Quantum computing for HEP:
Digital twins for quantum states, some goals, and qudits

Christine Muschik

Quantum Interactions

www.quantum-interactions.com

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TQT Transformative Quantum Technologies

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Let’s see what your near future holds…

Overview

1. Digital twins for quantum states
2. Some QuantHEP goals
3. Qudits for the win

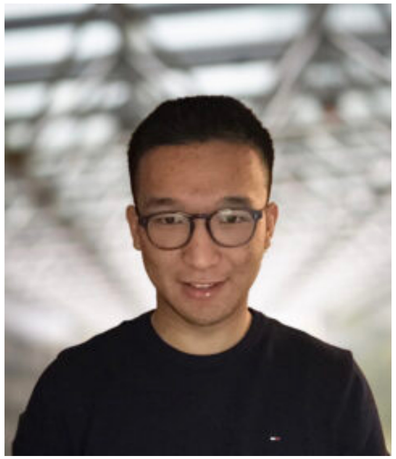
Neural shadow quantum state tomography

Towards digital twins for quantum states

Phys. Rev. Research 6, 023250 (2024).

Neural shadow quantum state tomography

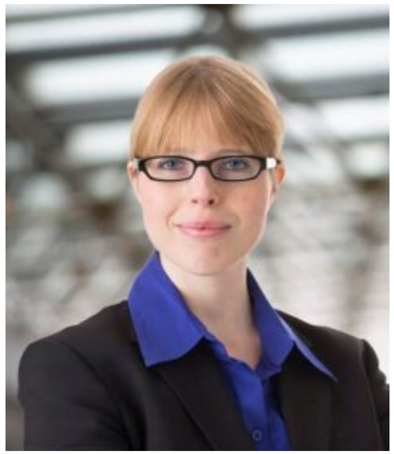
Victor Wei, W. A. Coish, Pooya Ronagh, Christine A. Muschik
Phys. Rev. Research 6, 023250 (2024).



Victor Wei
Stanford

Bill Coish
McGill

Pooya Ronagh
IQC, PI, UWaterloo



Christine Muschik
IQC, PI, UWaterloo

Digital twins of quantum states

$$|\Psi\rangle \xrightarrow{\text{Train}} \psi_\lambda$$

Physical quantum state

Neural network
quantum state ansatz

Why digital twins of quantum states?

Quantum Computer

Classical Computer

Some quantum states are hard to find…

…but easy to store classically.

Why digital twins of quantum states?

Quantum Computer

Classical Computer

Measure now

...decide later!

Why digital twins of quantum states?

Quantum Computer

Classical Computer

NISQ hardware

Post-process classically

Clean up the state: error mitigation

Why digital twins of quantum states?

Quantum Computer

Classical Computer

NISQ hardware

Post-process classically
Further optimization

Combine quantum computing with traditional LGT calculations

Quantum Computer

Classical Computer

Finite density calculation, or time evolution

…regular lattice gauge theory computation.

Combine quantum computing with traditional LGT calculations

Quantum Computer

Zohreh Davoudi

Classical Computer

Finite density calculation, or time evolution

…regular lattice gauge theory computation.

Digital twins for quantum states

- Digital circuit
- Analog simulation
- Hybrid, etc…

- Local measurements
- Entangling measurements
- POVMs, etc…

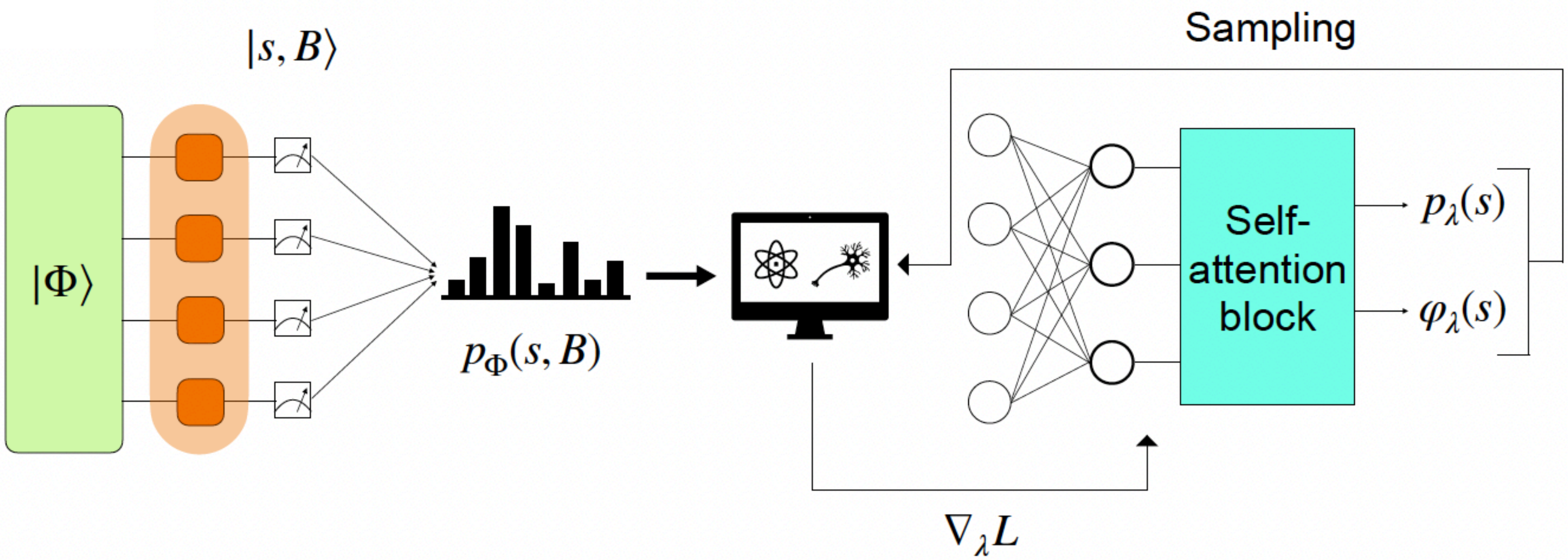
- Neural network
- Matrix Product States
- Other models…

Neural network quantum state tomography

Giacomo Torlai, Guglielmo Mazzola, Juan Carrasquilla, Matthias Troyer, Roger Melko & Giuseppe Carleo

Nature Physics **14**, 447–450 (2018)

