Quantum simulating QCD in the Large Nc limit

Christian Bauer
Theory Group Leader
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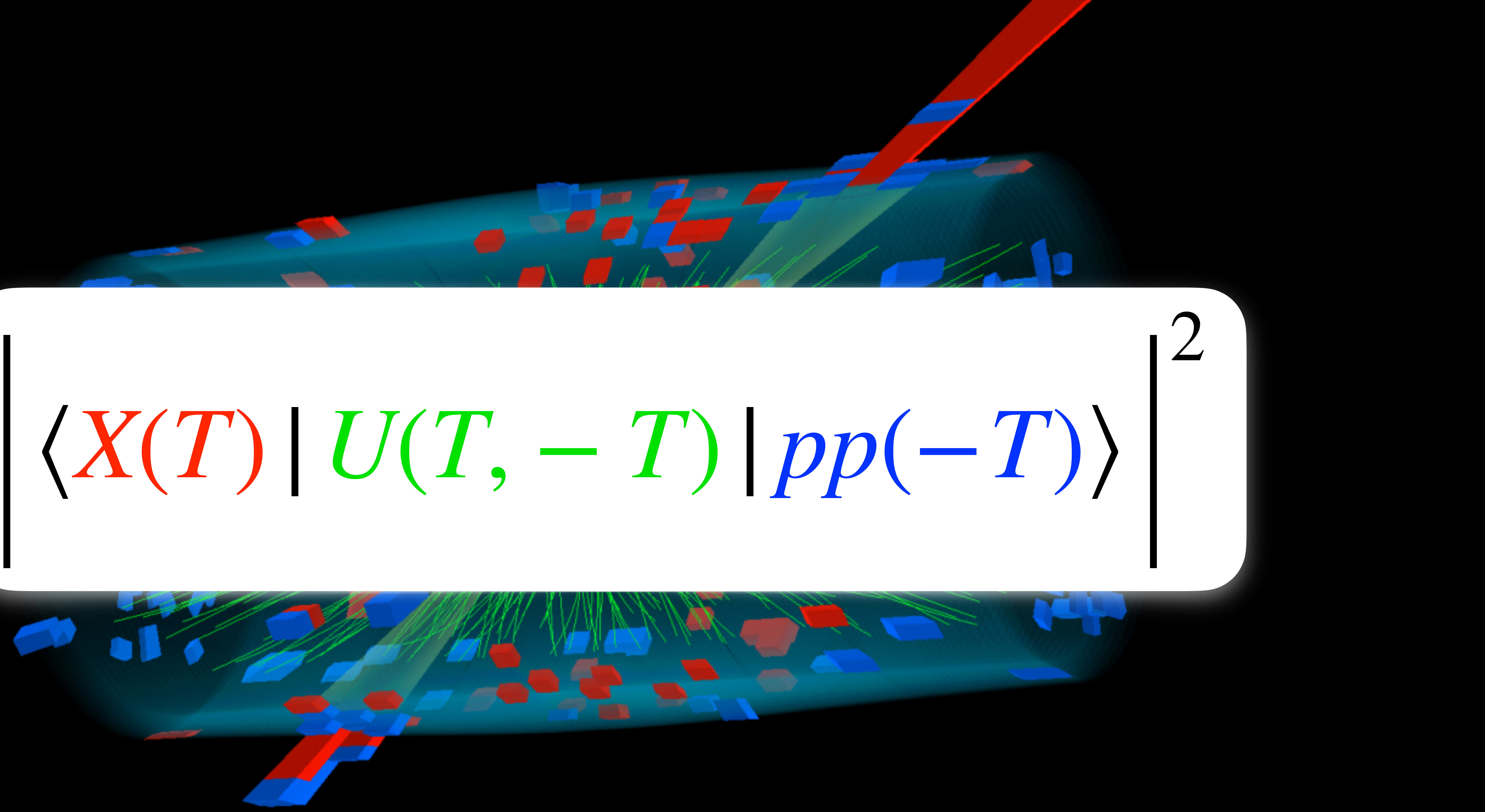


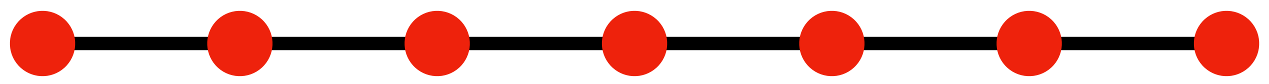
$$\left| \langle X(T) \,|\, U(T,-T) \,|\, pp(-T) \rangle \right|^2$$

CMS Run/Event 262548/458269

QCD simulations in more that 1 dimension is a major goal of the community, and finding the most efficient theory formulations is crucial

To make progress, typically start with simpler theories

1D



$U(1)$

Bajerjee et al, PRL 109, 175302 (2012)
Hauke et al, PRX 3, 041018 (2013)
Martinez et al, Nature 534, 516 (2016)
Klco et al, PRA 98, 032331 (2018)
…

$SU(2)$

Atas et al, Nat. Comm. 12, 6499 (2021)

$SU(3)$

Farrell et al, PRD 107, 054512 (2023)
Farrell et al, PRD 107, 054513 (2023)
Atas et al, PRR 5, 033184 (2022)
Ciavarella, PRD 108, 094513 (2023)

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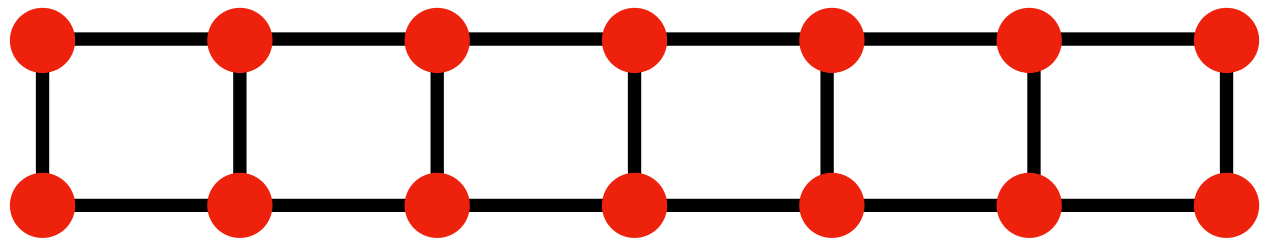
1D+



$U(1)$

$SU(2)$

Klco et al, PRD 101, 074512 (2020)
Rahman et al, PRD 104, 0345051 (2021)
Rahman et al, PRD 106, 074502 (2022)

$SU(3)$

Ciavarella et al, PRD 103, 094501 (2021)
Ciavarella et al, PRD 105, 074504 (2022)

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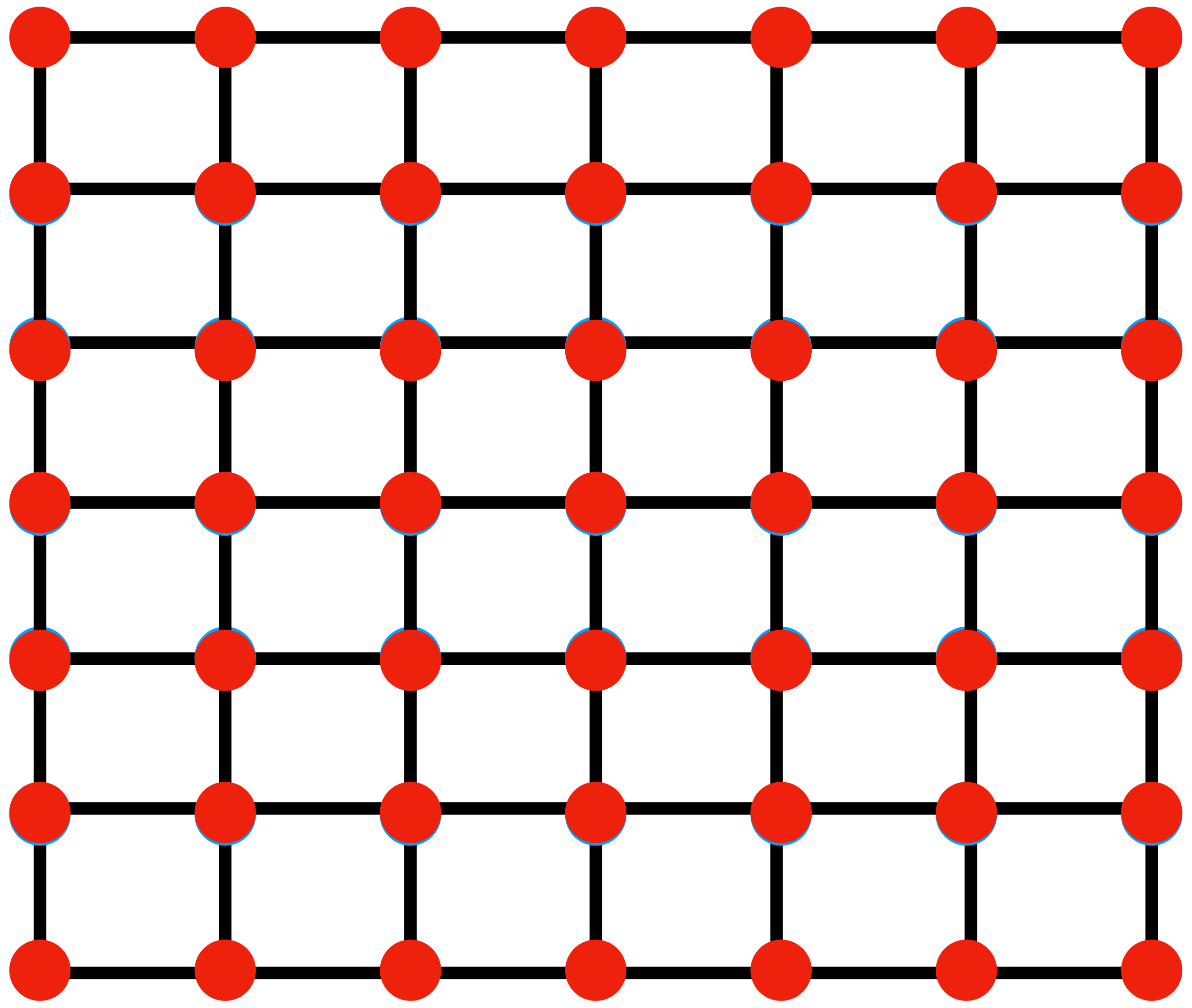
$U(1)$

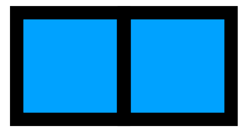
2D



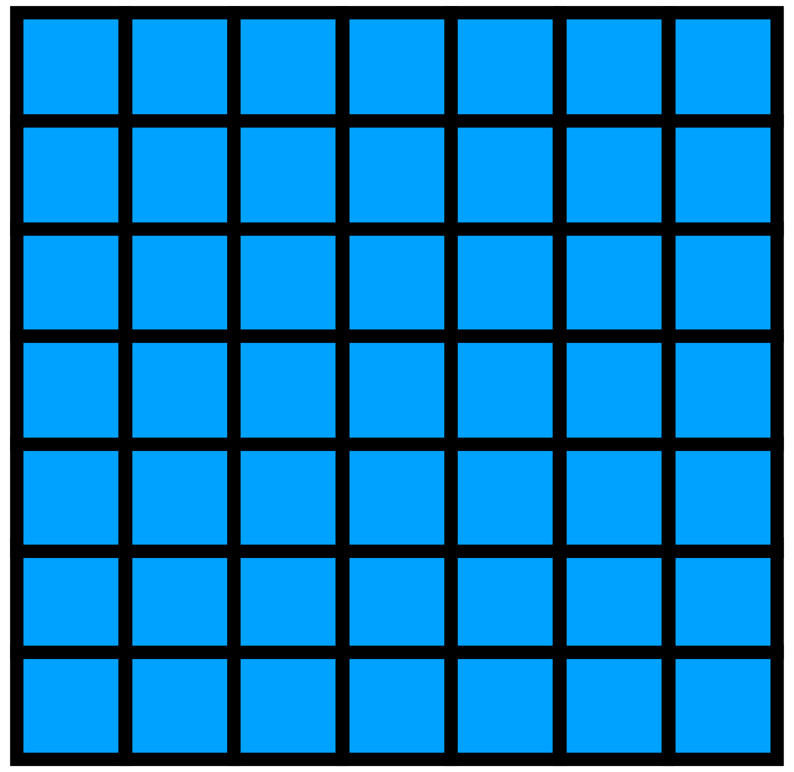
$SU(2)$ Turro, PRD 109, 114511 (2024) (4 plaquettes)

$SU(3)$ This talk

QCD simulations in more that 1 dimension is a major goal of the community, and finding the most efficient theory formulations is crucial



Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis,
2101.10227



Quantum Simulation of SU(3) Lattice Yang Mills Theory at Leading Order in Large N,
2402.10265

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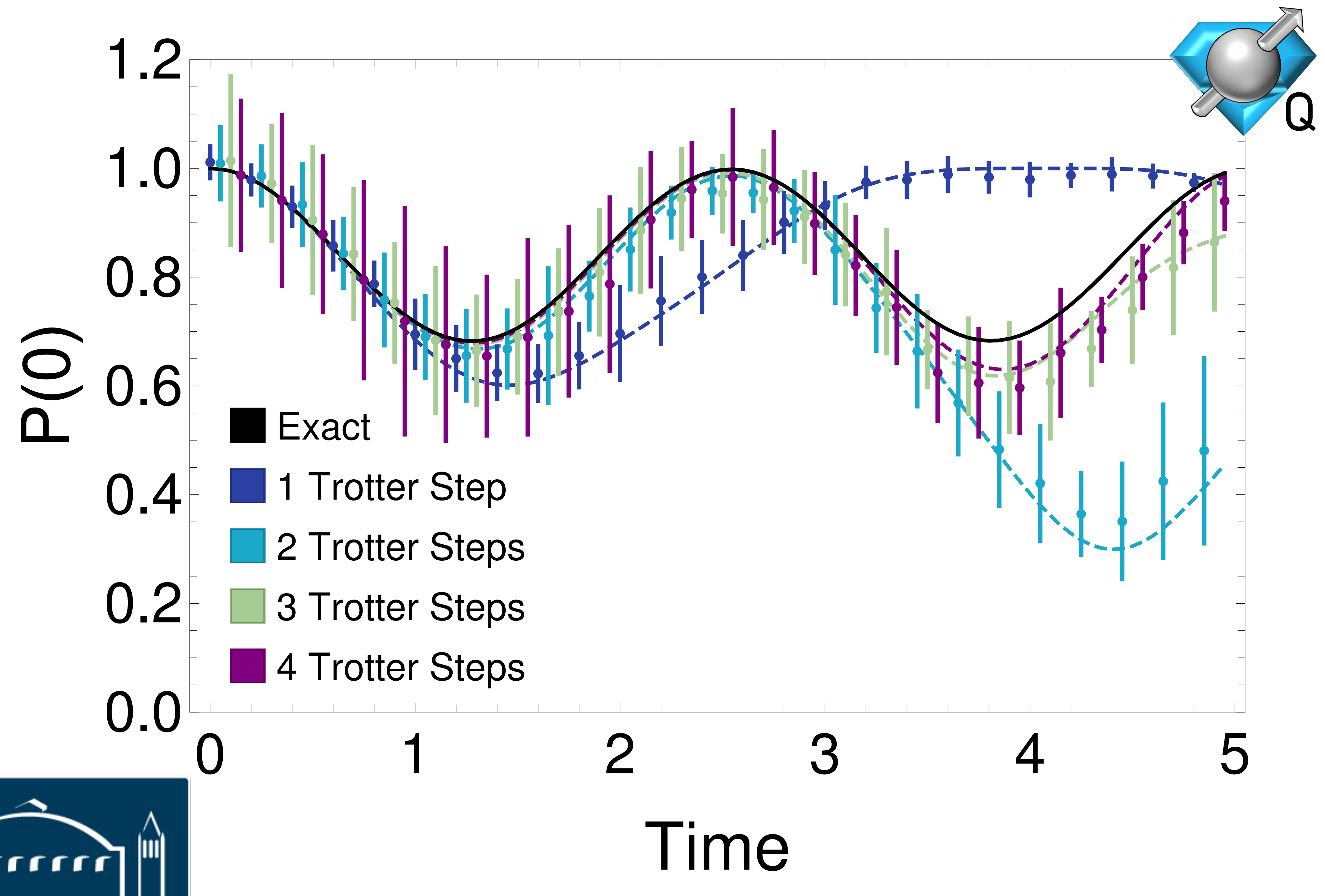
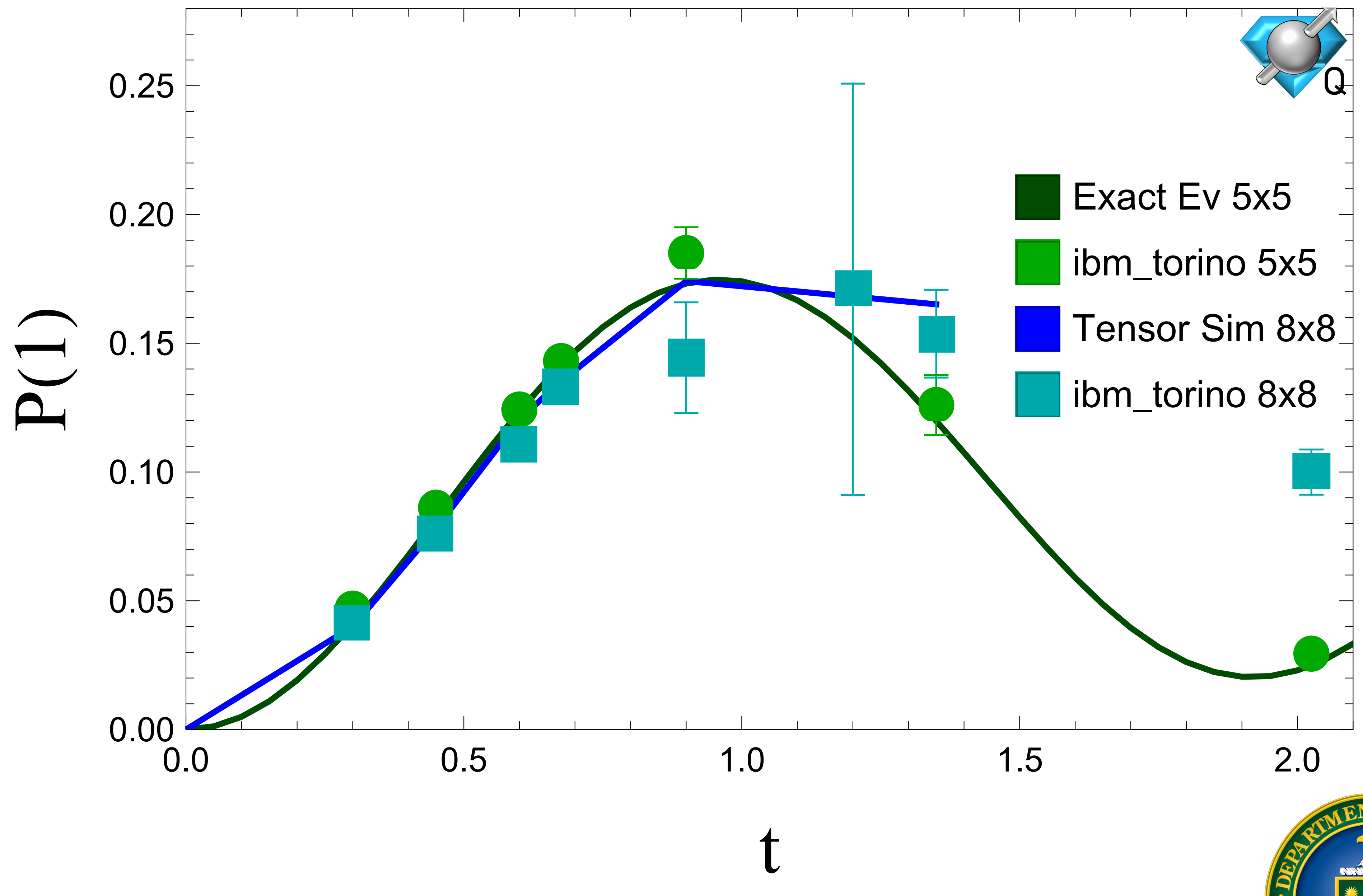




