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CROIT

Center for Research in Quantum Information and Technology



Quantum Technologies and Computation for High Energy Physics

Hamiltonian Simulation of Non-Abelian Gauge Theories Using Loops Strings and Hadrons

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Birla Institute of Technology and Science Pilani, K K Birla Goa Campus

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Our role

- Identifying the physics problem that would benefit from quantum computation
- Reformulating the problem suitable for quantum computation
- NISQ-era quantum simulation algorithms: analog and digital
- Think beyond NISQ era...

**Lattice gauge theory calculations without sign problem:
Real time dynamics**

Classical Computation Era

Lattice gauge theory calculations

Quantum Computation Era

- Requires different theoretical framework.
- Addressed different objectives
- Computational Methods are entirely different.

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

Intermediate steps:

- Suitable development and choice of framework.
- Suitable choice of variables/basis.
- Algorithm development for various tasks- classical/quantum/hybrid.
- Quantum information theoretic understanding - connection to physics of QCD
- Quantum advantage - knowledge generation in fundamental laws of nature.

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

Current Efforts: For simpler models such as Schwinger model, discrete gauge groups, low dimensional SU(2)/SU(3) gauge theory

Intermediate steps:

- Suitable development and choice of framework. ✓
- Suitable choice of variables/basis.
- Algorithm development for various tasks- classical/quantum/hybrid. ✓
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Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

Current Efforts: For arbitrary dimensional SU(2)/SU(3) gauge theories

Intermediate steps:

- Suitable development and choice of framework. ✓
- Suitable choice of variables/basis. ✓
- Algorithm development for various tasks- classical/quantum/hybrid. ✓
- Quantum information theoretic understanding - connection to physics of QCD ✓
- Quantum advantage - knowledge generation in fundamental laws of nature.

Framework: Hamiltonian Formalism

PHYSICAL REVIEW D

VOLUME 11, NUMBER 2

15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†

*Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel*

and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

Gauge Theories are always Difficult !

Hamiltonian framework and the principle of gauge invariance:

a set of local constraints to satisfy.

Tasks:

Choice of a basis: either a electric/strong coupling basis or a group element basis.

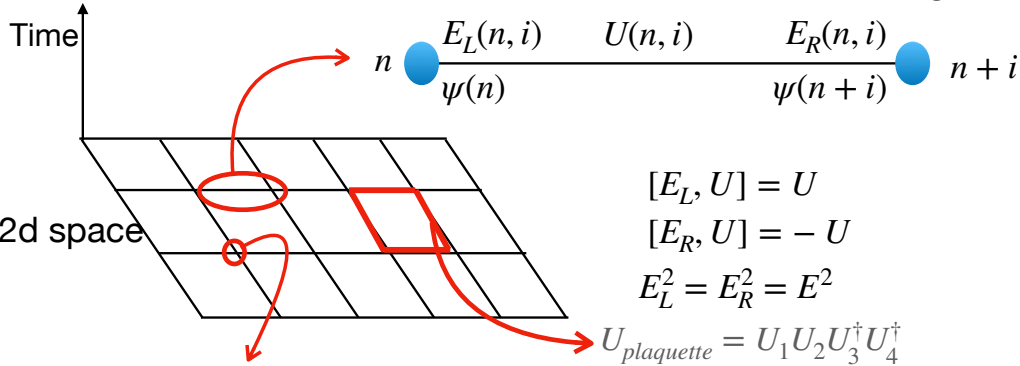
Dealing with gauge redundancy: via gauge fixing / finding gauge invariant solution.

Qubitization and Quantum computation/simulation of Hamiltonian dynamics:

in NISQ devices and beyond - benchmarking the quantum computation by classical computation

Framework: Hamiltonian Formalism

Kogut-Susskind '74



$$\begin{aligned}
 [E_L, U] &= U \\
 [E_R, U] &= -U \\
 E_L^2 &= E_R^2 = E^2 \\
 U_{\text{plaquette}} &= U_1 U_2 U_3^\dagger U_4^\dagger
 \end{aligned}$$

Gauss' law constraint:

$$\begin{aligned}
 G(n) | \Psi_{\text{phys}} \rangle &= 0 \\
 [H, G(n)] &= 0 \quad \forall n
 \end{aligned}$$

$$G(n) = \sum_I [E_L(n, I) - E_R(n - I, I)] - \rho(n)$$

$$H = H_E + H_M + H_I + H_B$$

$$\frac{g^2 a}{2} \sum_{n, I} E^2(n, I)$$

$$m \sum (-1)^n \psi^\dagger(n) \psi(n)$$

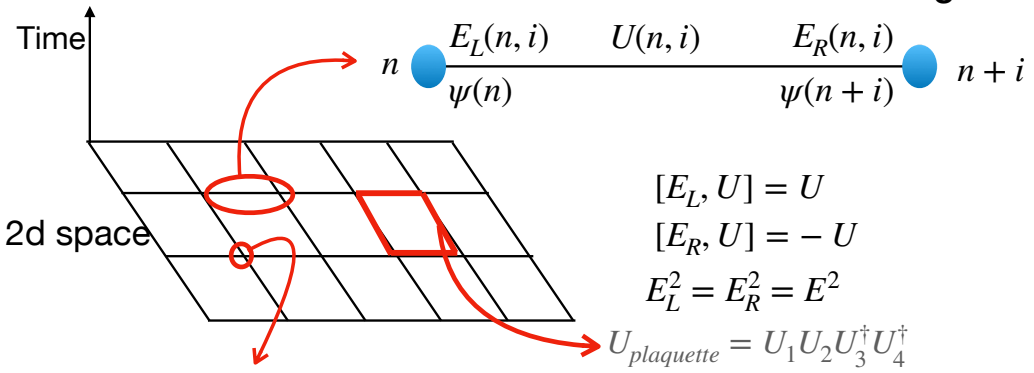
Staggered fermion

$$\frac{1}{2a} \sum_{n, I} (-1)^n \psi^\dagger(n) U(n, I) \psi(n + I)$$

$$\frac{2a}{g^2} \sum_{\text{plaquettes}} [\text{Tr} U_{\text{plaquette}} + h.c.]$$

Framework: Hamiltonian Formalism

Kogut-Susskind '74



Gauss' law constraint:

$$G(n) |\Psi_{phys}\rangle = 0$$

$$[H, G(n)] = 0 \quad \forall n$$

$$G(n) = \sum_I [E_L(n, I) - E_R(n - I, I)] - \rho(n)$$

U(1): $U(n, I) = e^{i\theta(n, I)}$

Schwinger Model : U(1) in 1+1d, H_B term absent

SU(2):
 $E \rightarrow E^a, \quad a = 1, 2, 3$
 $U \rightarrow U_{\alpha\beta}, \quad \alpha, \beta = 1, 2$
 $\psi \rightarrow \psi_\alpha, \quad \alpha = 1, 2$

$$G(n) \rightarrow G^a(n) = \sum_I [E_L^a(n, I) + E_R^a(n - I, I)] + \psi(n)^\dagger \frac{\sigma^a}{2} \psi(n)$$

SU(3): $a = 1, 2, 3, \dots, 8.$

$$H = H_E + H_M + H_I + H_B$$

$$\frac{g^2 a}{2} \sum_{n, I} E^2(n, I)$$

$$m \sum (-1)^n \psi^\dagger(n) \psi(n)$$

Staggered fermion

$$\frac{1}{2a} \sum_{n, I} (-1)^n \psi^\dagger(n) U(n, I) \psi(n + I)$$

$$\frac{2a}{g^2} \sum_{\text{plaquettes}} [\text{Tr} U_{\text{plaquette}} + h.c.]$$

A promising solution:

Loop-string-hadron (LSH)
framework

Features:

LSH occupation number basis: a strong coupling basis.

Non-Abelian gauge redundancy: solved analytically.

LSH basis states are explicitly annihilated by the onsite Gauss law generators.

A physical state is 1-sparse in the LSH occupation basis

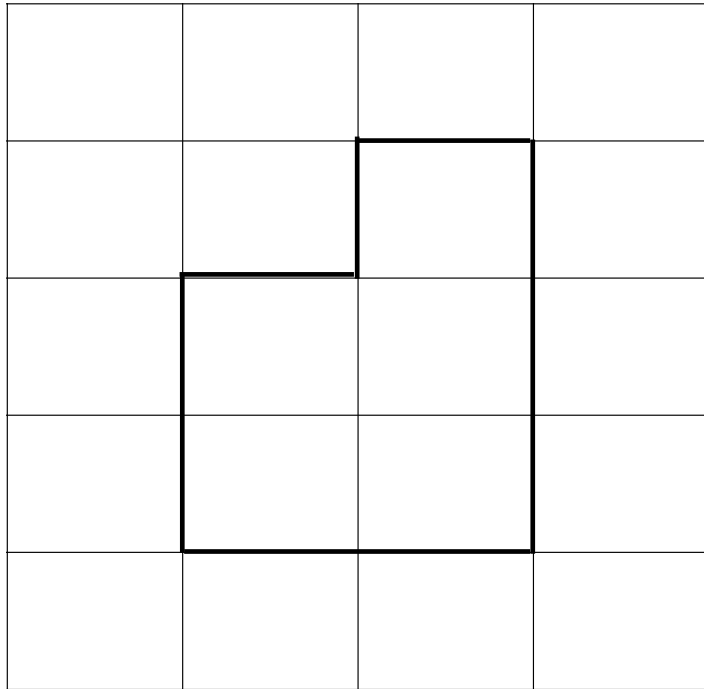
Qubitization: with Bosonic degrees of freedom

- allows analyzing and minimizing cut-off effect.

Hamiltonian dynamics: Spectrum is identical to Kogut-Susskind Hamiltonian

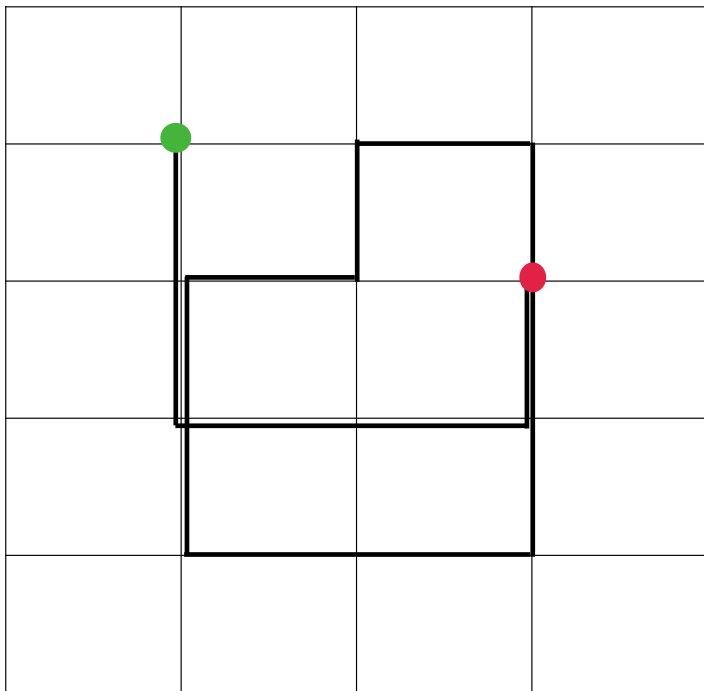
Exact diagonalization, Tensor network calculations;
algorithm for analog and digital quantum hardware.

Loops-Strings-Hadrons : SU(2)



Wilson loops

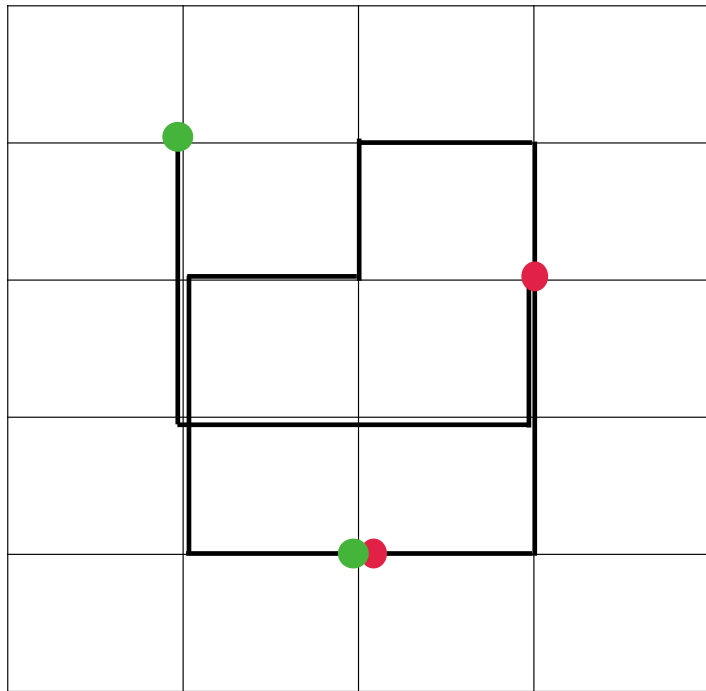
Loops-Strings-Hadrons : SU(2)



Wilson loops

Strings/mesons

Loops-Strings-Hadrons : SU(2)



Wilson loops

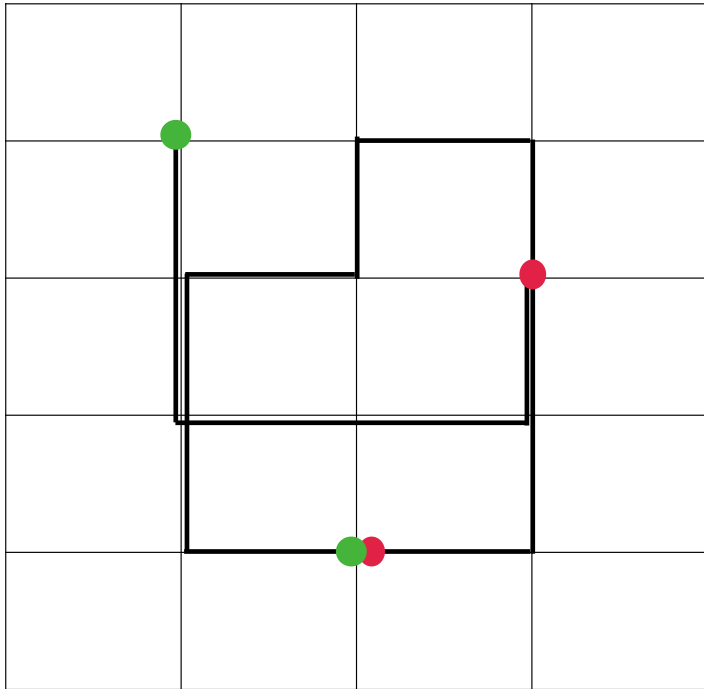
Strings

Hadrons

Loops-Strings-Hadrons : SU(2) in 1+1 d



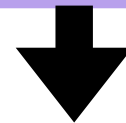
Loops-Strings-Hadrons : SU(2)



Wilson loops
Strings
Hadrons

Gauge invariance leads to non-locality

On site snapshots of gauge invariant configurations



LSH basis

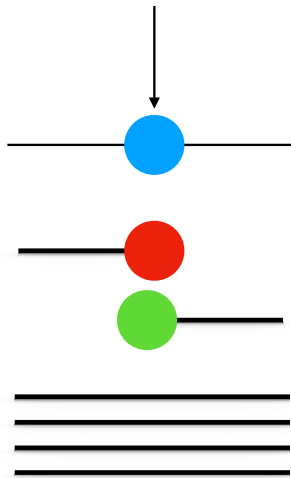
Loops-Strings-Hadrons : SU(2) in 1+1 d



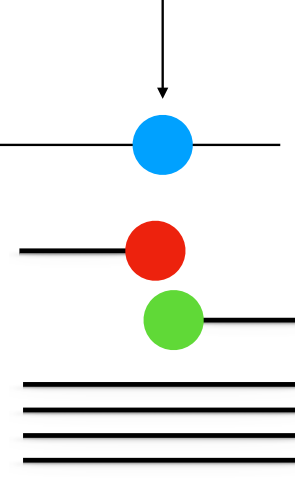
Loops-Strings-Hadrons : SU(2) in 1+1 d

On site snapshots of gauge invariant configurations

staggered site x

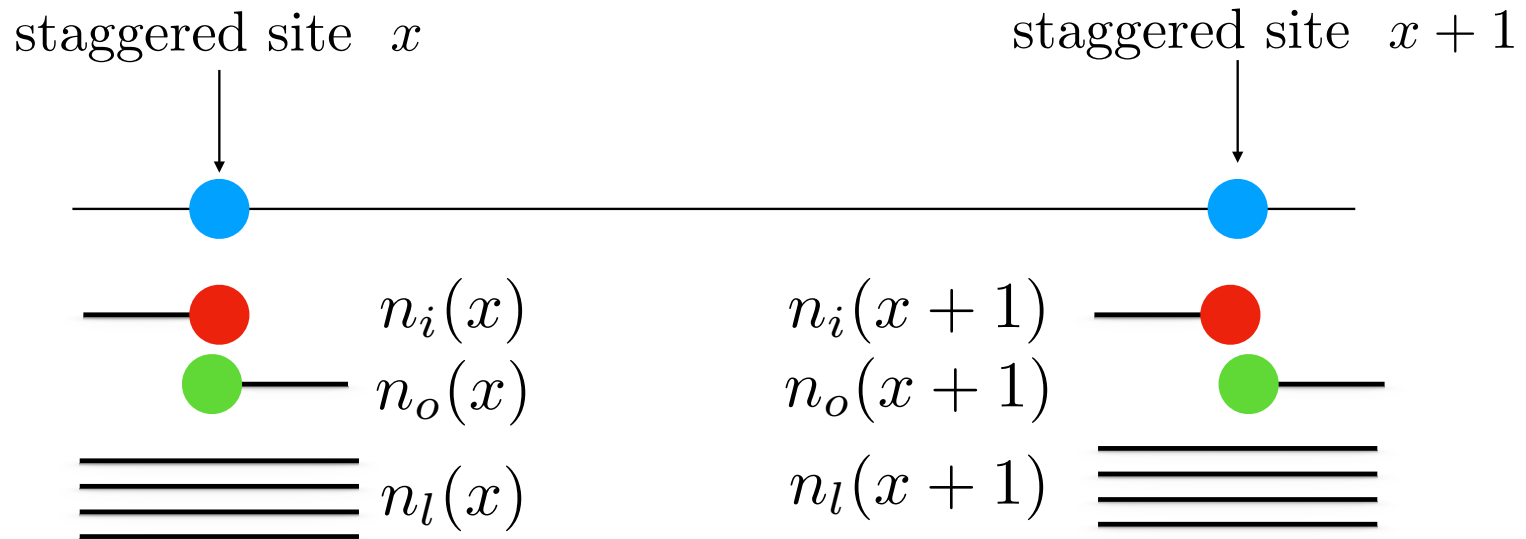


staggered site $x + 1$



Loops-Strings-Hadrons : SU(2) in 1+1 d

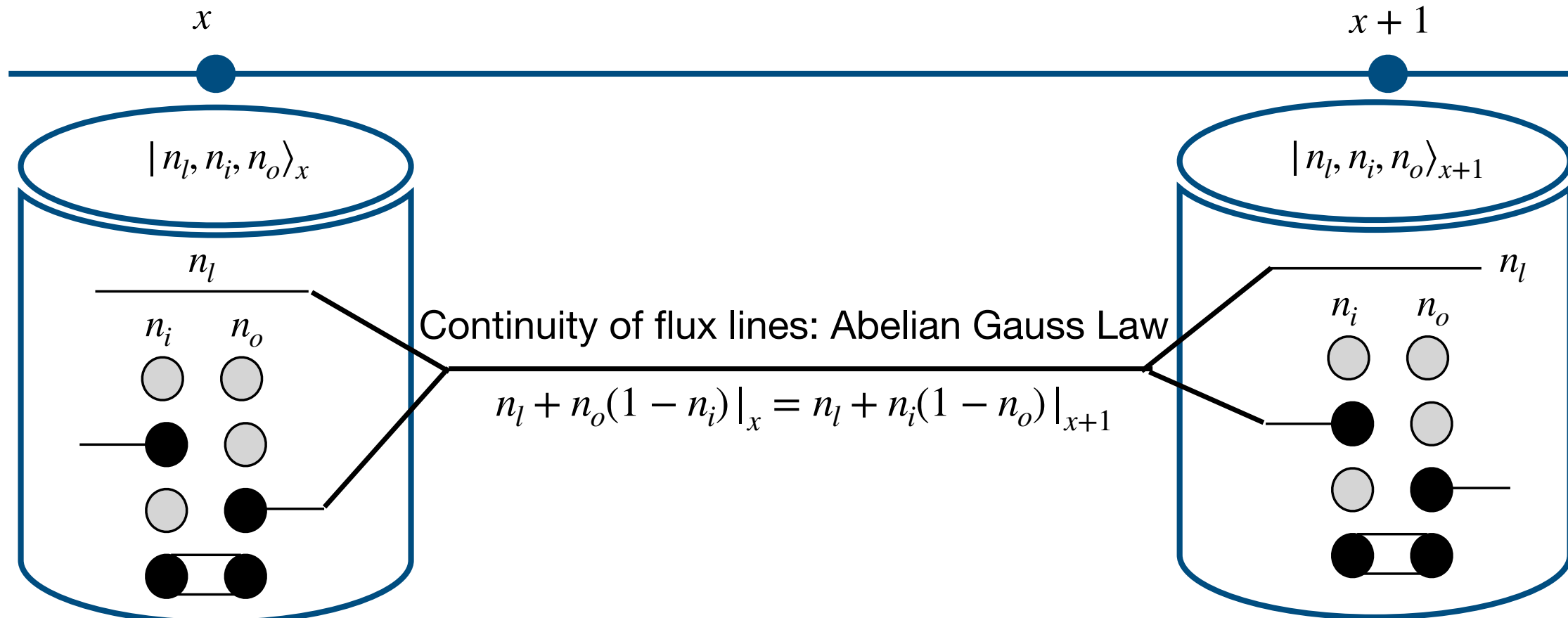
On site snapshots of gauge invariant configurations



