Efficient simulation of lattice gauge theories with qudit quantum computers

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QuantHEP, 5 September 2024

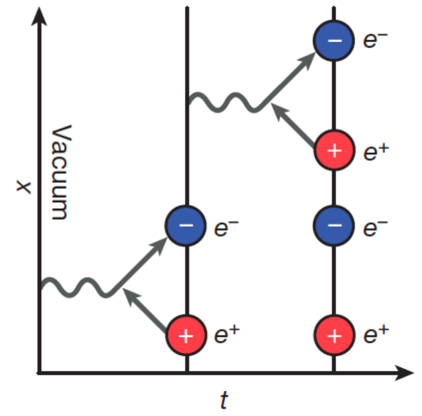
Outline

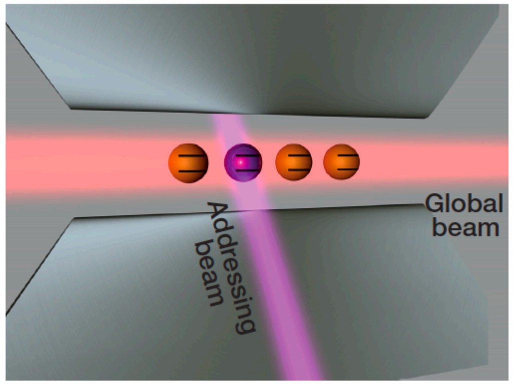
- Motivation
- Using qudits to simulate matter and gauge fields
- Using qudits to refine truncation effects
- Summary & Outlook

