Thermalization of gauge theories

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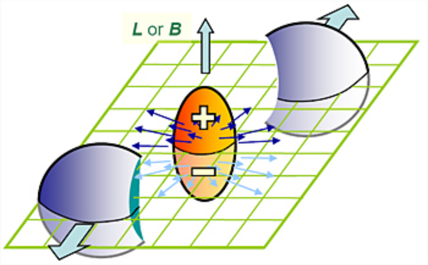
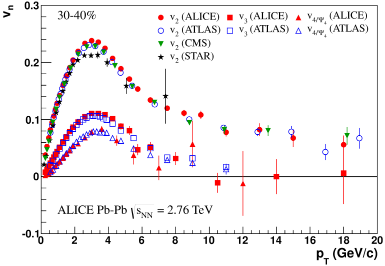
- Introduction
  - experiments (just two examples):
    1.) High energy heavy ion collisions at LHC
    2.) Ultracold atoms
  - AdS/CFT duality
- A concrete question: Does 1+2 dim SU(N) gauge theory show ETH behavior? arXiv:2308.16202, 2312.13408, 2401.15184

- Over decades very powerful techniques to describe QCD and non-abelian gauge theories in general have been developed: peturbative QCD, lattice QCD, AdS/CFT duality, tensor networks, MERA ...
- But some important questions concerning non-perturbative, dynamical processes remain, e.g. concerning thermalization and decoherence.
  Is quantum computing competitive to answer these?
- We focus on thermalization/entanglement for high-energy heavy ion collisions, e.g. at LHC.
  Note: QCD has unitary time evolution (time-reversal symmetry). Thus, no entropy is produced and the reaction products are in a highly entangled state.

Key questions of relativistic heavy ion physics: In which sense does the quark gluon plasma thermalize? Is "hydrodynamization" equivalent to thermalization? What are the relevant time scales?





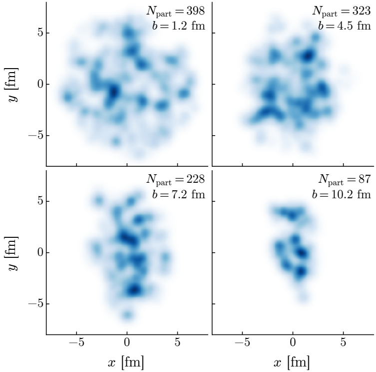
Observable: Elliptic flow $v_n \sim \cos(n\phi)$ with $n = 2$

How can transverse communication happen in less than 1fm/c?

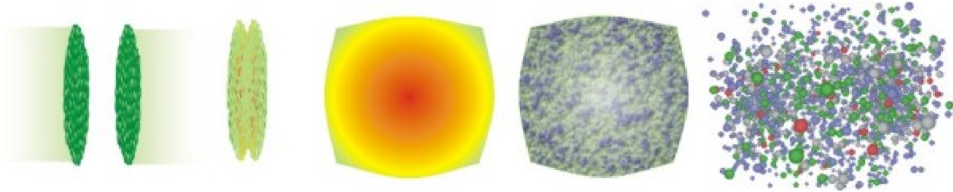
$\gamma(Pb) > 2500$ giving it a width of $11\mathrm{fm}/2500 = 0.004\mathrm{fm}$

In QCD the transverse color coherence length is of order $1/Q_s < 0.2$ fm which is much smaller than the transverse size.