Loops-Strings-Hadrons : SU(2) in 1+1 d

Global LSH states are constructed by imposing Abelian Gauss Law constraints

Non-locality remains crucial, but is then care by Abelian constraints



Hamiltonian, describing dynamics of loops, strings and hadrons.

$$H^{(\text{LSH})} = H_I^{(\text{LSH})} + H_E^{(\text{LSH})} + H_M^{(\text{LSH})}$$

$$H_I^{(\text{LSH})} = \frac{1}{2a} \sum_n \left\{ \frac{1}{\sqrt{\hat{n}_l(x) + \hat{n}_o(x)(1 - \hat{n}_i(x)) + 1}} \right. \\ \times \left[\hat{S}_o^{++}(x) \hat{S}_i^{+-}(x+1) + \hat{S}_o^{+-}(x) \hat{S}_i^{--}(x+1) \right] \\ \left. \times \frac{1}{\sqrt{\hat{n}_l(x+1) + \hat{n}_i(x+1)(1 - \hat{n}_o(x+1)) + 1}} + \text{h.c.} \right\},$$

$$H_E^{(\text{LSH})} = \frac{g^2 a}{2} \sum_n \left[\frac{\hat{n}_l(x) + \hat{n}_o(x)(1 - \hat{n}_i(x))}{2} \right. \\ \left. \times \left(\frac{\hat{n}_l(x) + \hat{n}_o(x)(1 - \hat{n}_i(x))}{2} + 1 \right) \right],$$

$$H_M^{(\text{LSH})} = m \sum_n (-1)^x (\hat{n}_i(x) + \hat{n}_o(x)),$$

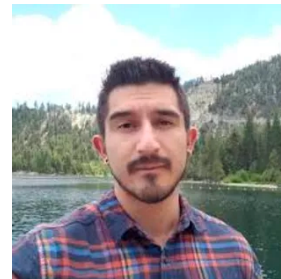
$$\hat{S}_o^{++} = \hat{\chi}_o^+(\lambda^+) \hat{n}_i \sqrt{\hat{n}_l + 2 - \hat{n}_i}, \\ \hat{S}_o^{--} = \hat{\chi}_o^-(\lambda^-) \hat{n}_i \sqrt{\hat{n}_l + 2(1 - \hat{n}_i)}, \\ \hat{S}_o^{+-} = \hat{\chi}_i^+(\lambda^-)^{1 - \hat{n}_o} \sqrt{\hat{n}_l + 2\hat{n}_o}, \\ \hat{S}_o^{-+} = \hat{\chi}_i^-(\lambda^+)^{1 - \hat{n}_o} \sqrt{\hat{n}_l + 1 + \hat{n}_o},$$

$$\hat{S}_i^{+-} = \hat{\chi}_o^-(\lambda^+)^{1 - \hat{n}_i} \sqrt{\hat{n}_l + 1 + \hat{n}_i}, \\ \hat{S}_i^{-+} = \hat{\chi}_o^+(\lambda^-)^{1 - \hat{n}_i} \sqrt{\hat{n}_l + 2\hat{n}_i}, \\ \hat{S}_i^{--} = \hat{\chi}_i^-(\lambda^-) \hat{n}_o \sqrt{\hat{n}_l + 2(1 - \hat{n}_o)}, \\ \hat{S}_i^{++} = \hat{\chi}_i^+(\lambda^+) \hat{n}_o \sqrt{\hat{n}_l + 2 - \hat{n}_o}.$$

The strong-coupling vacuum of the LSH Hamiltonian is given by

$$n_l(x) = 0, \text{ for all } x, \\ n_i(x) = 0, n_o(x) = 0, \text{ for } x \text{ even}, \\ n_i(x) = 1, n_o(x) = 1, \text{ for } x \text{ odd}.$$

Collaborators:



Jesse Stryker

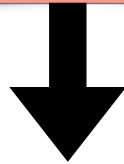
Spectrum is identical to Kogut Susskind Hamiltonian

Loops-Strings-Hadrons Framework

Local non-Abelian constraints are solved analytically by construction: LSH formalism is manifestly $SU(2)/SU(3)$ invariant

Local constraint structure:

$U(1)$, always as in 1d even for higher dimensional LSH



Global symmetry structure:

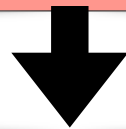
Multiple $U(1)$ and discrete symmetries

Loops-Strings-Hadrons Framework

Local non-Abelian constraints are solved analytically by construction: LSH formalism is manifestly $SU(2)/SU(3)$ invariant

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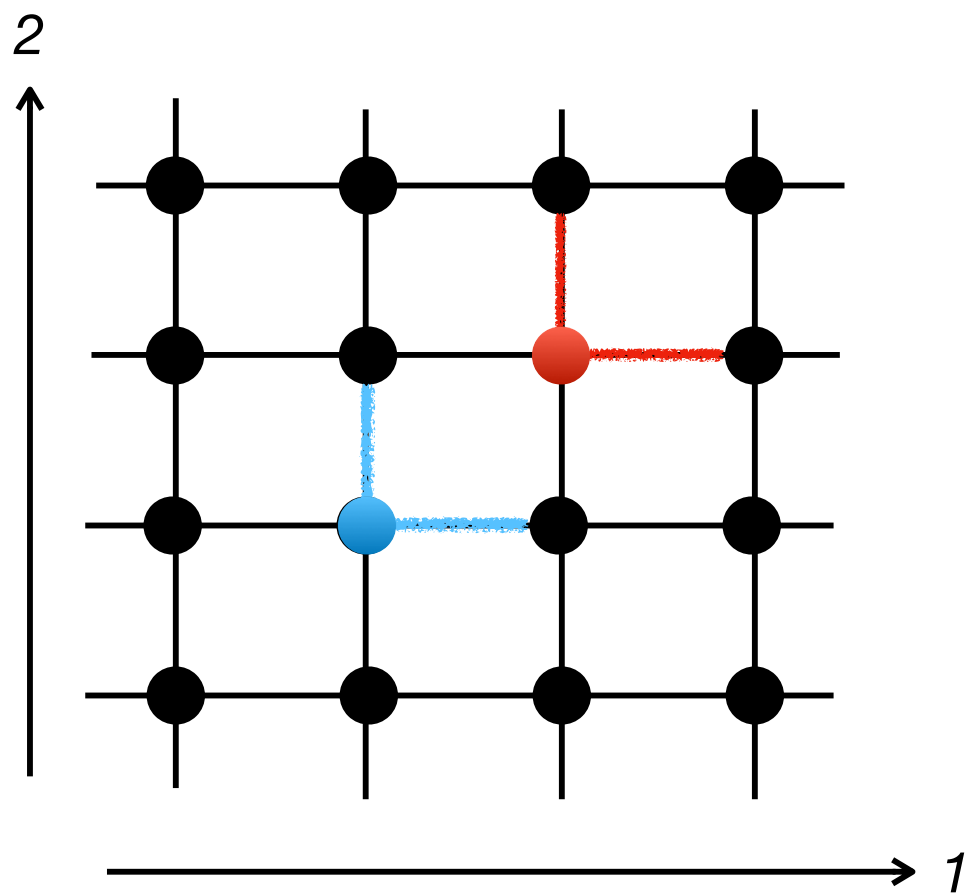
Global symmetry structure:

Multiple $U(1)$ and discrete symmetries

Useful for both theoretical analysis and classical/quantum computation

Underlying complicated route via prepotentials/Schwinger bosons can be bypasses for application/implementation

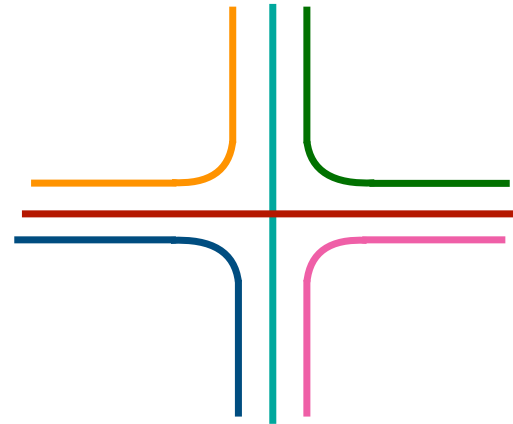
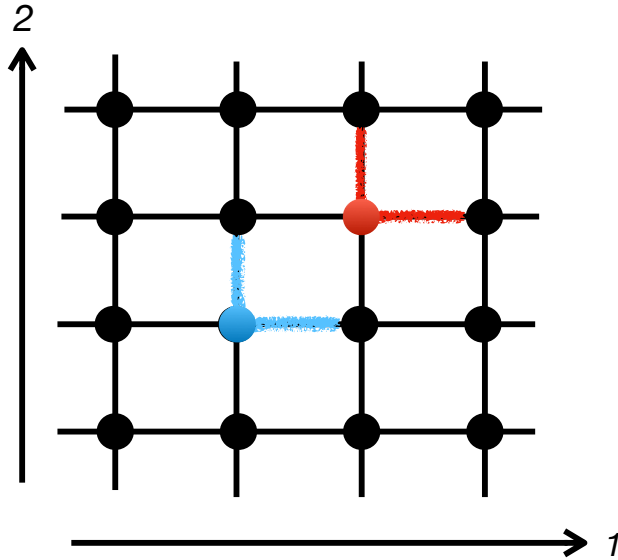
SU(2) LSH framework in $d > 1$



SU(2) LSH framework in $d = 2 + 1$

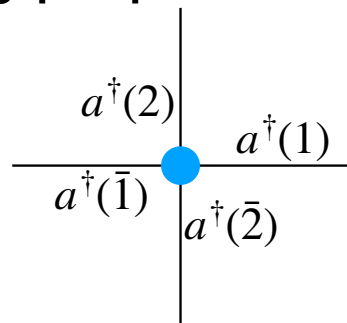
Focussing at a site for pure gauge theory

On-site contribution from Wilson loops:



Overcomplete
loop basis

Underlying prepotential construction:



Local Loop Operators

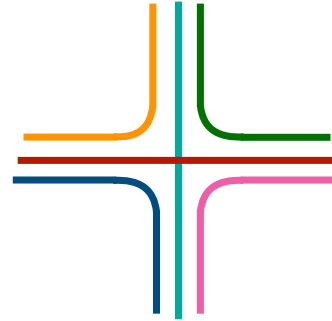
$$\mathcal{L}_{ij}^{++} = \epsilon^{\alpha\beta} a_\alpha^\dagger(i) a_\beta^\dagger(j)$$

Local loop state:

$$|l_{12}, l_{1\bar{1}} l_{1\bar{2}}, l_{2\bar{1}}, l_{2\bar{2}}, l_{\bar{1}\bar{2}}\rangle$$

$$\equiv \prod_{ij} \left(\mathcal{L}_{ij}^{++} \right)^{l_{ij}} |0\rangle$$

SU(2) LSH framework in $d = 2 + 1$

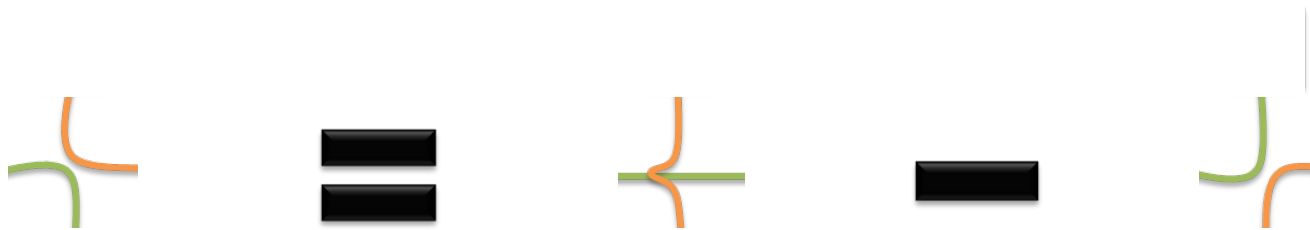


Overcomplete basis

3 physical d.o.f = 6 (local loop quantum numbers in 2d)

- 2(Abelian Gauss' law constraint along 2 link directions)

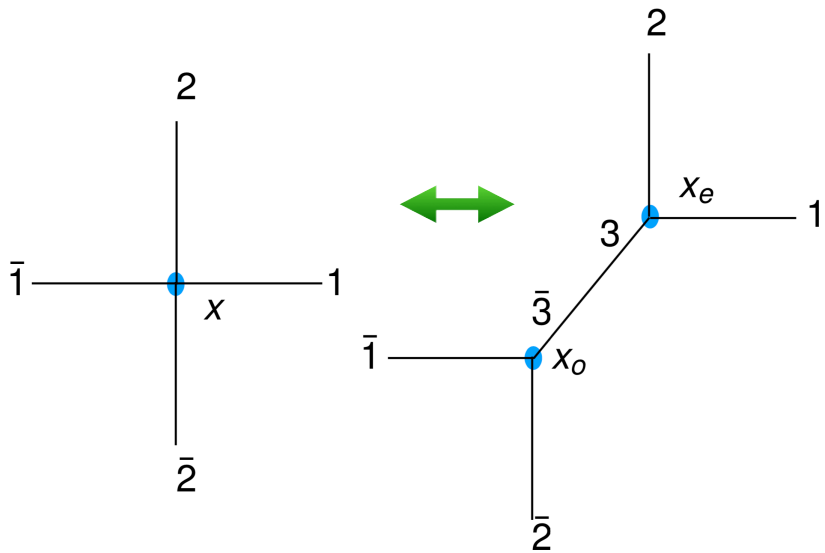
-1 (**Mandelstam constraint**)



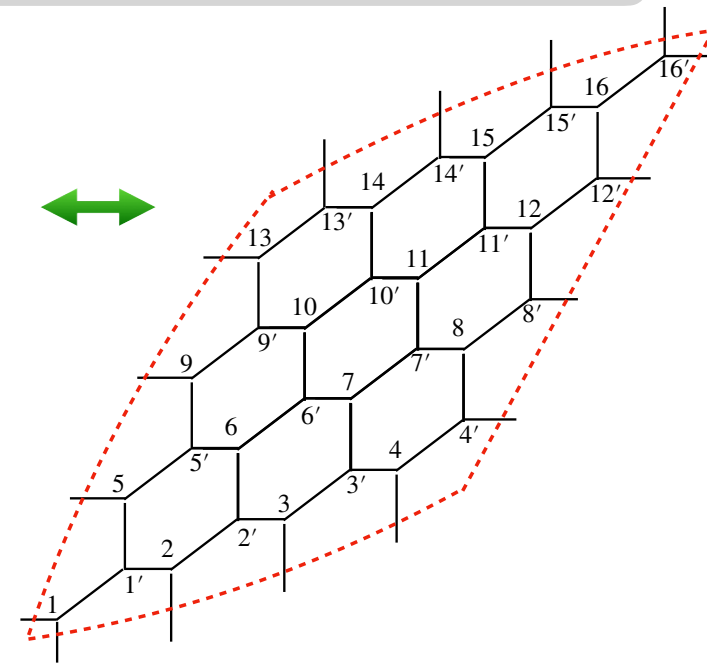
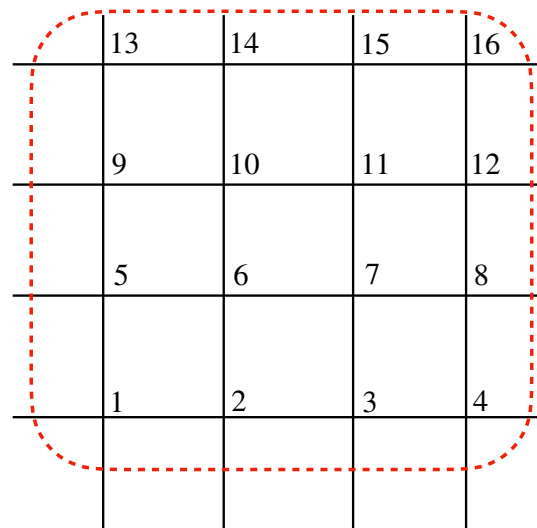
Non-linear constraints, become increasingly complicated with increasing dimensions

