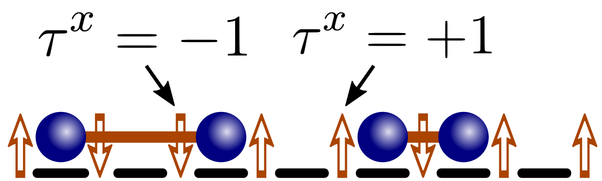
QUANTHEP, 3 September 2024

Confinement in a one-dimensional $\mathbb{Z}_2$ lattice gauge theory at finite temperature



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Motivation

Lattice gauge theories - "high energy physics"

- The standard model has local gauge symmetry: $SU(3) \times SU(2) \times U(1)$. Example:
  - ✓ Strong interaction: QCD - $SU(3)$ gauge theory.
  - ✓ Electromagnetic interaction: QED - $U(1)$ gauge theory.
- Non-perturbative problems: ***lattice*** gauge theory (LGT).
  - ✓ Analytics: lattice allows for regularization.
  - ✓ Numerics: Monte Carlo simulations **sign problem**
- Study of the **confinement-deconfinement** transition.
- Studying "simpler" toy models: $\mathbb{Z}_N$ LGTs?

Kogut, Rev. of Mod. Phys., 51, 659 (1979)

Global symmetry

$\hat{c}_j^\dagger \rightarrow V\hat{c}_j^\dagger$

$V \rightarrow V_j$ **Gauge transf.**

Interactions

Local symmetry $V_j^\dagger V_j = I$

$\hat{c}_j^\dagger \rightarrow V_j\hat{c}_j^\dagger$

Minimal coupling of matter to gauge fields:

Peirls phase, gauge invariance:

$\hat{c}_j^\dagger \hat{c}_{j+1} \rightarrow \hat{c}_j^\dagger \hat{U}_{j,j+1}\hat{c}_{j+1}$

$U_{j,j+1} \rightarrow V_j U_{j,j+1} V_{j+1}$

Zohar et al., Rep. Prog. Phys. 79 (2006).

→ **Quantum simulation?!**

Motivation

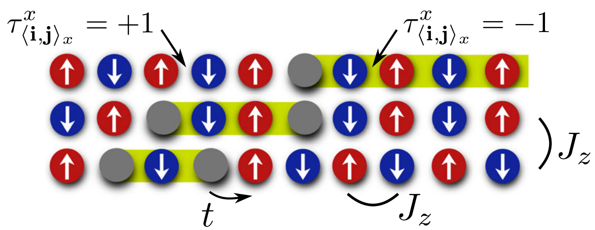
Lattice gauge theories - condensed matter

- Study of the **confinement-deconfinement** transition.
- Emergent low energy theories in strongly correlated systems:
  - ✓ connection to high temperature superconductivity,
  - ✓ topological spin liquids,
  - ✓ fractional excitations, ...
- Direct mapping between a simple 1D $\mathbb{Z}_2$ LGT and a $t - J_z$ model.
- Many degrees of freedom: hard to solve!
  - ✓ Analytical and numerical calculations (DMRG, QMC).

Quantum simulation?!

Kogut, Rev. of Mod. Phys., 51, 659 (1979)
Wilson, PRD 10, 2445 (1974)

Kitaev, Ann. of Phys. 303, 2 (2003).
Senthil et al., PRB 62, (2000).
Subir Sachdev Rep. Prog. Phys. 82 (2019).



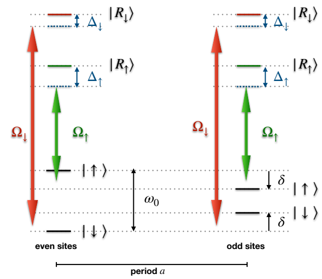
Grusdt et al., PRL 125, (2020).

Motivation

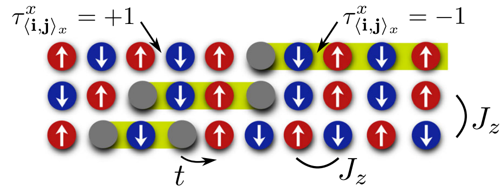
Lattice gauge theories - quantum simulation

- Many proposals:
  - ✓ Direct mapping between a simple 1D $\mathbb{Z}_2$ LGT and a $t - J_z$ model.