Motivation

Quantum simulation of QCD in (3+1)D

QED in (1+1)D

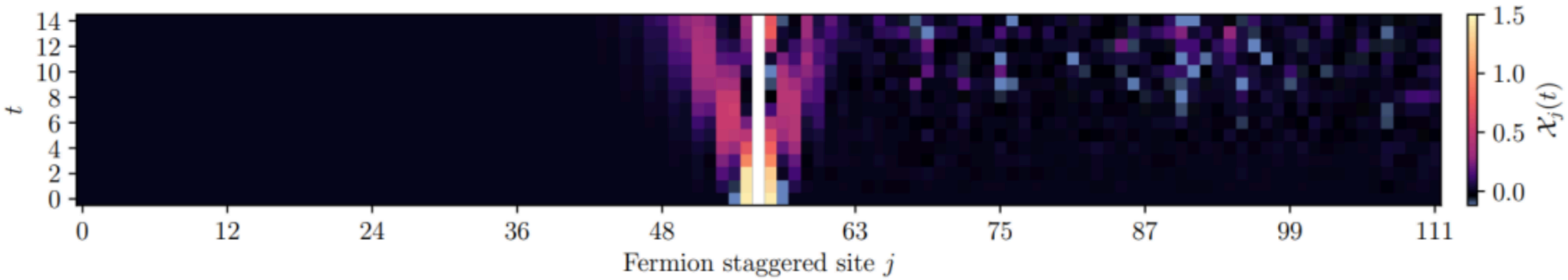


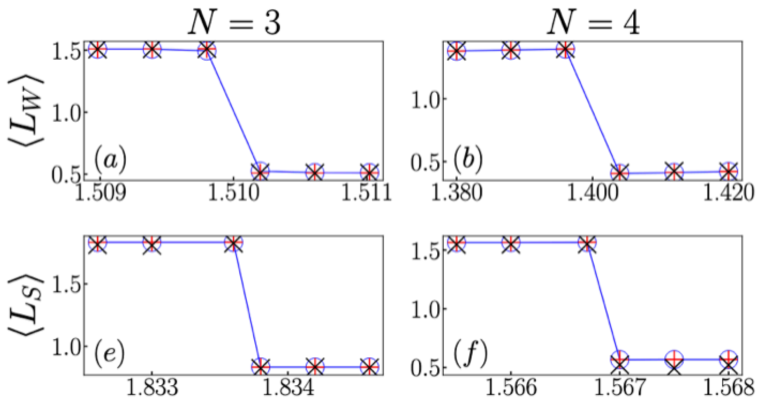


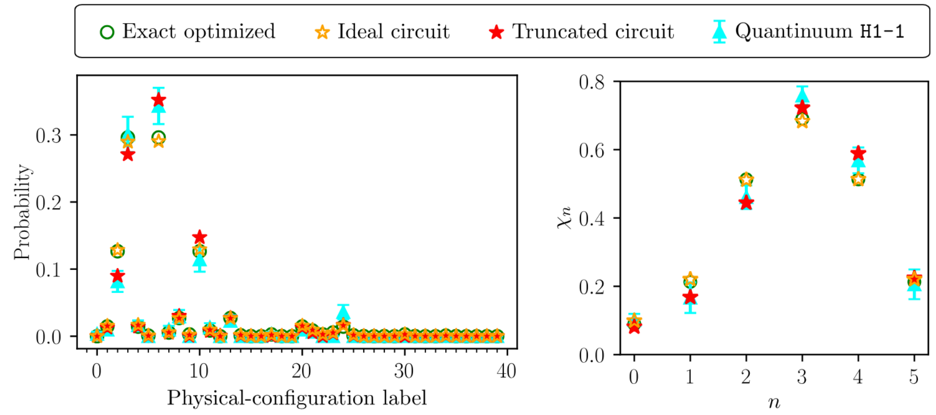
E. A. Martinez et al., Nature 534, 516 (2016)

Motivation

