Jackiw-Teitelboim Gravity with matter on quantum computer

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based on a joint work in progress with

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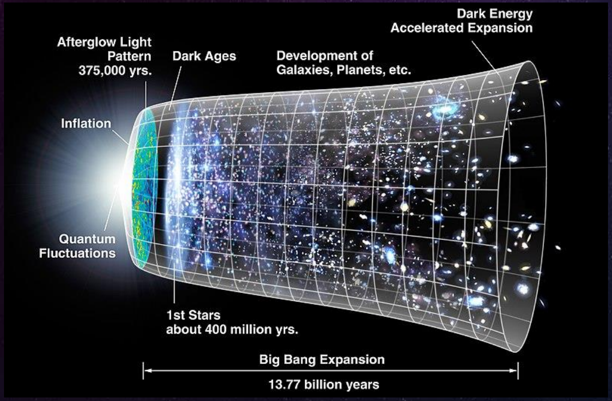
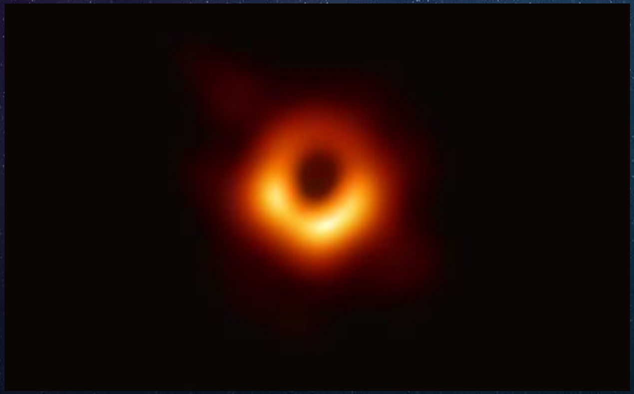
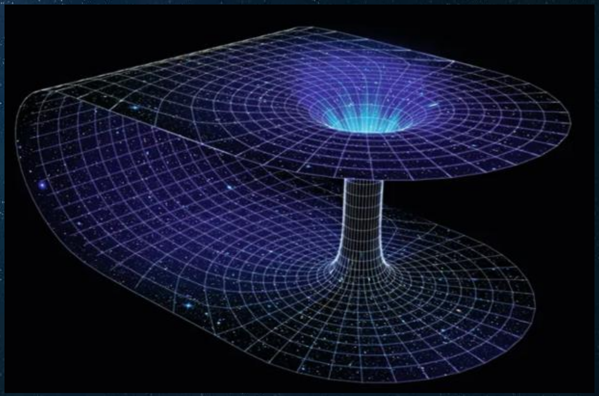
Quantum Gravity ?

- Quantum effect of gravity is expected to be important in very early universe
- Even finding a consistent formulation is challenging $(d \geq 4)$
- related to ultimate questions (e.g. what is spacetime?)
- $^{\exists}$ exciting objects: black holes, wormholes







[credits: NASA/WMAP science team, Event Horizon Telescope, Getty Images]

This talk:

Quantum Gravity on Quantum Computer

(QG) (QC)

⇩

Real time physics of QG

sign problem!

∃ Indirect approaches:

[cf. Garcia-Alvarez-Egusquiza-Campo-Sonner-Solano '16, Gharibyan-Hanada-MH-Liu '20, etc...]

Holography, Matrix theory (e.g. SYK, BFSS, BMN...)