SU(2) LSH framework in $d = 2 + 1$

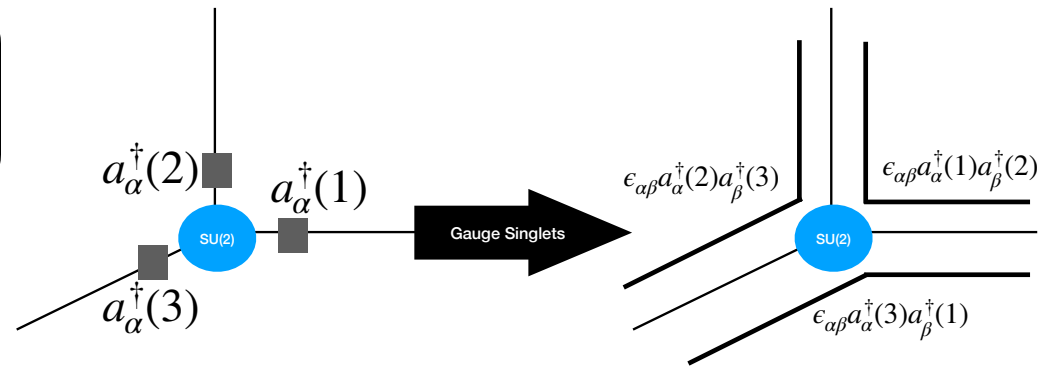
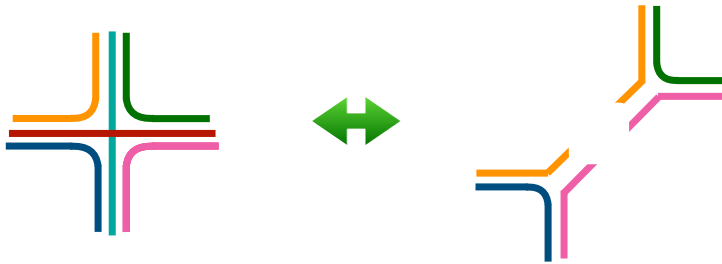
Solution of Mandelstam constraint via a virtual point splitting scheme in pre potential framework



2-d LSH on square lattice is effectively on hexagonal lattice

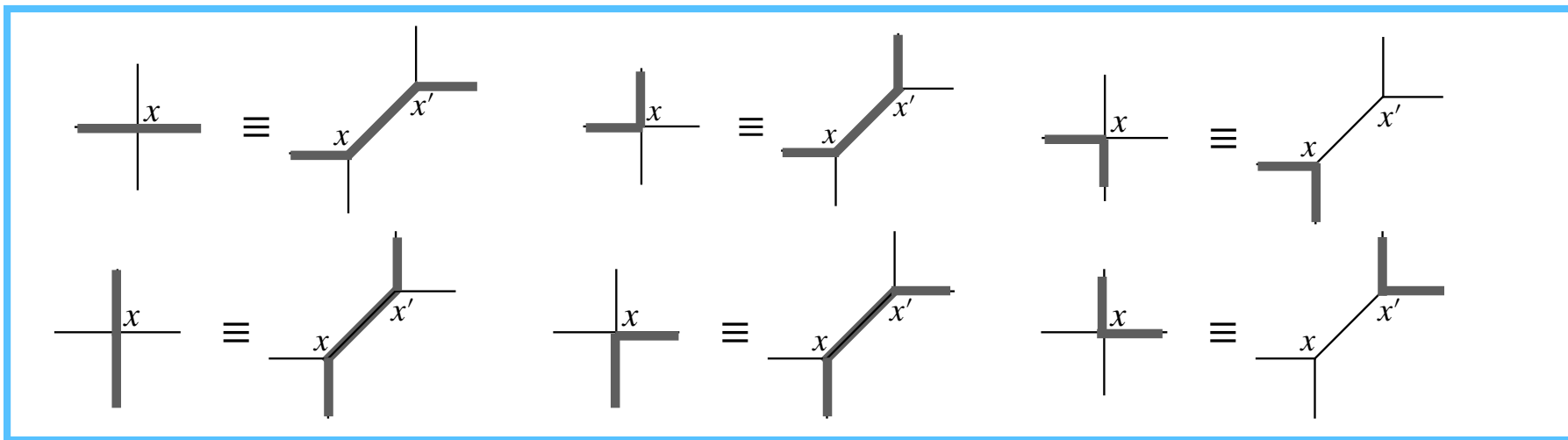
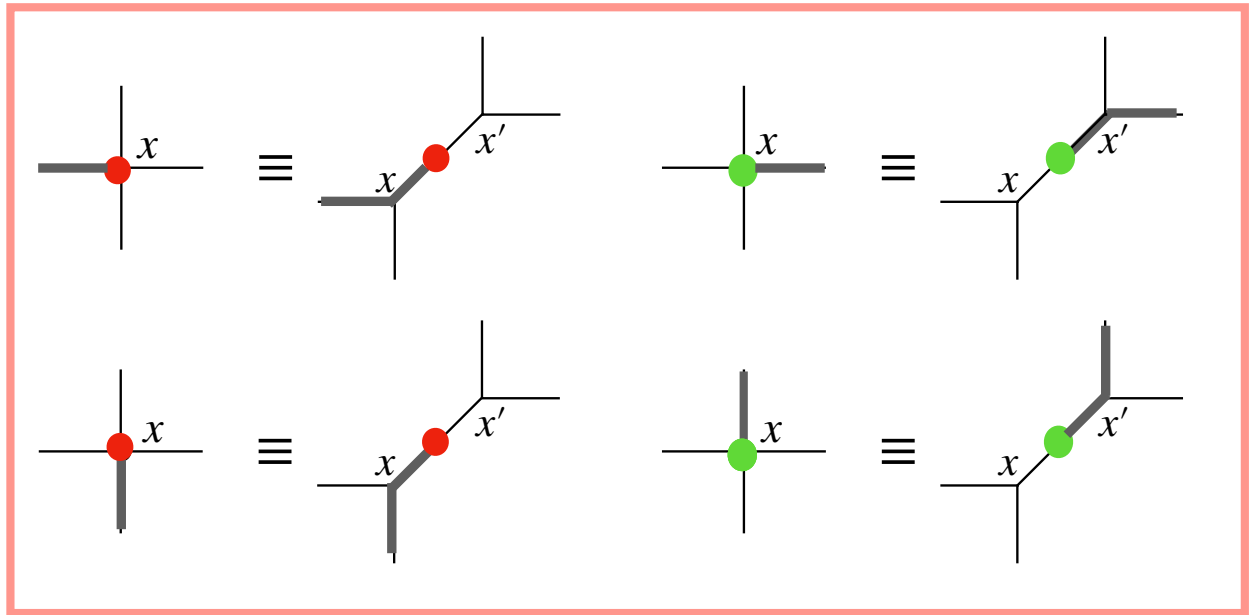
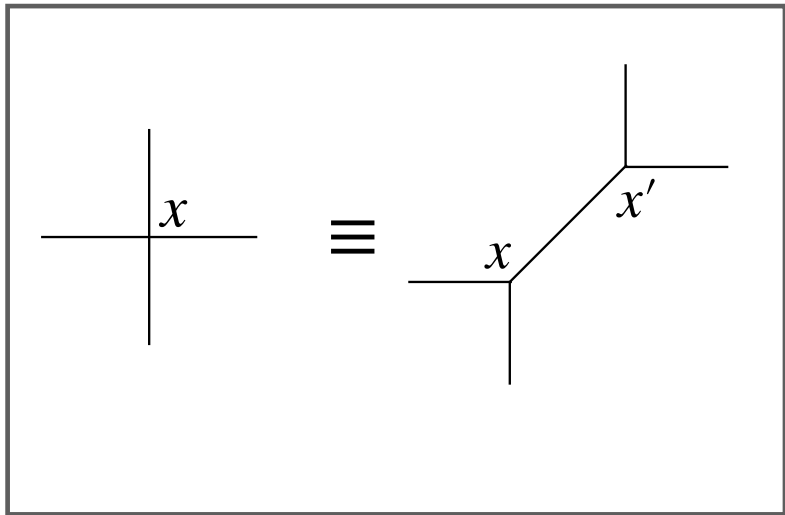


SU(2) LSH framework in $d = 2 + 1$

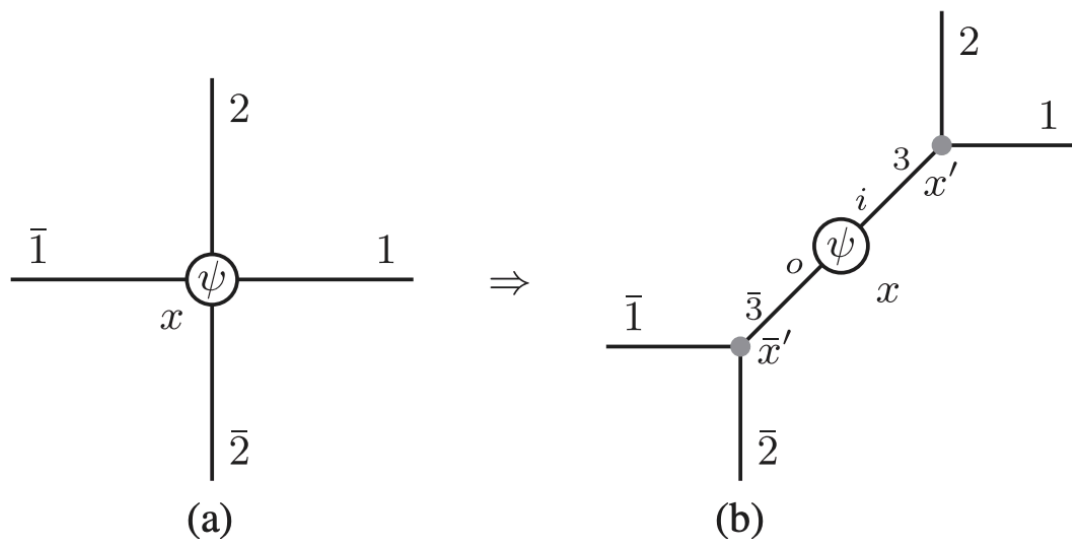


3 physical d.o.f = 2×3 (local loop quantum numbers in 2d)
 - 3 (Abelian Gauss' law constraint)
 + **0 (Mandelstam constraint)**

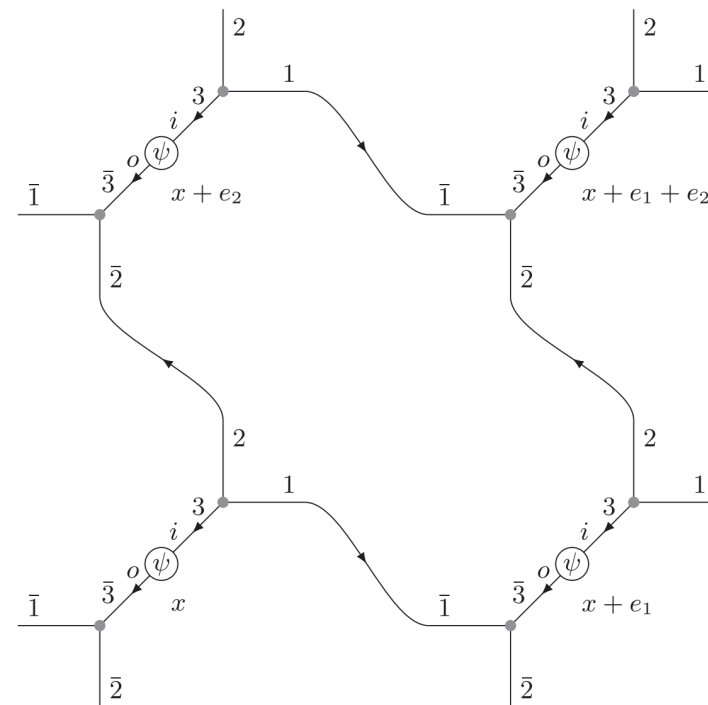
Generalized for arbitrary dimensions!
 Generalized to include matter!



SU(2) LSH Formalism: 2+1 d



Matter-Gauge interactions
are same as in 1d



SU(2) LSH Formalism: 3+1 d

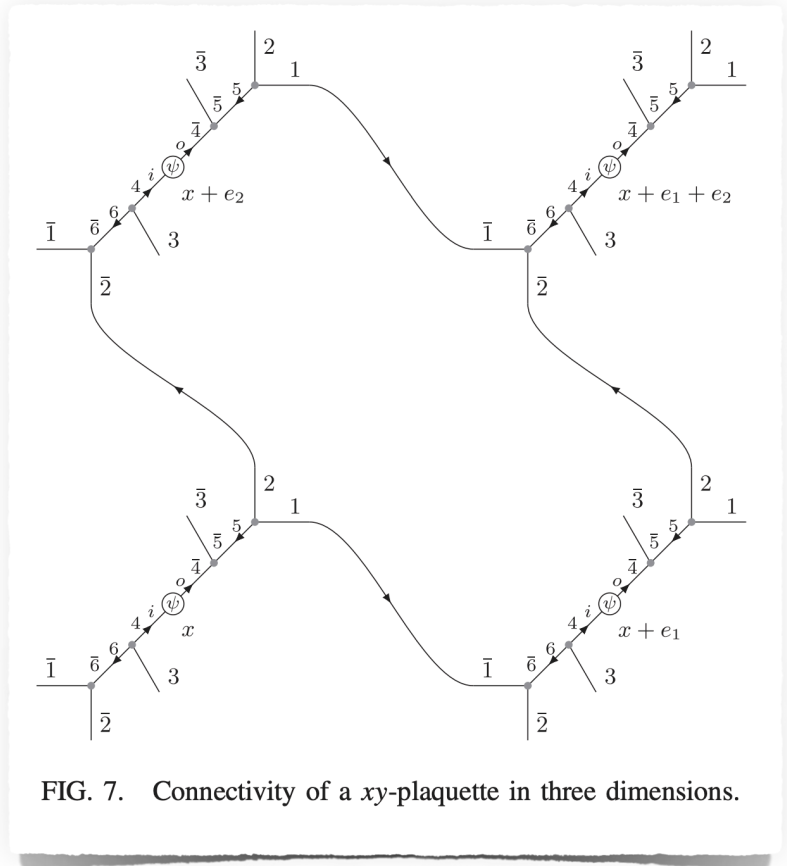
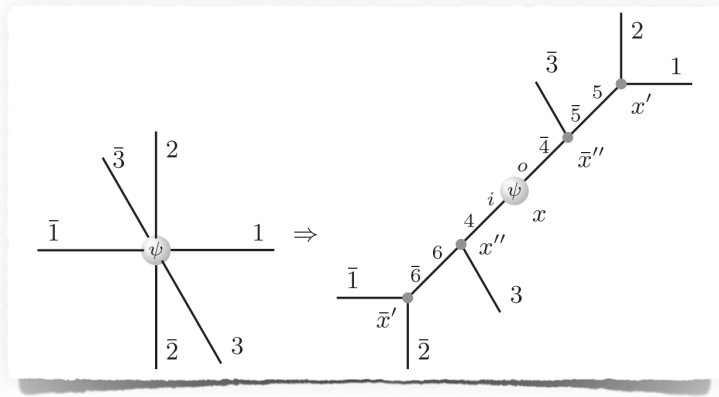


FIG. 7. Connectivity of a xy -plaquette in three dimensions.

- Matter-Gauge interactions are same as in 1+1d
- Pure gauge interactions are same as in 2+1d

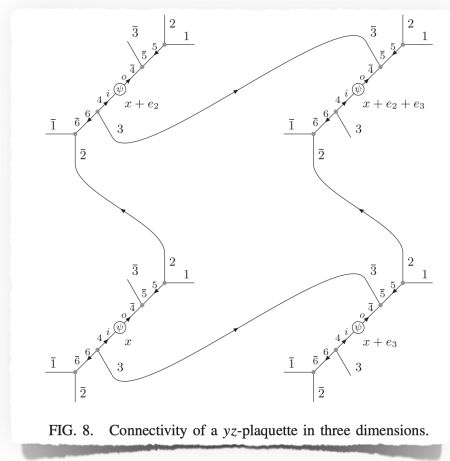


FIG. 8. Connectivity of a yz -plaquette in three dimensions.

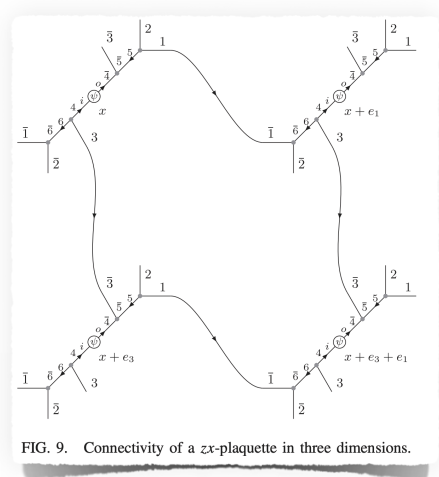
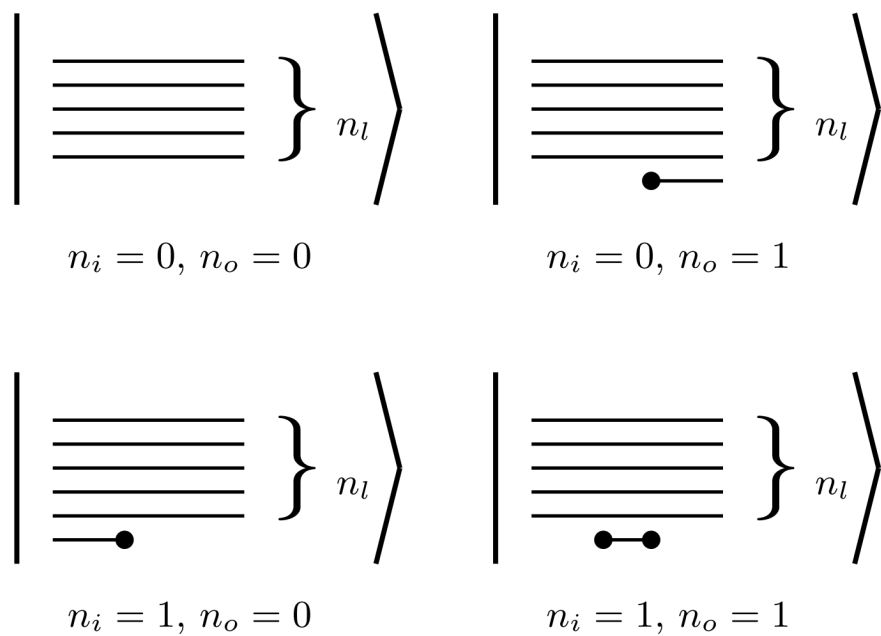
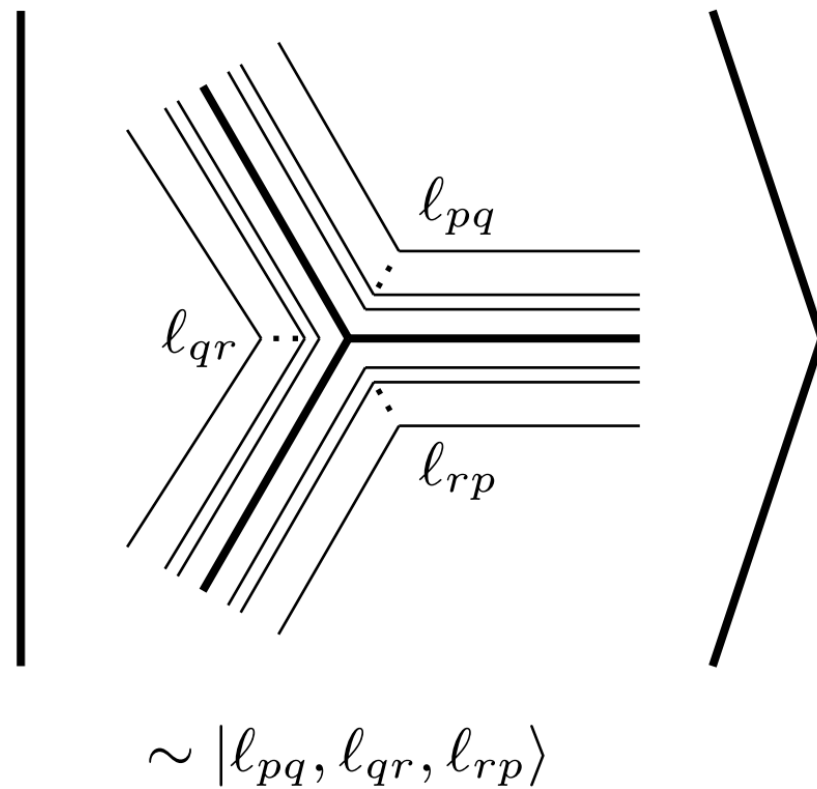


FIG. 9. Connectivity of a zx -plaquette in three dimensions.