Nuclear fluctuations are large. arXiv:1605.03954



The naive description of a heavy ion collision



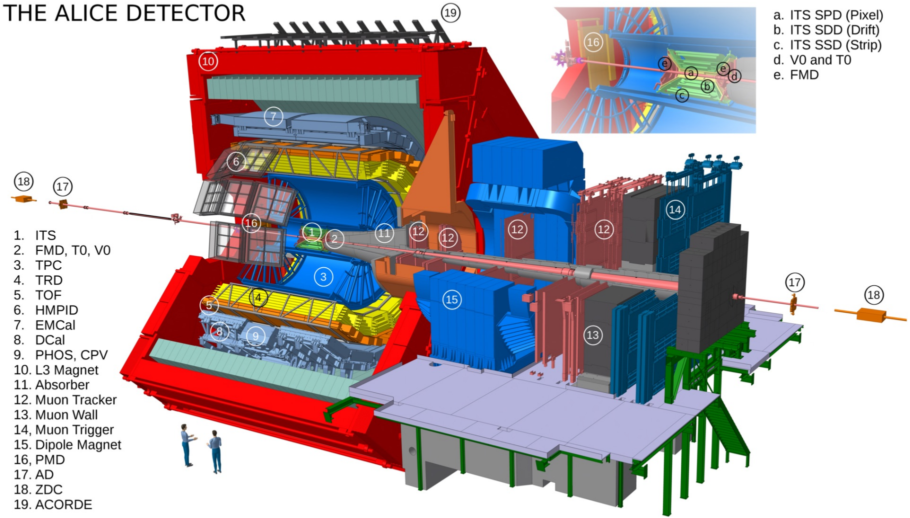
But: Only a fraction of the information can be experimentally observed. Thus the measurement process leads to information loss and thus entropy production.

An advanced description of a heavy ion collision, taking entanglement into account, has to be strongly theory based.

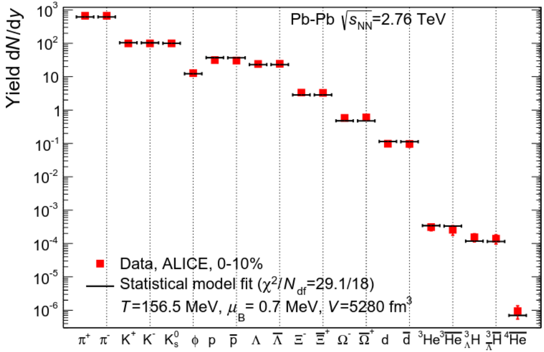
AdS/CFT clarified that hydrodynamization (of local obervables) is very fast, taking less than 1fm/c.

Hydrodynamization is not thermalization.

There is very much high precision data, e.g. from ALICE.



but some do not fit to the "naive" interpretation



R(rms , ${}^3_\Lambda H$)=10.6 fm~ $2R_{\mathrm{Pb}}$; $-B = 0.4\mathrm{MeV} << 156$ MeV the yield should be suppressed

Upshot: The theory of HICs is extremely difficult, having many subtle aspects

Therefore: We focus on ETH and entanglement entropy (so far)

Eigenstate Thermalization Hypothesis (ETH)

D'Alesio, Kafri, Polkovnikov, Rigol 1509.06411

$$O_{mn} = \langle m|\hat{O}|n\rangle = O(\bar{E})\delta_{mn} + e^{-S(\bar{E})/2} f_O(\bar{E},\omega) R_{mn}$$

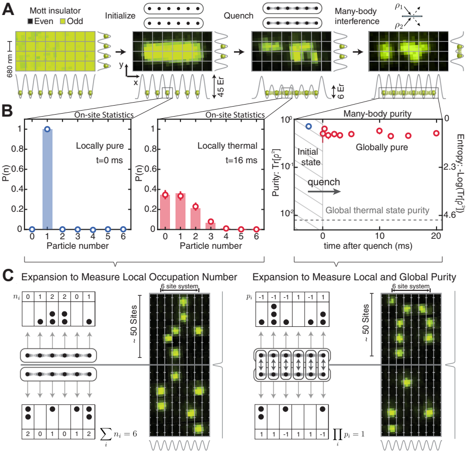
$\bar{E} = (E_m + E_n)/2$, $\omega = E_m - E_m)$, $S(\bar{E})$ thermodynamic entropy at energy $\bar{E}$, $O(\bar{E})$ and $f_O(\bar{E},\omega)$ are smooth functions, $O(\bar{E})$ is identical to the expectation value of the microcanonical ensemble at energy $\bar{E}$, and R_{mn} is a strongly fluctuating matrix (in the sense of RMT?)