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Here,

directly put QG on QC

[cf. loop quantum gravity: Cohen etal. '20, Shah '21]

This talk:

put JT gravity (w/ matter) on QC

&

make quantum algorithm to study real time physics

Pure JT:

- holographic dual of SYK model
- no propagating d.o.f. → d.o.f. only at boundaries
- analytically solvable (but should be a good first step)

With matter:

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Pure JT:

- holographic dual of SYK model
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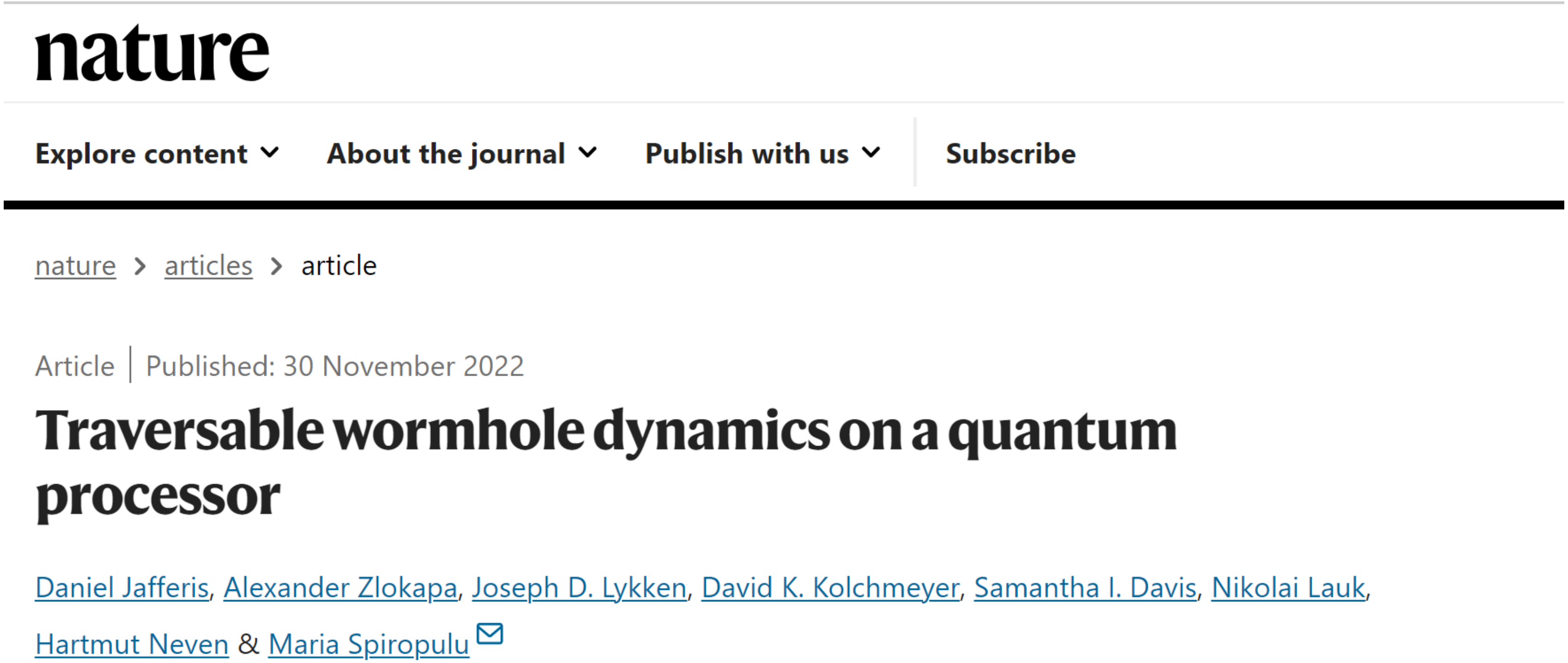
With matter:

- $^{\exists}$propagating d.o.f.
- generically unsolvable $\rightarrow$ new physical results?