Matter site



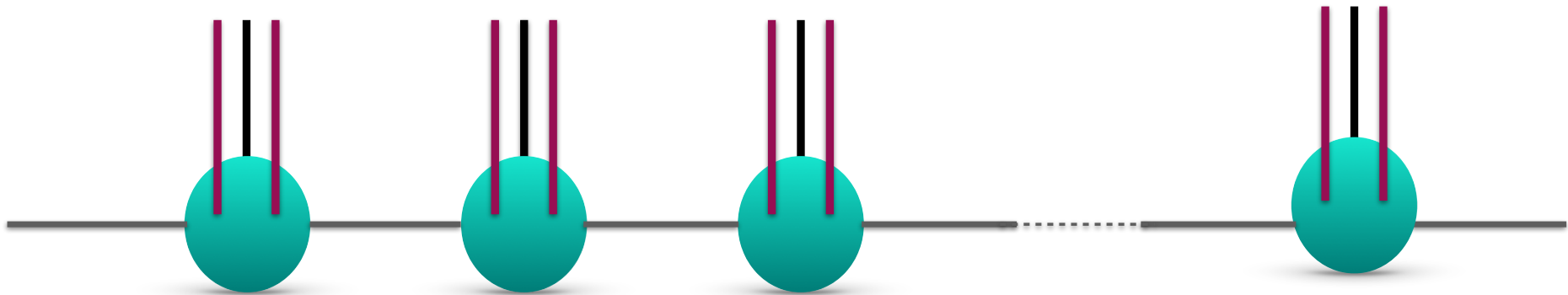
Gluonic site



Application of the $SU(2)$ LSH framework

Tensor network calculations for non-Abelian gauge theories

Matrix Product State Ansatz for LSH in one spatial dimension

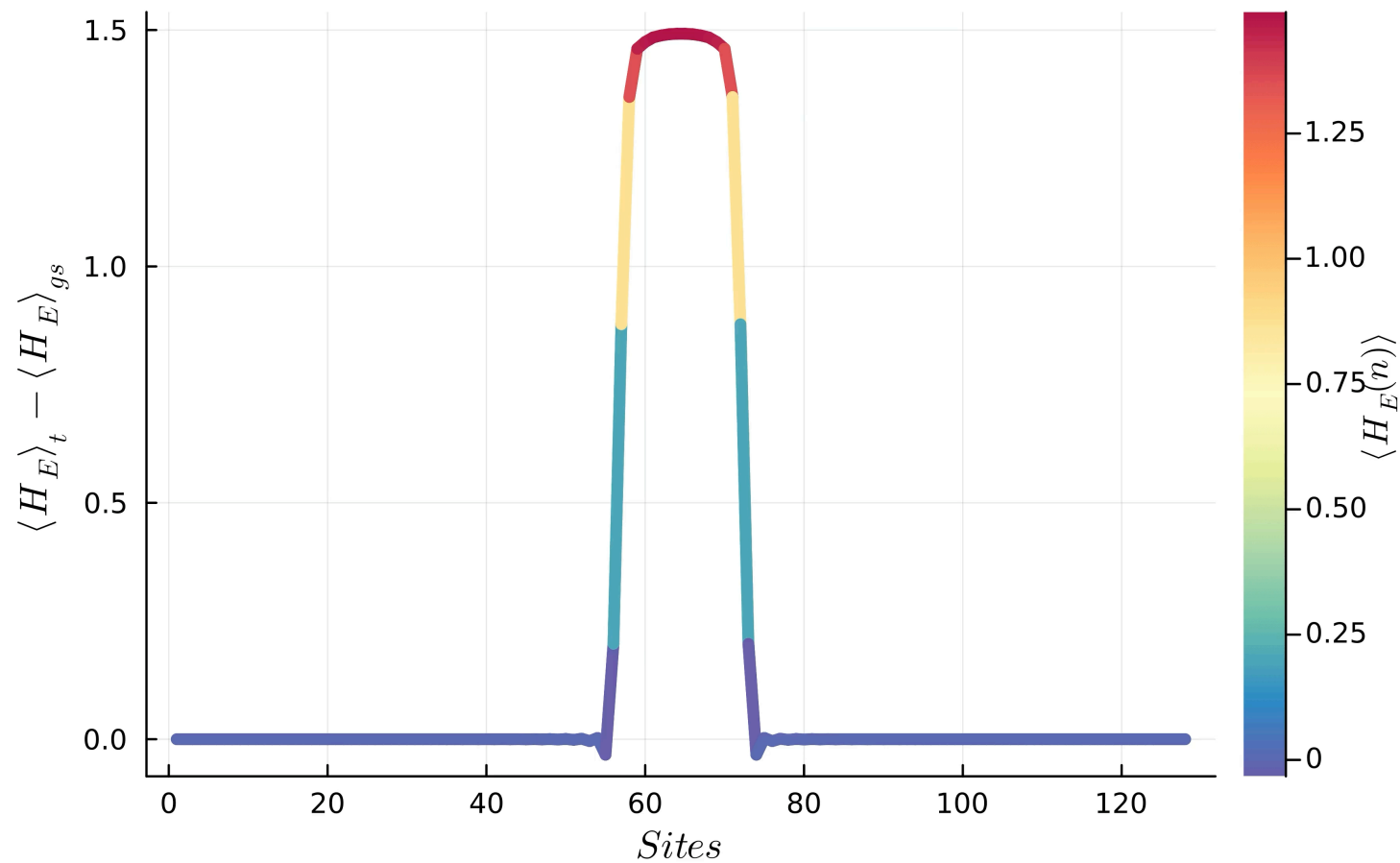


On-site tensor with three physical indices:
1 bosonic and 2 fermionic

MPS Calculations using LSH framework

Time-evolution of a string state on the interacting vacuum

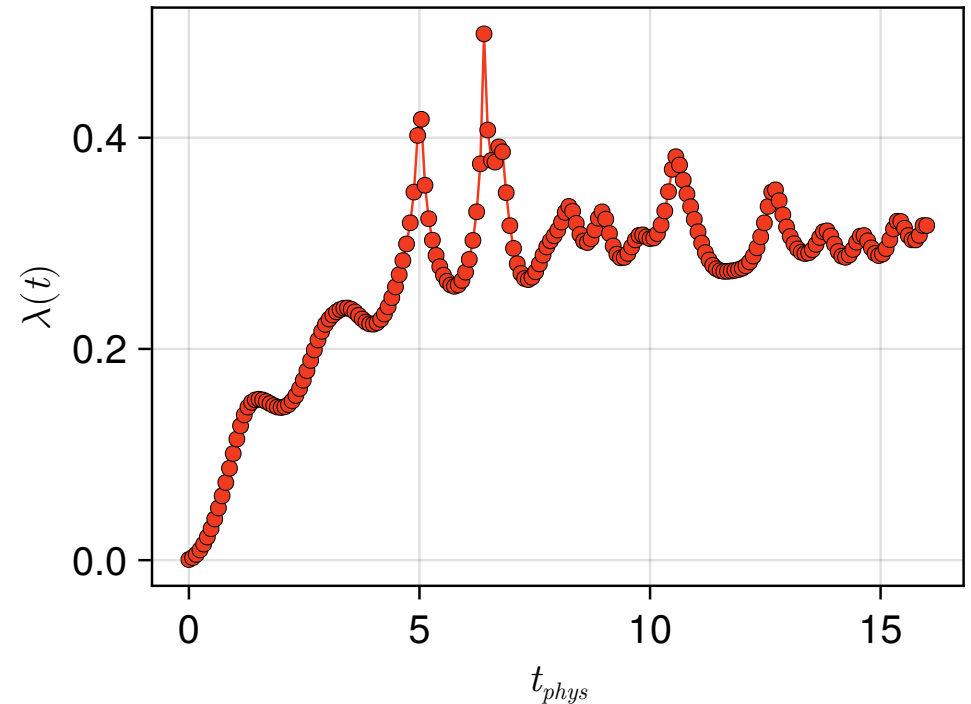
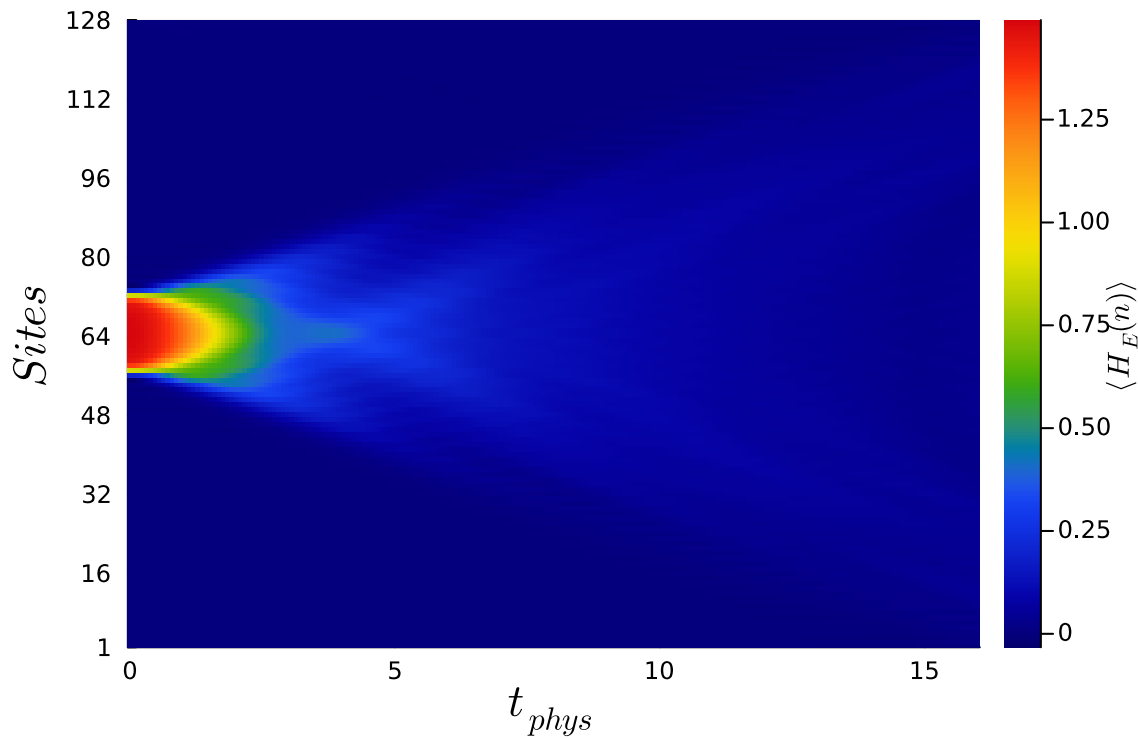
Time Step: 1