Abelian LGT in (1+1)D with dynamical matter







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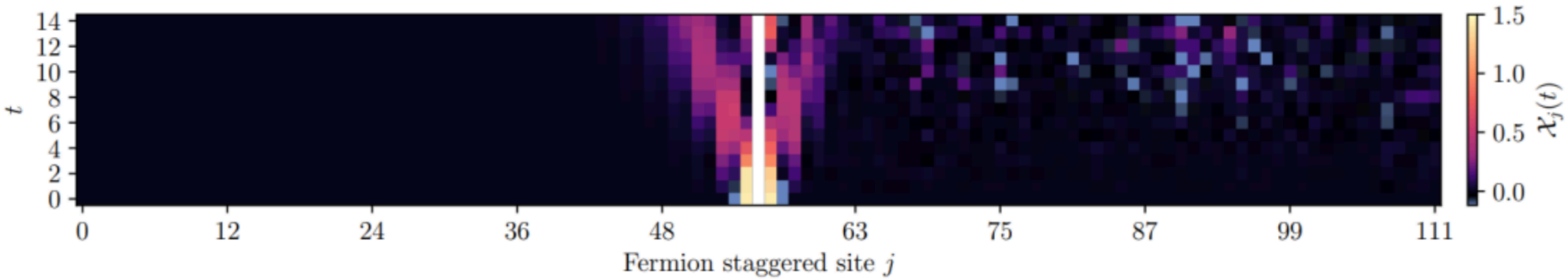
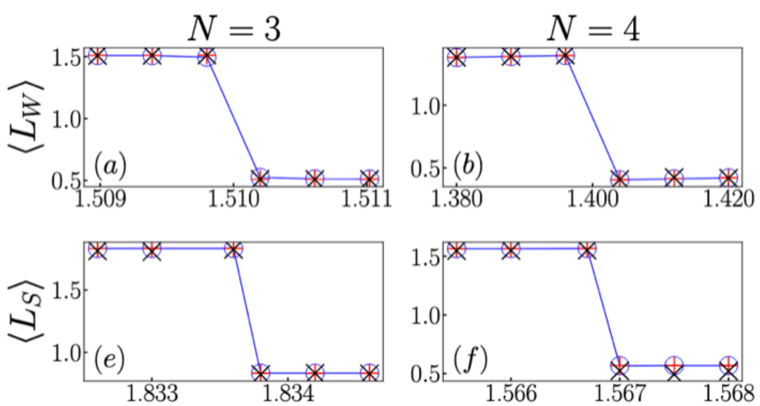
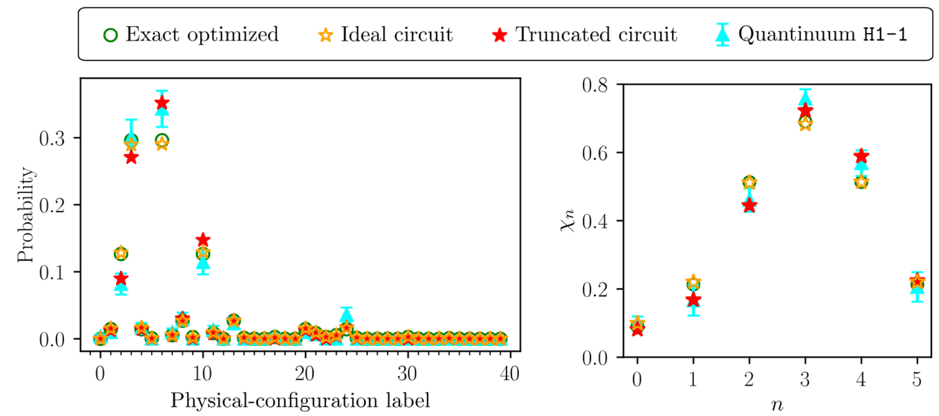
Abelian LGT in (1+1)D with dynamical matter