Neural network quantum state tomography



Neural quantum state

Transformer neural network quantum state ansatz

Takes a bit-string $s = (s_1, s_2, \ldots, s_n) \in \{0,1\}^n$

$$\psi_\lambda(s) = \sqrt{p_{\lambda_1}(s)}e^{i\varphi_{\lambda_2}(s)}$$

Parametrised by $\lambda = (\lambda_1, \lambda_2)$

Neural quantum state

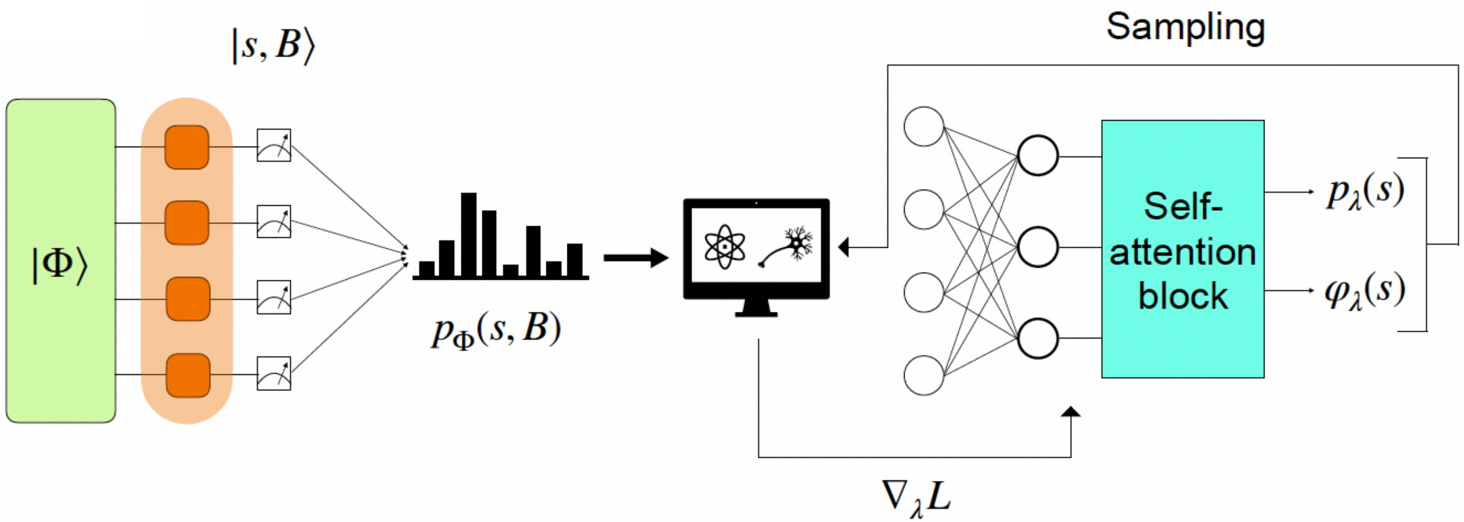
Measured bit-string s (corresponding to $|s\rangle$)

$$\psi_\lambda(s) = \sqrt{p_{\lambda_1}(s)}e^{i\varphi_{\lambda_2}(s)}$$

Complex-valued amplitude $\langle s|\psi_\lambda\rangle$

Pure state ansatz!

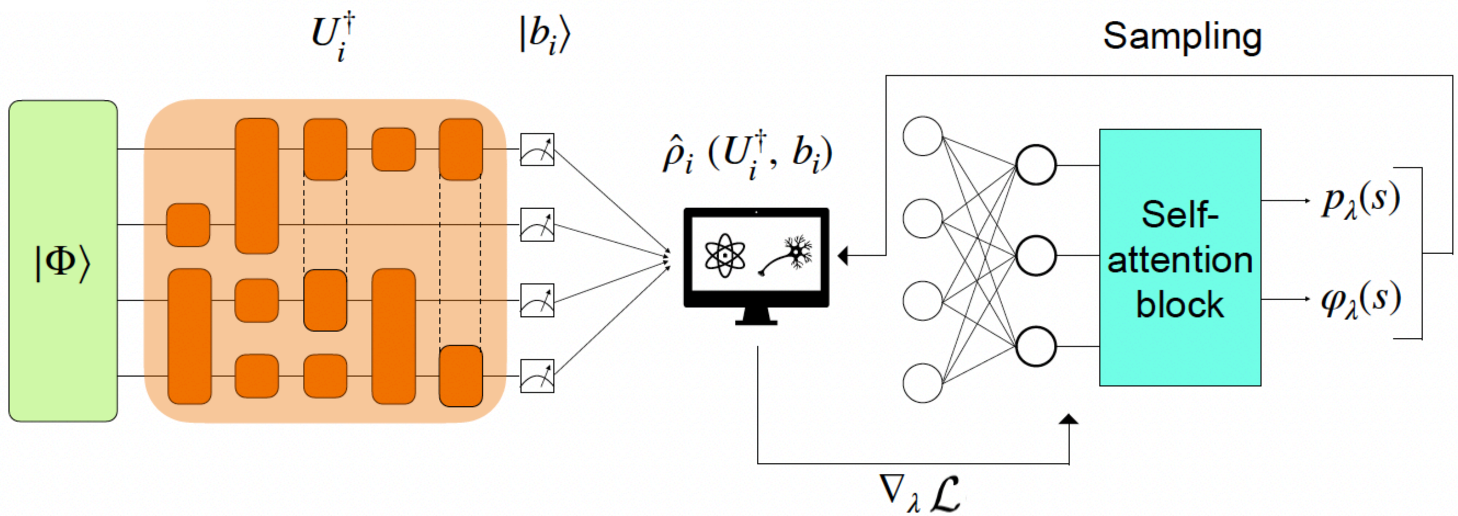
Neural network quantum state tomography



Minimize: cross-entropy loss function

$$L_\lambda = -\frac{1}{|\mathcal{B}|} \sum_{B \in \mathcal{B}} \sum_{s \in \{0,1\}^n} p_\Phi(s, B) \ln p_{\psi_\lambda}(s, B).$$

Neural shadow quantum state tomography: NSQST



Collect classical shadows

Minimize the infidelity

$$\mathcal{L}_\lambda(\mathcal{E}) := 1 - |\langle\psi_\lambda|\Phi\rangle|^2$$

Classical shadows:

S. Aaronson, Shadow tomography of quantum states, in *Proceedings of the 50th annual ACM SIGACT symposium on theory of computing* (2018) pp. 325–338.

H.-Y. Huang, R. Kueng, and J. Preskill, Predicting many properties of a quantum system from very few measurements, Nature Physics **16**, 1050 (2020).

Neural shadow quantum state tomography: NSQST

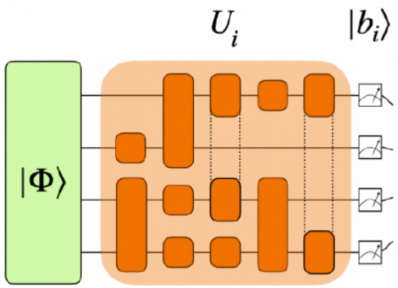
$$\psi_\lambda(s) = \sqrt{p_{\lambda_1}(s)}\, e^{i\varphi_{\lambda_2}(s)}$$

1. Use classical shadows
2. New cost function: fidelity

1. Improved phase information
2. Noise robustness

Neural shadow quantum state tomography: NSQST

- Perform random Clifford tails U_i and measure bit strings $|b_i\rangle$.
- Collect stabilizer states: $|\phi_i\rangle = U_i^\dagger |b_i\rangle$.
- Average effect of the Clifford twirling is a depolarizing noise channel $\mathcal{M}$ with strength $(2^n + 1)^{-1}$.
- Classical shadows: $\rho_i = \mathcal{M}^{-1}(|\phi_i\rangle\langle\phi_i|)$.
- Target state: $|\Phi\rangle\langle\Phi| = \mathbb{E}[\mathcal{M}^{-1}(|\phi_i\rangle\langle\phi_i|)]$.
- New loss:

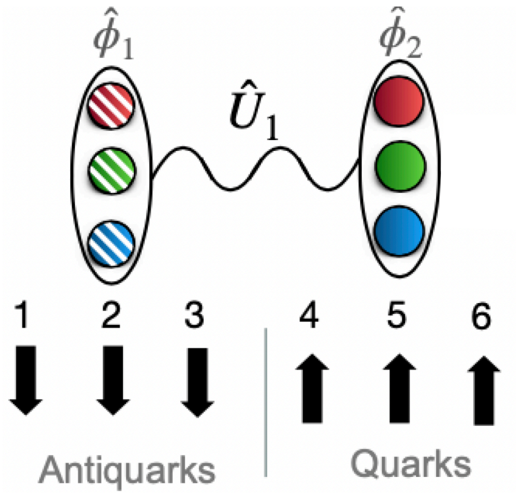


$$1 - |\langle\psi_\lambda|\Phi\rangle| \approx 1 - \frac{1}{N}\sum_i^N Tr(O_\lambda \rho_i) = 1 - \frac{1}{2^n}\left(1 - \frac{1}{f}\right) - \frac{1}{N}\sum_i^N |\langle\phi_i|\psi_\lambda\rangle|^2$$

