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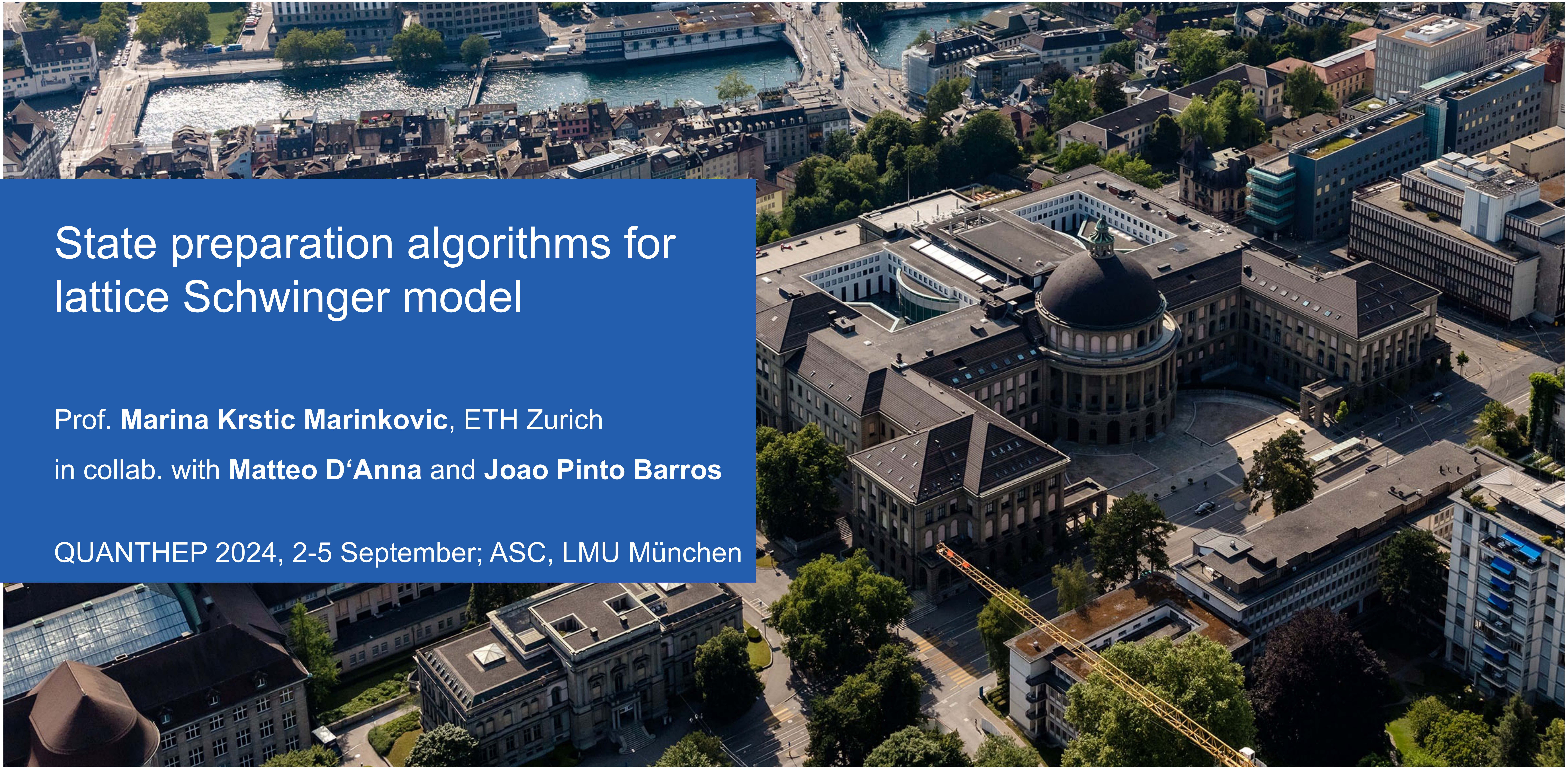
Institute for Theoretical Physics



State preparation algorithms for lattice Schwinger model

Prof. **Marina Krstic Marinkovic**, ETH Zurich

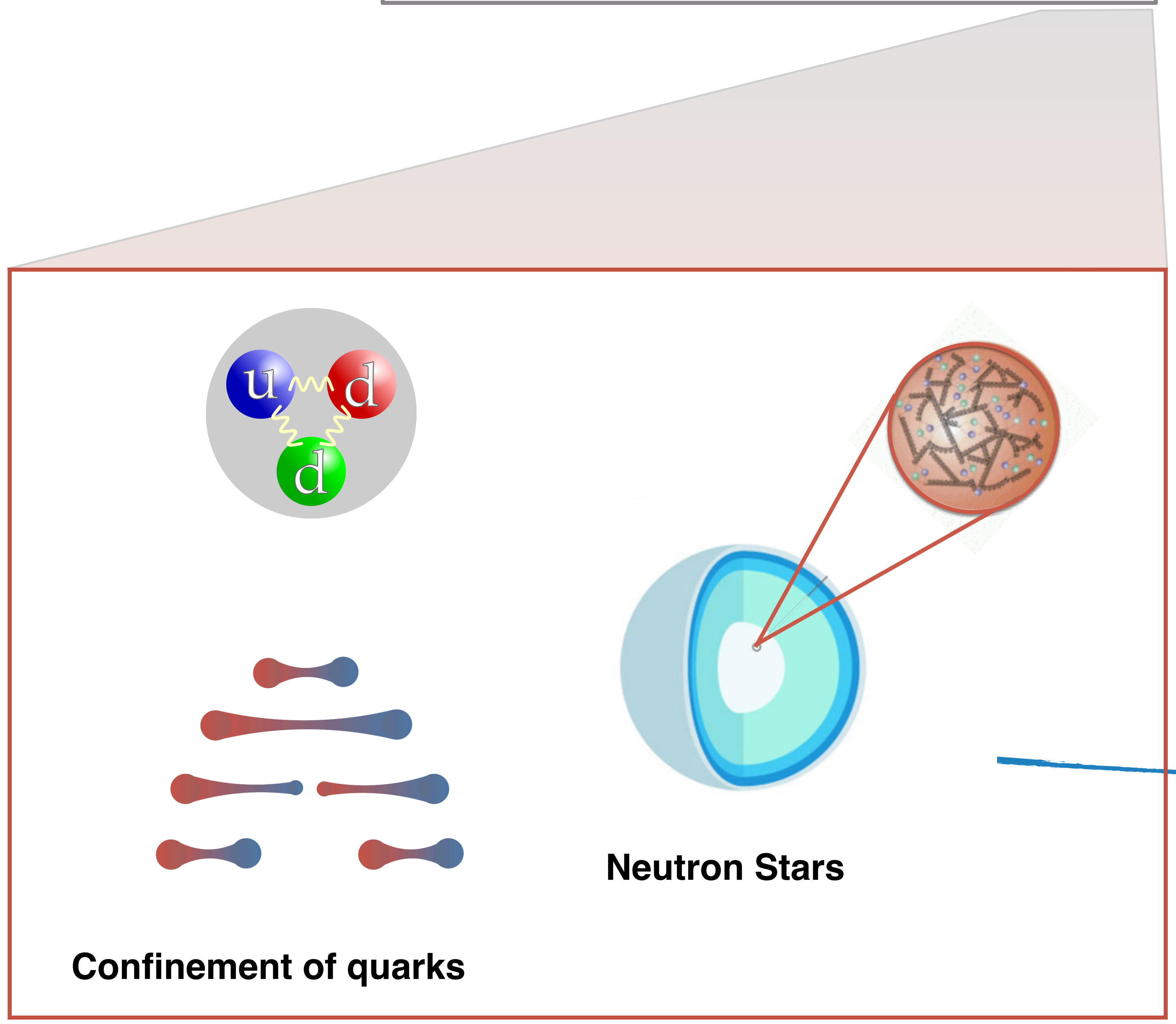
in collab. with **Matteo D‘Anna** and **Joao Pinto Barros**

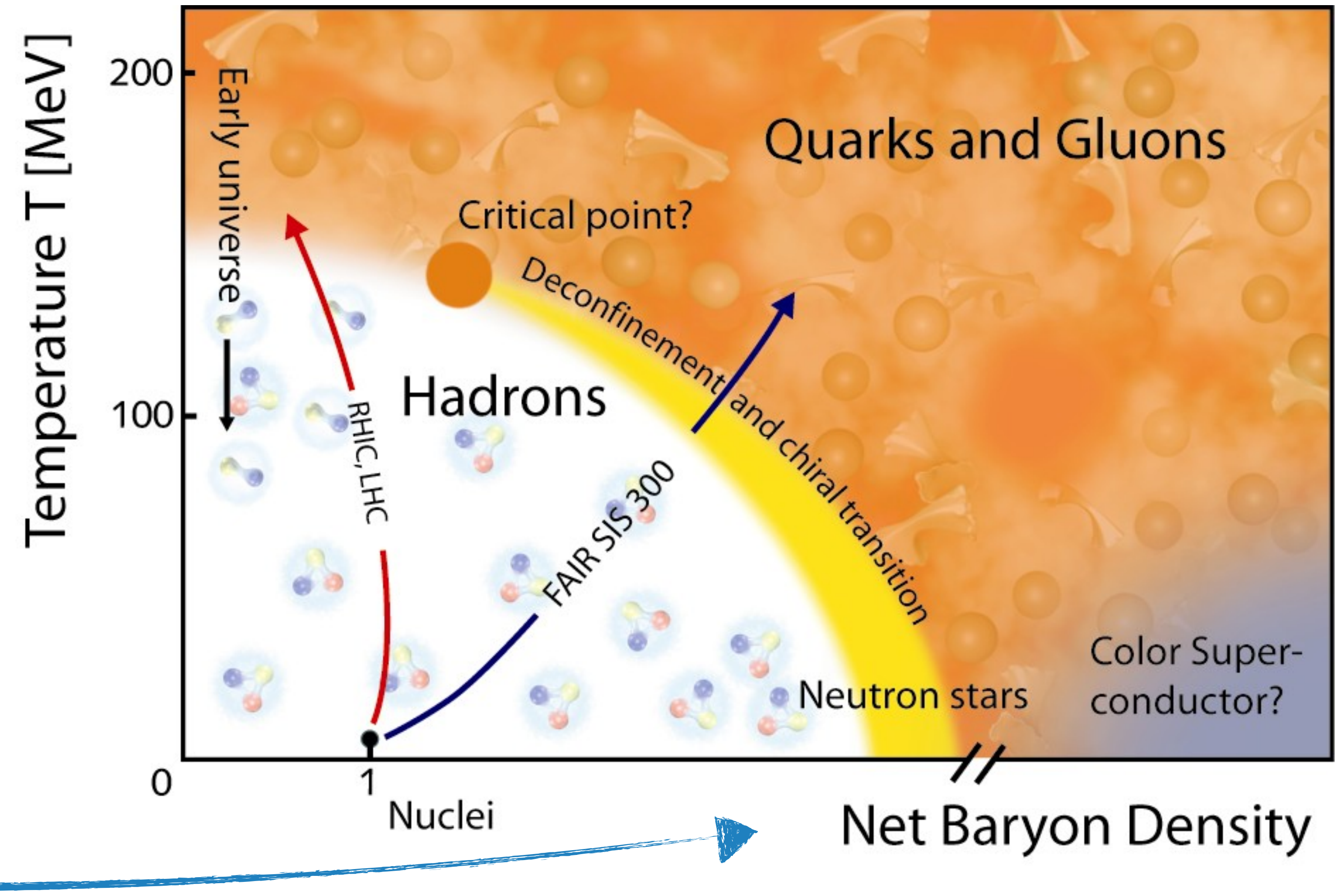
QUANTHEP 2024, 2-5 September; ASC, LMU München