Another motivation: experiment by Google Sycamore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Traversable wormhole dynamics on a quantum processor

Daniel Jafferis, Alexander Zlokapa, Joseph D. Lykken, David K. Kolchmeyer, Samantha I. Davis, Nikolai Lauk, Hartmut Neven & Maria Spiropulu

[Submitted on 15 Feb 2023]

Comment on "Traversable wormhole dynamics on a quantum processor"

Bryce Kobrin, Thomas Schuster, Norman Y. Yao

[Submitted on 27 Mar 2023]

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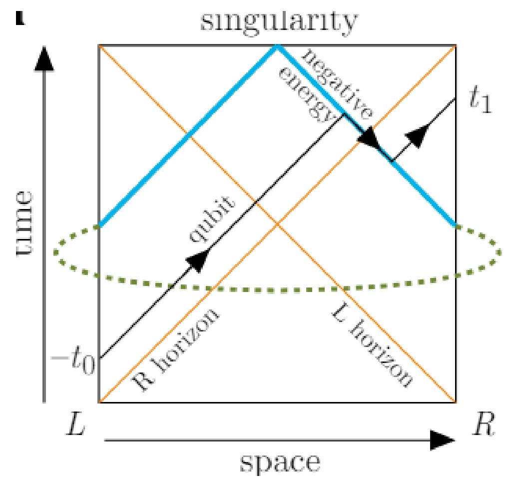
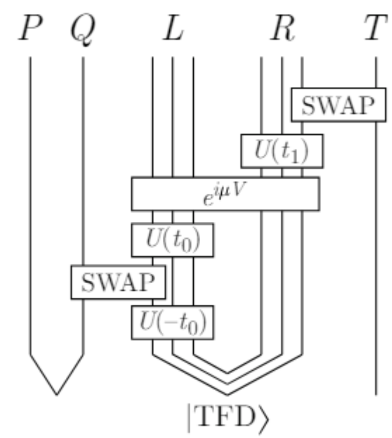
"Wormhole experiment" by Google Sycamore:

[Nature, Jafferis-Zlokapa-Lykken-Kolchmeyer-Davis-Lauk-Neven-Spiropulu '23]





- simulation of sparse SYK model assuming holography
- based on various nontrivial assumptions

  → contravarcy on whether wormhole was really made

Can we make it directly on the gravity side?

Contents

Jackiw-Teitelboim (JT) gravity

Fundamental fields:

[cf. Jafferis-Kolchmeyer '19, Penington-Witten '23]

$g_{\mu\nu}$: metric, Φ: dilaton (scalar)

Action :

(γ: induced metric, K: extrinsic curvature)

$$I_{\rm JT} = \int_M d^2x \sqrt{-g}\,\Phi(R+2) + 2\int_{\partial M} \sqrt{|\gamma|}\,\Phi(K-1) + \cdots$$

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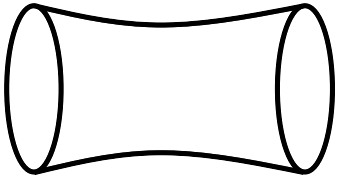
$$I_{\mathrm{JT}} = \int_M d^2x\,\sqrt{-g}\,\Phi(R+2) + 2\int_{\partial M}\sqrt{|\gamma|}\,\Phi(K-1) + \cdots$$

⇓ integrate Φ out

$$I_{\partial M} = 2\int_{\partial M}\sqrt{|\gamma|}\,\Phi(K-1)\,, \qquad R=-2 \to \mathrm{AdS}_2 \text{ (locally)}$$



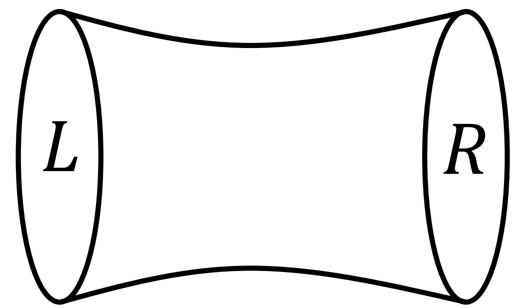
Coordinate :

$$ds^2 = -\cosh^2\sigma\, dT^2 + d\sigma^2 \qquad (\sigma, T \in \boldsymbol{R})$$