MPS Calculations using LSH framework

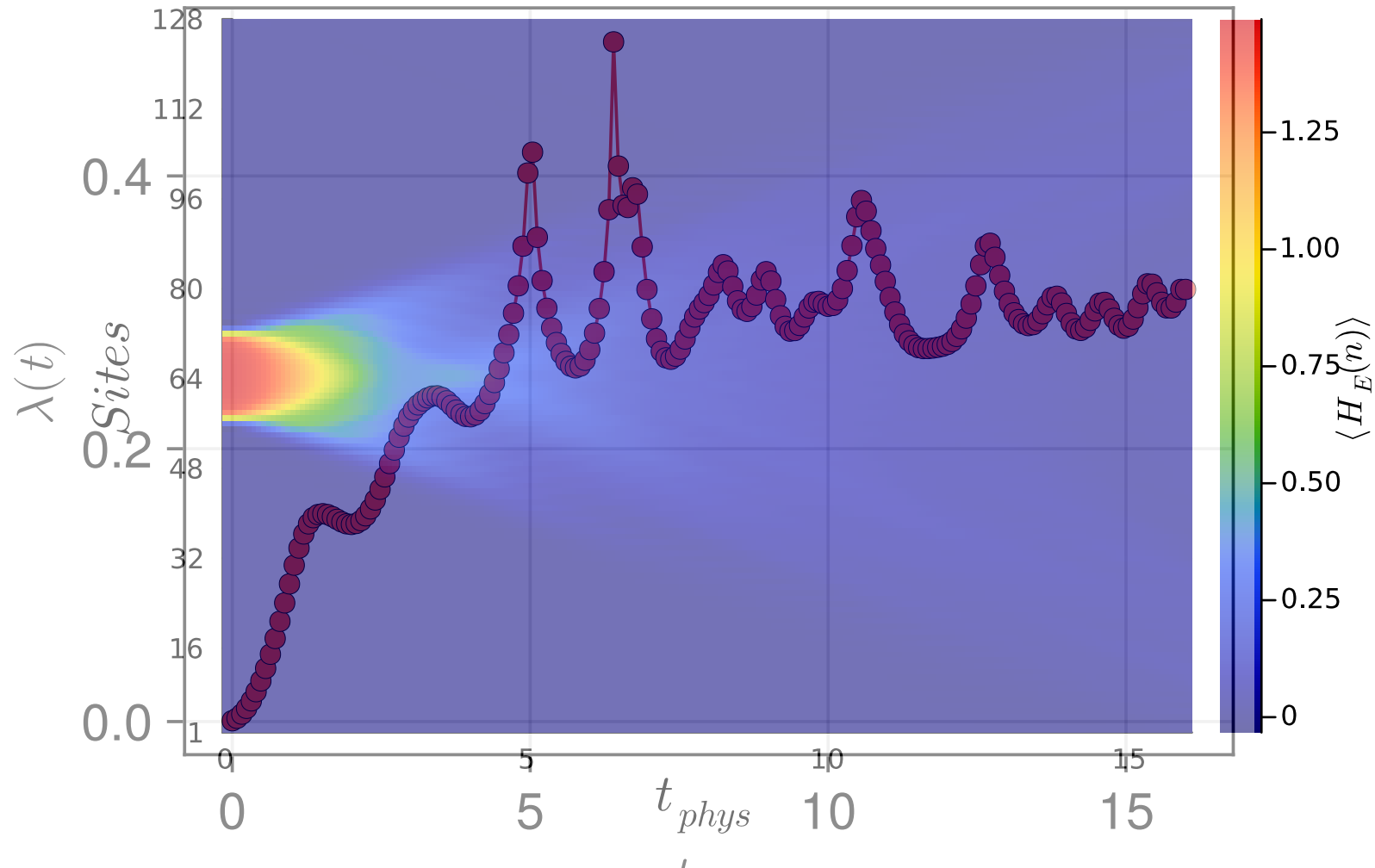
MPS preparation of interacting vacuum

Time-evolution of a string state on the interacting vacuum



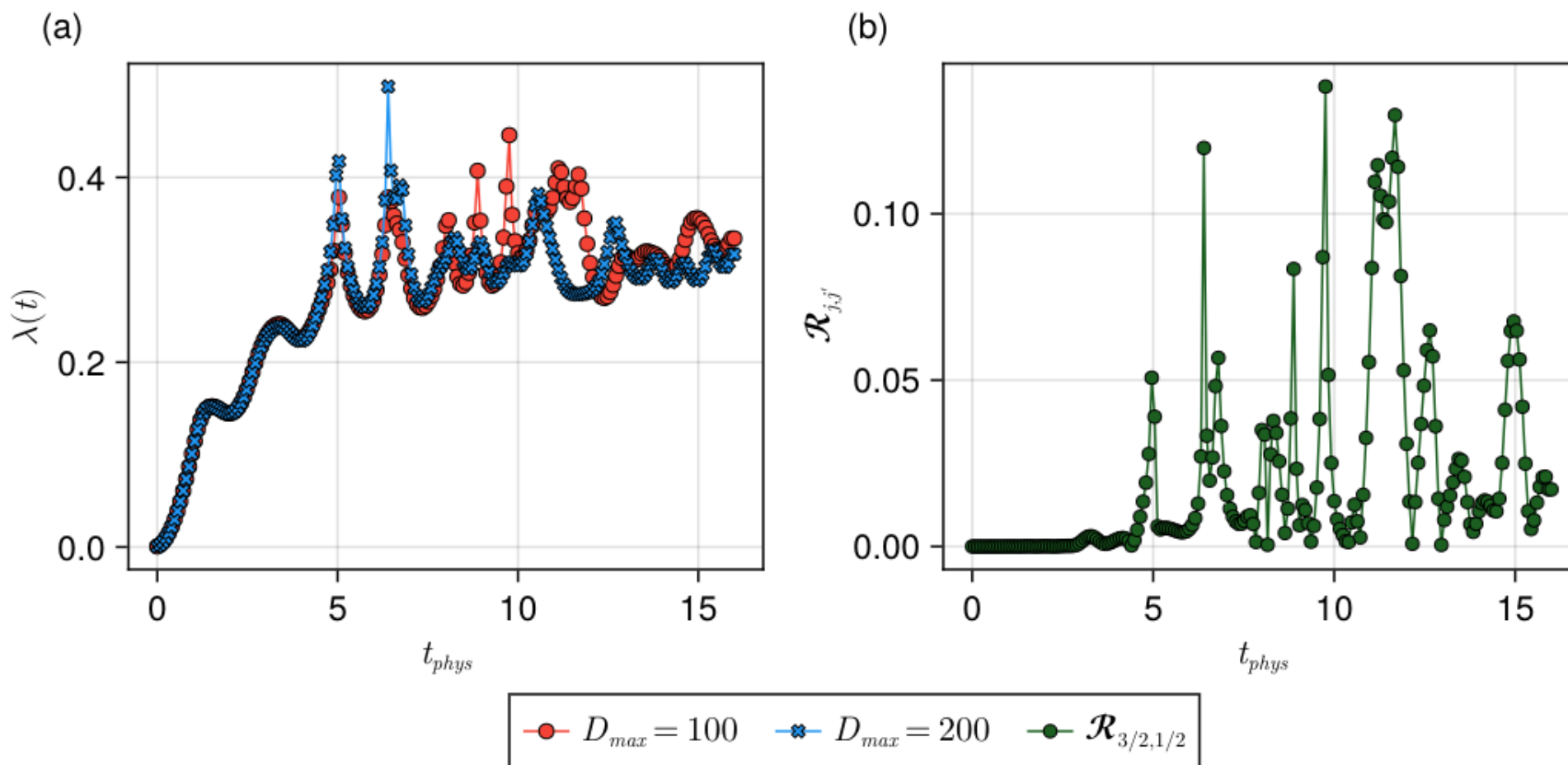
MPS Calculations using LSH framework

Time-evolution of a string state on the interacting vacuum



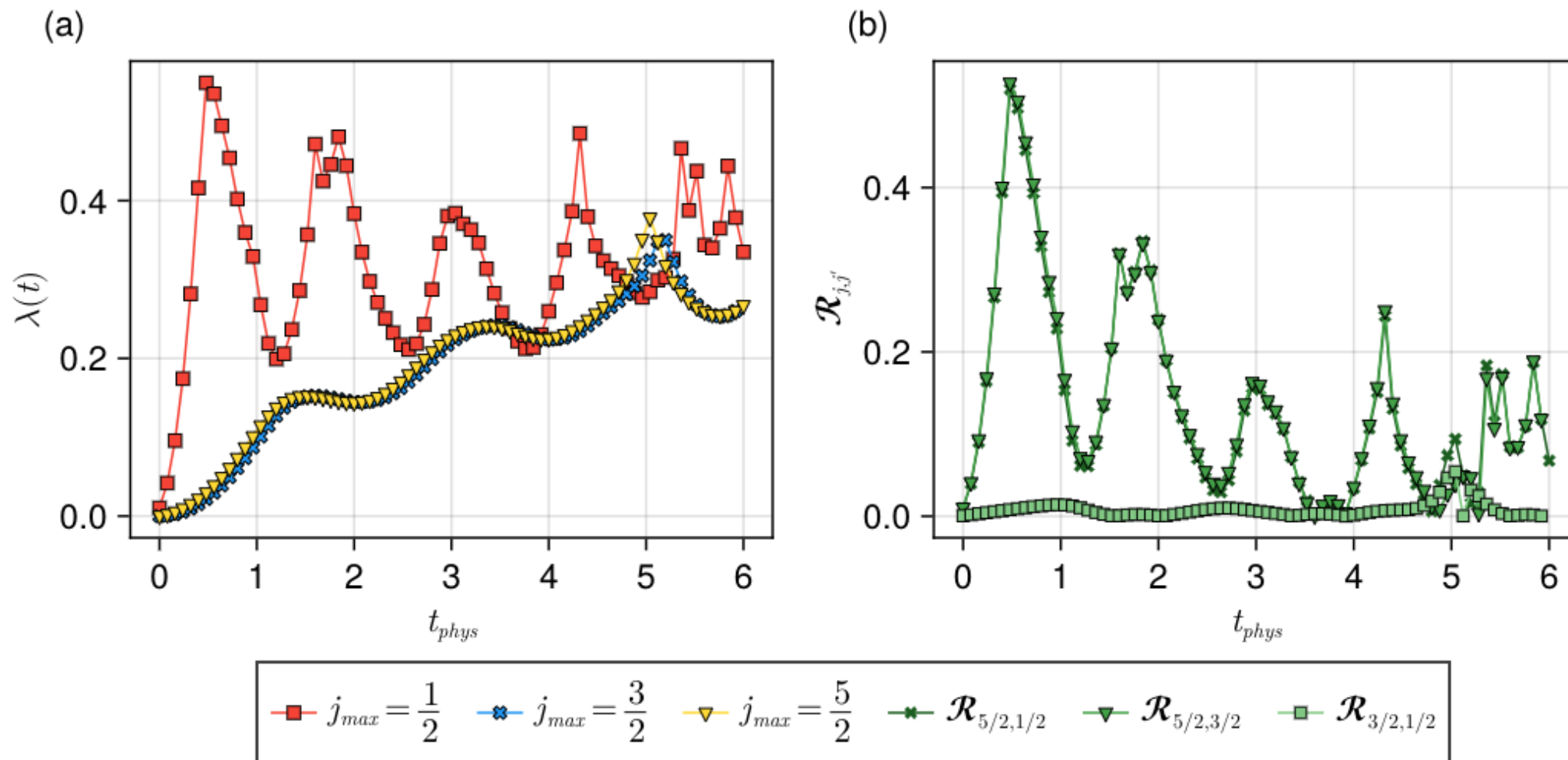
MPS Calculations using LSH framework

Probing effect of finite bond dimension: $N=128$



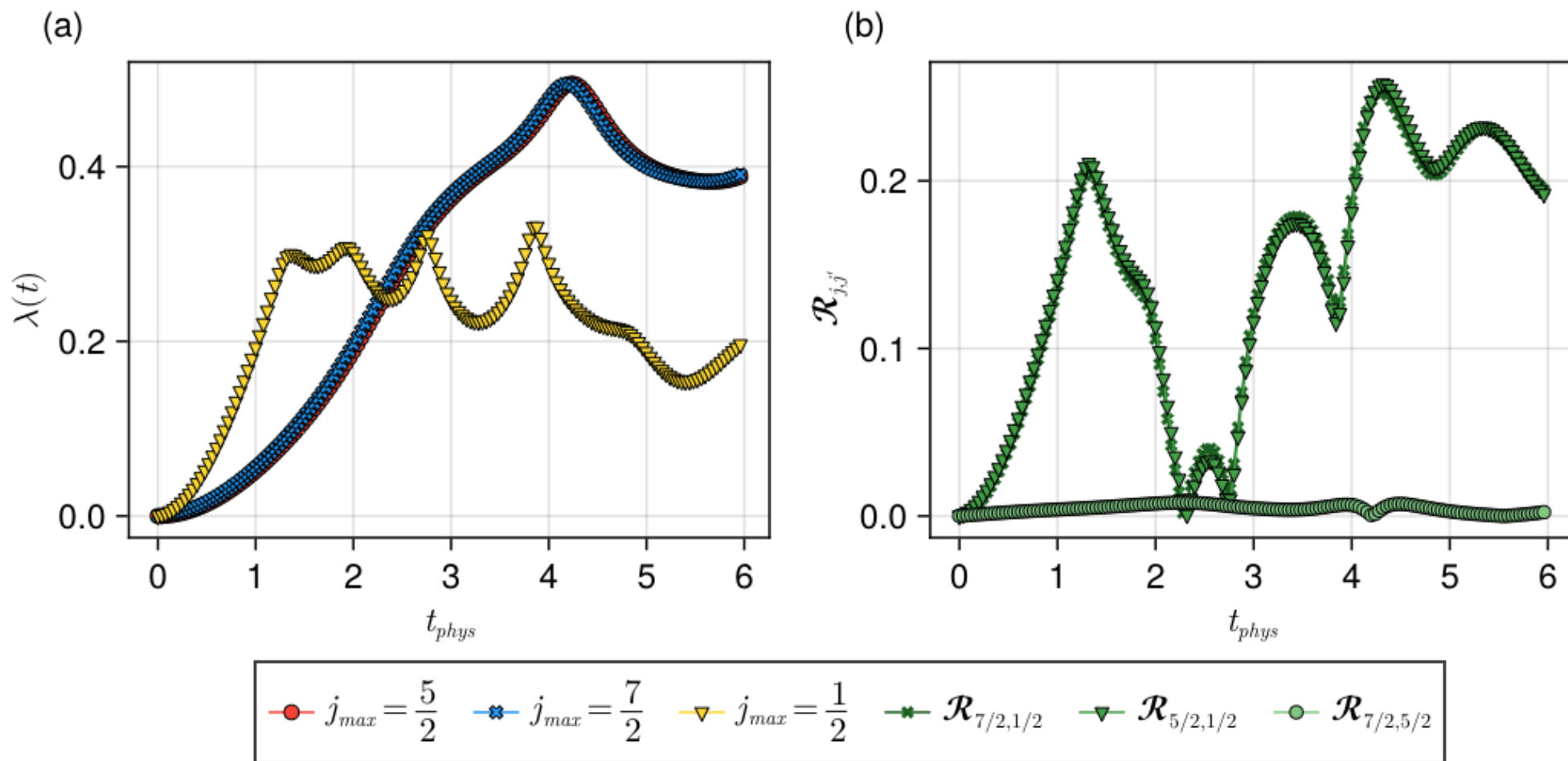
MPS Calculations using LSH framework

Probing cut-off dependence in dynamics: $N=128$



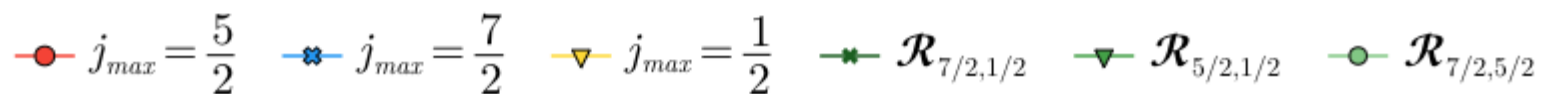
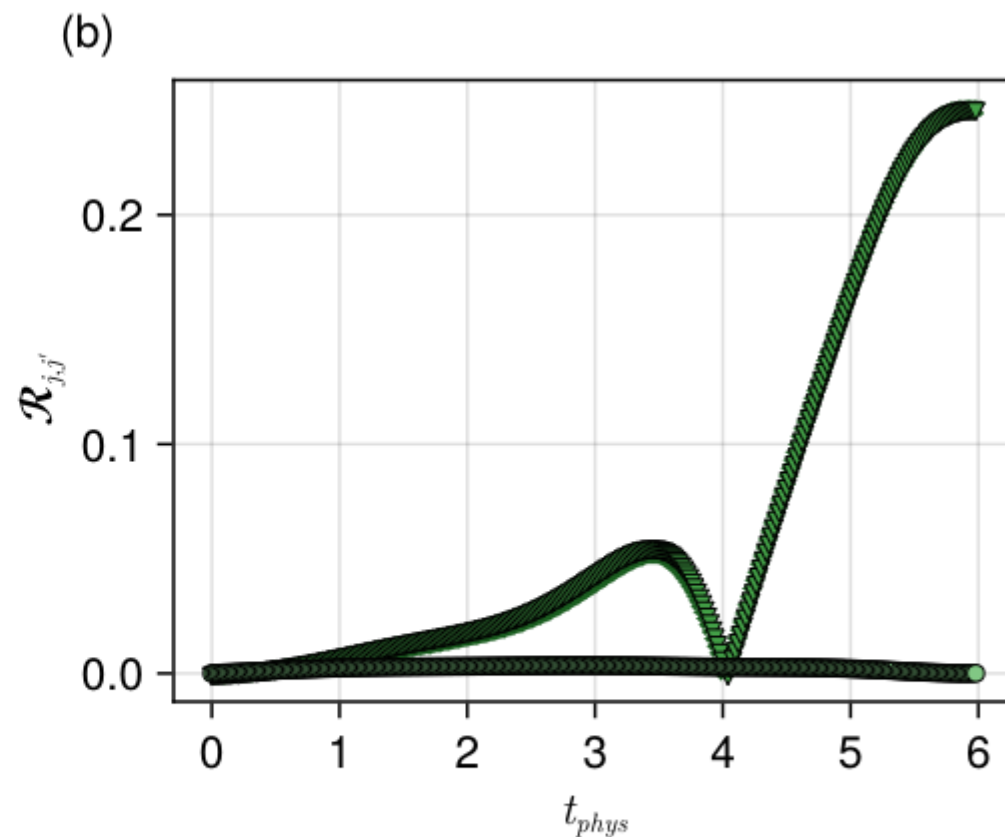
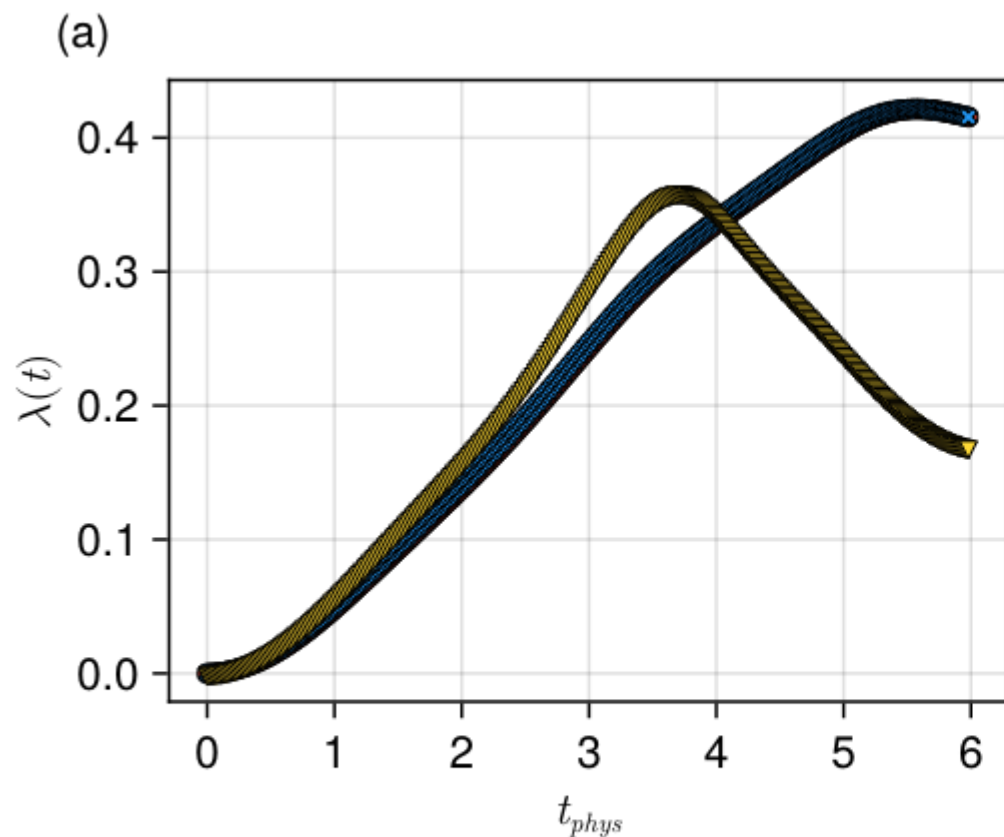
MPS Calculations using LSH framework

Probing cut-off dependence in dynamics: $N=64$



MPS Calculations using LSH framework

Probing cut-off dependence in dynamics: $N=32$



MPS Calculations using LSH framework

Probing the continuum limit of dynamics

....in progress
(both in static and dynamical observables)

Tensor Network Approach to LSH

Collaborators:



Emil Mathew



Navya Gupta



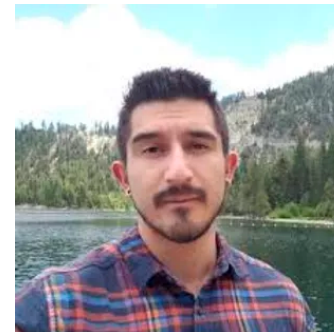
Aniruddha Bapat



Saurabh Kadam



Zohreh Davoudi



Jesse Stryker



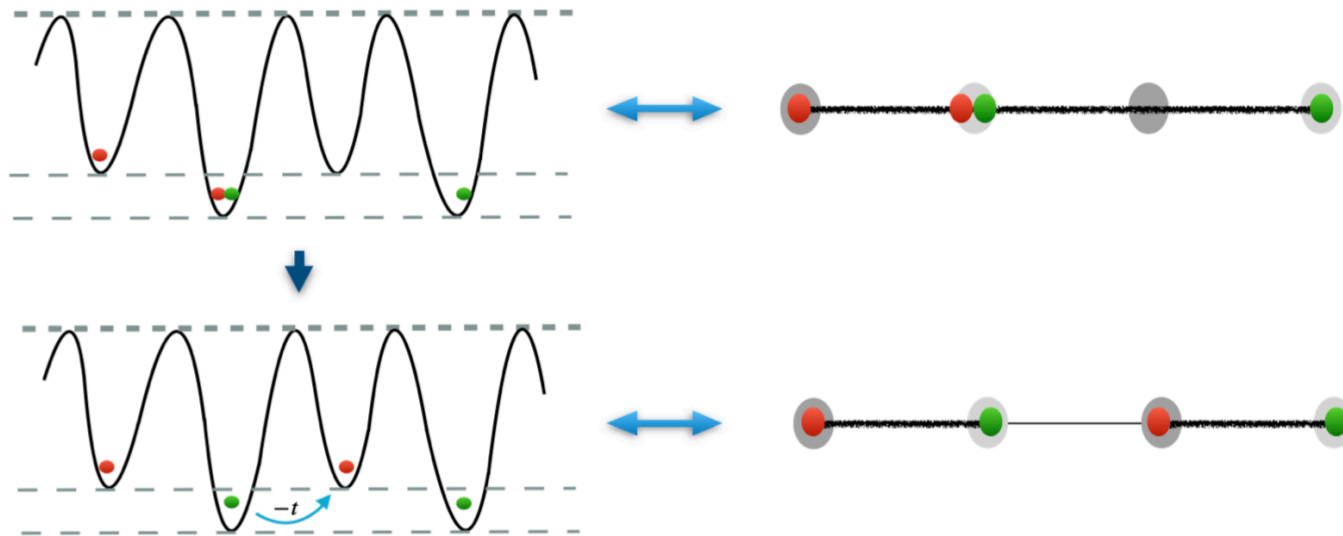
Nikko Pomata

Analog Quantum Simulation

PHYSICAL REVIEW A **105**, 023322 (2022)

Cold-atom quantum simulator for string and hadron dynamics in non-Abelian lattice gauge theory

Raka Dasgupta ^{1,*} and Indrakshi Raychowdhury ^{2,3,†}



Digital Quantum Computation

PHYSICAL REVIEW RESEARCH 2, 033039 (2020)

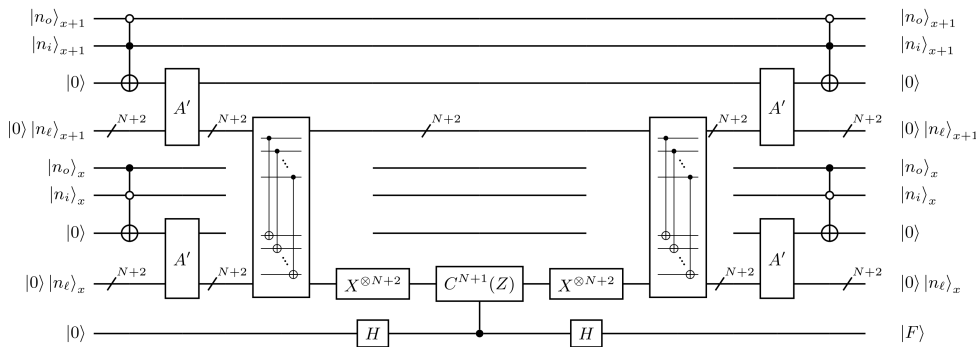
Solving Gauss's law on digital quantum computers with loop-string-hadron digitization

Indrakshi Raychowdhury*

Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA

Jesse R. Stryker[†]

Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA



near-term

| m/g | $\Delta_{\text{Trot.}}$ | x | L | η | t/a_s | Schwinger bosons | | | LSH | | |
|-------|-------------------------|-----|-----|--------|---------|------------------|----------|--------------------------|--------|----------|-----------------------|
| | | | | | | Qubits | Min. s | Min. CNOTs | Qubits | Min. s | Min. CNOTs |
| 1 | 10% | 0.1 | 10 | 2 | 1 | 92 | 186 | 4.8613×10^6 | 40 | 63 | 2.63088×10^5 |
| 1 | 10% | 0.1 | 10 | 2 | 5 | 92 | 2072 | 5.41538×10^7 | 40 | 702 | 2.93155×10^6 |
| 1 | 10% | 0.1 | 10 | 4 | 1 | 164 | 433 | 5.21403×10^8 | 60 | 136 | 1.64261×10^6 |
| 1 | 10% | 0.1 | 10 | 4 | 5 | 164 | 4841 | 5.82936×10^9 | 60 | 1519 | 1.83465×10^7 |
| 1 | 10% | 0.1 | 20 | 2 | 1 | 192 | 262 | 1.44561×10^7 | 80 | 89 | 7.84624×10^5 |
| 1 | 10% | 0.1 | 20 | 2 | 5 | 192 | 2929 | 1.61611×10^8 | 80 | 993 | 8.75429×10^6 |
| 1 | 10% | 0.1 | 20 | 4 | 1 | 344 | 613 | 1.55832×10^9 | 120 | 193 | 4.92111×10^6 |
| 1 | 10% | 0.1 | 20 | 4 | 5 | 344 | 6846 | 1.74034×10^{10} | 120 | 2149 | 5.47952×10^7 |

far-term

| m/g | x | η | L | t/a_s | Δ | $\alpha_{\text{Trot.}}$ | $\alpha_{\text{Newt.}}$ | Schwinger bosons | | LSH | |
|-------|-----|--------|------|---------|----------|-------------------------|-------------------------|------------------|--------------------------|--------|--------------------------|
| | | | | | | | | Qubits | T gates | Qubits | T gates |
| 1 | 1 | 4 | 100 | 1 | 0.01 | 90% | 9% | 2626 | 8.19713×10^{11} | 1319 | 3.91817×10^{10} |
| 1 | 1 | 4 | 100 | 1 | 0.001 | 90% | 9% | 2704 | 3.09951×10^{12} | 1397 | 1.5172×10^{11} |
| 1 | 1 | 4 | 100 | 10 | 0.01 | 90% | 9% | 2704 | 3.0993×10^{13} | 1397 | 1.51643×10^{12} |
| 1 | 1 | 4 | 100 | 10 | 0.001 | 90% | 9% | 2808 | 1.2146×10^{14} | 1475 | 5.76229×10^{12} |
| 1 | 1 | 4 | 1000 | 1 | 0.01 | 90% | 9% | 18904 | 3.12769×10^{13} | 6797 | 1.53099×10^{12} |
| 1 | 1 | 4 | 1000 | 1 | 0.001 | 90% | 9% | 19008 | 1.22564×10^{14} | 6875 | 5.81562×10^{12} |
| 1 | 1 | 4 | 1000 | 10 | 0.01 | 90% | 9% | 19008 | 1.22564×10^{15} | 6875 | 5.81468×10^{13} |
| 1 | 1 | 4 | 1000 | 10 | 0.001 | 90% | 9% | 19086 | 4.48657×10^{15} | 6979 | 2.29217×10^{14} |

General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory

Zohreh Davoudi,^{1,2,*} Alexander F. Shaw,^{3,†} and Jesse R. Stryker^{1,‡}

“The loop-string-hadron formulation further retains the non-Abelian gauge symmetry despite the inexactness of the digitized simulation, without the need for costly controlled operations. Such theoretical and algorithmic considerations are likely to be essential in quantumly simulating other complex theories of relevance to nature.”