Motivation: Strong Interaction in Standard Model of Particle Physics

… understanding the phase diagram and real time dynamics

Quantum Chromodynamics

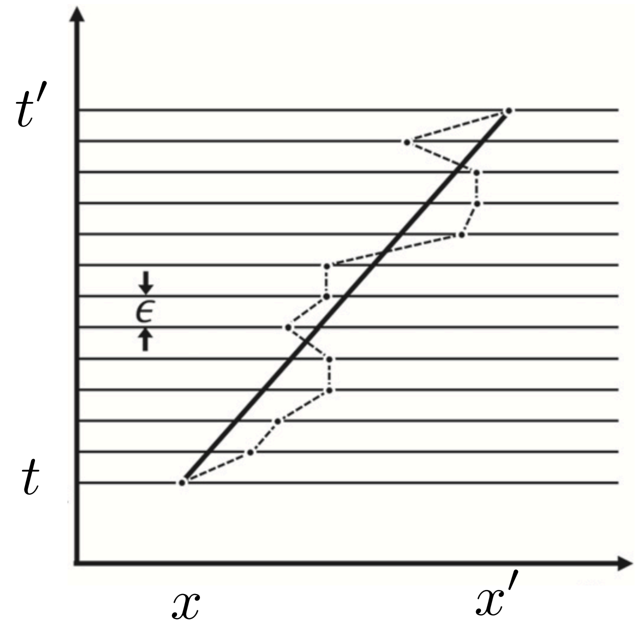




[Credit: Peter Senger (p.senger@gsi.de)]

Lattice Gauge Theories beyond Euclidean Monte Carlo Approach

Path Integrals



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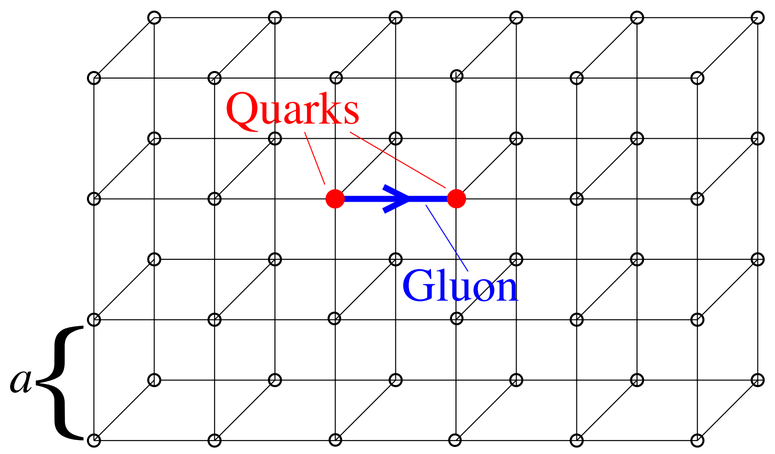
Euclidean space-time

$\epsilon \longrightarrow -i\,a$

+

Quantum Chromodynamics (QCD)



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Lattice QCD

Monte Carlo (MC) Simulations

FLAG Review 2021

[Aoki et al. Eur.Phys.J.C 82 (2022) 10, 869]

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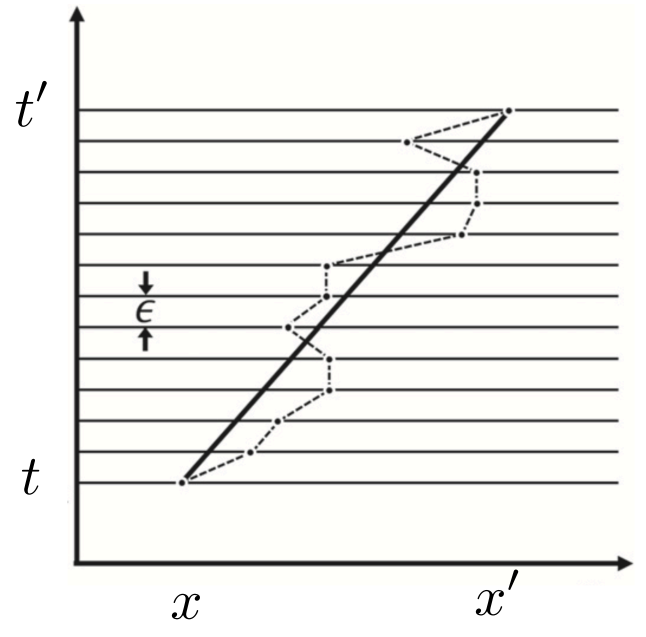

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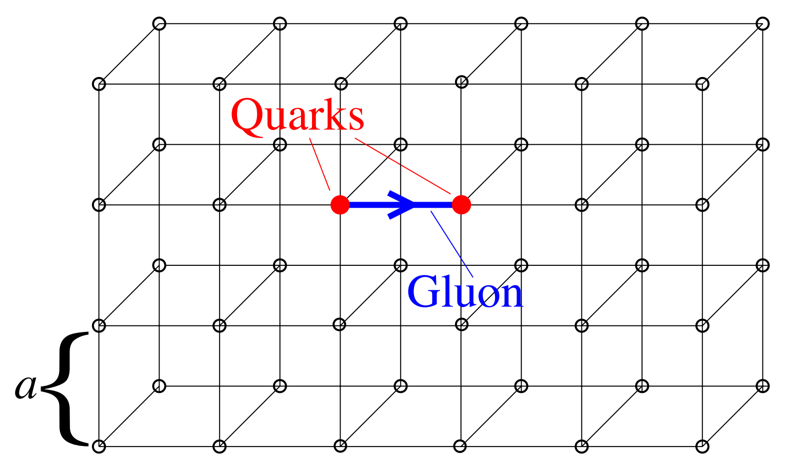

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Lattice QCD

Monte Carlo (MC) Simulations

Inaccessible with conventional MC approach:

- **Real time dynamics**
- **Topological θ-term**
- **Finite chemical potential**

$$\langle O[A,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int D[A]\, \det D[A] e^{-S_G[A]}$$

Complex Weight W

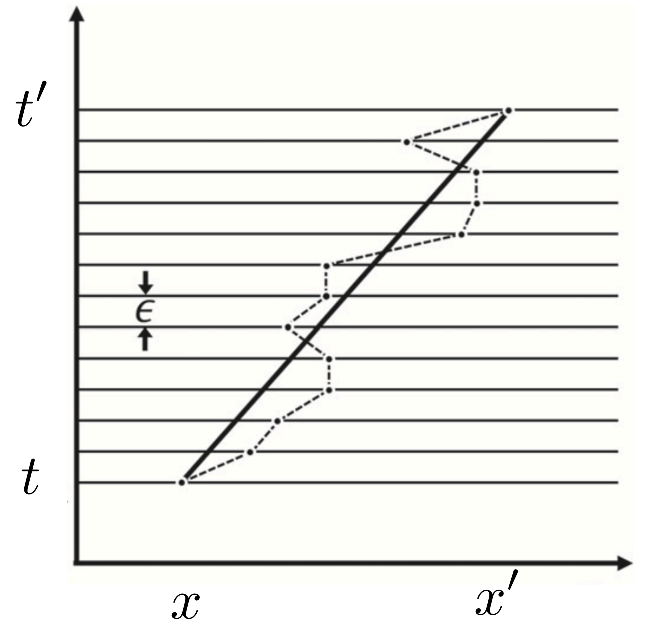
Sign Problem

$$\langle O\rangle = \frac{\sum_C O_C \cdot W_C}{\sum_C W_C} = \frac{\sum_C O_C \cdot \mathrm{sign}(W_C)\cdot |W_C|}{\sum_C \mathrm{sign}(W_C)\cdot |W_C|}$$

$$= \frac{\langle \mathrm{sign}\cdot O\rangle_{|W|}}{\langle \mathrm{sign}\rangle_{|W|}}$$

Lattice Gauge Theories beyond Euclidean Monte Carlo Approach

Path Integrals



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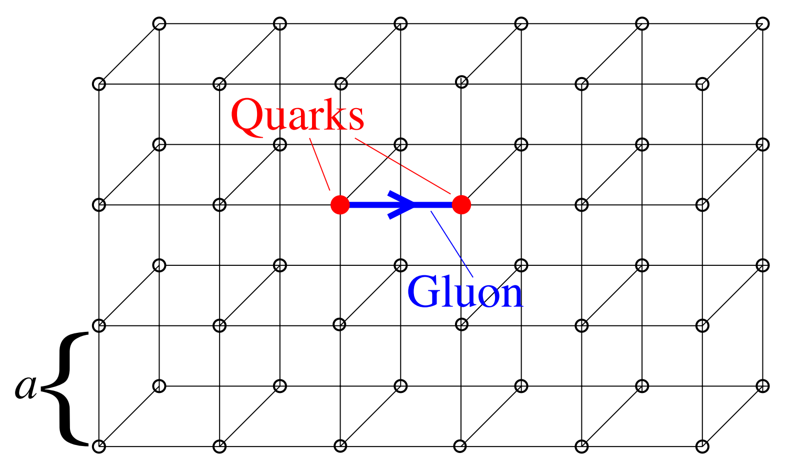

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Quantum Simulations

Toy model for QCD in 3+1D: QED in 1+1D

Schwinger model [Schwinger, Physical Review 125, 397 (1962)]

Chiral symmetry, confinement, string breaking …

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Quantum Electrodynamics in 1+1D

- **Schwinger Model** (QED2) **Lagrangian with topological θ angle**

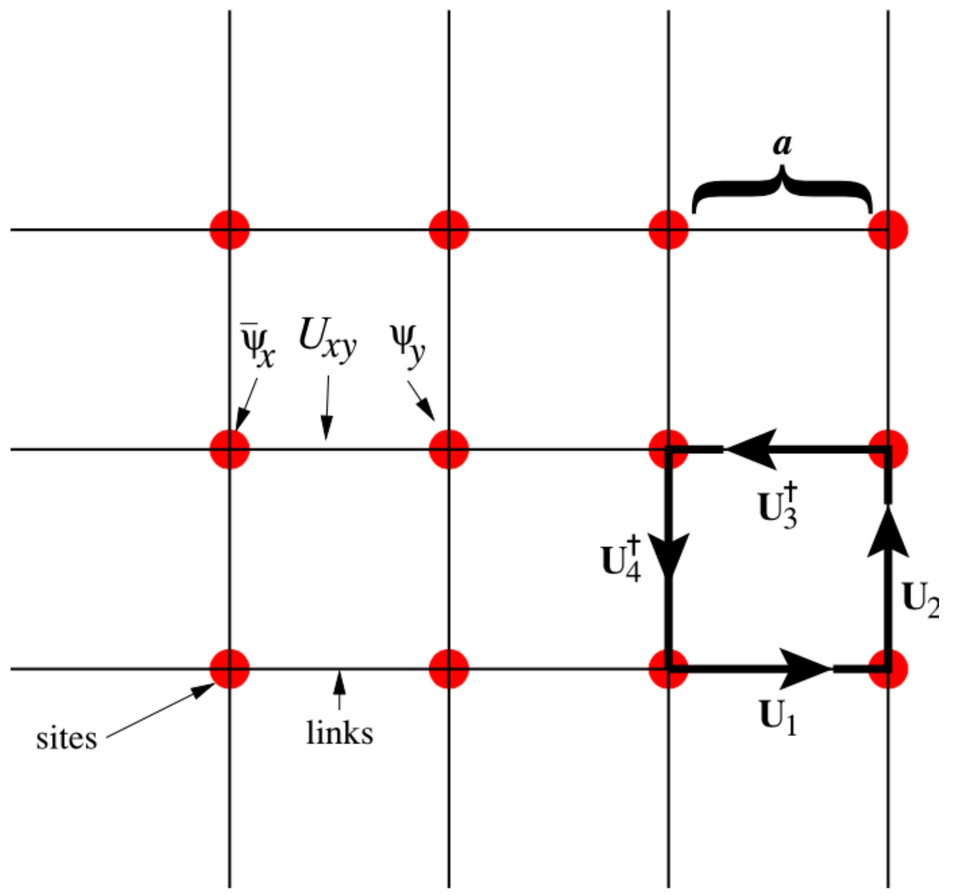
[Schwinger, Physical Review 125, 397 (1962)]

$$\mathscr{L}_{QED2} = -\frac{1}{4}\,\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \frac{g\theta}{4\pi}\varepsilon_{\mu\nu}\mathrm{F}^{\mu\nu} + \mathrm{i}\bar{\psi}\gamma^{\mu}\left(\partial_{\mu} + \mathrm{i}gA_{\mu}\right)\psi - m\bar{\psi}\psi$$