NSQST for concrete examples

1D-QCD: time-evolved state
Heisenberg antiferromagnet
Phase shifted GHZ state

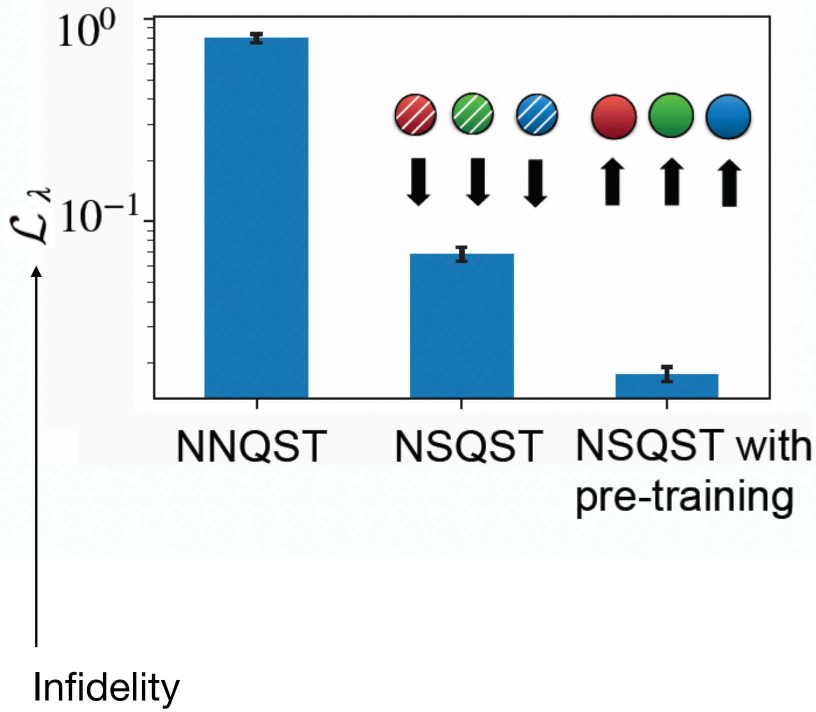
NSQST for a concrete example

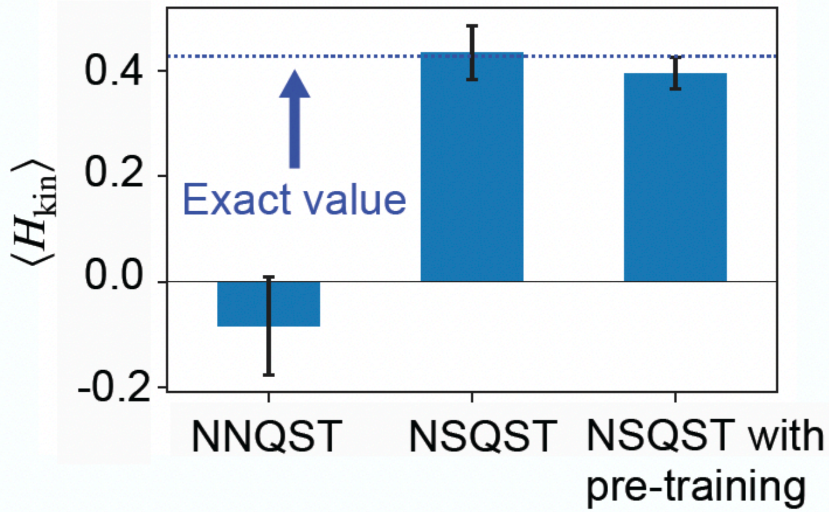


Simulating one-dimensional quantum chromodynamics on a quantum computer: Real-time evolutions of tetra- and pentaquarks

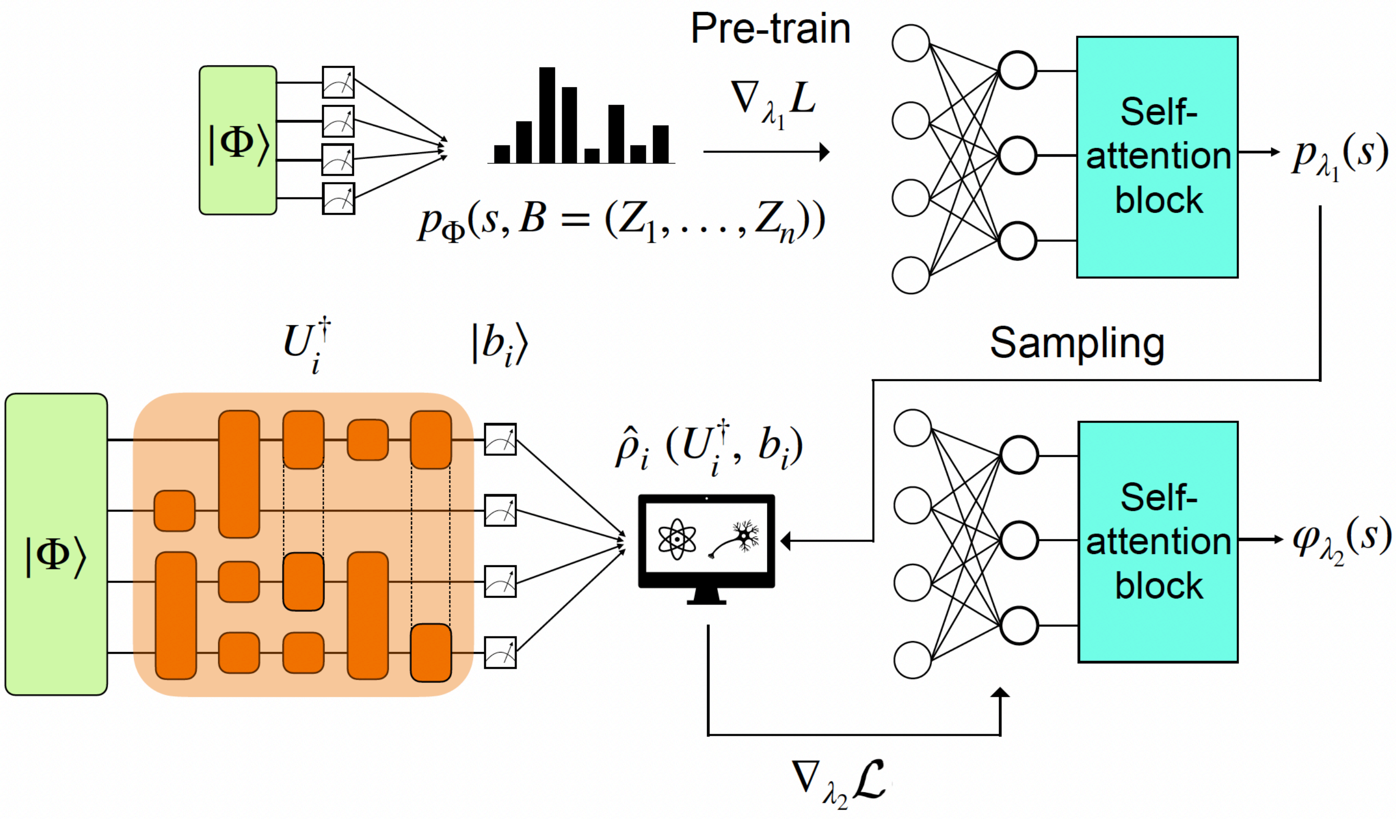
Yasar Y. Atas*,[1,2,†] Jan F. Haase*,[1,2,3,‡] Jinglei Zhang,[1,2,§] Victor Wei,[1,4]
Sieglinde M.-L. Pfaendler,[5] Randy Lewis,[6] and Christine A. Muschik[1,2,7]