Christian Bauer

There are many different parts of the theory that need to be worked out when formulating a Hamiltonian lattice gauge theory

1. How to formulate a lattice theory that reproduces SU(3) in the limit of vanishing lattice spacing

- Whether to add any additional expansions in the theory

2. What basis to choose for the Hilbert space

3. How to implement gauge invariance

4. How to truncate the theory (how to choose a discrete set of field values)

Goal is a Hamiltonian Lattice theory that reproduces QCD in continuum limit

A Trailhead for Quantum Simulation of SU(3) Yang-Mills Lattice Gauge Theory in the Local Multiplet Basis

Anthony Ciavarella,[1,*] Natalie Klco,[2,†] and Martin J. Savage[1,‡]

[1] *InQubator for Quantum Simulation (IQuS), Department of Physics, University of Washington, Seattle, WA 98195, USA*

[2] *Institute for Quantum Information and Matter (IQIM) and Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena CA 91125, USA*

(Dated: February 23, 2021 - 1:41)

arXiv:2101.10227v2

Phys.Rev.D 103 (2021) 9

Part 1: What lattice Hamiltonian to use in the without truncation.

In this case the Kogut-Susskind Hamiltonian is used

$$\hat{H} = \frac{g^2}{2a^{d-2}} \sum_{b,\text{links}} |\hat{\mathbf{E}}^{(b)}|^2 + \frac{1}{2a^{4-d}g^2} \sum_{\text{plaquettes}} \left[6 - \hat{\Box}(\mathbf{x}) - \hat{\Box}^\dagger(\mathbf{x}) \right]$$

Part 2: How to represent basis to choose for Hilbert space

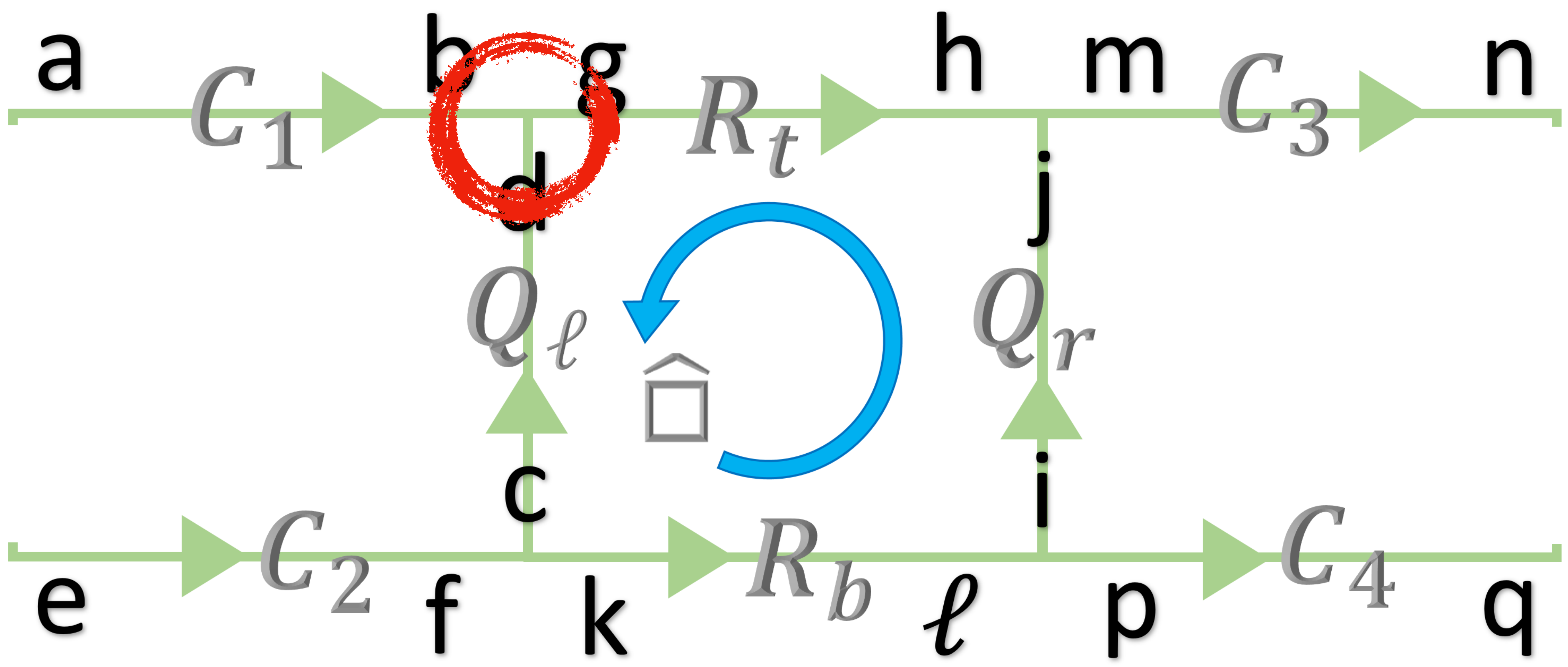
In this case a basis in representation of SU(3) was chosen in which electric Hamiltonian is diagonal

$$\sum_b |\hat{\mathbf{E}}^{(b)}|^2 \, |p, q\rangle = \frac{p^2 + q^2 + pq + 3p + 3q}{3} \, |p, q\rangle$$

$$\dim(p, q) = \frac{(p+1)(q+1)(p+q+2)}{2}$$

Part 3: How to implement gauge invariance

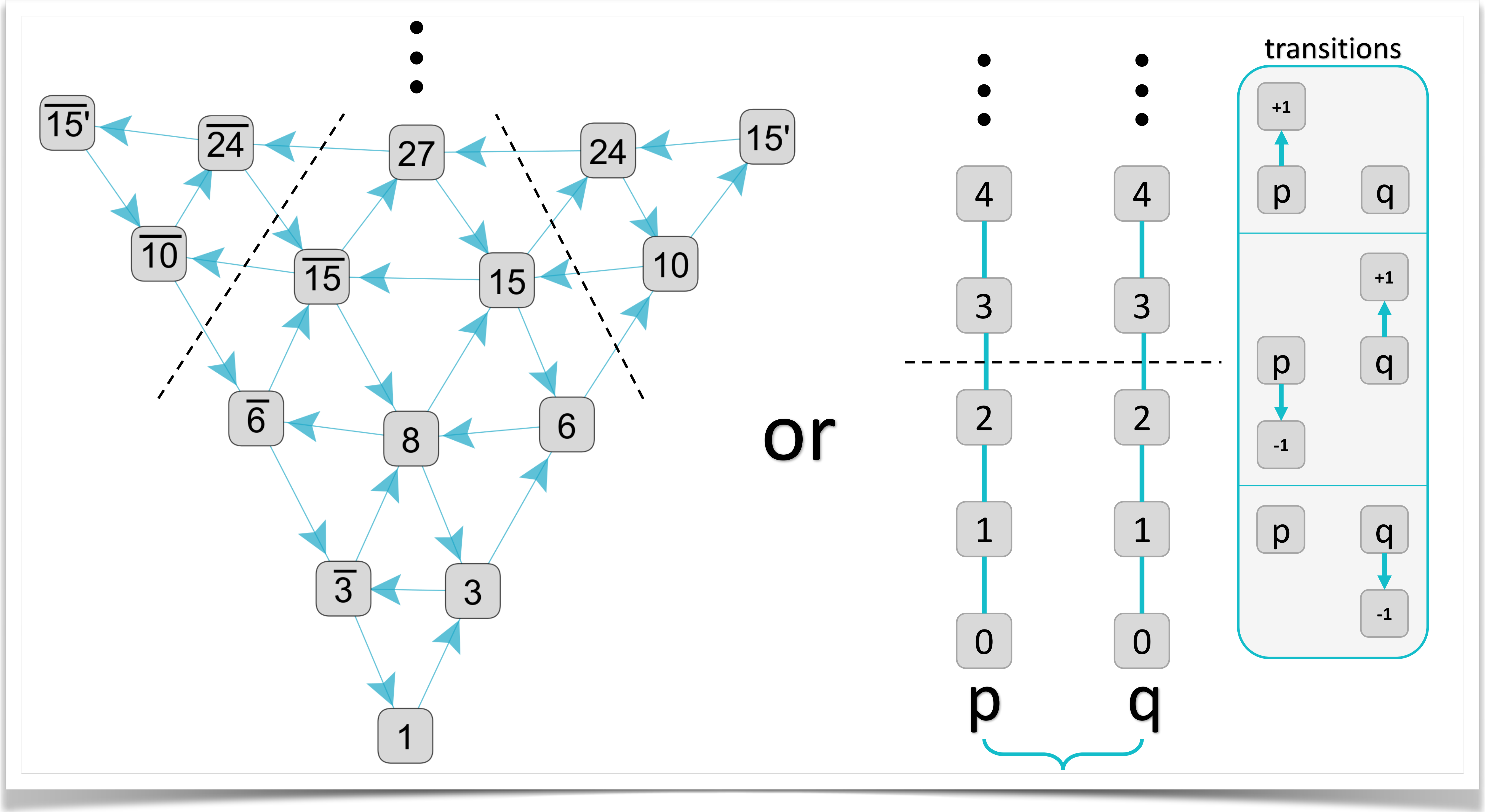
In this case gauge invariance is implemented by requiring that representations satisfy Gauss' law, therefore putting restrictions on each plaquette



$$|\psi_{3pt}\rangle \sim \sum_{b,g,d,\Gamma} \langle \mathbf{C}_1, b, \overline{\mathbf{R}}_t, g | \overline{\mathbf{Q}}_\ell, d\rangle_\Gamma \; |\mathbf{C}_1, a, b\rangle |\mathbf{Q}_\ell, c, d\rangle |\mathbf{R}_t, g, h\rangle$$

Part 4: How to truncate the theory

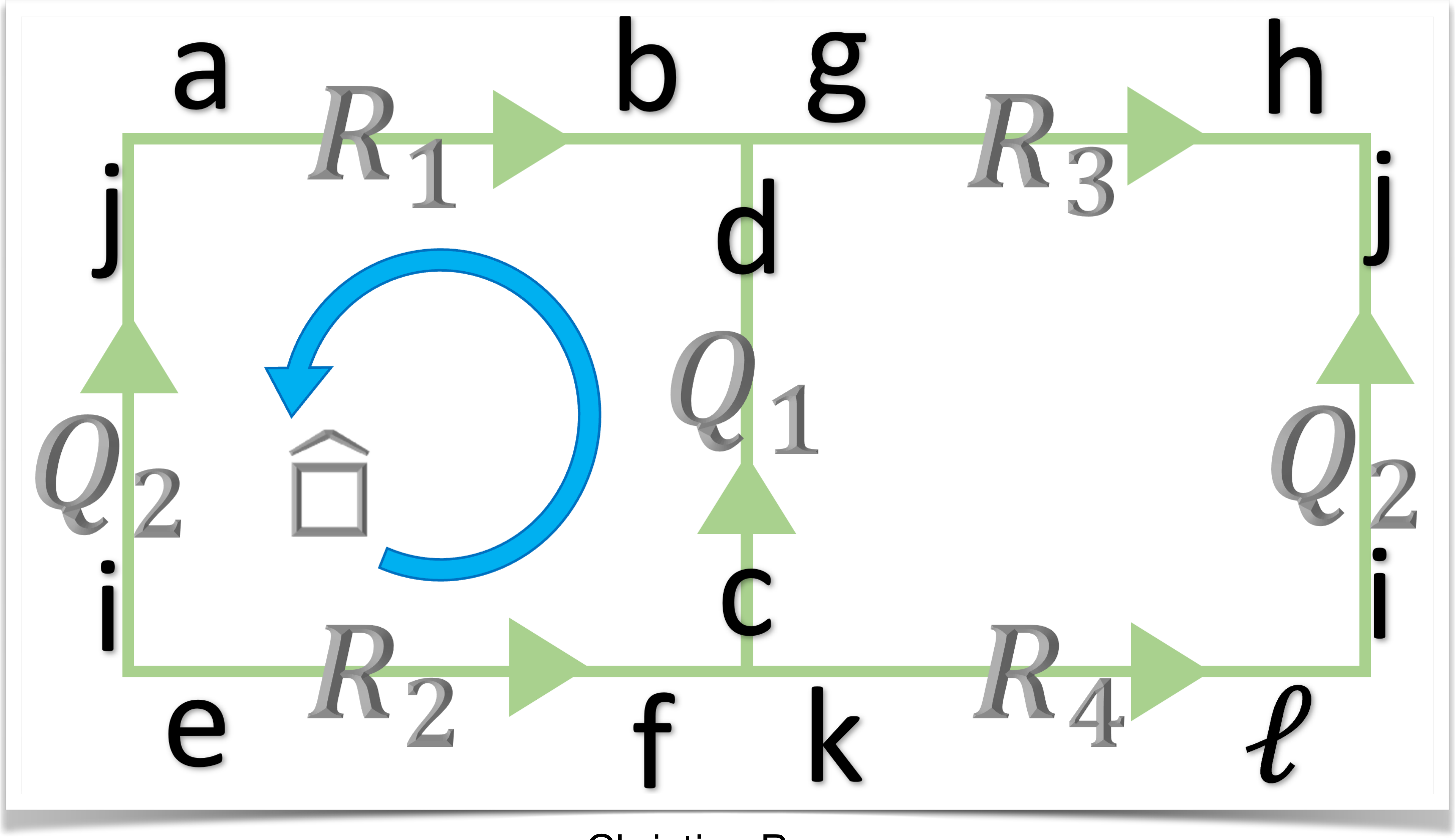
In this case theory is truncated by the maximum allowed p and q values of the representation at each link