- **Lattice Schwinger Model Hamiltonian:**

$$H_{QED2} = -\mathrm{i}\sum_{n=1}^{N-1}\left(w - (-1)^n\frac{m}{2}\sin\theta\right)\left(\chi_n^{\dagger}\chi_{n+1} - \chi_{n+1}^{\dagger}\chi_n\right) + m\cos\theta\sum_{n=1}^{N}(-1)^n\chi_n^{\dagger}\chi_n + J\sum_{n=0}^{N-1}L_n^2,$$

$$L_n = \sum_{k=1}^{n}\left(\chi_k^{\dagger}\chi_k - \frac{1-(-1)^k}{2}\right)$$ **Gauss' law**



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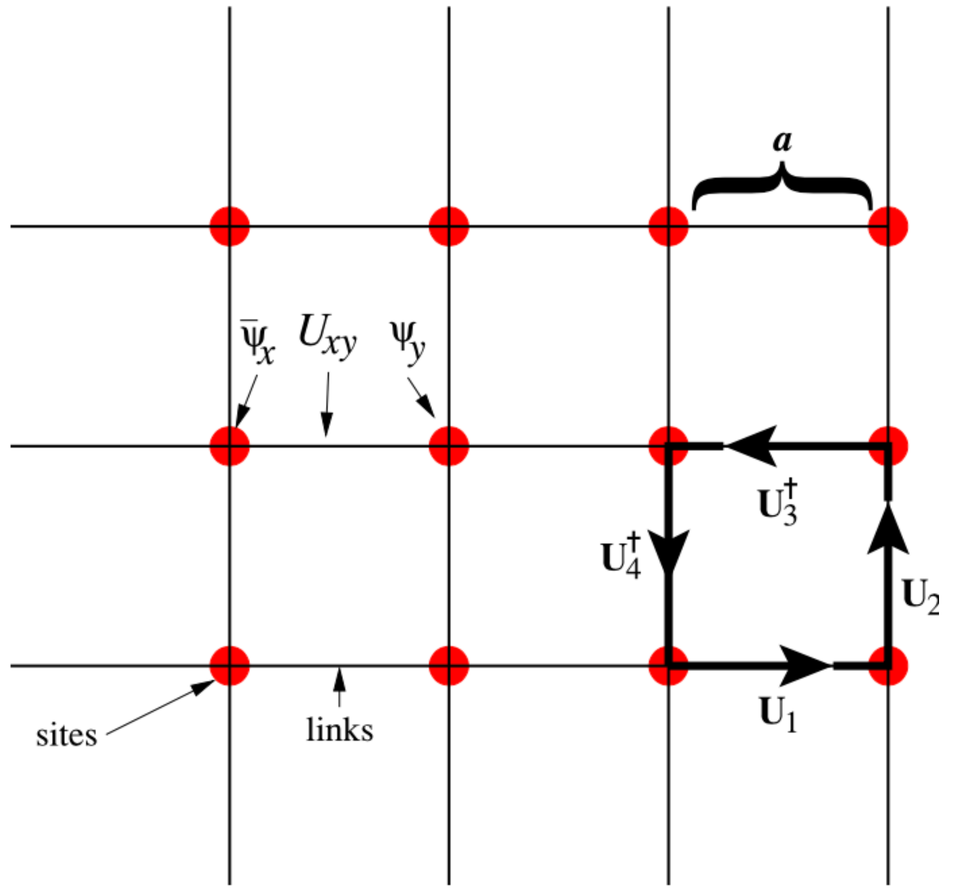
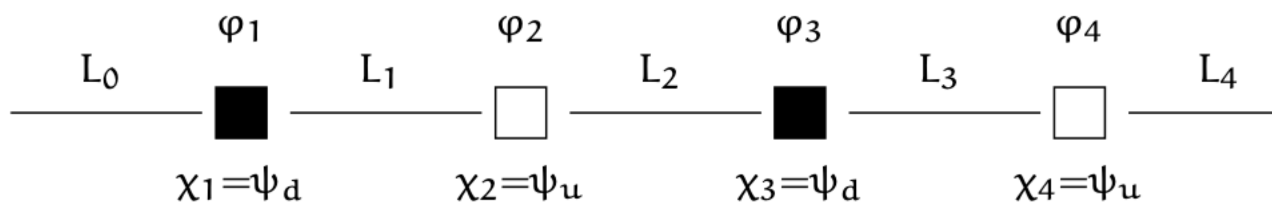


- Technicalities — starting from $\mathscr{L}_{QED2}$ to get H_{QED2}:
    - ➡ Chiral $U(1)_A$ rotation: $\psi \rightarrow e^{\mathrm{i}\frac{\theta\,\gamma_5}{2}}\psi$; timelike axial gauge: $A_0 \equiv 0$
    - ➡ Staggered fermion discretization

[Kogut, Susskind, Phys. Rev. D 11, 395 (1975)]



Staggered Fermions

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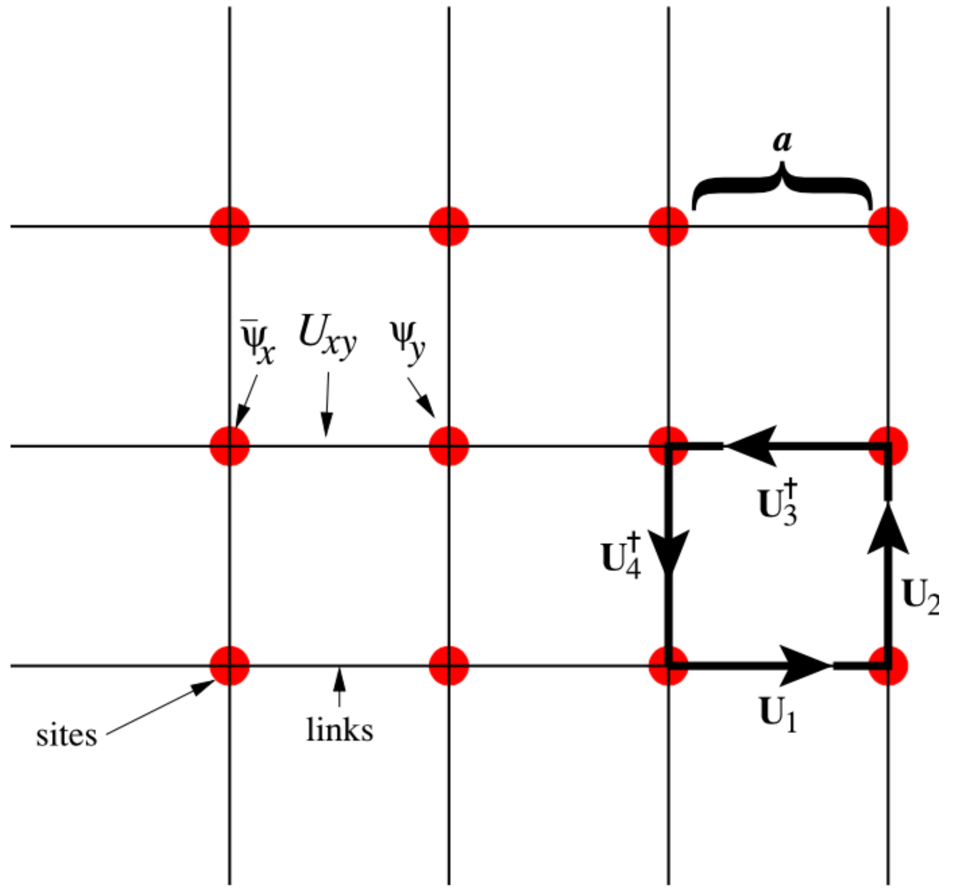

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- **Digital Quantum Simulations of QED2: non-exhaustive list**

[Martinez et al. Nature 534, 516–519 (2016)]
[Klco et al. Phys. Rev. A 98, 032331 (2018)]
[Kokail et al. Nature 569, 355–360 (2019)]
[Jong et al. Phys. Rev. D 106, 054508 (2022)]
[Nguyen et al. Quantum 3, 020324 (2022)]
[Chakraborty et al. Phys. Rev. D 105, (2022) 94503]
[Farell et al., PRX Quantum 5, 020315(2024)]
[Ghim et al., 2404.14788]
[Guo et al, 2407.15629] . . .

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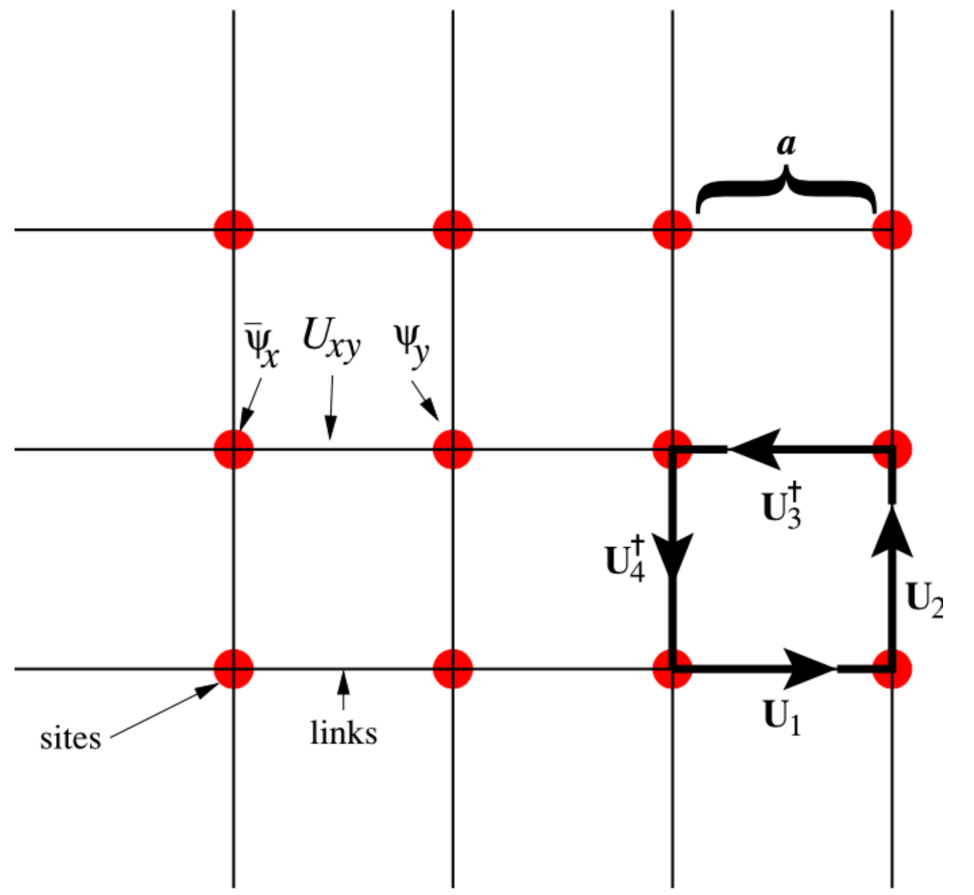

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- **Tensor Network approaches: alternative to/augmenting quantum simulations**

[Banlus et al. Physical Review D 93, (2016) 094512]
[Buyens et al. Physical Review D 94, (2016) 085018]
[Buyens et. al. Physical Review X 6, 041040 (2016)]
[Butt et al. Physical Review D 101, (2020) 094509]
[Funcke et al. Phys. Rev. D 101, (2020) 054507] . . .

