notation 
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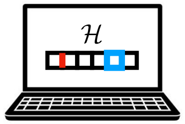
To simulate $\mathcal{H}$  ▶ 

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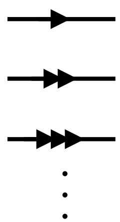
Arrow notation 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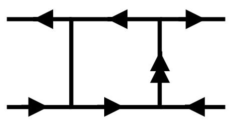
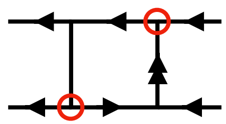
**Gauge generator** $\hat{G}$ ▶ $\hat{S}^z_i + \hat{S}^z_k - \hat{S}^z_j$

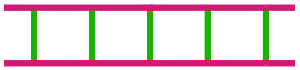

Allowed ✓


Not allowed ✖

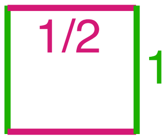
# 2. The pure Lattice QED on ladder geometry

**Spin representation**

Translation and reflection inv.



**Lowest** spin irrep assignments which admits gauge inv. configs:



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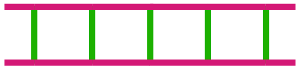

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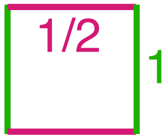
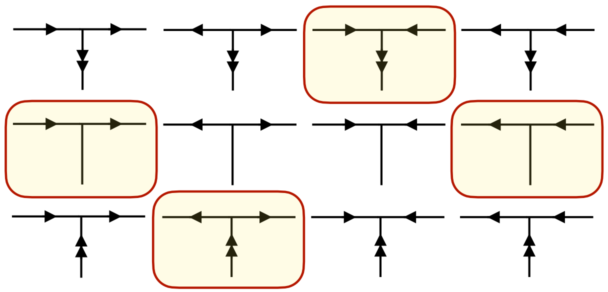



**Gauss' Law on vertices**



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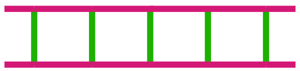

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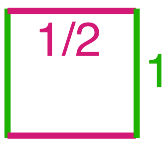


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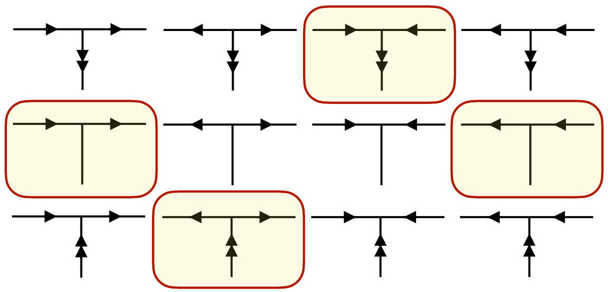
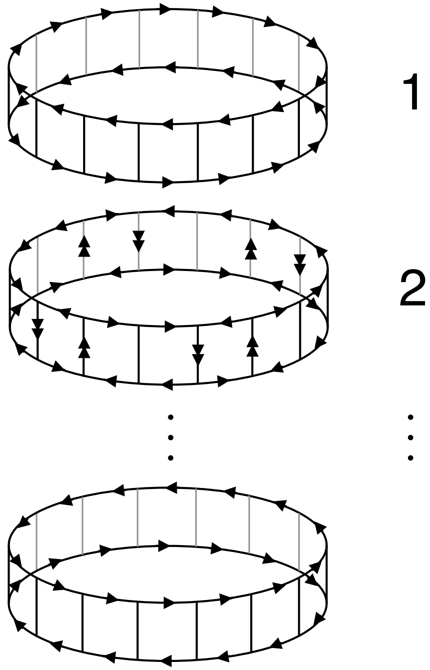


$L$ **plaquettes ladder in PBC**



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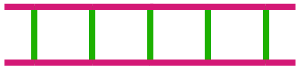

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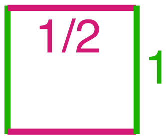


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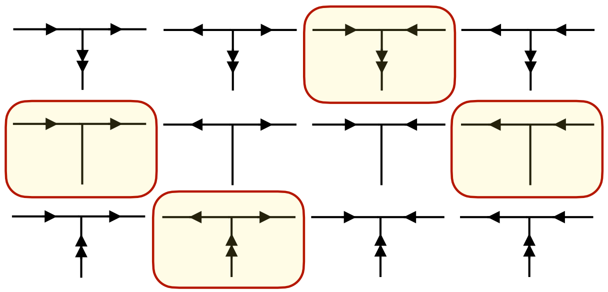

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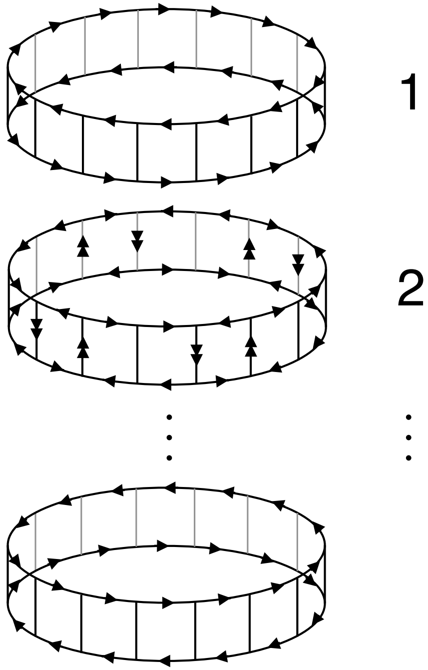


$$\dim \mathcal{H} = 2^L + 2$$

Exp scaling

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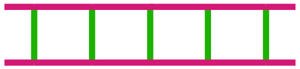

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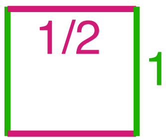


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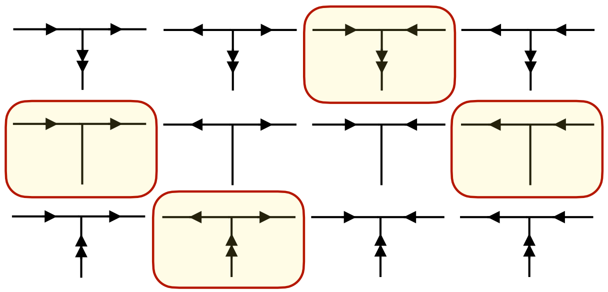

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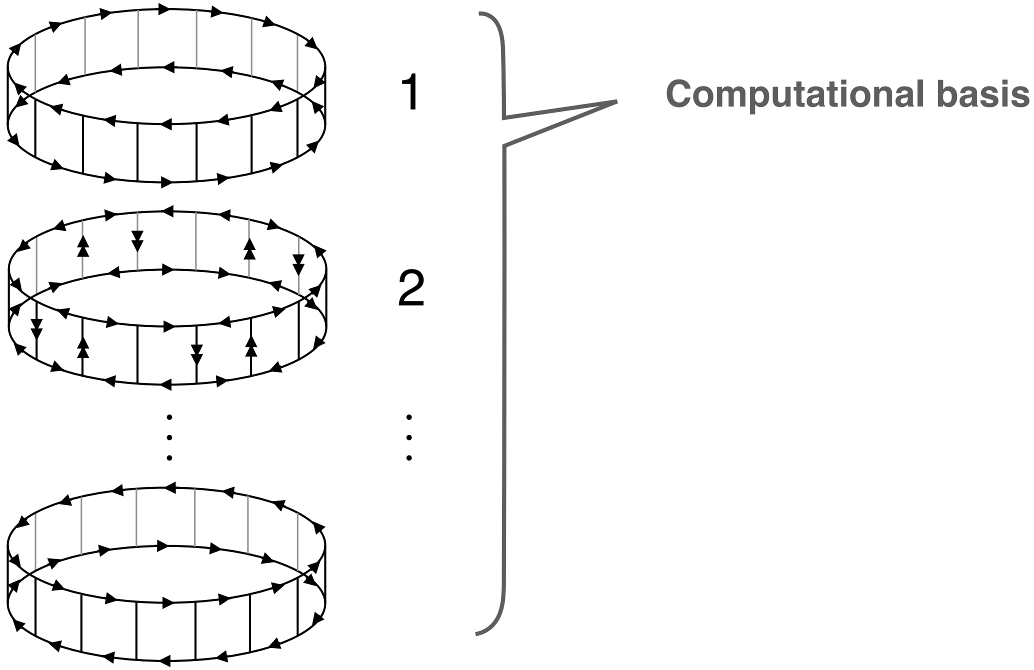


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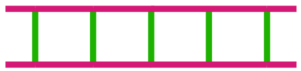

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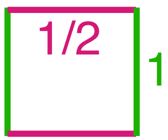


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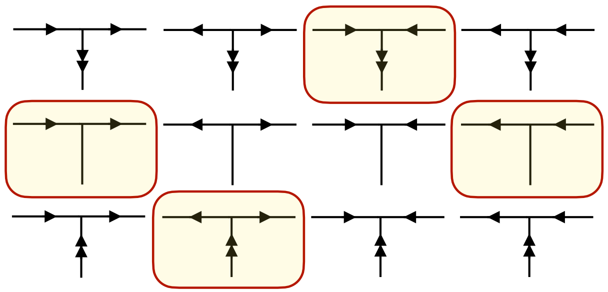

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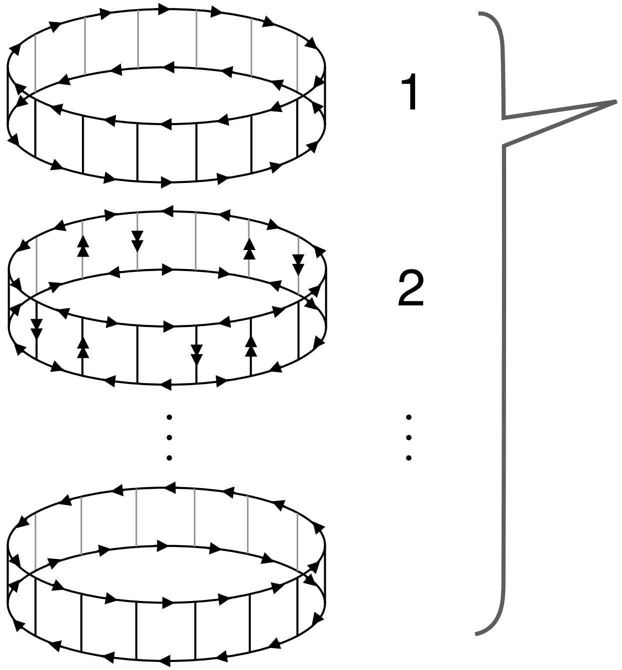


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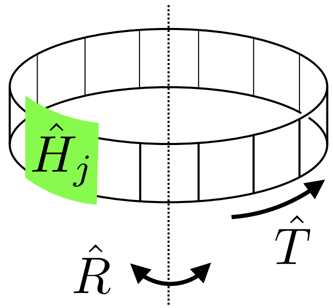
Exp scaling

**Construct operators**

**Computational basis**



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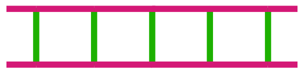

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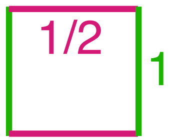


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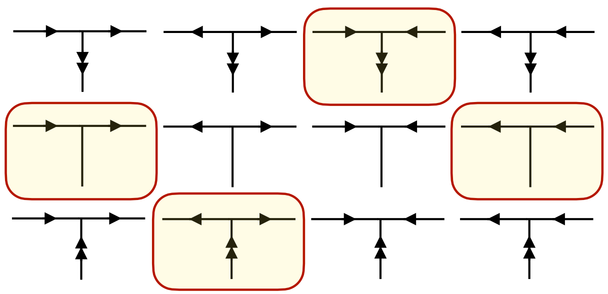
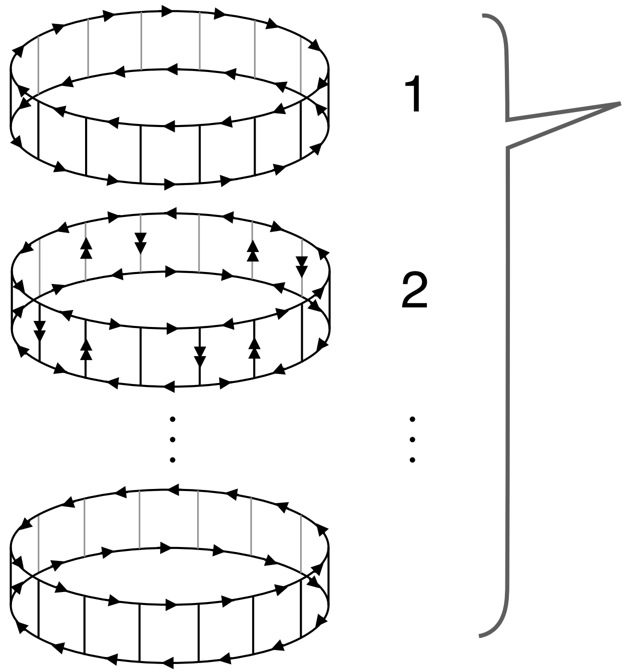


***L* plaquettes ladder in PBC**

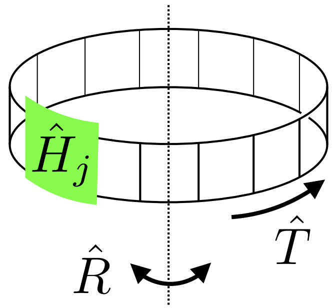



$$\dim \mathcal{H} = 2^L + 2$$

↑ Exp scaling

**Construct operators**

**Computational basis**




$$[\hat{H}, \hat{T}] = 0$$

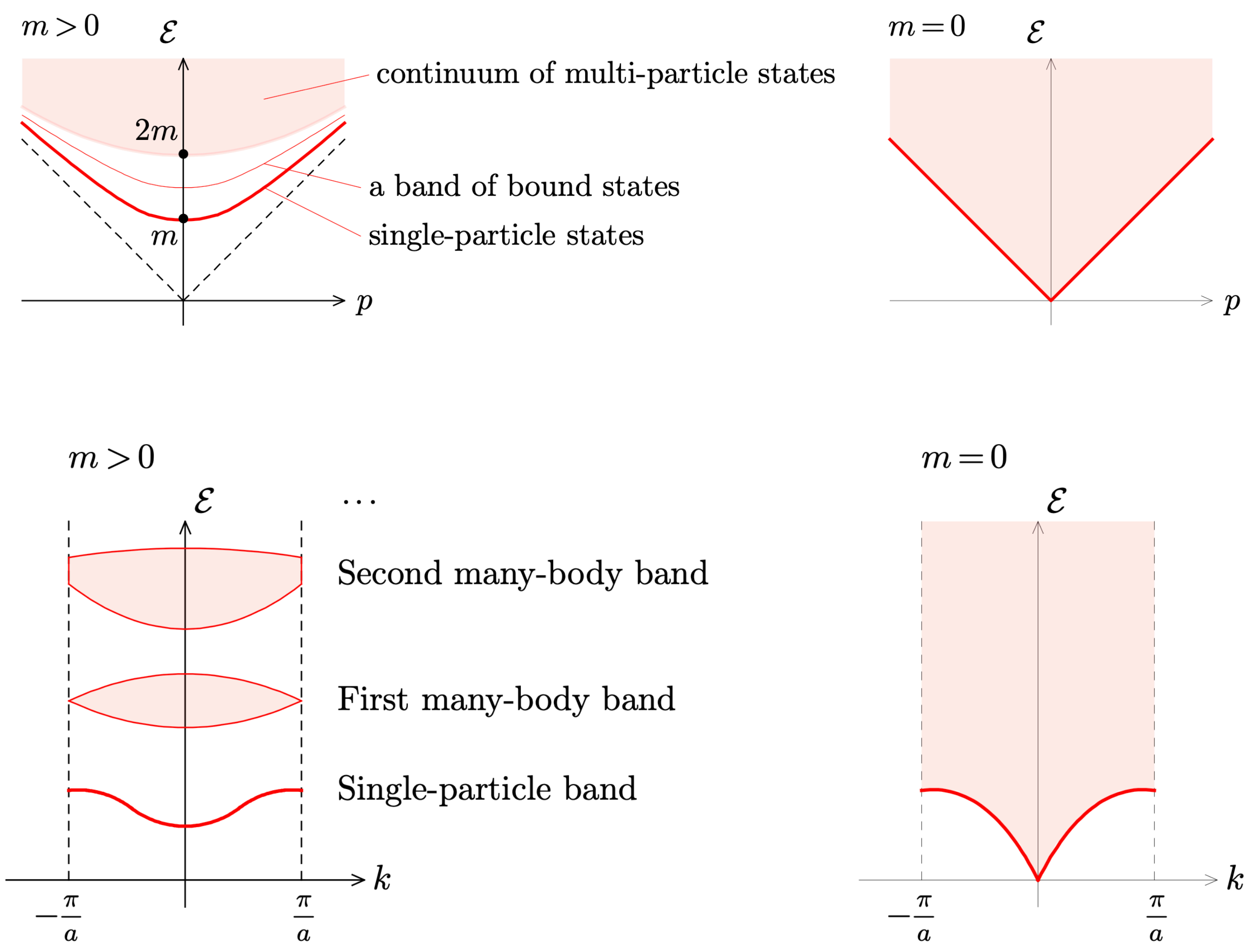
**Dispersion relation!**

# 2. The pure Lattice QED on ladder geometry$(\frac{1}{2}, 1)$

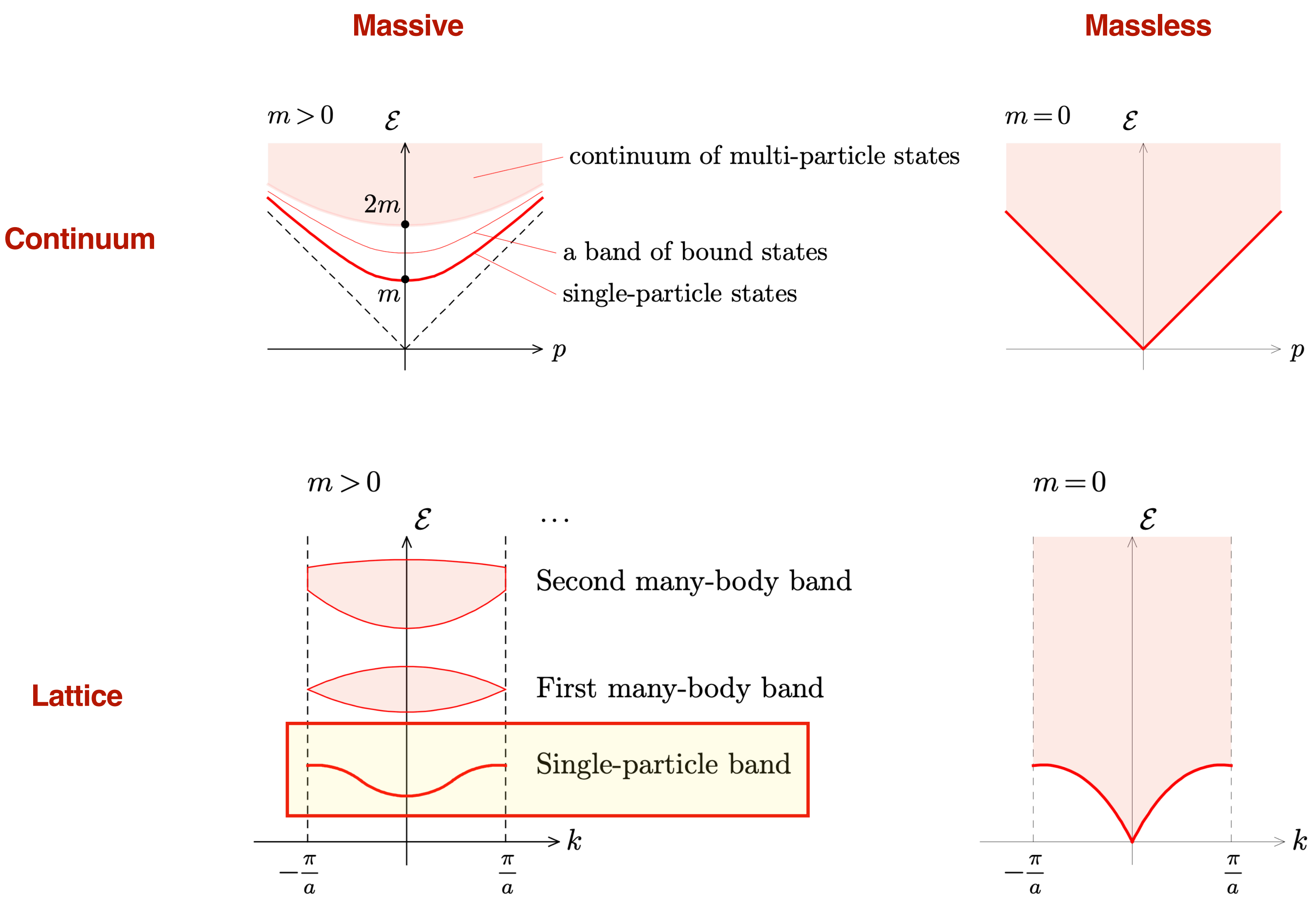
**Massive** | **Massless**

**Continuum**

**Lattice**



# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$



# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$

**Assuming**

$[\hat{H}, \hat{T}] = 0$ **Translational invar. (PBC)**

$L = 13$ **Intermediate system size**

1/2  1 **Lowest spin rep.**

$a = 1$ **Unit lattice spacing**

( varying the coupling $g$ )

# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$

**Assuming**

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( varying the coupling $g$ )

**Simultaneous *exact* diagonalization**
**(computationally difficult step)**

$$\dim \mathcal{H} = 8194$$

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**Dispersion relation**
**in our (interacting!) model:**

# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$

**Assuming**

$[\hat{H}, \hat{T}] = 0$ **Translational invar. (PBC)**

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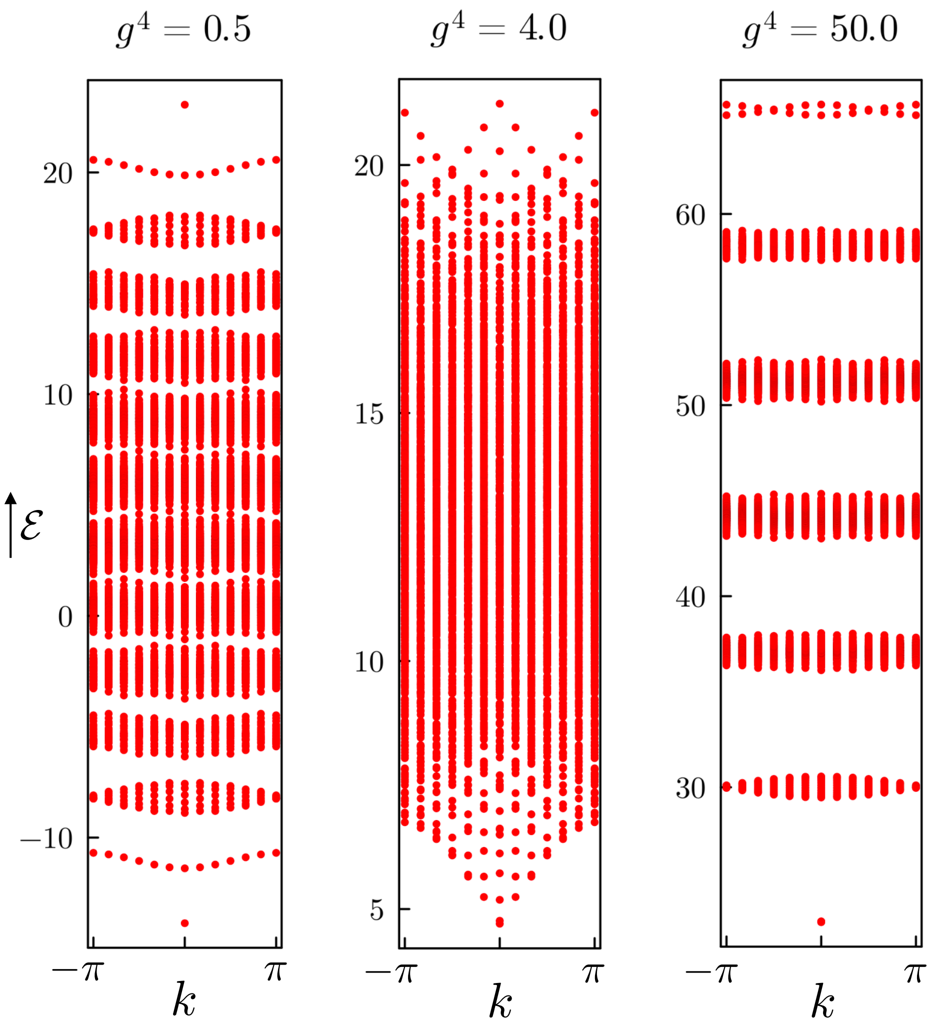
( varying the coupling $g$ )

**Simultaneous *exact* diagonalization (computationally difficult step)**

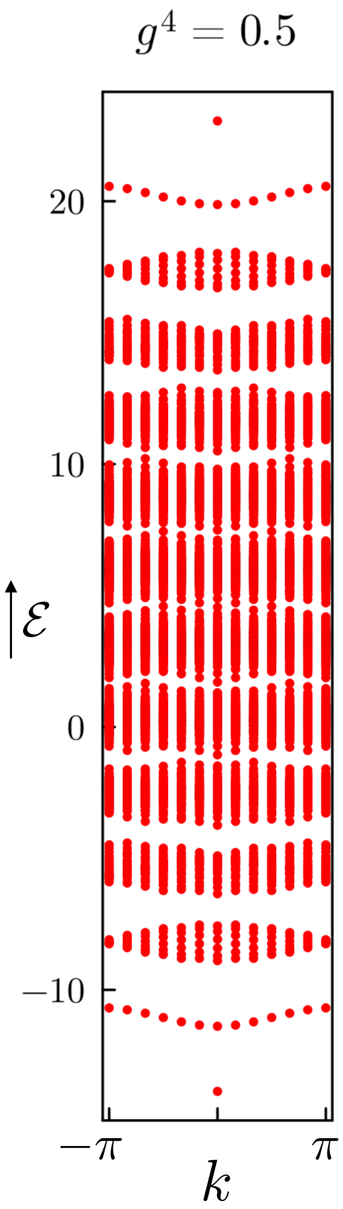
$\dim \mathcal{H} = 8194$

**Dispersion relation in our (interacting!) model:**




Gapped phase (massive photon) | Critical point (massless photon) | Gapped SSB phase (degenerate g.s.)

# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$



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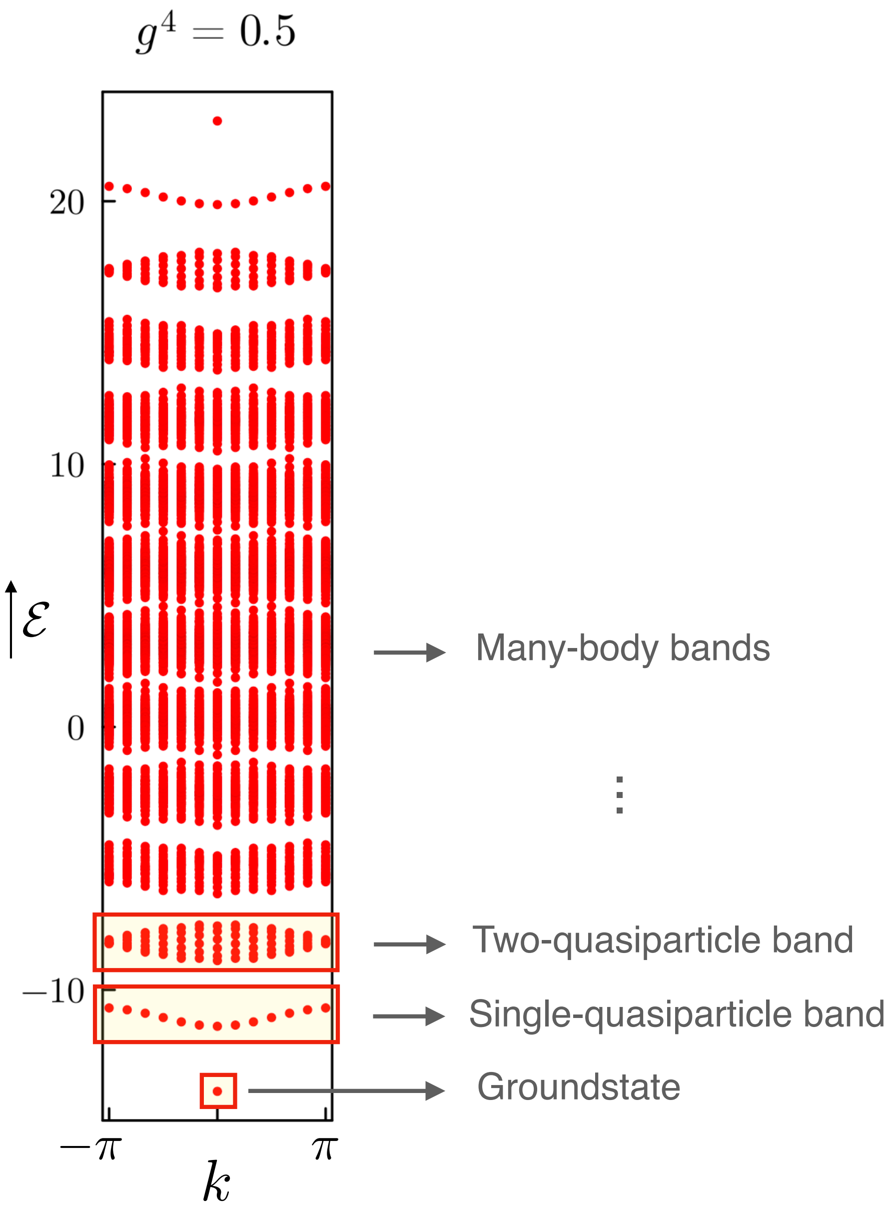

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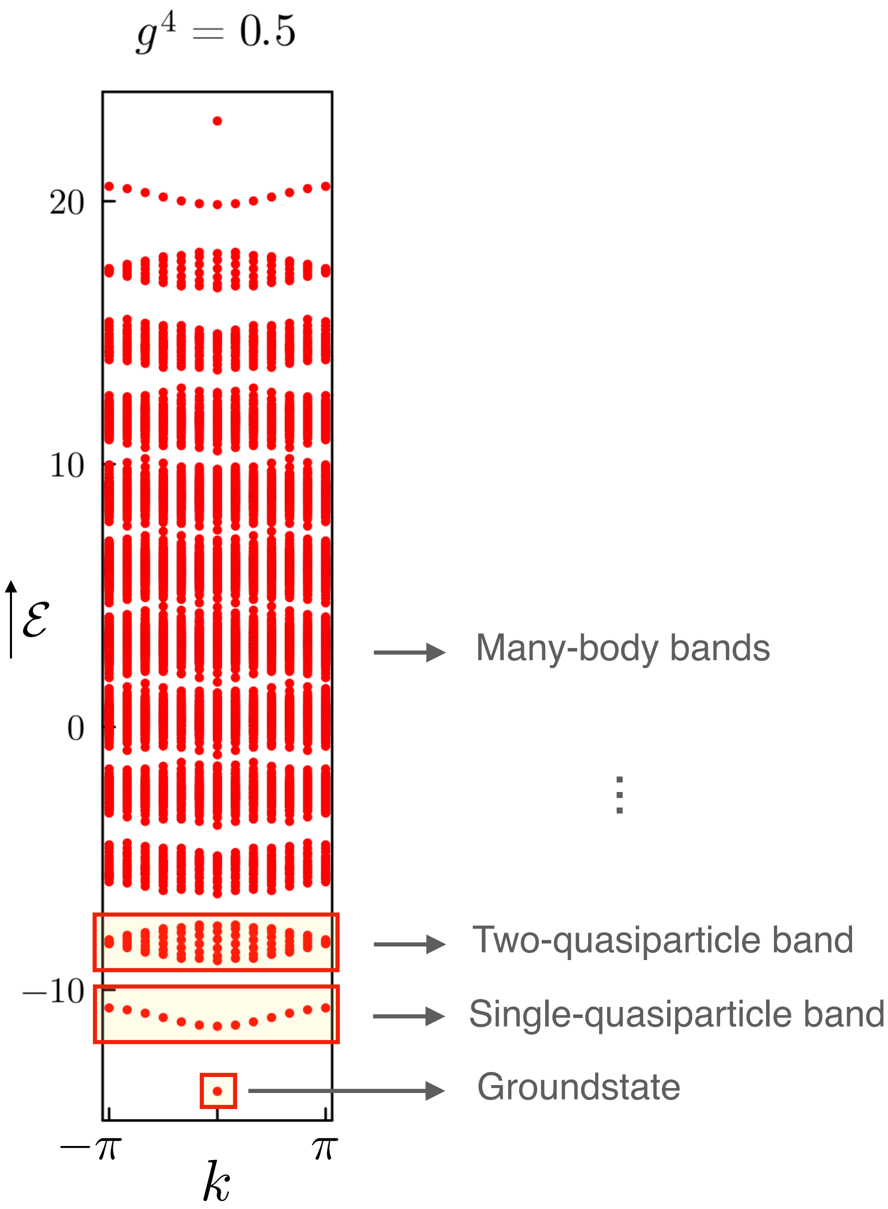