NSQST for a 1D-QCD time evolution

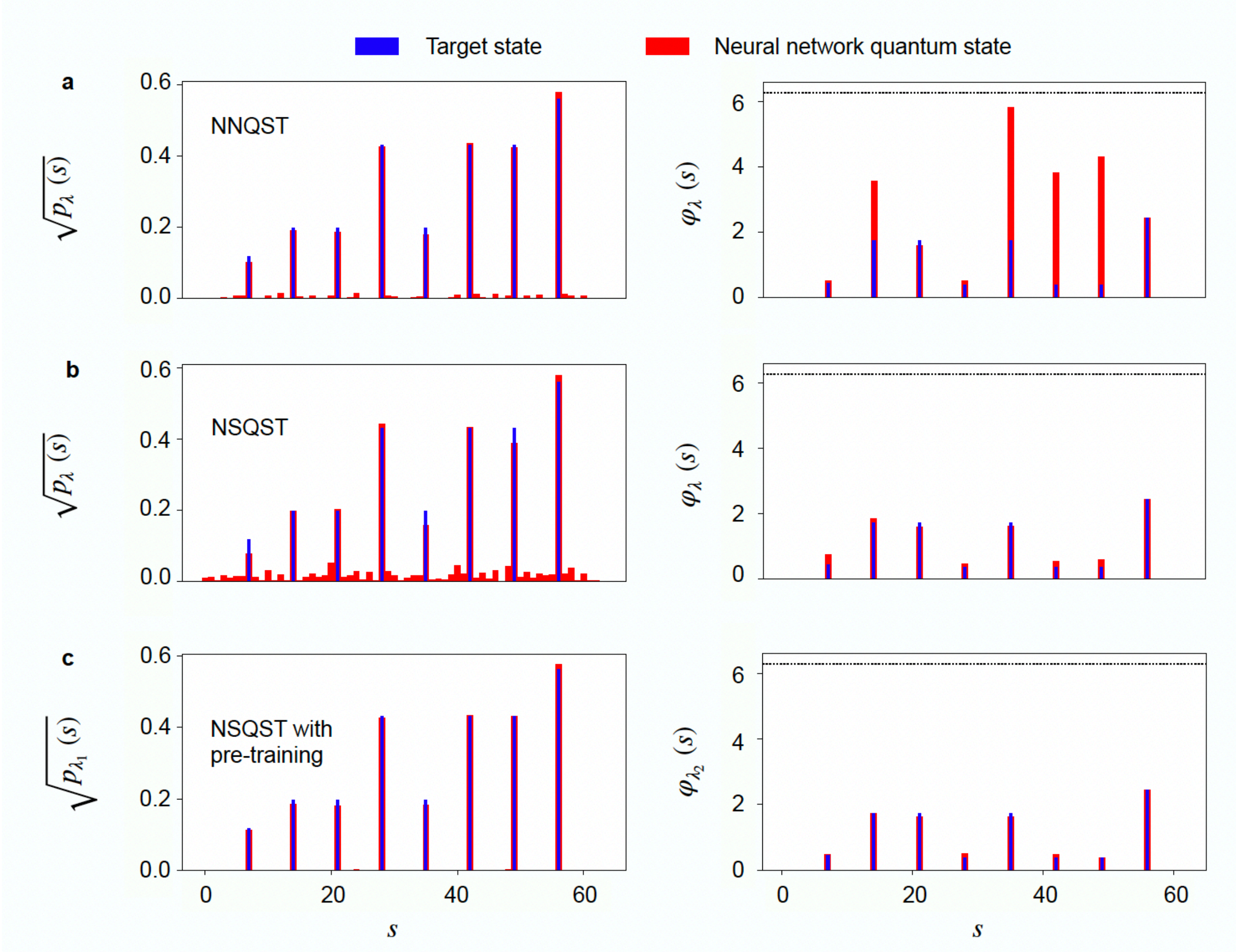




NSQST with pre-training



NSQST for a 1D-QCD time evolution



NSQST: Robustness to noise

Phys. Rev. Research 6, 023250 (2024).

Outlook

Food for thought

Quantum experiments are difficult to perform:
- Expensive
- Time-intensive

No-cloning theorem:
- How to capture experiments in a re-usable fashion
- Make the results of an experiment available to the community

Neural network trained on quantum data:
- Schemes like NSQST can be more useful than a list of measurements
- in the same way GPT is more useful than a text on the web

The neural network representation is a **digital twin** of the quantum state
- Shift the value from quantum experiments to **quantum data**
- Inference from the digital twin us **cheaper** than re-running quantum experiments
- The digital town is more **malleable and easier to interface with** than a quantum computer

Combine quantum computing with traditional LGT calculations

Quantum Computer

Classical Computer

Finite density calculation, or time evolution

…regular lattice gauge theory computation.

Hamiltonian learning

$$H = \sum_i \left(J^x X_i X_{i+1} + J^y Y_i Y_{i+1} + J^z Z_i Z_{i+1} + h_i X_i\right)$$