Quantum-Simulation-Ready $QED2$ Hamiltonian

- **Jordan-Wigner transformation (space discrete, time continuous):**

[Chakraborty et al. Physical Review D 105, (2022) 94503]

$$H_{QED2} = H_{ZZ} + H_{\pm} + H_Z$$

$$H_{ZZ} = \frac{J}{2} \sum_{n=2}^{N-1} \sum_{1 \le k < \ell \le n} Z_k Z_\ell$$

$$H_{\pm} = \frac{1}{2} \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left(X_n X_{n+1} + Y_n Y_{n+1} \right)$$

$$H_Z = \frac{m\cos\theta}{2} \sum_{n=1}^{N} (-1)^n Z_n + J \sum_{n=1}^{N-1} v(n; n_+, n_-) \sum_{\ell=1}^{n} Z_\ell$$

- ➡ $w = \frac{1}{2a}, \quad J = \frac{g^2 a}{2}$
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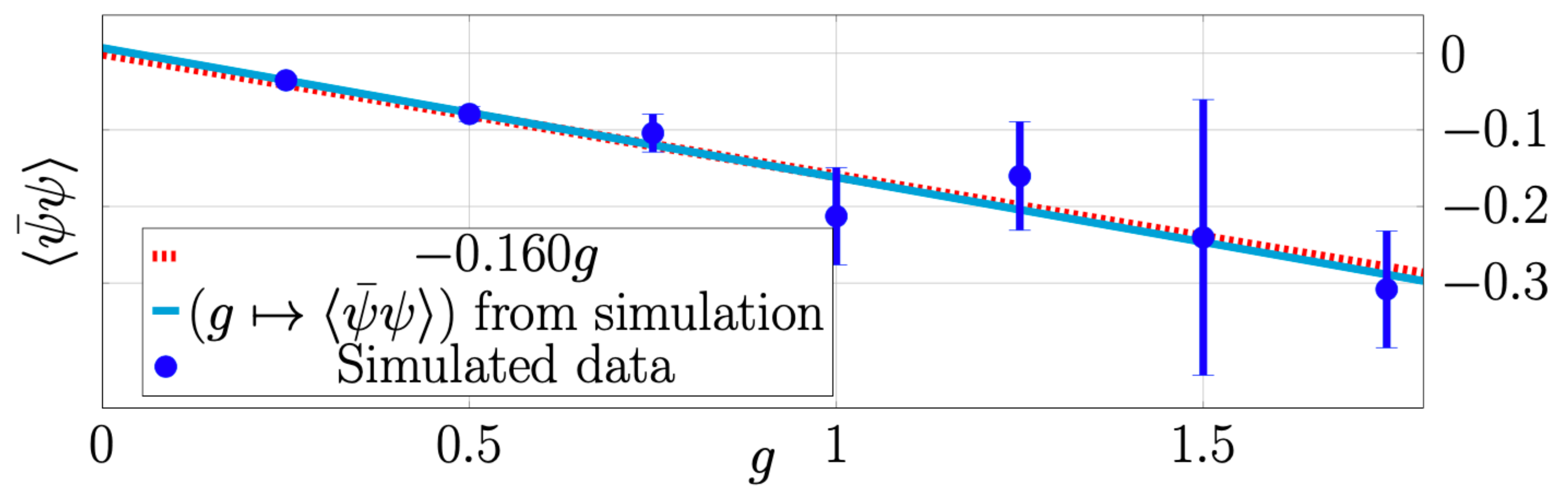
Fixed-Q Adiabatic State Preparation



- **Suzuki-Trotter decomposition**

[Trotter, Proc. Am. Math. Soc. 10 (1959),
Suzuki Commun. Math. Phys. 51 (1976)]

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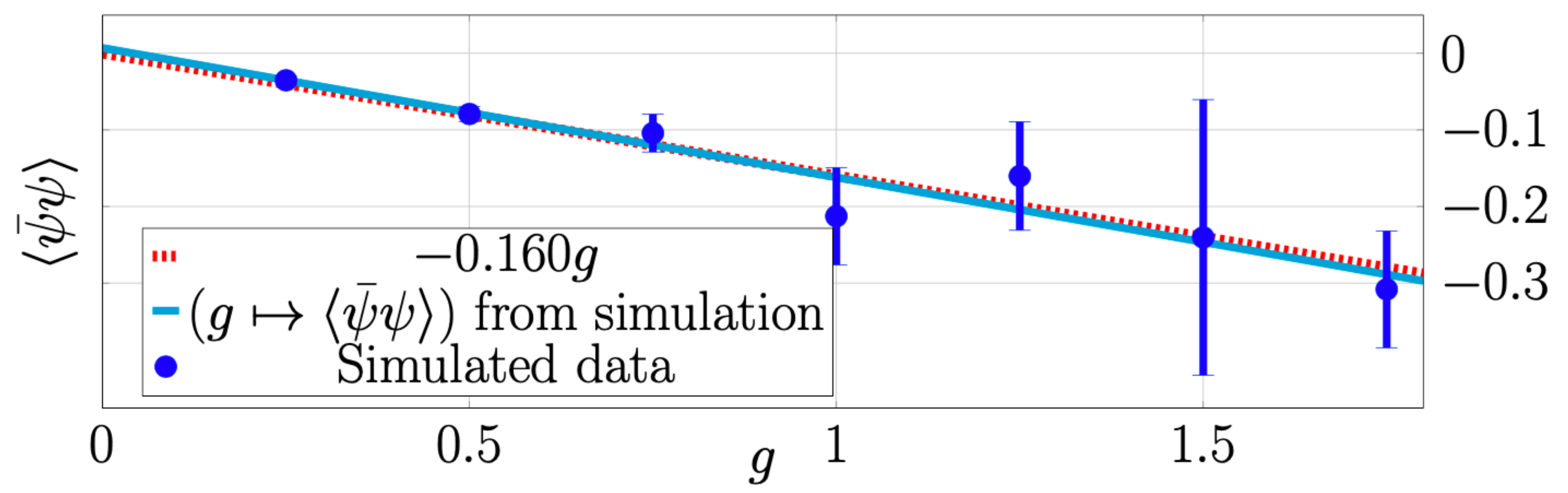

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- **Quantum simulations require efficient quantum state preparation algorithms**

[Jordan, Lee, Preskill, Science 336 (2012) 1130-1133]
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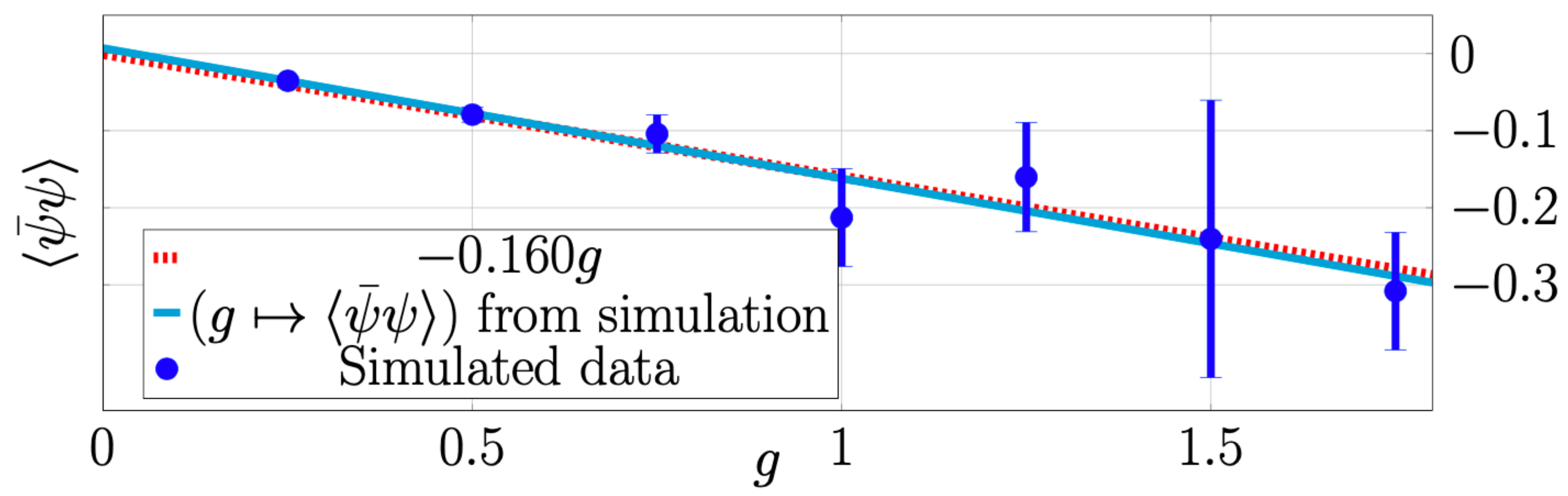

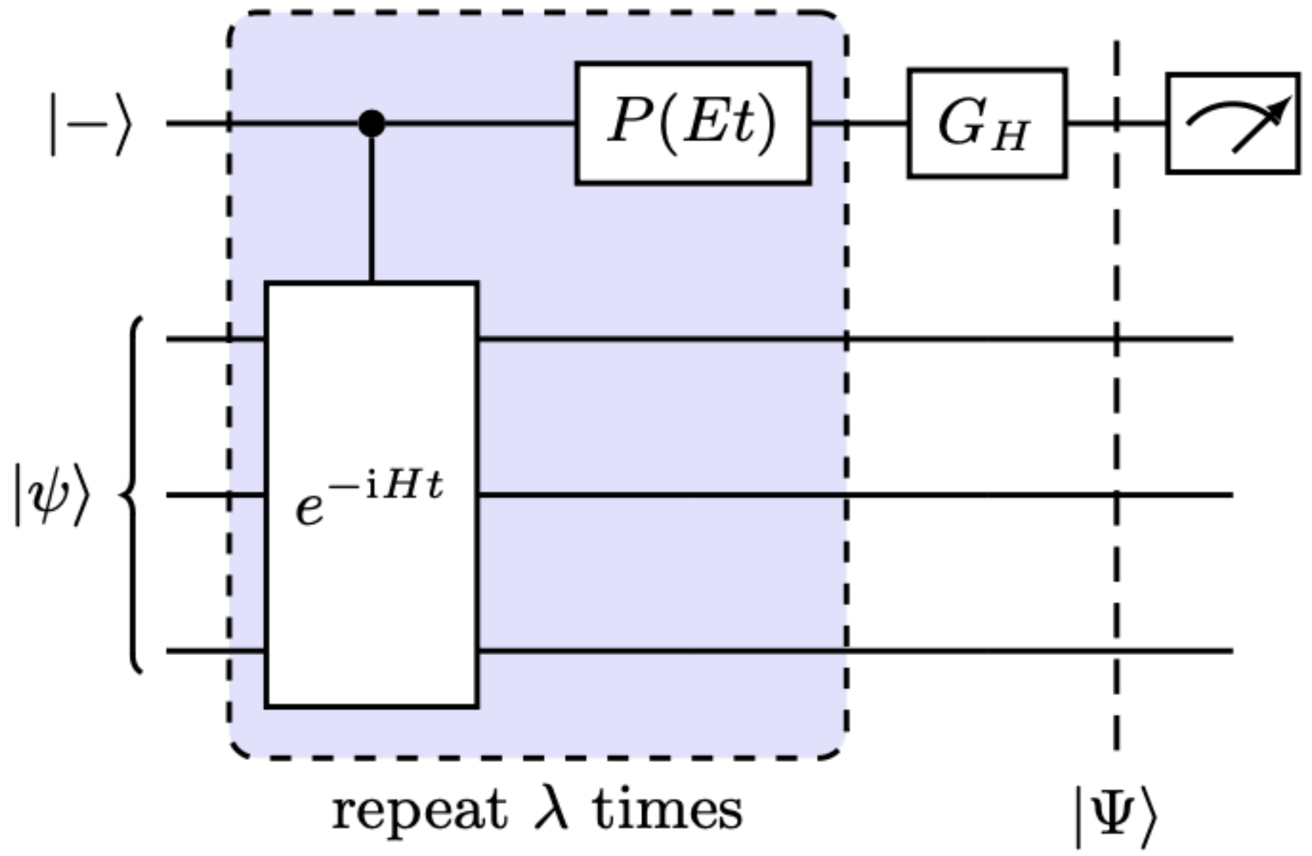

Alternatives to ASP: Rodeo Algorithm

[Choi et al. PRL 127(4 (2021) 040505]