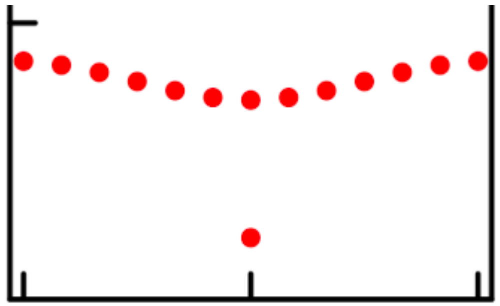

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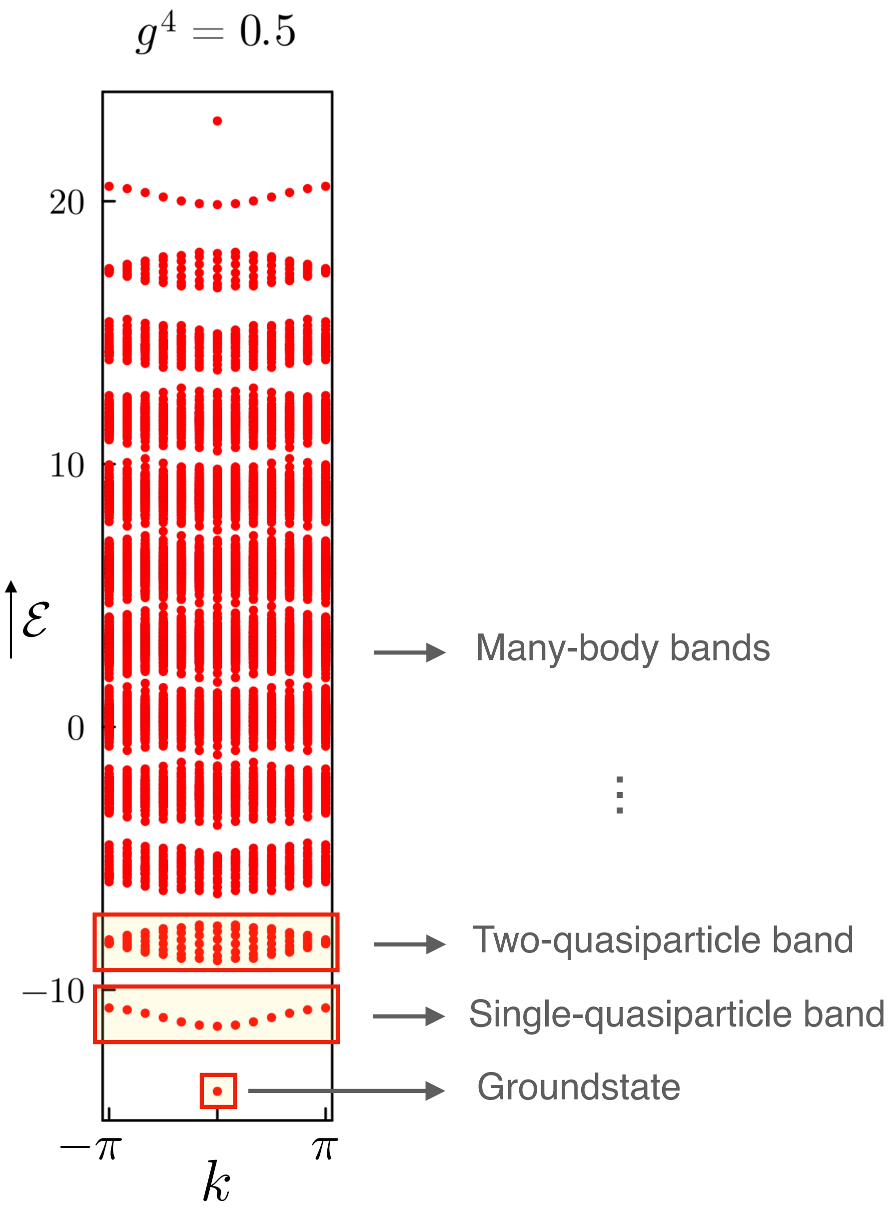


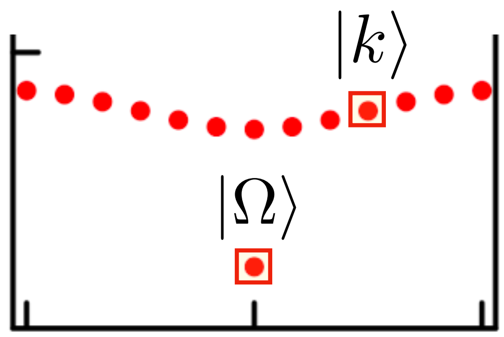

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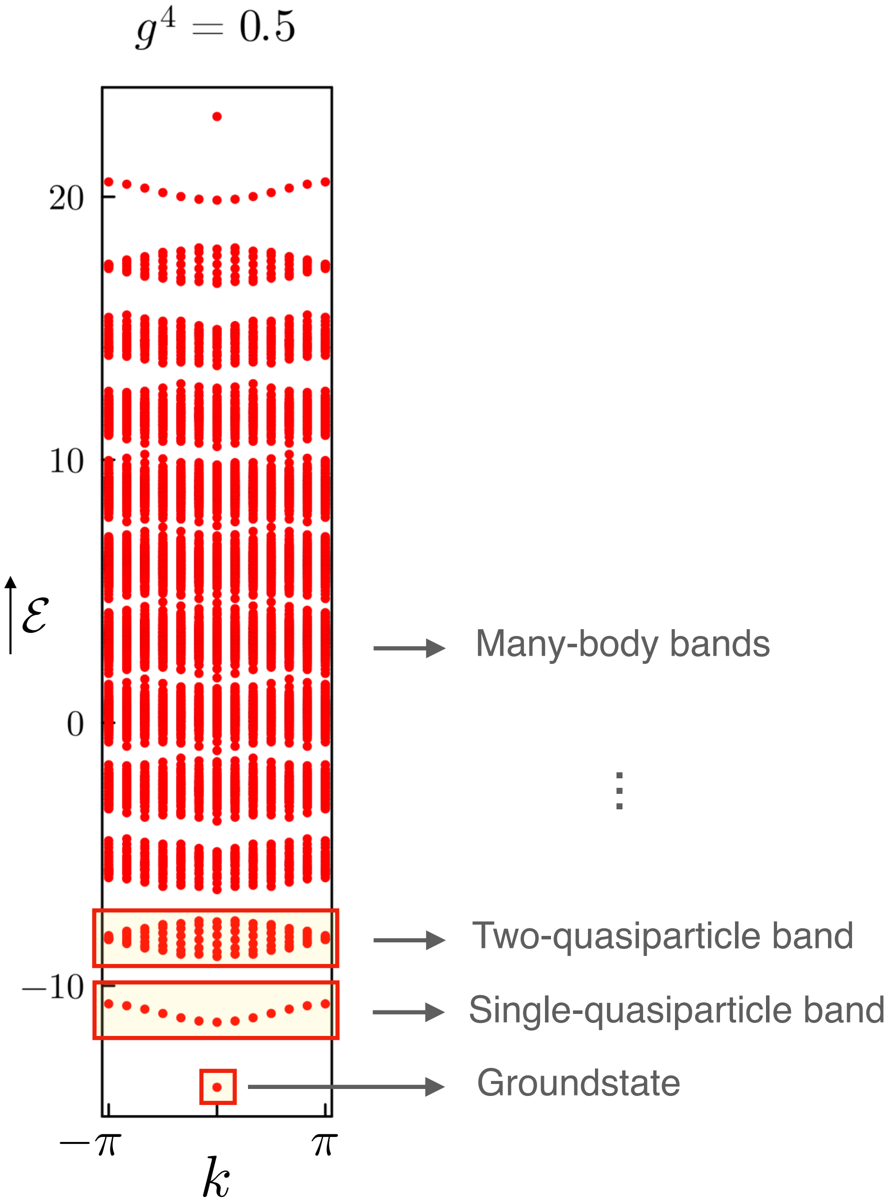
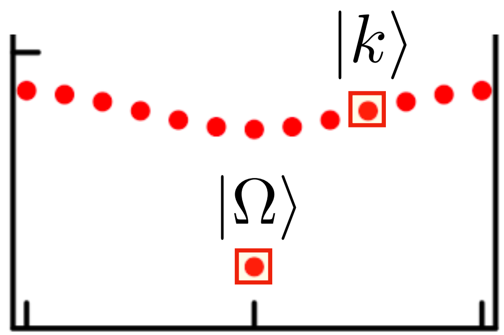
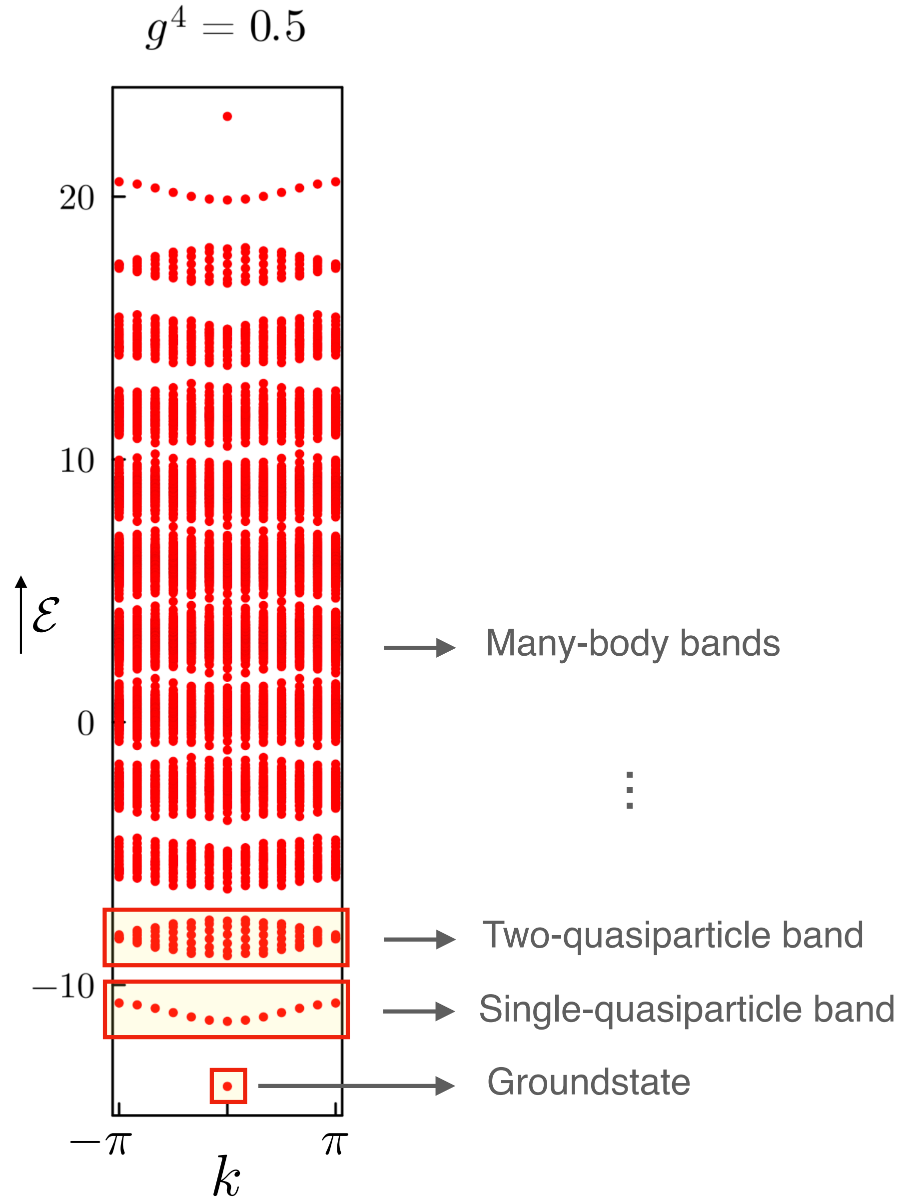


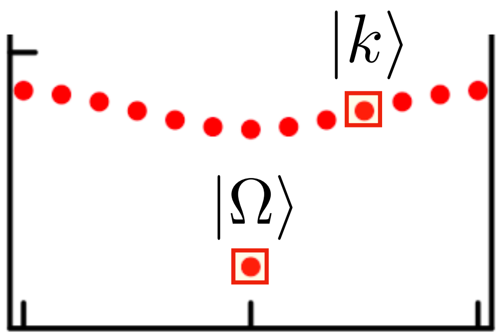




$$|\phi\rangle = \sum a_k |k\rangle$$

**Single-particle state (Bloch basis)**

# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$

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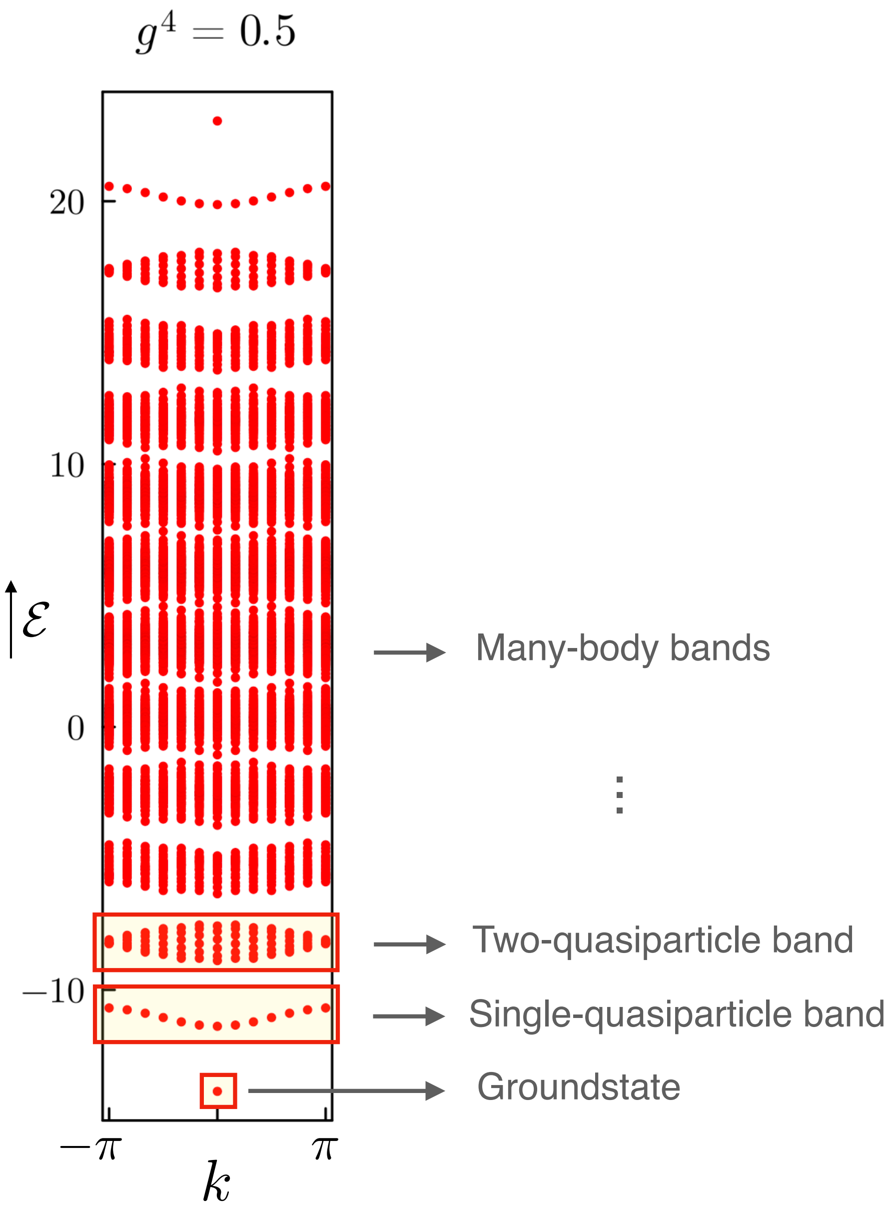
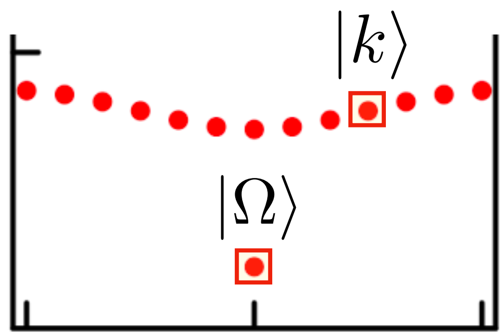
**Single-particle state (Bloch basis)**

**Wannier Functions (real-space basis)**

# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$



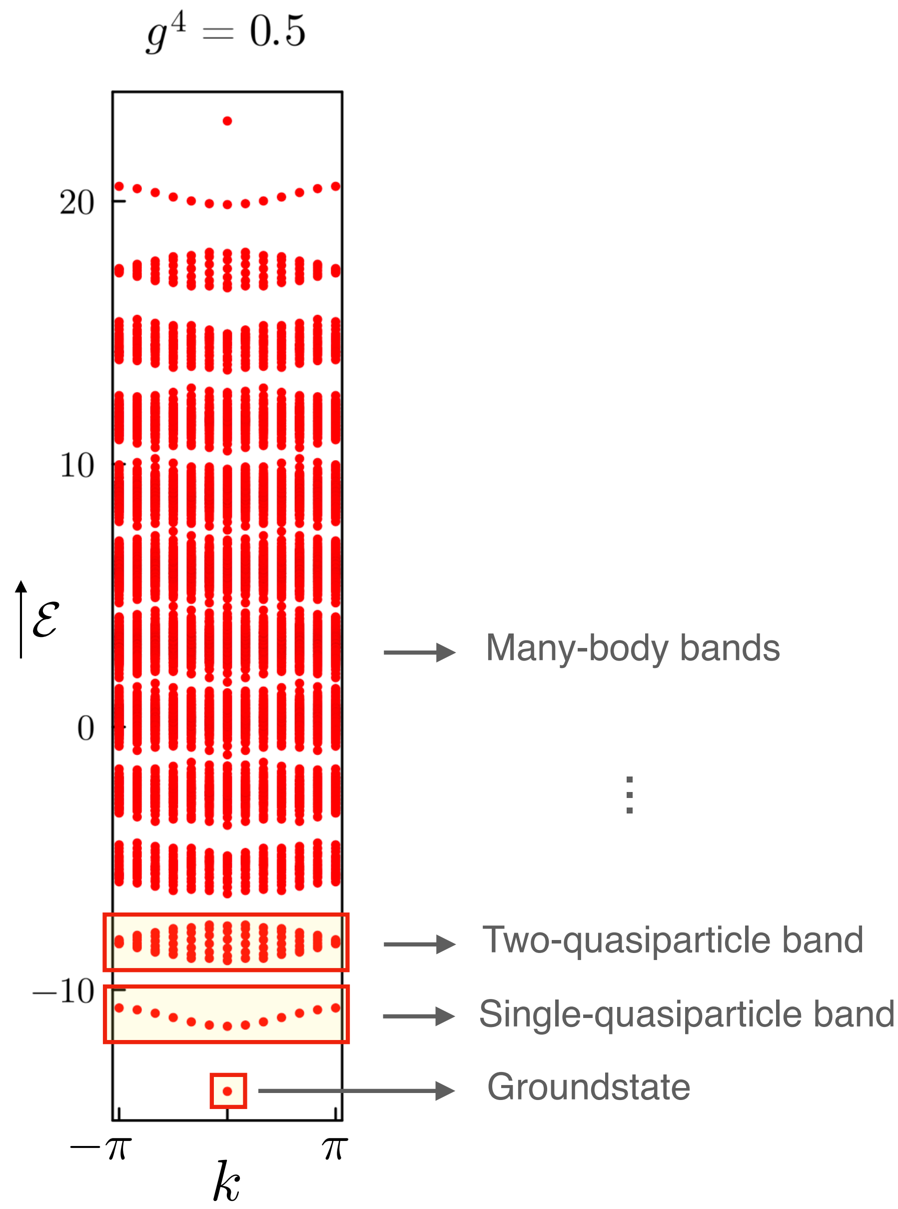

$$|\phi\rangle = \sum a_k |k\rangle$$

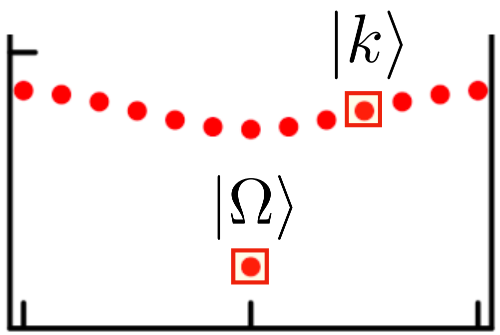
**Single-particle state (Bloch basis)**

**Wannier Functions (real-space basis)**

**Wave-packet state**

# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$




**Program**

$$|\phi\rangle = \sum a_k |k\rangle$$

**Single-particle state (Bloch basis)**

**Wannier Functions (real-space basis)**

**Wave-packet state**

# 3. Construction of the wavepackets

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**Wannier states**

$$|W(\theta_k)\rangle = \frac{1}{\sqrt{L}} \sum_k e^{i\theta_k} |k\rangle$$

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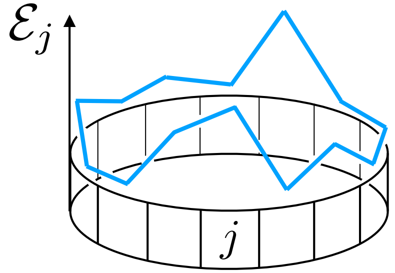


**Energy density**

$$\mathcal{E}_j[\phi] = \langle\phi|\hat{H}_j|\phi\rangle$$

# 3. Construction of the wavepackets

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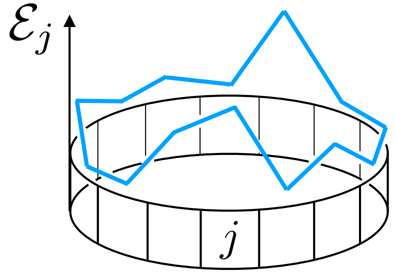


**Energy density**

$$\mathcal{E}_j[\phi] = \langle \phi | \hat{H}_j | \phi \rangle$$

**Spread functional**

$$\sigma^2[\phi] \equiv \frac{\sum_j j^2 \mathcal{E}_j[\phi]}{\sum_j \mathcal{E}_j[\phi]} - \left( \frac{\sum_j j \mathcal{E}_j[\phi]}{\sum_j \mathcal{E}_j[\phi]} \right)^2$$

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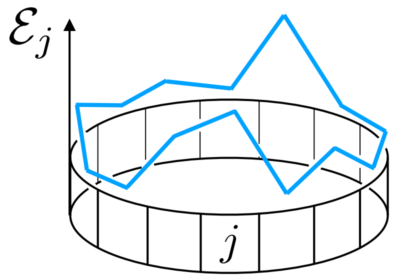


$\theta$ ▶ $\bar{\theta}$

Minimize $\sigma^2$

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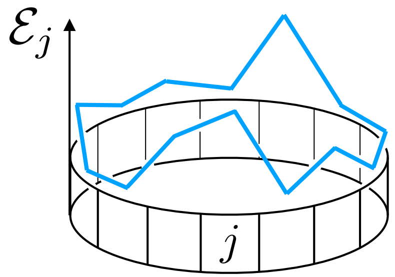
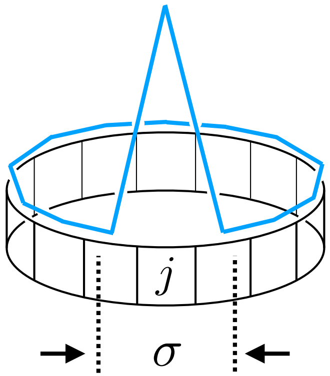



**Maximally localized Wannier states**

$$|W_j\rangle \equiv |W_j(\bar{\theta})\rangle$$

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# 3. Construction of the wavepackets

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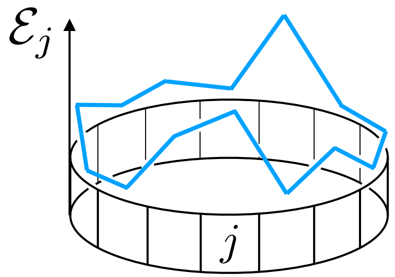
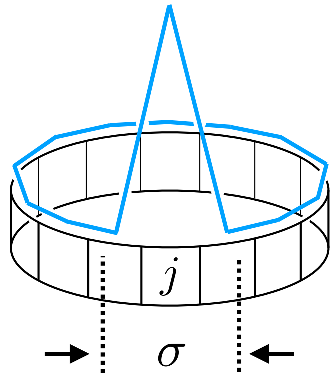



$\theta$ ▶ $\bar{\theta}$

Minimize $\sigma^2$

### Maximally localized Wannier states

$$|W_j\rangle \equiv |W_j(\bar{\theta})\rangle$$



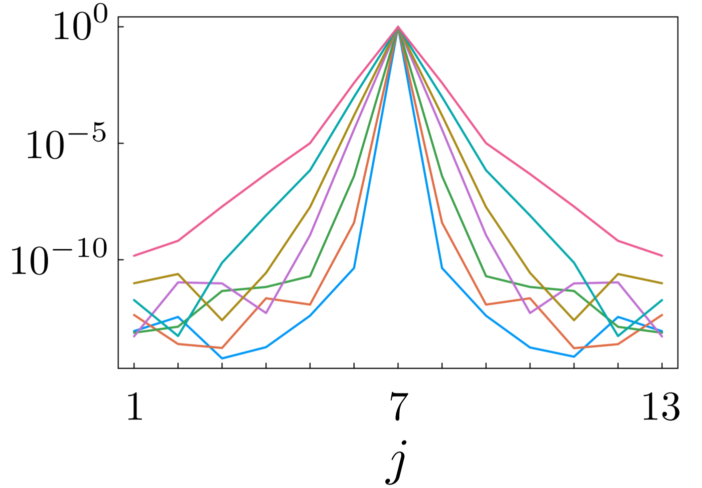

### Energy density

$$\mathcal{E}_j[\phi] = \langle\phi|\hat{H}_j|\phi\rangle$$

### Spread functional

$$\sigma^2[\phi] \equiv \frac{\sum_j j^2 \mathcal{E}_j[\phi]}{\sum_j \mathcal{E}_j[\phi]} - \left(\frac{\sum_j j \mathcal{E}_j[\phi]}{\sum_j \mathcal{E}_j[\phi]}\right)^2$$

**Energy density after minimization**



# 3. Construction of the wavepackets

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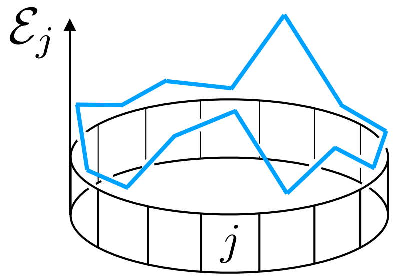
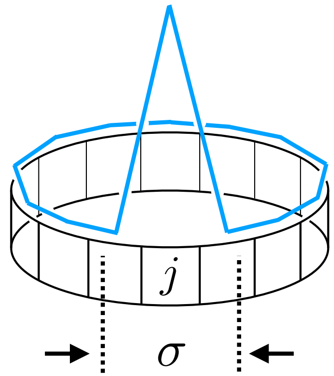



$\theta \rightarrow \bar{\theta}$

Minimize $\sigma^2$

**Maximally localized Wannier states**

$$|W_j\rangle \equiv |W_j(\bar{\theta})\rangle$$

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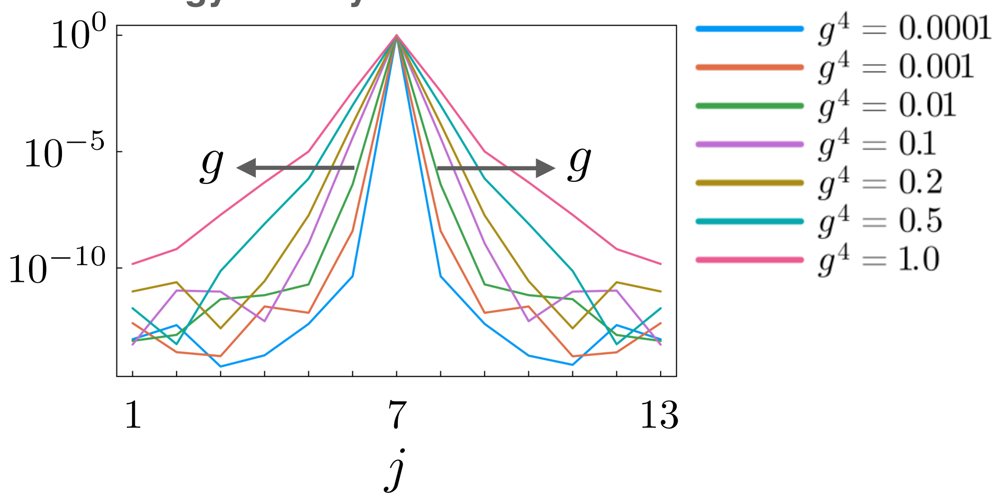
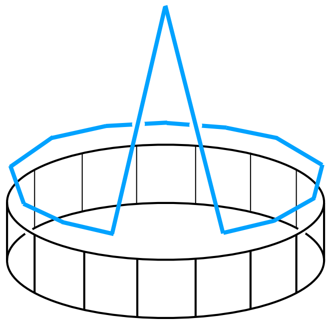
**Energy density after minimization**



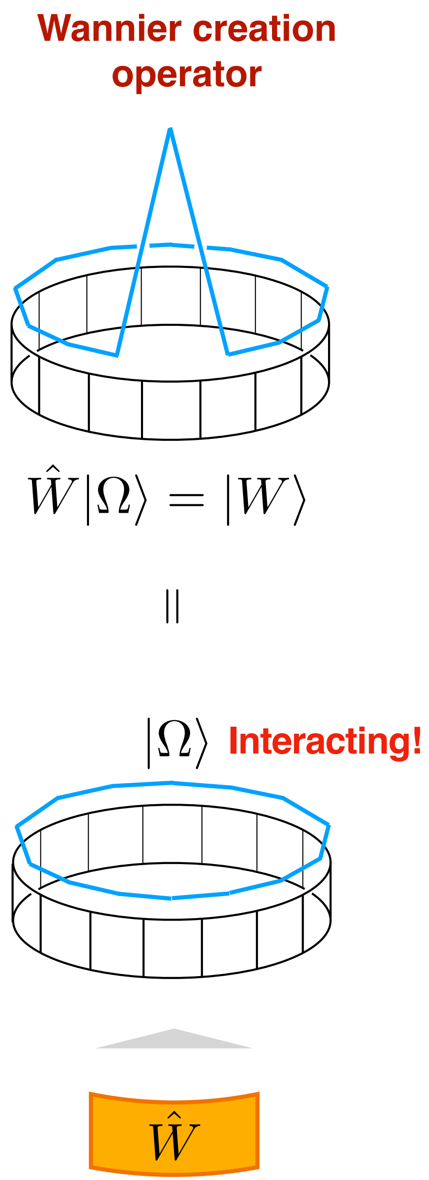
# 3. Construction of the wavepackets

**Wannier creation operator**



$$\hat{W}|\Omega\rangle = |W\rangle$$

# 3. Construction of the wavepackets



# 3. Construction of the wavepackets

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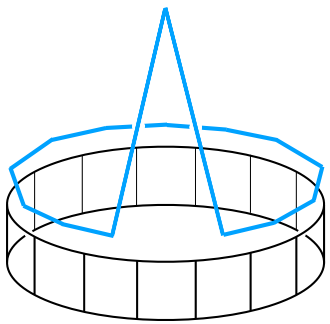
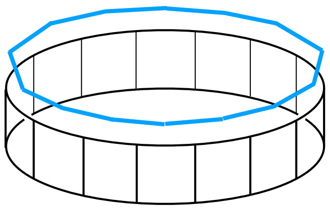
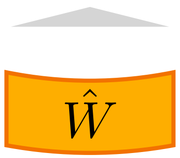


$$\hat{W}|\Omega\rangle = |W\rangle$$

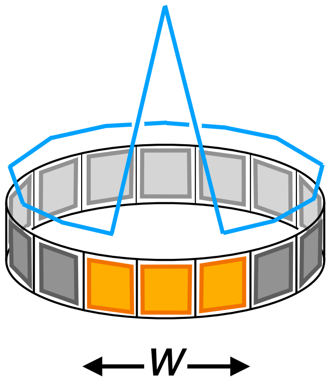
$$||$$

$|\Omega\rangle$ **Interacting!**

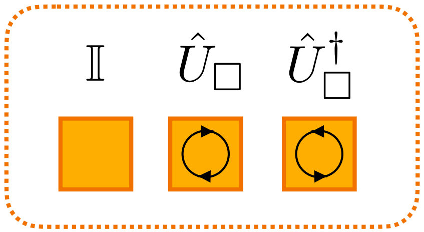





**Wannier creation ansatz**

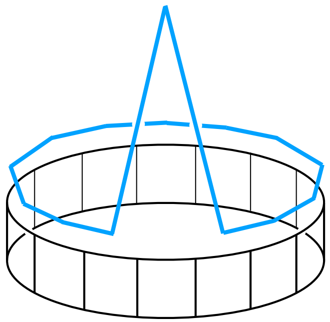



choose $L_\alpha$



# 3. Construction of the wavepackets

**Wannier creation operator**



$$\hat{W}|\Omega\rangle = |W\rangle$$

$$||$$

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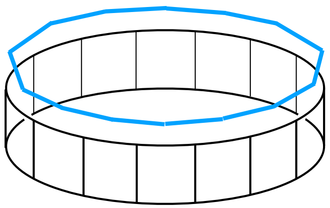

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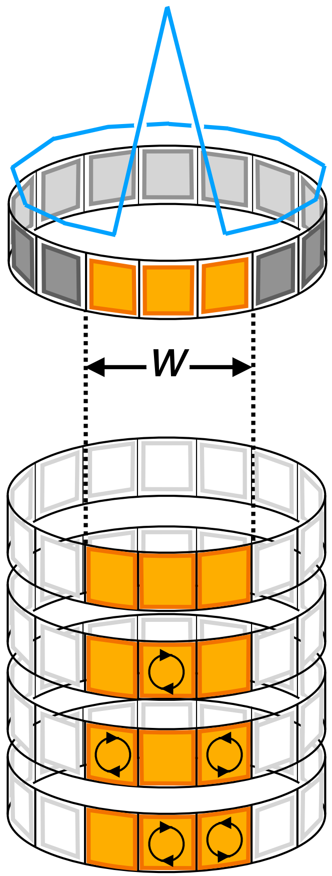
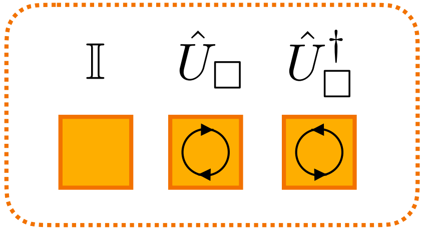
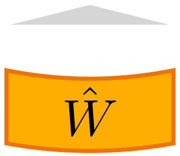



$$c_1 \mathbb{I}$$
$$+ c_2 \hat{U}^\dagger_{\square,2}$$
$$+ c_3 \hat{U}^\dagger_{\square,1} \hat{U}_{\square,3}$$
$$+ c_3 \hat{U}^\dagger_{\square,2} \hat{U}_{\square,3}$$