    Spin-dependent super-lattice potential:



MK et al., PRL. 127, (2021)

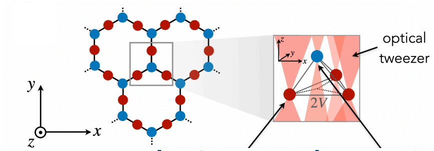


Grusdt et al., PRL 125, (2020).

  - ✓ Superconducting qubits,

Homeier et al., PRB 104, (2021)

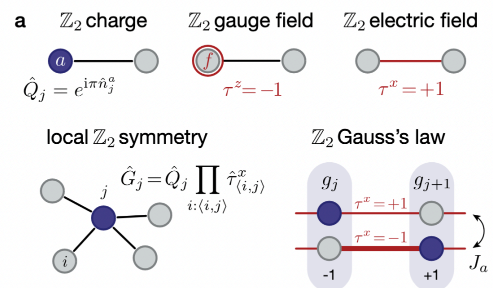
  - ✓ Rydberg tweezers, ...



Homeier et al., Comm. Phys. 6, (2023).

- Experimental **realization** using Floquet scheme.

Schweizer et al., Nature Phys. 15, (2019).



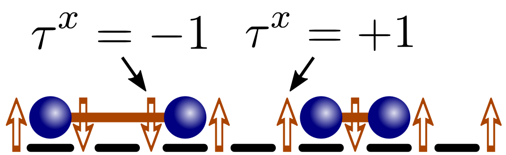
Görg et al., Nature Phys. 15, (2019).

Barbiero et al., Sci. Adv. 5 (2019)

Overview of this talk

1D $\mathbb{Z}_2$ Lattice gauge theory coupled to dynamical matter

- Motivation
- The Hamiltonian and confinement of partons into mesons.
- Phase diagrams at different fillings.
- Confinement at finite temperature.
  - ✓ Smooth crossover from confined to thermally deconfined regime.
  - ✓ Simple experimental probe of confinement: Friedel oscillations and string length histograms.
  - ✓ Dynamical of confined mesons.
- Conclusion



MK et al., Phys. Rev. B 109, (2024).

One-dimensional $\mathbb{Z}_2$ lattice gauge theory

1D $\mathbb{Z}_2$ lattice gauge theory

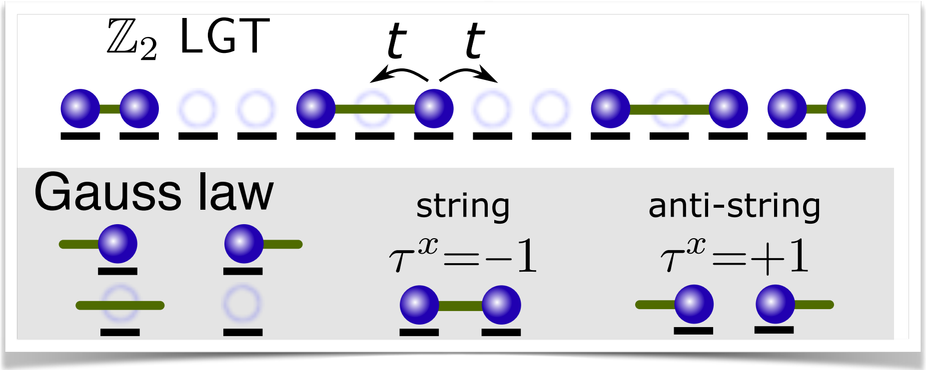
- $\hat{H} = -t\sum_{\langle i,j\rangle}\left(\hat{a}_i^{\dagger}\hat{\tau}^z_{\langle i,j\rangle}\hat{a}_j + \mathrm{h.c.}\right) - h\sum_{\langle i,j\rangle}\hat{\tau}^x_{\langle i,j\rangle} + V\sum_j \hat{n}_j\hat{n}_{j+1}$
- Gauss law:

$$\hat{G}_i|\psi\rangle = \hat{\tau}^x_{\langle i-1,i\rangle}\hat{\tau}^x_{\langle i,i+1\rangle}(-1)^{\hat{n}_i}|\psi\rangle = \pm|\psi\rangle \longrightarrow g_i = +1$$

$$[\hat{H}, \hat{G}_j] = 0, \quad [\hat{G}_j, \hat{G}_i] = 0.$$

Prosko et al., PRB 96, (2017).