Boundary condition & Schwarzian theory

Boundary condition: $(\epsilon \ll 1)$

$$\begin{cases} \gamma_{tt} = -\dfrac{1}{\epsilon^2} \quad \rightarrow \quad e^{-\sigma_L} = \dfrac{2}{\epsilon}\dfrac{1}{\dot{T}_L}, \ e^{\sigma_R} = \dfrac{2}{\epsilon}\dfrac{1}{\dot{T}_R} \\ \Phi|_{\partial M} = \dfrac{\phi_b}{\epsilon} \end{cases}$$



$\epsilon \rightarrow 0$ limit:

$$I_{\partial M} = \phi_b \int dt \left[-\dot{T}_R^2 + \left(\frac{\ddot{T}_R}{\dot{T}_R} \right)^2 \right] + (\mathrm{L} \leftrightarrow \mathrm{R})$$

2 copies of the Schwarzian theory

Switching to operator formalism

$$I_{\partial M} = \phi_b \int dt \left[-\dot{T}_R^2 + \left(\frac{\ddot{T}_R}{\dot{T}_R} \right)^2 \right] + (\mathrm{L} \leftrightarrow \mathrm{R})$$

introduce auxiliary fields

$$S = \phi_b \int dt \left[p_{T_R} \dot{T}_R + p_{\chi_R} \dot{\chi}_R - \frac{1}{2\phi_b} \left(\frac{p_{\chi_R}^2}{2} + p_{T_R} e^{\chi_R} + \frac{e^{2\chi_R}}{2} \right) \right] + (\mathrm{L} \leftrightarrow \mathrm{R})$$

Hamiltonian:

$$H = \frac{1}{2\phi_b} \left(\frac{p_{\chi_R}^2}{2} + p_{T_R} e^{\chi_R} + \frac{e^{2\chi_R}}{2} \right) + (\mathrm{L} \leftrightarrow \mathrm{R})$$

This is not a whole story…

We need to impose conditions on physical states

Conditions on physical states

[cf. Penington-Witten '23]

① Invariance under AdS_2 isometry: $\left(J_a^L + J_a^R\right)|\mathrm{phys}\rangle = 0$

$$\left(\begin{array}{ll} J_1^L = p_{T_L}, & J_1^R = p_{T_R} \\ J_2^L = -\cos T_L\, p_{T_L} + \sin T_L\, p_{\chi_L} - e^{\chi_L}\cos T_L - \frac{i\sin T_L}{2}, & J_2^R = +\cos T_R\, p_{T_R} - \sin T_R\, p_{\chi_R} + e^{\chi_R}\cos T_R + \frac{i\sin T_R}{2} \\ J_3^L = -\sin T_L\, p_{T_L} - \cos T_L\, p_{\chi_L} - e^{\chi_L}\sin T_L + \frac{i\cos T_L}{2}, & J_3^R = +\sin T_R\, p_{T_R} + \cos T_R\, p_{\chi_R} + e^{\chi_R}\sin T_R - \frac{i\cos T_R}{2} \end{array}\right)$$

②

Conditions on physical states

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② Spacelike separation of the bdy's (causality)

$$\langle T_L, \chi_L, T_R, \chi_R|\mathrm{phys}\rangle = 0 \qquad \text{for } |T_L - T_R| \geq \pi$$

$$\Downarrow \qquad H = \frac{1}{4\phi_b}\left(\eta^{ab} J_a^R J_a^R - \frac{1}{4}\right) + (L \leftrightarrow R)$$

$$\left\{\begin{array}{l} H = \dfrac{1}{\phi_b}\left(\dfrac{p_\ell^2}{2} + \dfrac{1}{2}e^{-\ell}\right) \\ \ell = -\chi_R - \chi_L + \log\left(\dfrac{1+\cos(T_L - T_R)}{2}\right) \end{array}\right.$$

"renormalized geodesic length"

$$H = \frac{1}{\phi_b}\left(\frac{p_\ell^2}{2} + \frac{1}{2}e^{-\ell}\right) \qquad [\ell, p_\ell\,] = i$$