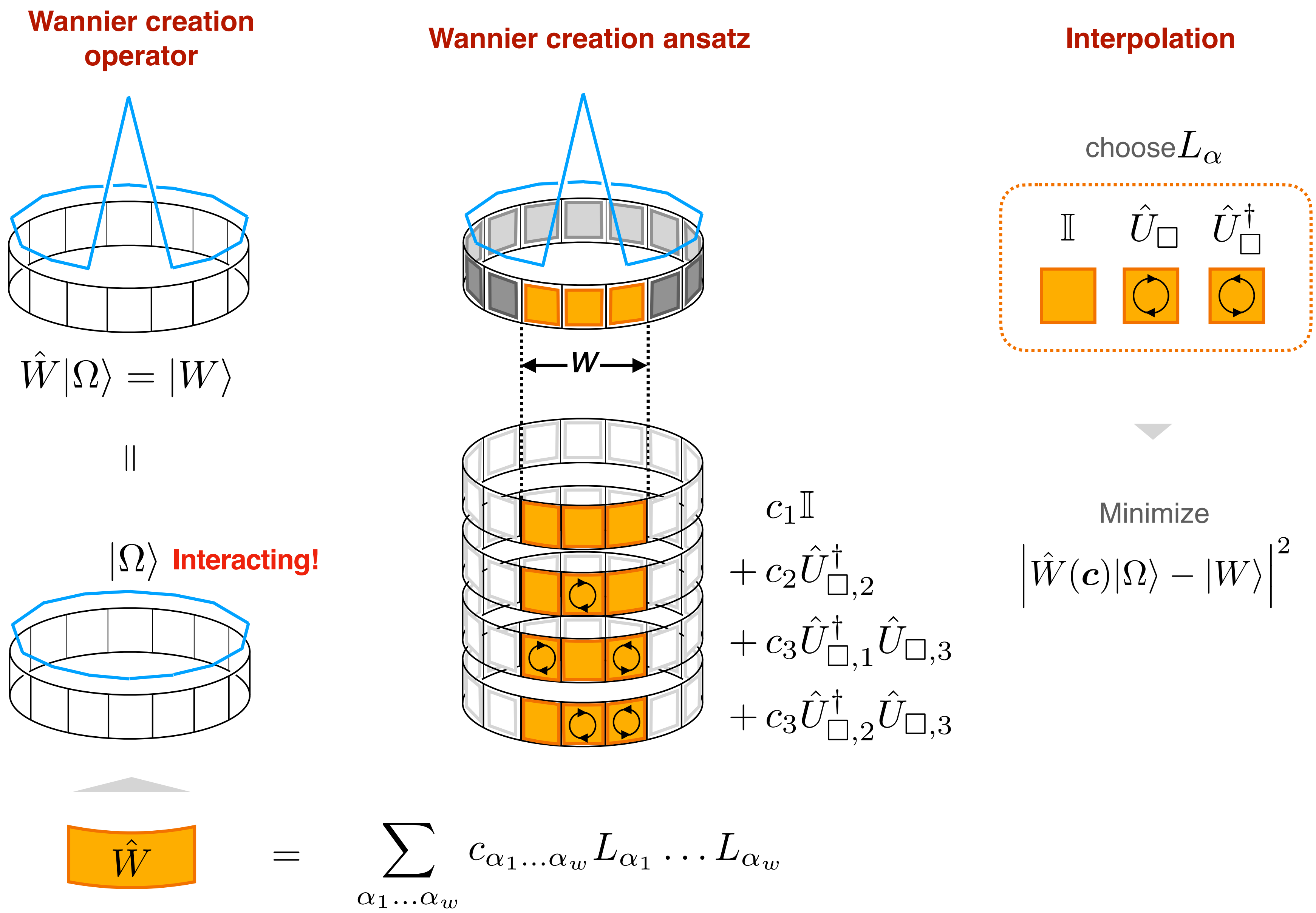
choose $L_\alpha$




$$= \sum_{\alpha_1 \ldots \alpha_w} c_{\alpha_1 \ldots \alpha_w} L_{\alpha_1} \ldots L_{\alpha_w}$$

# 3. Construction of the wavepackets



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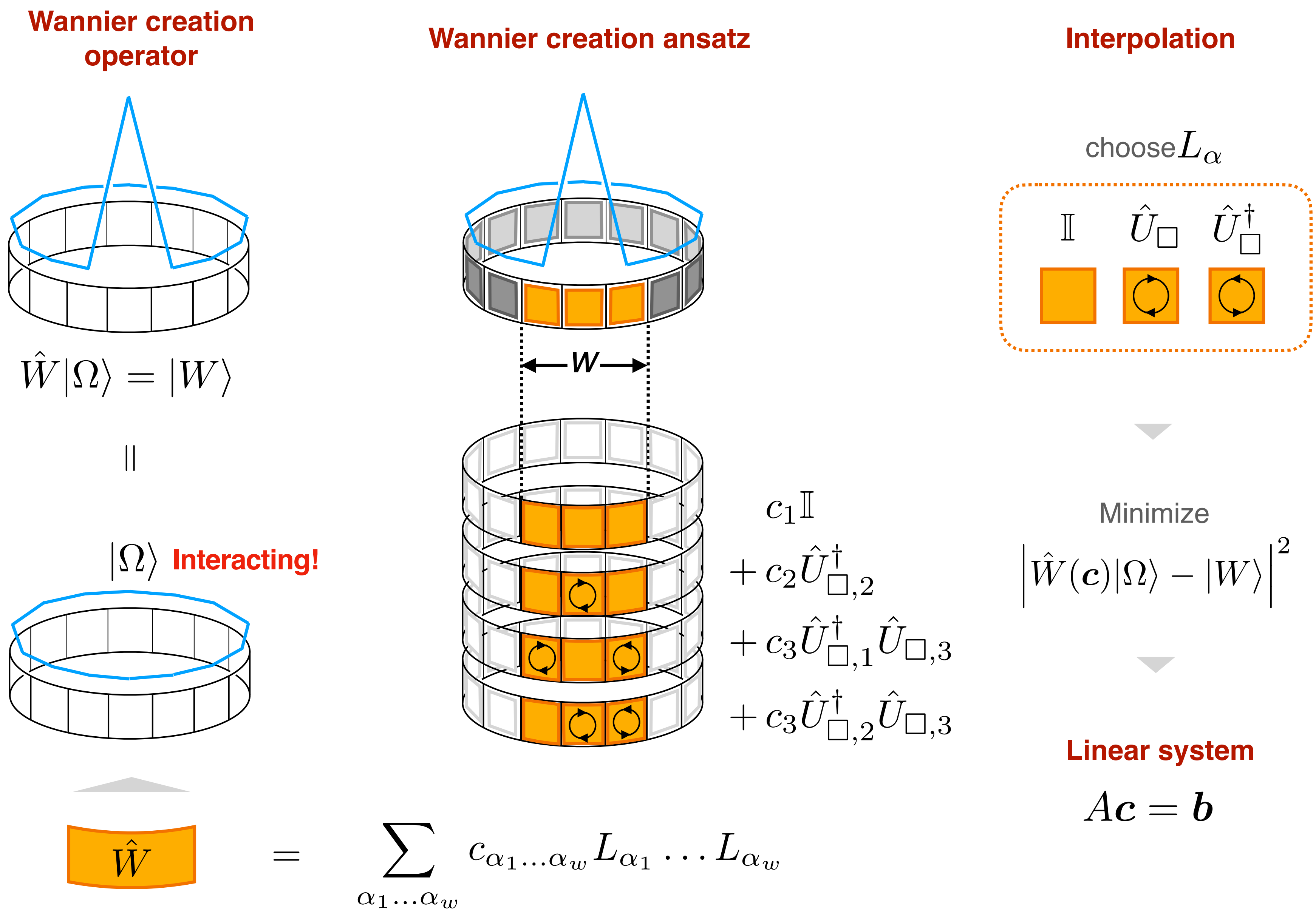
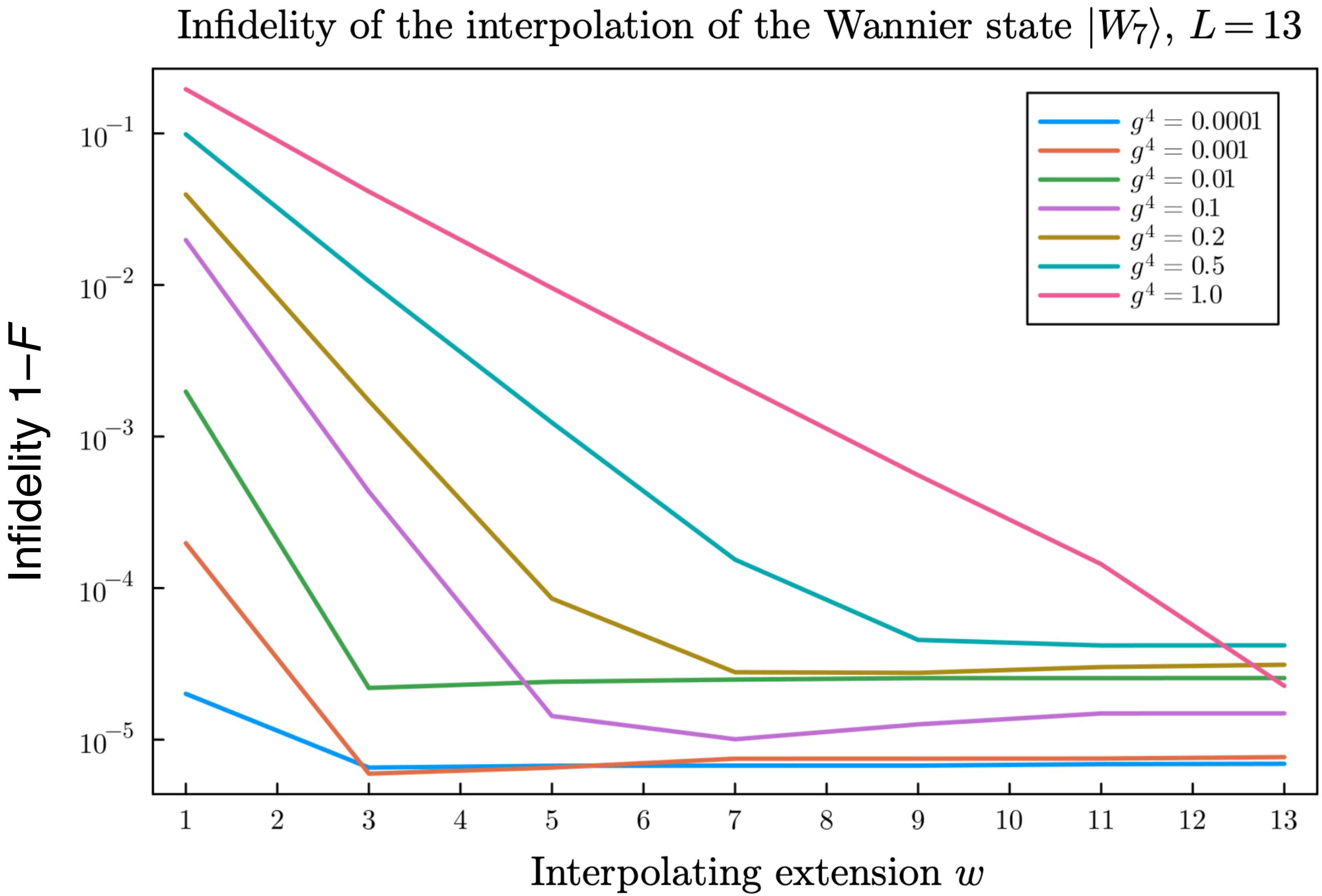


# 3. Construction of the wavepackets



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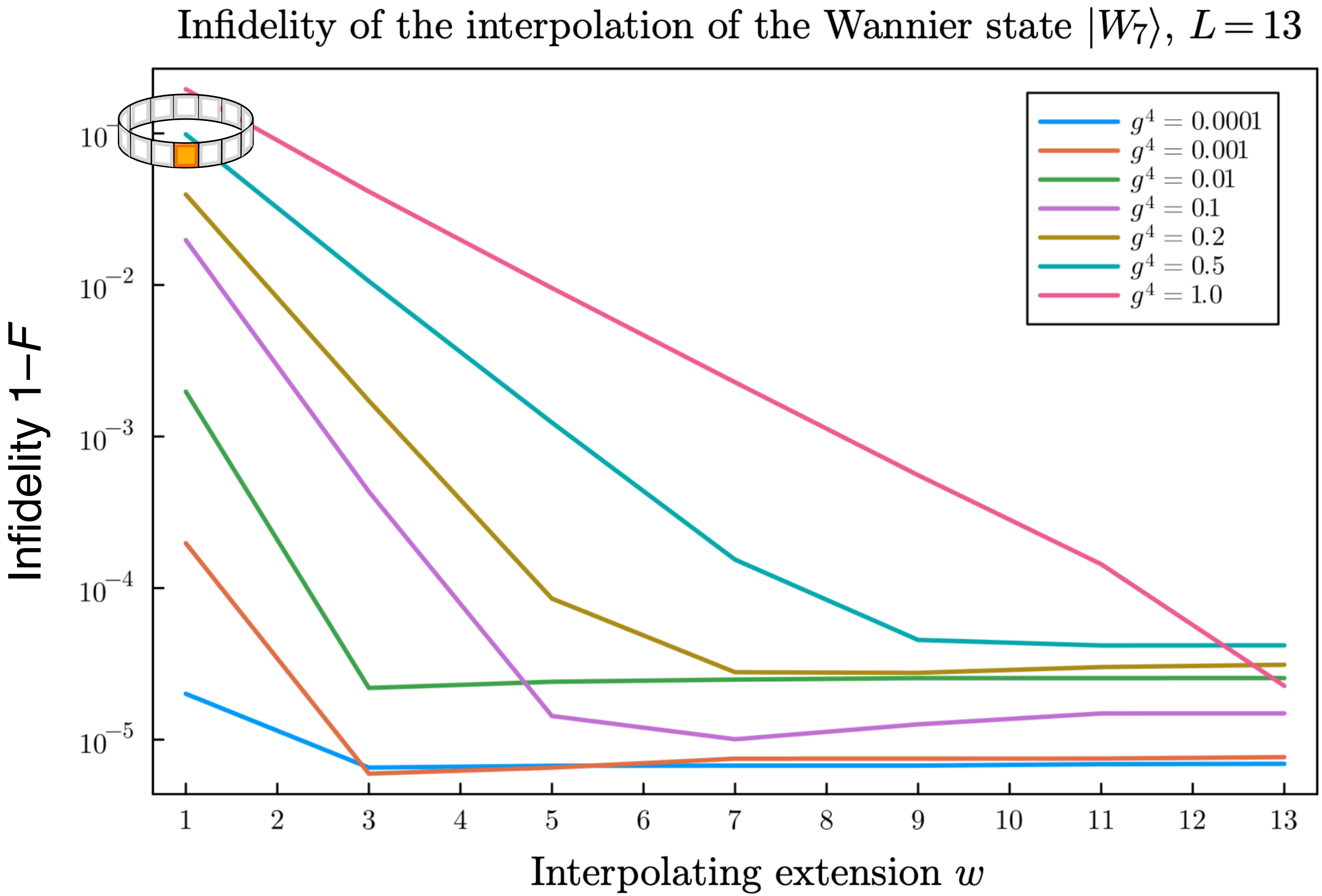

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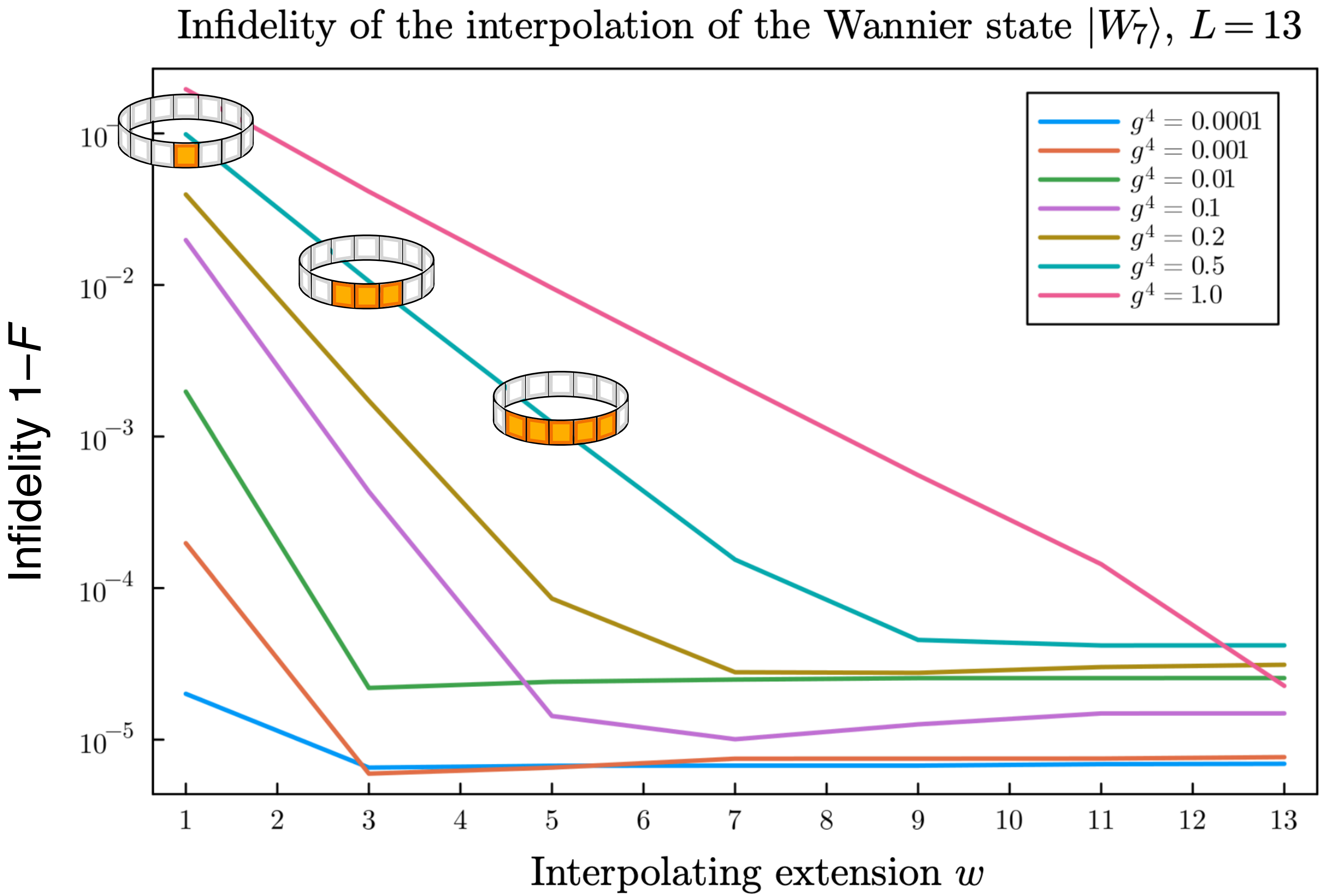

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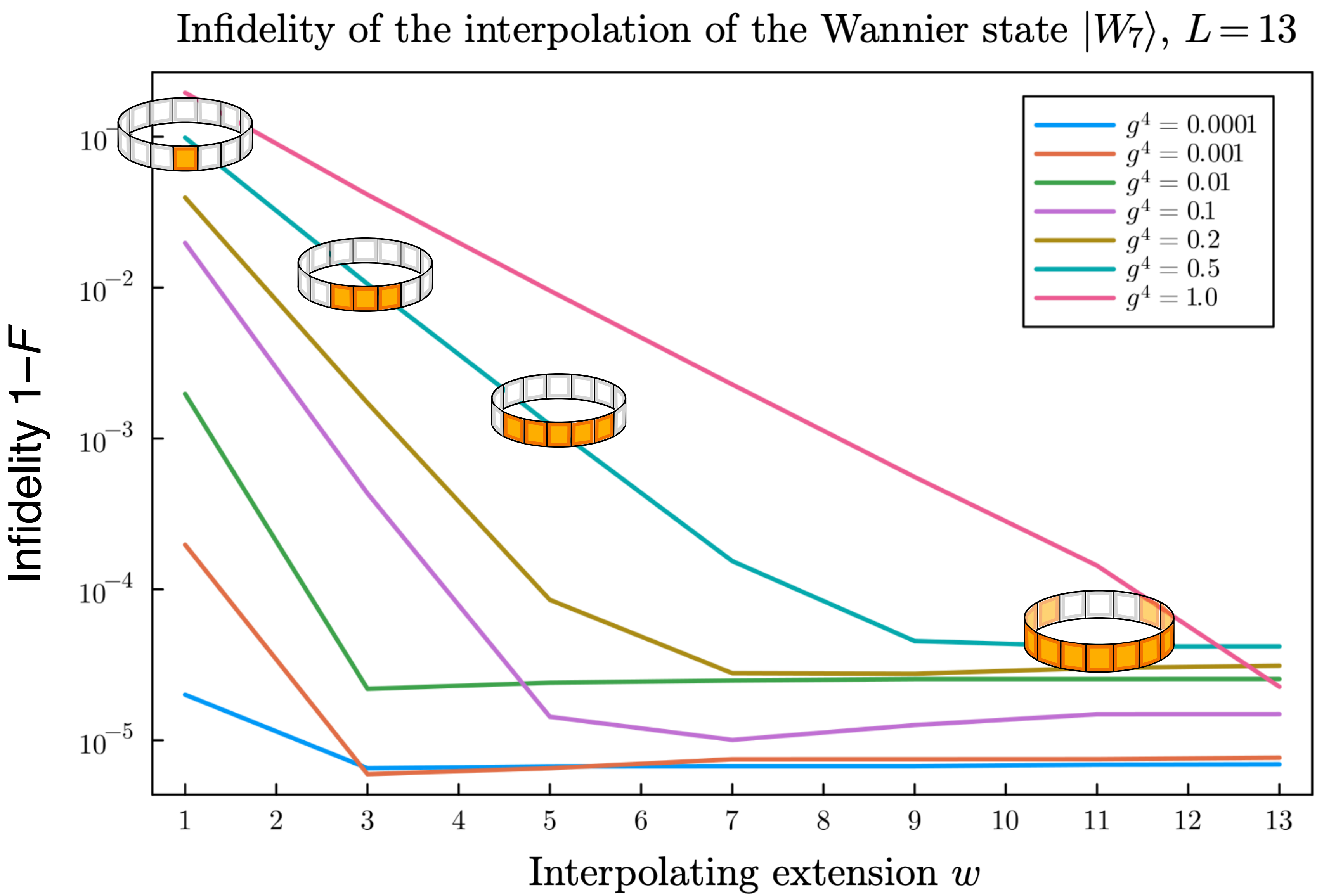
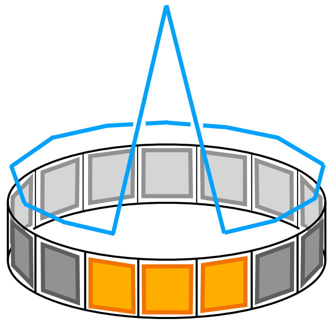
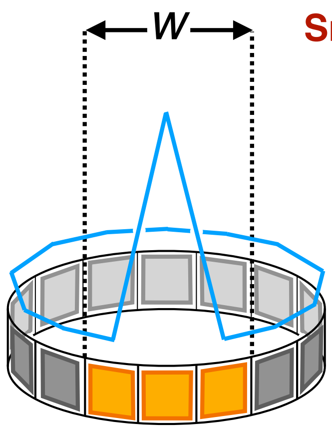


# 3. Construction of the wavepackets



**Intermediate** $L \sim 15$

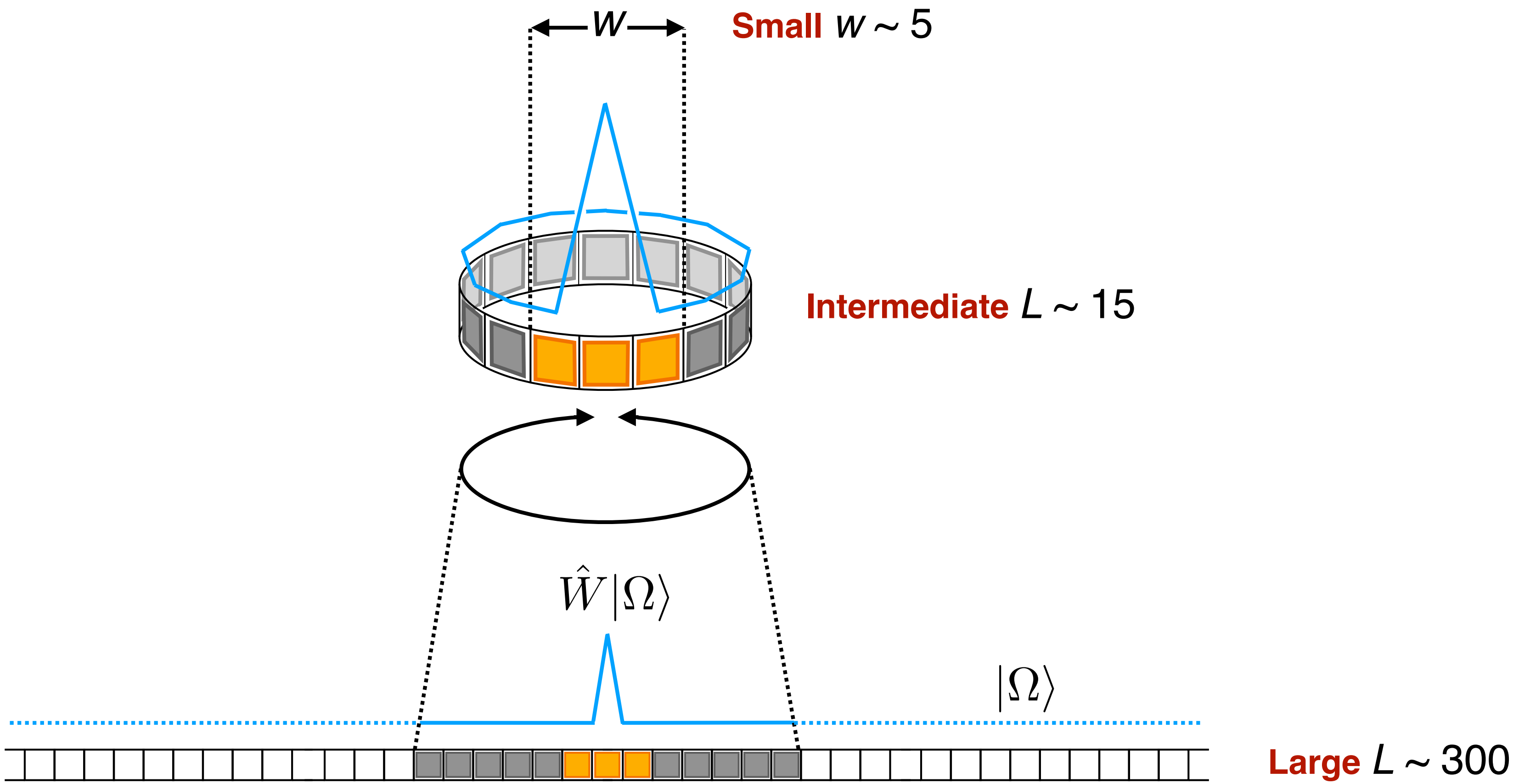
# 3. Construction of the wavepackets




**Small** $w \sim 5$

**Intermediate** $L \sim 15$

# 3. Construction of the wavepackets



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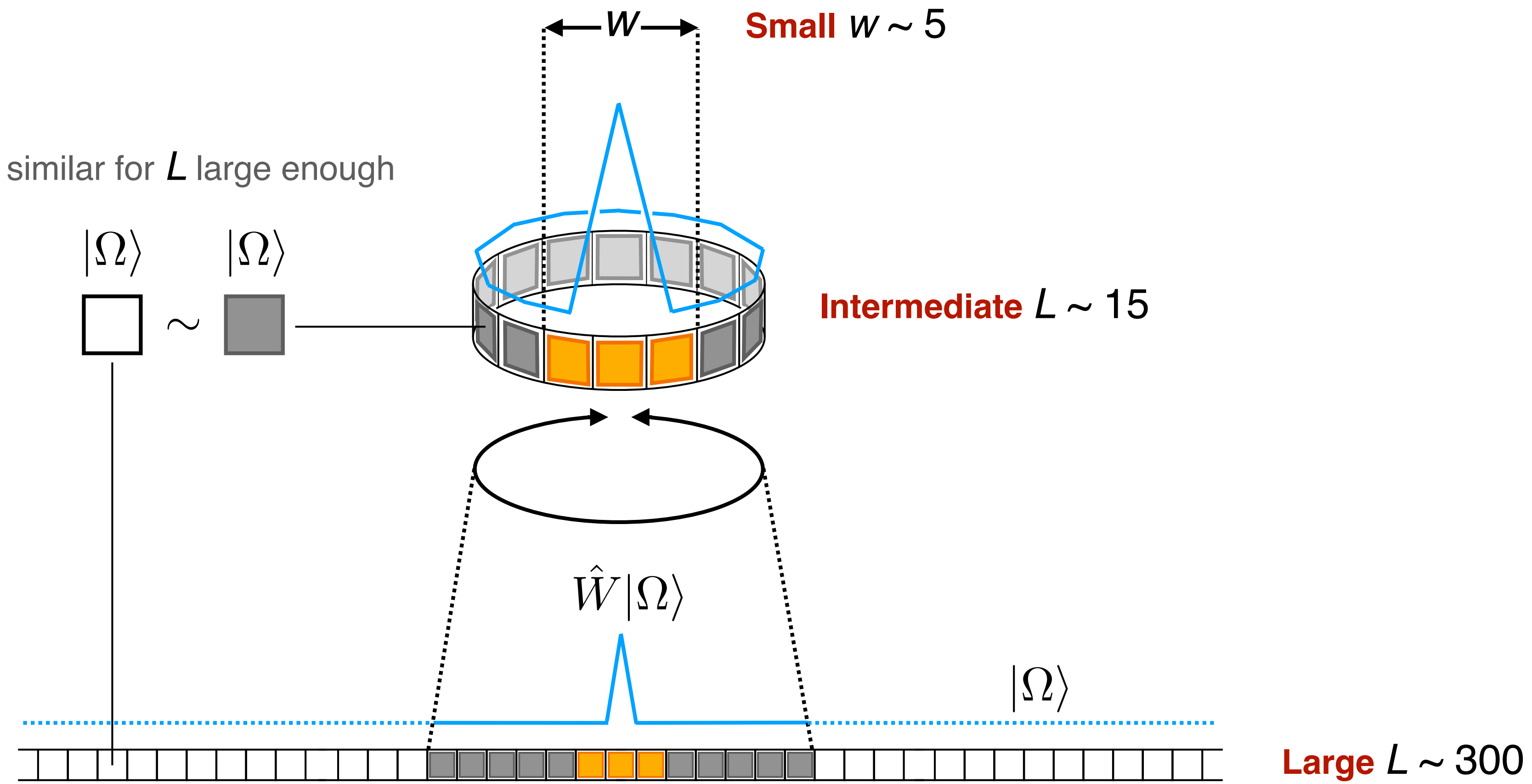
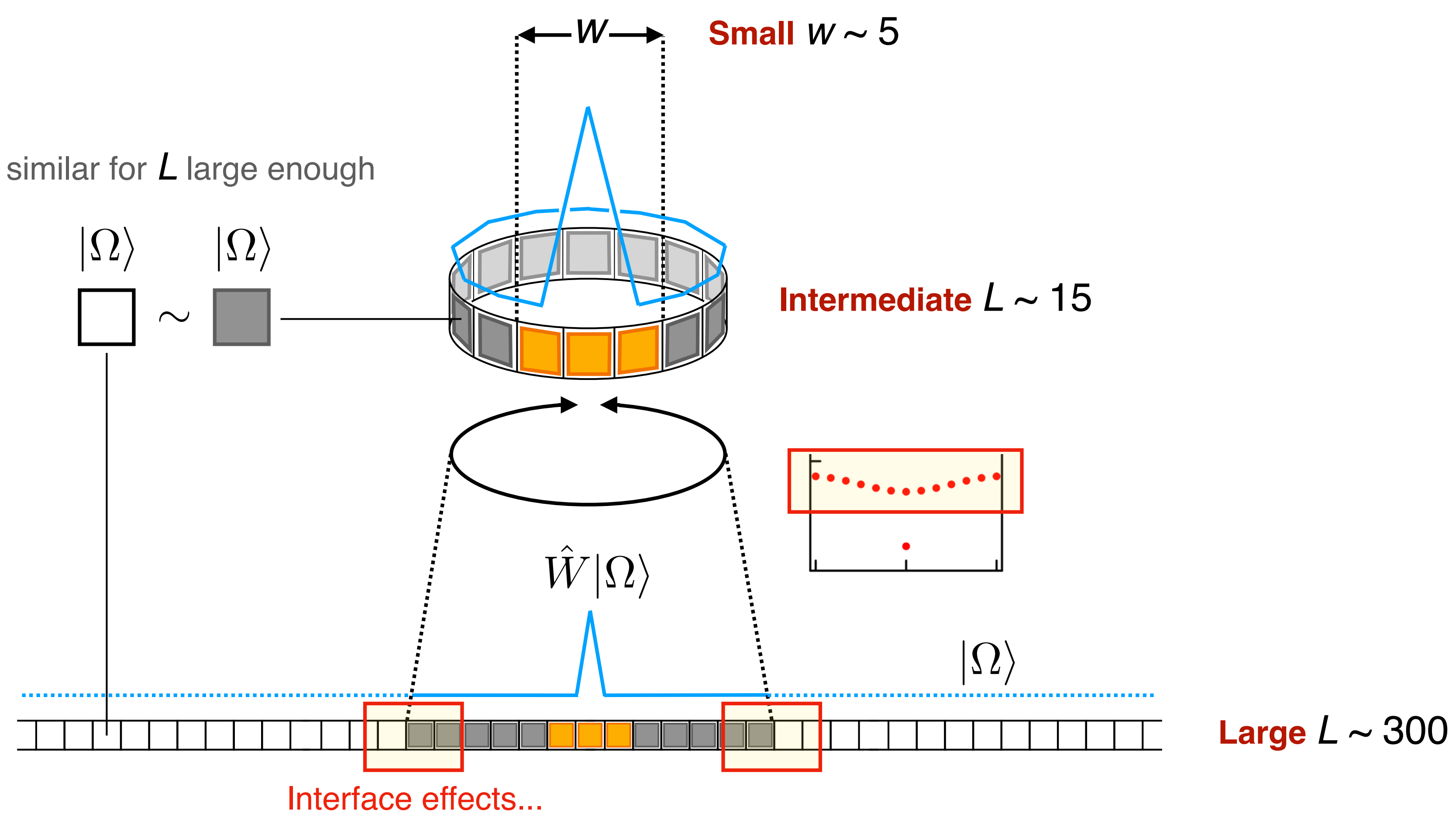