All details in place ⇒ theoretical framework. Now needs to work out efficient quantum algorithms and get results from hardware

Paper presented above was first (and essentially still only one) that could do real SU(3) calculations in 2+1D on quantum hardware

Results could be obtained on a 3x2 lattice



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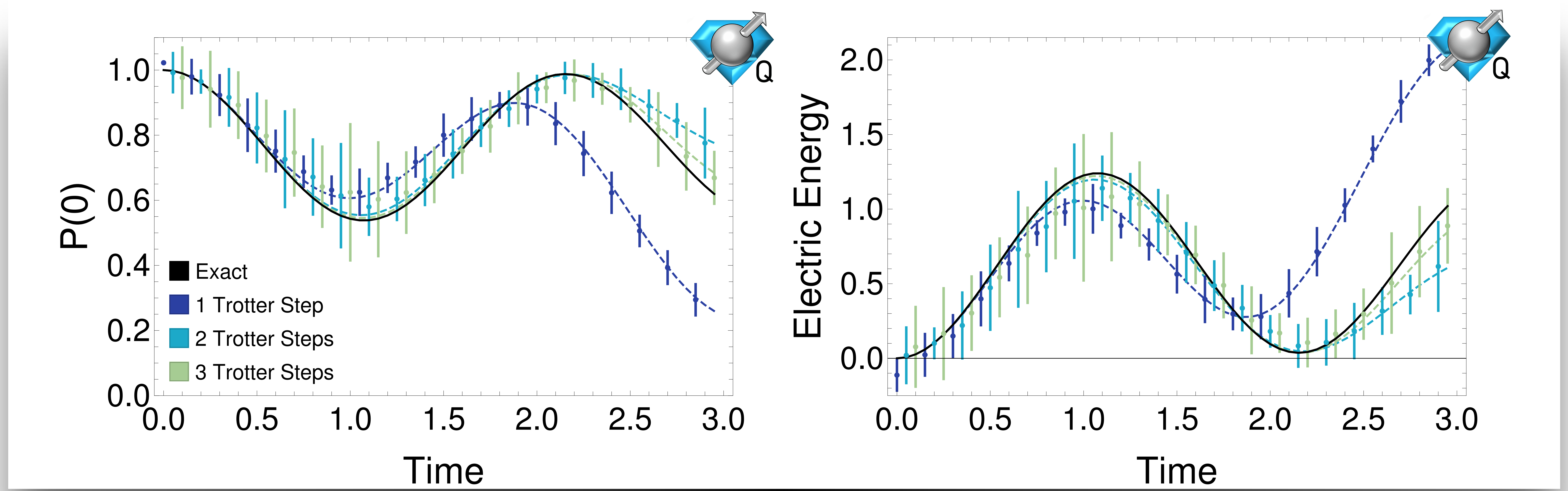
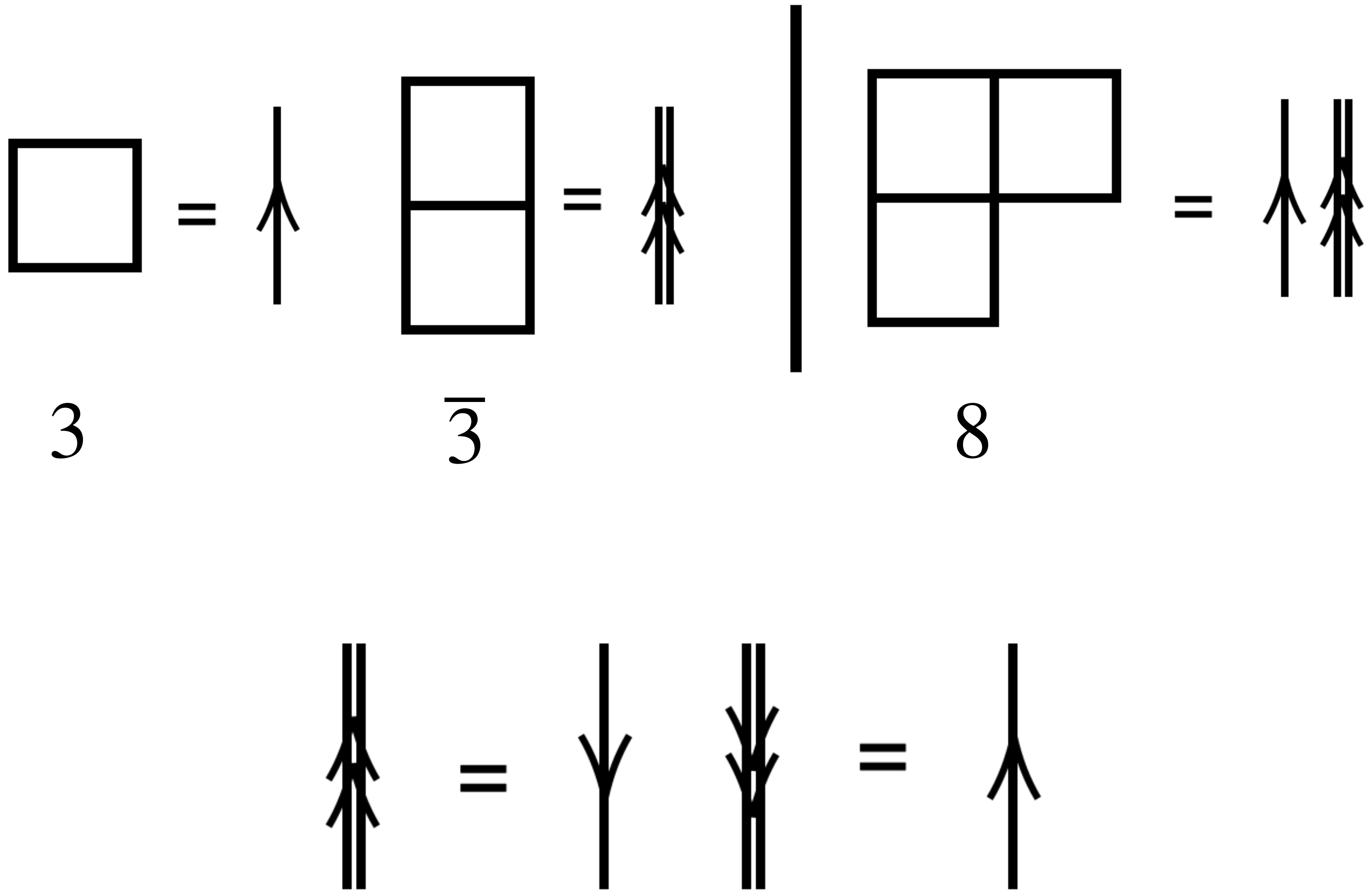


We very recently realized that adding an additional expansion can lead to dramatic simplifications in the lattice theory

1. How to formulate a lattice theory that reproduces SU(3) in the limit of vanishing lattice spacing
   - Whether to add any additional expansions in the theory
2. What basis to choose for the Hilbert space
3. How to implement gauge invariance
4. How to truncate the theory (how to choose a discrete set of field values)
5. …

The basic idea of the 1/Nc expansion is to expand the amplitudes of states and terms in the Hamiltonian in powers of 1/Nc

Step 1: Represent representations of SU(3) graphically through double line notation



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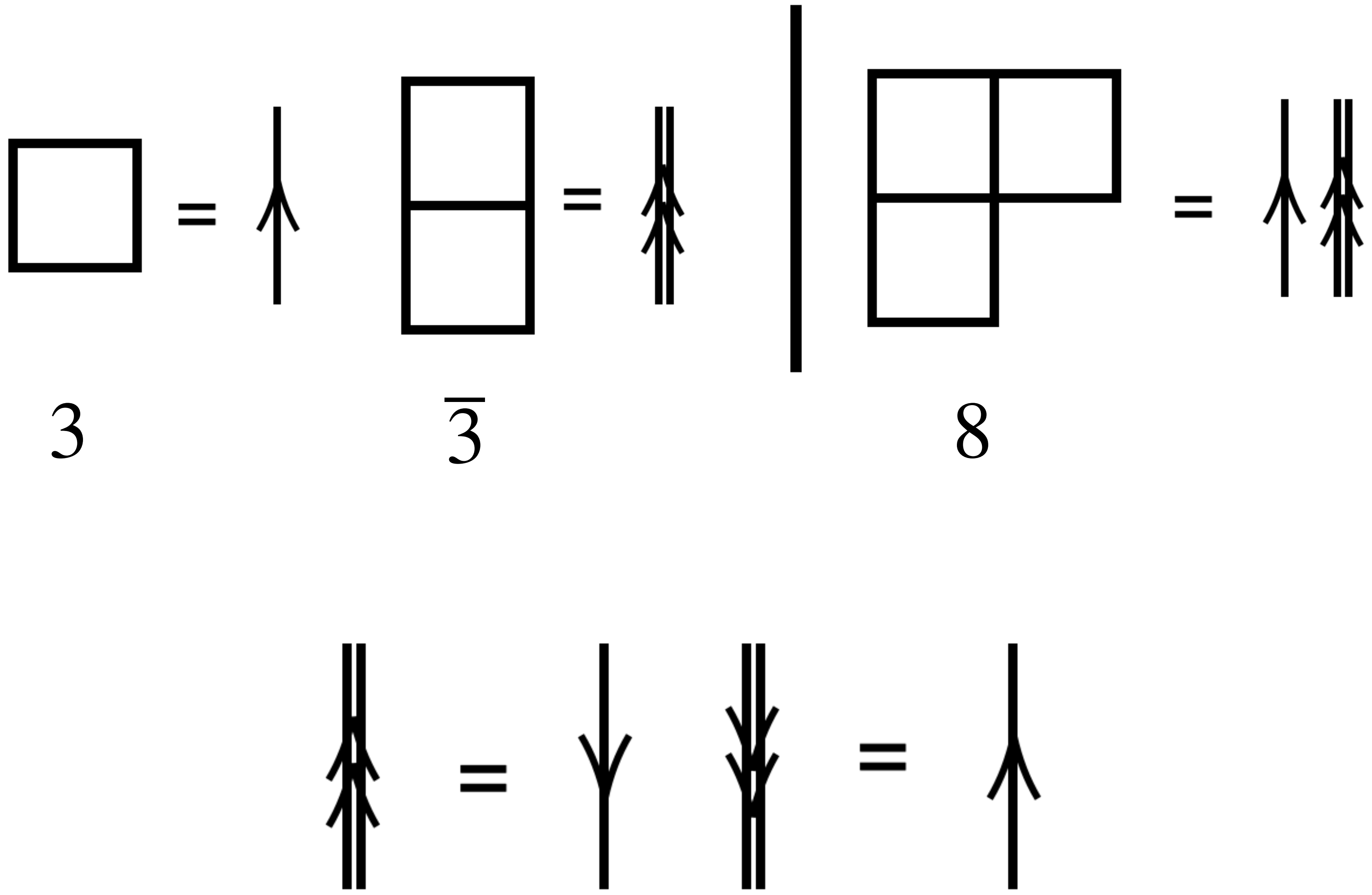
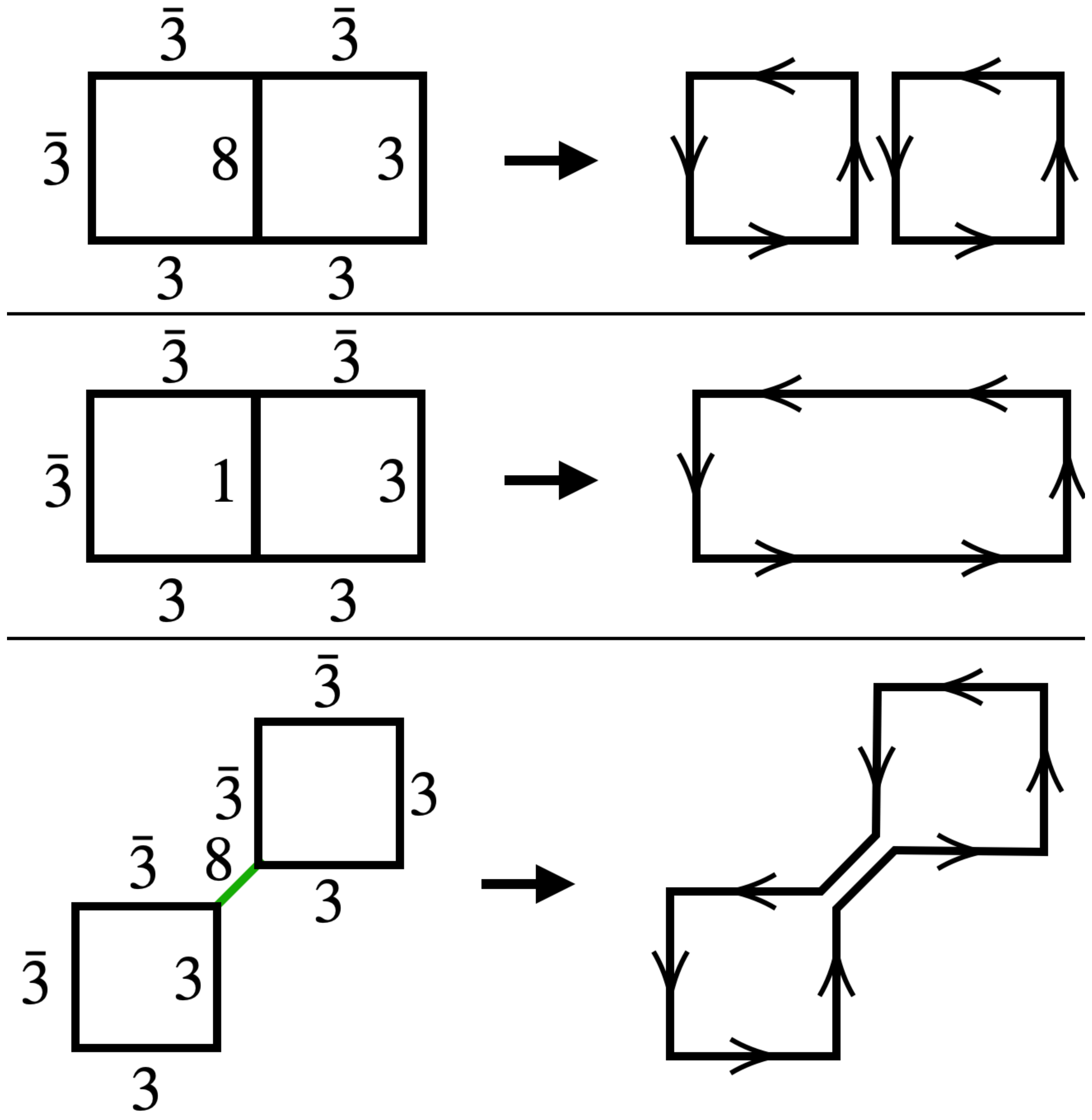


Step 2: Represent states on the lattice through loops and combinations of overlapping links



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Step 3: Ground state in interacting theory obtained by acting with electric (diagonal) or plaquette ops on electric vacuum

$$|\Omega\rangle = \exp\left[-i\int_0^T \mathrm{d}t\left(H_E + \lambda(t)H_B\right)\right]|0\rangle_E \qquad \lambda(0)=0,,\lambda(T)=1$$

Excited states (with simplest topology) require additional action on interacting ground state with more electric or plaquettes ops on ground state

General state in interacting theory (simplest topology) can therefore be written as

$$\left|\{P_p, \bar{P}_p\}\right\rangle \equiv \prod_p \hat{\Box}_p^{P_p} \hat{\Box}_p^{\dagger \bar{P}_p} |0\rangle$$