Learn Hamiltonian parameters

Time evolution

Local measurements on time-evolved state

Matrix Product States

Overview

1. Digital twins for quantum states
2. Some QuantHEP goals
3. Qudits for the win

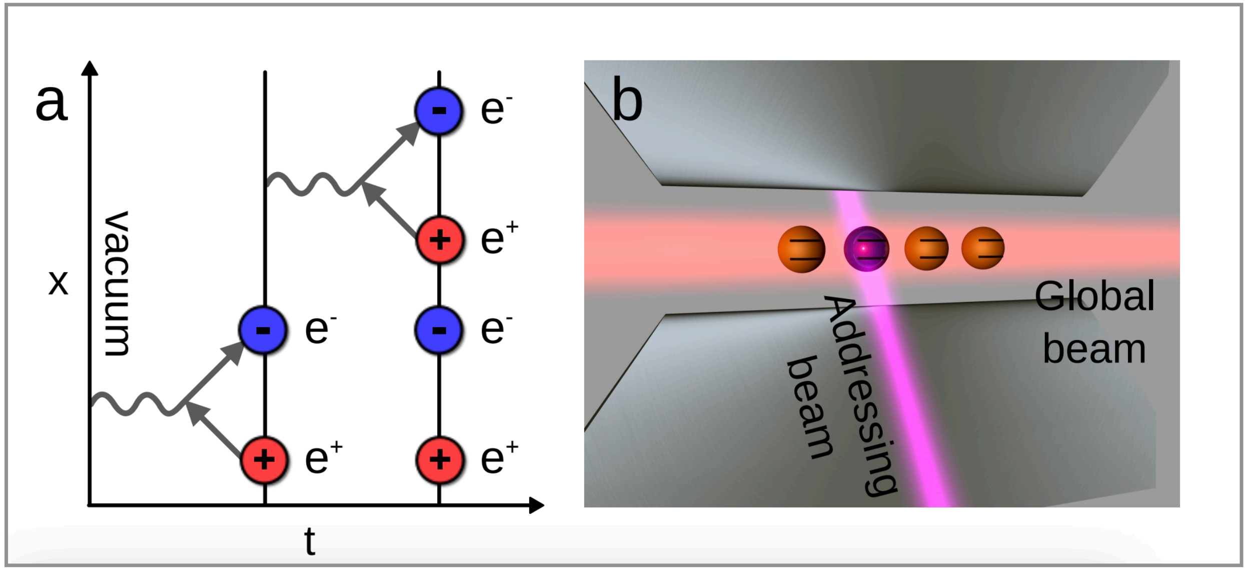
Some goals

Past → Present → Future

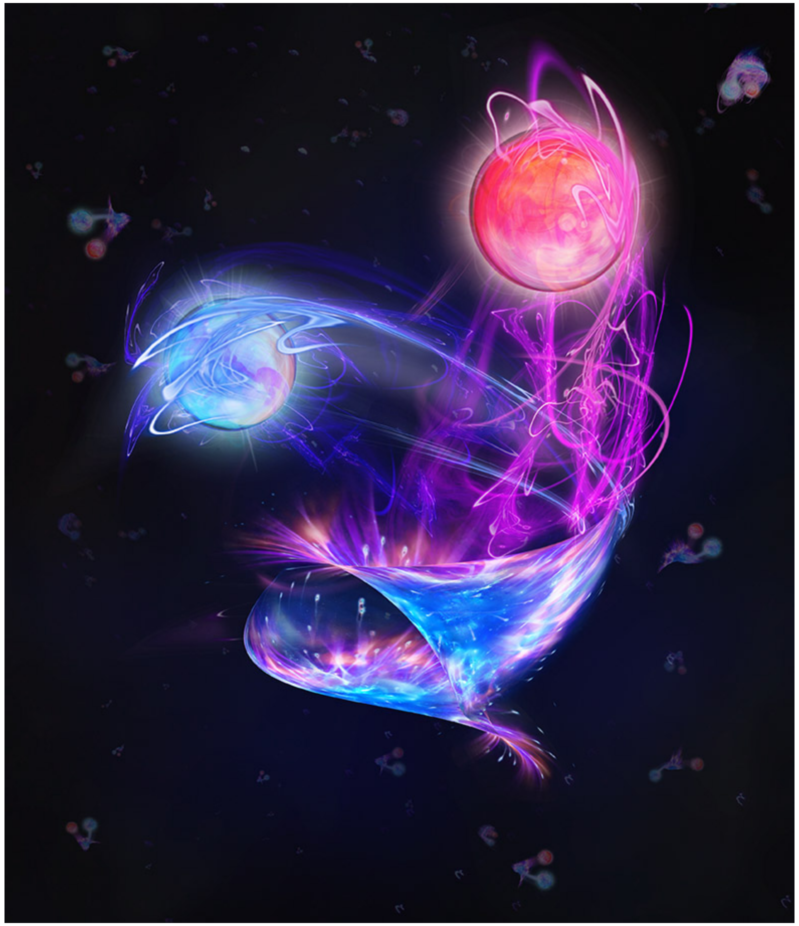
Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Nature **534**, 516–519 (2016)

➡ Quantum simulation of 1D-QED



Theory + Experiment



Top 10 breakthroughs in physics 2016

Physics World

SCIENTIFIC AMERICAN

nature

COMPUTING

In a First, Quantum Computer Simulates High-Energy Physics

The technique could allow quantum computers to address otherwise-intractable problems in particle physics

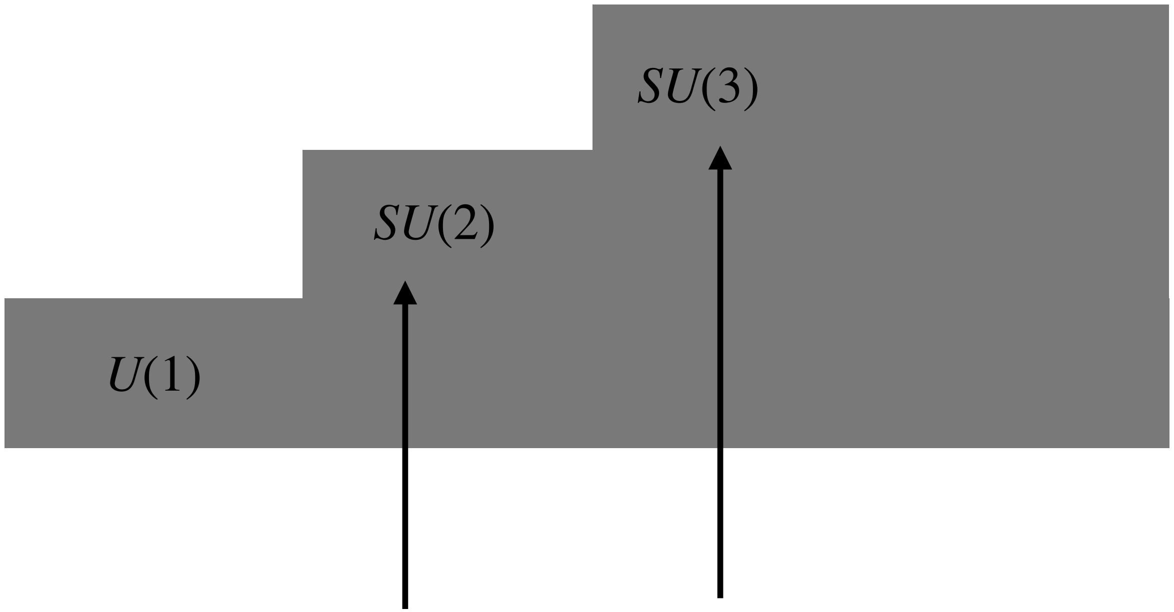
Some goals

Towards 3D

Represent gauge fields

Non-Abelian

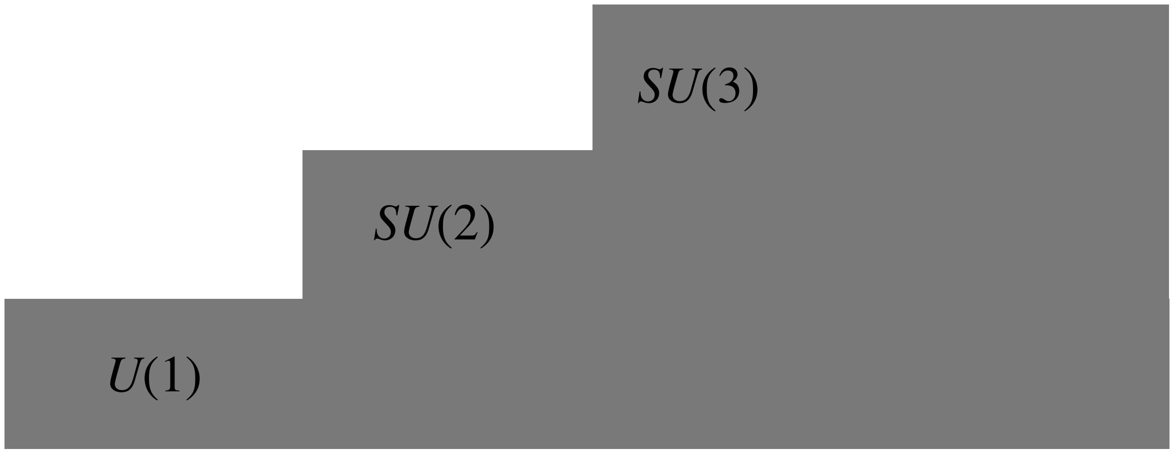
Gauge group of QCD



1D-SU(2)
Experiment on IBM
Nature Communications **12**, 6499 (2021).