A detailed analysis establishes benefits of using LSH framework on universal quantum computers both in

Way to a complete SU(3) LSH framework

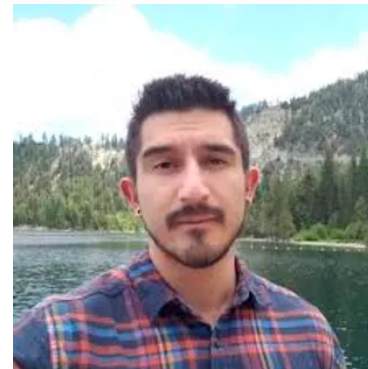
PHYSICAL REVIEW D **107**, 094513 (2023)

Loop-string-hadron formulation of an SU(3) gauge theory with dynamical quarks

Saurabh V. Kadam,^{1,*} Indrakshi Raychowdhury^{2,†} and Jesse R. Stryker^{1,‡}

¹*Maryland Center for Fundamental Physics, University of Maryland, College Park, Maryland 20742, USA*

²*Department of Physics, BITS-Pilani, K K Birla Goa Campus, Zuarinagar, Goa 403726, India*



Jesse Stryker



Saurabh Kadam

Way to a complete SU(3) LSH framework

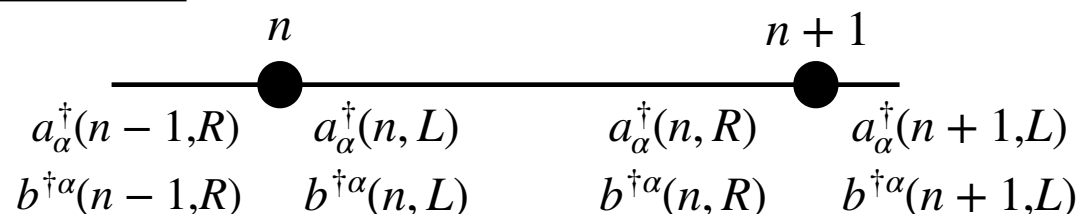
- Sneak-peak into the underlying pre potential framework: SU(2) vs SU(3)

e.g. in 1 spatial dimension



Irrep by Schwinger boson construction: monomial

$$|j, m_L, m_R\rangle \equiv a_1^\dagger(n, L)^{j+m_L} a_2^\dagger(n, L)^{j-m_L} a_1^\dagger(n, R)^{j+m_R} a_2^\dagger(n, R)^{j-m_R} |0,0,0,0\rangle$$



Irrep by Schwinger boson construction: Not a monomial

An irrep must be traceless: $a \cdot b \approx 0$

Irreducible Schwinger Bosons for SU(3):

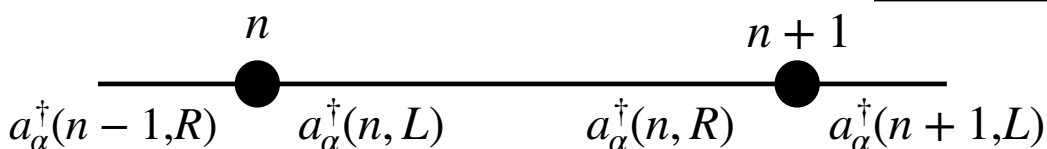
$$a_\alpha^\dagger, b^{\dagger\alpha} \rightarrow A_\alpha^\dagger, B^{\dagger\alpha}$$

Irrep by irreducible Schwinger boson construction: monomials

Way to a complete SU(3) LSH framework

- Sneak-peak into the underlying pre potential framework: SU(2) vs SU(3)

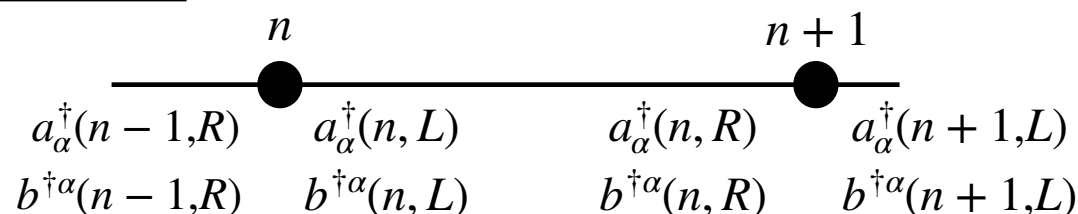
e.g. in 1 spatial dimension



Irrep by Schwinger boson construction: monomial

$$|j, m_L, m_R\rangle \equiv a_1^\dagger(n, L)^{j+m_L} a_2^\dagger(n, L)^{j-m_L} a_1^\dagger(n, R)^{j+m_R} a_2^\dagger(n, R)^{j-m_R} |0, 0, 0, 0\rangle$$

**Couple with fermions
for both SU(2) and SU(3)**



Irrep by Schwinger boson construction: Not a monomial

An irrep must be traceless: $a \cdot b \approx 0$

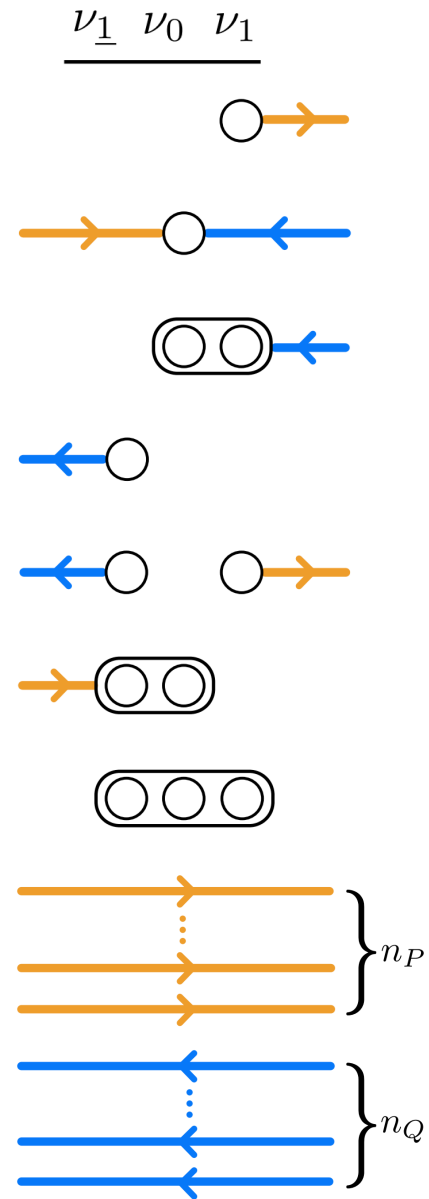
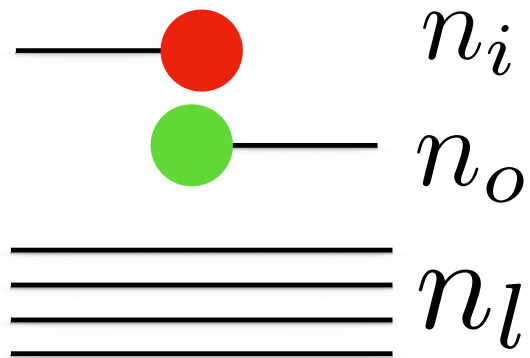
Irreducible Schwinger Bosons for SU(3):

$$a_\alpha^\dagger, b^{\dagger\alpha} \rightarrow A_\alpha^\dagger, B^{\dagger\alpha}$$

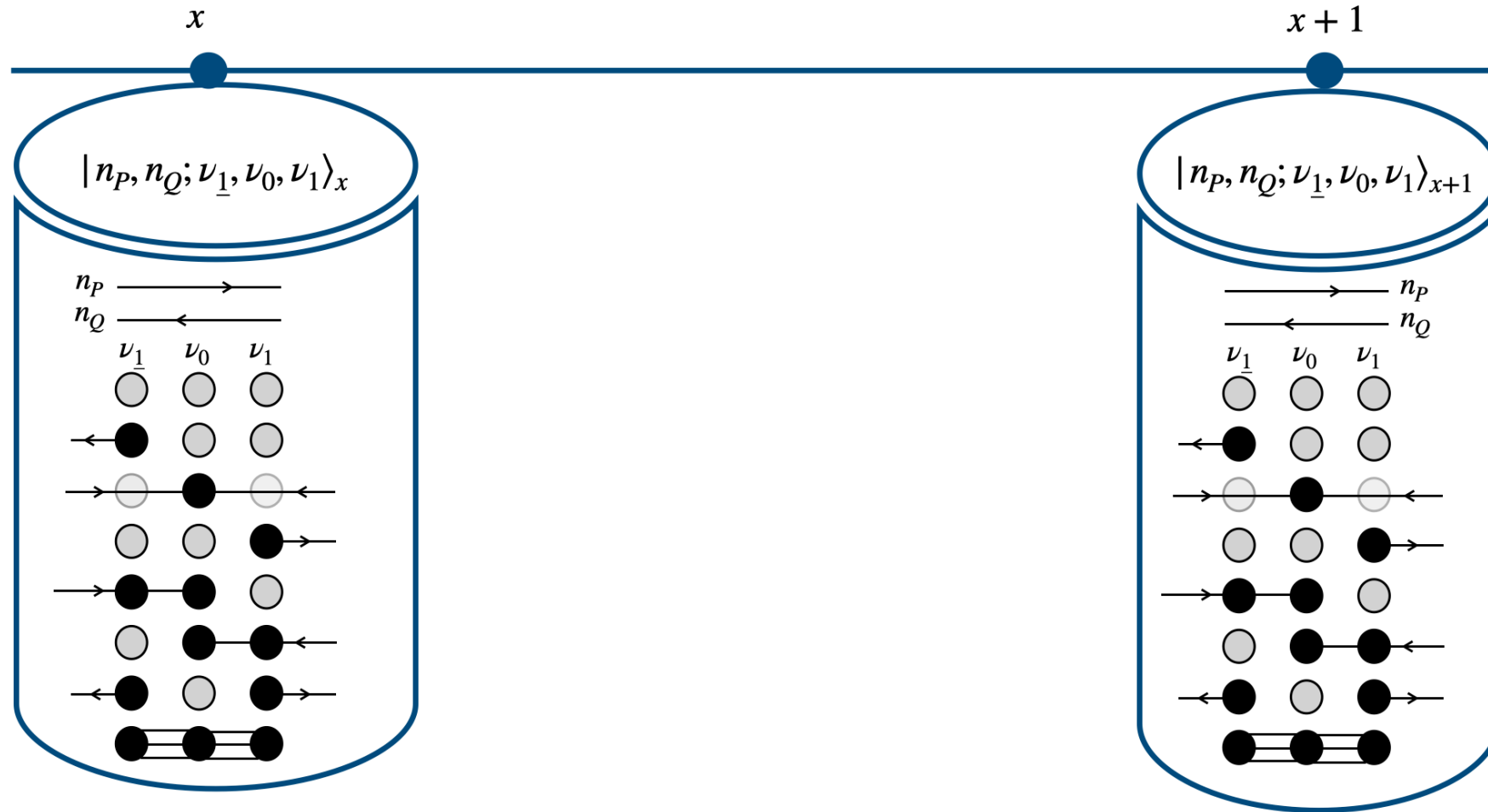
Irrep by irreducible Schwinger boson construction: monomials

Local LSH states

SU(2) vs SU(3): 1d spatial lattice

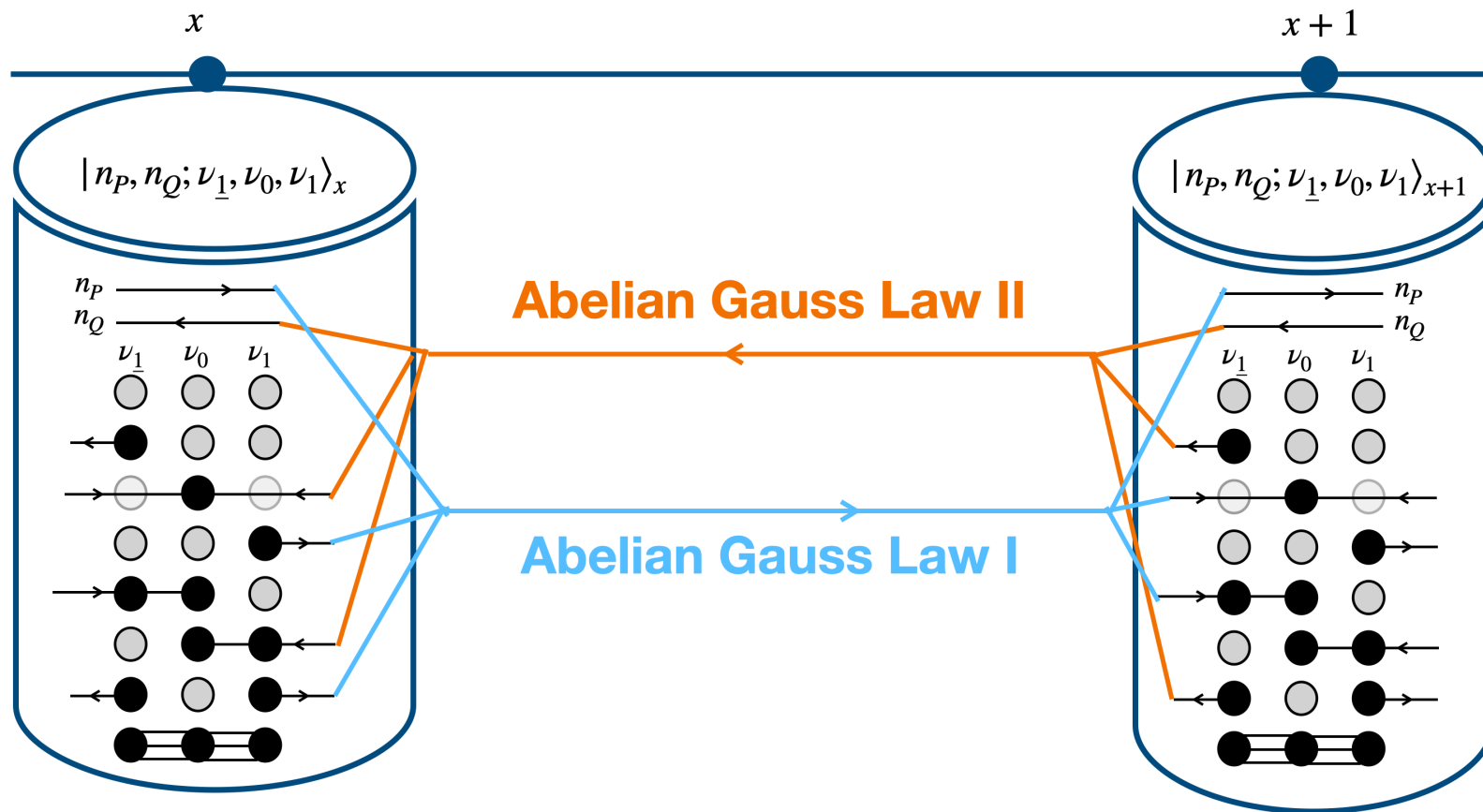


Loops-Strings-Hadrons : SU(3) in 1+1 d



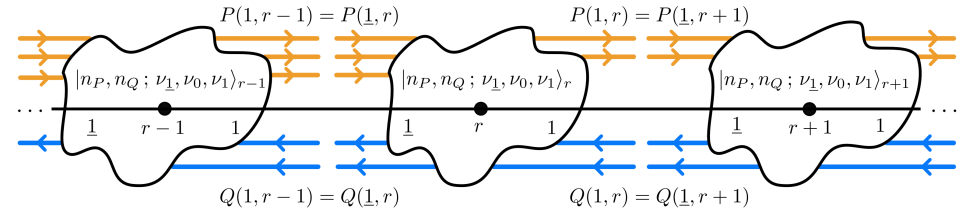
Loops-Strings-Hadrons : SU(2) in 1+1 d

Global LSH states are constructed by imposing a pair of Abelian Gauss Law constraints



LSH Formulation for SU(3): first step in 1+1 dimension is complete

Local LSH state:



$$|n_P, n_Q; \nu_{\underline{1}}, \nu_0, \nu_1\rangle_r, \quad n_P, n_Q \in \{0, 1, 2, \dots\}, \quad \nu_{\underline{1}}, \nu_0, \nu_1 \in \{0, 1\},$$

Abelian Gauss laws

$$P(1, r) = P(\underline{1}, r + 1) \quad \text{and} \quad Q(1, r) = Q(\underline{1}, r + 1)$$

$$P(\underline{1}, r) = n_P(r) + \nu_0(r) (1 - \nu_1(r))$$

$$P(1, r) = n_P(r) + \nu_1(r) (1 - \nu_0(r))$$

$$Q(\underline{1}, r) = n_Q(r) + \nu_{\underline{1}}(r) (1 - \nu_0(r))$$

$$Q(1, r) = n_Q(r) + \nu_0(r) (1 - \nu_{\underline{1}}(r))$$

Normalized ladder operators in LSH basis

$$|n_P, n_Q; \nu_{\underline{1}}, \nu_0, \nu_1\rangle_r$$

$$|n_P, n_Q; \nu_{\underline{1}}, \nu_0, \nu_1\rangle = (\hat{\Gamma}_P^\dagger)^{n_P} (\hat{\Gamma}_Q^\dagger)^{n_Q} (\hat{\chi}_{\underline{1}}^\dagger)^{\nu_{\underline{1}}} (\hat{\chi}_0^\dagger)^{\nu_0} (\hat{\chi}_1^\dagger)^{\nu_1} |0, 0; 0, 0, 0\rangle$$

$$\hat{\Gamma}_P(r) = \sum_{n_P=1}^{\infty} \sum_{n_Q, \nu_{\underline{1}}, \nu_0, \nu_1} |n_P - 1, n_Q; \nu_{\underline{1}}, \nu_0, \nu_1\rangle \langle n_P, n_Q; \nu_{\underline{1}}, \nu_0, \nu_1|_r$$

$$\hat{\Gamma}_Q(r) = \sum_{n_Q=1}^{\infty} \sum_{n_P, \nu_{\underline{1}}, \nu_0, \nu_1} |n_P, n_Q - 1; \nu_{\underline{1}}, \nu_0, \nu_1\rangle \langle n_P, n_Q; \nu_{\underline{1}}, \nu_0, \nu_1|_r$$

$$\hat{\chi}_{\underline{1}}(r) = \sum_{n_P, n_Q, \nu_0, \nu_1} |n_P, n_Q; 0, \nu_0, \nu_1\rangle \langle n_P, n_Q; 1, \nu_0, \nu_1|_r$$

$$\hat{\chi}_0(r) = \sum_{n_P, n_Q, \nu_{\underline{1}}, \nu_1} |n_P, n_Q; \nu_{\underline{1}}, 0, \nu_1\rangle \langle n_P, n_Q; \nu_{\underline{1}}, 1, \nu_1|_r (-1)^{\nu_{\underline{1}}}$$

$$\hat{\chi}_1(r) = \sum_{n_P, n_Q, \nu_{\underline{1}}, \nu_0} |n_P, n_Q; \nu_{\underline{1}}, \nu_0, 0\rangle \langle n_P, n_Q; \nu_{\underline{1}}, \nu_0, 1|_r (-1)^{\nu_{\underline{1}} + \nu_0}$$

$\psi^\dagger \cdot B^\dagger(1), \psi^\dagger \cdot A(\underline{1})$: apply $\hat{\chi}_1^\dagger$,

$\psi^\dagger \cdot B^\dagger(\underline{1}), \psi^\dagger \cdot A(1)$: apply $\hat{\chi}_{\underline{1}}^\dagger$,

$\psi^\dagger \cdot A^\dagger(1) \wedge B(1), \psi^\dagger \cdot A^\dagger(\underline{1}) \wedge B(\underline{1}), \psi^\dagger \cdot A^\dagger(\underline{1}) \wedge A^\dagger(1)$: apply $\hat{\chi}_0^\dagger$,

$A^\dagger(\underline{1}) \cdot B^\dagger(1)$: raise \hat{n}_P by one,

$B^\dagger(\underline{1}) \cdot A^\dagger(1)$: raise \hat{n}_Q by one.