Can represent this state as superposition of basis of loop states

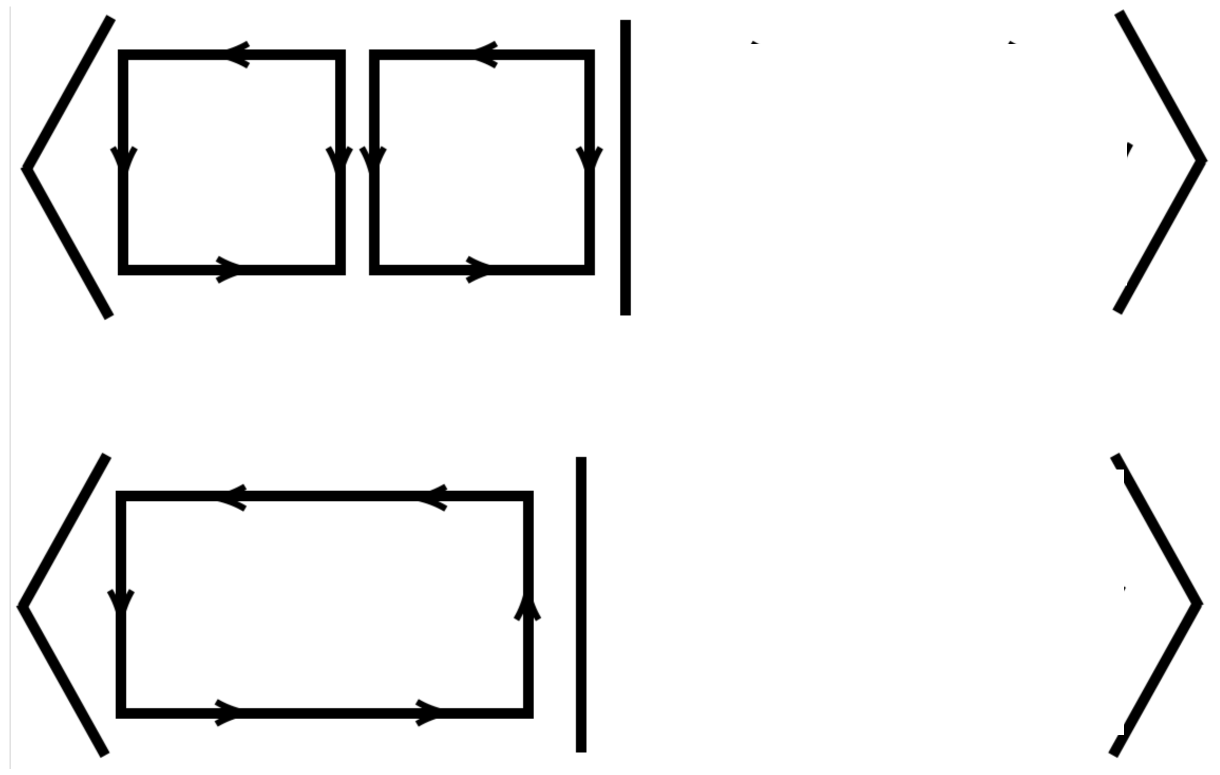
$$\left|\{P_p, \bar{P}_p\}\right\rangle = \sum_{\{L_i, a_\ell\}} \left\langle \{L_i, a_\ell\} \middle| \{P_p, \bar{P}_p\} \right\rangle |L_i, a_\ell\rangle$$

The basic idea of the 1/Nc expansion is to expand the amplitudes of states and terms in the Hamiltonian in powers of 1/Nc

Step 4: Expand overlap of $\langle \{L_i, a_\ell\} \,|\, \{P_p, \bar{P}_p\} \rangle$ using general formula

$$\int dU \prod_{n=1}^{q} U_{i_n j_n} U^*_{i'_n j'_n} = \frac{1}{N_c} \sum_{\text{permsk}} \prod_{n=1}^{q} \delta_{i_n i'_{k_n}} \delta_{j_n j'_{k_n}} + \mathcal{O}(1/N_c)$$

Expansion can be done graphically



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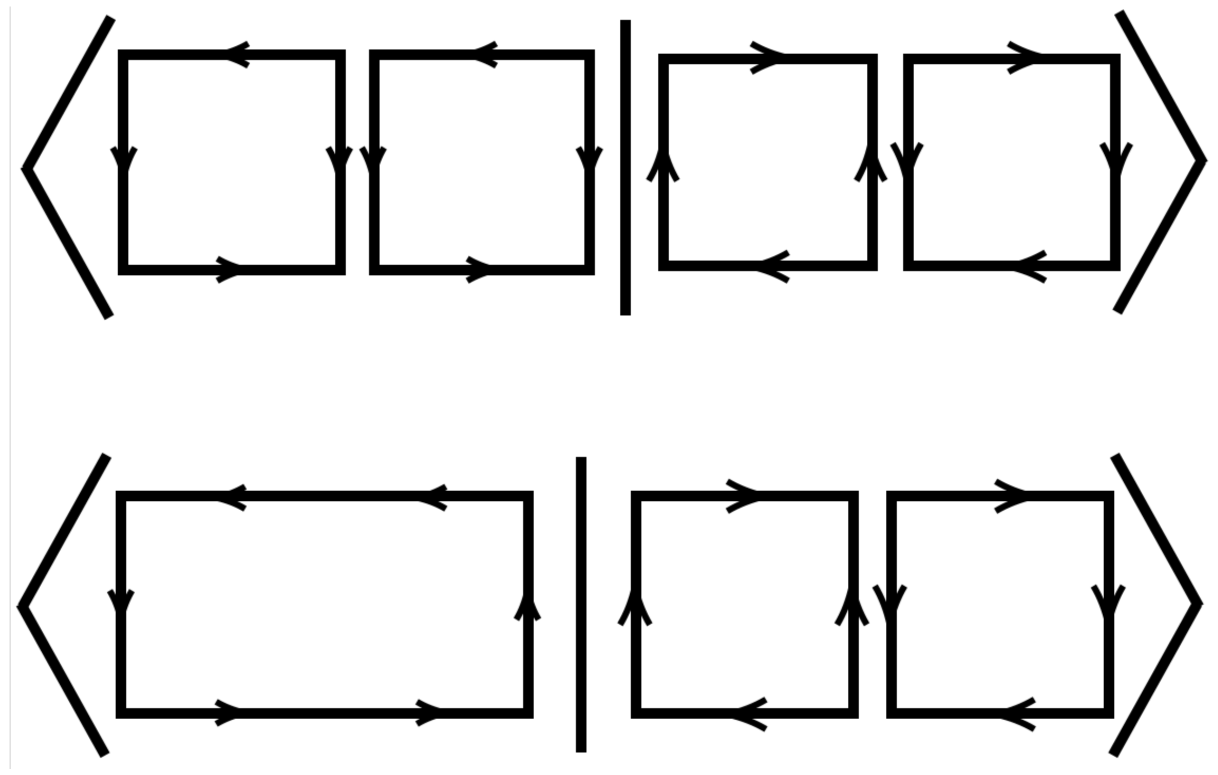

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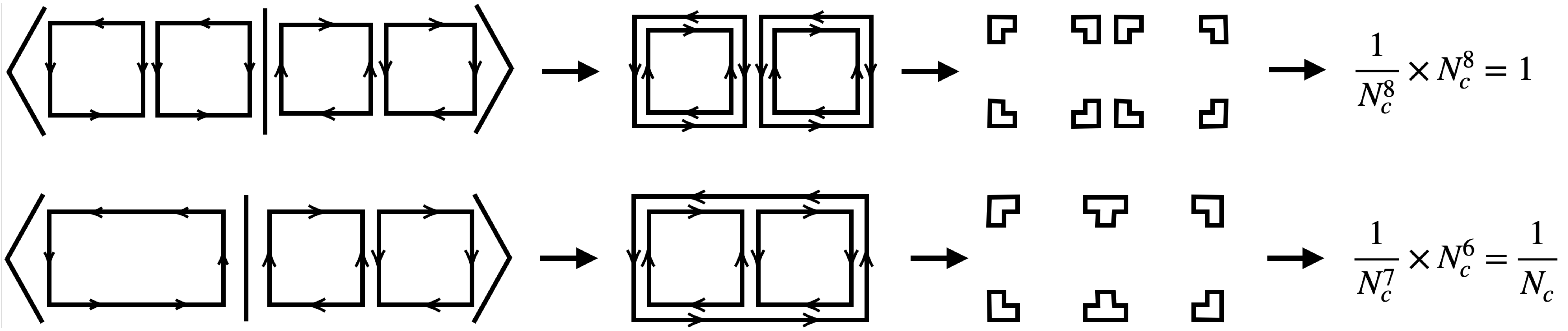


The basic idea of the 1/Nc expansion is to expand the amplitudes of states and terms in the Hamiltonian in powers of 1/Nc

Step 4: Expand overlap of a given loop state with the state $|\{P_p, \bar{P}_p\}\rangle$ using general formula

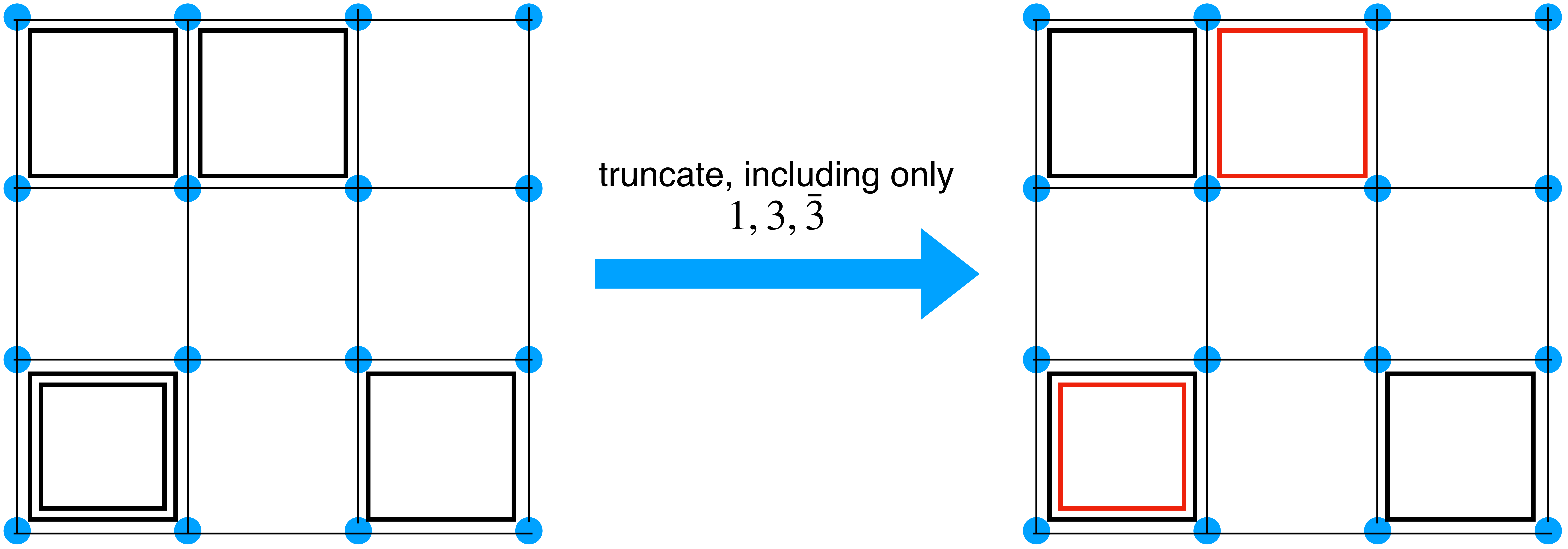
Putting results together, can show that final result is given by

$$\langle\{L_i, a_\ell\}|\{P_p, \bar{P}_p\}\rangle \propto \prod_i N_c^{1-m_i}$$

$\prod_i$: multiplying contributions from each loop. m_i : number of plaquettes encircled by each loop

To leading order in $1/N_c$ only have states with loop encircling a single plaquette

Now that we know that only loops encircling single plaquettes are allowed at large N_c, add truncation of Hilbert space



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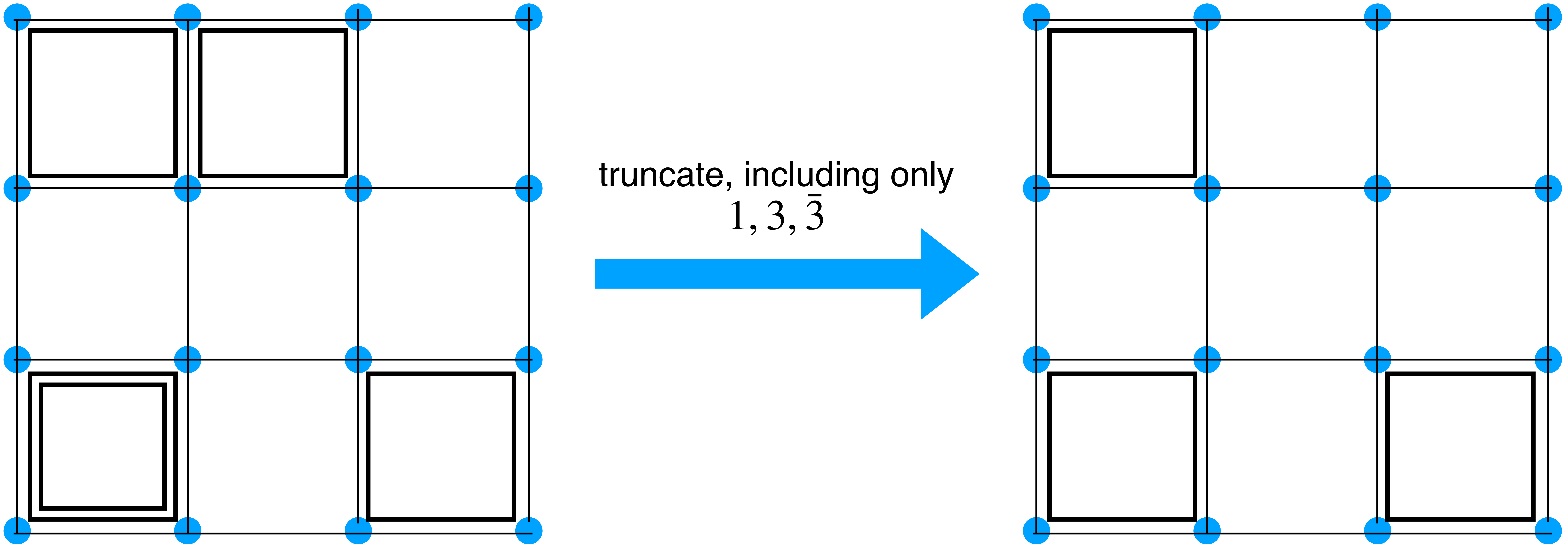
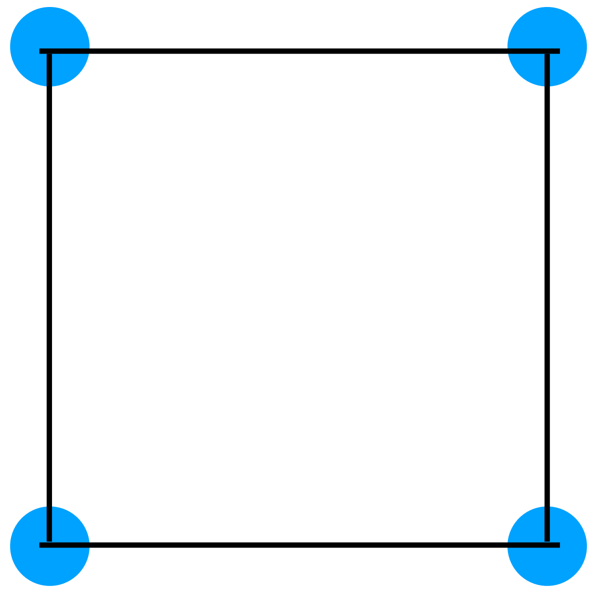
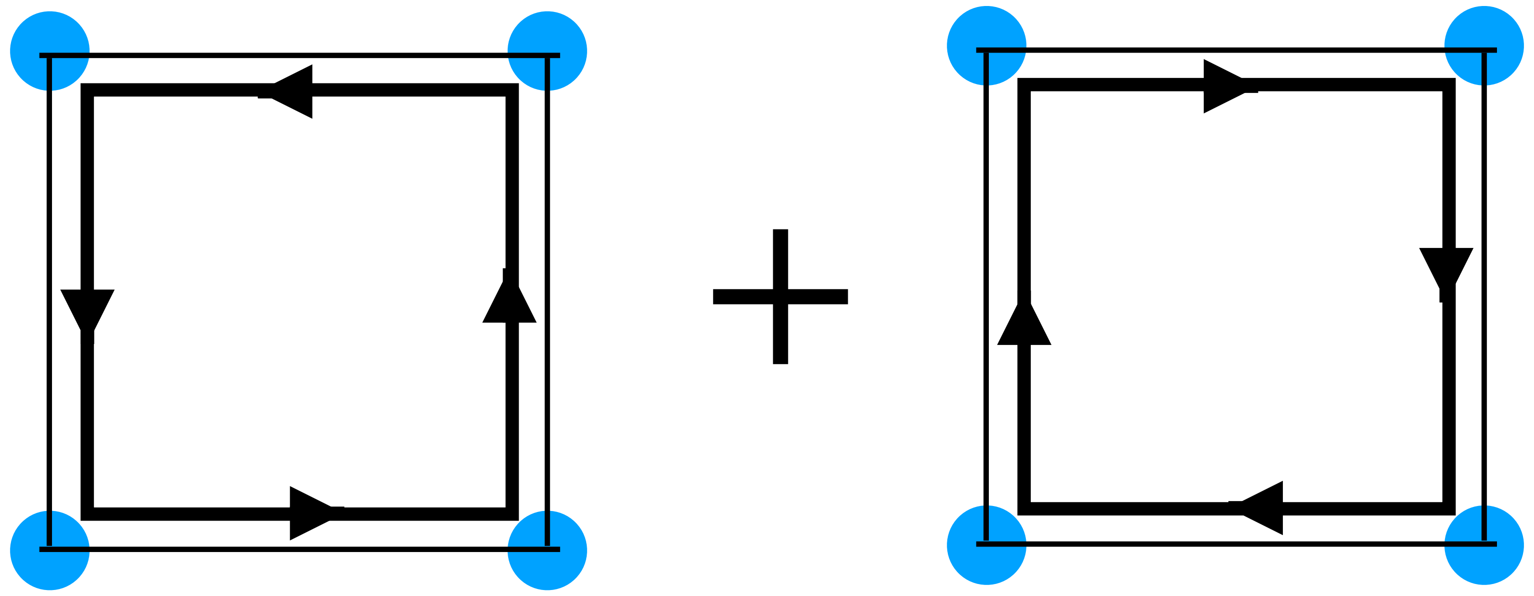