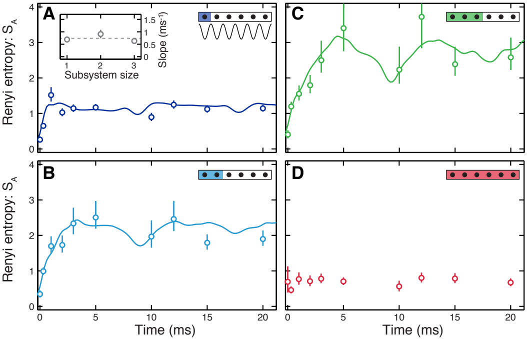
$\Rightarrow$ If only a small subsystem is measured it looks thermal
An explanation for the ${}^{3}_{\Lambda}H$ puzzle?

Questions: For which operators does ETH apply? Does it apply to SU(N)?

The Page curve

The experiment arXiv:1603.04409 “Quantum thermalization through entanglement in an isolated many-body system”



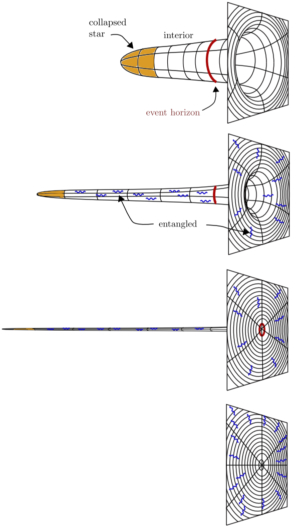


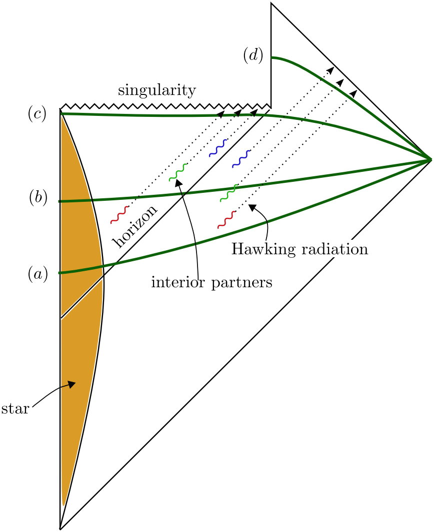
Subsystem entropy $\quad S_A = -\log(Tr[\rho_A^2])$

off-diagonal matrix elements are relevant

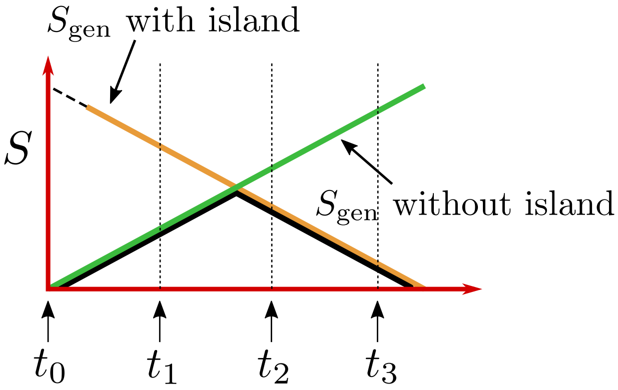
Holography (AdS/CFT) describes the intimate connection between QFT and gravity.