$\psi^\dagger \cdot B^\dagger(\underline{1}) \psi^\dagger \cdot B(1)$: apply $\hat{\chi}_{\underline{1}}^\dagger \hat{\chi}_1^\dagger$

$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(1)$: apply $\hat{\chi}_0^\dagger \hat{\chi}_1^\dagger$

$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(\underline{1})$: apply $\hat{\chi}_{\underline{1}}^\dagger \hat{\chi}_0^\dagger$

$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger$: apply $\hat{\chi}_{\underline{1}}^\dagger \hat{\chi}_0^\dagger \hat{\chi}_1^\dagger$

Irreducible Prepotentials to LSH

$$\psi^\dagger \cdot B^\dagger(\underline{1}) \mapsto \hat{\chi}_1^\dagger(\hat{\Gamma}_P^\dagger)^{\hat{\nu}_0} \sqrt{\hat{n}_P + 2 - \hat{\nu}_0} \sqrt{\frac{\hat{n}_P + \hat{n}_Q + 3 + \hat{\nu}_0}{\hat{n}_P + \hat{n}_Q + 2 + \hat{\nu}_1 + \hat{\nu}_0}}$$

$$\psi \cdot B(\underline{1}) \mapsto \hat{\chi}_1(\hat{\Gamma}_P)^{\hat{\nu}_0} \sqrt{\hat{n}_P + 2(1 - \hat{\nu}_0)} \sqrt{\frac{\hat{n}_P + \hat{n}_Q + 3}{\hat{n}_P + \hat{n}_Q + 2 + \hat{\nu}_1}}$$

$$\psi^\dagger \cdot B^\dagger(\underline{1}) \mapsto \hat{\chi}_1^\dagger(\hat{\Gamma}_Q^\dagger)^{\hat{\nu}_0} \sqrt{\hat{n}_Q + 2 - \hat{\nu}_0} \sqrt{\frac{\hat{n}_P + \hat{n}_Q + 3 + \hat{\nu}_0}{\hat{n}_P + \hat{n}_Q + 2 + \hat{\nu}_1 + \hat{\nu}_0}}$$

$$\psi \cdot B(\underline{1}) \mapsto \hat{\chi}_1(\hat{\Gamma}_Q)^{\hat{\nu}_0} \sqrt{\hat{n}_Q + 2(1 - \hat{\nu}_0)} \sqrt{\frac{\hat{n}_P + \hat{n}_Q + 3}{\hat{n}_P + \hat{n}_Q + 2 + \hat{\nu}_1}}$$

$$\psi^\dagger \cdot A(\underline{1}) \mapsto \hat{\chi}_1^\dagger(\hat{\Gamma}_Q)^{1-\hat{\nu}_0} \sqrt{\hat{n}_Q + 2\hat{\nu}_0} \sqrt{\frac{\hat{n}_P + \hat{n}_Q + 2 + \hat{\nu}_1}{\hat{n}_P + \hat{n}_Q + 2}}$$

$$\psi \cdot A^\dagger(\underline{1}) \mapsto \hat{\chi}_1(\hat{\Gamma}_Q^\dagger)^{1-\hat{\nu}_0} \sqrt{\hat{n}_Q + 1 + \hat{\nu}_0} \sqrt{\frac{\hat{n}_P + \hat{n}_Q + 3 - \hat{\nu}_0 + \hat{\nu}_1}{\hat{n}_P + \hat{n}_Q + 3 - \hat{\nu}_0}}$$

$$\psi^\dagger \cdot A(\underline{1}) \mapsto \hat{\chi}_1^\dagger(\hat{\Gamma}_P)^{1-\hat{\nu}_0} \sqrt{\hat{n}_P + 2\hat{\nu}_0} \sqrt{\frac{\hat{n}_P + \hat{n}_Q + 2 + \hat{\nu}_1}{\hat{n}_P + \hat{n}_Q + 2}}$$

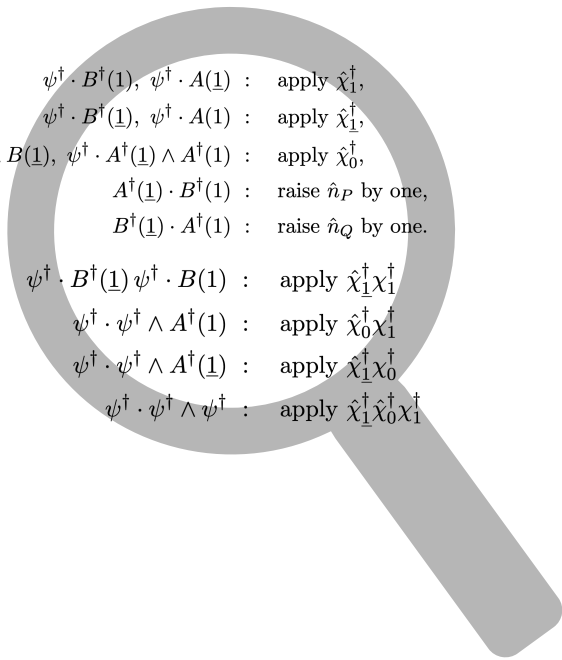
$$\psi \cdot A^\dagger(\underline{1}) \mapsto \hat{\chi}_1(\hat{\Gamma}_P^\dagger)^{1-\hat{\nu}_0} \sqrt{\hat{n}_P + 1 + \hat{\nu}_0} \sqrt{\frac{\hat{n}_P + \hat{n}_Q + 3 - \hat{\nu}_0 + \hat{\nu}_1}{\hat{n}_P + \hat{n}_Q + 3 - \hat{\nu}_0}}$$

$$\psi^\dagger \cdot A^\dagger(\underline{1}) \wedge B(\underline{1}) \mapsto -\hat{\chi}_0^\dagger(\hat{\Gamma}_P)^{1-\hat{\nu}_1}(\hat{\Gamma}_Q^\dagger)^{\hat{\nu}_1} \sqrt{\hat{n}_P + 2\hat{\nu}_1} \sqrt{\hat{n}_Q + 2 - \hat{\nu}_1}$$

$$\psi \cdot B^\dagger(\underline{1}) \wedge A(\underline{1}) \mapsto \hat{\chi}_0(\hat{\Gamma}_P^\dagger)^{1-\hat{\nu}_1}(\hat{\Gamma}_Q)^{\hat{\nu}_1} \sqrt{\hat{n}_P + 1 + \hat{\nu}_1} \sqrt{\hat{n}_Q + 2(1 - \hat{\nu}_1)}$$

$$\psi^\dagger \cdot A^\dagger(\underline{1}) \wedge B(\underline{1}) \mapsto \hat{\chi}_0^\dagger(\hat{\Gamma}_P^\dagger)^{\hat{\nu}_1}(\hat{\Gamma}_Q)^{1-\hat{\nu}_1} \sqrt{\hat{n}_P + 2 - \hat{\nu}_1} \sqrt{\hat{n}_Q + 2\hat{\nu}_1}$$

$$\psi \cdot B^\dagger(\underline{1}) \wedge A(\underline{1}) \mapsto -\hat{\chi}_0(\hat{\Gamma}_P)^{\hat{\nu}_1}(\hat{\Gamma}_Q^\dagger)^{1-\hat{\nu}_1} \sqrt{\hat{n}_P + 2(1 - \hat{\nu}_1)} \sqrt{\hat{n}_Q + 1 + \hat{\nu}_1}$$



$\psi^\dagger \cdot B^\dagger(\underline{1}), \psi^\dagger \cdot A(\underline{1})$: apply $\hat{\chi}_1^\dagger$,
 $\psi^\dagger \cdot B^\dagger(\underline{1}), \psi^\dagger \cdot A(\underline{1})$: apply $\hat{\chi}_1^\dagger$,
 $\psi^\dagger \cdot A^\dagger(\underline{1}) \wedge B(\underline{1}), \psi^\dagger \cdot A^\dagger(\underline{1}) \wedge A^\dagger(\underline{1})$: apply $\hat{\chi}_0^\dagger$,
 $A^\dagger(\underline{1}) \cdot B^\dagger(\underline{1})$: raise \hat{n}_P by one,
 $B^\dagger(\underline{1}) \cdot A^\dagger(\underline{1})$: raise \hat{n}_Q by one.
 $\psi^\dagger \cdot B^\dagger(\underline{1}) \psi^\dagger \cdot B(\underline{1})$: apply $\hat{\chi}_1^\dagger \hat{\chi}_1$
 $\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(\underline{1})$: apply $\hat{\chi}_0^\dagger \hat{\chi}_1^\dagger$
 $\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(\underline{1})$: apply $\hat{\chi}_1^\dagger \hat{\chi}_0^\dagger$
 $\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger$: apply $\hat{\chi}_1^\dagger \hat{\chi}_0^\dagger \hat{\chi}_1^\dagger$

Diagonal functions are in terms of

$$\hat{n}_l |n_P, n_Q; \nu_1, \nu_0, \nu_1\rangle = n_l |n_P, n_Q; \nu_1, \nu_0, \nu_1\rangle,$$

$$\hat{\nu}_f |n_P, n_Q; \nu_1, \nu_0, \nu_1\rangle = \nu_f |n_P, n_Q; \nu_1, \nu_0, \nu_1\rangle.$$

$$l \in \{P, Q\}$$

$$f \in \{\underline{1}, 0, 1\}$$

LSH Hamiltonian for SU(3)

$$H = H_M + H_E + H_I$$

$$H_M = \sum_{r=1}^N H_M(r) \equiv \mu \sum_{r=1}^N (-1)^r (\hat{\nu}_1(r) + \hat{\nu}_0(r) + \hat{\nu}_1(r))$$

$$H_E = \sum_{r=1}^{N'} H_E(r) \equiv \sum_{r=1}^{N'} \frac{1}{3} \left(\hat{P}(1, r)^2 + \hat{Q}(1, r)^2 + \hat{P}(1, r)\hat{Q}(1, r) \right) + \hat{P}(1, r) + \hat{Q}(1, r)$$

$$\begin{aligned} H_I = \sum_{r=1}^{N'} H_I(r) \equiv \sum_r x & \left[\hat{\chi}_1^\dagger (\hat{\Gamma}_P^\dagger)^{\hat{\nu}_0} \sqrt{1 - \hat{\nu}_0 / (\hat{n}_P + 2)} \sqrt{1 - \hat{\nu}_1 / (\hat{n}_P + \hat{n}_Q + 3)} \right]_r \otimes \left[\sqrt{1 + \hat{\nu}_0 / (\hat{n}_P + 1)} \sqrt{1 + \hat{\nu}_1 / (\hat{n}_P + \hat{n}_Q + 2)} \hat{\chi}_1 (\hat{\Gamma}_P^\dagger)^{1 - \hat{\nu}_0} \right]_{r+1} \\ & + x \left[\hat{\chi}_1^\dagger (\hat{\Gamma}_Q)^{1 - \hat{\nu}_0} \sqrt{1 + \hat{\nu}_0 / (\hat{n}_Q + 1)} \sqrt{1 + \hat{\nu}_1 / (\hat{n}_P + \hat{n}_Q + 2)} \right]_r \otimes \left[\sqrt{1 - \hat{\nu}_0 / (\hat{n}_Q + 2)} \sqrt{1 - \hat{\nu}_1 / (\hat{n}_P + \hat{n}_Q + 3)} \hat{\chi}_1 (\hat{\Gamma}_Q)^{\hat{\nu}_0} \right]_{r+1} \\ & + x \left[\hat{\chi}_0^\dagger (\hat{\Gamma}_P)^{1 - \hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{\hat{\nu}_1} \sqrt{1 + \hat{\nu}_1 / (\hat{n}_P + 1)} \sqrt{1 - \hat{\nu}_1 / (\hat{n}_Q + 2)} \right]_r \otimes \left[\sqrt{1 - \hat{\nu}_1 / (\hat{n}_P + 2)} \sqrt{1 + \hat{\nu}_1 / (\hat{n}_Q + 1)} \hat{\chi}_0 (\hat{\Gamma}_P)^{\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{1 - \hat{\nu}_1} \right]_{r+1} \end{aligned}$$

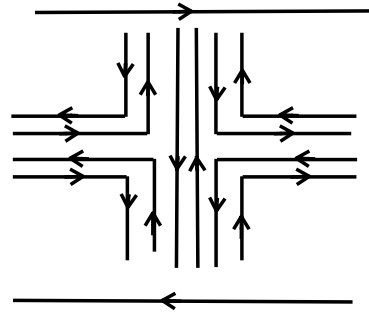
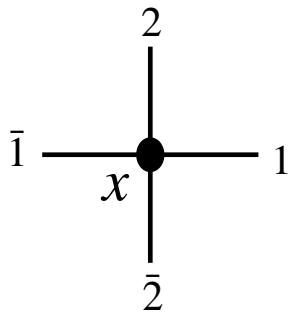
Numerically benchmarked with completely gauge fixed (pure gauge) Hamiltonian

Structurally identical to the SU(2) LSH construction

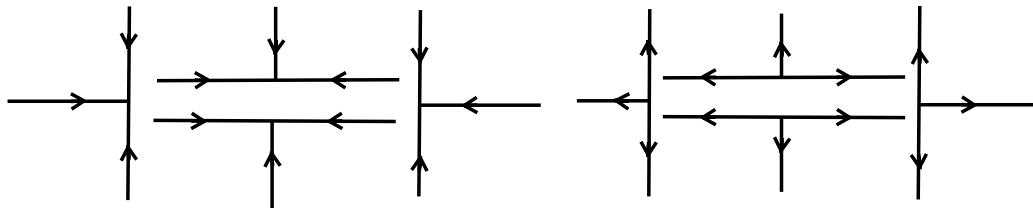
Loops-Strings-Hadrons : SU(3) beyond 1+1 d

Many surprises!