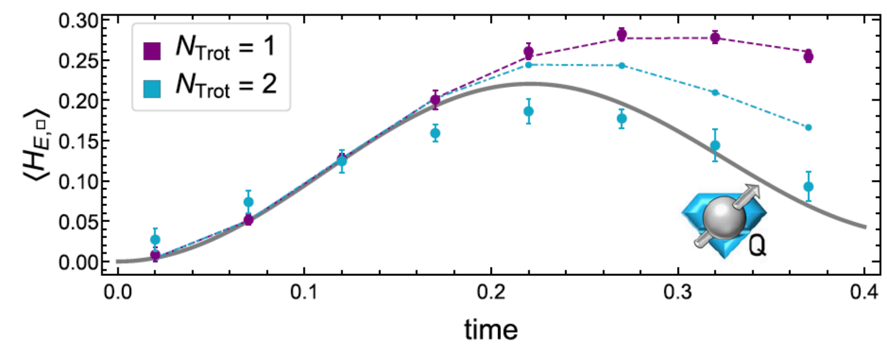




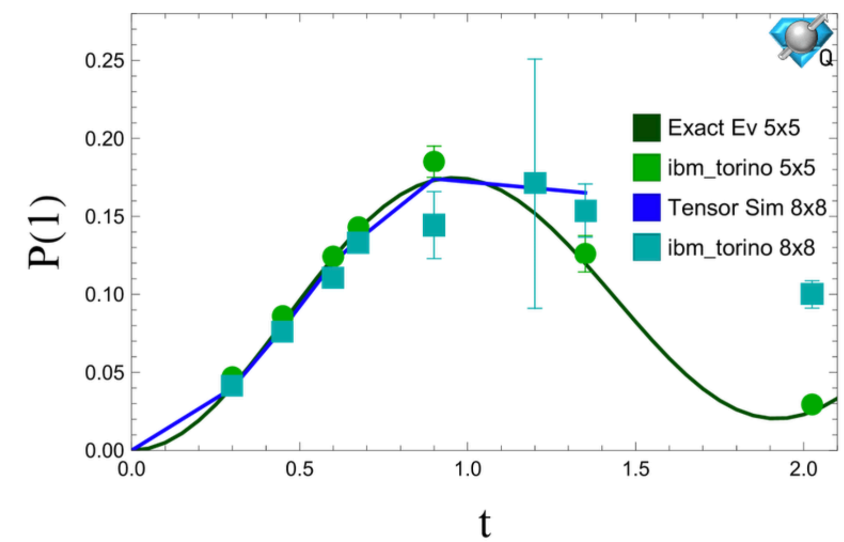


Pure gauge non-Abelian theory

Quasi-1D

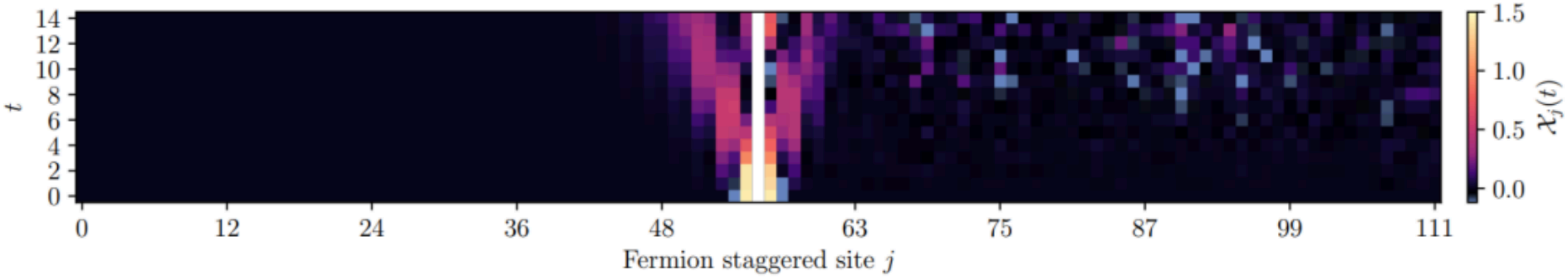


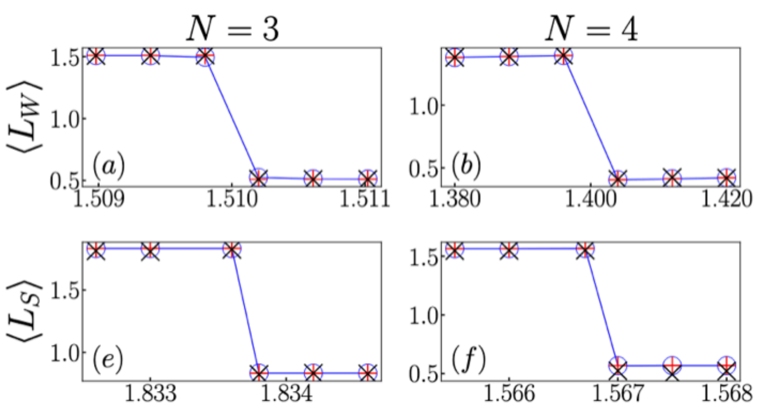
2D

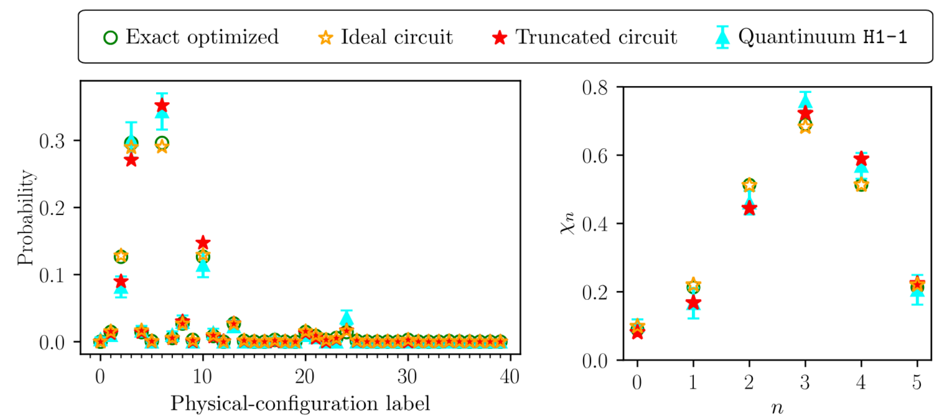