1D-SU(3)
Experiment on IBM
Phys. Rev. Research **5**, 033184 (2023).

Gauge group of QCD



Loop-String-Hadron Formulation for SU(2) and SU(3): Indrakshi Raychowdhury, Jesse Stryker
General quantum algorithms for non-Abelian LGTs: Zohreh Davoudi, Alexander F. Shaw, Jesse R. Stryker
Trailhead for SU(3): Anthony Ciavarella, Natalie Klco, Martin J. Savage
Digitizing SU(2) Gauge Fields: T. Hartung, T. Jakobs, K. Jansen, J. Ostmeyer, C. Urbach
3 new quantum simulation ideas for non-Abelian gauge theories: Torsten Zache, Daniel González-Cuadra, Peter Zoller

Simulating 1D-QCD on a quantum computer

Theory + Experiment:
- Tetraquarks
- Real-time evolution

Simulate quarks with three colours, i.e. the gauge group of QCD

Simulating 1D-QCD on a quantum computer

Related work:

Preparations for Quantum Simulations of Quantum Chromodynamics in 1+1 Dimensions: (I) Axial Gauge

Roland C. Farrell, Ivan A. Chernyshev, Sarah J. M. Powell, Nikita A. Zemlevskiy, Marc Illa, Martin J. Savage

Tools necessary for quantum simulations of $1+1$ dimensional quantum chromodynamics are developed. When formulated in axial gauge and with two flavors of quarks, this system requires 12 qubits per spatial site with the gauge fields included via non-local interactions. Classical computations and D-Wave's quantum annealer Advantage are used to determine the hadronic spectrum, enabling a decomposition of the masses and a study of quark entanglement. Color edge states confined within a screening length of the end of the lattice are found. IBM's 7-qubit quantum computers, ibmq_jakarta and ibm_perth, are used to compute dynamics from the trivial vacuum in one-flavor QCD with one spatial site. More generally, the Hamiltonian and quantum circuits for time evolution of $1+1$ dimensional $SU(N_c)$ gauge theory with N_f flavors of quarks are developed, and the resource requirements for large-scale quantum simulations are estimated.

M. Savage,
University of Washington

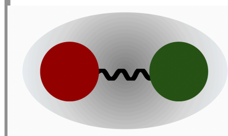
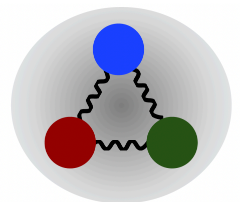
Experimental demonstration of color neutral objects (gauge singlets/color singlets).

Color neutral states of SU(3): invariant under arbitrary rotations in color space

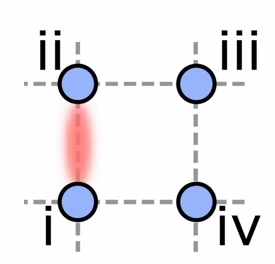
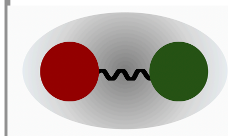
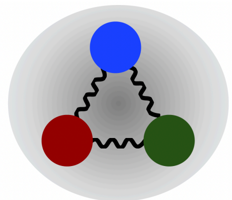
Involve all the color charges available in the theory i.e. red, green, and blue (and their anticolors).
In contrast to Abelian quantum electrodynamics (QED), where a singlet state involves electron-positron pairs only.

Color singlet states are the relevant physical states,
→ important step towards the understanding, description and prediction of more complex and realistic experiments

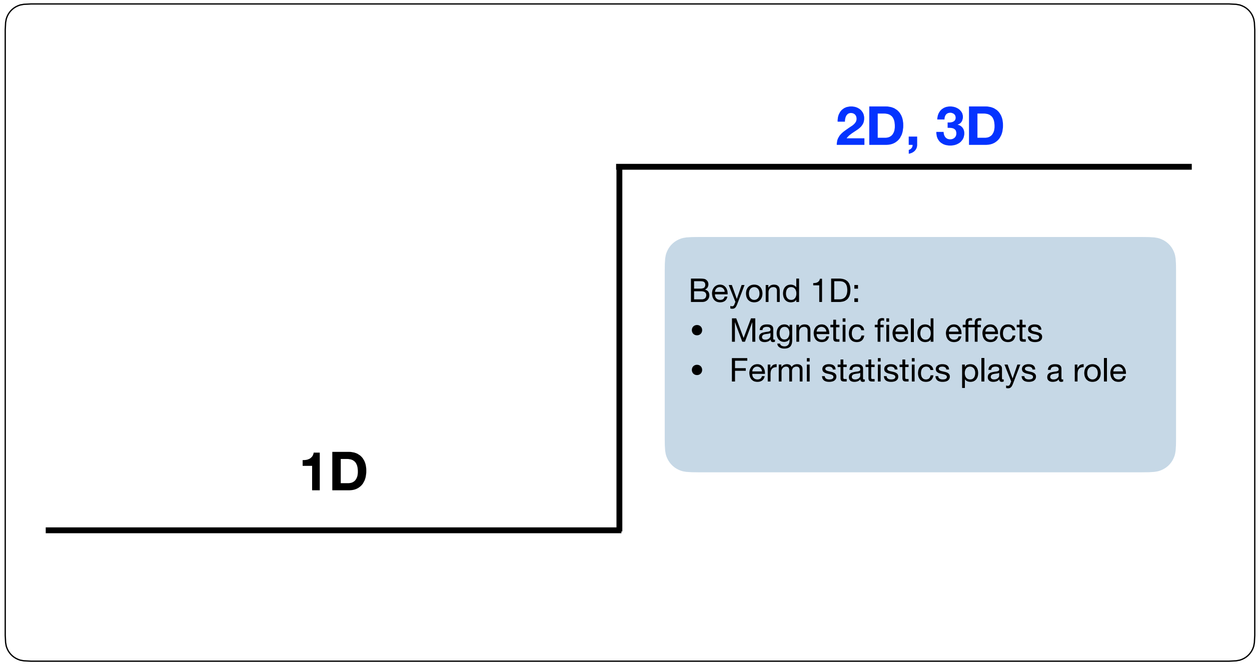
Some goals

Towards 3D	Represent gauge fields	Non-Abelian
		1D-SU(2) Experiment on IBM  1D-SU(3) Experiment on IBM 

Some goals

Towards 3D	Represent gauge fields	Non-Abelian
2D-U(1) Experiment with trapped ions 		1D-SU(2) Experiment on IBM  1D-SU(3) Experiment on IBM 

Gauge theories for particle physics beyond 1D



Experimental demonstrations

Gauge theories for particle physics beyond 1D

- N. Klco, M. J. Savage, and J. R. Stryker, Phys. Rev. D **101**, 074512 (2020).
- A. Ciavarella, N. Klco, and M. J. Savage, Phys. Rev. D **103**, 094501 (2021).
- S. A Rahman, R. Lewis, E. Mendicelli, and S. Powell, Phys. Rev. D **104**, 034501 (2021).
- A. N. Ciavarella and I. A. Chernyshev, Phys. Rev. D **105**, 074504 (2022).
- S. A Rahman, R. Lewis, E. Mendicelli, and S. Powell, Phys. Rev. D **106**, 074502 (2022).
- S. A. Rahman, R. Lewis, E. Mendicelli, and S. Powell, "Real time evolution and a traveling excitation in SU(2) pure gauge theory on a quantum computer," (2022), arXiv:2210.11606 [hep-lat].
- A. N. Ciavarella, "Quantum Simulation of Lattice QCD with Improved Hamiltonians," (2023), arXiv:2307.05593 [hep-lat].