Finally, theory has charge conjugation symmetry

$$\mathrm{C-even:} \quad \left(|\circlearrowright\rangle + |\circlearrowleft\rangle\right)/\sqrt{2} \qquad \mathrm{C-odd:} \quad \left(|\circlearrowright\rangle - |\circlearrowleft\rangle\right)/\sqrt{2}$$

Final structure of Hilbert space at leading order in $1/N_c$ and simplest non-trivial truncation is very simple. Needs one qubit per plaquette

$|0\rangle =$ 

$|1\rangle =$ 

Neighboring plaquettes can not be excited at the same time

One can now work out the Hamiltonians. H_E is diagonal, while H_B contains plaquette operators that can excite plaquettes if neighboring plaquettes unexcited

$$\hat{H} = \sum_p \left(\frac{8}{3} g^2 - \frac{1}{2g^2} \right) \hat{P}_{1,p} - \frac{1}{\sqrt{2}g^2} \hat{P}_{0,p+x} \hat{P}_{0,p-x} \hat{P}_{0,p+y} \hat{P}_{0,p-y} \hat{X}_p$$

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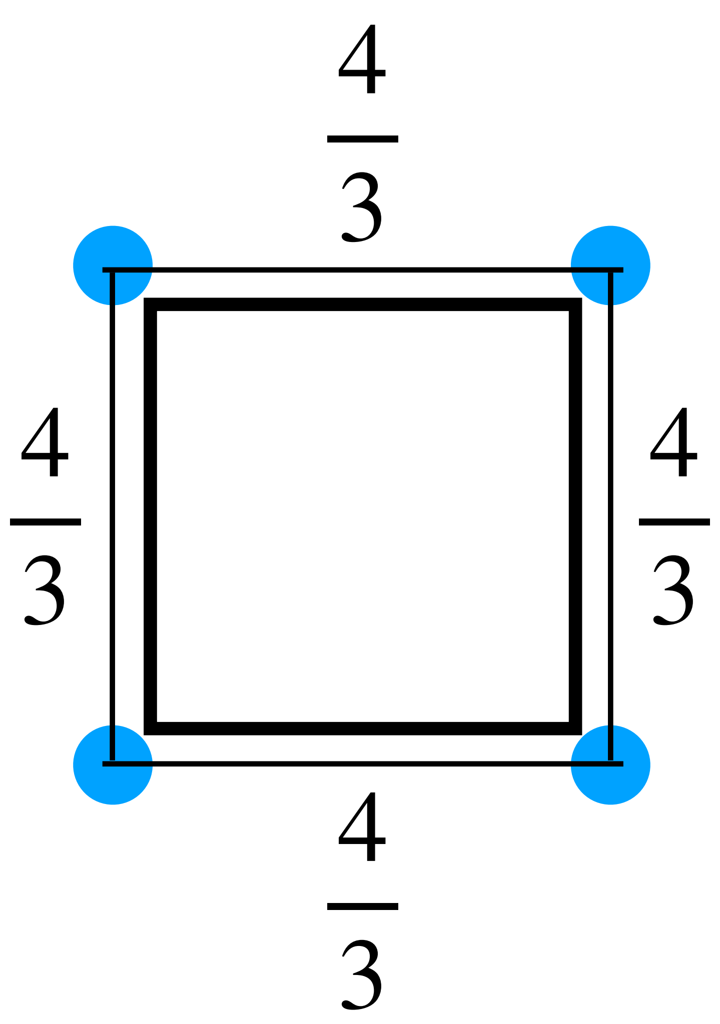

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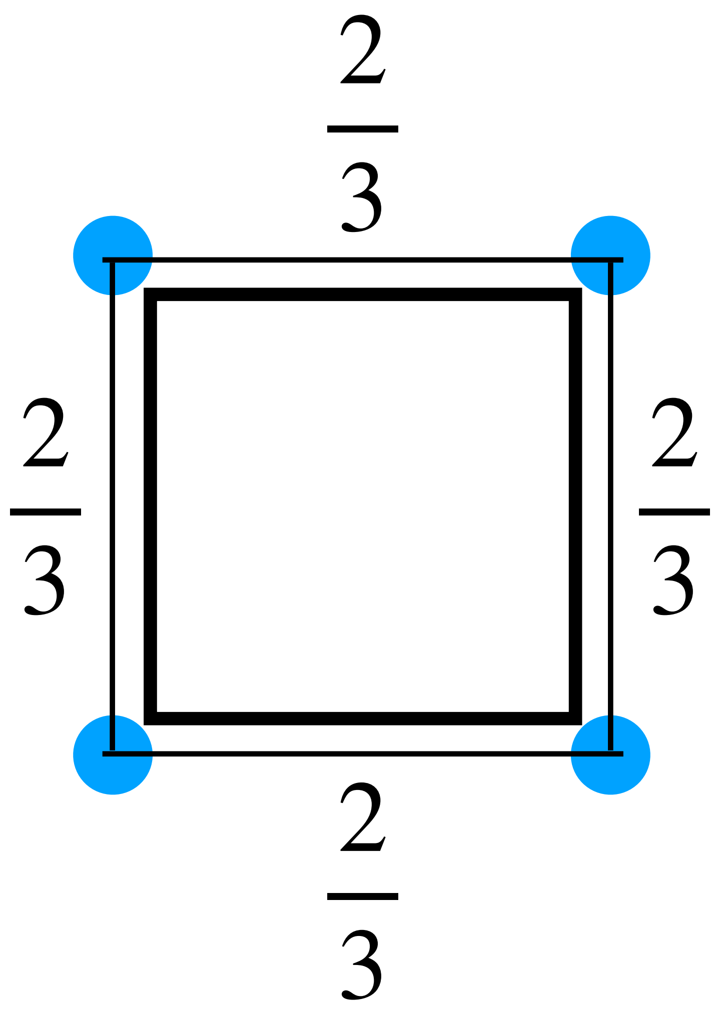

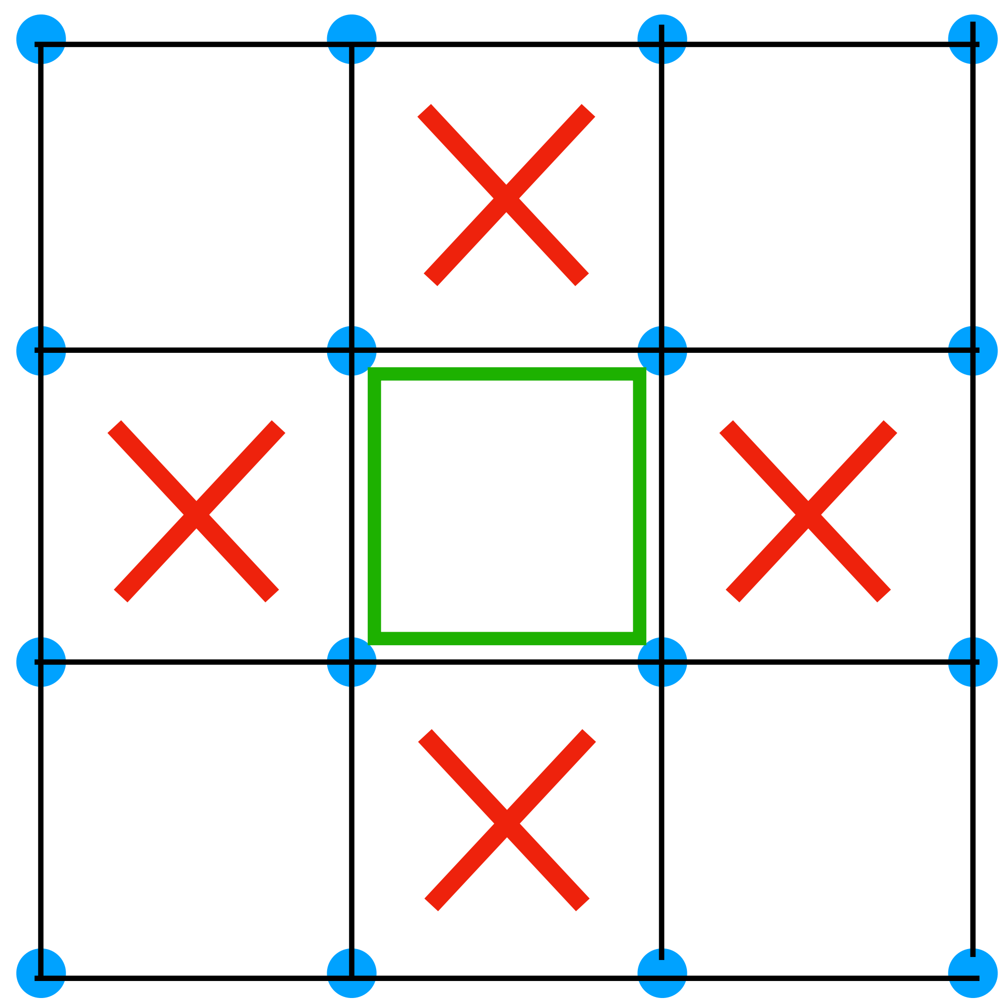

To implement this system, we need to deal with the realities of existing quantum hardware. Most difficult term is multicontrolled term in Hamiltonian

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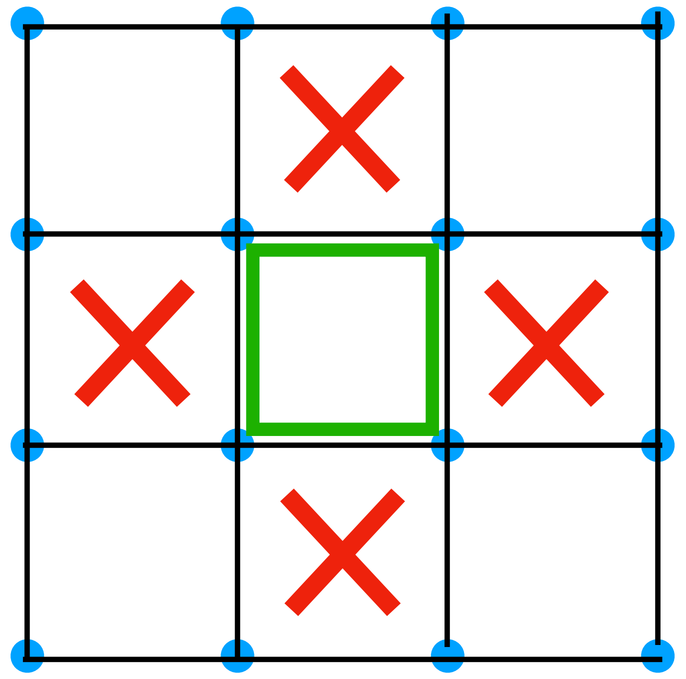
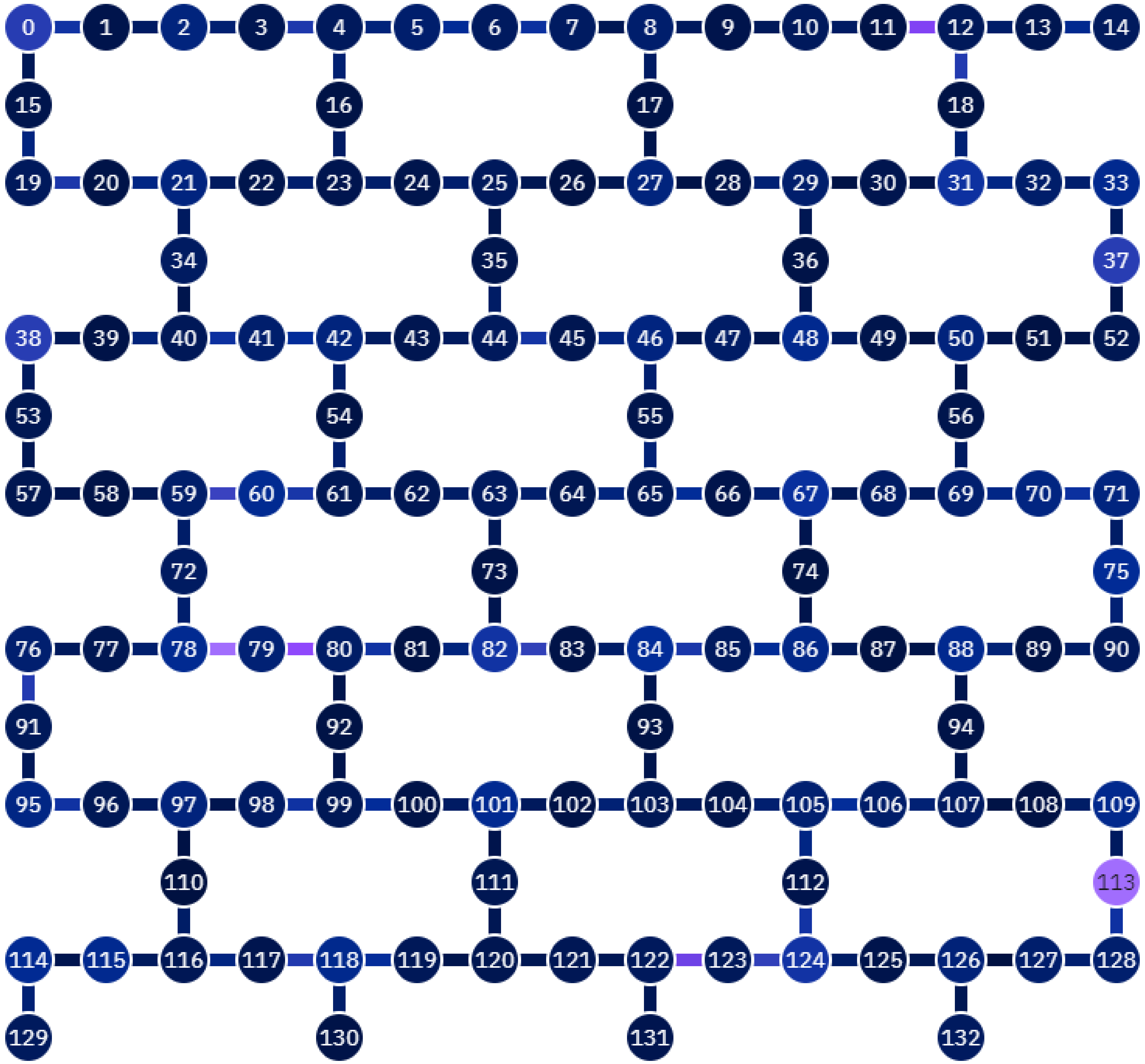


Multicontrolled operations difficult, and typically don't have correct connectivity