Borla et al., Phys. Rev. Lett. 124, (2020).



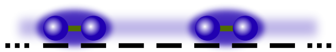
MK et al., Phys. Rev. Lett. 127, (2021).

Confinement

- Linear confining potential $h\sum_{\langle i,j\rangle}\hat{\tau}^x_{\langle i,j\rangle}$.
- Green's function decays exponentially for $h > 0$

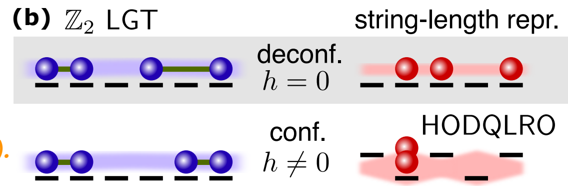
Borla et al., Phys. Rev. Lett. 124, (2020)

→ ***Confinement of partons into mesons!***



Solving the confinement problem

- String-length basis:



MK et al., Phys. Rev. Lett. 127, (2021).

Broken translational symmetry in the new basis → confinement!

Ground state properties of the $\mathbb{Z}_2$ lattice gauge theory

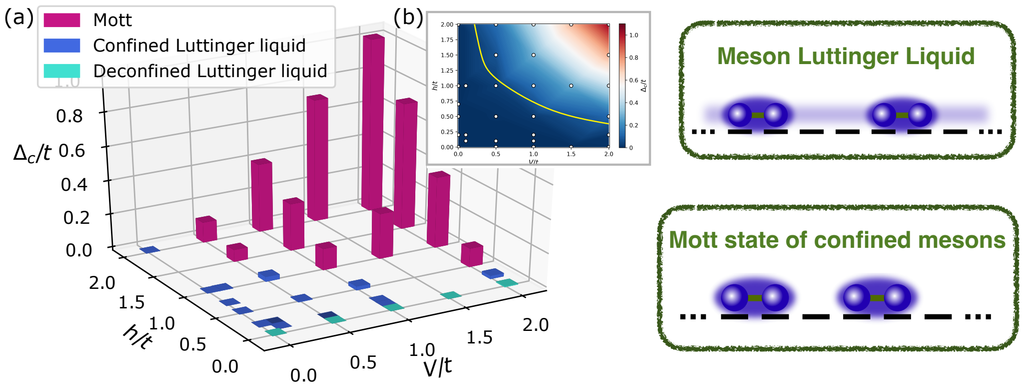
Phase Diagrams

$$\hat{H} = -t\sum_{\langle i,j\rangle}\left(\hat{a}_i^\dagger \hat{\tau}^z_{\langle i,j\rangle}\hat{a}_j + \mathrm{h.c.}\right) - h\sum_{\langle i,j\rangle}\hat{\tau}^x_{\langle i,j\rangle} + V\sum_j \hat{n}_j\hat{n}_{j+1}$$

- Luttinger liquid for generic values of h and V.
- Different Mott transitions at fillings $n = 2/3$ and $n = 1/2$.

Two-thirds filling

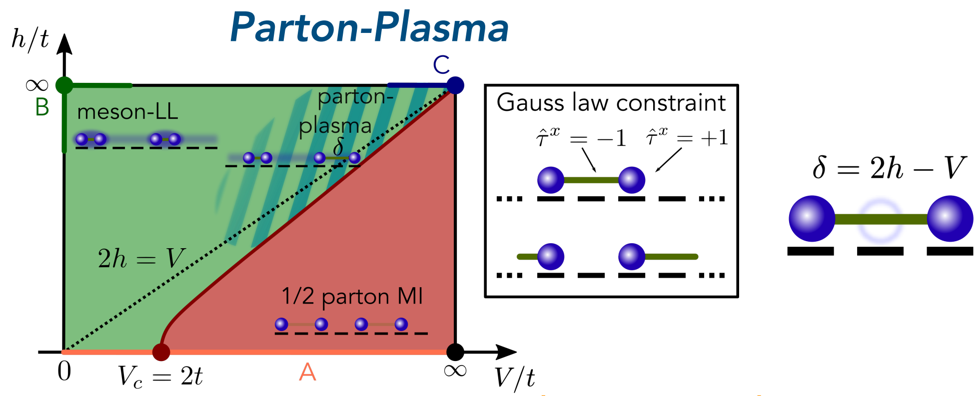
- Mott state of confined dimers stabilized by V.