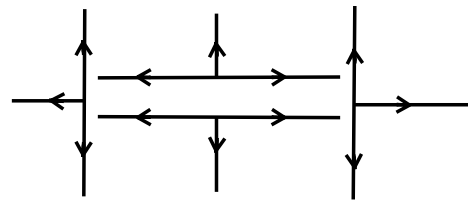
LSH Formulation for SU(3) in 2+1 dimension and beyond: generalising SU(2) construction



(a)



(b)

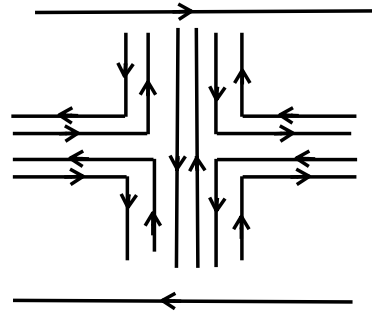
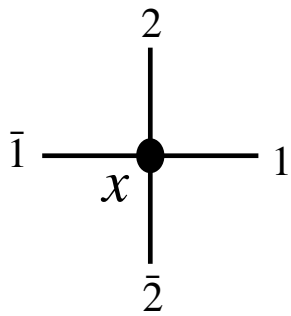


(c)

Loop configurations for 2d lattice

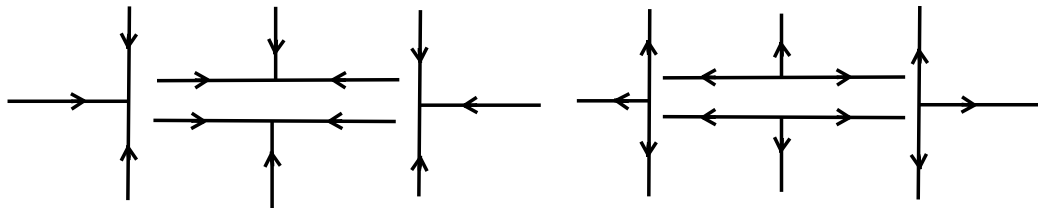
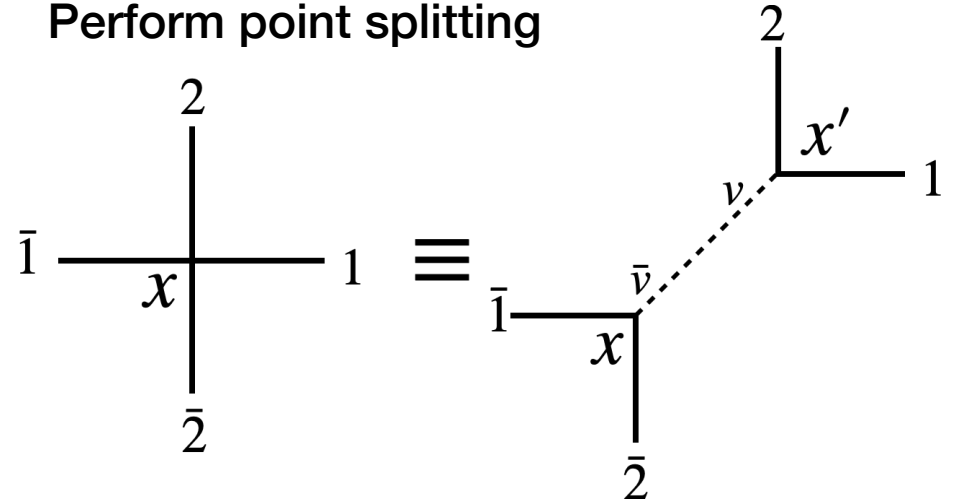
Too many redundant loop degrees of freedom

LSH Formulation for SU(3) in 2+1 dimension and beyond: generalising SU(2) construction



(a)

Perform point splitting



(b)

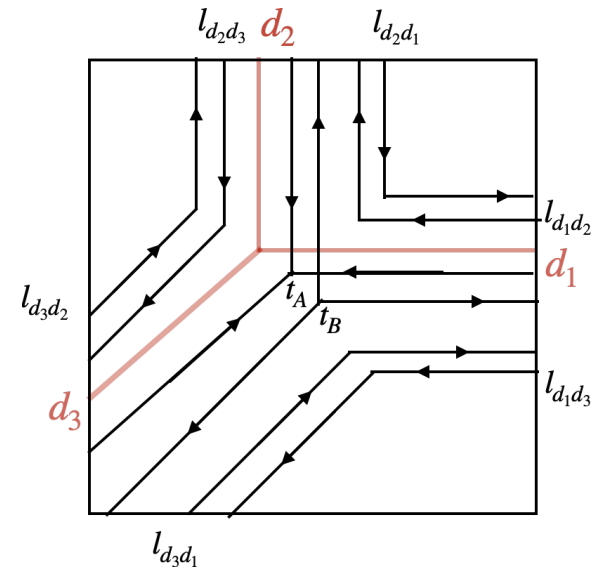
(c)

Loop configurations for 2d lattice

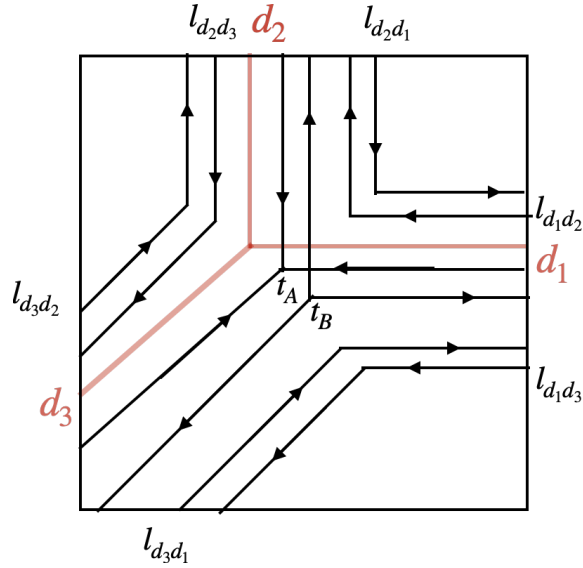
Too many redundant loop degrees of freedom

Loops on triangular lattice

Redundancy still persists



- Sneak-peak into the underlying SU(3) singlets



$$\hat{L}_{IJ} = \sum_{\alpha} A_{\alpha}^{\dagger}[I] B^{\dagger\alpha}[J] \text{ for } I \neq J$$

$$\hat{T}_{IJK}^A = \sum_{\alpha, \beta, \gamma} \epsilon^{\alpha\beta\gamma} A_{\alpha}^{\dagger}[I] A_{\beta}^{\dagger}[J] A_{\gamma}^{\dagger}[K]$$

$$\hat{T}_{IJK}^B = \sum_{\alpha, \beta, \gamma} \epsilon_{\alpha\beta\gamma} B^{\dagger\alpha}[I] B^{\dagger\beta}[J] B^{\dagger\gamma}[K]$$

Mandelstam constraint:

$$\hat{T}^A \hat{T}^B = \hat{L}_{12} \hat{L}_{2v} \hat{L}_{v1} + \hat{L}_{12} \hat{L}_{2v} \hat{L}_v$$

$$\begin{aligned} & |l_{12}, l_{2v}, l_{v1}, l_{21}, l_{v2}, l_{1v}; t_A, t_B\rangle \\ &= \left(\hat{L}_{12}\right)^{l_{12}} \left(\hat{L}_{2v}\right)^{l_{2v}} \left(\hat{L}_{v1}\right)^{l_{v1}} \\ & \left(\hat{L}_{12}\right)^{l_{21}} \left(\hat{L}_{2v}\right)^{l_{v2}} \left(\hat{L}_{v1}\right)^{l_{1v}} \\ & \left(\hat{T}^A\right)^{t_A} \left(\hat{T}^B\right)^{t_B} |0, 0, 0, 0, 0, 0; 0, 0\rangle \end{aligned}$$

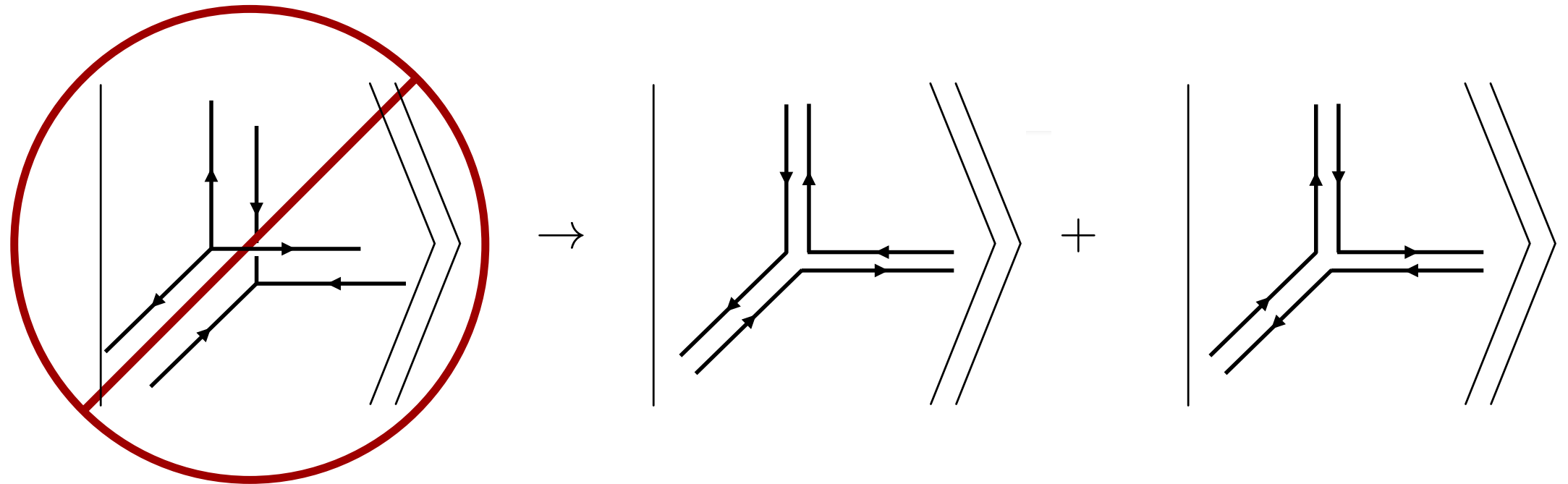
Occupation number basis:

not a set of orthogonal quantum numbers

$$p_1 + p_2 + p_v = q_1 + q_2 + q_v + 3(t_A - t_B)$$

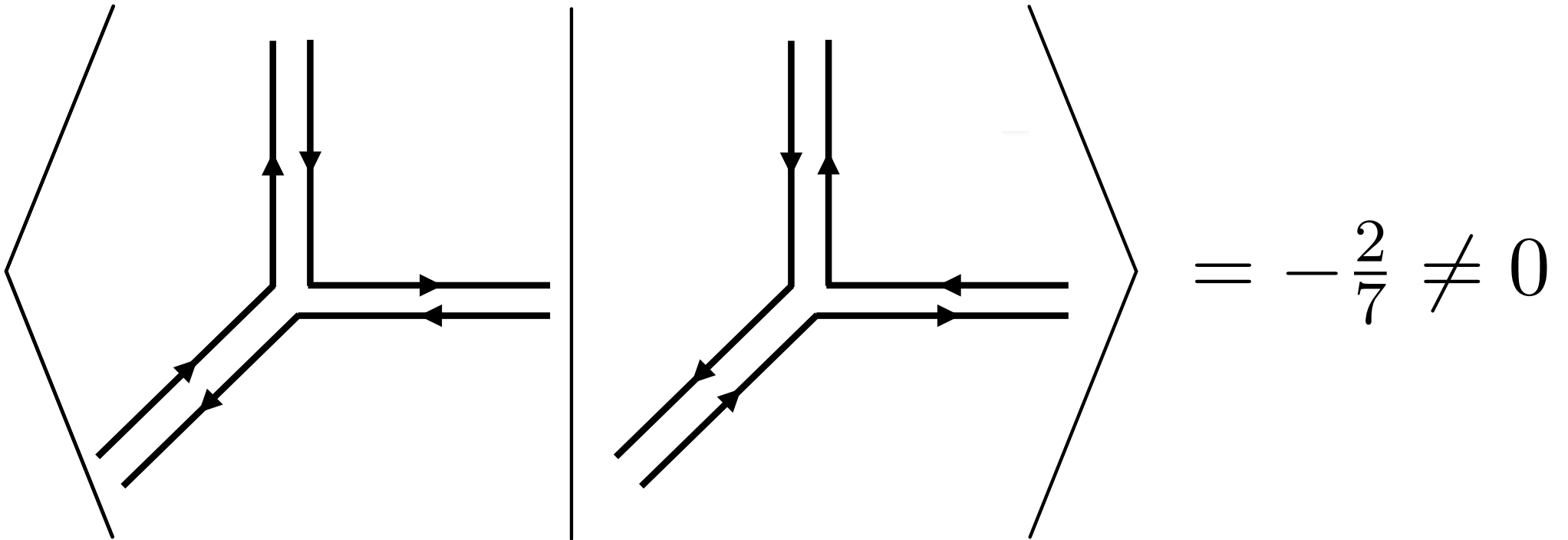
Search for a seventh Casimir quantum number begins...

Remnant Mandelstam Constraint

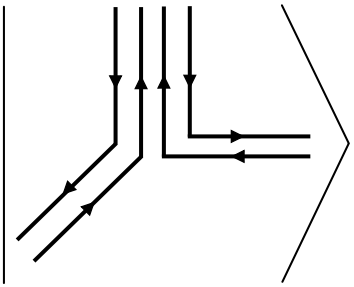


$$T_A^\dagger T_B^\dagger |0\rangle = |\ell_{12} = \ell_{23} = \ell_{31} = 1\rangle\rangle + |\ell_{21} = \ell_{32} = \ell_{13} = 1\rangle\rangle$$

Surprise: Non-orthogonality

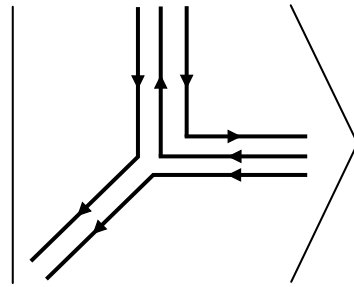


Attempt to systematically study: orthogonal subspaces



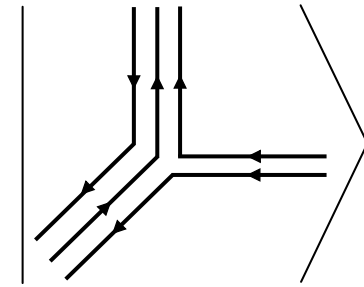
$$= |\ell_{12} = \ell_{23} = \ell_{21} = \ell_{32} = 1\rangle$$

Class I



$$= |\ell_{12} = \ell_{23} = \ell_{21} = \ell_{13} = 1\rangle$$

Class IIa



$$= |\ell_{12} = \ell_{23} = \ell_{32} = \ell_{13} = 1\rangle$$

Class IIb

Proposed Seventh Casimir to characterise the degenerate subspace

$$C_T \equiv (T_A T_B)^\dagger T_A T_B$$

arXiv:2407.19181v1

IQuS@UW-21-086

**Loop-string-hadron approach to SU(3) lattice Yang-Mills theory:
Gauge invariant Hilbert space of a trivalent vertex**

Saurabh V. Kadam,^{1,*} Aahiri Naskar,^{2,†} Indrakshi Raychowdhury,^{2,3,‡} and Jesse R. Stryker^{4,5,§}

...in progress

Ongoing work in Hamiltonian construction;
Effort in making $SU(3)$ LSH accessible for application.

Collaborators:



Aahiri Naskar,
Grad student,
BITS Goa



Jesse Stryker



Saurabh Kadam

...in progress

Thermalization

Thermalization of gauge theories



 Sep 4, 2024, 3:30 PM

 30m

 B052 (ASC)

Speaker

 Prof. Andreas Schäfer (University of Regensburg)

Description

Elucidating the microscopic mechanisms leading to thermalization of gauge theories is often advocated as an especially promising future application of quantum computing. What is usually not appreciated, however, is that many other approaches, e.g. lattice QCD, AdS/CFT duality, hydrodynamics, tensor networks, eigenstate thermalization (ETH) have already provided many complementary partial explanations and continue doing so. The talk will try to convey an impression of the existing web of methods and results, in which quantum computing still has to find its precise role. The talk will then focus on one very specific question, namely whether $SU(2)$ gauge theory shows ETH properties. The answer, based on numerical simulations on classical, digital computers, turns out to be affirmative.

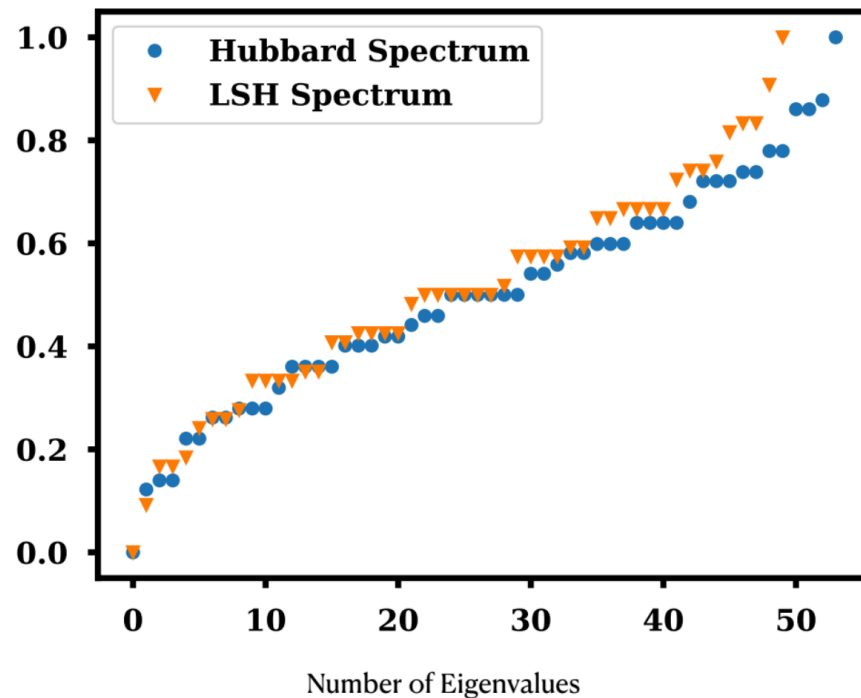
...in progress

Analog simulation

1+1d dynamics:
dynamics of SU(3)
strings and hadrons
in LSH framework

Continuity of strings are guaranteed
by AGL: protected by global
symmetries

Simulated by SU(3)
Fermi-Hubbard
model



Collaborators: Madhumita Kabiraj, Emil Mathew, Raka Dasgupta

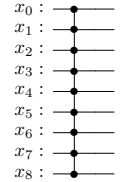
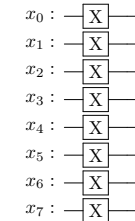
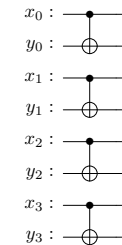
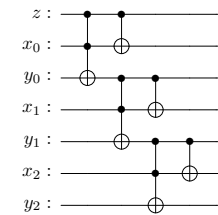
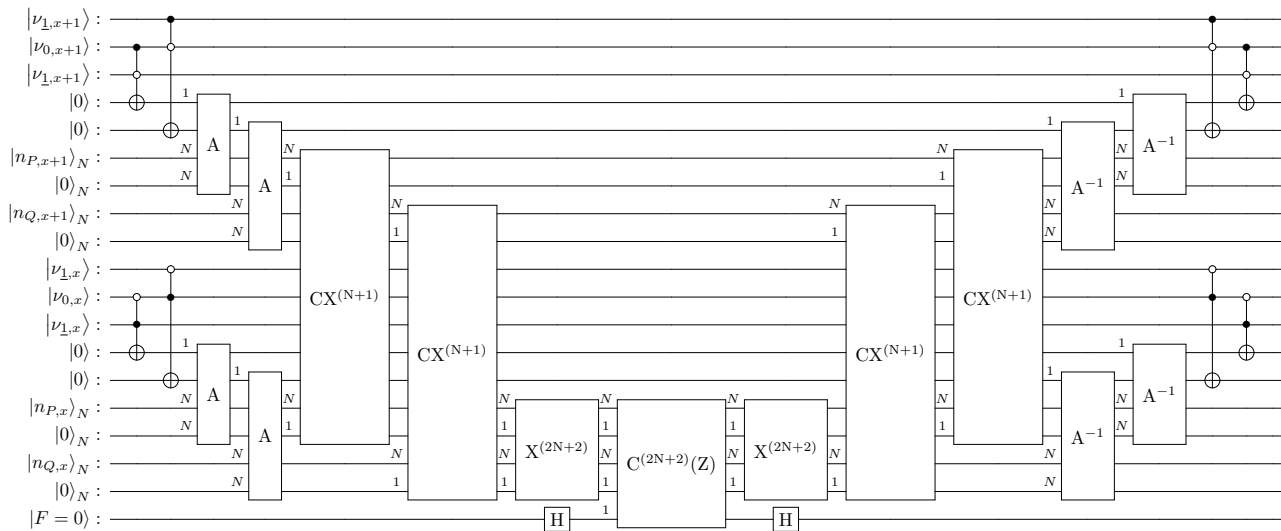
...in progress

Digital Quantum Computing

Fran Ilčić,
Grad. student,
BITS Goa



Physicality Oracle: Preliminary construction



Useful component for state preparation algorithms such as QAOA
and error detection in a simulation

Summary and Outlook

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⇒ Considerably less progress in quantum simulating the same using angular momentum basis within KS framework

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LSH framework in 3+1 dimension including multiple quark flavours: QCD



BITS Pilani
PILANI | DUBAI | GOA | HYDERABAD

CROIT

Center for Research in Quantum Information and Technology

Thank You

Collaborators:



Lawrence Berkeley
National Laboratory

W

UNIVERSITY of
WASHINGTON



Universität Regensburg

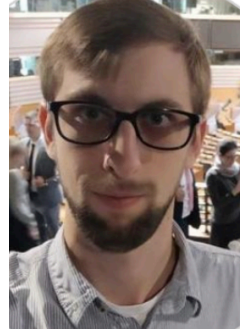
IQuS InQubator for Quantum Simulation



Emil Mathew
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