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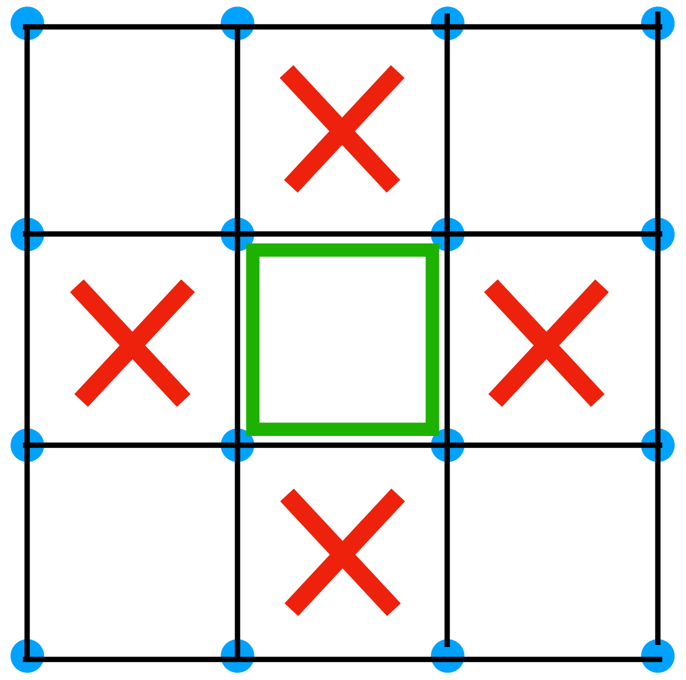

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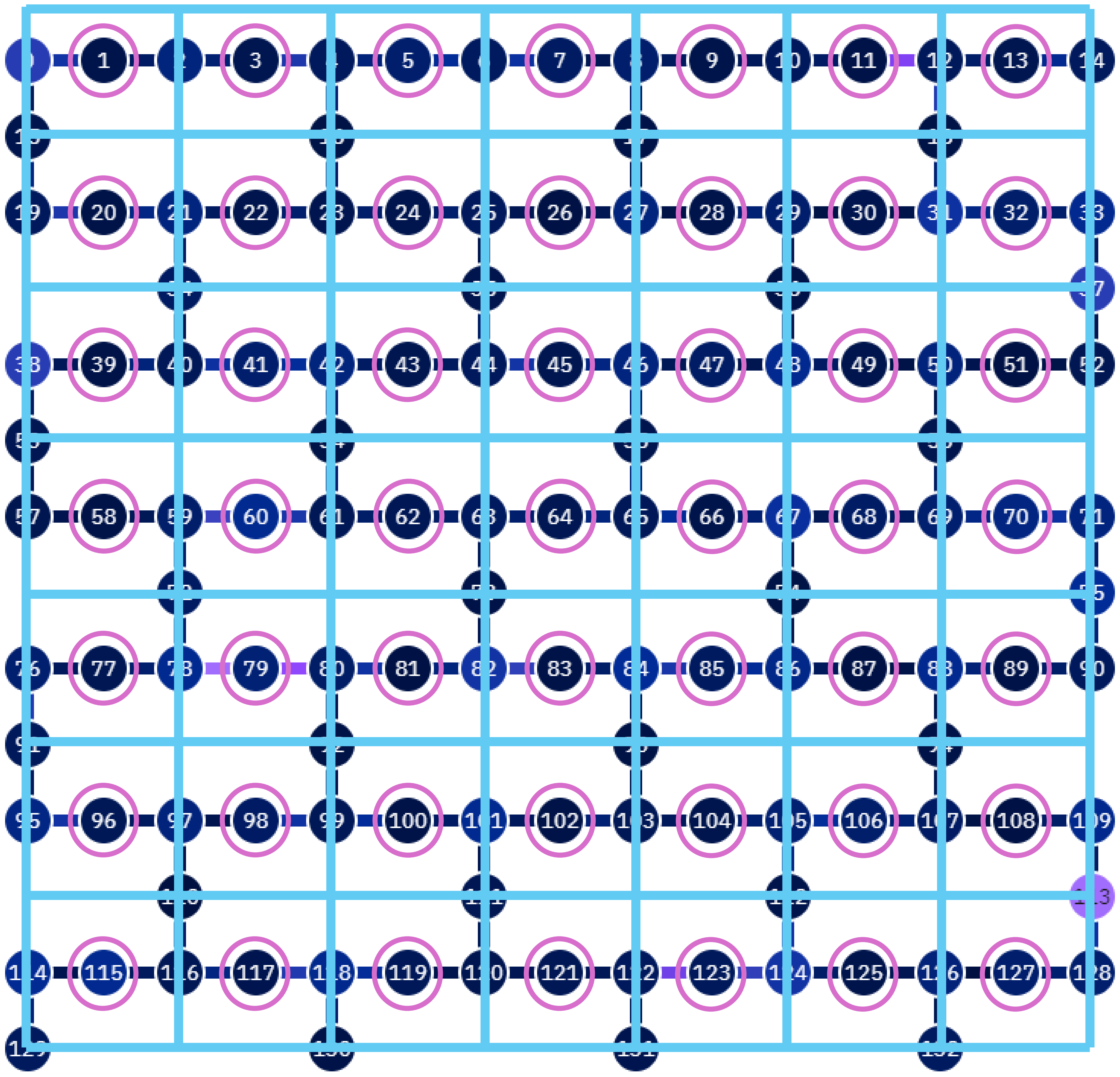

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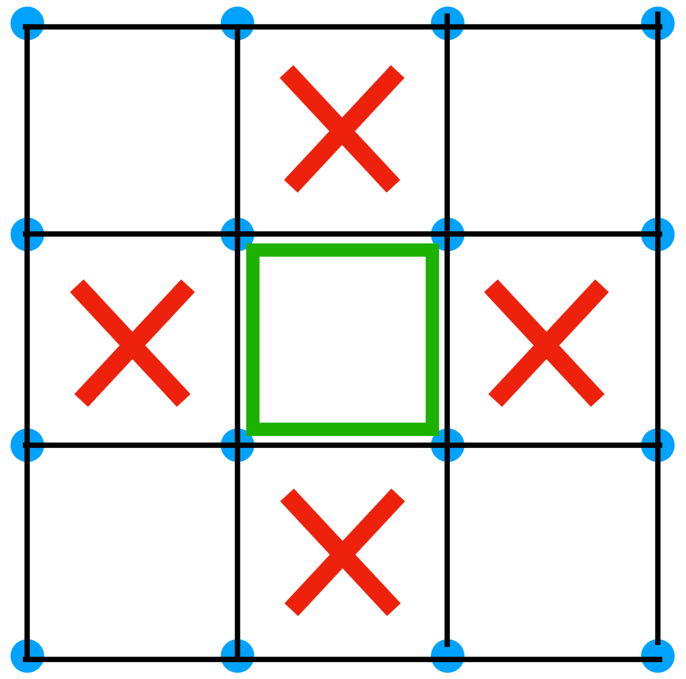

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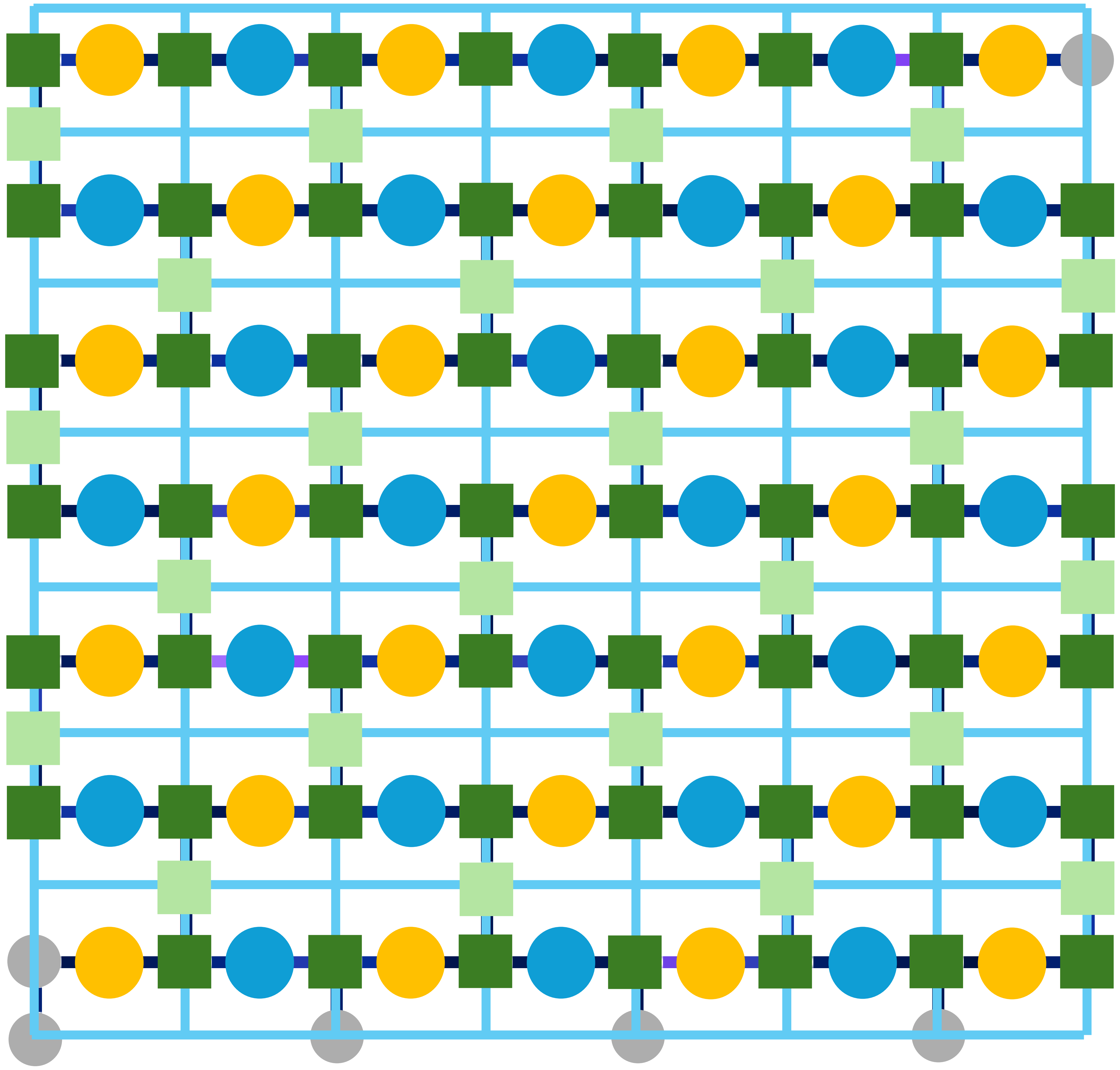

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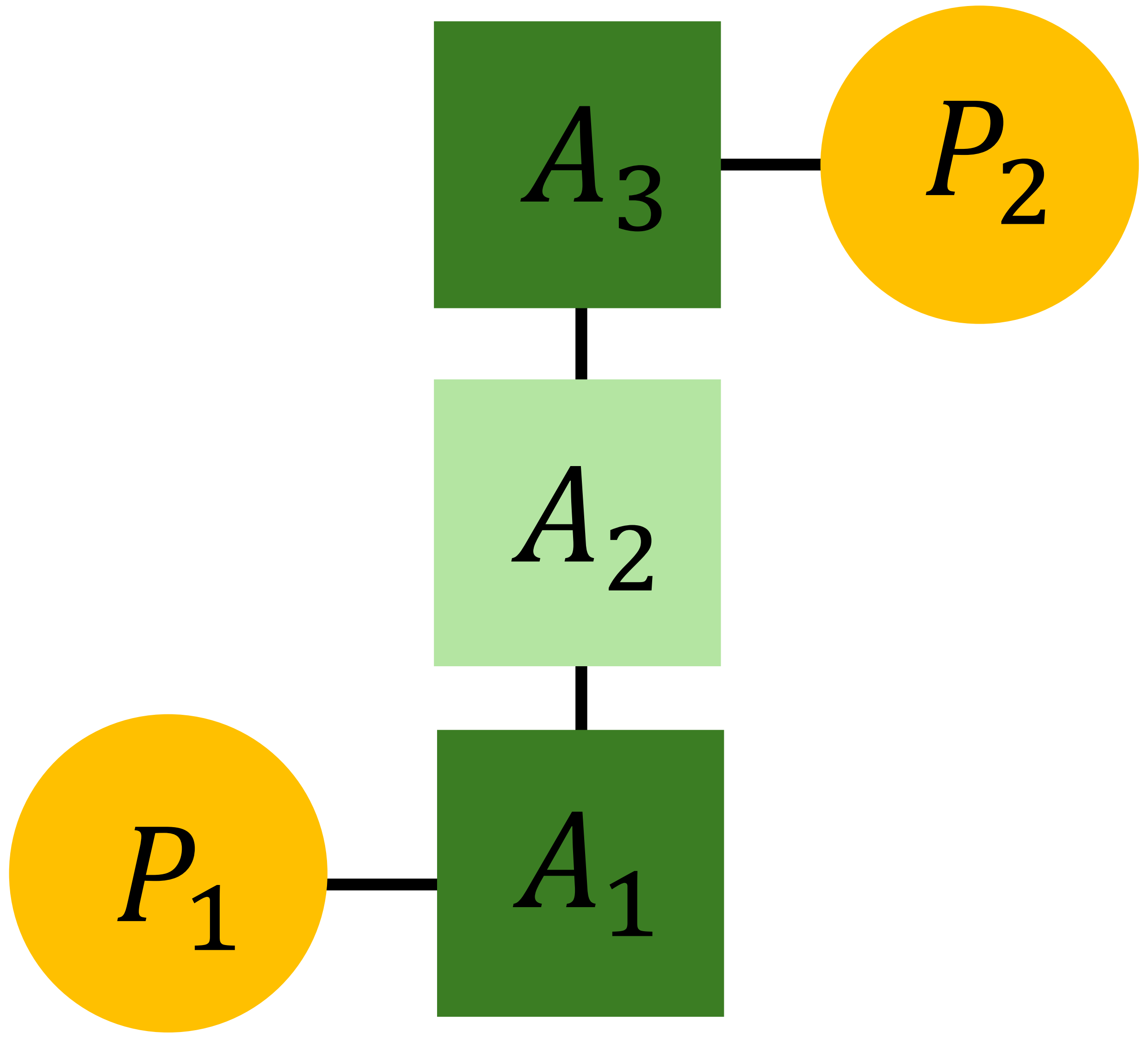

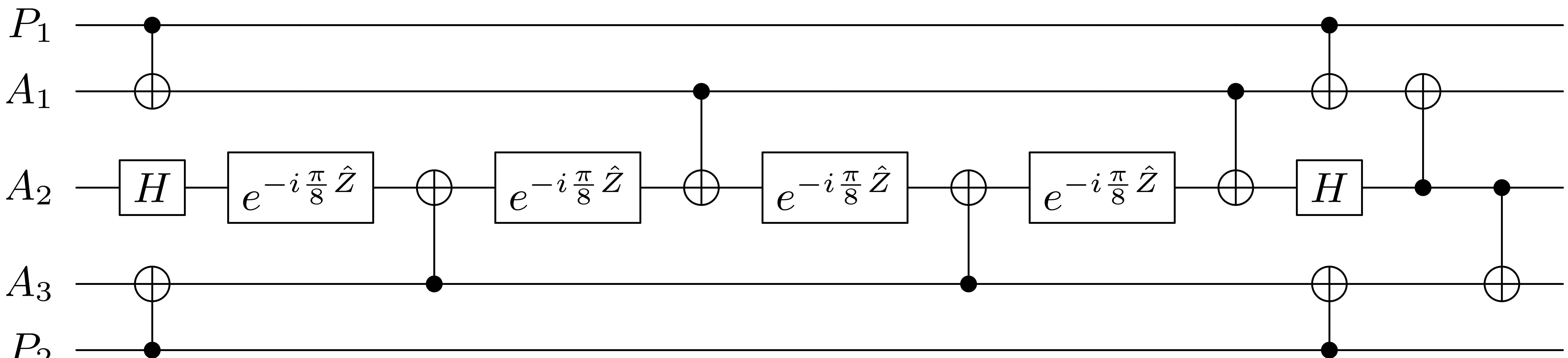

(b) Ancilla Preparation Circuit

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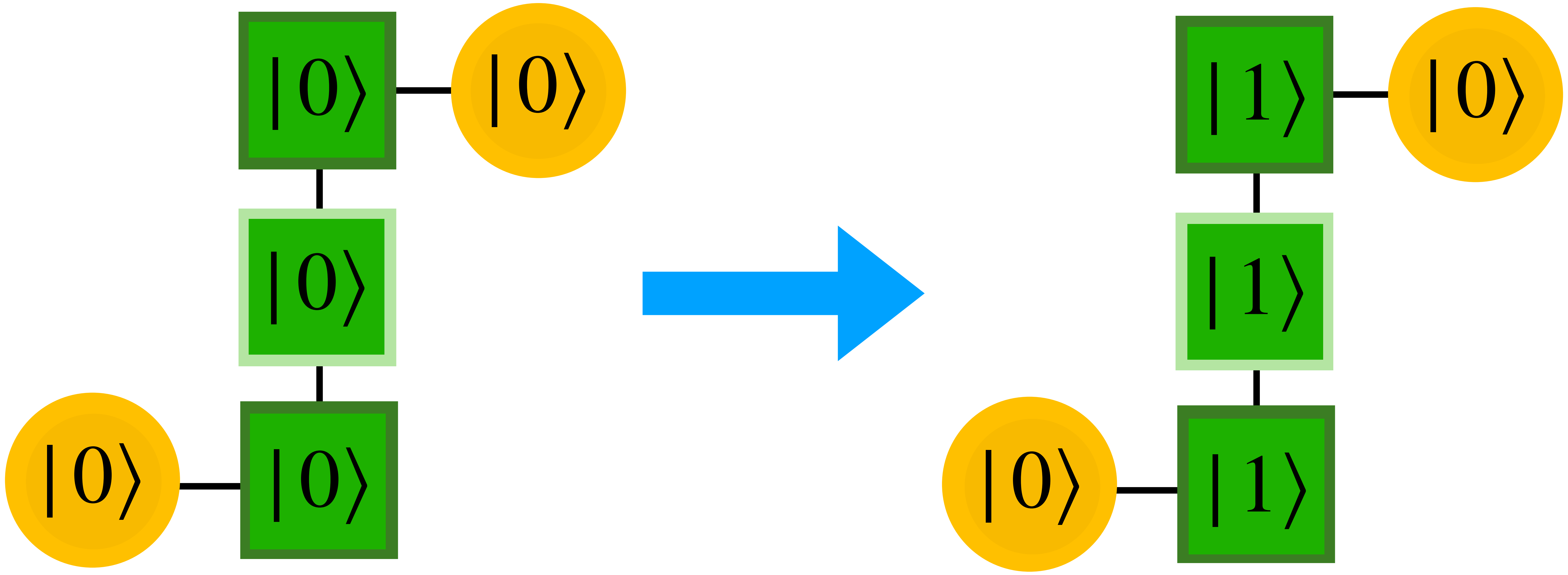