MK et al., Phys. Rev. Lett. 127, (2021)

Half filling

- Highly degenerate regime when $h, V \gg t$ and $2h \approx V$.

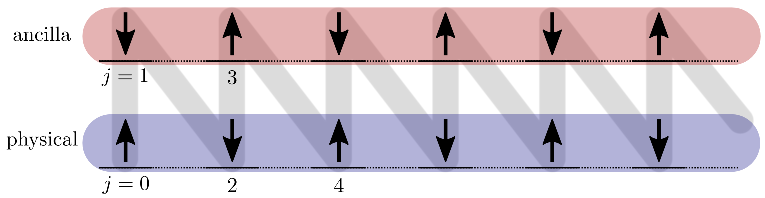


MK et al 2023 New J. Phys. 25 013035

Numerical calculations of finite temperature LGT

Method

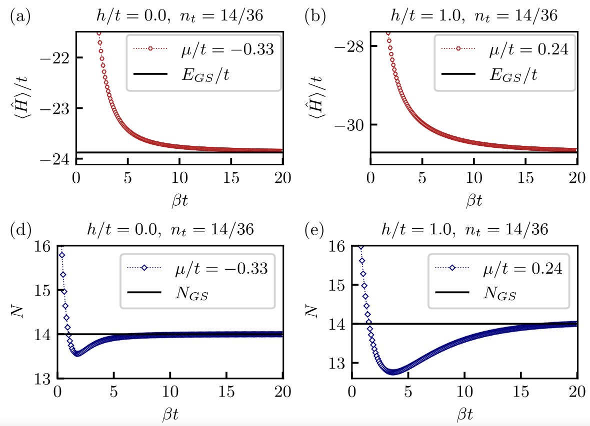
- DMRG for the ground state calculations
- Quantum purification scheme to obtain finite temperature results.
- Ancilla site added to every physical lattice site.



- Maximally entangled state between ancilla and physical lattice sites is the infinite temperature state $\beta = 1/T = 0$

Nocera et al., PRB, 93, 045 137, (2016).
Zwolak et al., PRL., 93, 207 205, (2004).
Feiguin et al., PRB, 72, 220 401, (2005).

$$\hat{H} = -t \sum_{\langle i,j \rangle} \left(\hat{a}_i^\dagger \hat{\tau}^z_{\langle i,j \rangle} \hat{a}_j + \mathrm{h.c.} \right) - h \sum_{\langle i,j \rangle} \hat{\tau}^x_{\langle i,j \rangle}$$



- By performing imaginary time evolution we "cool" the system

$$|\psi(\beta)\rangle = e^{-\beta \hat{\mathscr{H}}/2} |\psi(0)\rangle \,.$$

- Physical observables are calculated as

$$\langle \hat{\mathcal{O}} \rangle = \frac{\langle \psi(\beta) | \hat{\mathcal{O}} | \psi(\beta) \rangle}{\langle \psi(\beta) | \psi(\beta) \rangle} \,.$$

Confinement in the $\mathbb{Z}_2$ LGT at finite temperature

Green's function

- $\mathbb{Z}_2$ invariant Green's function - probe of confinement:

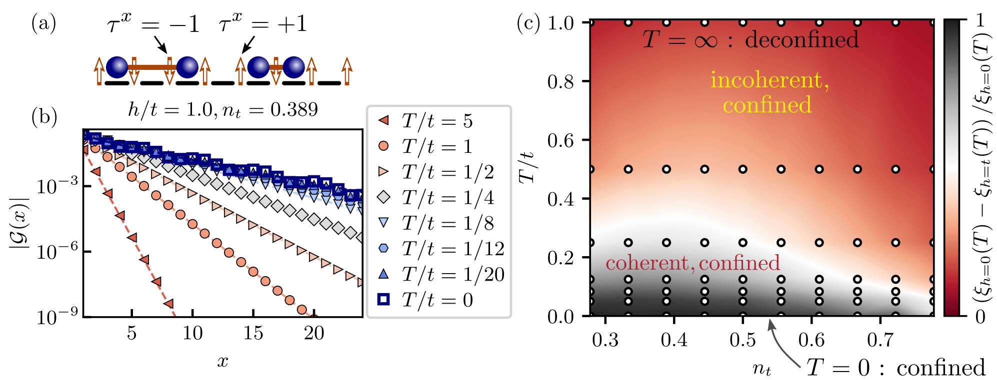
$$\mathcal{G}(i-j) = \langle \hat{a}_i^\dagger \left(\prod_{i \leq \ell < j} \hat{\tau}^z_{l,l+1} \right) \hat{a}_j \rangle \,.$$

- Power-law decay in deconfined and exponential decay in the confined phase.
- We fit the Green's function with an exponential function to extract the correlation length ξ

$$f_G(x) = A x^{-\alpha} e^{-x/\xi} \,.$$

- Comparison of $\big(\xi(h/t=1) - \xi(h/t=0)\big)/\xi(h/t=0)$.
- Quantum purification scheme via MPS.

$$\hat{H} = -t \sum_{\langle i,j \rangle} \left(\hat{a}_i^\dagger \hat{\tau}^z_{\langle i,j \rangle} \hat{a}_j + \mathrm{h.c.} \right) - h \sum_{\langle i,j \rangle} \hat{\tau}^x_{\langle i,j \rangle}$$



MK et al., Phys. Rev. B 109, (2024).

Smooth crossover!

$T_c/t \approx 0.25$

- We uncover a smooth crossover region between confined and thermally deconfined regime as a function of temperature.

1+1D $\mathbb{Z}_2$ LGT at finite temperature

Friedel oscillations

- Period of the Friedel oscillations doubles in the confined regime: $2\pi n \to \pi n\,.$

Borla et al., Phys. Rev. Lett. 124, (2020)

Parton Luttinger liquid $h = 0$

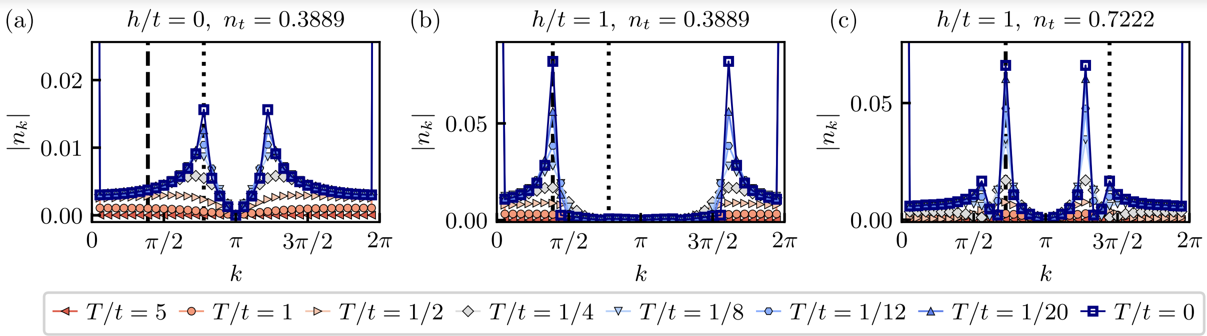
Meson Luttinger liquid $h > 0$

Another signature of confinement!

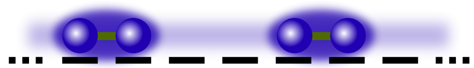
$$\hat{H} = -t\sum_{\langle i,j\rangle}\left(\hat{a}_i^\dagger \hat{\tau}^z_{\langle i,j\rangle}\hat{a}_j + \mathrm{h.c.}\right) - h\sum_{\langle i,j\rangle}\hat{\tau}^x_{\langle i,j\rangle}$$

- Frequency remains constant for $h \neq 0$ regime as a function of T.