Pure JT gravity = solvable quantum mechanics

In this sense,

no “theoretical” motivation to simulate the pure JT

But,

worth to realize it on real quantum device

&

useful for putting more complicated theories on QC

(such as JT w/ matter)

Contents

"Regularization" of Hilbert space

Hilbert space of QFT is typically ∞ dimensional

⟶ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)
  - Putting on spatial lattice, Hilbert sp. is finite dimensional
- scalar
  - Hilbert sp. at each site is ∞ dimensional (need truncation or additional regularization)
- gauge field (w/ kinetic term)
  - no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)
  - ∞ dimensional Hilbert sp. in higher dimensions

Truncation of pure JT Hilbert space

$$H = \frac{1}{\phi_b}\left(\frac{p_\ell^2}{2} + \frac{1}{2}e^{-\ell}\right) \qquad [\ell, p_\ell\,] = i$$

∞ dimensional Hilbert space → Truncate it!

$\exists$ choices of basis: ℓ, p_ℓ, harmonic osc., energy, etc…

Case for energy basis:

$$H = \int_{E_0}^{\infty} dE\;\; E|E\rangle\langle E| = \int_{s_0}^{s_\infty} ds\;\; E'(s)E(s)|E(s)\rangle\langle E(s)|$$

↓ truncation

$$H_\Lambda = \sum_{n=0}^{\Lambda} \Delta s\, E'(s_n)E(s_n)|n\rangle\langle n|$$

$$\Delta s := \frac{s_\infty - s_0}{\Lambda}, \qquad s_n := \mathrm{s}_0 + \mathrm{n}\Delta s$$

Map to spin system (q-qubits)

$$\begin{cases} \Lambda \coloneqq 2^q - 1 \\ n = \mathrm{b_{q-1}} 2^{\mathrm{q-1}} + \mathrm{b_{q-2}} 2^{\mathrm{q-2}} + \cdots + b_0 2^0 \end{cases}$$

(binary representation)

Given n, we assign q-qubits state:

$$|n\rangle := |b_{q-1}\rangle |b_{q-2}\rangle \cdots |b_0\rangle$$

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(binary representation)

Given n, we assign q-qubits state:

$$|n\rangle := |b_{q-1}\rangle |b_{q-2}\rangle \cdots |b_0\rangle$$

Then,

$$H_\Lambda = \sum_{n=0}^{\Lambda} \omega_{\mathrm{n}} |n\rangle\langle n| = \sum_{\{b_j = 0,1\}} \omega_n |b_{q-1} \cdots b_0\rangle\langle b_{q-1} \cdots b_0|$$

Spin system!

$$\left(\begin{array}{ll} |0\rangle\langle 0| = \frac{1_2 - \sigma_z}{2}, & |1\rangle\langle 1| = \frac{1_2 + \sigma_z}{2}, \\ |0\rangle\langle 1| = \frac{\sigma_x + i\sigma_y}{2}, & |1\rangle\langle 0| = \frac{\sigma_x - i\sigma_y}{2} \end{array} \right)$$

Time evolution is easily implemented:

$$e^{-iH_\Lambda t} = \prod_{\{b_j\}} e^{-i\omega_n t |b_{q-1} \cdots b_0\rangle\langle b_{q-1} \cdots b_0|}$$

Construction of Hartle-Hawking state

[cf. Jafferis-Kolchmeyer '19]

Hartle-Hawking state:

$$|\Psi_\beta\rangle := \int_0^\infty du\,\sqrt{2u\sinh(2\pi u)}\;e^{-\frac{\beta}{2}E(u)}|E(u)\rangle \qquad \left(E(u)=\frac{u^2}{\phi_b}\right)$$