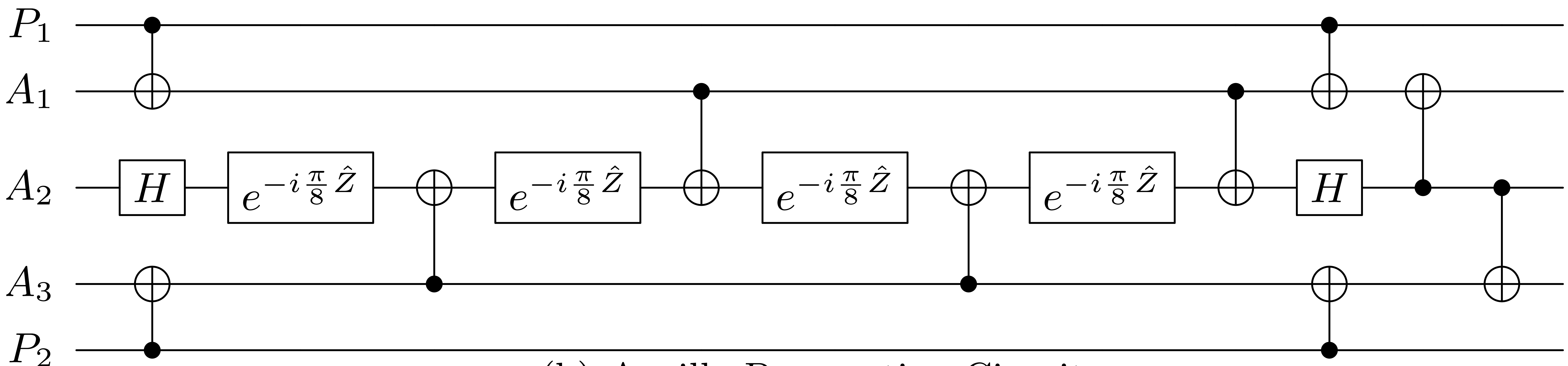

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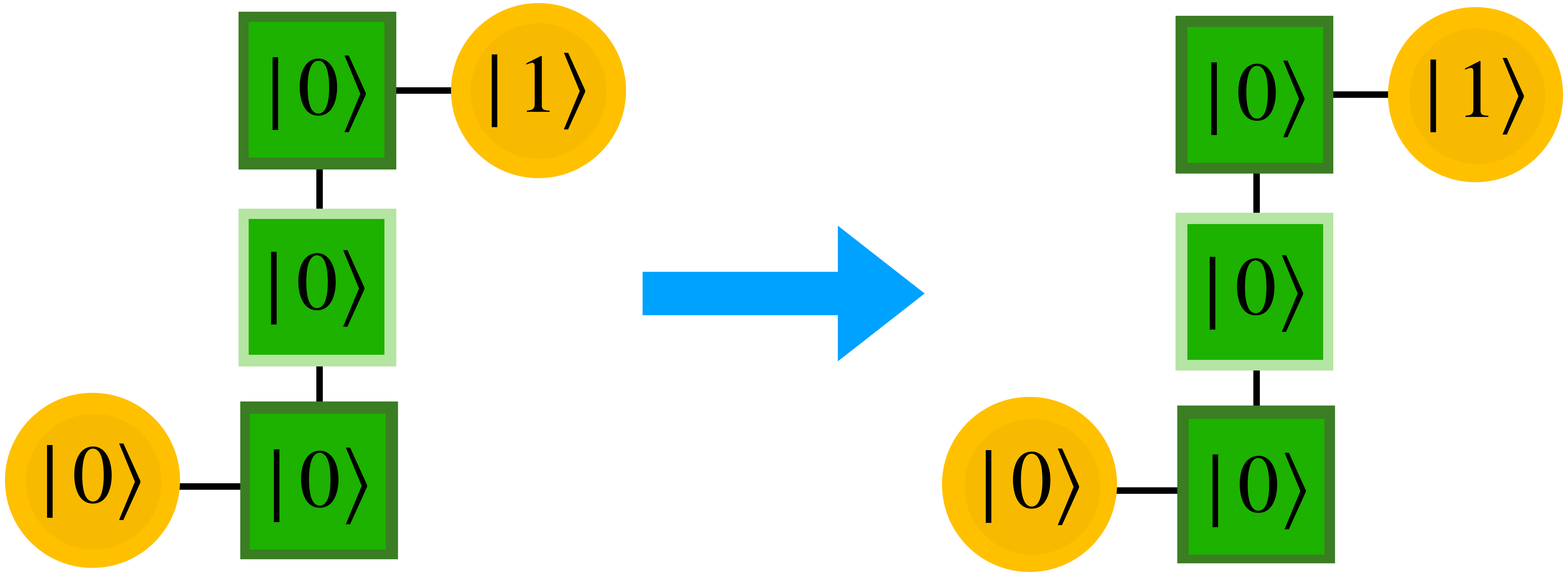

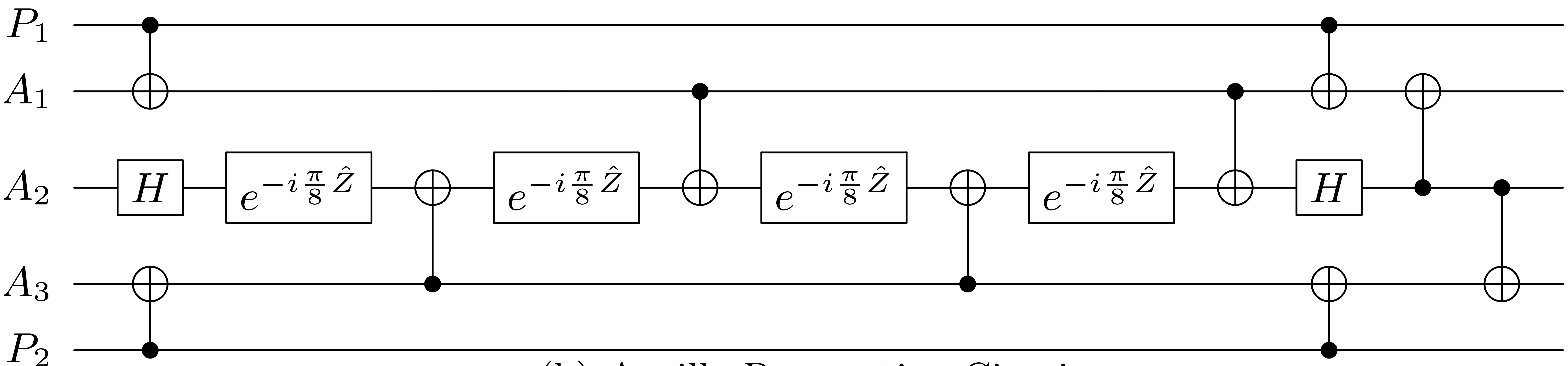

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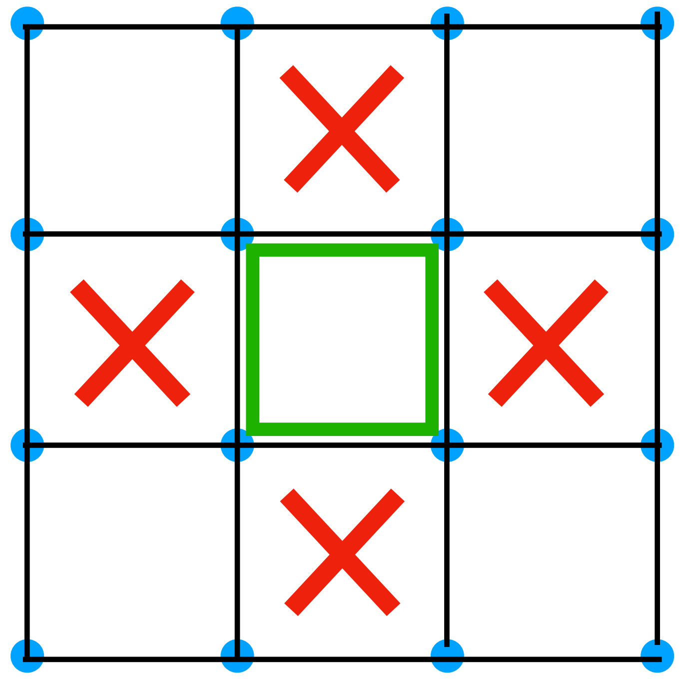

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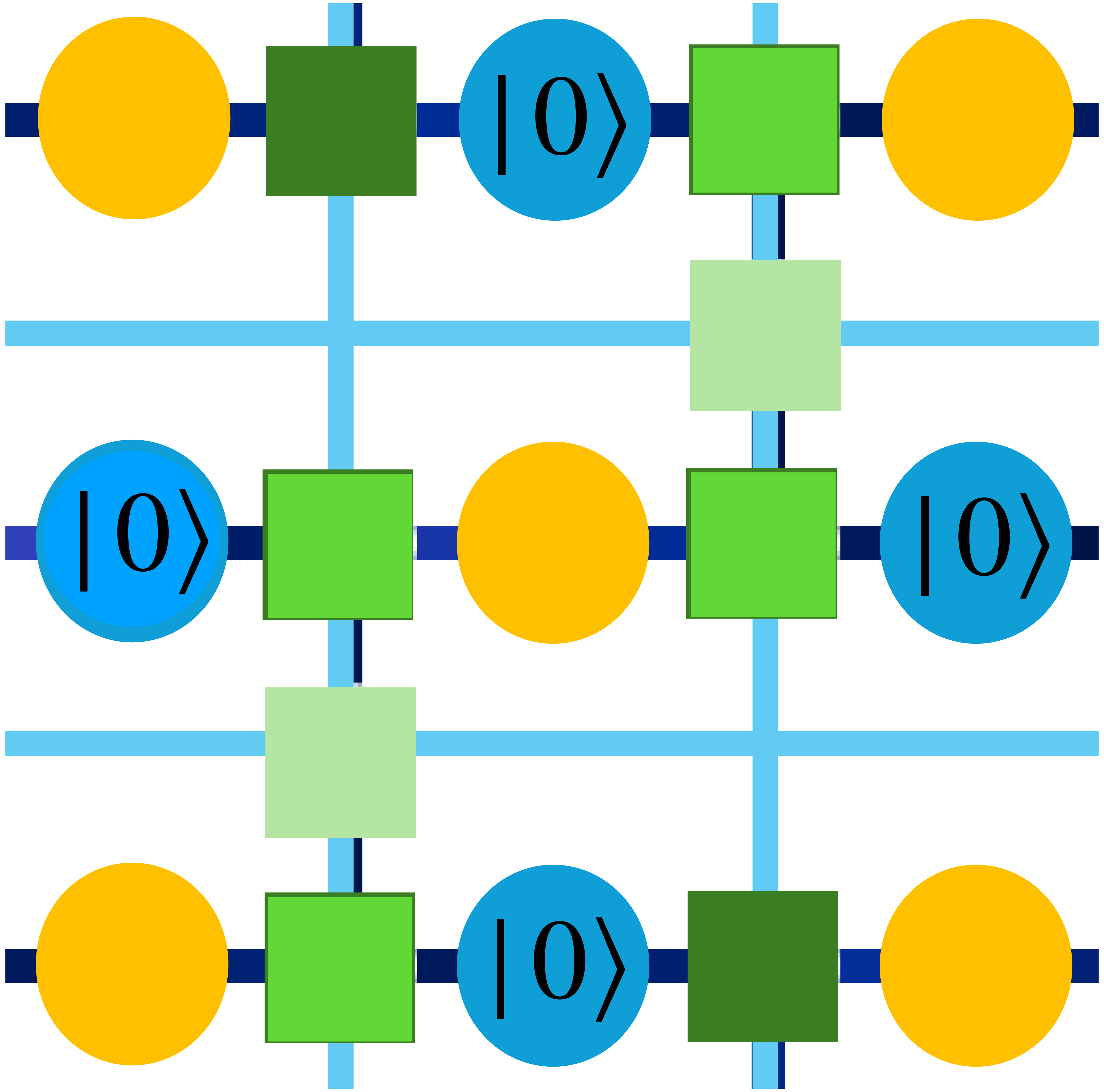
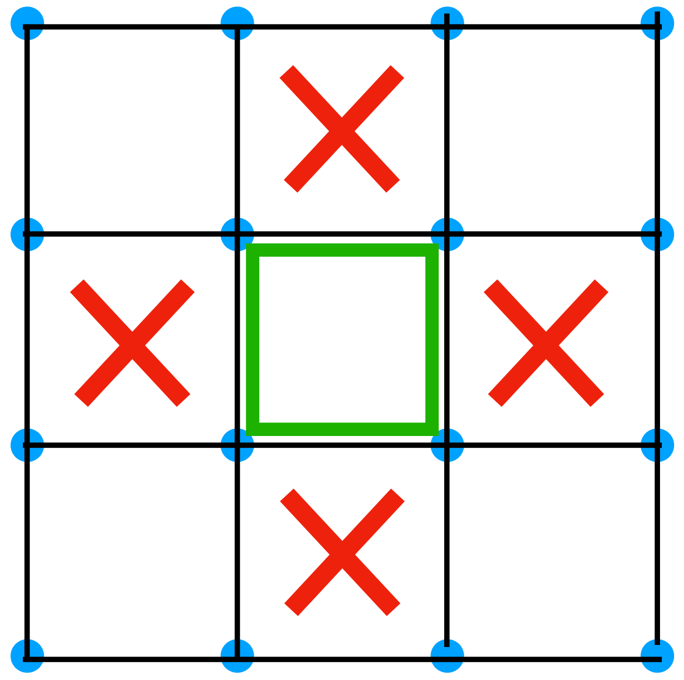
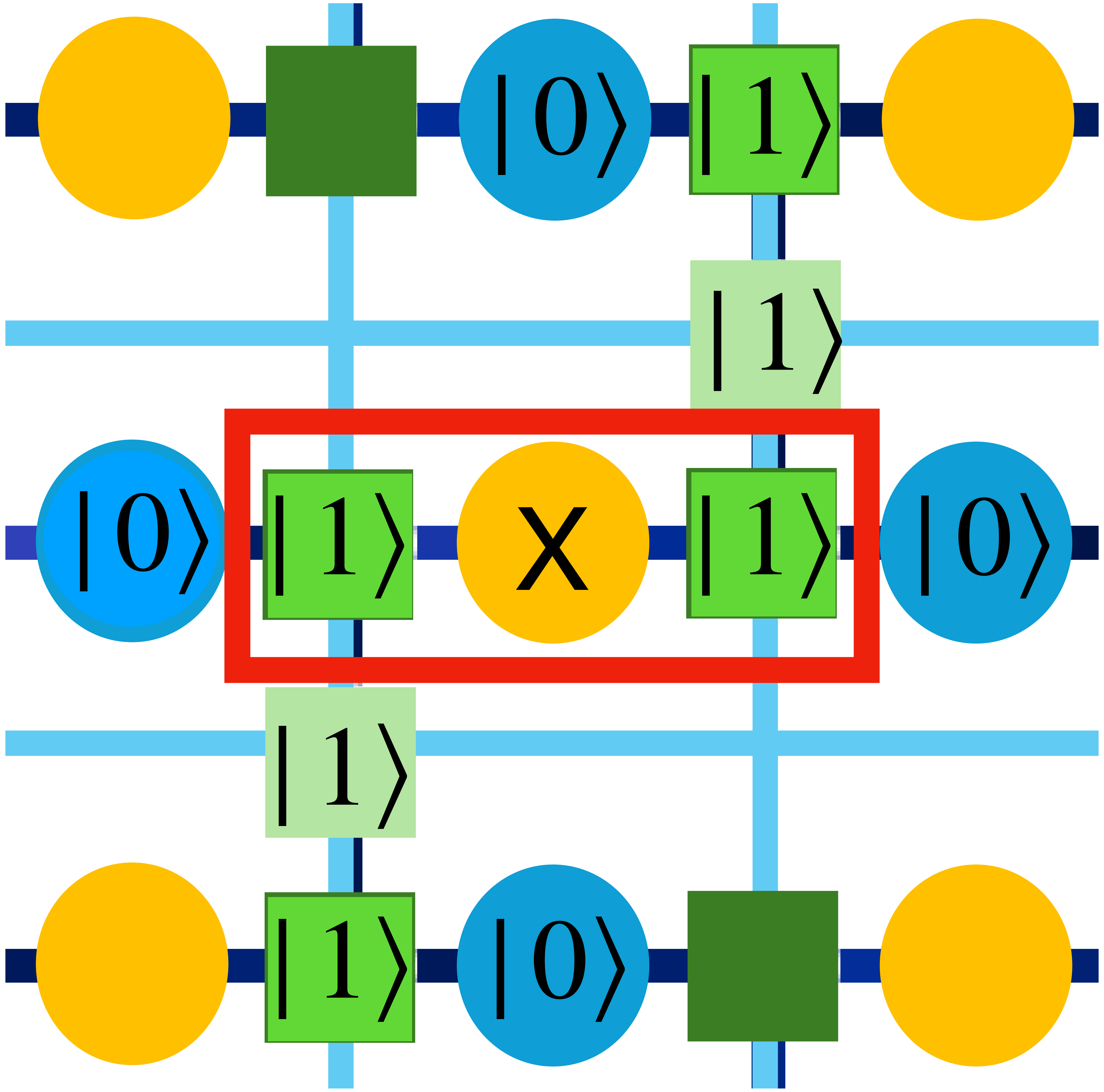