# 3. Construction of the wavepackets



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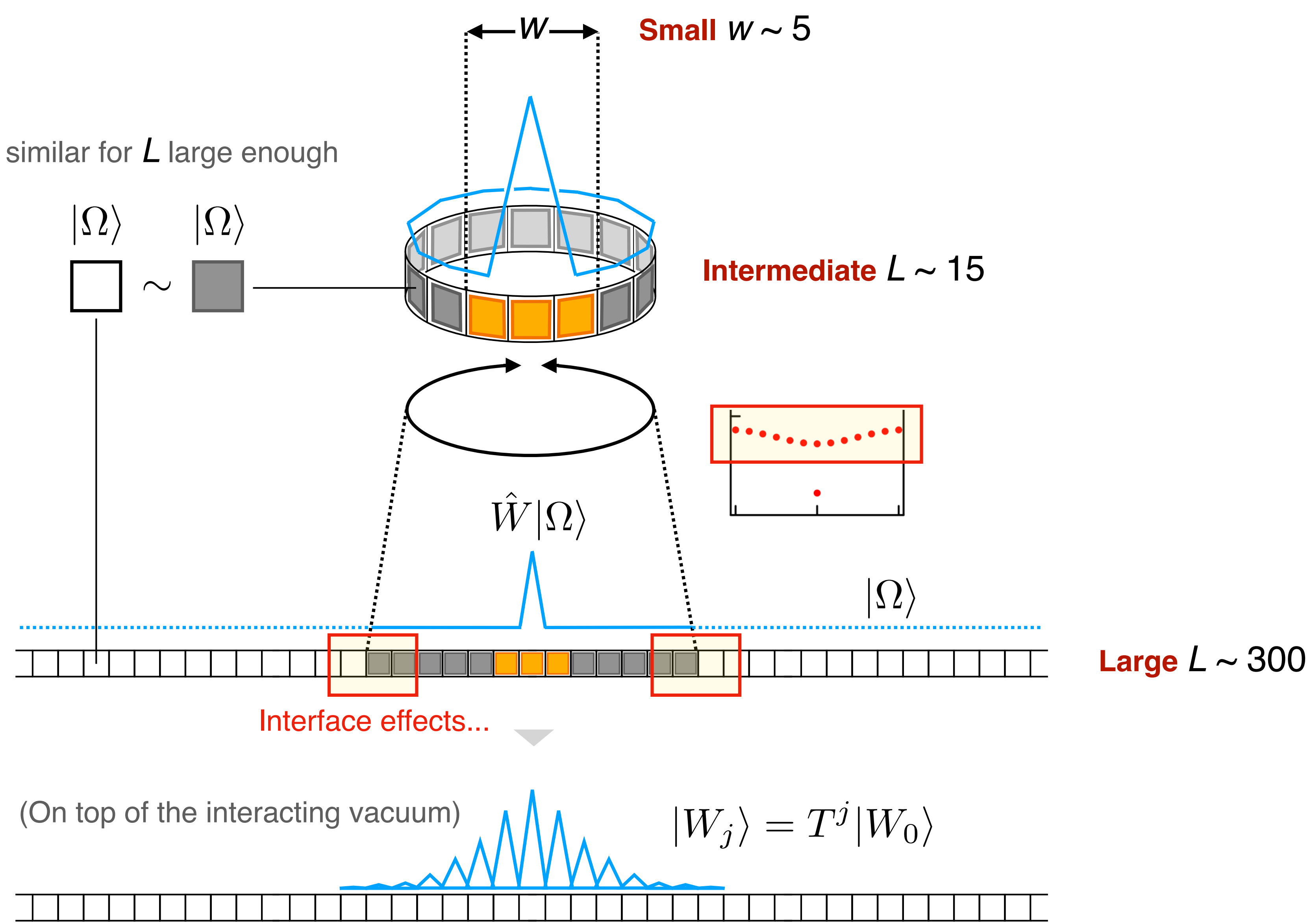
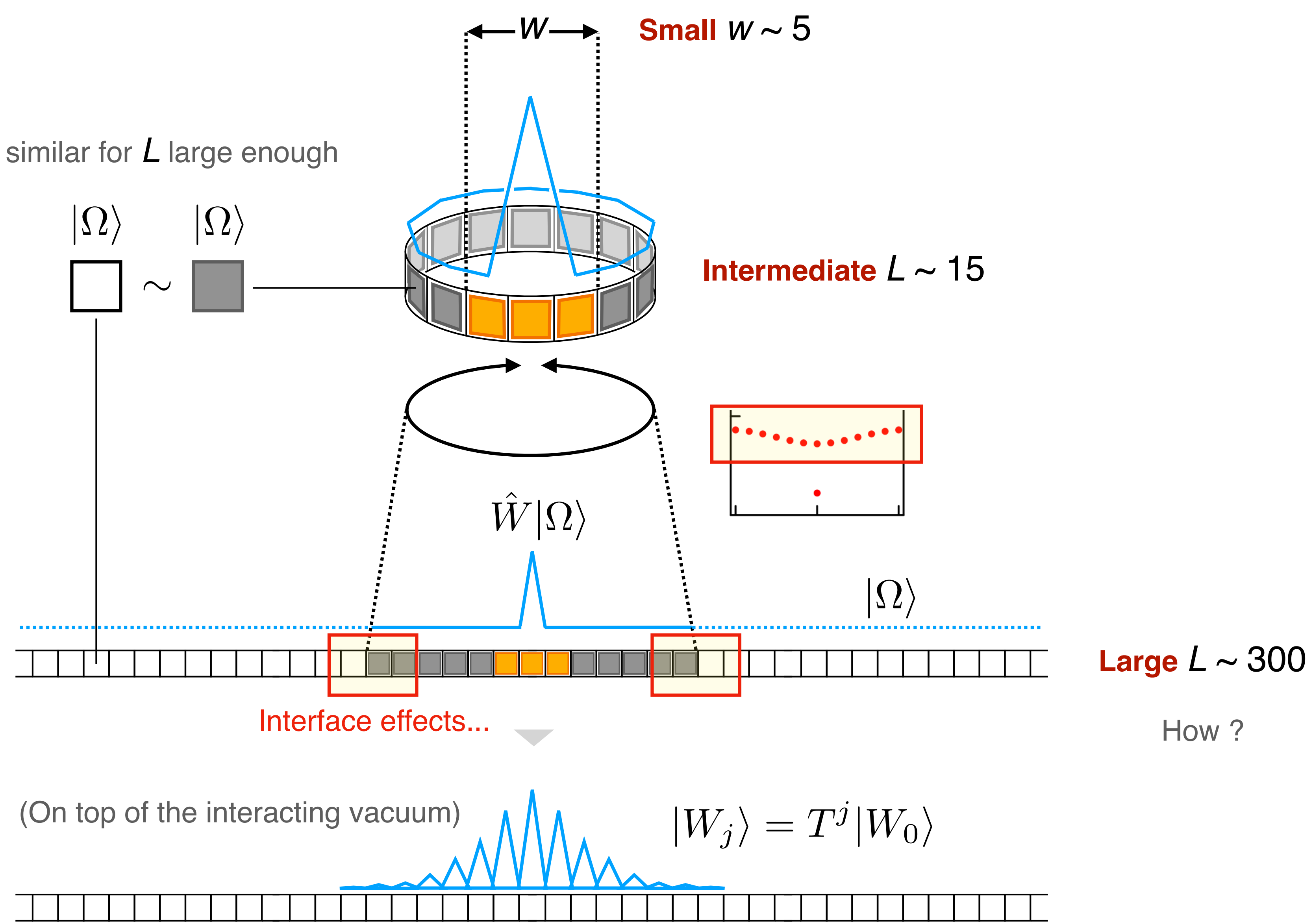



**Wannier function** → **building blocks** to construct any single-(quasi)particle state.

# 3. Construction of the wavepackets

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# 4. Tensor Network methods in a nutshell

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**Tensor Networks notation:** each tensor $T$ (node) has $n$ indices $i$ (links). Each index has a dimension $d$ (size).

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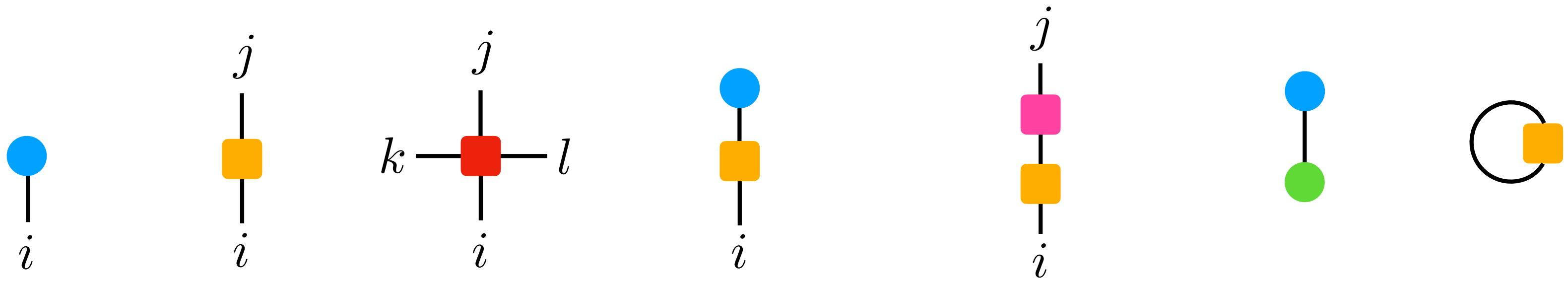


| vectors | matrices | tensors | vector-matrix | matrix-matrix | scalar prod. | trace |
|---|---|---|---|---|---|---|
| $V_i$ | $M_{ij}$ | $T_{ijkl}$ | $M_{ij}V_j$ | $M_{ik}N_{kj}$ | $V_iW_i$ | $M_{ii}$ |

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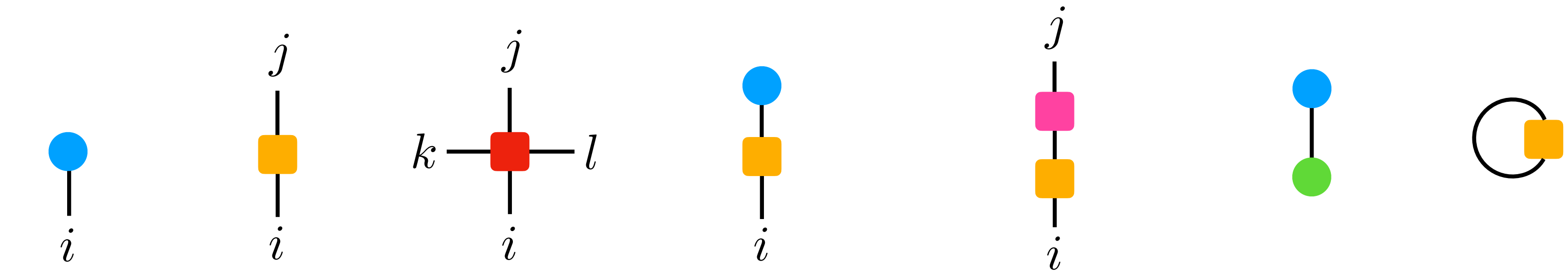


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**Singular Value Decomposition (SVD)**


*M*

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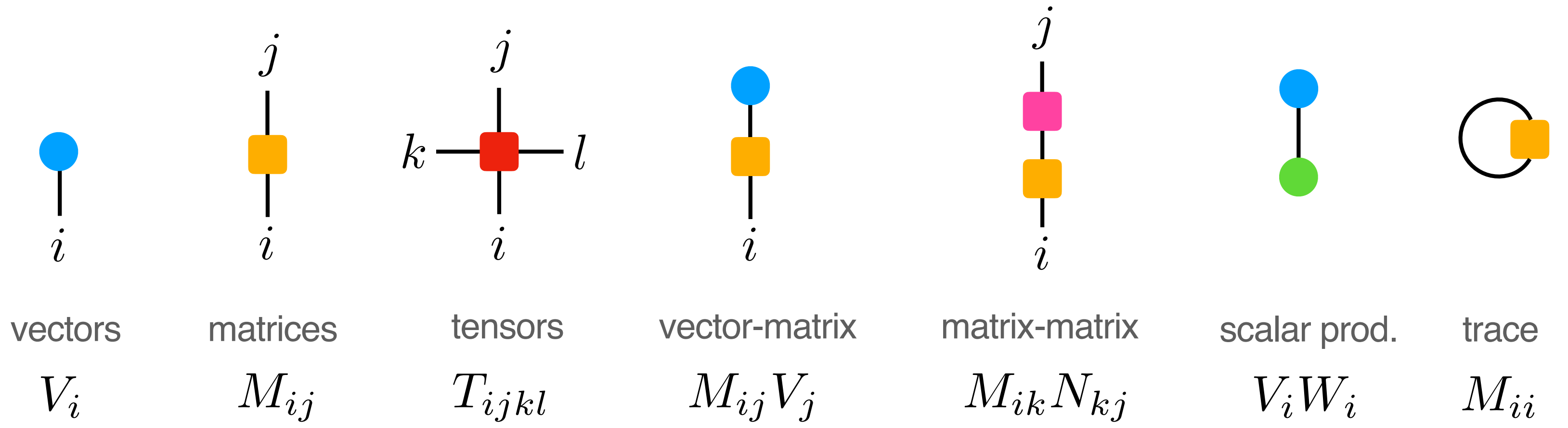
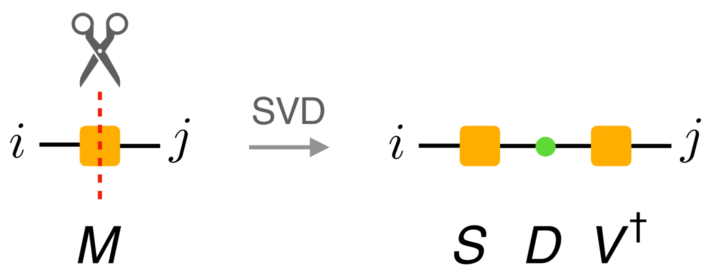



**Singular Value Decomposition (SVD)**



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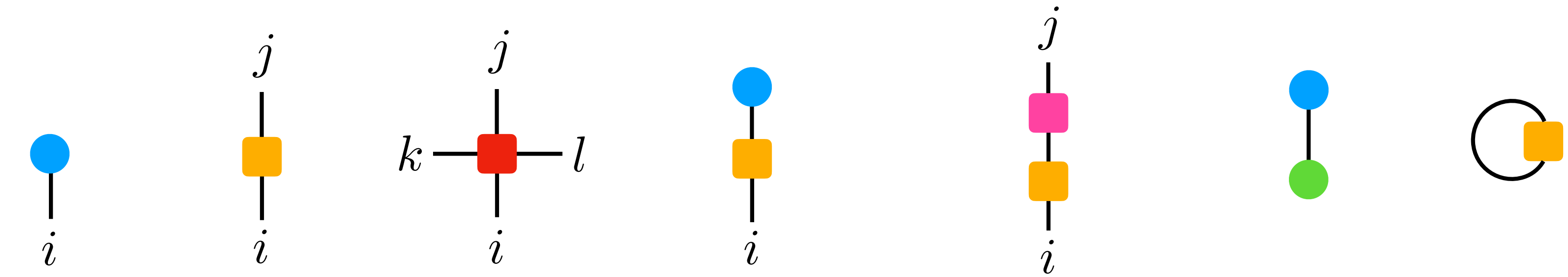


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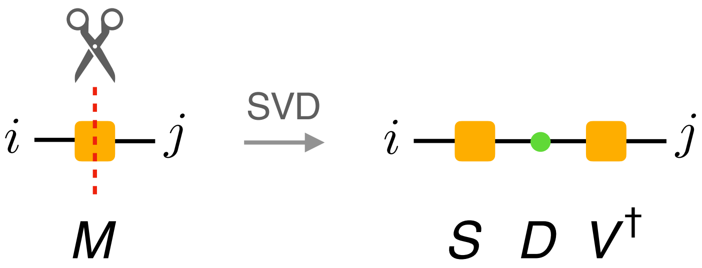
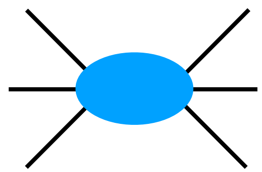
**Singular Value Decomposition (SVD)**



6-rank tensor

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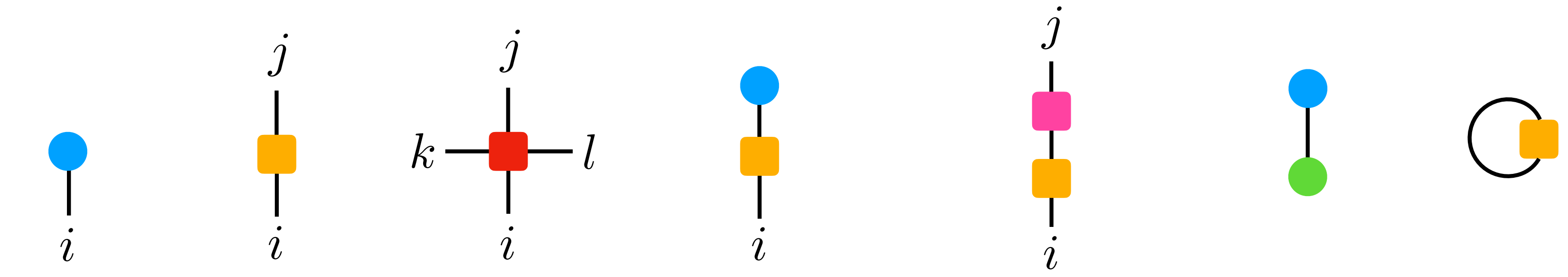


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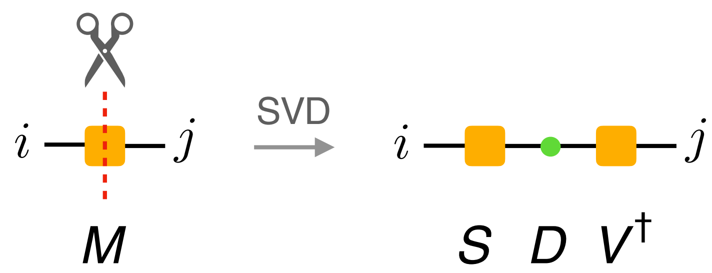
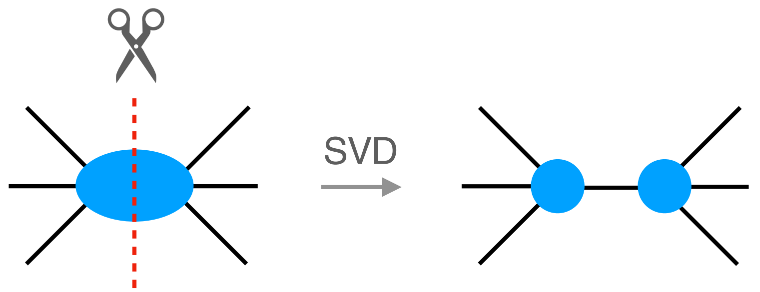



6-rank tensor

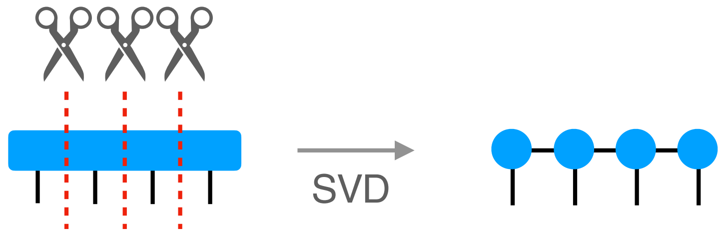
two 4-rank tensors

# 4. Tensor Network methods in a nutshell

**Quantum MB states**

# 4. Tensor Network methods in a nutshell

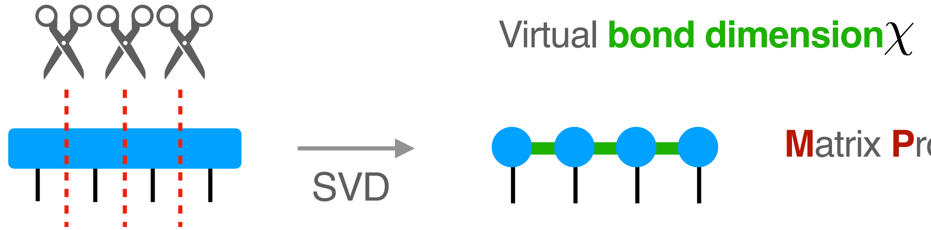
**Quantum MB states**




**M**atrix **P**roduct **S**tates

# 4. Tensor Network methods in a nutshell

**Quantum MB states**




**M**atrix **P**roduct **S**tates

# 4. Tensor Network methods in a nutshell

Physical links with **local dimension** $d$

Virtual **bond dimension** $\chi$

**Quantum MB states**


**M**atrix **P**roduct **S**tates

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Physical links with **local dimension** $d$

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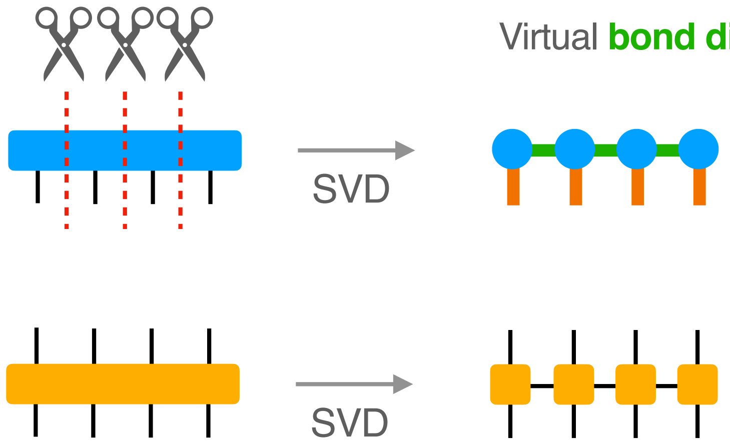
**Quantum MB Operators**




**M**atrix **P**roduct **S**tates

**M**atrix **P**roduct **O**perator

# 4. Tensor Network methods in a nutshell

Physical links with **local dimension** $d$

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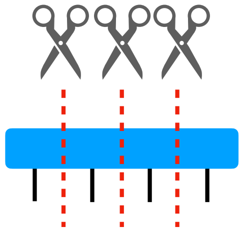
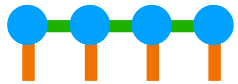
**Quantum MB states**

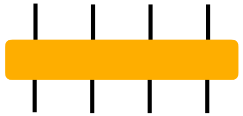


SVD

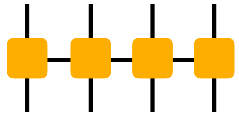


**M**atrix **P**roduct **S**tates

**Quantum MB Operators**



SVD



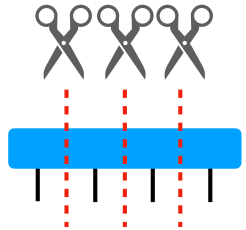
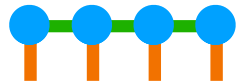
**M**atrix **P**roduct **O**perator

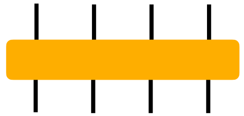
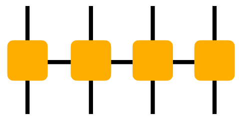
**Computational resources** $\mathcal{O}(d^L)$ → $\mathcal{O}(L)$ ( We can menage really high $L$ )

# 4. Tensor Network methods in a nutshell

Physical links with **local dimension** $d$

Virtual **bond dimension** $\chi$

**Quantum MB states**  → SVD  **M**atrix **P**roduct **S**tates

**Quantum MB Operators**  → SVD  **M**atrix **P**roduct **O**perator

**Computational resources** $\mathcal{O}(d^L)$ → $\mathcal{O}(L)$ ( We can menage really high $L$ )

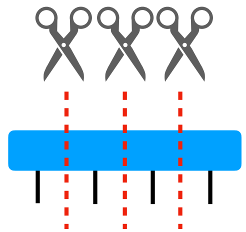
**Is this representation efficient?**

# 4. Tensor Network methods in a nutshell

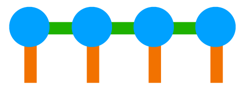
Physical links with **local dimension** $d$

Virtual **bond dimension** $\chi$

**Quantum MB states**

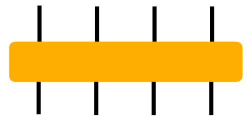


SVD

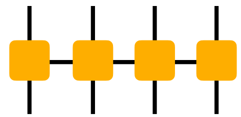


**M**atrix **P**roduct **S**tates

**Quantum MB Operators**



SVD

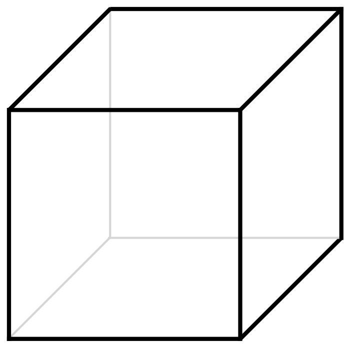


**M**atrix **P**roduct **O**perator

**Computational resources** $\mathcal{O}(d^L)$ → $\mathcal{O}(L)$ ( We can menage really high $L$ )

**Local Hamiltonian**

**Is this representation efficient?**



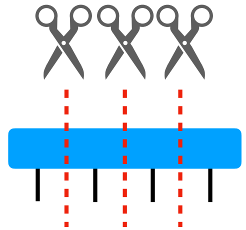
Hilbert space $\mathcal{H}$

# 4. Tensor Network methods in a nutshell

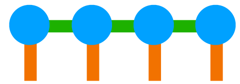
Physical links with **local dimension** $d$

Virtual **bond dimension** $\chi$

**Quantum MB states**

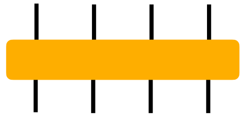


→ SVD

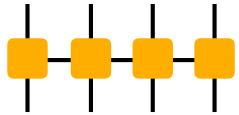


**M**atrix **P**roduct **S**tates

**Quantum MB Operators**



→ SVD



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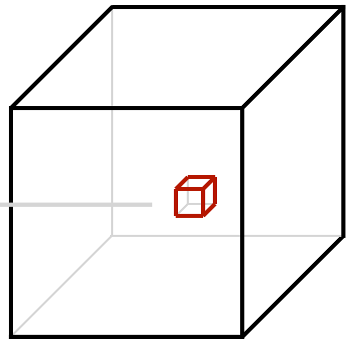
**Local Hamiltonian**

▼

**Is this representation efficient?** **Area law states**



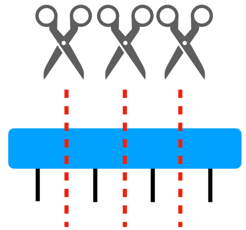
Hilbert space $\mathcal{H}$

# 4. Tensor Network methods in a nutshell

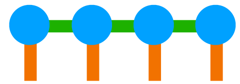
Physical links with **local dimension** $d$

Virtual **bond dimension** $\chi$

**Quantum MB states**

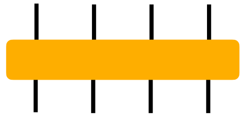


→ SVD



**M**atrix **P**roduct **S**tates

**Quantum MB Operators**



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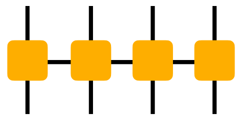


**M**atrix **P**roduct **O**perator

**Computational resources** $\mathcal{O}(d^L)$ → $\mathcal{O}(L)$ ( We can menage really high $L$ )

**Is this representation efficient?**

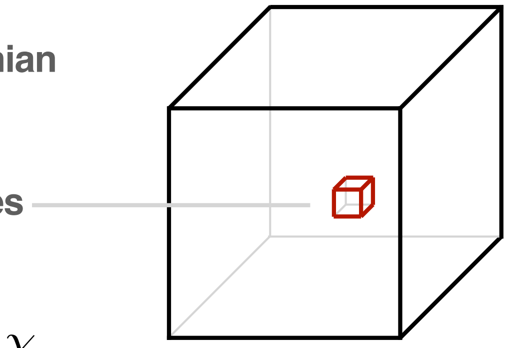
**Local Hamiltonian**

▼

**Area law states**

▼

**MPS with low** $\chi$



Hilbert space $\mathcal{H}$

# 4. Tensor Network methods in a nutshell

**Useful MPS and MPO Algorithms**

# 4. Tensor Network methods in a nutshell

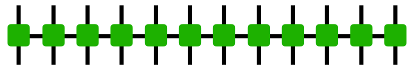
## Useful MPS and MPO Algorithms

### Ground-state search

# 4. Tensor Network methods in a nutshell

## Useful MPS and MPO Algorithms

**Ground-state search** **D**ensity **M**atrix **R**enormalization **G**roup



**MPO Hamiltonian**

# 4. Tensor Network methods in a nutshell

**Useful MPS and MPO Algorithms**

**Ground-state search** **D**ensity **M**atrix **R**enormalization **G**roup

# 4. Tensor Network methods in a nutshell

**Useful MPS and MPO Algorithms**

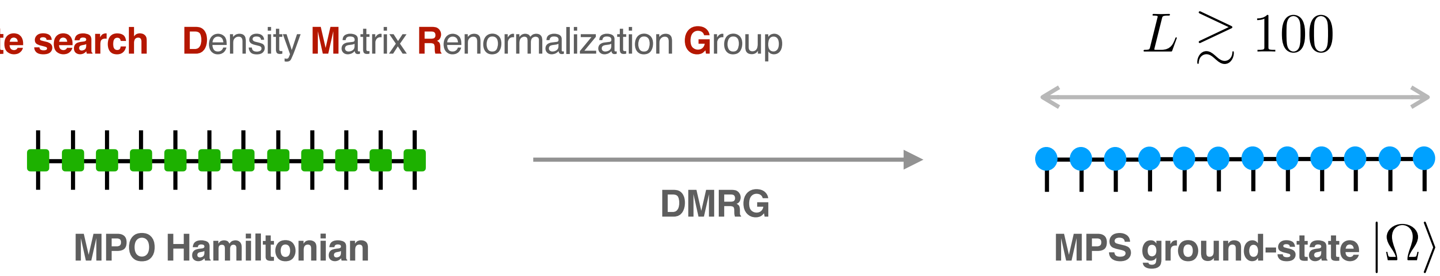
**Ground-state search** **D**ensity **M**atrix **R**enormalization **G**roup

# 4. Tensor Network methods in a nutshell

**Useful MPS and MPO Algorithms**

Large systems
(with just a laptop!)

**Ground-state search** **D**ensity **M**atrix **R**enormalization **G**roup




**Time evolution for MPS**

# 4. Tensor Network methods in a nutshell

**Useful MPS and MPO Algorithms**

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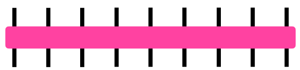
**Ground-state search** **D**ensity **M**atrix **R**enormalization **G**roup


**Time evolution for MPS** **T**ime **E**volving **B**lock **D**ecimation (…but also **TDVP**)



# 4. Tensor Network methods in a nutshell

**Useful MPS and MPO Algorithms**

Large systems
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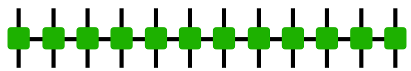
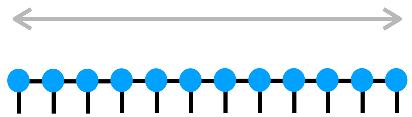
**Ground-state search** **D**ensity **M**atrix **R**enormalization **G**roup

$L \gtrsim 100$

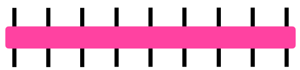


**DMRG**

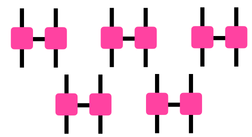


**MPO Hamiltonian** **MPS ground-state** $|\Omega\rangle$

**Time evolution for MPS** **T**ime **E**volving **B**lock **D**ecimation (…but also **TDVP**)



**Suzuki-Trotter** $\delta t$



**Time evolution operator** **Iteration of gates**

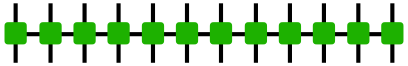
# 4. Tensor Network methods in a nutshell

**Useful MPS and MPO Algorithms**

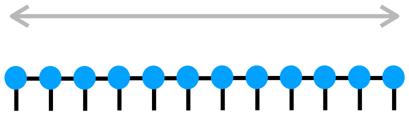
Large systems
(with just a laptop!)

**Ground-state search** **D**ensity **M**atrix **R**enormalization **G**roup

$L \gtrsim 100$



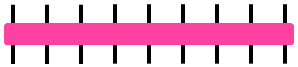
**DMRG**



**MPO Hamiltonian**

**MPS ground-state** $|\Omega\rangle$

**Time evolution for MPS** **T**ime **E**volving **B**lock **D**ecimation (…but also **TDVP**)



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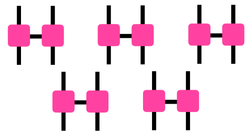


**Time evolution operator**

**Iteration of gates**

**Operator to MPO conversion**

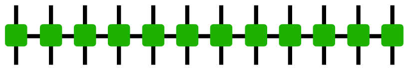
# 4. Tensor Network methods in a nutshell

## Useful MPS and MPO Algorithms

Large systems
(with just a laptop!)