Motivation

Abelian LGT in (1+1)D with dynamical matter







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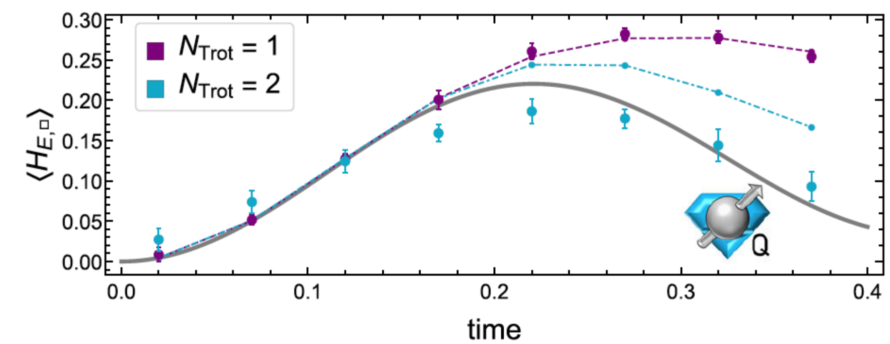

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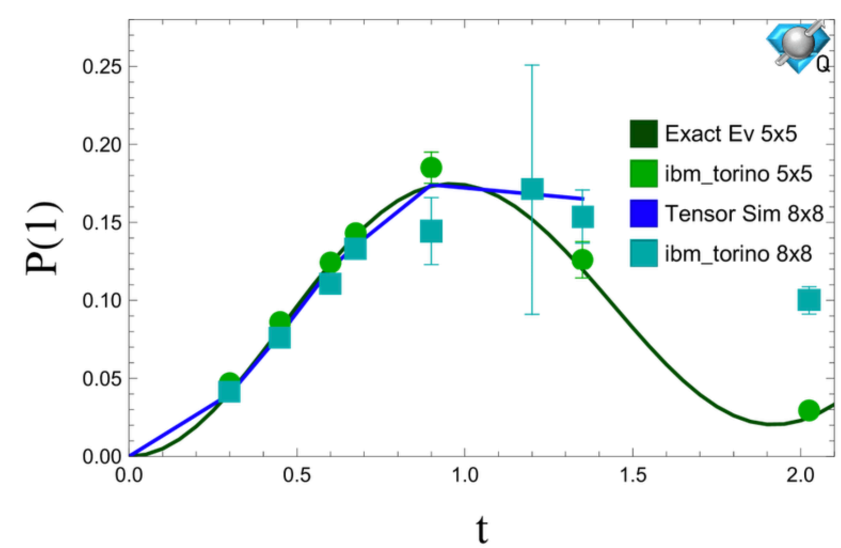