Island mechanism of BH evaporation Almheiri et al. 2006.06872





The Hawking radiation is entangled with an “island”.
This results in the Page curve



AdS/CFT

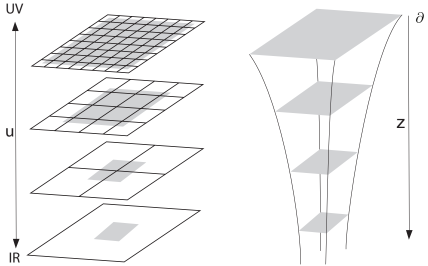
Classical: geometry, $G_{\mu\nu}(x) \quad = \quad 8\pi G T_{\mu\nu}$ energy momentum
AdS/CFT: ART& string theory $\quad = \quad$ QFT

Example: Renormalization flow (Ramallo 1310.4319)

$$V(x,a) \quad \to \quad V(x,2a) \quad \to \quad V(x,4a) \quad \to \quad \ldots$$

$$u = a,\ 2a,\quad 4a; \qquad \frac{\partial}{\partial \log u}\, g(u) \; = \; \beta(u)$$

$$J|_{UV} \qquad = \qquad \Phi|_{\partial}$$



geometric interpretation of new coordinate called z

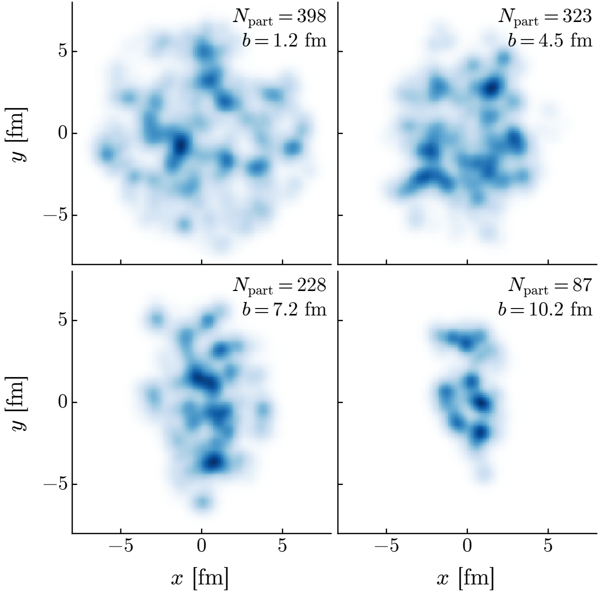
$$ds^2 = \Omega^2(z)\,[dt^2 - dx^i dx^i - dz^2]$$

The properties of the renormalization flow is only simple for conformal theories.

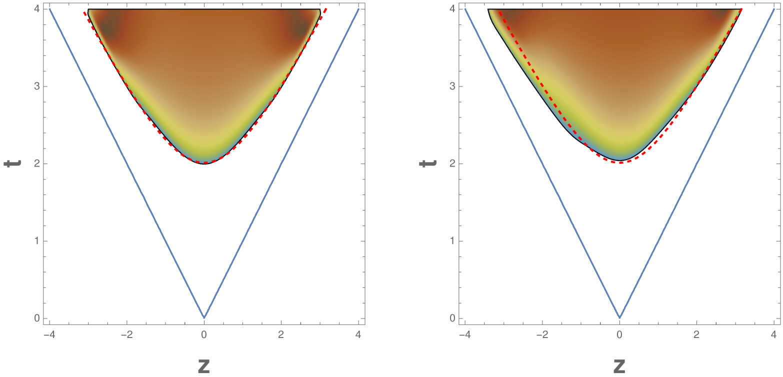
$$\begin{aligned} z &\to \lambda z \\ \Omega(z) &= \frac{L}{z} \to \lambda^{-1}\Omega(z) \\ ds^2 &= \frac{L^2}{z^2}\,[dt^2 - dx^i dx^i - dz^2] \qquad \mathrm{AdS-metric} \end{aligned}$$