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Adiabatic State Preparation (I)

- **Ground state preparation for H_{QED2}**

1. **Initial state $|\psi_0\rangle$: ground state of $H_A(0) \equiv H_0$**

2. **"Evolve" $H_A(t)$: $H_A(T) \equiv H_{QED2}$**

3. **Ground state at time T approximated by**

$$|\psi_T\rangle = \mathcal{T}\{e^{-\mathrm{i}\int_0^T \mathrm{d}t\, H_A(t)}\}|\psi_0\rangle \approx \underbrace{U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)}_{M \text{ steps}}|\psi_0\rangle$$

➡ $U(\tau) = e^{-\mathrm{i}H_A(\tau)\delta t}$, $\delta t = \dfrac{T}{M}$, T finite, $\int$ discretized

➡ Applied to $QED2$ [Chakraborty et al. Physical Review D 105, (2022) 094503]

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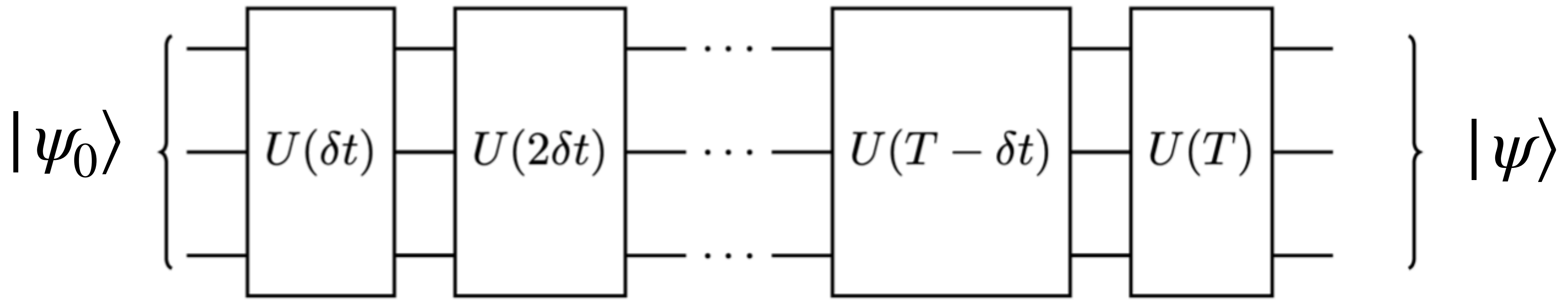

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Adiabatic Theorem:

$H_A(t)$ gapped, unique ground state $\Longrightarrow$

$$|\psi_T\rangle = \lim_{T\to\infty} \mathcal{T}\{e^{-\mathrm{i}\int_0^T \mathrm{d}t\, H_A(t)}\}\,|\psi_0\rangle$$

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Symmetries of the *QED2* Hamiltonian

- **Charge conservation:**

$$Q = \frac{g}{2}\sum_n Z_n \qquad \left[Q, H_{QED2}\right] = 0$$

➡ Q: charge operator on the lattice
➡ states belong to different charge sectors

- **Schwinger Model is toy model of QCD, exhibits confinement and string breaking**

- **Full (θ, m) phase diagram inaccessible to conventional MC simulations**

[Coleman, Annals of Physics 101 (1976) 239]

[Coleman, Jackiw, Susskind, Annals of Physics 93 (1975) 267]

[Thompson, Siopsis, Quantum Science&Technology (2021)7]

- Schwinger model **is known to have a phase transition for $\theta = \pi$ and $m/g > m_c/g \approx 0.33$**

- $QED2$ **has a ground state in nonzero charge sector for $\theta \in [0.8\pi, 1.5\pi]$ and certain values of m**

- **String breaking not accessible with state preparation algorithms that keep Q fixed**

Multi-Q Adiabatic State Preparation is desirable

Adiabatic State Preparation (II)

- **Fixed-Q ASP Algorithm (A1): single charge sector of** H_{QED2}

$$H_{A1}(t) = H_{QED2}\Big|_{m=m(t),w=w(t),\theta=\theta(t)}$$

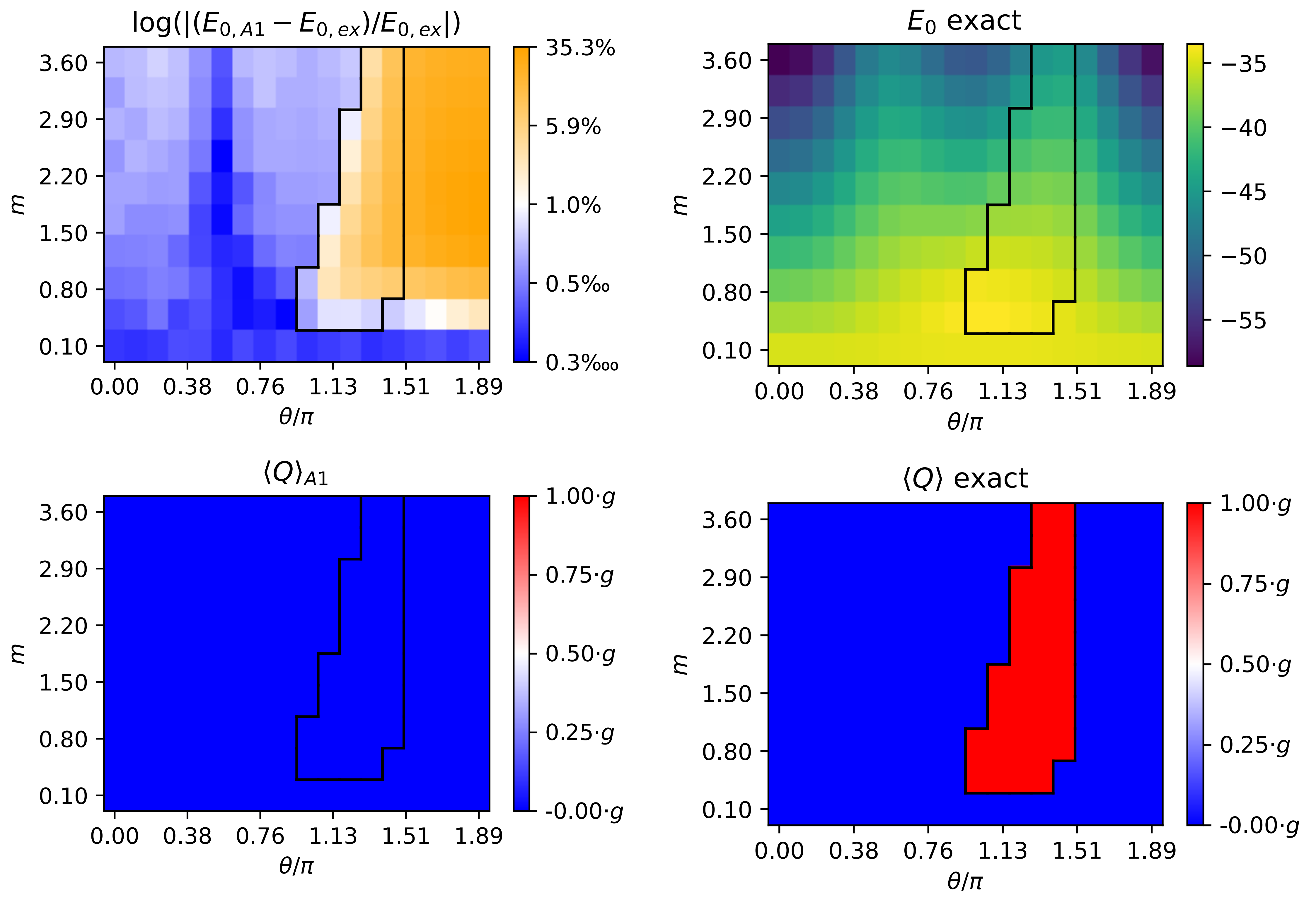
$$m(t) = m_0\left(1-\frac{t}{T}\right) + m\frac{t}{T}; \qquad w(t) = w\frac{t}{T}; \qquad \theta(t) = \theta\frac{t}{T}$$

- **Charge remains constant:**

  ➡ $[Q, H_{A1}(t)] = 0\,,\ \forall t$

- **Challenging to determine the ground state for all** (θ, m)

Fixed-Q ASP Algorithm: **Exact Diagonalization:**



Adiabatic State Preparation (III)

- **Multi-Q algorithm: arbitrary charge sector of H_{QED2}**

$$H_{A2}(t) = \left(1 - \frac{t}{T}\right)\beta \sum_{n=1}^{N} f(n) X_n + \frac{t}{T} H_{QED2} \quad \textbf{for} \quad f(n) \in \{-1,1\}$$

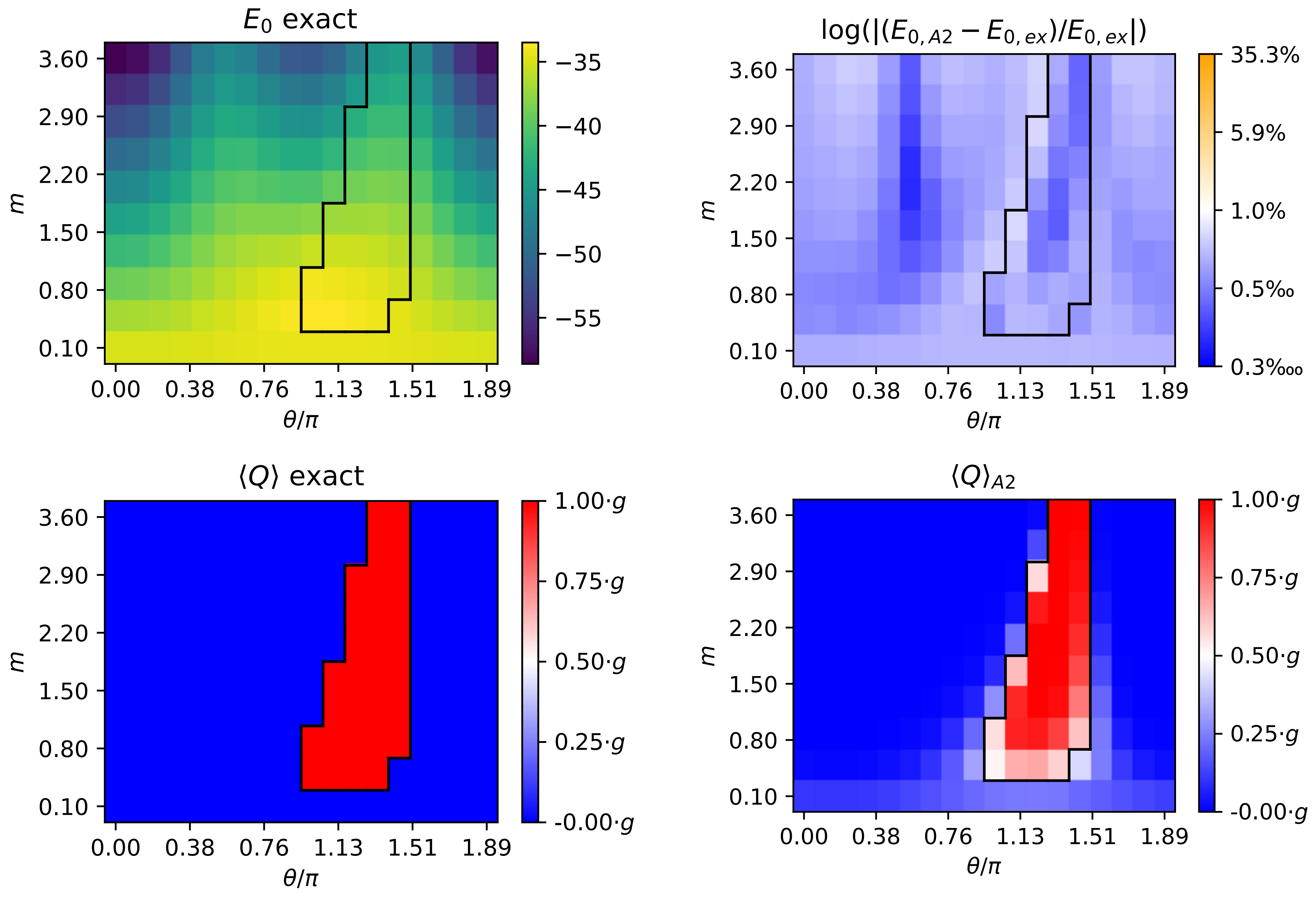
- **Several choices for $f(n)$ probed, best thus far: $f(n) = (-1)^n$**