QED in (2+1)D with dynamical matter

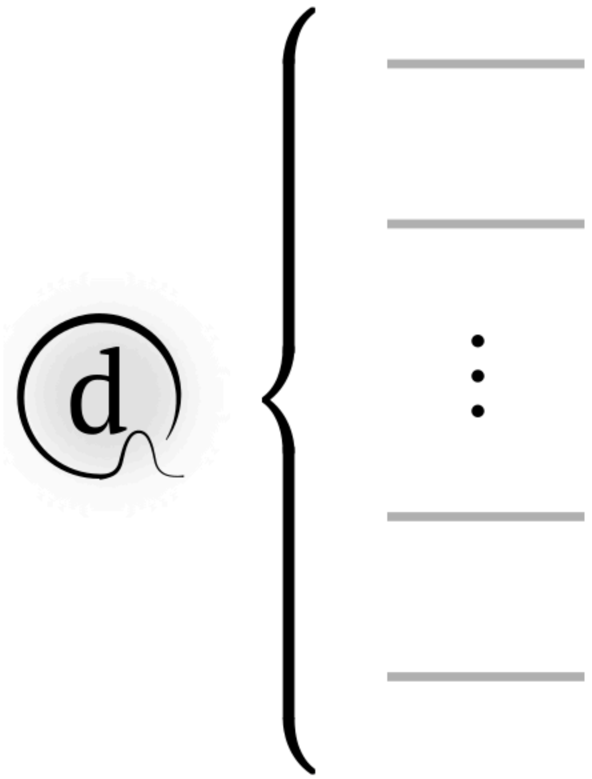
arXiv:2310.12110

Simulating 2D lattice gauge theories on a qudit quantum computer

Michael Meth,[1] Jan F. Haase,[2,3,4] Jinglei Zhang,[2,3] Claire Edmunds,[1] Lukas Postler,[1] Andrew J. Jena,[2,3] Alex Steiner,[1] Luca Dellantonio,[2,3,5] Rainer Blatt,[1,6,7] Peter Zoller,[8,6] Thomas Monz,[1,7] Philipp Schindler,[1] Christine Muschik*,[2,3,9] and Martin Ringbauer*[1]

Motivation

- **How do we describe systems beyond 1D?**
  Magnetic field effects, gauge fields not trivial anymore
- **How to best utilise quantum resources?**
  Hardware-efficient, short-depth circuit



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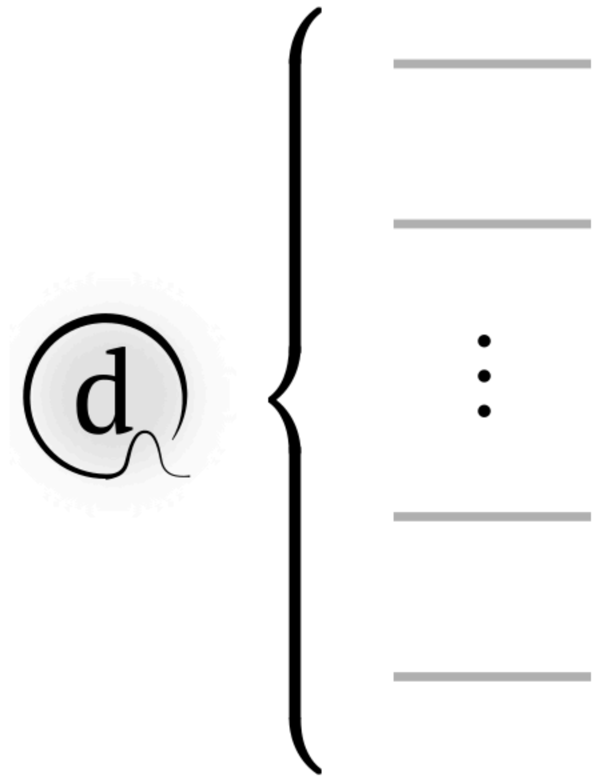
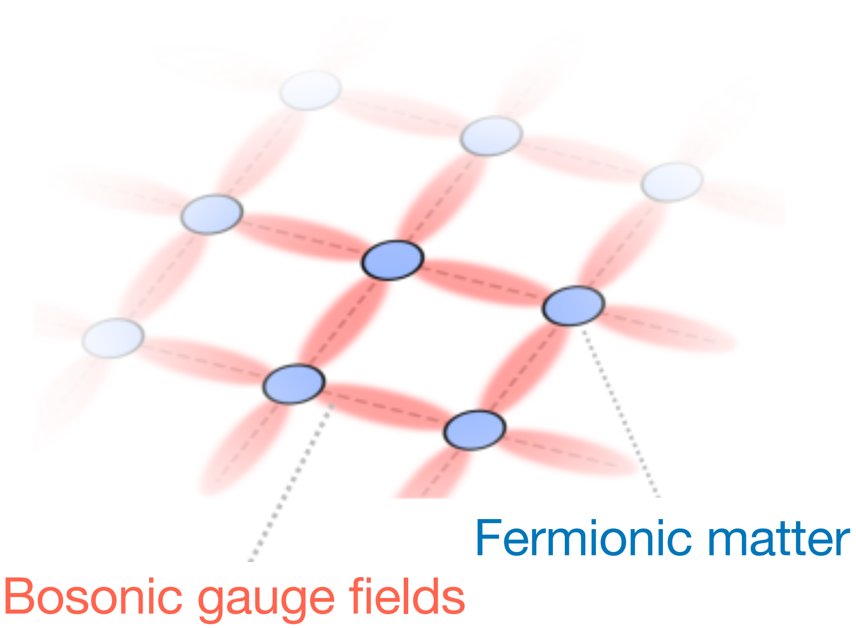