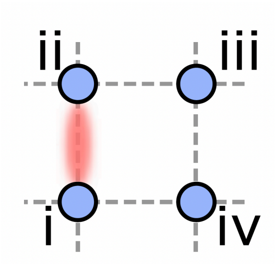
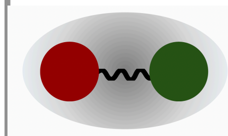
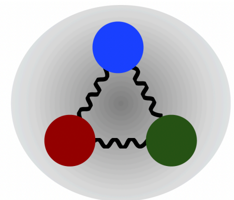
→ **Impressive Advances!**
(so far gauge fields or matter fields were trivial)

Experimental demonstration

Gauge theories for particle physics beyond 1D

Including both - dynamical gauge and matter fields

Some goals

Towards 3D	Represent gauge fields	Non-Abelian
2D-U(1) Experiment with trapped ions 		1D-SU(2) Experiment on IBM  1D-SU(3) Experiment on IBM 

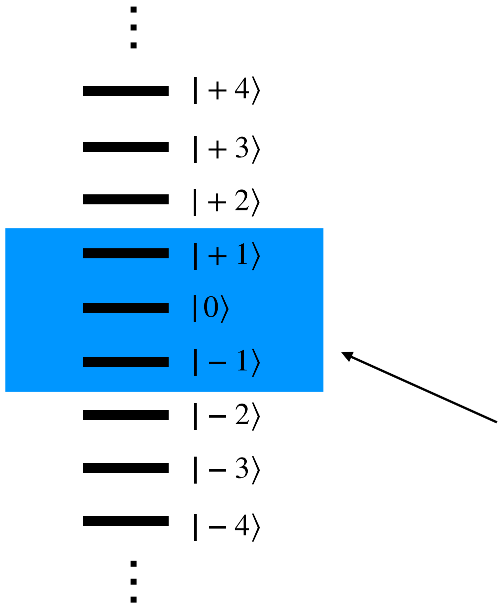
Representing the gauge fields

Gauge fields

Example: lattice QED

$$\hat{E}\,|E\rangle = E\,|E\rangle,$$

$$E = 0,\, \pm 1,\, \pm 2, \dots$$



Classical simulations and finite dimensional quantum hardware require a **truncation**:

Minimal truncation: quTrit
- field in positive direction
- no flux
- field in negative direction

Truncations for bosonic systems:

- Schwinger boson representation
- Holstein-Primakoff-representation
- Dysen-Maleev transformation
- Highly occupied boson model

Truncations for qubit systems:

$$\hat{E} \longmapsto \hat{S}^z,$$
$$\hat{U} \longmapsto \hat{V}^-,$$

$$\hat{V}^- \equiv \begin{bmatrix} 0 & \dots & \dots & 0 \\ 1 & \dots & \dots & 0 \\ 0 & \ddots & \vdots & 0 \\ 0 & \dots & 1 & 0 \end{bmatrix}$$

Alternative:

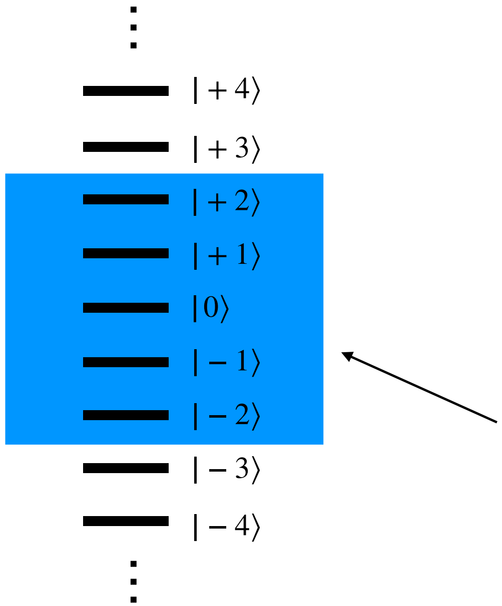
$$\hat{V}^- \equiv \hat{S}^- / |l|$$
$$\hat{S}^- = \hat{S}^x - i\hat{S}^y$$

Gauge fields

Example: lattice QED

$$\hat{E}\,|E\rangle = E\,|E\rangle,$$

$$E = 0,\, \pm 1,\, \pm 2, \ldots$$



Classical simulations and finite dimensional quantum hardware require a **truncation**:

Better truncation: quQuint
- field in positive direction
- no flux
- field in negative direction

Truncations for bosonic systems:

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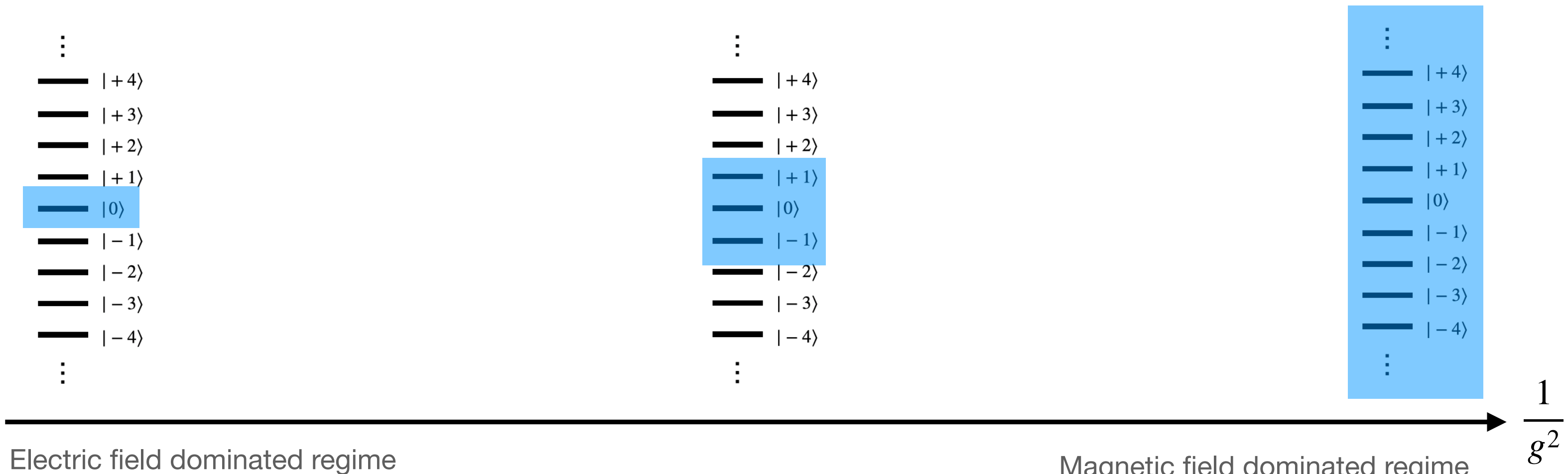
Hilbert space truncation
with a regularisation of the gauge group



$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2} \hat{H}_B$$

Our solution:

- cQED → ccQED (doubly comp[act QED)
- Approximate U(1) by Z_{2L+1}
- Introduce double scaling (scale L with g)

(*) Our scheme was further improved in: C. W. Bauer and D. M. Grabowska, Phys. Rev. D 107, L031503 (2023).