$SU(N)$, $\mathcal{N} = 4$ is conformal

The AdS/CFT picture of HICs was very successful, e.g., remember



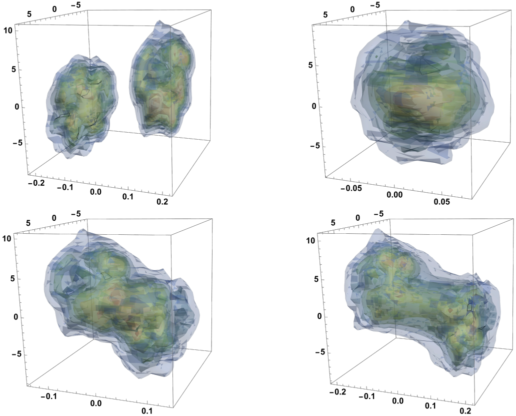
Also this can be described by AdS/CFT 1906.05086 Waeber, Yaffe et al.



answer: Hydrodynamization occurs at **fixed eigenzeit** $\Rightarrow$ basically not boost dependent, geometric mean

criterium: $\Delta = \frac{1}{p}\sqrt{\delta T^{\mu\nu}\ \delta T_{\mu\nu}} < 0.15$ with $\delta T^{\mu\nu} = T^{\mu\nu} - T^{\mu\nu}_{\mathrm{hydro}}$

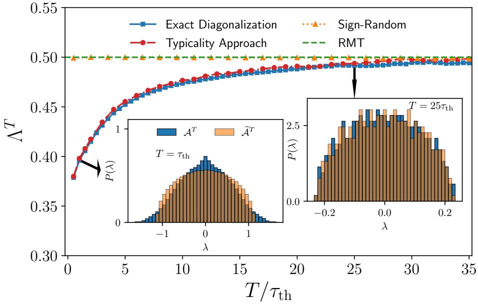
S. Waeber and L. Yaffe have tremendously improved the numerics arXiv:2211.09190



energy density

ETH provides detailed information on thermalization dynamics, see e.g. Wang, Lamann, Richter, Steinigeweg, Dymarsky, 2110.04085

The time needed to establish ETH behavior depends on the observable. Here for an Ising spin chain.



$$\Lambda^T = \frac{\mathcal{M}_2^2}{\mathcal{M}_4}; \qquad \mathcal{M}_k = \mathrm{Tr}[(\mathcal{O}_c^T)^k]/d; \qquad \mathcal{O}_c^T = \mathcal{O}^T - \mathrm{Tr}(\mathcal{O}^T)/d$$

energy window $\left[-\frac{\pi}{T}, \frac{\pi}{T}\right]$

The tentative sequence of stages for HICs

Initial state	→	AdS/CFT free streaming	→	hydrodynamics AdS/CFT, ETH	⇢	complete RMT thermal equilibrium

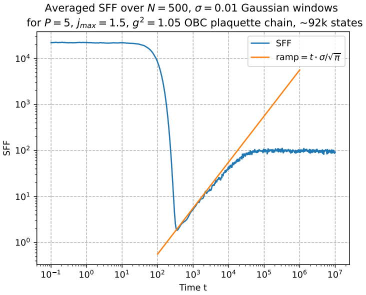
? ↑ ?

hadronization

very long time:
dip-ramp-plateau
spectral formfactor

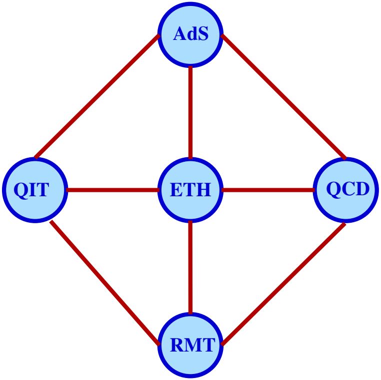
$SFF(T,f) =$

$\langle \sum_{n,m} f(E_m) f(E_n) e^{i(E_m - E_n)T} \rangle$



T_{Thouless} $T_{\text{Heisenberg}}$

Summary: The microscopic description of HICs is kind of messy. Only combining many different approaches can give a complete picture. Quantum Computing and ETH are just two elements.



On the other hand: QCD in the ultra vacuum of the LHC is really what you have to understand, not the Ising model!