Qudits for LGT simulation

J. Zhang et al., Quantum 7, 1148 (2023)
E. J. Gustafson and H. Lamm, arXiv:2301.10207
T. V. Zache et al., Quantum 7, 1140 (2023)
G. Calajò et al., arXiv:2402.07987
P. P. Popov et al., PRR 6, 013202 (2024)

The model: 2D-QED



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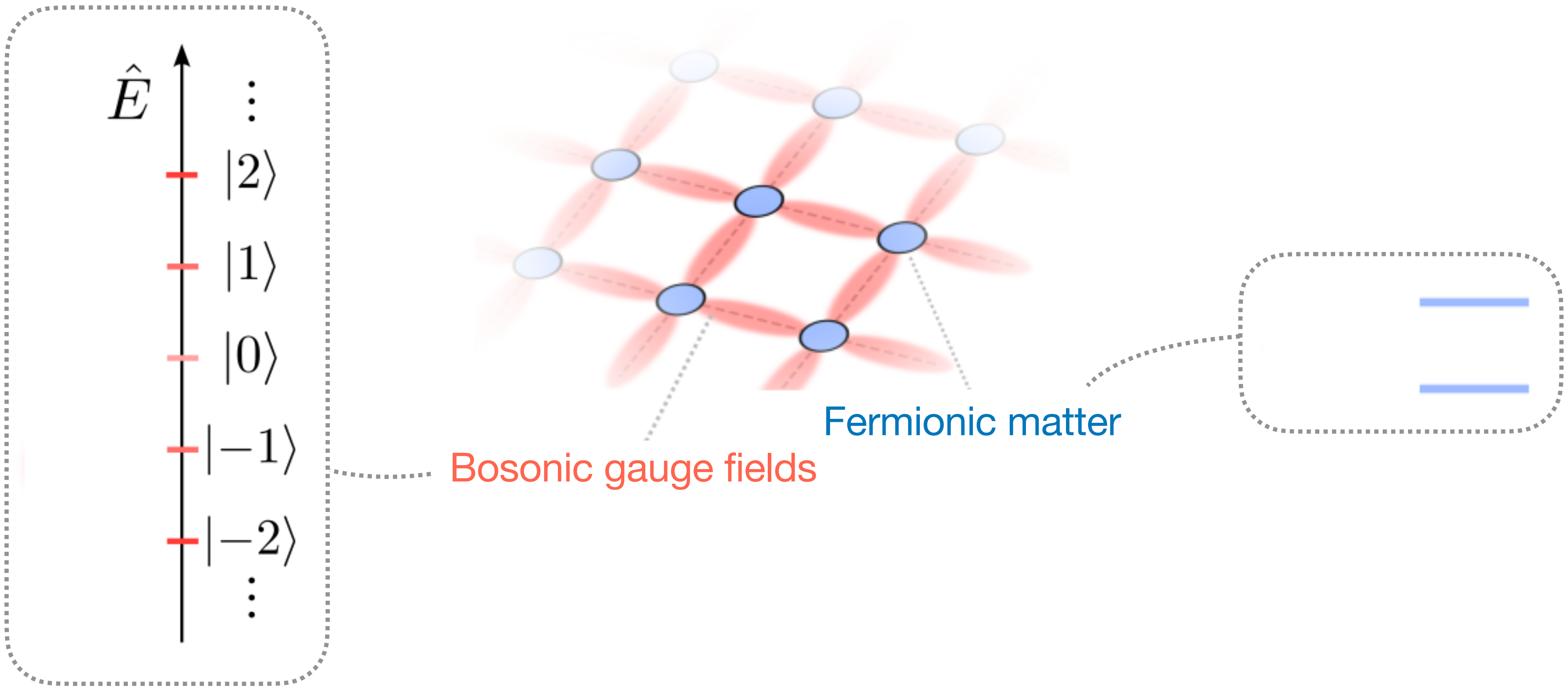