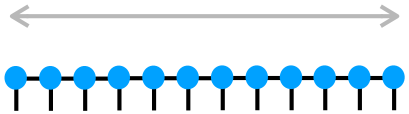
$L \gtrsim 100$

**Ground-state search** Density Matrix Renormalization Group



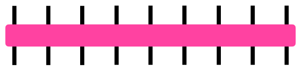
MPO Hamiltonian

DMRG



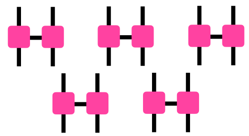
MPS ground-state $|\Omega\rangle$

**Time evolution for MPS** Time Evolving Block Decimation (…but also **TDVP**)



Time evolution operator

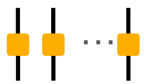
Suzuki-Trotter $\delta t$



Iteration of gates

**Operator to MPO conversion**

$$\sum_\alpha \hat{L}_\alpha = \sum$$



Sum of compositions of local operators

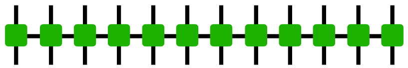
# 4. Tensor Network methods in a nutshell

## Useful MPS and MPO Algorithms

Large systems
(with just a laptop!)

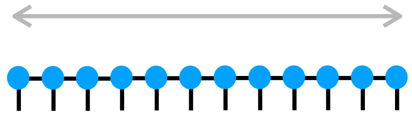
**Ground-state search** **D**ensity **M**atrix **R**enormalization **G**roup

$L \gtrsim 100$



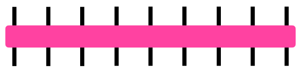
MPO Hamiltonian

DMRG



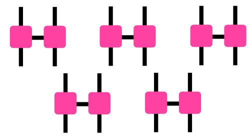
MPS ground-state $|\Omega\rangle$

**Time evolution for MPS** **T**ime **E**volving **B**lock **D**ecimation (…but also **TDVP**)



Time evolution operator

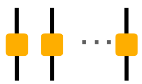
Suzuki-Trotter $\delta t$



Iteration of gates

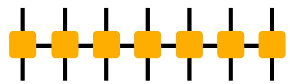
**Operator to MPO conversion**

$$\sum_\alpha \hat{L}_\alpha = \sum$$



Sum of compositions of local operators

**Automata procedure**



Matrix Product Operator

# 4. Tensor Network methods in a nutshell

**Automata procedure** Fröwis et al. Phys. Rev. A 81 (2010) 062337

# 4. Tensor Network methods in a nutshell

**Automata procedure**

1. **Operator picture** $$\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$$

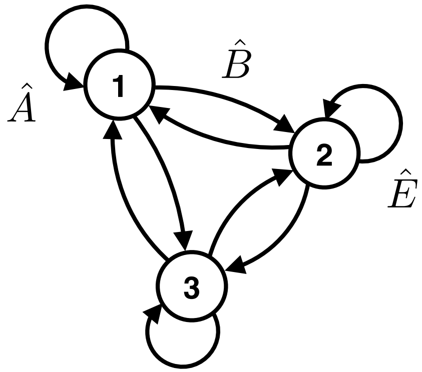
# 4. Tensor Network methods in a nutshell

**Automata procedure**

1 Operator picture

$$\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$$

2 Automata picture



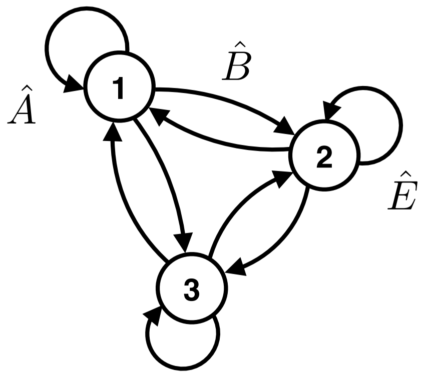
# 4. Tensor Network methods in a nutshell

**Automata procedure**

1. Operator picture

$$\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$$

2. Automata picture




3. Matrix picture

$$\begin{pmatrix} \hat{A} & \hat{B} & \hat{C} \\ \hat{D} & \hat{E} & \hat{F} \\ \hat{G} & \hat{H} & \hat{I} \end{pmatrix}$$

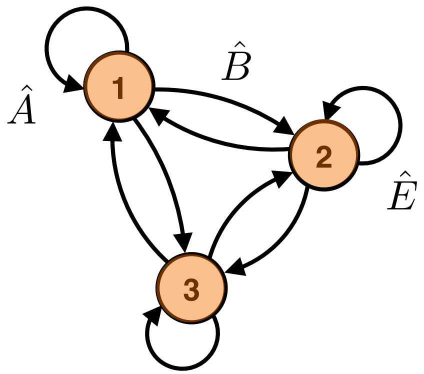
# 4. Tensor Network methods in a nutshell

**Automata procedure**

1 **Operator picture**

$$\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$$

2 **Automata picture**




3 **Matrix picture**

$$\begin{pmatrix} \hat{A} & \hat{B} & \hat{C} \\ \hat{D} & \hat{E} & \hat{F} \\ \hat{G} & \hat{H} & \hat{I} \end{pmatrix}$$

# 4. Tensor Network methods in a nutshell

**Automata procedure**

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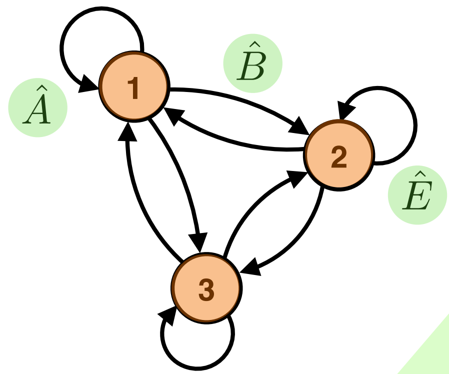
$$\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$$

2. **Automata picture**




3. **Matrix picture**

$$\begin{pmatrix} \hat{A} & \hat{B} & \hat{C} \\ \hat{D} & \hat{E} & \hat{F} \\ \hat{G} & \hat{H} & \hat{I} \end{pmatrix}$$

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

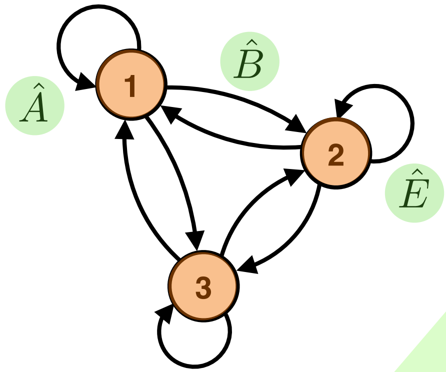
# 4. Tensor Network methods in a nutshell

**Automata procedure**

1. **Operator picture**

$$\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$$

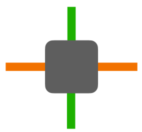
2. **Automata picture**




3. **Matrix picture**

$$\begin{pmatrix} \hat{A} & \hat{B} & \hat{C} \\ \hat{D} & \hat{E} & \hat{F} \\ \hat{G} & \hat{H} & \hat{I} \end{pmatrix} \qquad \hat{C} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

4. **TN picture**



# 4. Tensor Network methods in a nutshell

**Automata procedure**

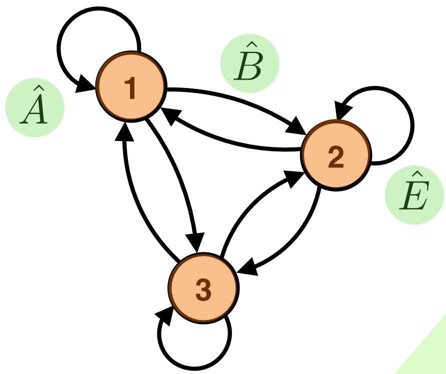
**Wannier creation operator**

1 **Operator picture**

$$\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$$

$$\hat{W} = \sum_{\alpha_1 \ldots \alpha_w} c_{\alpha_1 \ldots \alpha_w} L_{\alpha_1} \ldots L_{\alpha_w}$$

2 **Automata picture**

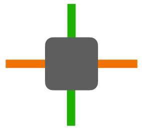



3 **Matrix picture**

$$\begin{pmatrix} \hat{A} & \hat{B} & \hat{C} \\ \hat{D} & \hat{E} & \hat{F} \\ \hat{G} & \hat{H} & \hat{I} \end{pmatrix}$$

$$\hat{C} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

4 **TN picture**



# 4. Tensor Network methods in a nutshell

**Automata procedure** | **Wannier creation operator**

**1 Operator picture**

$$\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$$

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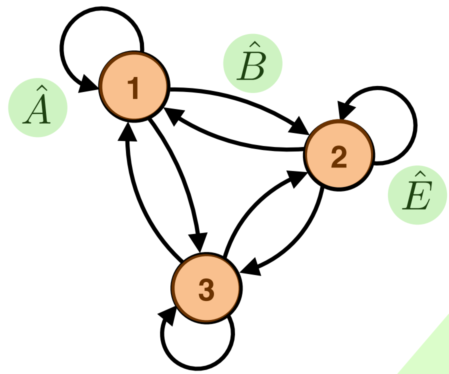
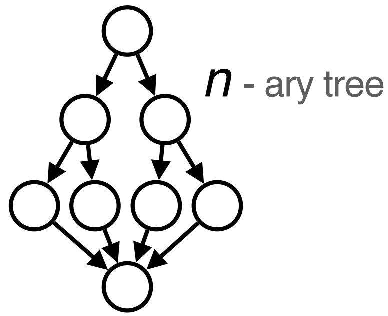
**2 Automata picture**



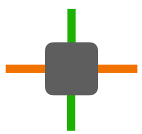




**3 Matrix picture**

$$\begin{pmatrix} \hat{A} & \hat{B} & \hat{C} \\ \hat{D} & \hat{E} & \hat{F} \\ \hat{G} & \hat{H} & \hat{I} \end{pmatrix}$$

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

**4 TN picture**



# 4. Tensor Network methods in a nutshell

**Automata procedure** | **Wannier creation operator**

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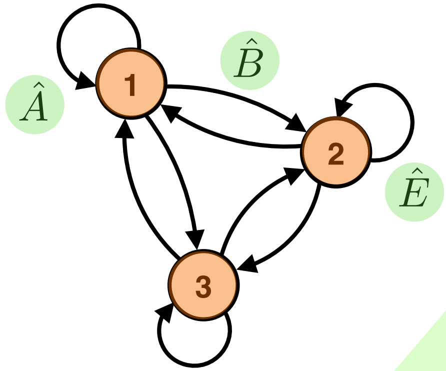
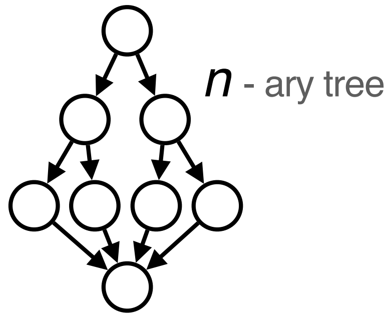
**2 Automata picture**



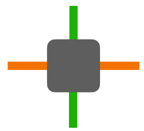




**3 Matrix picture**

$$\begin{pmatrix} \hat{A} & \hat{B} & \hat{C} \\ \hat{D} & \hat{E} & \hat{F} \\ \hat{G} & \hat{H} & \hat{I} \end{pmatrix} \qquad \hat{C} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \; \mathcal{O}(e^{w})$$

**4 TN picture**



# 4. Tensor Network methods in a nutshell

**Automata procedure** | **Wannier creation operator**

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$$\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$$

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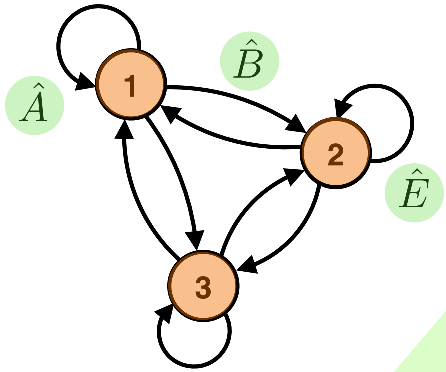
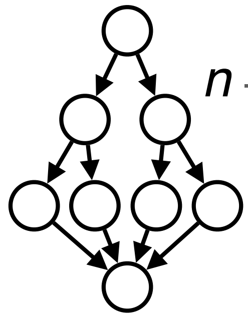
**2 Automata picture**




**3 Matrix picture**

$$\begin{pmatrix} \hat{A} & \hat{B} & \hat{C} \\ \hat{D} & \hat{E} & \hat{F} \\ \hat{G} & \hat{H} & \hat{I} \end{pmatrix}, \quad \hat{C} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

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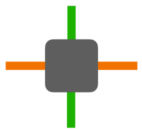
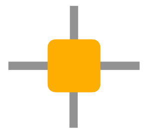




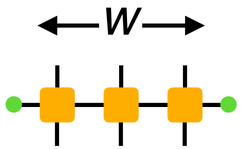
"Wannier Ansätze"

# 5. Back to Wavepackets

“Wannier Ansätze”

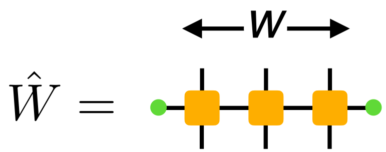
# 5. Back to Wavepackets

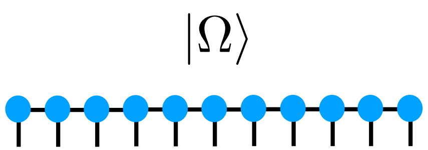
**Wannier creation operator Ansätze**

$\hat{W} =$ 

# 5. Back to Wavepackets

**Wannier creation operator Ansätze**

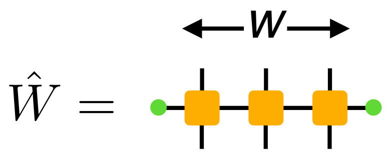


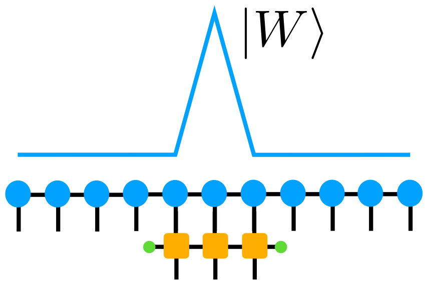



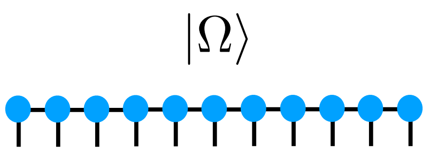
Large *L* vacuum state found with DMRG

# 5. Back to Wavepackets

**Wannier creation operator Ansätze**



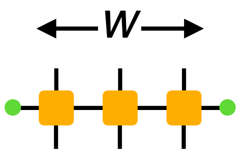



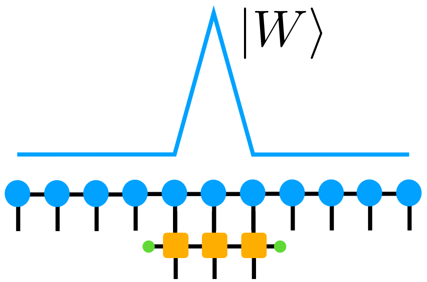




Large $L$ vacuum state
found with DMRG

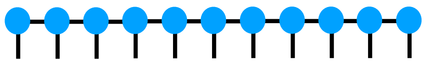
# 5. Back to Wavepackets

**Wannier creation operator Ansätze**

$\hat{W} =$ 

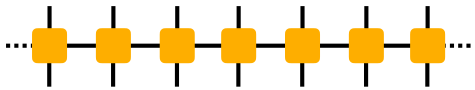




$|\Omega\rangle$



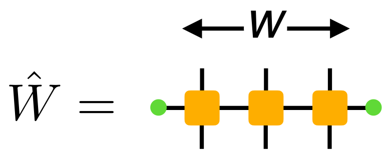
Large $L$ vacuum state
found with DMRG

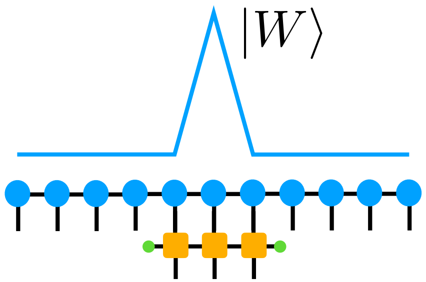
**Wavepacket Ansatz**

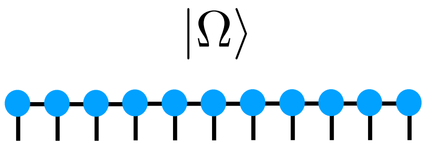
$\hat{\Phi} =$ 

# 5. Back to Wavepackets

**Wannier creation operator Ansätze**




Large $L$ vacuum state
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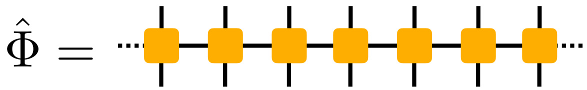
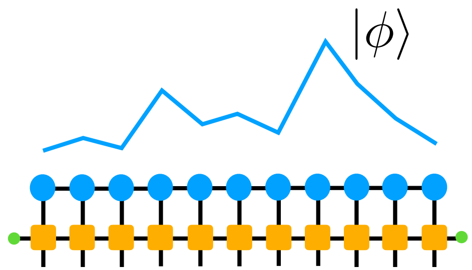
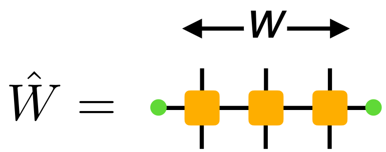
**Wavepacket Ansatz**

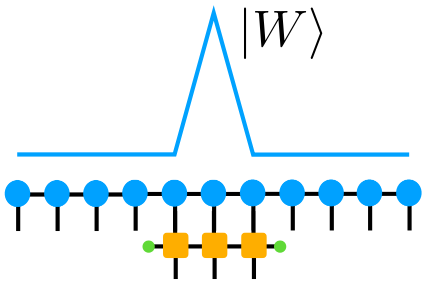


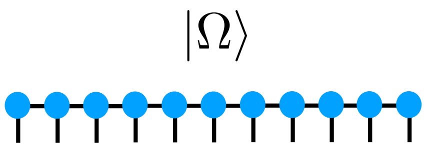



# 5. Back to Wavepackets

**Wannier creation operator Ansätze**

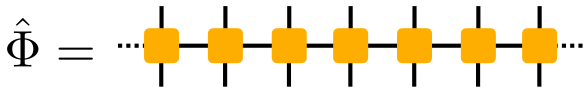


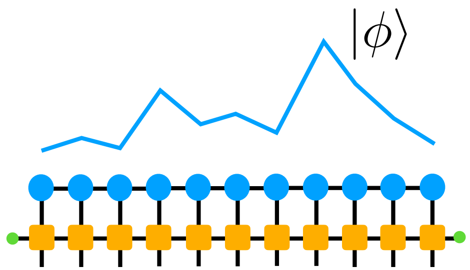




Large *L* vacuum state found with DMRG

**Wavepacket Ansatz**



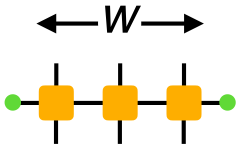



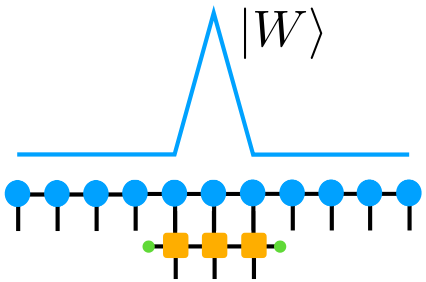

No scaling of $\chi$ with size!

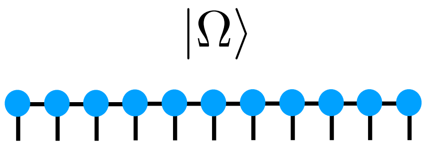
$$\chi = \mathcal{O}(e^w, \cancel{L})$$

# 5. Back to Wavepackets

**Wannier creation operator Ansätze**

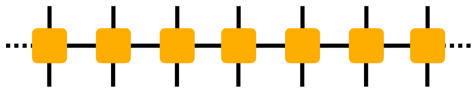
$\hat{W} =$ 

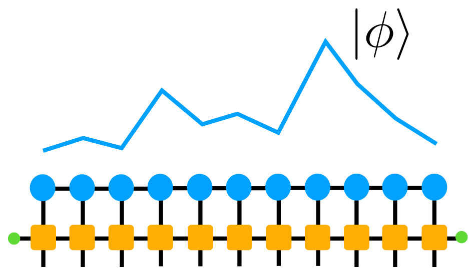




Large *L* vacuum state found with DMRG

**Wavepacket Ansatz**

$\hat{\Phi} =$ 




No scaling of $\chi$ with size!

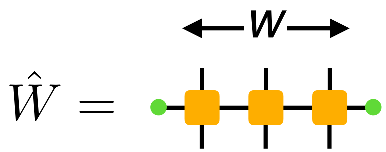
$$\chi = \mathcal{O}(e^w, \cancel{L})$$

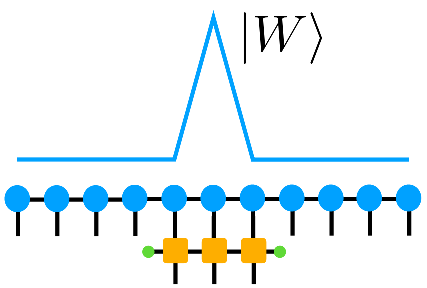
**Gaussian wavepacket**

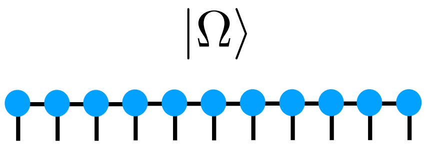
$$\frac{1}{\mathcal{N}} e^{-\frac{(j-j_0)^2}{2\sigma}} e^{ijk_0}$$

# 5. Back to Wavepackets

**Wannier creation operator Ansätze**

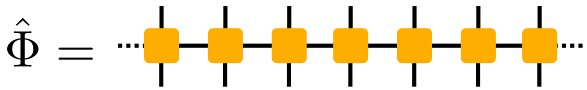


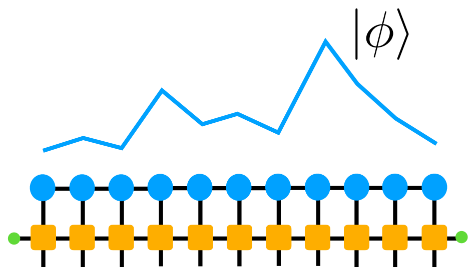




Large *L* vacuum state found with DMRG

**Wavepacket Ansatz**



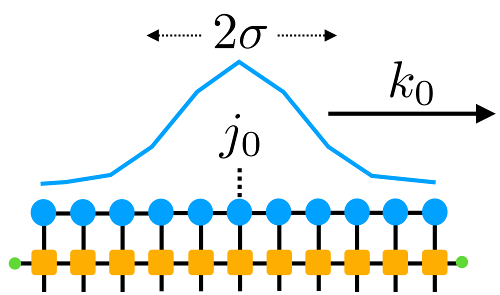

No scaling of $\chi$ with size!

$$\chi = \mathcal{O}(e^w, \not{L})$$

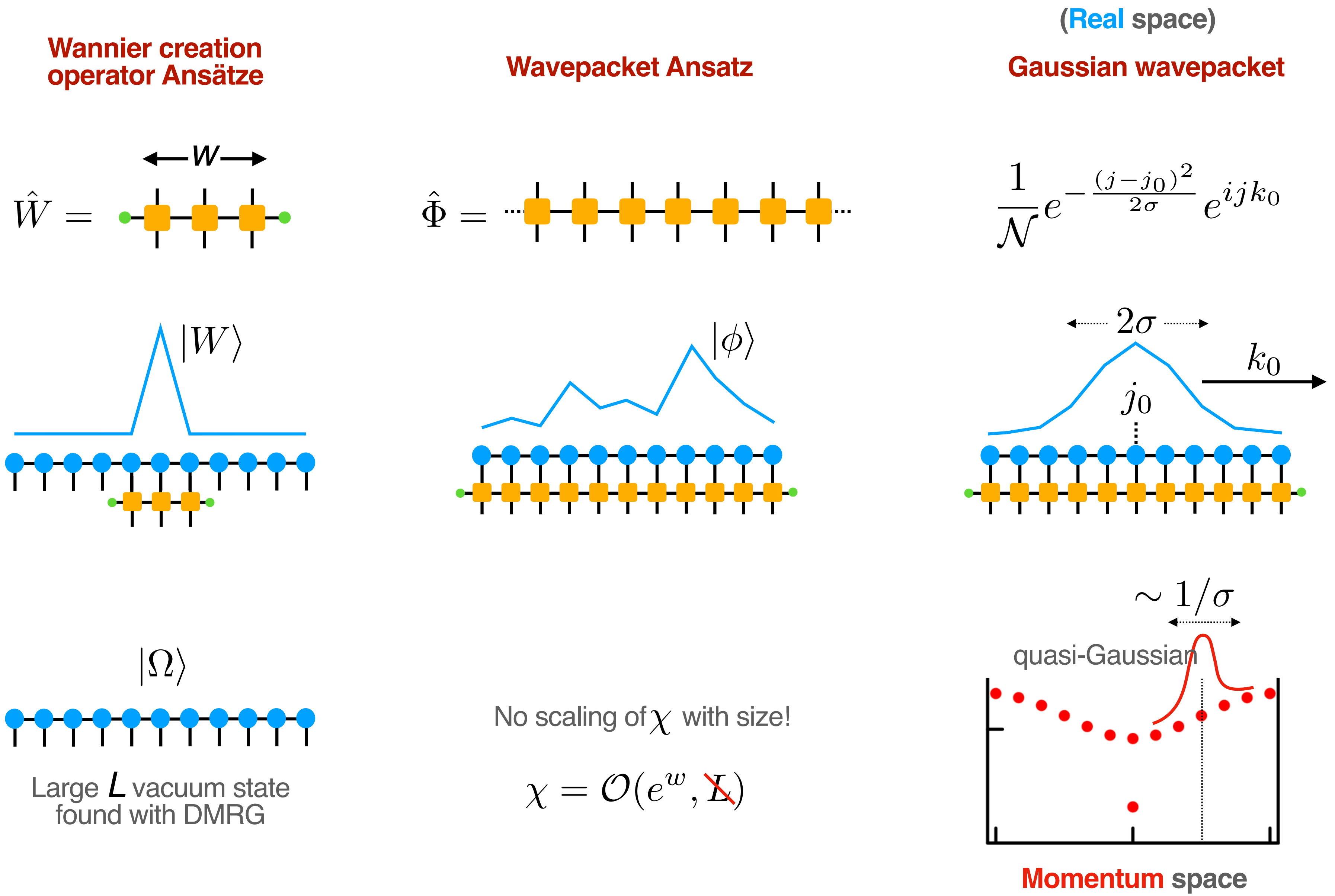
(**Real** space)

**Gaussian wavepacket**

$$\frac{1}{\mathcal{N}} e^{-\frac{(j-j_0)^2}{2\sigma}} e^{ijk_0}$$



# 5. Back to Wavepackets



# 6. Numerical simulations

**System parameters**

$$a = 1$$
$$L = 100$$
$$g^4 = 0.1$$

(Dirichlet OBC)

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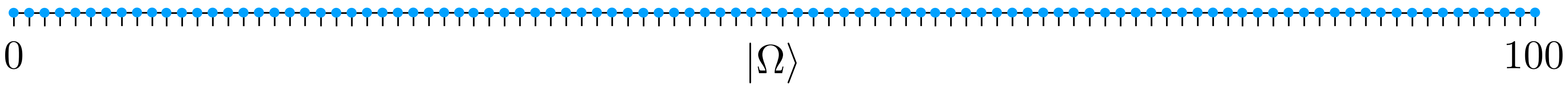

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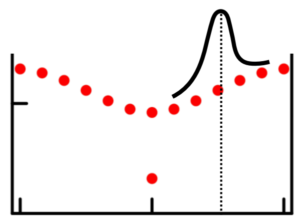

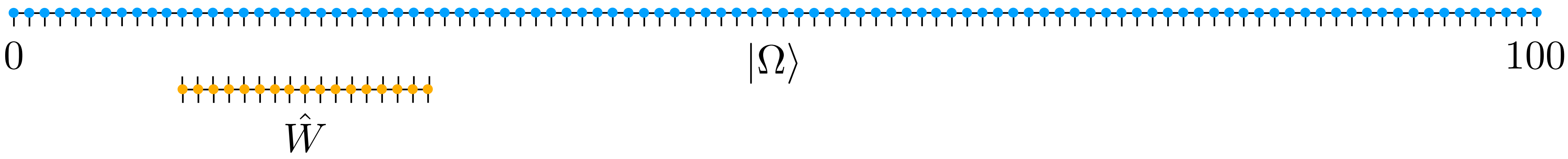

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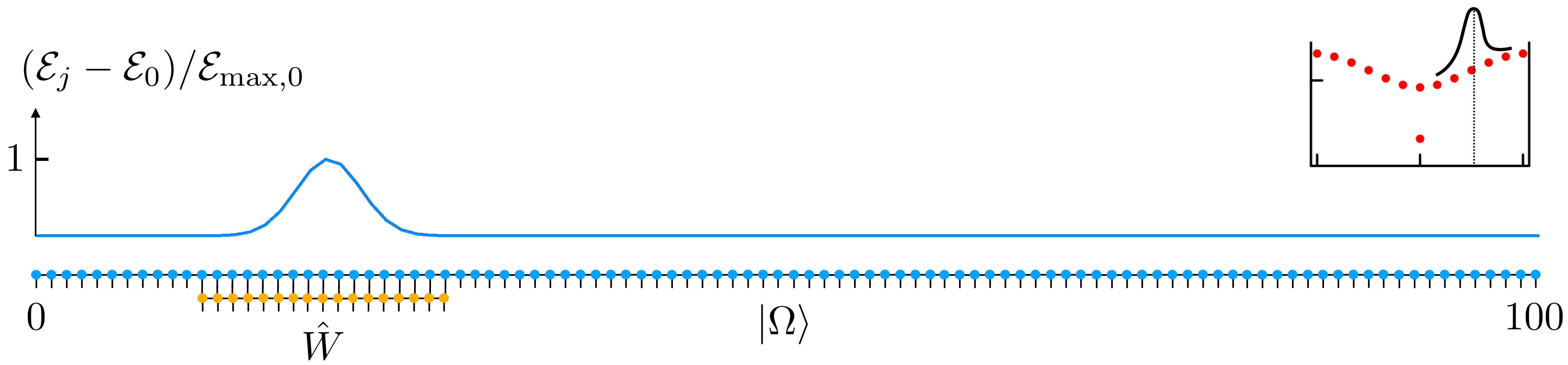

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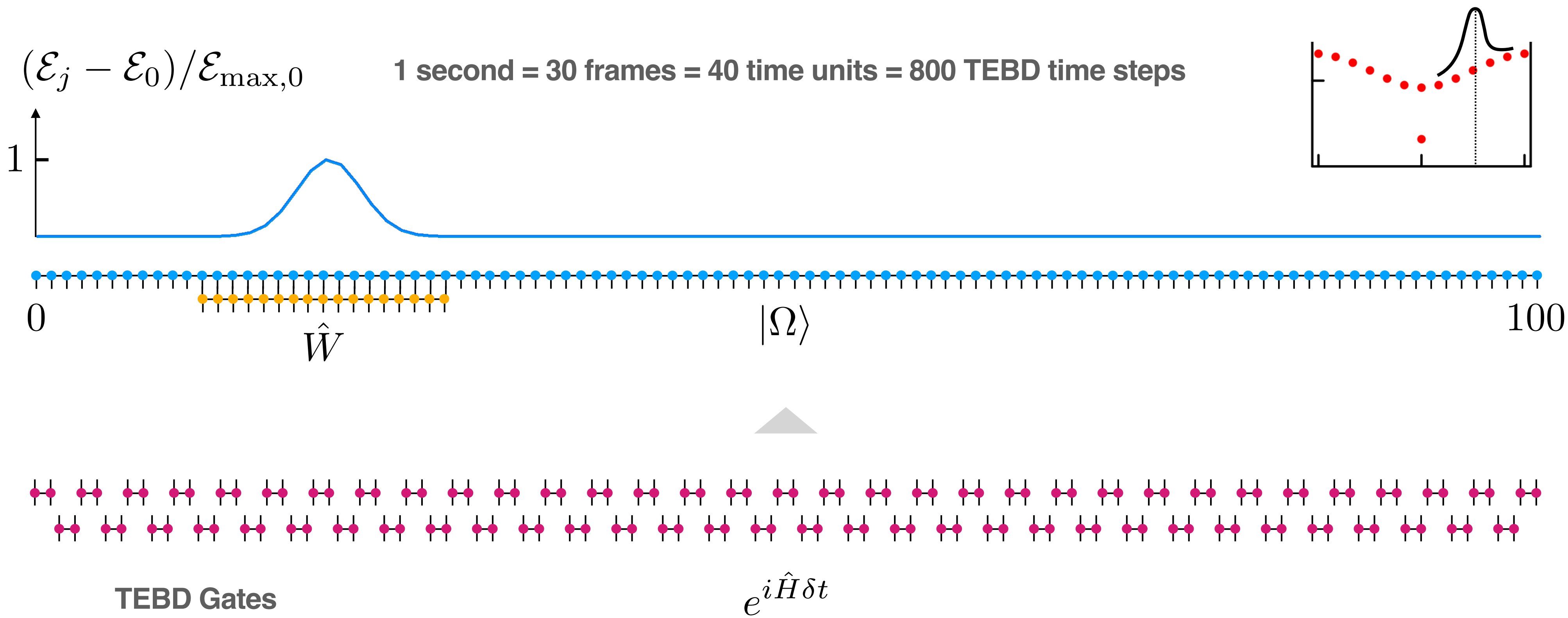