Pre-formed mesons at high temperature



Meson Luttinger, $h > 0$

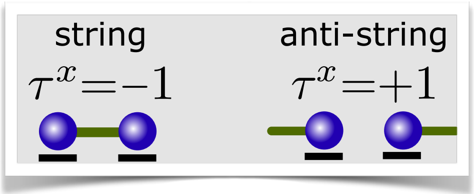


MK et al., Phys. Rev. B 109, (2024).

1+1D $\mathbb{Z}_2$ LGT at finite temperature

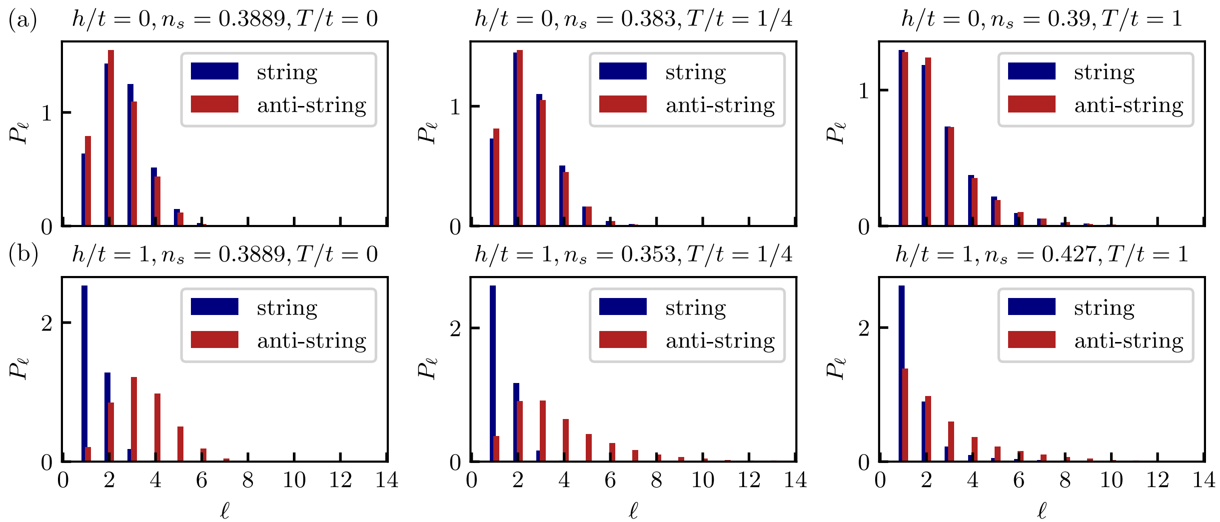
Snapshots

- Green's function is difficult to access in cold-atom experiments.
- Snapshots sampled directly from MPS.
- String lengths can be easily accessible from projective measurements.
- Histograms of string and anti-string lengths.



Bimodal distribution in the confined regime.

Robust measure of confinement!