  ➡ $\beta \approx \dfrac{|E_0|}{N}$

  ➡ **Total charge no longer a symmetry of $H_{A2}(t)$**

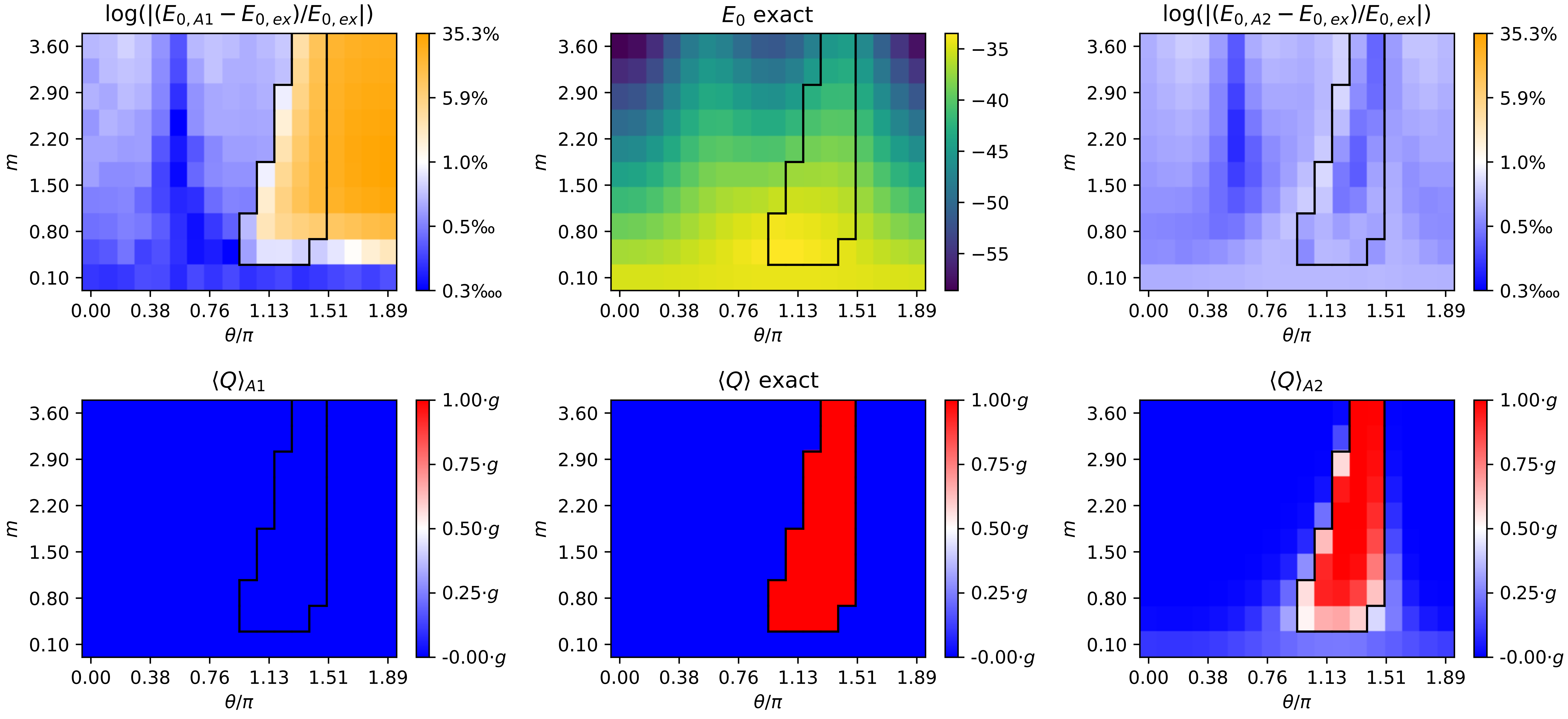
  ➡ **Ground state of $H_{A2}(0)$ mixes states within different charge sectors**

Exact Diagonalization: **Multi-Q ASP Algorithm:**



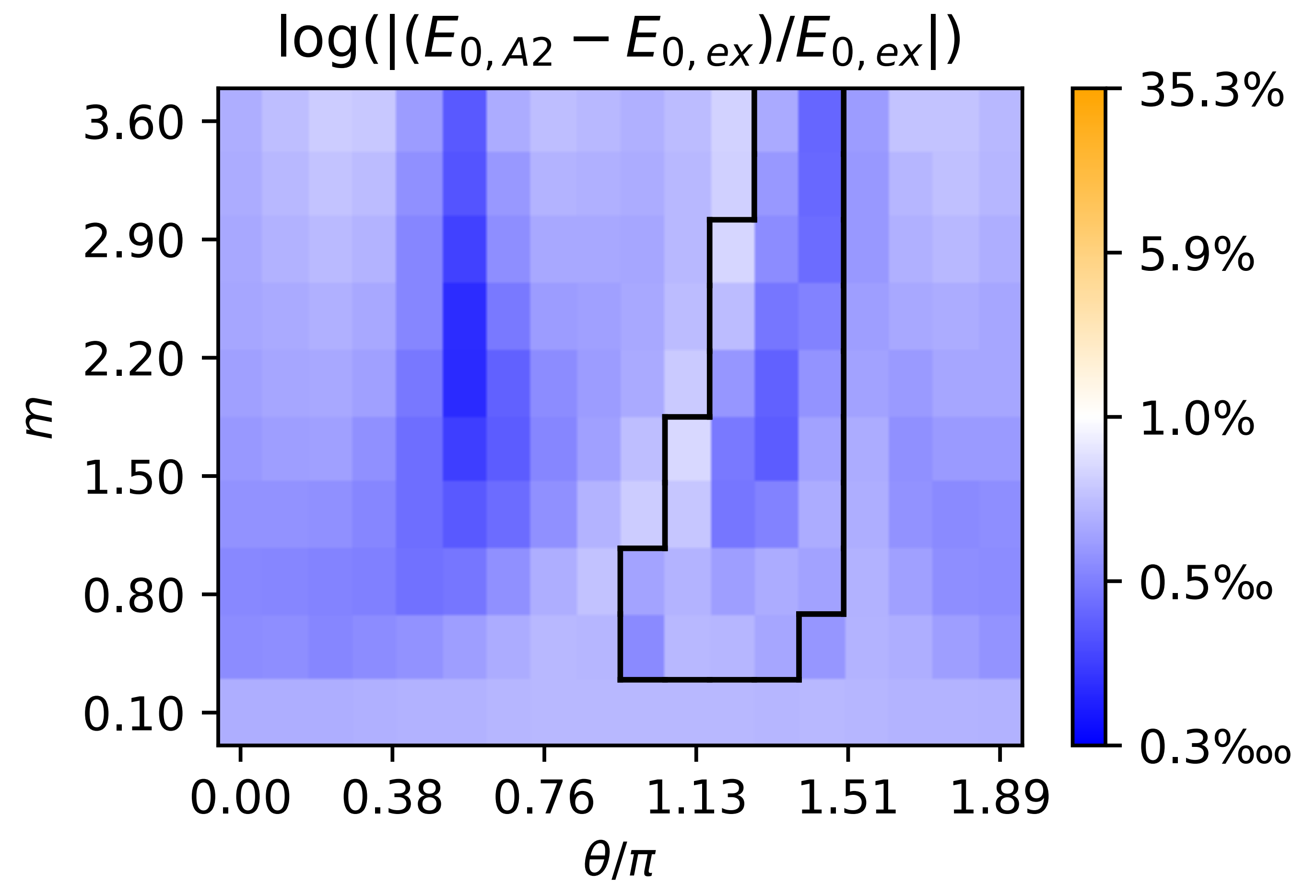
Comparison of State Preparation Algorithms (I)

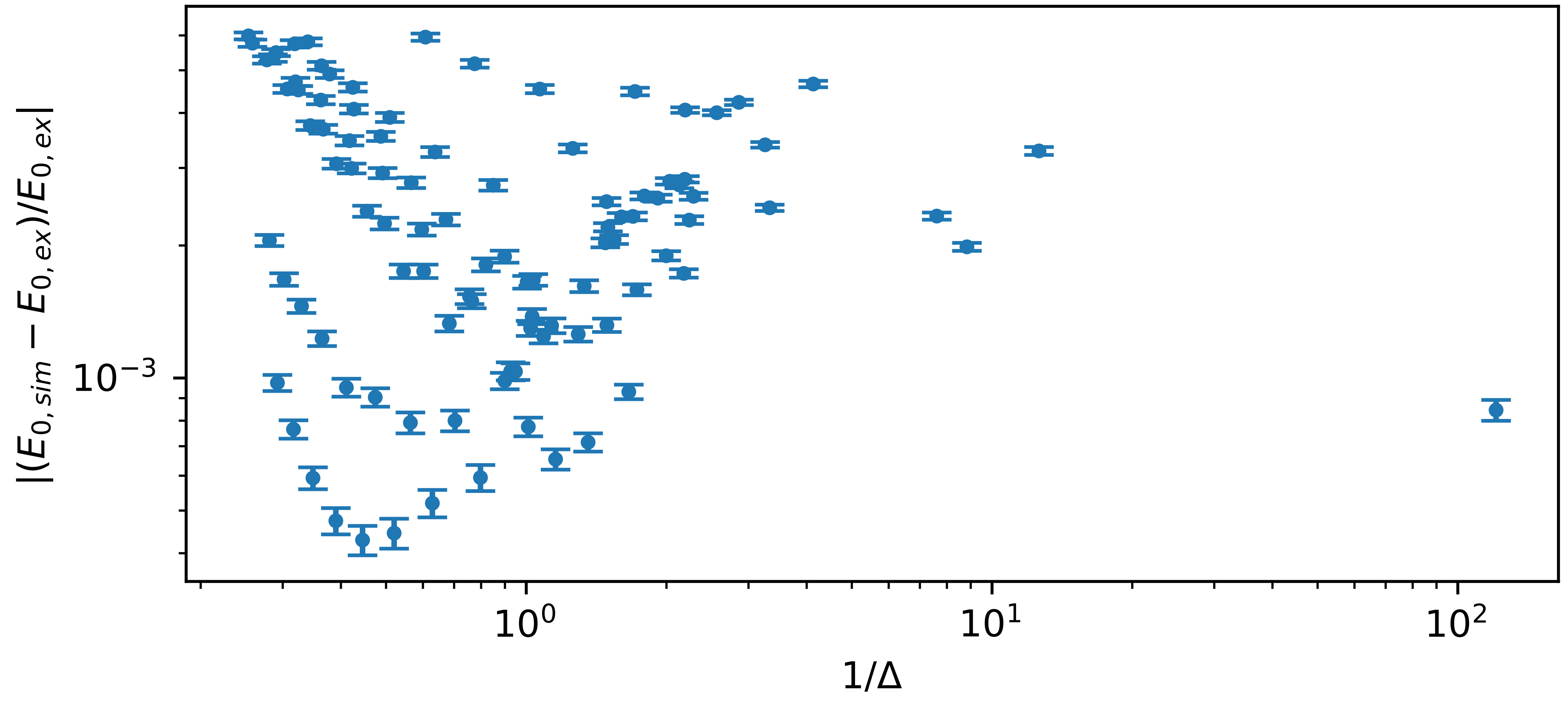
Fixed-Q ASP Algorithm: **Exact Diagonalization:** **Multi-Q ASP Algorithm:**