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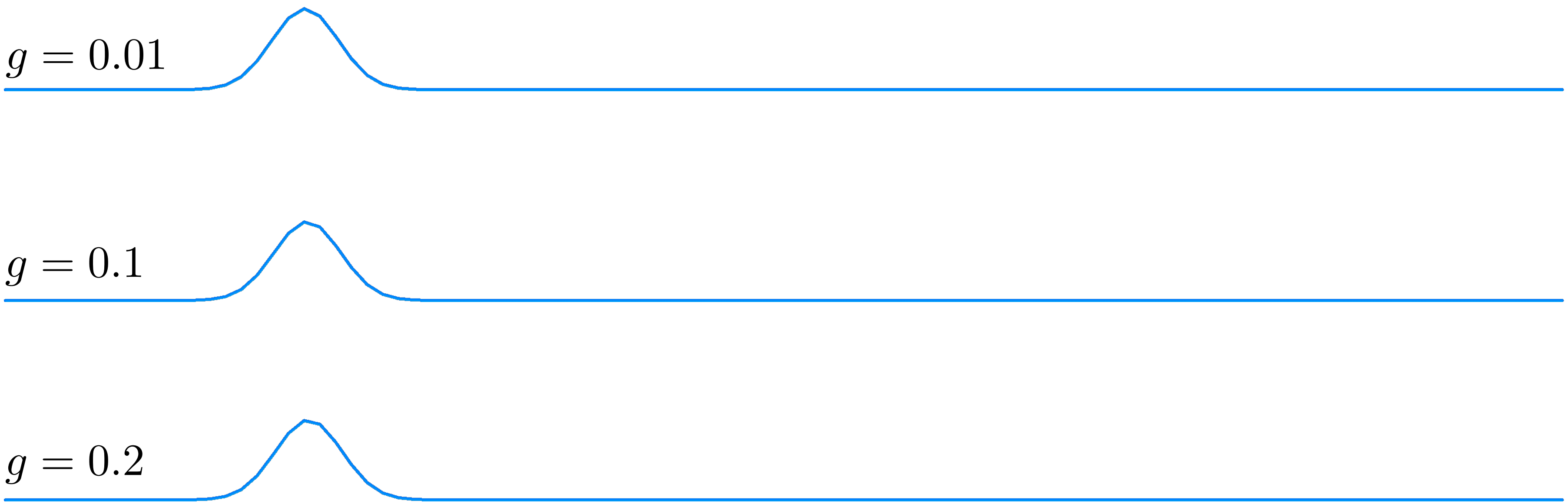

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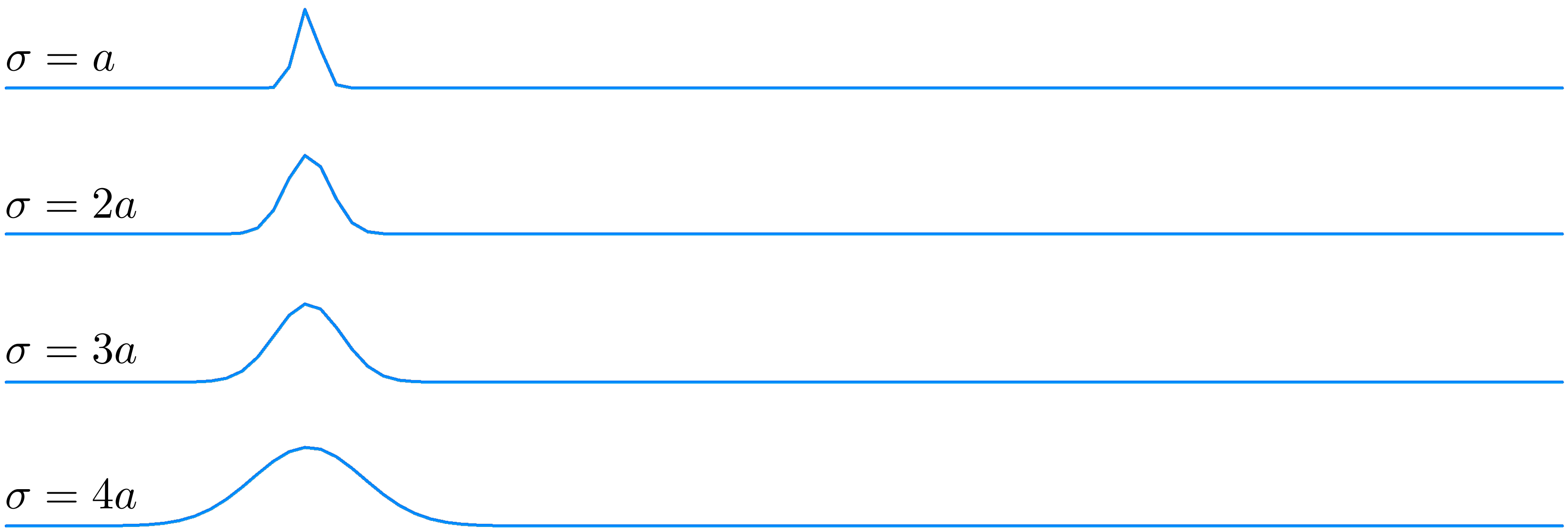

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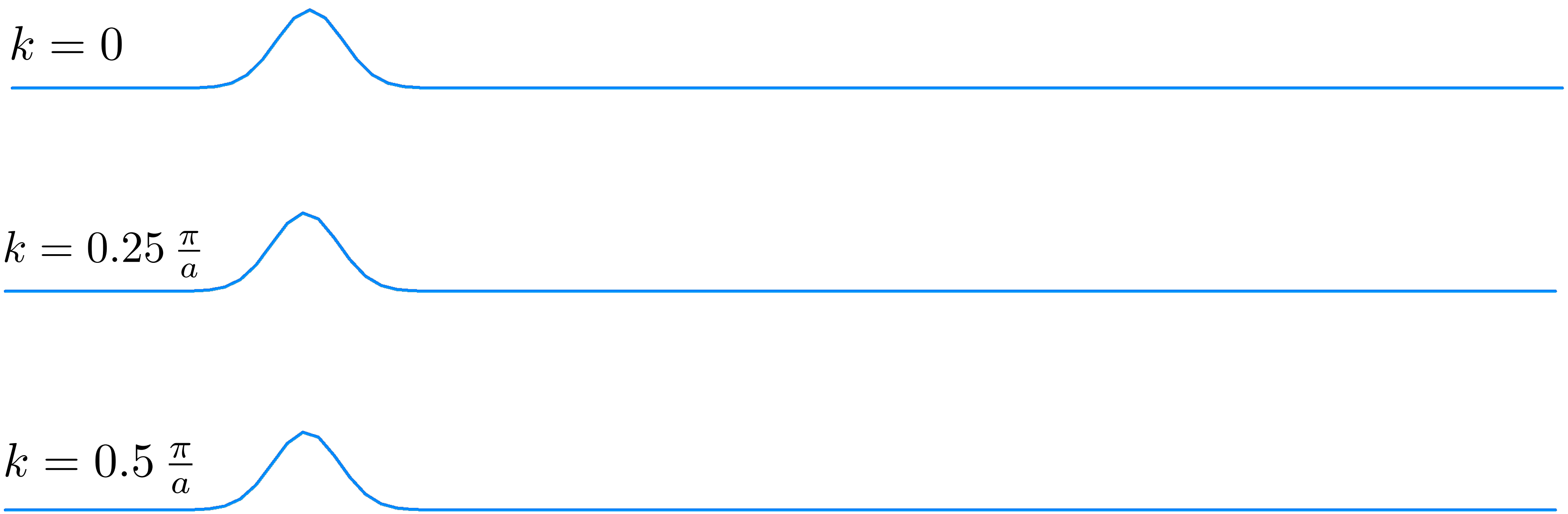

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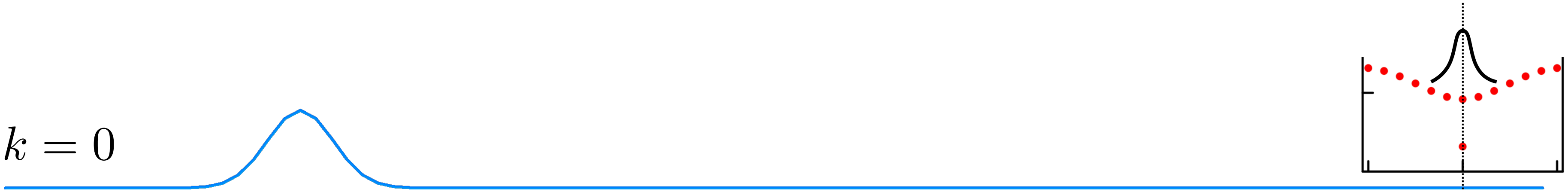
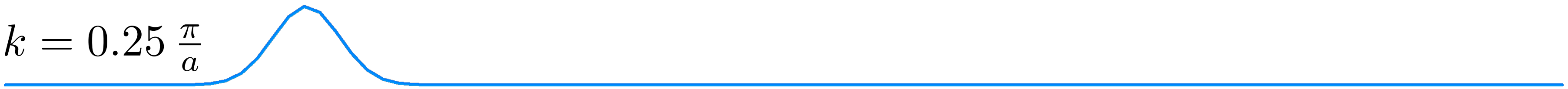


maximum dispersion (curvature), null group velocity

$k = 0.25\,\frac{\pi}{a}$



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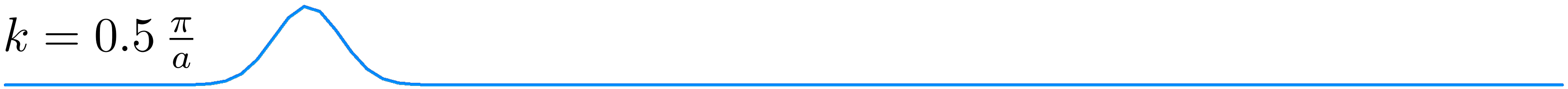

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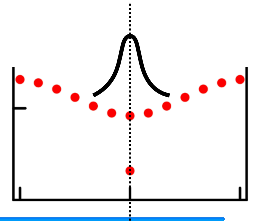

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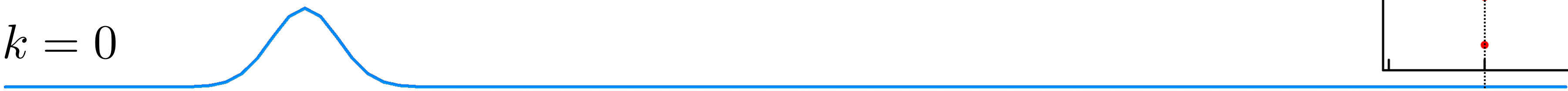

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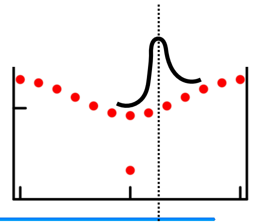

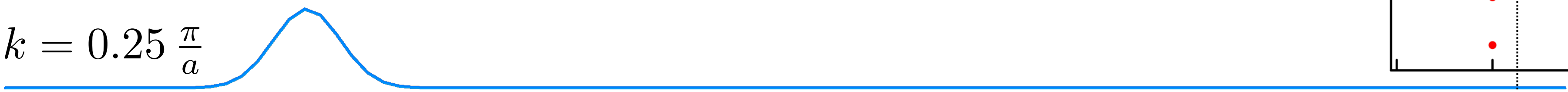

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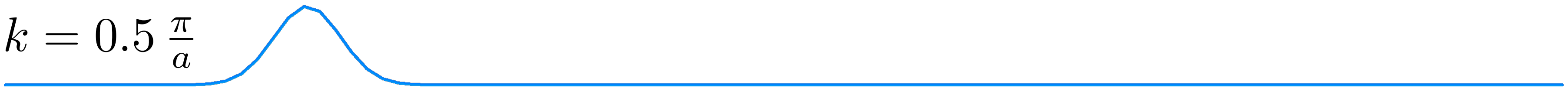

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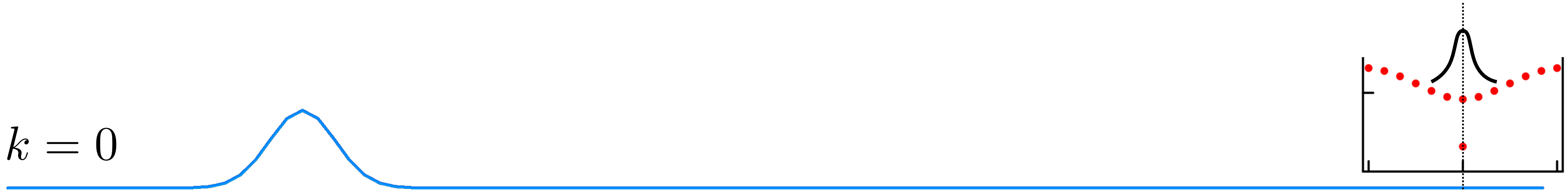


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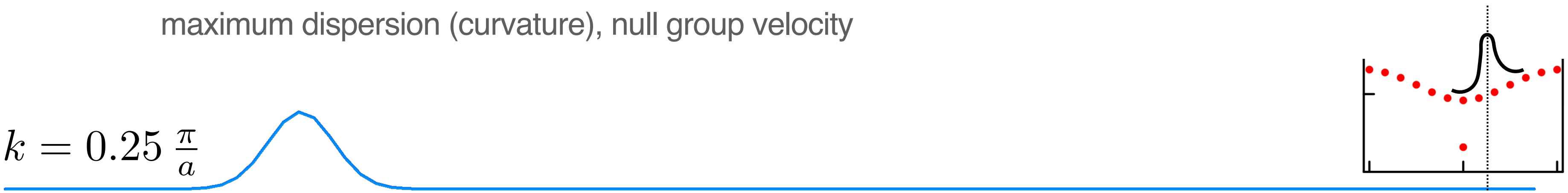


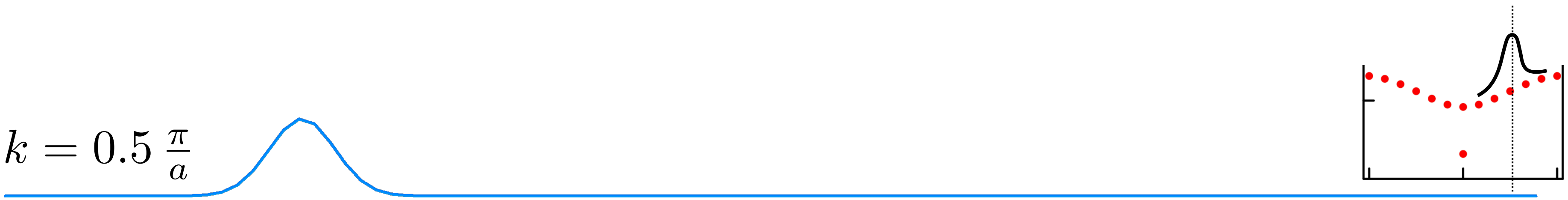


minimum dispersion (curvature), maximum group velocity  Closest case to a “real 3D photon”

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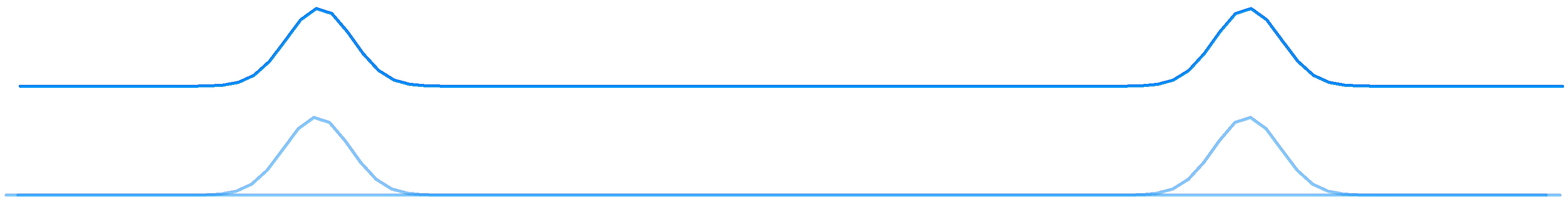
$\sigma = 3a \quad k = \pi/2a$



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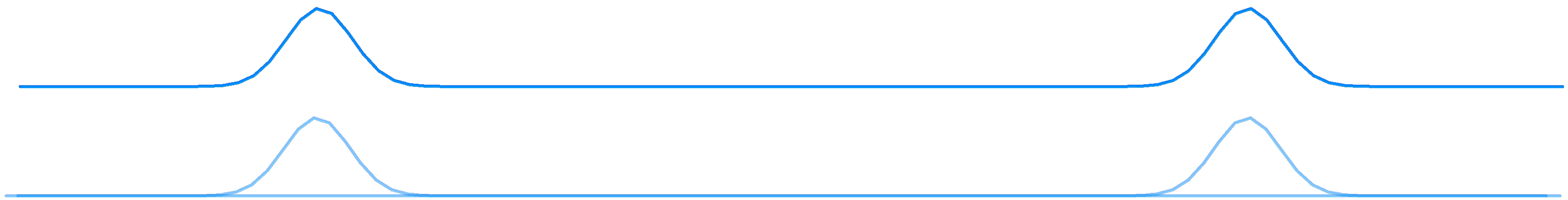

**Nothing happens: good, they are like photons, but bad, it's boring like this!**

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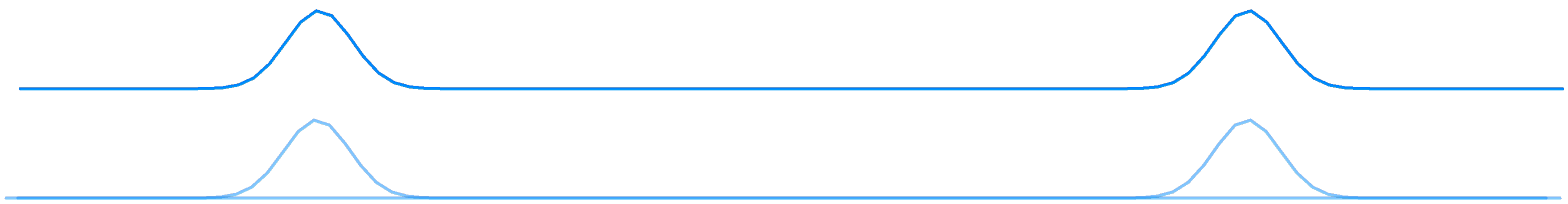

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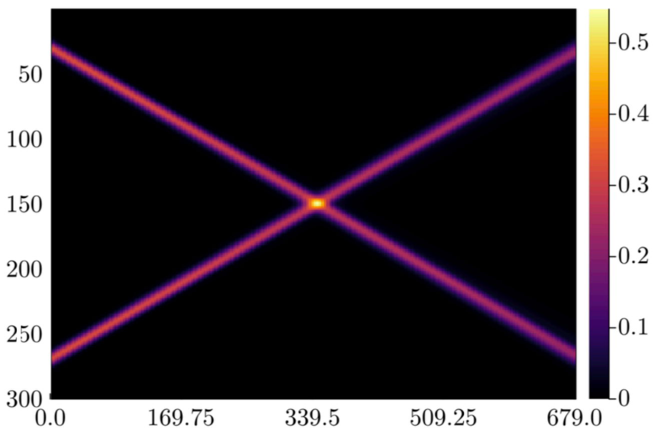
**It becomes less and less interacting as we approach the continuum limit:**

**Energy density**




$$L = 300$$

$$\sigma = 5a$$

# 6. Numerical simulations

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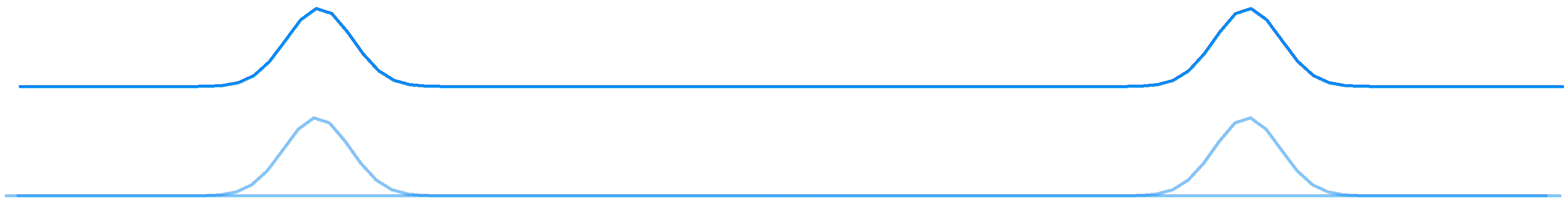

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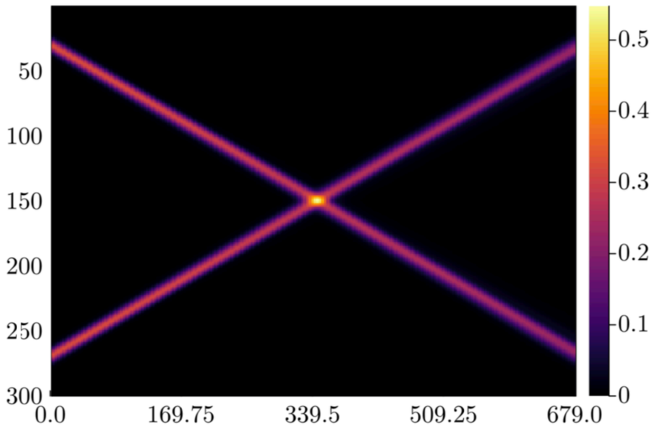
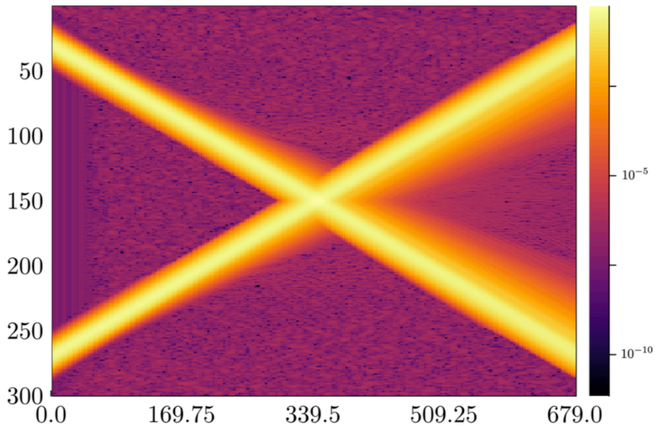



**Energy density in log**

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**Interaction effects:**
- **Lattice** artifacts
- **Finite size** effects
- **1D** lattice geometry

# 7. Conclusions

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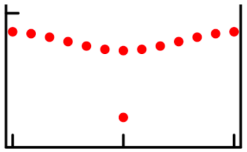
1 **Simultaneous exact diagonalization, finding the Bloch states**

$$[\hat{H}, \hat{T}] = 0$$

Computationally expensive step: **intermediate** system size



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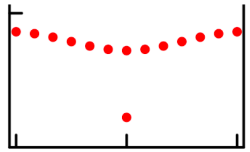

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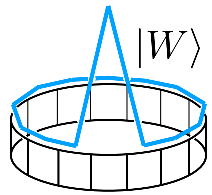
$$|W(\theta_k)\rangle = \frac{1}{\sqrt{L}} \sum_k e^{i\theta_k} |k\rangle$$



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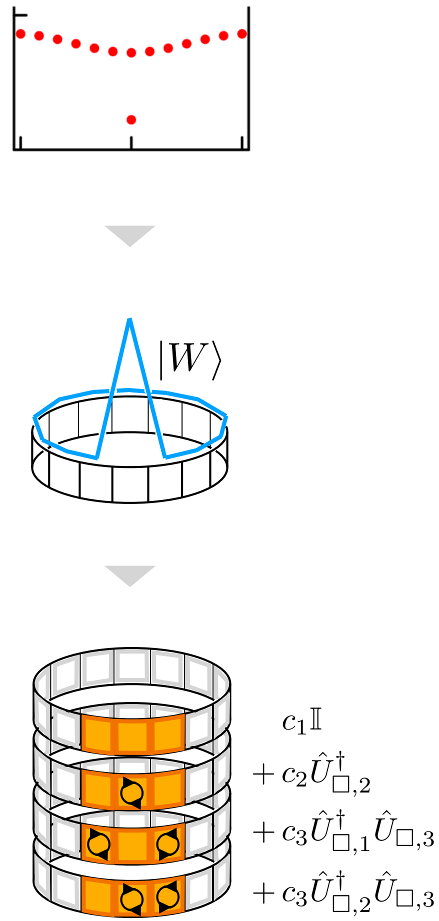
From the **interacting** vacuum



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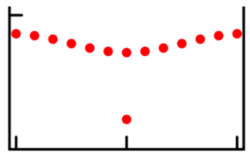

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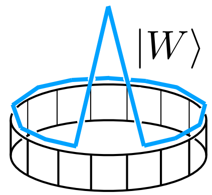


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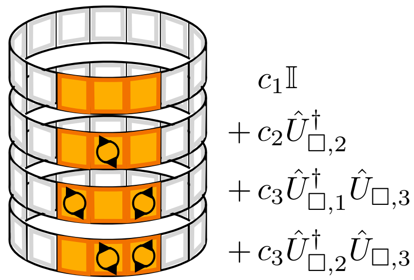

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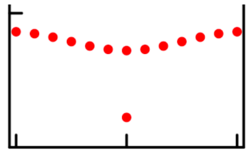

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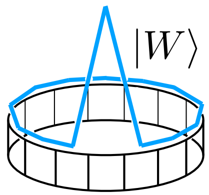


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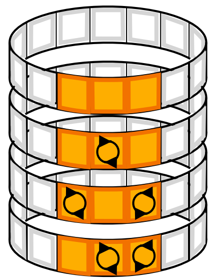


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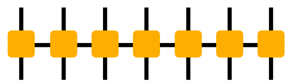

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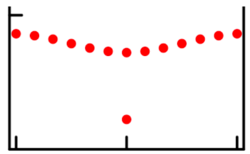

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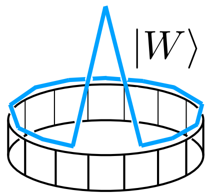


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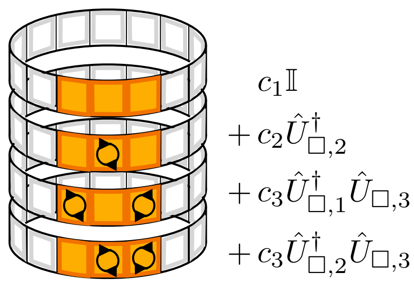


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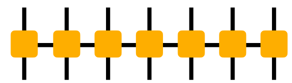

**Output: the MPO**

# 8. Outlook

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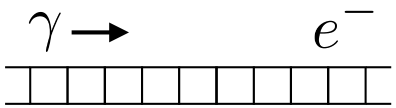
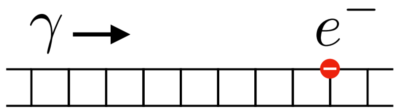
$$(1, 1),\ (\tfrac{3}{2}, 2),\ \ldots$$

**Higher spin representations**




**Adding matter d.o.f**

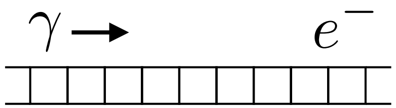



**Background charge**

# 8. Outlook

$$(1,1),\ (\tfrac{3}{2},2),\ \ldots$$

**Higher spin representations**

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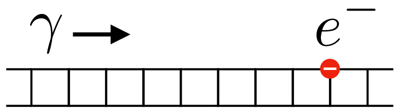


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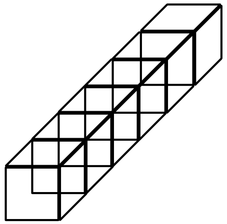
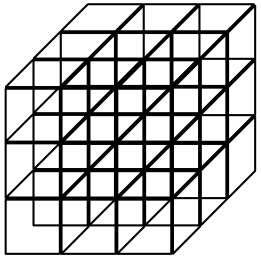


**Different geometries**



**Higher dimension**

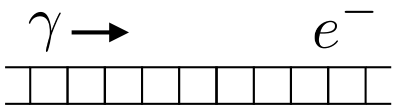
$$\begin{pmatrix} \alpha_1 & \beta_2 & & & & 0 \\ \beta_2 & \alpha_2 & \beta_3 & & & \\ & \beta_3 & \alpha_3 & \ddots & & \\ & & \ddots & \ddots & \beta_{m-1} & \\ & & & \beta_{m-1} & \alpha_{m-1} & \beta_m \\ 0 & & & & \beta_m & \alpha_m \end{pmatrix}$$

**Try to overcome exact diagonalization**

# 8. Outlook

$$(1,1),\ (\tfrac{3}{2},2),\ \ldots$$

**Higher spin representations**

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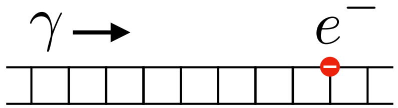
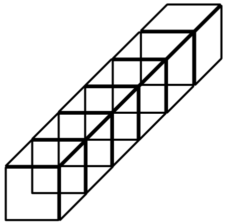



**Background charge**



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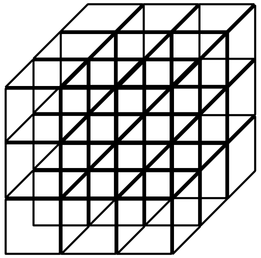

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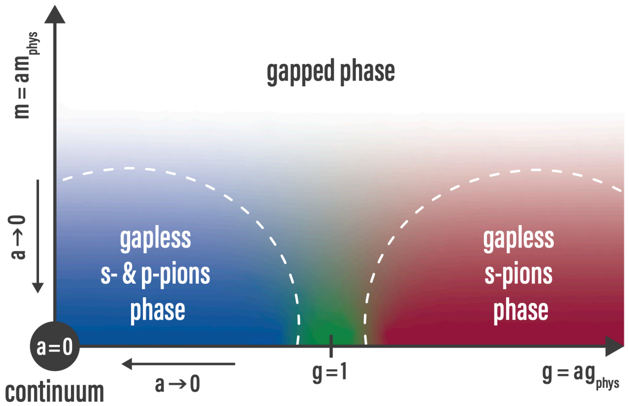
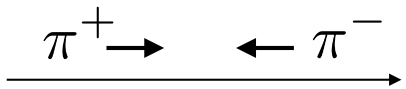
**Try to overcome exact diagonalization**




arXiv:2308.04488

**Last but not least**

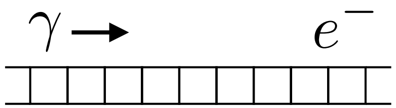
moving to **1D QCD with two flavors**, simulating **pion-pion scattering**



# 8. Outlook

$$(1,1),\ (\tfrac{3}{2},2),\ \ldots$$

**Higher spin representations**

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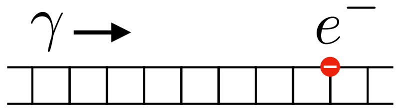


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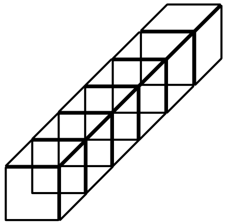

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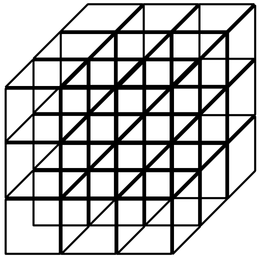

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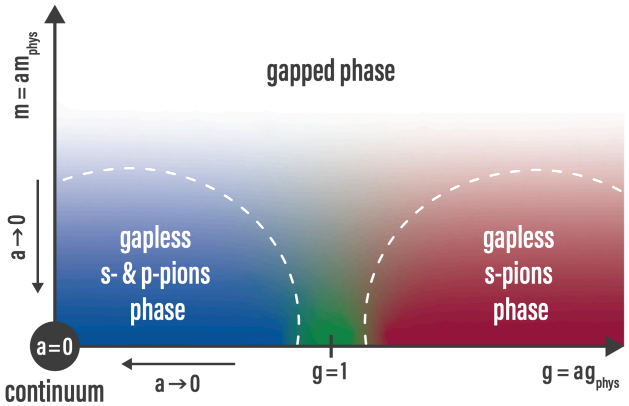


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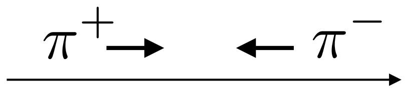



**Challenge**: $d = 54$ local dimension

Università degli Studi di Padova

Dipartimento di Fisica e Astronomia Galileo Galilei

QUANTUM Information and Matter

INFN PADOVA

PASQuanS2

QUANTUM FLAGSHIP

QUANTERA

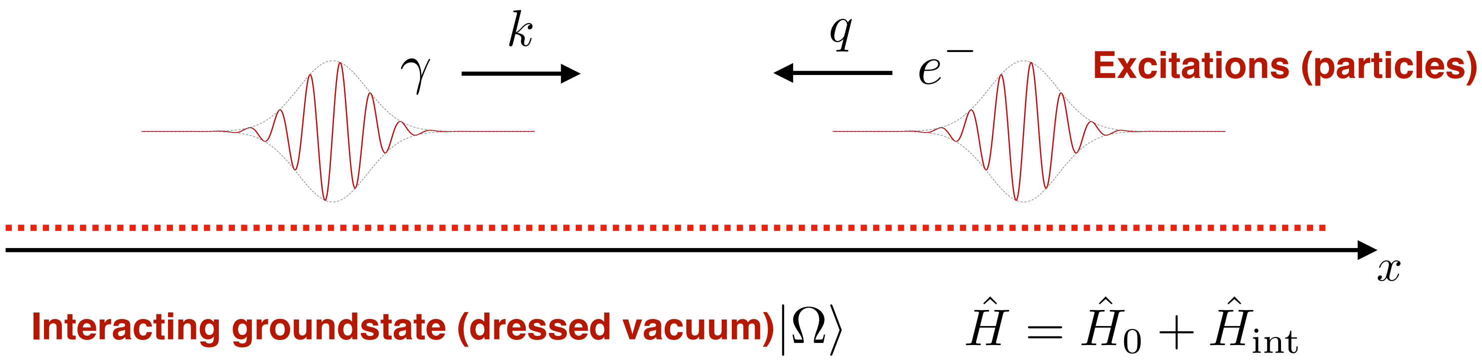
ICSC Centro Nazionale di Ricerca in HPC, Big Data and Quantum Computing

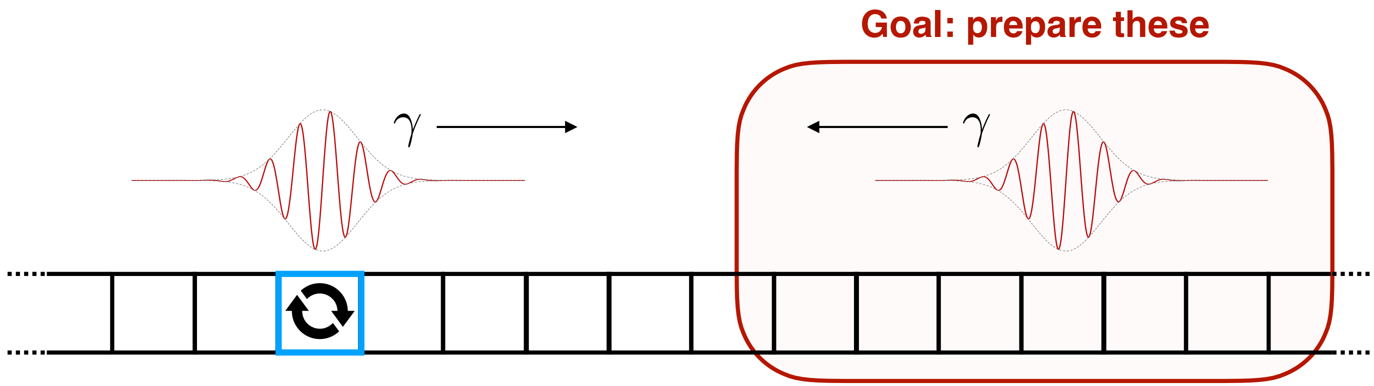
QUANTUM COMPUTING AND SIMULATION CENTER

QuantHEP

T-NiSQ Tensor Networks in Simulation of Quantum Matter

# Introduction



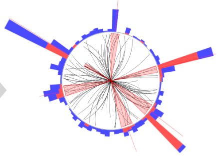




**Toy Model: (quasi)-1D Hamiltonian Lattice Pure Quantum Electro Dynamics on Ladder Geometries**

- (quasi)-1D — first step
- Hamiltonian — quantum tech
- Lattice — computable
- Pure — first step (still interacting)
- Quantum — quantum tech
- Electro — first step
- Dynamics — Scattering
- on Ladder Geometries — photon propagation

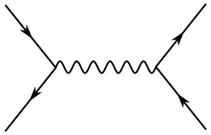
# Motivation: problems and solutions

Quantum Physics



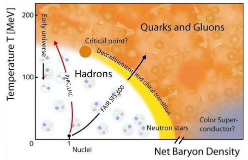
**HEP**

assumptions:



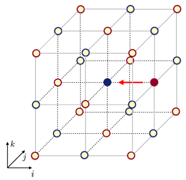
**Analytical**

QCD ?