↓ truncation

$$\left|\Psi_\beta^\Lambda\right\rangle = \frac{1}{2^{q/2}}\sum_{n=0}^{\Lambda}\mathcal{M}\big(E(s_n)\big)\,|n\rangle = \mathcal{M}(H_\Lambda)\frac{1}{2^{q/2}}\sum_{n=0}^{\Lambda}|n\rangle = \mathcal{M}(H_\Lambda)\prod_{j=0}^{q-1}H_j|0\rangle$$

$\mathcal{M}$: Hermitian, non-unitary operator

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$\mathcal{M}$: Hermitian, non-unitary operator

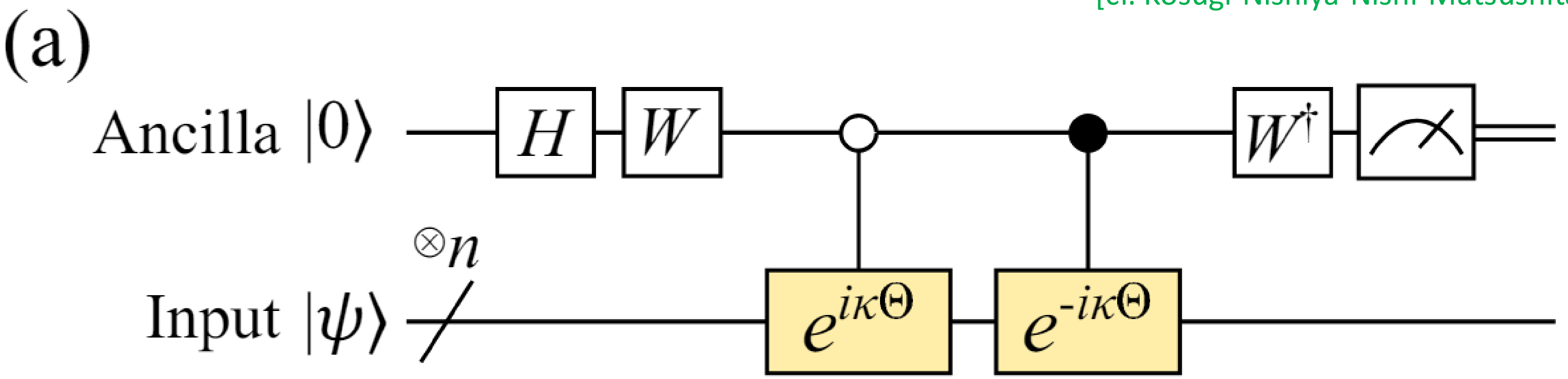
If it is realized, then we can compute interesting observables such as

$$P(t) := \left|\left\langle\Psi_\beta^\Lambda\right| e^{-iH_\Lambda t}\left|\Psi_\beta^\Lambda\right\rangle\right|^2 \longrightarrow$$ wormhole, black hole etc...?

How can we implement $\mathcal{M}$?

Algorithm to act Hermitian non-Unitary op.

[cf. Kosugi-Nishiya-Nishi-Matsushita '21]



$$\mathcal{M}^\dagger = \mathcal{M}, \qquad \Theta \equiv \arccos \frac{\mathcal{M} + \sqrt{1 - \mathcal{M}^2}}{\sqrt{2}}.$$

$$W \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}.$$

$$|\psi\rangle \otimes |0\rangle \longmapsto \mathcal{M}|\psi\rangle \otimes |0\rangle + \sqrt{1 - \mathcal{M}^2}|\psi\rangle \otimes |1\rangle,$$

Contents

Solving physical constraints

[cf. Penington-Witten '23]

Let's include a matter which doesn't couple directly to the dilation Φ

As in the pure case, we have the physical constraint:

$$\left(J_a^L + J_a^R + J_a^{\mathrm{mat}}\right)|\mathrm{phys}\rangle = 0 \qquad J_a^{\mathrm{mat}} \sim \int_{\mathrm{space}} \xi_a^\mu T_{t\mu}$$