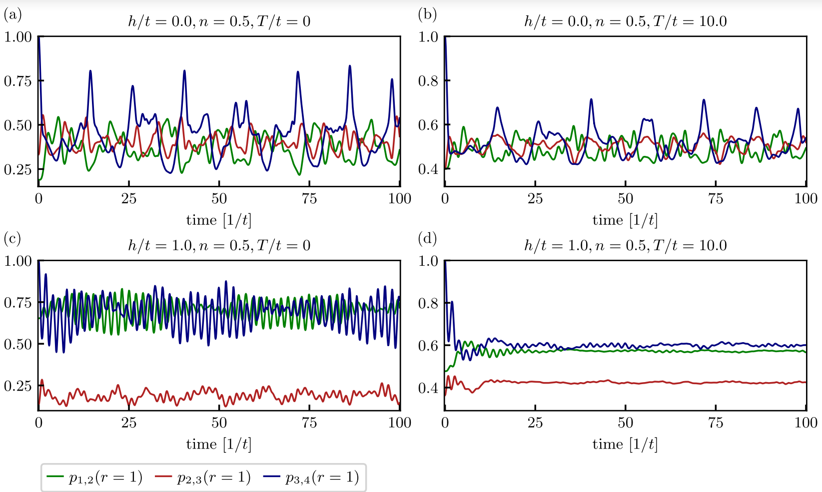


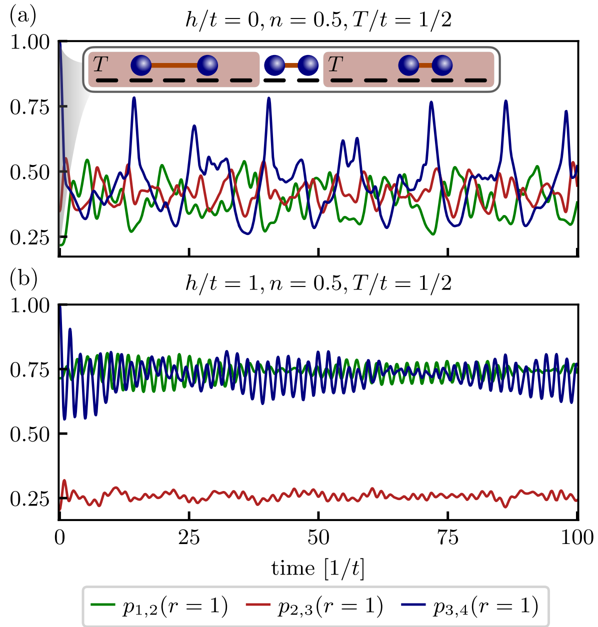
MK et al., Phys. Rev. B 109, (2024).

Quench dynamics

Time evolution

- Initial state with a meson in the center of the system.
- Probability of distance $r = 1$ between charges.
- For $h > 0$ the probability of well defined meson remains high at high temperature.



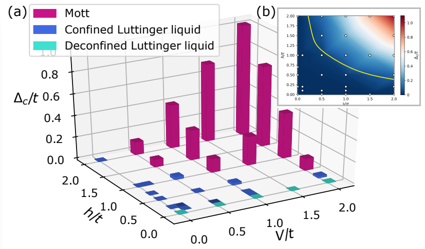


MK et al., Phys. Rev. B 109, (2024).

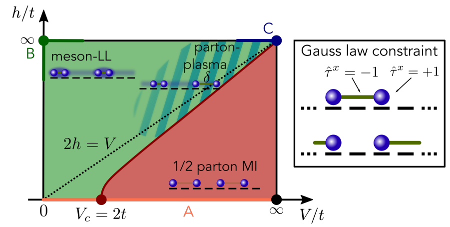
Conclusion

Summary

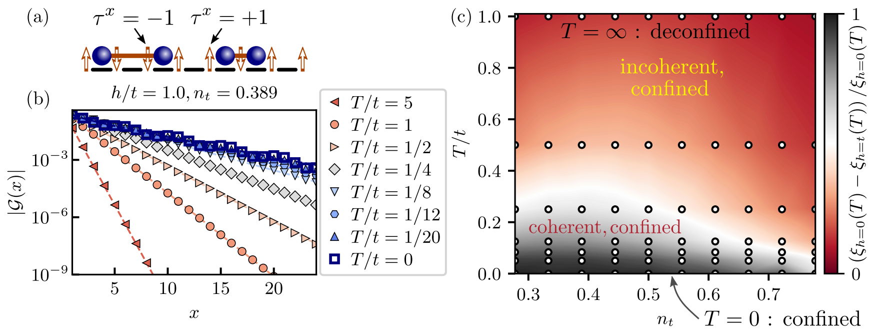
- Interplay of non-local confining field and local interaction result in rich phase diagrams.
- Crossover to deconfined regime at high temperature.
- Pre-formed mesons above the crossover temperature!
- Within the reach of current experimental setups.



MK et al., PRL, 127, (2021)



MK et al., New J. Phys. 25, (2023).



MK et al., Phys. Rev. B 109, (2024).

Outlook

- Confinement in LGTs with more complicated gauge structure.
- Confinement in the 2D $\mathbb{Z}_2$ LGT with matter?