To implement this system, we need to deal with the realities of existing quantum hardware. Most difficult term is multicontrolled term in Hamiltonian

$$\frac{1}{\sqrt{2}g^2}\hat{P}_{0,p+x}\hat{P}_{0,p-x}\hat{P}_{0,p+y}\hat{P}_{0,p-y}\hat{X}_p$$



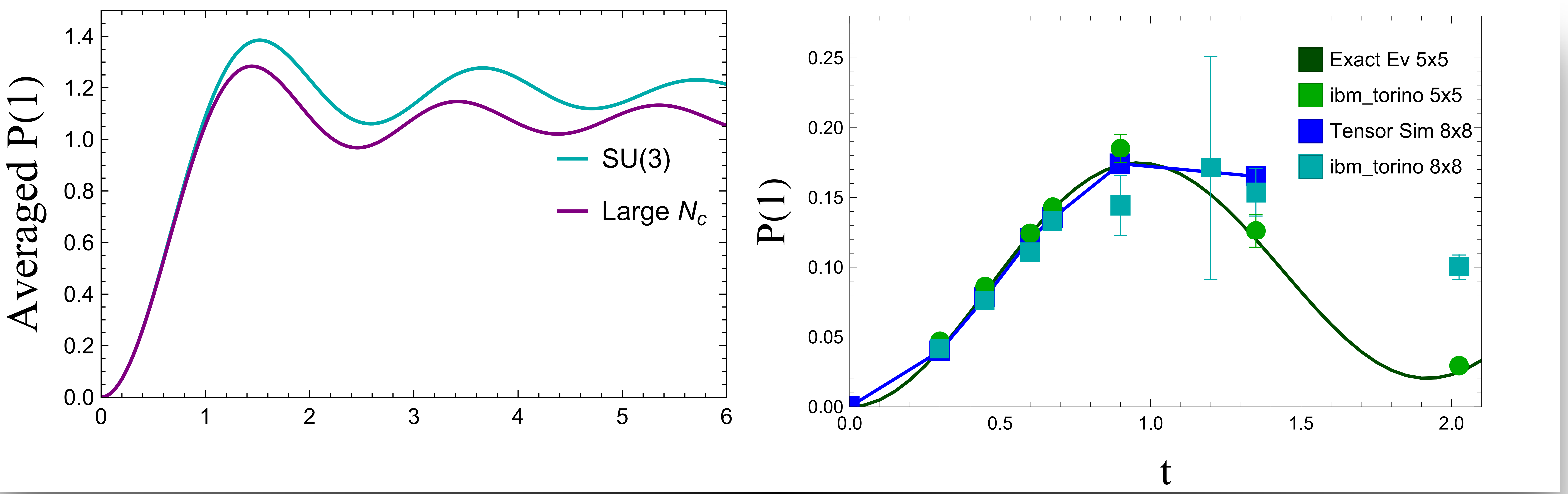
Multicontrolled operations difficult, and typically don't have correct connectivity



A $1/N_c$ expansion in QCD is quite standard in many classical applications. Can it help in quantum simulation?

Gives dramatic simplifications on the size of the allowed Hilbert space and dramatically simplifies interactions

Results obtained on 8x8 lattice (25 times more plaquettes than previous best)



Adding an expansion in $1/N_c$ dramatically reduces the available Hilbert space, and also dramatically simplies the Hamiltonian.

This has allowed for the first simulation of SU(3) YM theory on digital IBMQ quantum computers in 2+1D

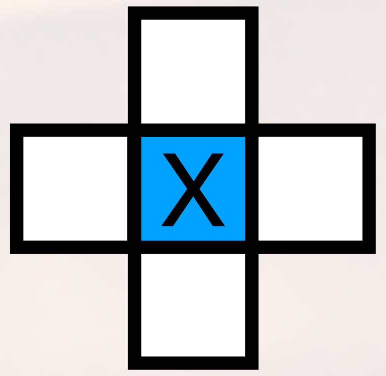
There is one more very interesting fact abot the Hamiltonian we have derived. It is very closely related to the Hamiltonian describing the dynamics of neutral rydberg atoms

The Hamiltonian has a relatively simple form

$$\hat{H} = \left(\frac{8}{3}g^2 - \frac{1}{2g^2}\right)\sum_p \hat{P}_{1,p} - \frac{1}{\sqrt{2}g^2}\sum_p \hat{P}_{0,p+x}\hat{P}_{0,p-x}\hat{P}_{0,p+y}\hat{P}_{0,p-y}\hat{X}_p$$

$$\hat{P}_{0,p} = |0\rangle_{pp}\langle 0|\,, \qquad \hat{P}_{1,p} = |1\rangle_{pp}\langle 1|$$

Second term applies an $\hat{X}$ gate if neighboring qubits are $|0\rangle$



Closely related to a different Hamiltonian

$$\hat{H} = -\Delta\sum_p \hat{P}_{1,p} + \frac{\Omega}{2}\sum_p \hat{X}_p - \sum_{pp'}\frac{1}{d^6_{pp'}}\hat{P}_{1,p}\hat{P}_{1,p'}$$

Article

Quantum phases of matter on a 256-atom programmable quantum simulator

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Can study QCD using analog neutral atom arrays

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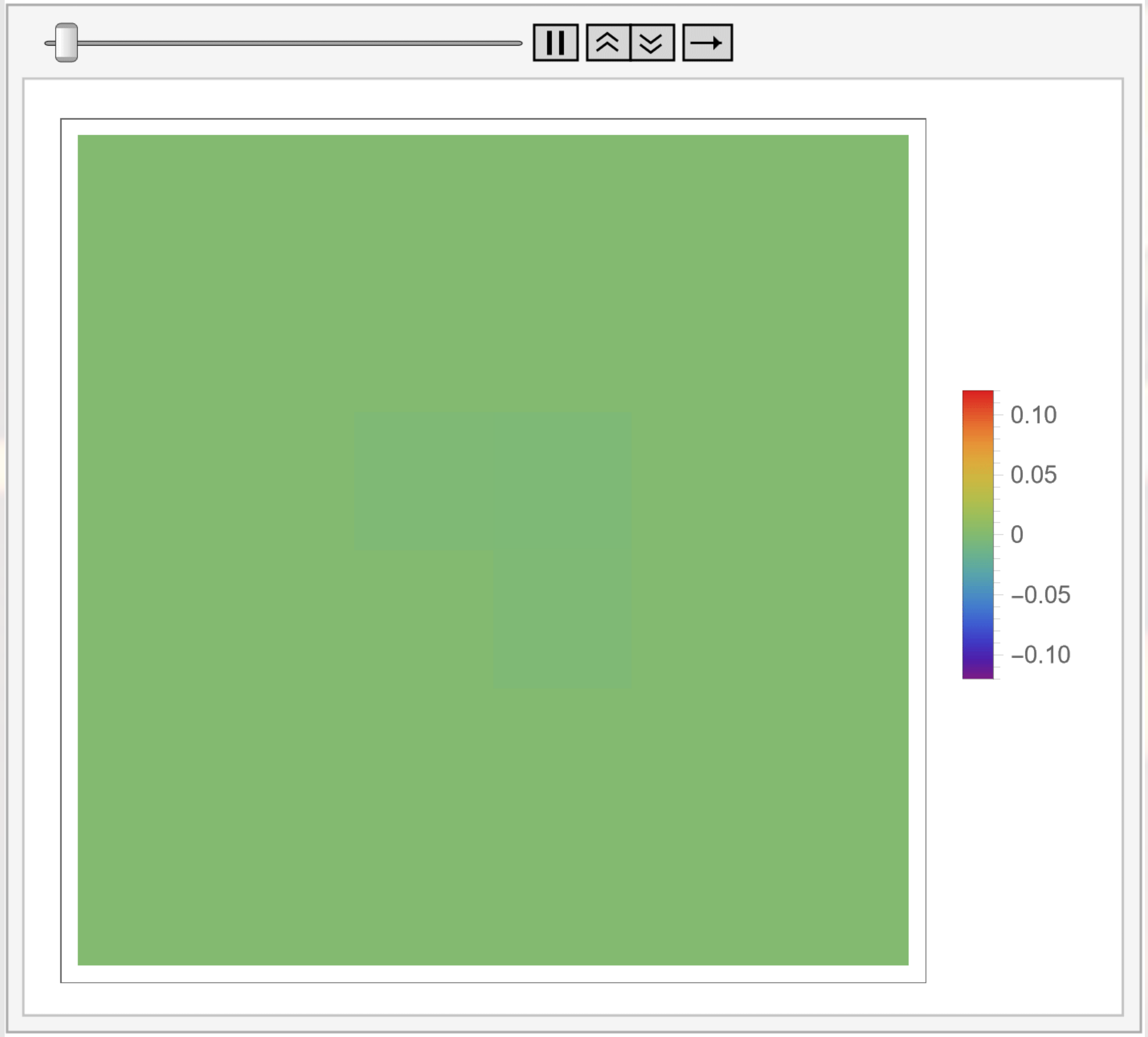
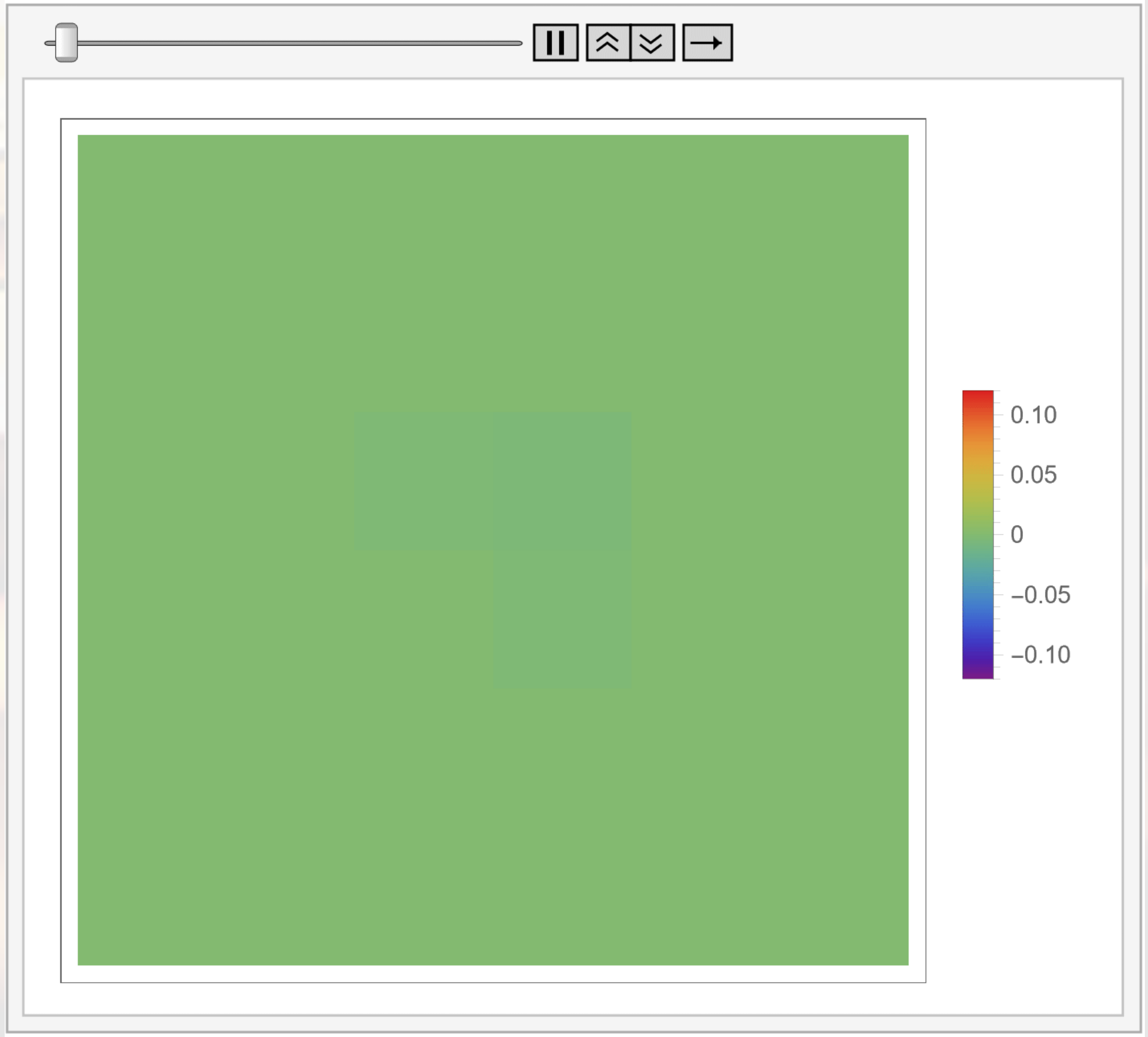
The Hamiltonian has a relatively simple form



Closely related to a different Hamiltonian



This is just the beginning. Hamiltonians describing HEP related problems have many features that make for interesting interactions

- Things get more complicated in 3 dimensions; Can we create systems that allow us to still do the required simulations?
- Going beyond the $1/N_c$ expansion will add other terms to the Hamiltonian; Can we still use analog systems to do the simulations?
- So far have focused on superconducint digital and analog neutral atoms; Are there other systems (trapped ions etc) that might be even better?
- We do have ideas how to use Rydberg blockade for digital simulations; Can we try that out and improve on the techniques?
- We are developing other formulations of QCD that might be better suited for some set of parameter values; Can we use similar techniques?

A $1/N_c$ expansion in QCD is quite standard in many classical applications. Can it help in quantum simulation?

Gives dramatic simplifications on the size of the allowed Hilbert space and dramatically simplifies interactions

Results obtained on 8x8 lattice (25 times more plaquettes than previous best)

I believe that this opens the door for quantum simulation of QCD through a systematic expansion, where higher order effects can be included as computing hardware improves

QUESTIONS?