Testing ETH for SU(2)

L.Ebner, B. Müller, AS, C. Seidl, X. Yao

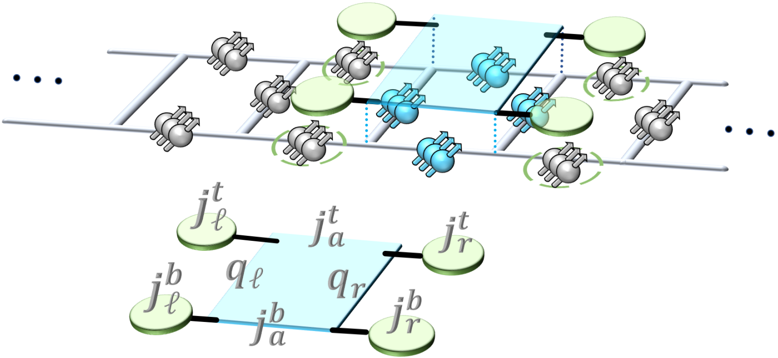
Time dependence from Hamiltonian lattice gauge theory would be the ideal tool but requires quantum computing.

$$\hat{H} = \frac{g^2}{2}\sum_{\text{links}} \hat{E}^2 - \frac{1}{2g^2}\sum_{\square}\left(\hat{\square} + \hat{\square}^\dagger\right)$$

$$\hat{\square} = \sum_{\alpha,\beta,\gamma,\delta=\frac{1}{2}}^{\frac{1}{2}} \hat{U}_{\alpha\beta}\hat{U}_{\beta\gamma}\hat{U}_{\gamma\delta}\hat{U}_{\delta\alpha} \quad .$$

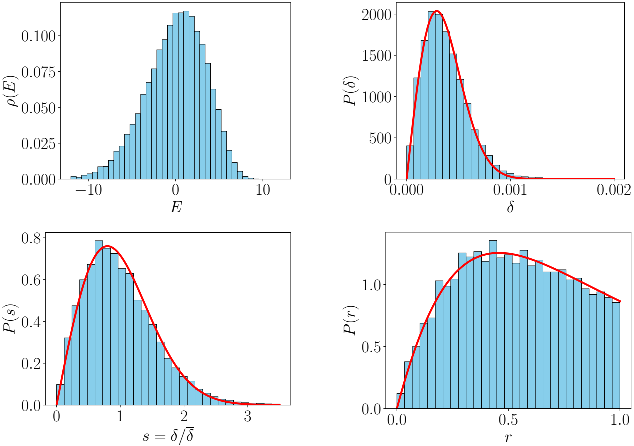
Does SU(2) in e.g. 1+2 dimension show ETH behaviour? It can be simulated on classical computers, expressing it by spin couplings!!!