⇓

$$\begin{cases} 2\phi_b H_L = \frac{1}{2}\left(p_\ell - \frac{1}{2}J_3^{\mathrm{mat}}\right)^2 - \frac{1}{2}\left(J_1^{\mathrm{mat}} + J_2^{\mathrm{mat}}\right)e^{-\frac{\ell}{2}} + \frac{1}{2}e^{-\ell} \\ 2\phi_b H_R = \frac{1}{2}\left(p_\ell + \frac{1}{2}J_3^{\mathrm{mat}}\right)^2 + \frac{1}{2}\left(J_1^{\mathrm{mat}} - J_2^{\mathrm{mat}}\right)e^{-\frac{\ell}{2}} + \frac{1}{2}e^{-\ell} \end{cases}$$

The total Hamiltonian is written as

$$H = H_{\mathrm{pure}} + H_{\mathrm{others}}$$

An example of regularization

$$\begin{cases} 2\phi_b H_L = \frac{1}{2}\left(p_\ell - \frac{1}{2}J_3^{\mathrm{mat}}\right)^2 - \frac{1}{2}\left(J_1^{\mathrm{mat}} + J_2^{\mathrm{mat}}\right)e^{-\frac{\ell}{2}} + \frac{1}{2}e^{-\ell} \\ 2\phi_b H_R = \frac{1}{2}\left(p_\ell + \frac{1}{2}J_3^{\mathrm{mat}}\right)^2 + \frac{1}{2}\left(J_1^{\mathrm{mat}} - J_2^{\mathrm{mat}}\right)e^{-\frac{\ell}{2}} + \frac{1}{2}e^{-\ell} \end{cases}$$

- JT part → truncation by energy eigenstate of pure JT

  —— H is no longer diagonal in this basis:

$$\mathcal{O}(\hat{\ell}) = \int d\ell\, \mathcal{O}(\ell)|\ell\rangle\langle\ell|$$

$$= \int d\ell dE dE'\, \mathcal{O}(\ell)|E\rangle\langle E|\ell\rangle\langle\ell|E'\rangle\langle E'|$$

$$\to \sum_{n,n'} c_{n,n'}|n\rangle\langle n'|$$

- matter part → put on lattice

  —— additional truncation is needed for bosons

Complexity for time evolution op.

$$\begin{cases} 2\phi_b H_L = \frac{1}{2}\left(p_\ell - \frac{1}{2}J_3^{\mathrm{mat}}\right)^2 - \frac{1}{2}\left(J_1^{\mathrm{mat}} + J_2^{\mathrm{mat}}\right)e^{-\frac{\ell}{2}} + \frac{1}{2}e^{-\ell} \\ 2\phi_b H_R = \frac{1}{2}\left(p_\ell + \frac{1}{2}J_3^{\mathrm{mat}}\right)^2 + \frac{1}{2}\left(J_1^{\mathrm{mat}} - J_2^{\mathrm{mat}}\right)e^{-\frac{\ell}{2}} + \frac{1}{2}e^{-\ell} \end{cases}$$

Suzuki-Trotter approximation:

$$e^{-iHt} = \left(e^{-\frac{it}{M}H}\right)^M = \left[e^{-\frac{it}{M}H_{\mathrm{pure}}} \cdot e^{-\frac{it}{M}H_{\mathrm{others}}}\right]^M + \mathcal{O}\left(\frac{1}{M}\right)$$

When the matter is a Dirac fermion regularized by a staggered fermion of N sites lattice,

$$\#(\text{gates}) = \mathcal{O}(M(N^2 + N\Lambda^2))$$

(good values of N & Λ are a priori nontrivial)

Summary & Outlook

Summary:

put JT gravity (w/ matter) on QC

&

make quantum algorithm to study wormhole physics

Outlook:

- algorithm to construct "Hartle-Hawking state" for the case w/ matter
- states corresponding to wormhole/baby universe
- implementation in simulator/real devise
- Is tensor network efficient for JT gravity?

Thanks!