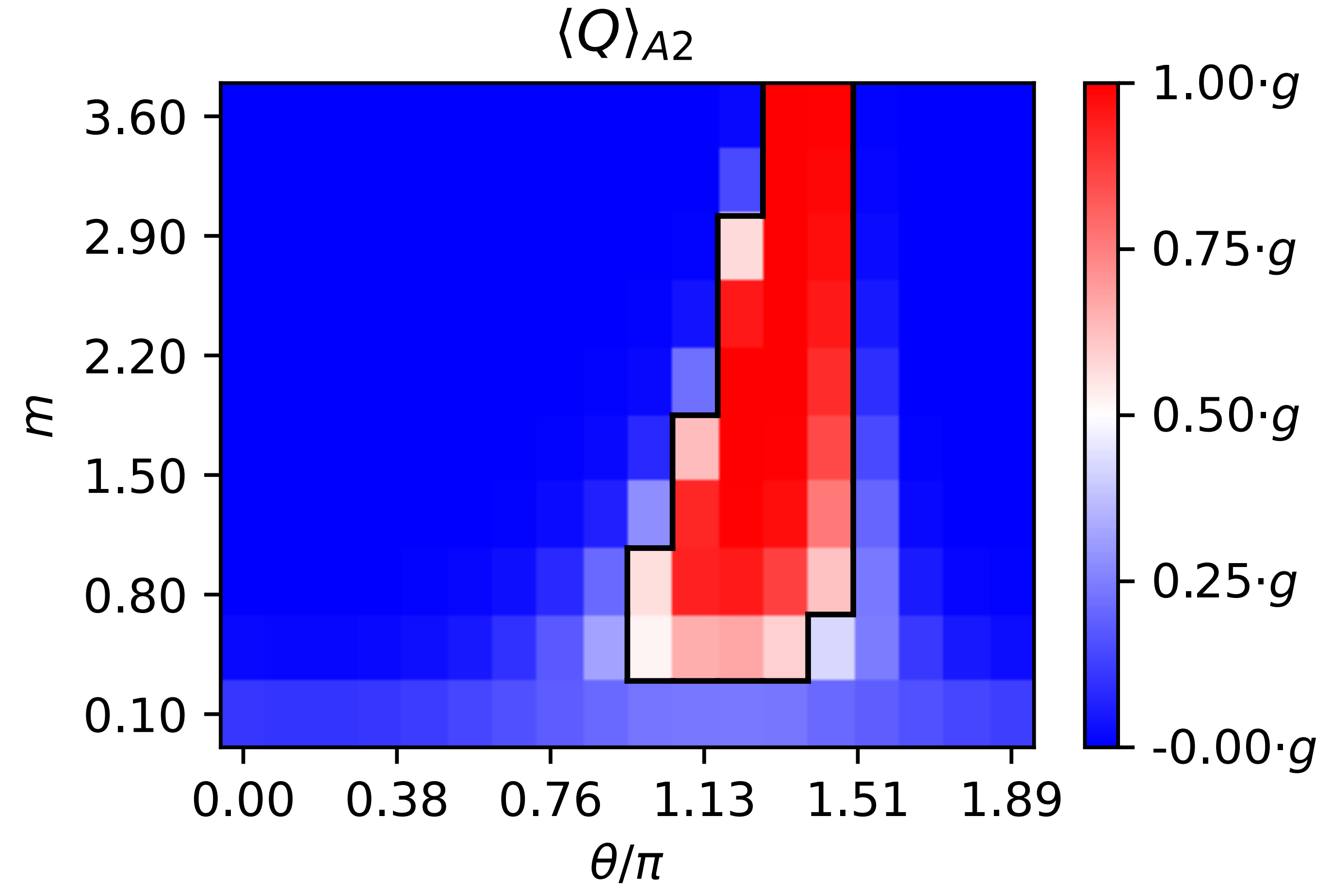


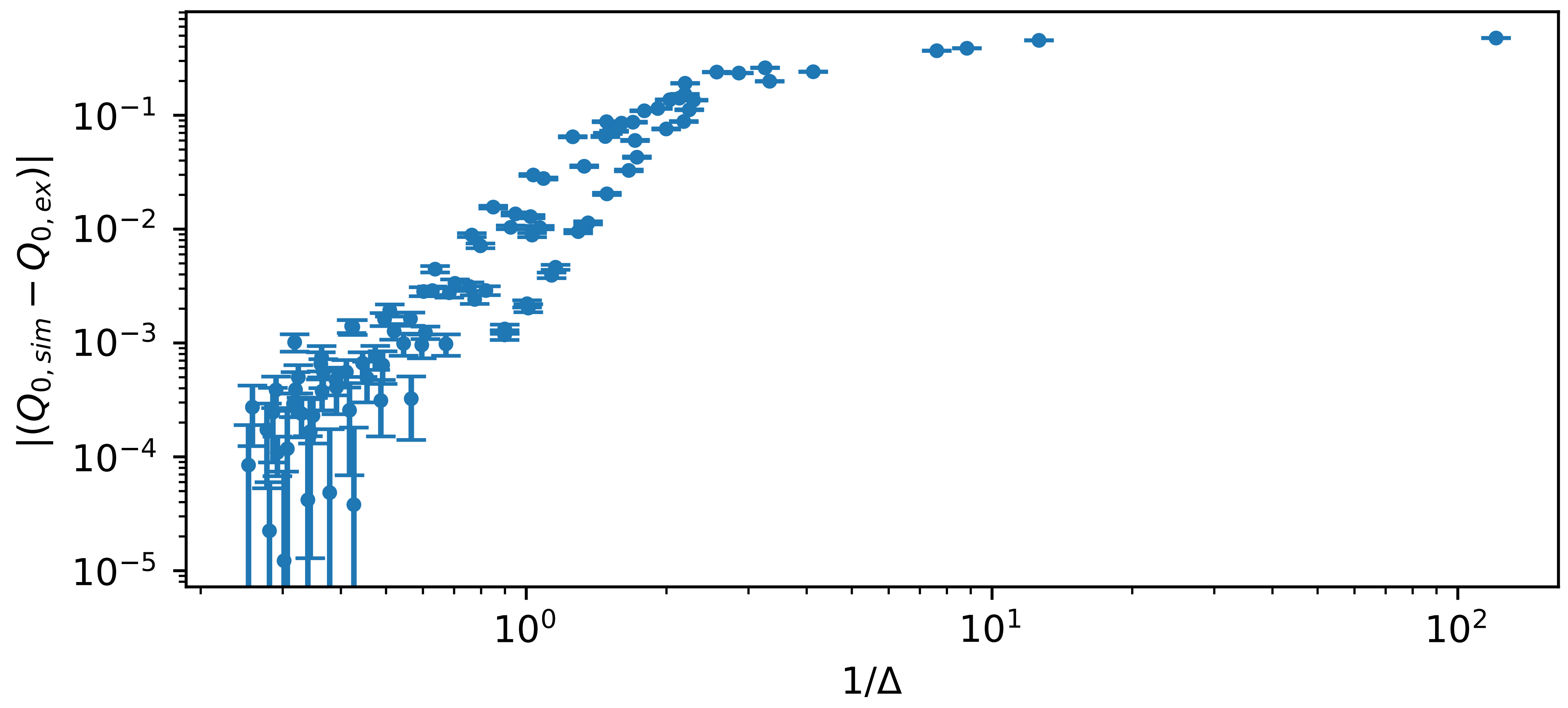
We acknowledge use of the IBM Q for this work. The views expressed are those of the authors and do not reflect the official policy or position of IBM or the IBM Q team.

Comparison of State Preparation Algorithms (II)