N. Klco, J. R. Stryker and M. J. Savage, arXiv:1803.03326


j_ℓ^t
j_a^t
j_r^t
j_ℓ^b
q_ℓ
q_r
j_r^b
j_a^b

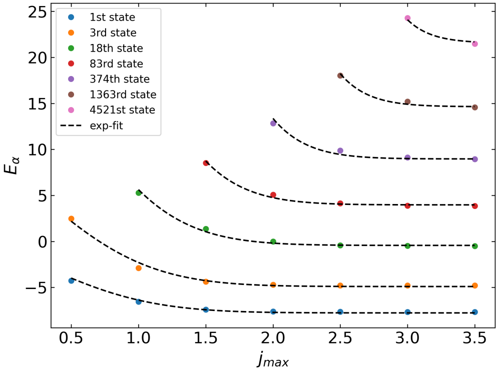
$$
\begin{aligned}
&\langle \chi_{\dots,j_\ell^{t,b},q_{\ell f},j_{af}^{t,b},q_{rf},j_r^{t,b},\dots} | \hat{\square} | \chi_{\dots,j_\ell^{t,b},q_{\ell i},j_{ai}^{t,b},q_{ri},j_r^{t,b},\dots} \rangle = \\
&\sqrt{\dim(j_{ai}^t)\dim(j_{af}^t)\dim(j_{ai}^b)\dim(j_{af}^b)} \\
&\times \sqrt{\dim(q_{\ell i})\dim(q_{\ell f})\dim(q_{ri})\dim(q_{rf})} \\
&\times (-1)^{j_\ell^t + j_\ell^b + j_r^t + j_r^b + 2(j_{af}^t + j_{af}^b - q_{\ell i} - q_{ri})} \\
&\times \begin{Bmatrix} j_\ell^t & j_{ai}^t & q_{\ell i} \\ \frac{1}{2} & q_{\ell f} & j_{af}^t \end{Bmatrix} \begin{Bmatrix} j_\ell^b & j_{ai}^b & q_{\ell i} \\ \frac{1}{2} & q_{\ell f} & j_{af}^b \end{Bmatrix} \begin{Bmatrix} j_r^t & j_{ai}^t & q_{ri} \\ \frac{1}{2} & q_{rf} & j_{af}^t \end{Bmatrix} \begin{Bmatrix} j_r^b & j_{ai}^b & q_{ri} \\ \frac{1}{2} & q_{rf} & j_{af}^b \end{Bmatrix}
\end{aligned}
$$

Test of GOE predictions:

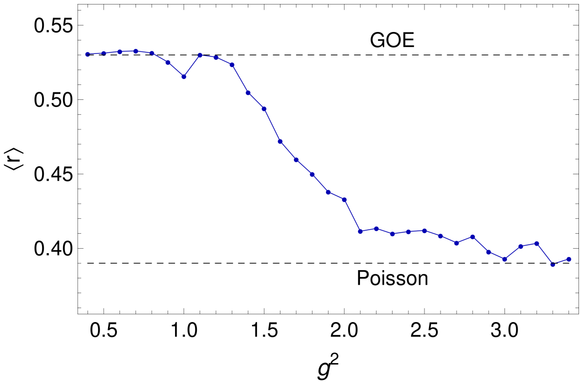


Density of eigenstates, distributions of gaps, rescaled gaps and gap ratios in the momentum $k_x = k_y = 1$ sector on the $N_x = 5, N_y = 4$ lattice for $g^2 = 0.75$.

Test of $j_{\max}$ convergence.

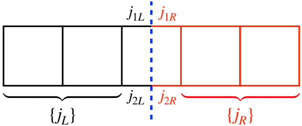


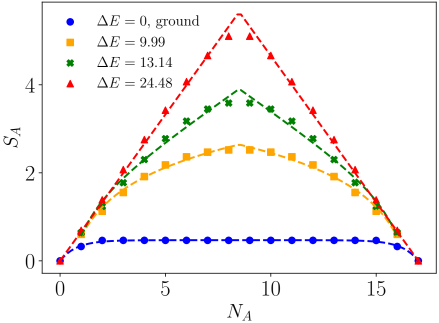
g^2 dependence of the restricted gap ratio $\langle r \rangle$. GOE predicts 0.53, Poisson predicts 0.39.



$$r = \frac{\min[\delta_\alpha, \delta_{\alpha-1}]}{\max[\delta_\alpha, \delta_{\alpha-1}]} \qquad \delta_\alpha = E_{\alpha+1} - E_\alpha$$