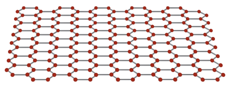



**Some problems remain intractable...**

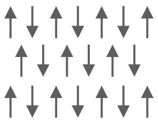
discretization:



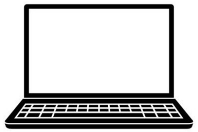
**LGT**



**CM**



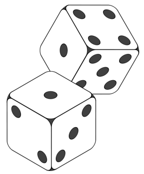
**Models**



**Numerical**

**problem:**

$$\dim \mathcal{H} = \mathcal{O}(d^L)$$



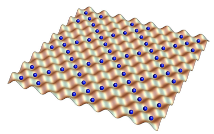
**Monte Carlo**

**sign problem...**

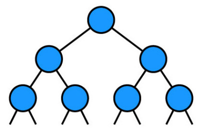
finite density (fermions)

**out-of-equilibrium** (scattering…)

**Quantum technologies**



**Q. Simulations**



**Tensor Networks**

# 1. Theoretical background

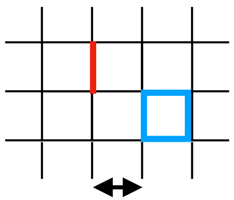
**(2+1)D pure EM Hamiltonian** $$H = \frac{1}{2}\int d^2x\,(E^2 + B^2)$$

Divergence of E → Vertex flux

Gauge invariance → Gauss' Law

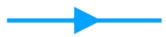
**1** (Quantum d.o.f survive!)

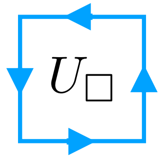
Space **Discretization**

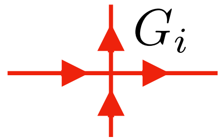


Spacing $a$

| Electric | Wilson line | Wilson loop | Vertex flux | Gauss' Law |
| --- | --- | --- | --- | --- |
| $E_l$ | $e^{-iA_l}$ | $U_\square$ | $G_i$ | $G_i = 0$ |







**2** **Canonical quantization**

$$[\hat{E}_l, \hat{U}_m] = \delta_{lm}\hat{U}_l \qquad \hat{U}_\square \qquad \hat{G}_i \qquad \hat{G}_i|\psi\rangle = 0$$

$\mathcal{H}$ Hilbert space — Electric op. — Comparator — Plaquette operator — Gauge generator — Physical states

**Kogut-Susskind Hamiltonian**

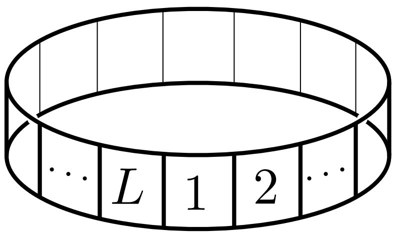
$$\hat{H} = \frac{g^2}{2a}\sum_l \hat{E}_l^2 + \frac{1}{2ag^2}\sum_\square\left(\hat{U}_\square + \text{H.c.}\right)$$

$$[\hat{G}, \hat{H}] = 0$$

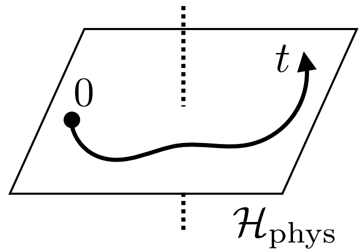
Coupling $g^2 = e^2 a$

Choose **geometry**, **topology**

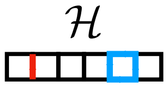
Size $L$ (# plaquettes)

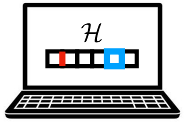


Physical Hilbert space



# 1. Theoretical background

To simulate 


We want $\mathcal{H}$ **finite**-dimensional!

**U(1) Lattice QED** → **U(1) Quantum Link Model (QLM)**

$\forall$ link Hilbert space $\mathcal{H}_{\text{link}}$ → $\text{SU}(2)$ irreducible representations $s \in \mathbb{N}/2$

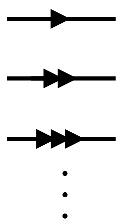
**Operators**

$\hat{E}$ → $\hat{S}^z$

$\hat{U}$ → $\hat{S}^+ / s$

$\hat{U}^\dagger$ → $\hat{S}^- / s$

Arrow notation



$s_z = 1/2$

$s_z = 1$

$s_z = 3/2$

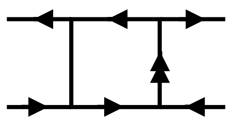
$\vdots$

$\sigma(\hat{E})$ → $\sigma(\hat{S}^z) = \{-s, \dots, s\}$ **finite!** ✓

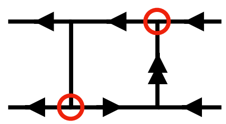
**CCR** $[\hat{E}, \hat{U}] = \hat{U}$ → $[\hat{S}^z, \hat{S}^+] = \hat{S}^+$ ✓

**Unitarity** $\hat{U}\hat{U}^\dagger = \mathbb{I}$ → $\hat{S}^+\hat{S}^- / s^2 \to \mathbb{I}$ ✓ **Kogut - Susskind limit** $s \to \infty$

**Gauge generator** $\hat{G}$ → $\hat{S}_i^z + \hat{S}_k^z - \hat{S}_j^z$



Allowed ✓

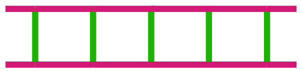


Not allowed ✖

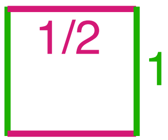
# 2. The pure Lattice QED on ladder geometry

**Spin representation**

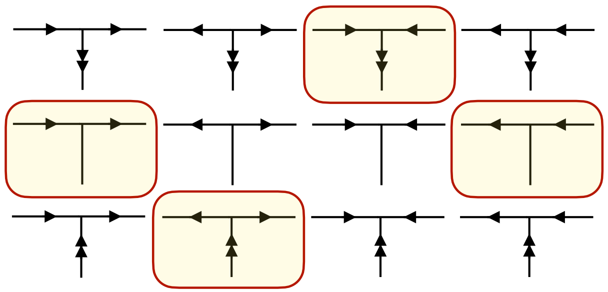
Translation and reflection inv.



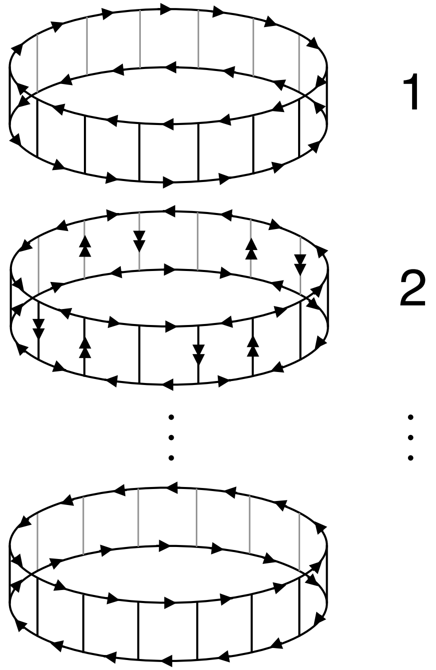
**Lowest** spin irrep assignments which admits gauge inv. configs:




**Gauss' Law on vertices**



***L* plaquettes ladder in PBC**

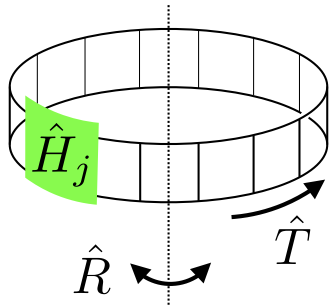



$$\dim \mathcal{H} = 2^L + 2$$

Exp scaling

**Construct operators**

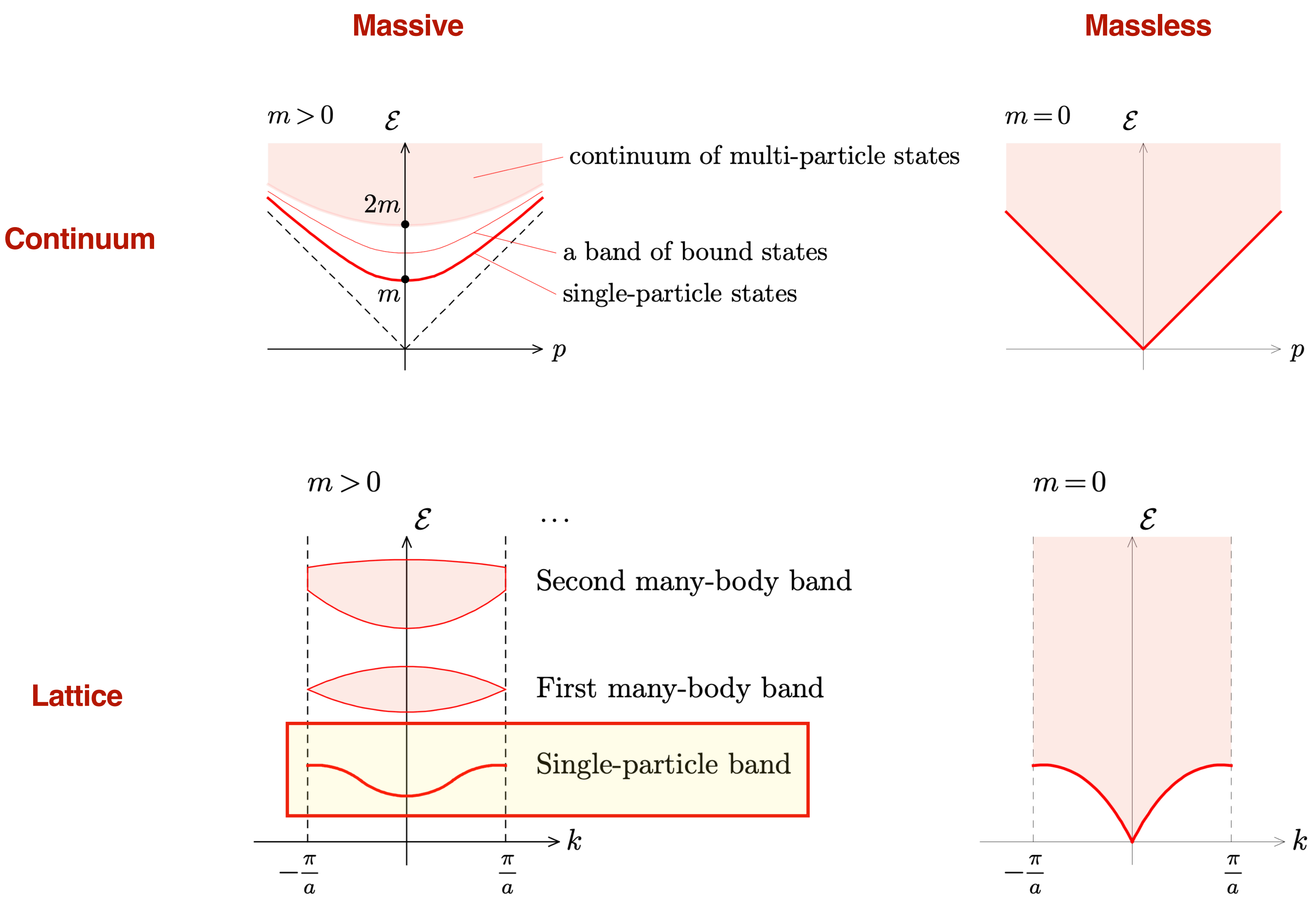
**Computational basis**




$$[\hat{H}, \hat{T}] = 0$$

**Dispersion relation!**

# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$



# 2. The pure Lattice QED on ladder geometry$(\frac{1}{2}, 1)$

**Assuming**

$[\hat{H}, \hat{T}] = 0$ **Translational invar. (PBC)**

$L = 13$ **Intermediate system size**

1/2 □ 1 **Lowest spin rep.**

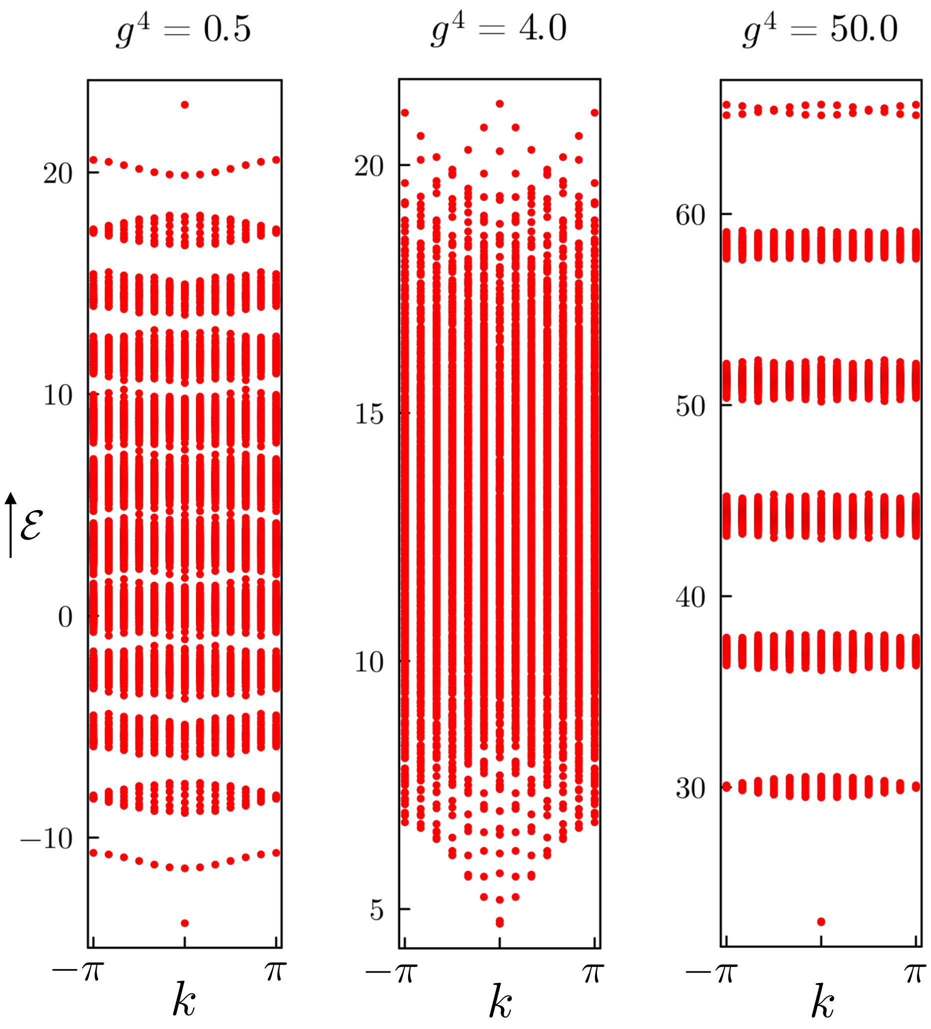
$a = 1$ **Unit lattice spacing**

( varying the coupling $g$ )

**Simultaneous *exact* diagonalization (computationally difficult step)**

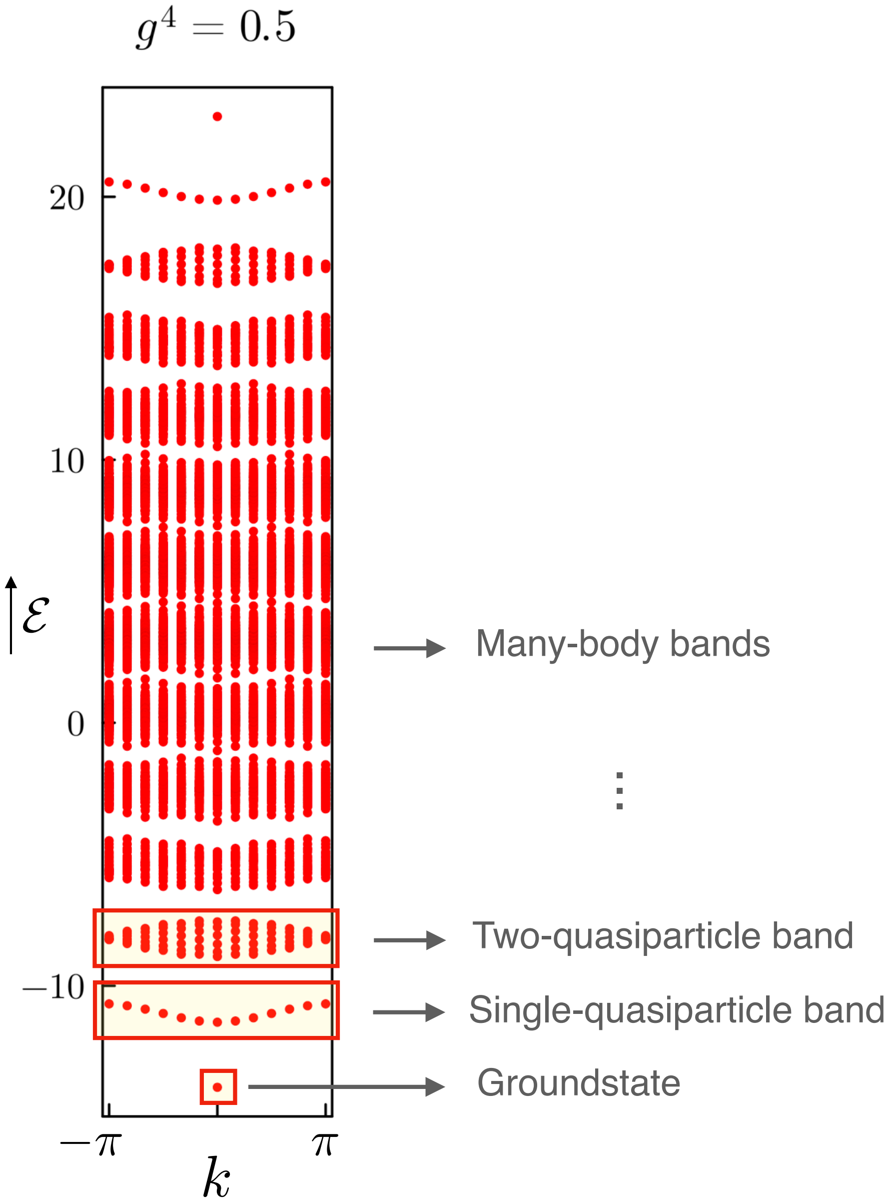
$\dim \mathcal{H} = 8194$

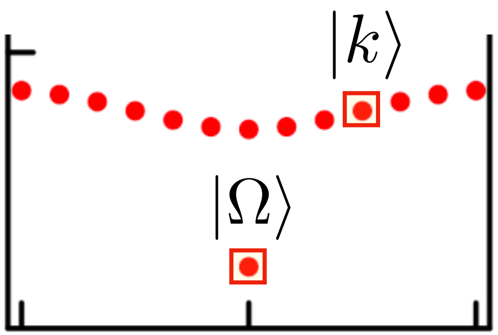
**Dispersion relation in our (interacting!) model:**




Gapped phase (massive photon) | Critical point (massless photon) | Gapped SSB phase (degenerate g.s.)

# 2. The pure Lattice QED on ladder geometry $(\frac{1}{2}, 1)$




**Program**

$$|\phi\rangle = \sum a_k |k\rangle$$

**Single-particle state (Bloch basis)**

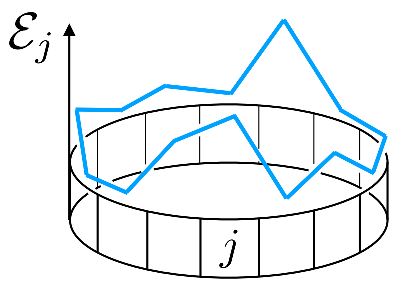
**Wannier Functions (real-space basis)**

**Wave-packet state**

# 3. Construction of the wavepackets

**Wannier states**

$$|W(\theta_k)\rangle = \frac{1}{\sqrt{L}} \sum_k e^{i\theta_k} |k\rangle$$

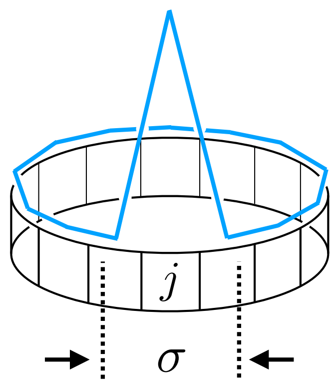



$\theta$ ▶ $\bar{\theta}$

Minimize $\sigma^2$

**Maximally localized Wannier states**

$$|W_j\rangle \equiv |W_j(\bar{\theta})\rangle$$



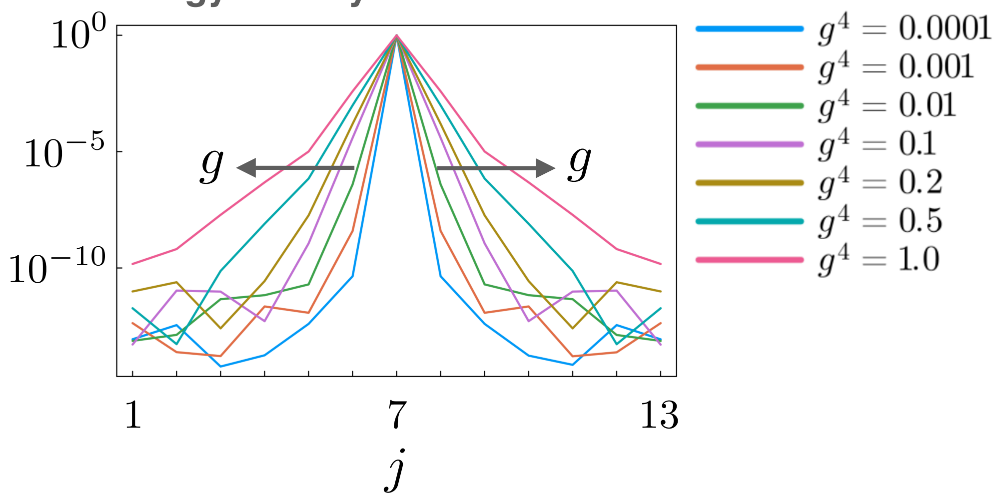

**Energy density**

$$\mathcal{E}_j[\phi] = \langle\phi|\hat{H}_j|\phi\rangle$$

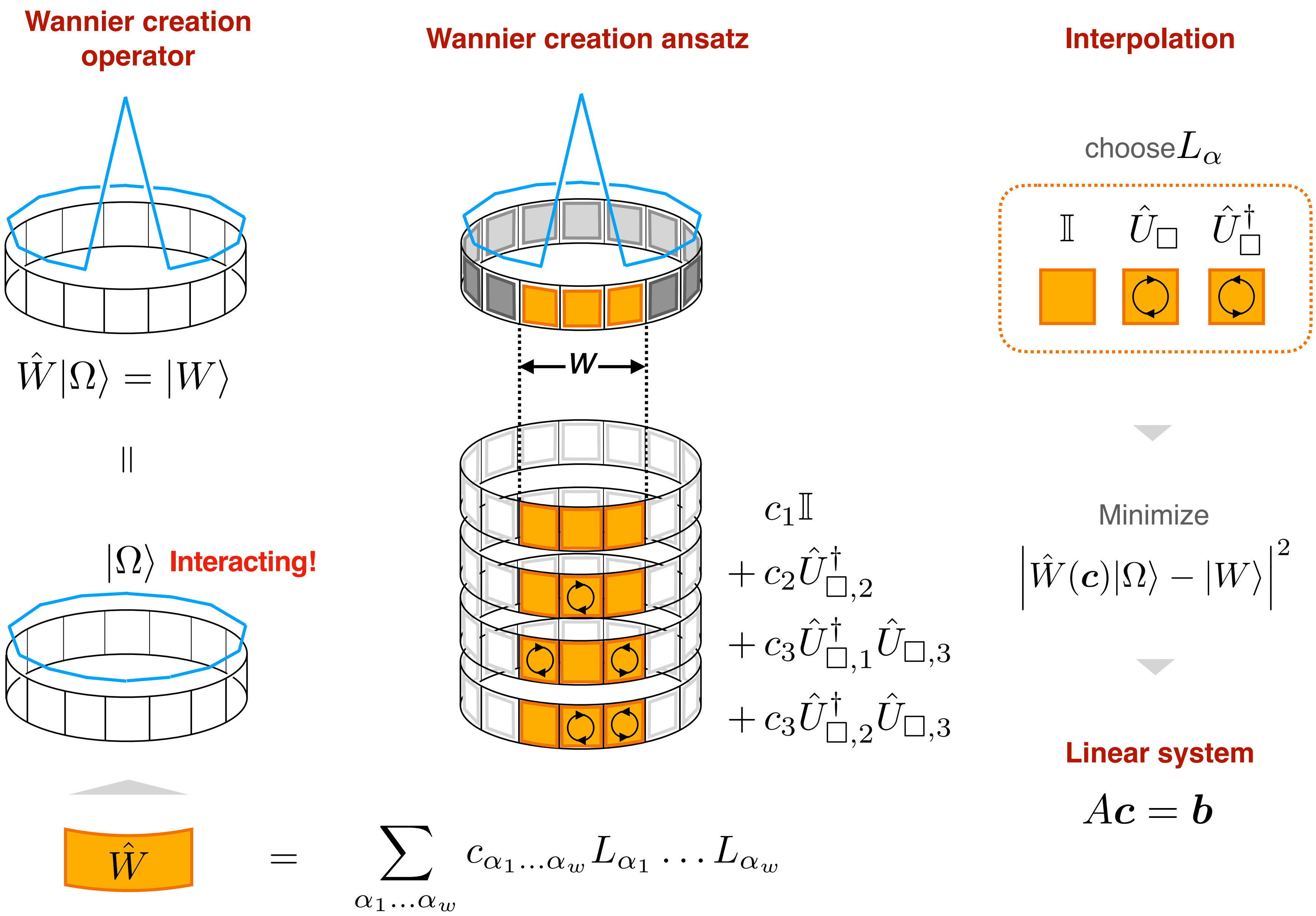
**Spread functional**

$$\sigma^2[\phi] \equiv \frac{\sum_j j^2 \mathcal{E}_j[\phi]}{\sum_j \mathcal{E}_j[\phi]} - \left(\frac{\sum_j j\mathcal{E}_j[\phi]}{\sum_j \mathcal{E}_j[\phi]}\right)^2$$