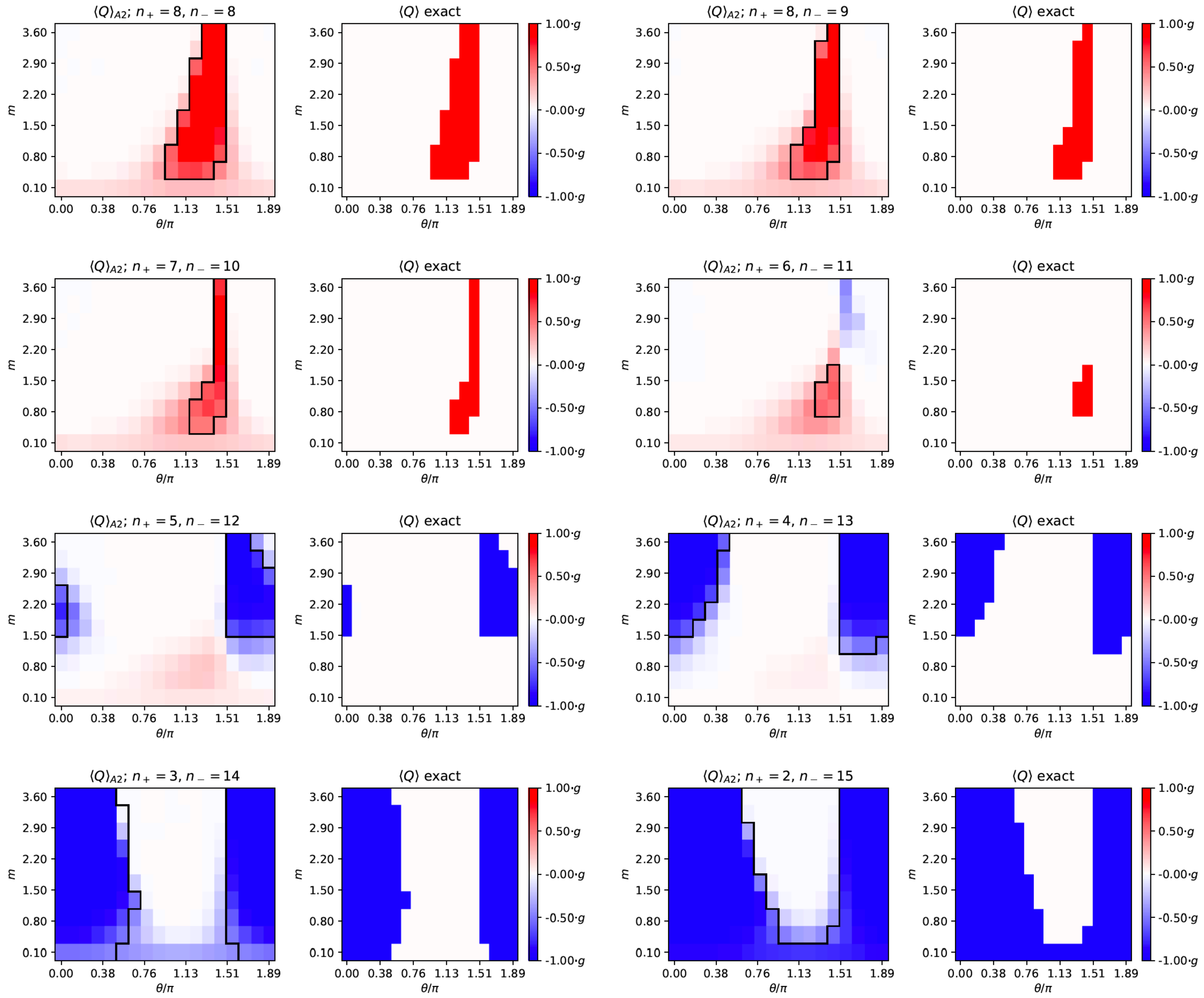




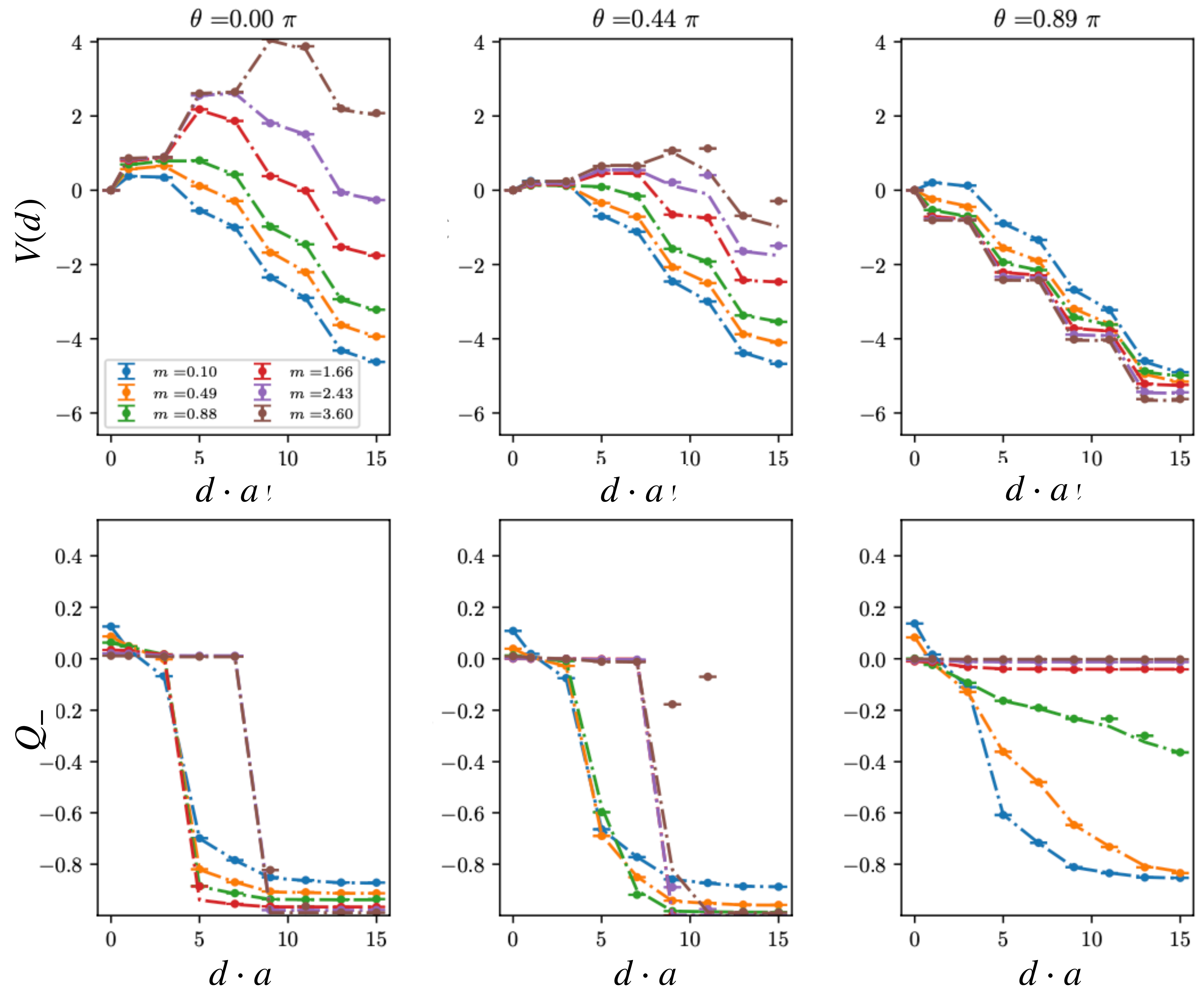


Relative error of the ground state energy (top) and the charge(bottom) as a function of the spectral gap $\Delta = E_1 - E_0$

$\langle Q \rangle_{A2}$ in the presence of static charges

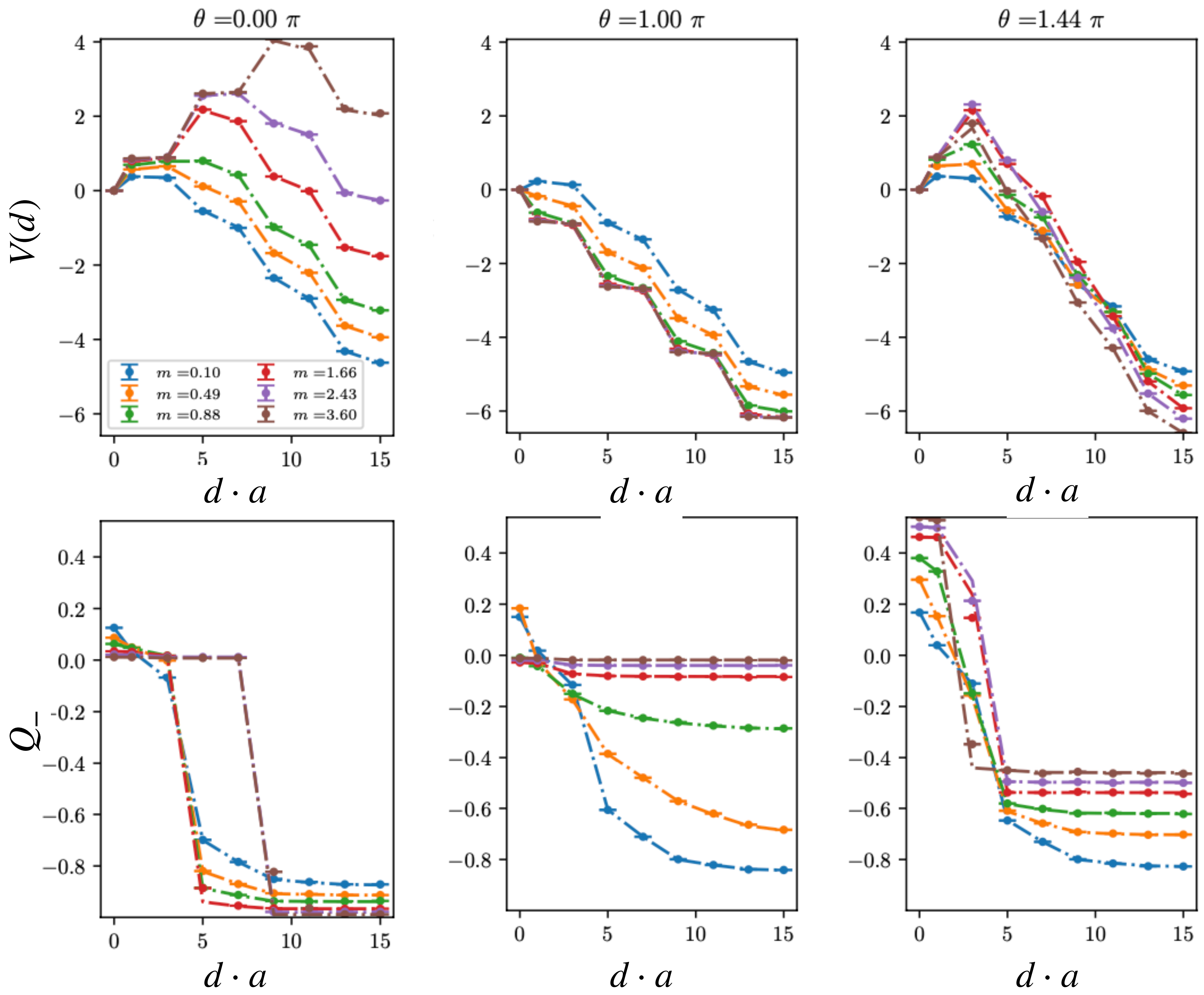


String Breaking (I)



- **Multi-charge ASP enables studies of confinement of charge in QED2**
- **Notation from** [Buyens et. al. Physical Review X 6, 041040 (2016)] $V(d) = E_0(d) - E_0(0)$
- $Q_- =\ -$ **operator sensitive to the string breaking**

String Breaking (II)



- **Multi-charge ASP enables studies of confinement of charge in QED2**
- $V(d) = E_0(d) - E_0(0) \leftarrow$ **static charge-anticharge potential: not a good indicator of string breaking for** $\theta \neq 0$
- $Q_- = \frac{g}{2}\sum_{n=1}^{N/2} Z_n$ $\leftarrow$ **operator sensitive to the string breaking**

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Summary & Outlook

[Banlus et al. Physical Review D 93, (2016) 094512]
[Buyens et al. Physical Review D 94, (2016) 085018]
[Butt et al. Physical Review D 101, (2020) 094509]
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[Nguyen et al. Quantum 3, 020324 (2022)]
[Chakraborty et al. Phys. Rev. D 105, (2022) 94503]
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- **Schwinger model $QED2$ toy model for strongly interacting quantum chromodynamics**
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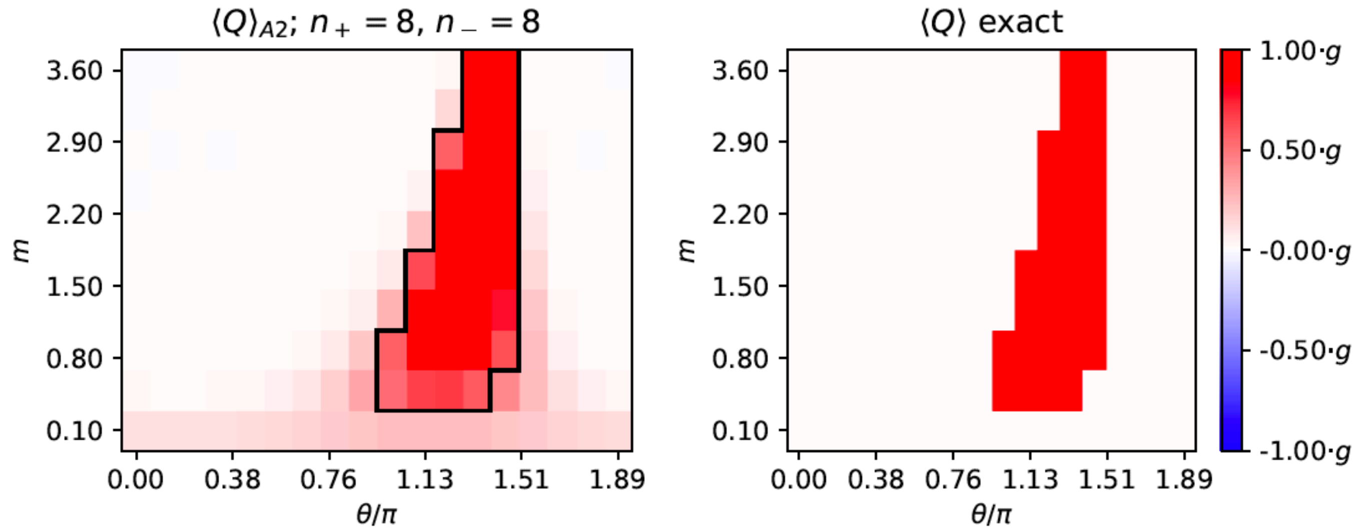
- **In progress:**
  - ➡ **Theoretical understanding of the algorithm; applications to topological cond. matter;**
  - ➡ **Multi-Q ASP applied to study the confinement & string breaking in QED2**



Thank you!

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