Hilbert space truncation
with a regularisation of the gauge group

	$1/g^2$	Standard truncation (electric basis)	Scaled $\mathbb{Z}_N$ truncation (electric and magnetic basis)
electric fields dominate	0.1	27	27
intermediate regime	10	2197	125
magnetic fields dominate	100	> 9261	27

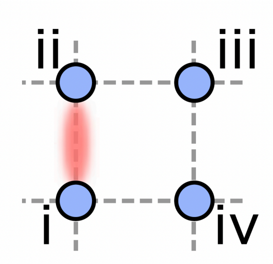
g = bare coupling

Number of states required to reach a 1% accuracy in the expectation value of the two-dimensional plaquette in QED

Some goals

Towards 3D

2D-U(1)
Experiment with trapped ions

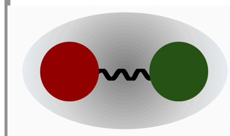


Represent gauge fields

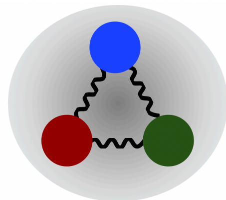
Introduced “B-rep”
Quantum 5, 393 (2021).

Non-Abelian

1D-SU(2)
Experiment on IBM



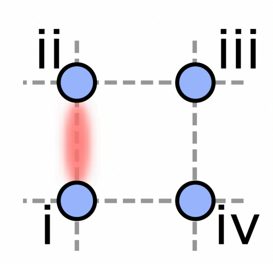
1D-SU(3)
Experiment on IBM



Some goals

Towards 3D

2D-U(1)
Experiment with trapped ions



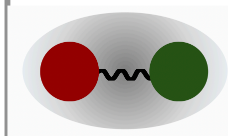
Represent gauge fields

Introduced “B-rep”
Quantum 5, 393 (2021).

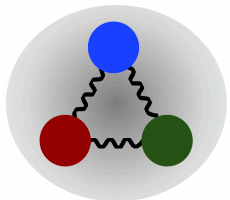
Developed qudit algorithms

Non-Abelian

1D-SU(2)
Experiment on IBM



1D-SU(3)
Experiment on IBM



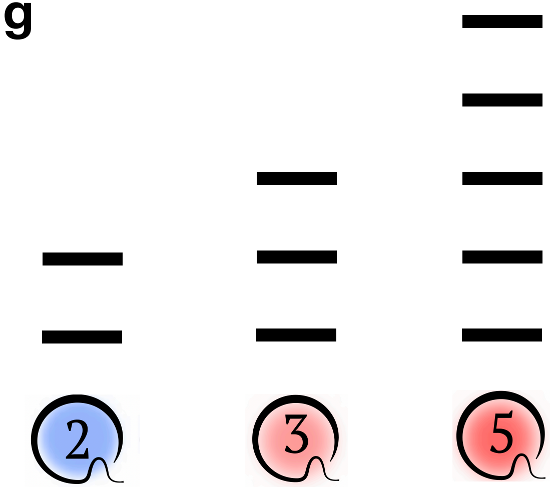
Overview

1. Digital twins for quantum states
2. Some QuantHEP goals
3. Qudits for the win

Beyond binary

Today's quantum hardware: **capable of qudit encoding**

- Trapped ions
- Superconducting architectures
- Rydberg atoms in optical tweezers
- Ultracold atoms in optical lattices
- Nuclear spins
- Photonic systems



Gauge fields represented by qudits

[Reference to Martin’s paper]

CIFAR

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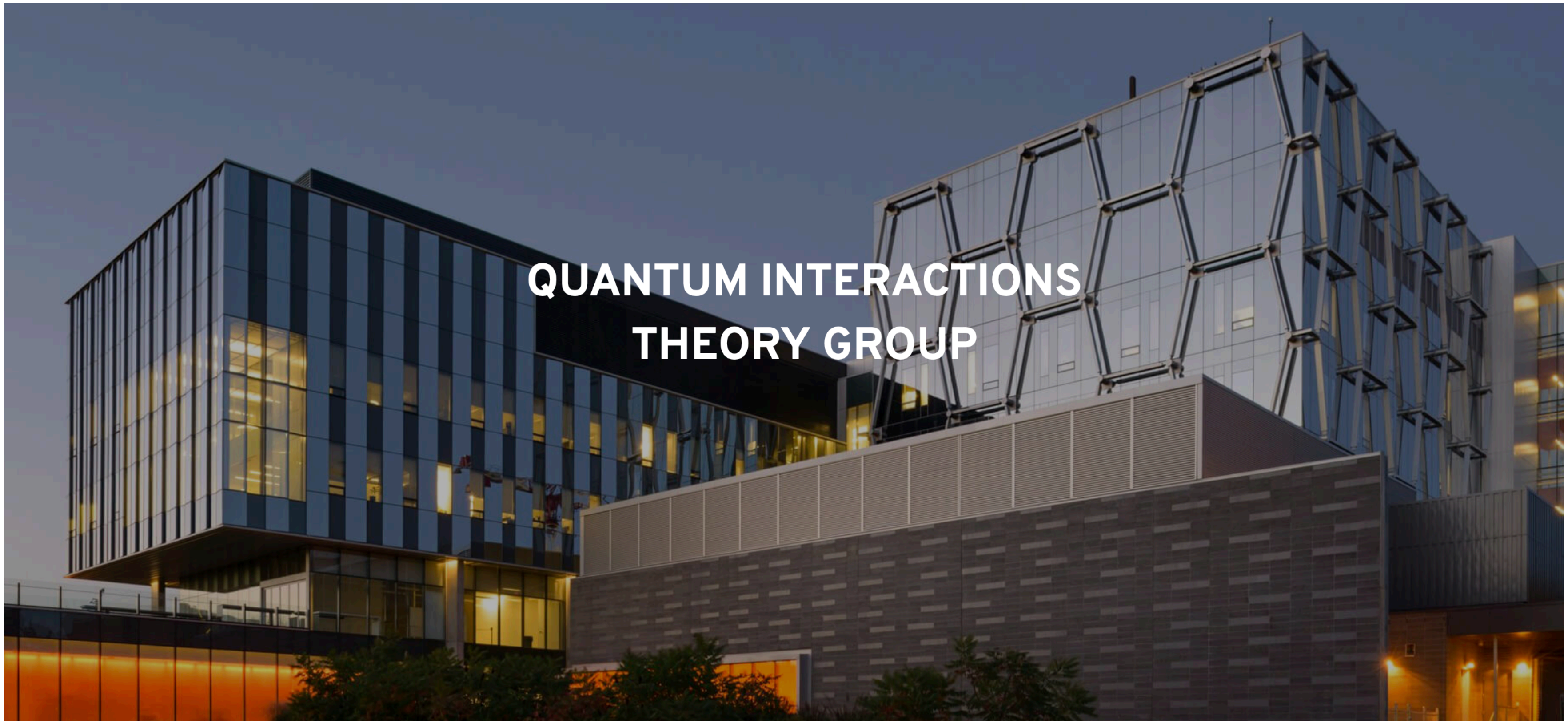
www.quantum-interactions.com



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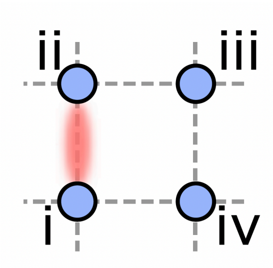
Thank you for your time

Towards 3D

2D-U(1)
Experiment with trapped ions



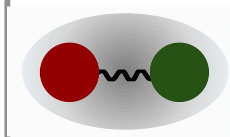
Represent gauge fields

Introduced “B-rep”
Quantum 5, 393 (2021).

Developed qudit algorithms

Non-Abelian

1D-SU(2)
Experiment on IBM



1D-SU(3)
Experiment on IBM