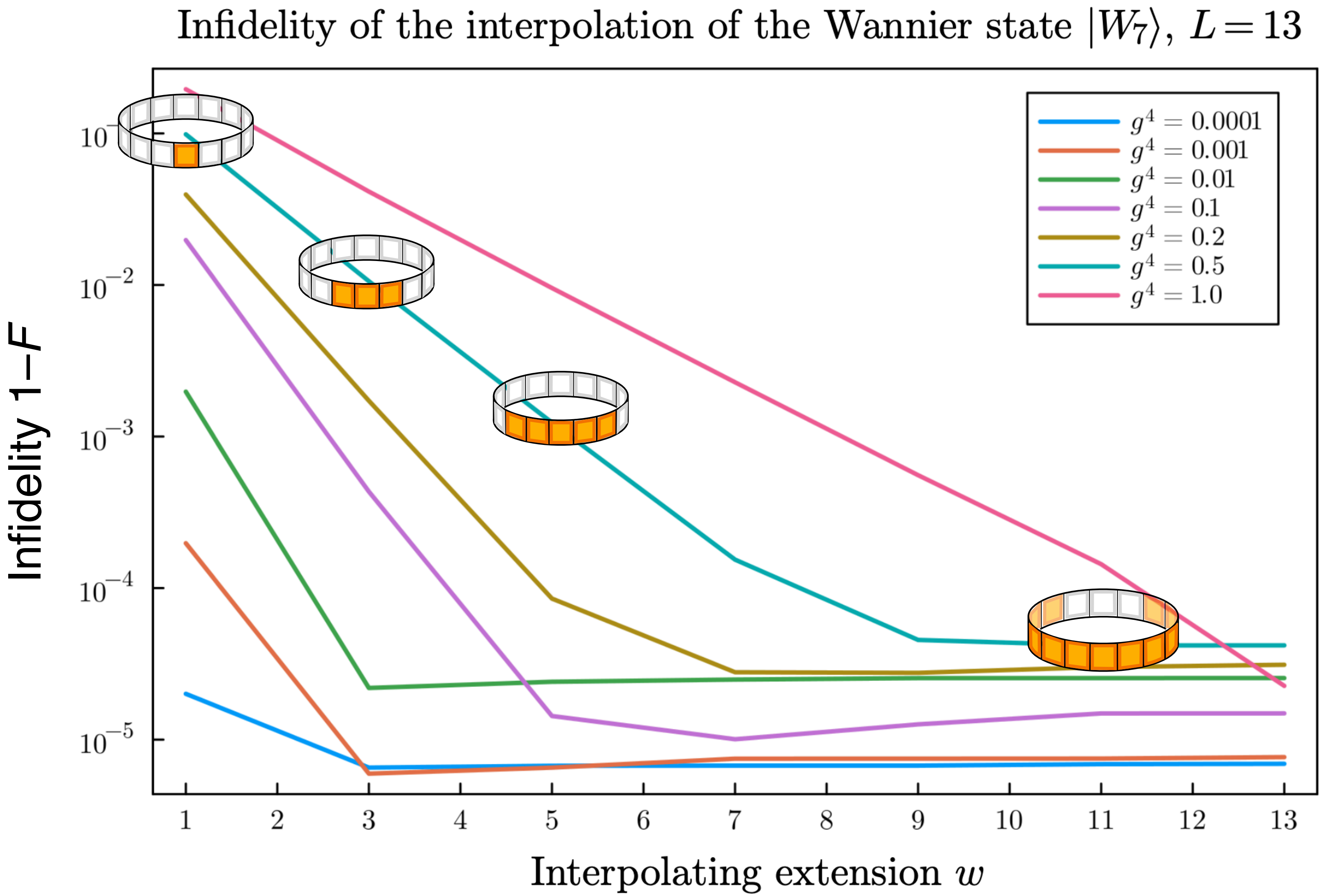
**Energy density after minimization**



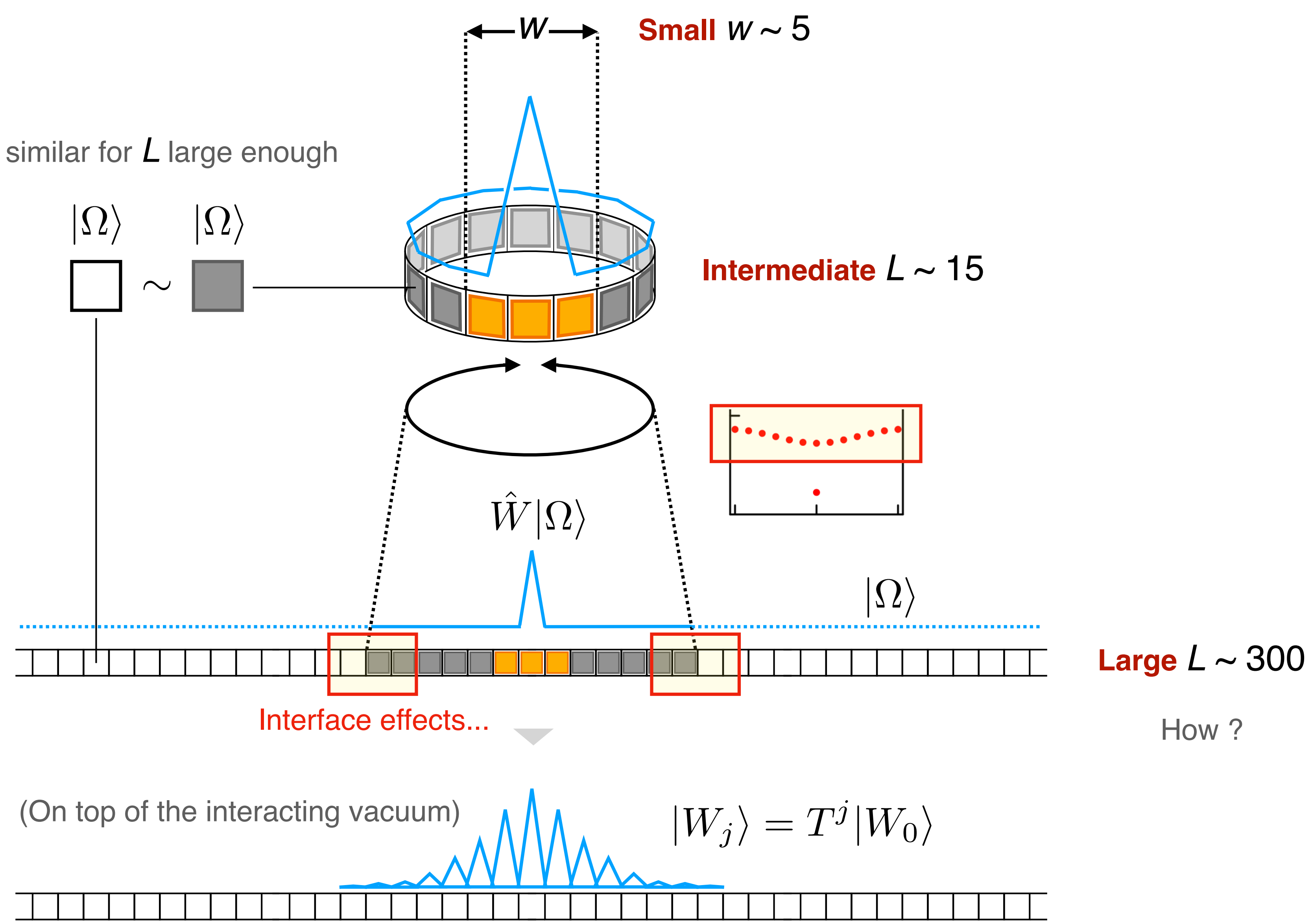
# 3. Construction of the wavepackets



# 3. Construction of the wavepackets



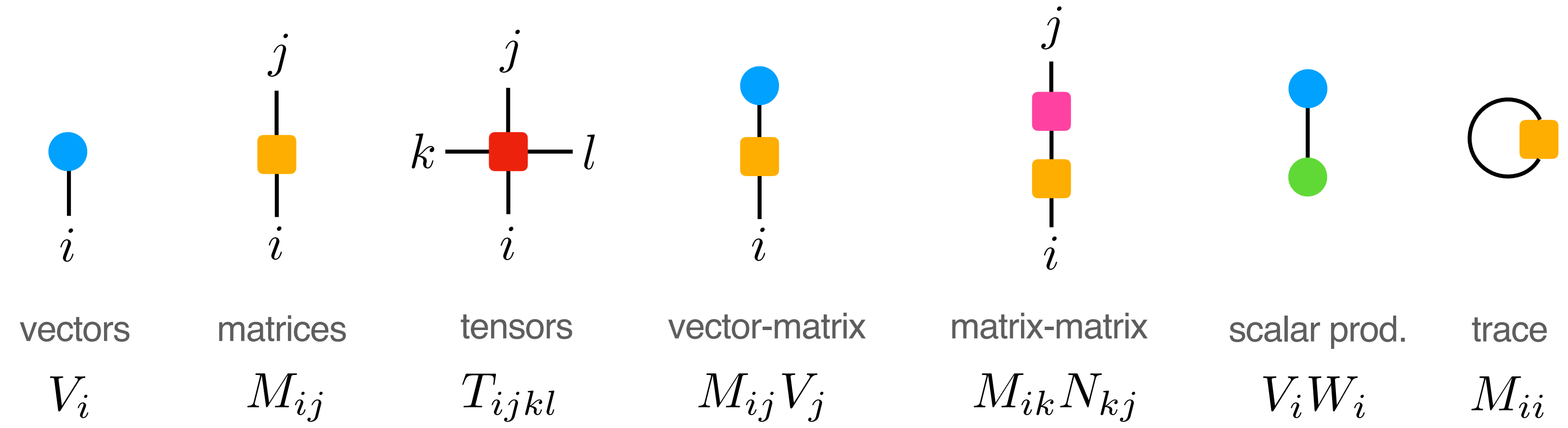
# 3. Construction of the wavepackets



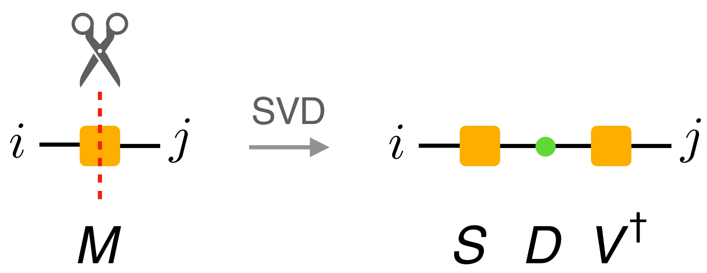

**Wannier function** → **building blocks** to construct any single-(quasi)particle state.

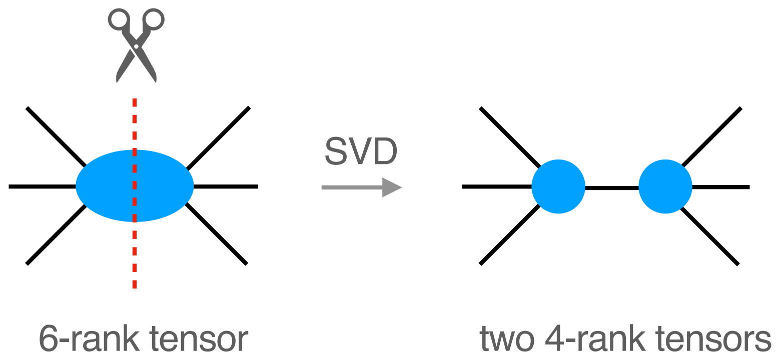
# 4. Tensor Network methods in a nutshell

**Tensor Networks notation:** each tensor *T* (node) has *n* indices *i* (links). Each index has a dimension *d* (size).




**Singular Value Decomposition (SVD)**



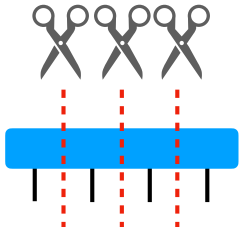



# 4. Tensor Network methods in a nutshell

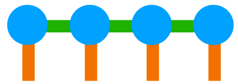
Physical links with **local dimension** $d$

Virtual **bond dimension** $\chi$

**Quantum MB states**

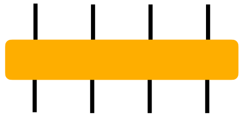


SVD

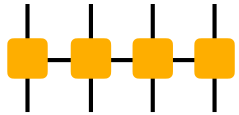


**M**atrix **P**roduct **S**tates

**Quantum MB Operators**



SVD



**M**atrix **P**roduct **O**perator

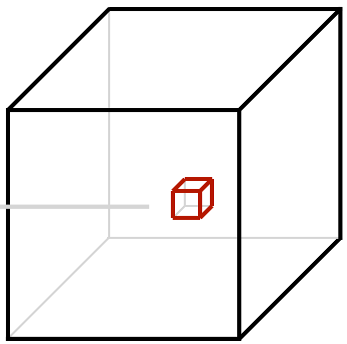
**Computational resources** $\mathcal{O}(d^L)$ → $\mathcal{O}(L)$ ( We can menage really high $L$ )

**Is this representation efficient?**

**Local Hamiltonian**

**Area law states**

**MPS with low** $\chi$



Hilbert space $\mathcal{H}$

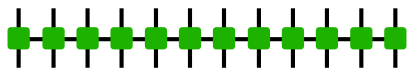
# 4. Tensor Network methods in a nutshell

**Useful MPS and MPO Algorithms**

Large systems
(with just a laptop!)

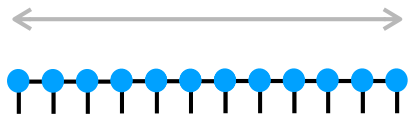
**Ground-state search** **D**ensity **M**atrix **R**enormalization **G**roup

$$L \gtrsim 100$$



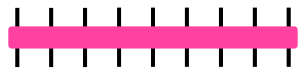
MPO Hamiltonian

DMRG



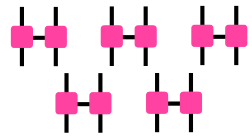
MPS ground-state $|\Omega\rangle$

**Time evolution for MPS** **T**ime **E**volving **B**lock **D**ecimation (…but also **TDVP**)



Time evolution operator

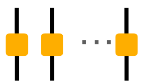
Suzuki-Trotter $\delta t$



Iteration of gates

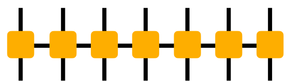
**Operator to MPO conversion**

$$\sum_\alpha \hat{L}_\alpha = \sum$$



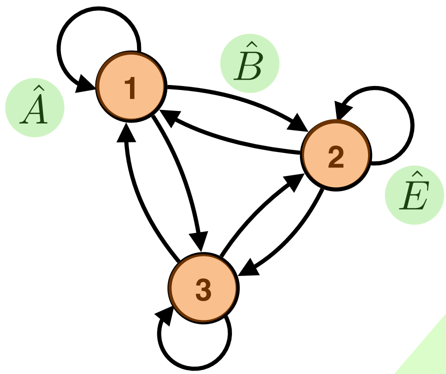
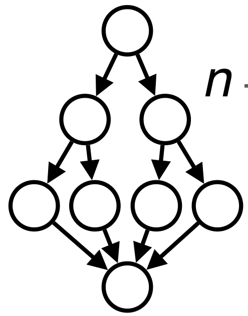
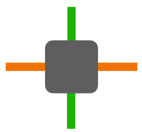
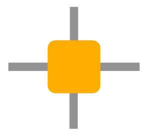
Sum of compositions of local operators

**Automata procedure**



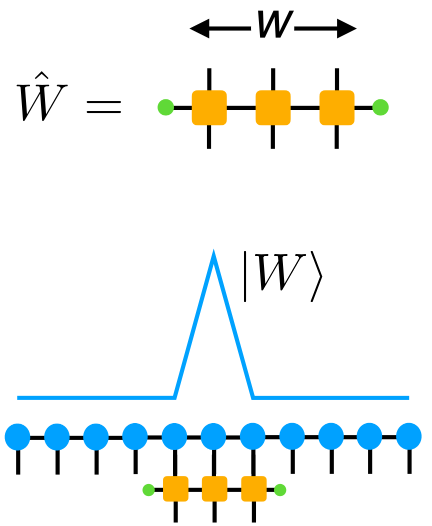
Matrix Product Operator

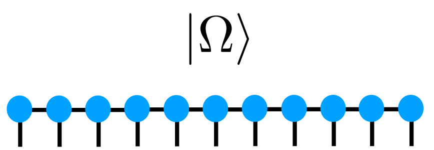
# 4. Tensor Network methods in a nutshell

| | **Automata procedure** | **Wannier creation operator** |
|---|---|---|
| 1 Operator picture | $\hat{O} = \sum_{\alpha} \hat{L}_{\alpha}$ | $\hat{W} = \sum_{\alpha_1 \ldots \alpha_w} c_{\alpha_1 \ldots \alpha_w} L_{\alpha_1} \ldots L_{\alpha_w}$ |
| 2 Automata picture |  |  |
| 3 Matrix picture | $\begin{pmatrix} \hat{A} & \hat{B} & \hat{C} \\ \hat{D} & \hat{E} & \hat{F} \\ \hat{G} & \hat{H} & \hat{I} \end{pmatrix}$, $\hat{C} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$ | $\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$ $\mathcal{O}(e^w)$ |
| 4 TN picture |  |  "Wannier Ansätze" |

# 5. Back to Wavepackets

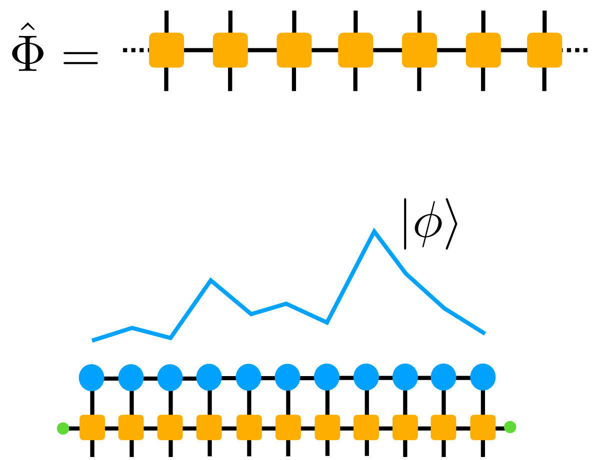
**Wannier creation operator Ansätze**




Large $L$ vacuum state found with DMRG

**Wavepacket Ansatz**



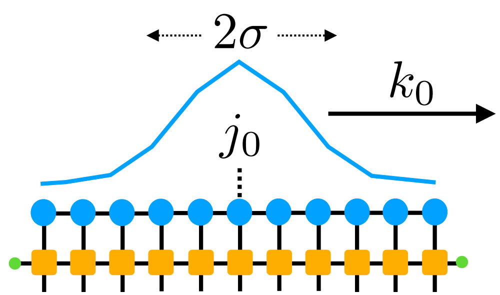

No scaling of $\chi$ with size!

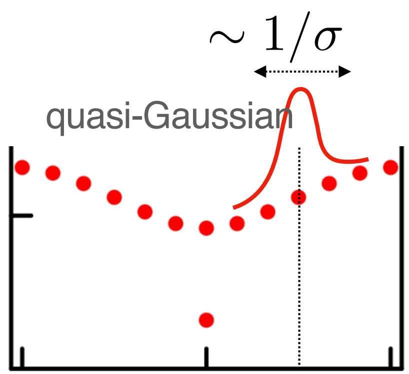
$$\chi = \mathcal{O}(e^{w}, \cancel{L})$$

(**Real** space)

**Gaussian wavepacket**

$$\frac{1}{\mathcal{N}} e^{-\frac{(j-j_0)^2}{2\sigma}} e^{ijk_0}$$




**Momentum space**

# 6. Numerical simulations

**System parameters**

$$a = 1$$
$$L = 100$$
$$g^4 = 0.1$$

(Dirichlet OBC)

**DMRG parameters**

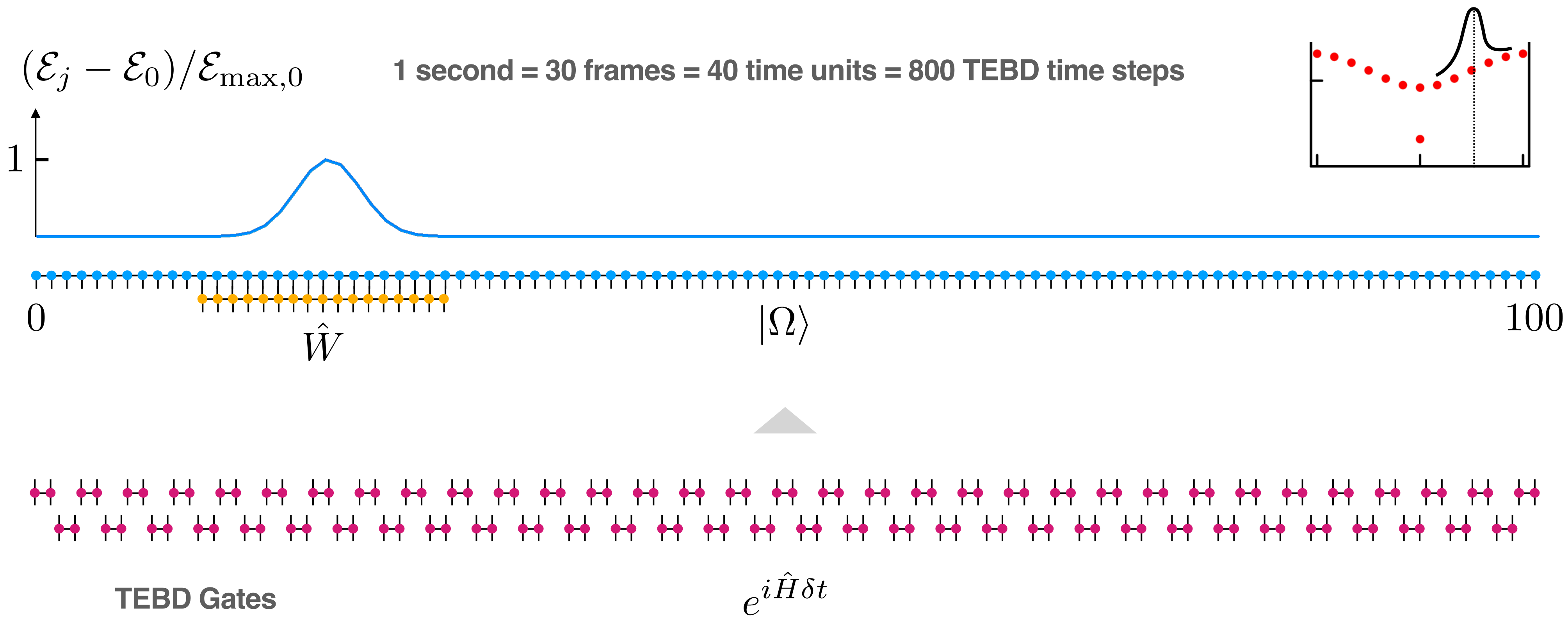
$$\chi_{\text{max}} = 200$$
$$n_{\text{sweeps}} = 50$$
$$\epsilon_{\text{SVD}} = 10^{-13}$$

**Wavepacket parameters**

$$\sigma = 3a$$
$$k = \pi/2a$$
$$j_0 = 20$$

**TEBD parameters**

$$\delta t = 0.05$$
$$\Delta t = 400$$
$$\epsilon_{\text{SVD}} = 10^{-10}$$
$$\chi_{\text{max}} = 50$$



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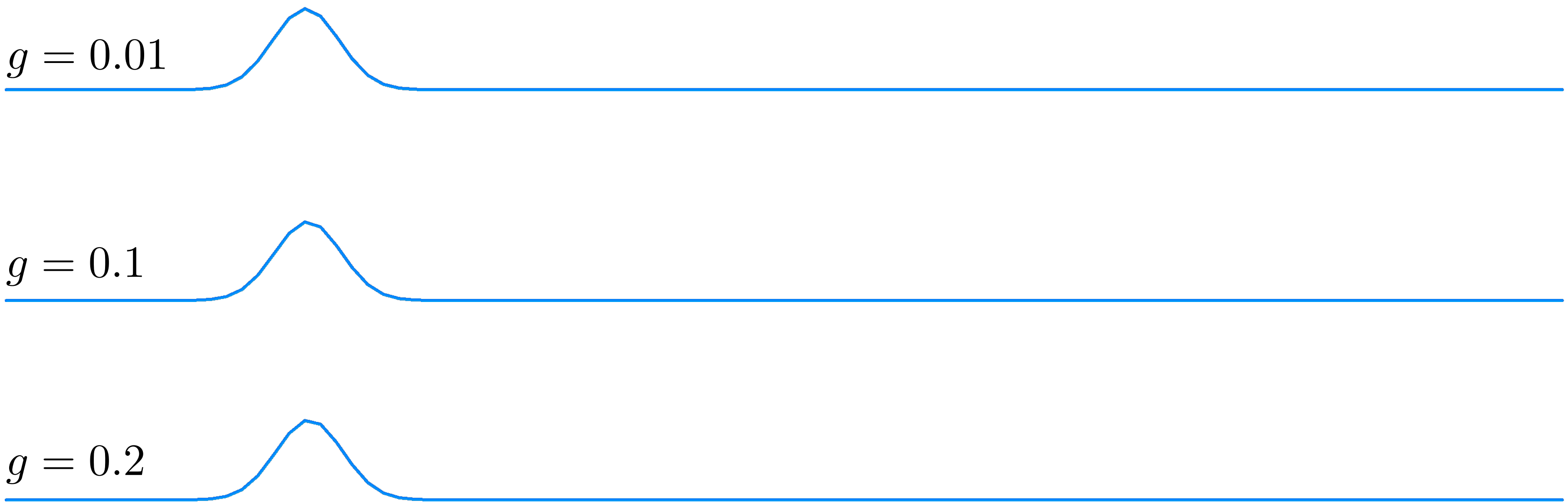

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$\chi_{\max} = 200$

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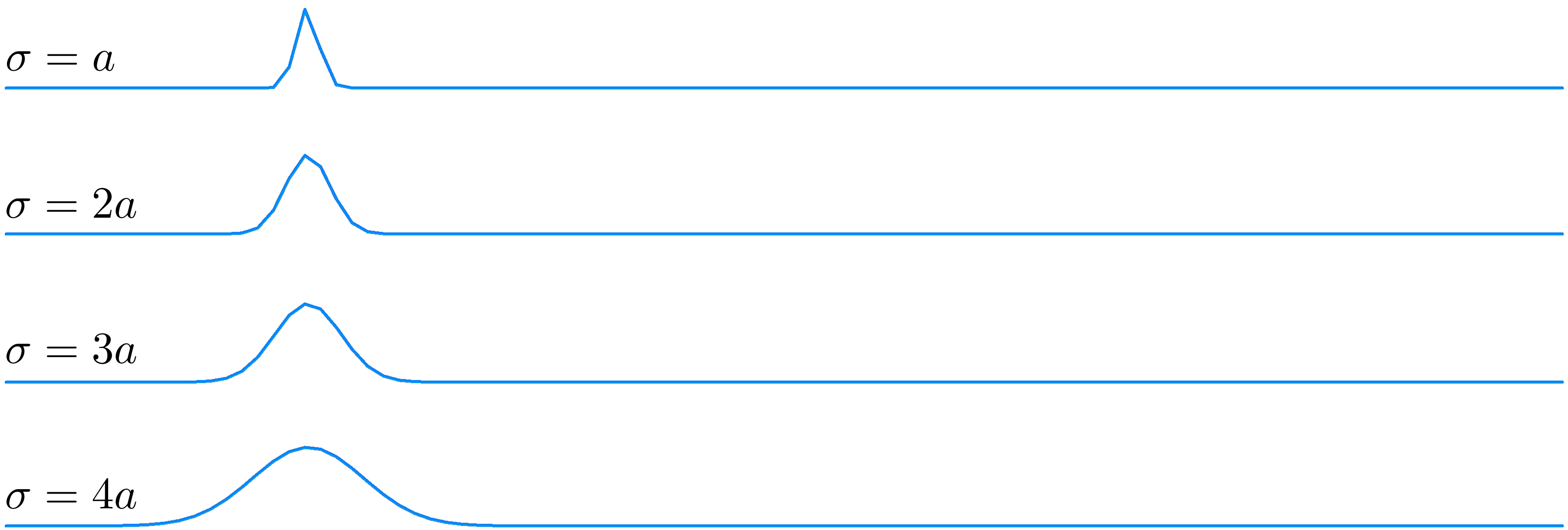
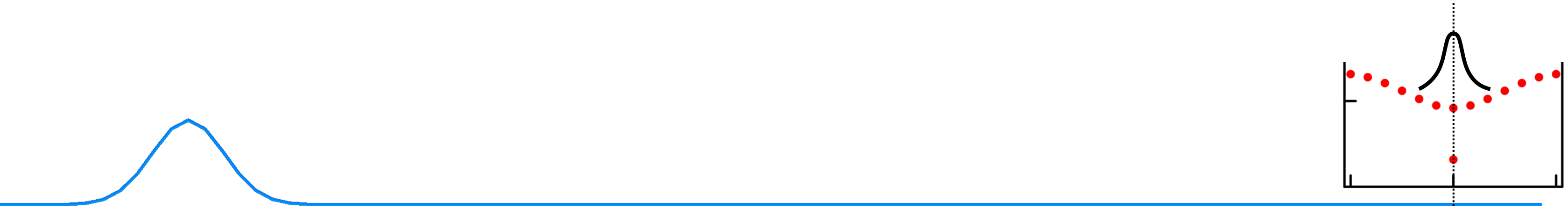


# 6. Numerical simulations

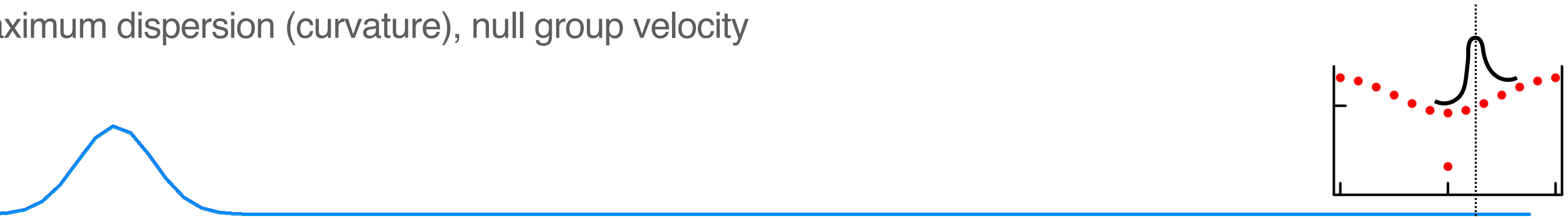
| System parameters | DMRG parameters | Wavepacket parameters | TEBD parameters |
|---|---|---|---|
| $a = 1$ | $\chi_{\max} = 200$ | $\sigma = 3a$ | $\delta t = 0.05$ |
| $L = 100$ | $n_{\text{sweeps}} = 50$ | $k = \pi/2a$ | $\Delta t = 400$ |
| $g^4 = 0.1$ | $\epsilon_{\text{SVD}} = 10^{-13}$ | $j_0 = 20$ | $\epsilon_{\text{SVD}} = 10^{-10}$ |
| (Dirichlet OBC) | | | $\chi_{\max} = 50$ |

$k = 0$

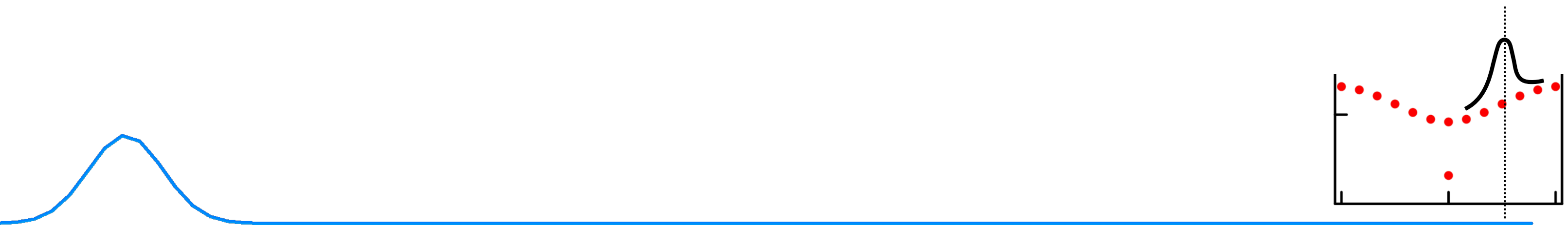


maximum dispersion (curvature), null group velocity

$k = 0.25\,\frac{\pi}{a}$



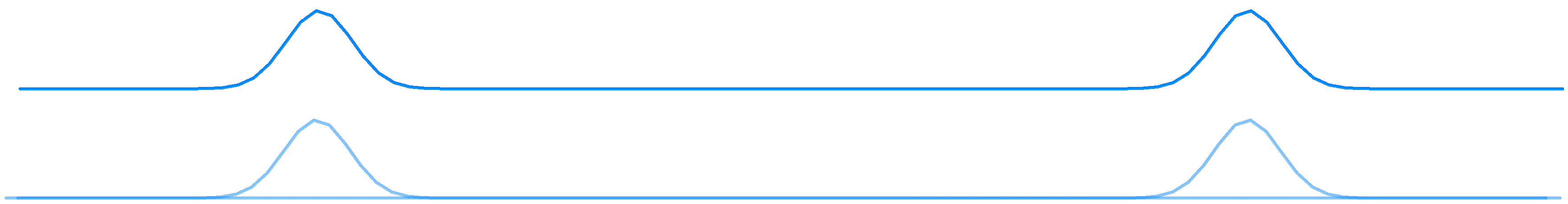
$k = 0.5\,\frac{\pi}{a}$



minimum dispersion (curvature), maximum group velocity   Closest case to a “real 3D photon”

# 6. Numerical simulations

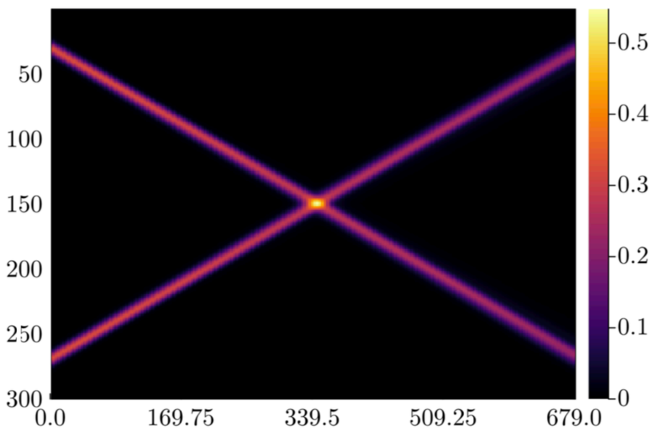
$$\sigma = 3a \quad k = \pi/2a$$



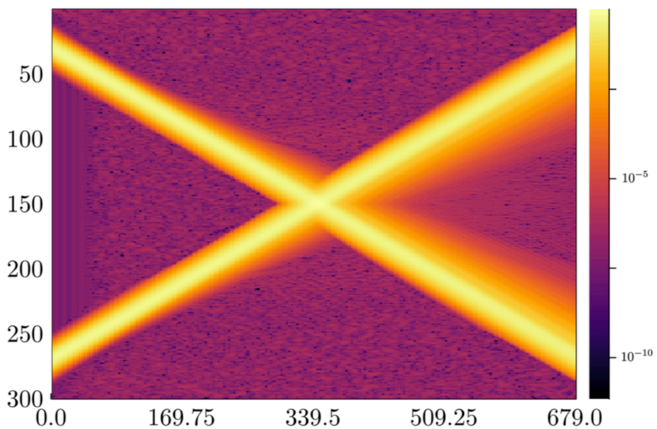
**Nothing happens: good, they are like photons, but bad, it's boring like this!**

**It becomes less and less interacting as we approach the continuum limit:**

**Energy density**




**Energy density in log**




$$L = 300$$

$$\sigma = 5a$$

**Interaction effects:**
- **Lattice** artifacts
- **Finite size** effects
- **1D** lattice geometry

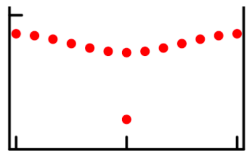
# 7. Conclusions

Inputs $\hat{H}, \hat{T}$ plus **few assumptions** (model independent)

**1 Simultaneous exact diagonalization, finding the Bloch states**

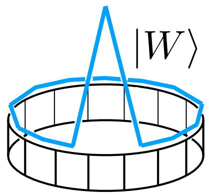
$$[\hat{H}, \hat{T}] = 0$$

Computationally expensive step: **intermediate** system size



**2 Computation of the maximally localized Wannier states**

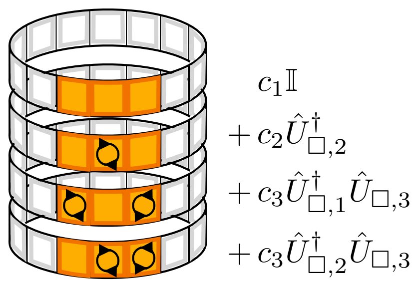
$$|W(\theta_k)\rangle = \frac{1}{\sqrt{L}} \sum_k e^{i\theta_k} |k\rangle$$




**3 Construction of the Wannier creation operator**

$$\hat{W}|\Omega\rangle = |W\rangle$$

From the **interacting** vacuum

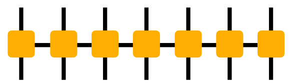



**4 Construction of the Wavepacket creation operator**

$$\hat{\Phi} = \sum_j \phi_j \hat{W}_j$$

We mostly choose a gaussian wavepacket

**5 Conversion of the Wavepacket creation operator to an MPO**

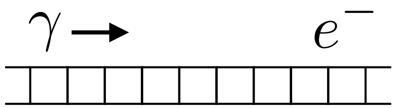


**Output: the MPO**

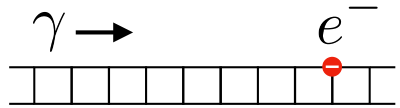
# 8. Outlook

$$(1,1),\ (\tfrac{3}{2},2),\ \ldots$$

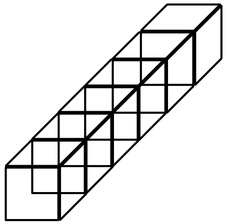
**Higher spin representations**



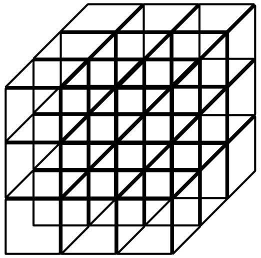

**Adding matter d.o.f**




**Background charge**



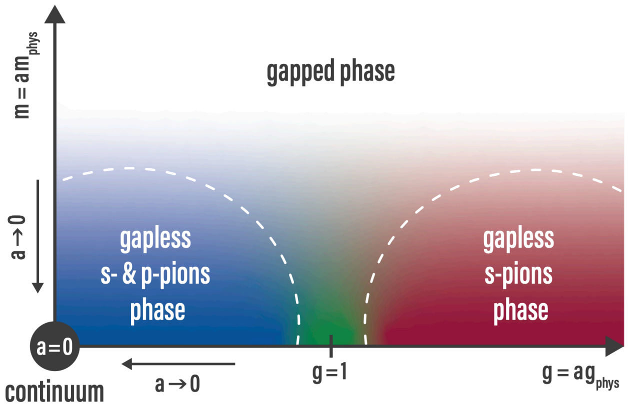
**Different geometries**



**Higher dimension**

$$\begin{pmatrix} \alpha_1 & \beta_2 & & & & 0 \\ \beta_2 & \alpha_2 & \beta_3 & & & \\ & \beta_3 & \alpha_3 & \ddots & & \\ & & \ddots & \ddots & \beta_{m-1} & \\ & & & \beta_{m-1} & \alpha_{m-1} & \beta_m \\ 0 & & & & \beta_m & \alpha_m \end{pmatrix}$$

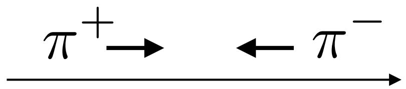
**Try to overcome exact diagonalization**




arXiv:2308.04488

**Last but not least**

moving to **1D QCD with two flavors**, simulating **pion-pion scattering**




**Challenge**: $d = 54$ local dimension