The Page curve for a chain of 17 plaquettes 2401.15184

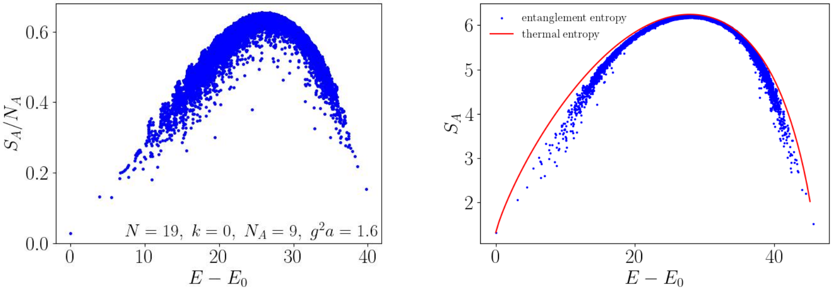




$$S_A = \mathrm{Tr}\Big(\rho_A \ln(\rho_A)\Big) \quad ; \quad \rho_A = \mathrm{Tr}\Big(|\psi_A\rangle \bigotimes |\Psi_{\overline{A}}\rangle\langle\psi_A| \bigotimes \langle\Psi_{\overline{A}}|\Big)$$

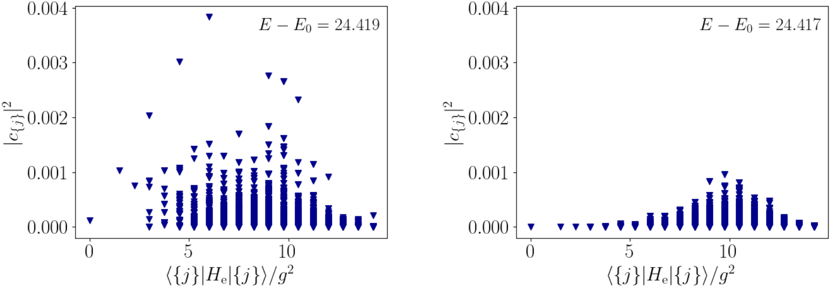
Does SU(2) have Quantum Many Body Scars (QMBS) ?

Spin chains suggest 'yes', $j_{\rm max} = 3/2$ SU(2) results suggest 'no'.

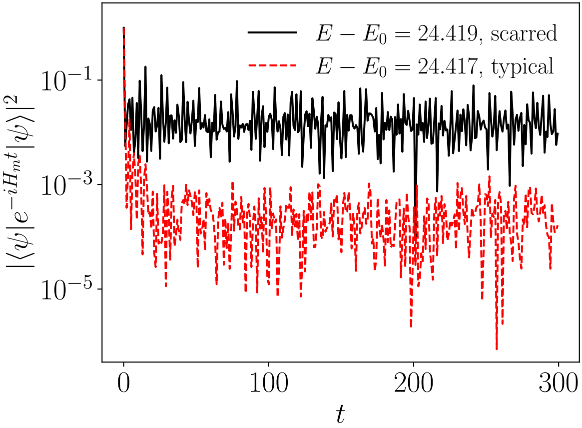


Left: Entanglement entropy for $j_{\rm max} = 1/2$ and $N = 19$, Right: The same for $j_{\rm max} = 3/2$ and $N = 6$

Further evidence



Wavefunction components of the scarred eigenstate and a typical eigenstate (Left: electric basis; $k = 0$ sector; periodic $N = 19$ plaquette chain; $j_{\max} = \frac{1}{2}$ and $g^2 a = 1.6$). Here $|\psi\rangle = c_{\{j\}}|\{j\}\rangle$ where $\{j\}$ is a collection of j values on all the links and implicitly summed over.



Recurrence probabilities of the scarred and typical eigenstate.
In the latter case it converges to 1/(size of Hilbert space).

Conclusions

- ETH, decoherence and thermalization of isolated quantum systems are topics of universal interest.
- Heavy Ion Collisions offer an ideal situation to study them.
- There exist many technically different approaches (classical nonlinear dynamics, RMT and ETH, Lattice QCD, AdS/CFT, QCD phenomenology, pQCD, hydrodynamics, quantum computing ...) which are expected to provide complementary, consistent information.
- We have started simulations on classical computers.
- So far everything is compatible with SU(2) fulfilling ETH.
- Presently we test together with Indrakshi Raychowdhury, Saurab Kadam, and Diptarka Das whether LSH shows ETH properties.