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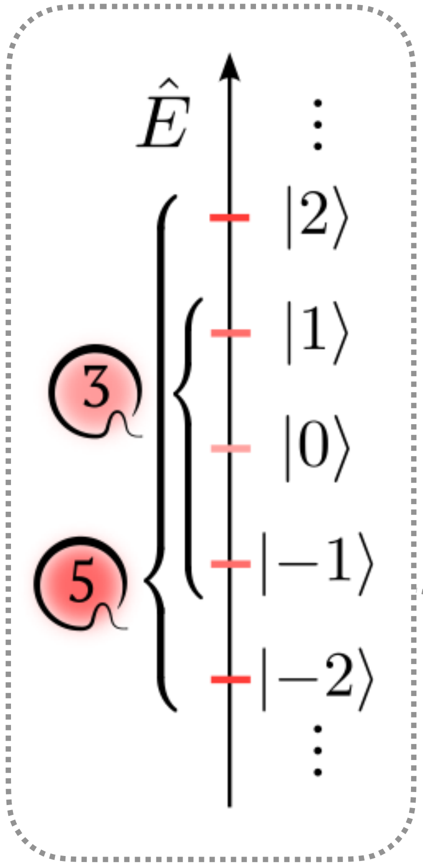
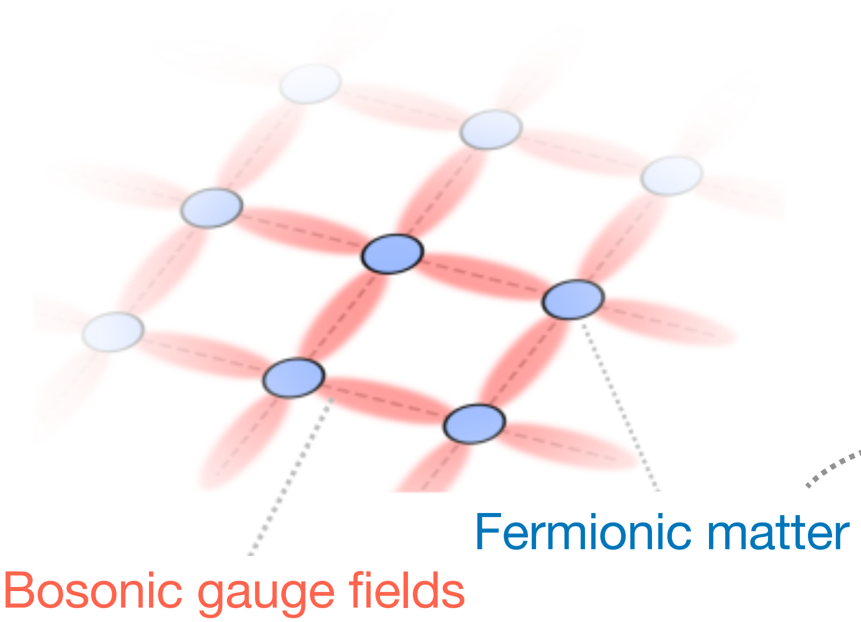


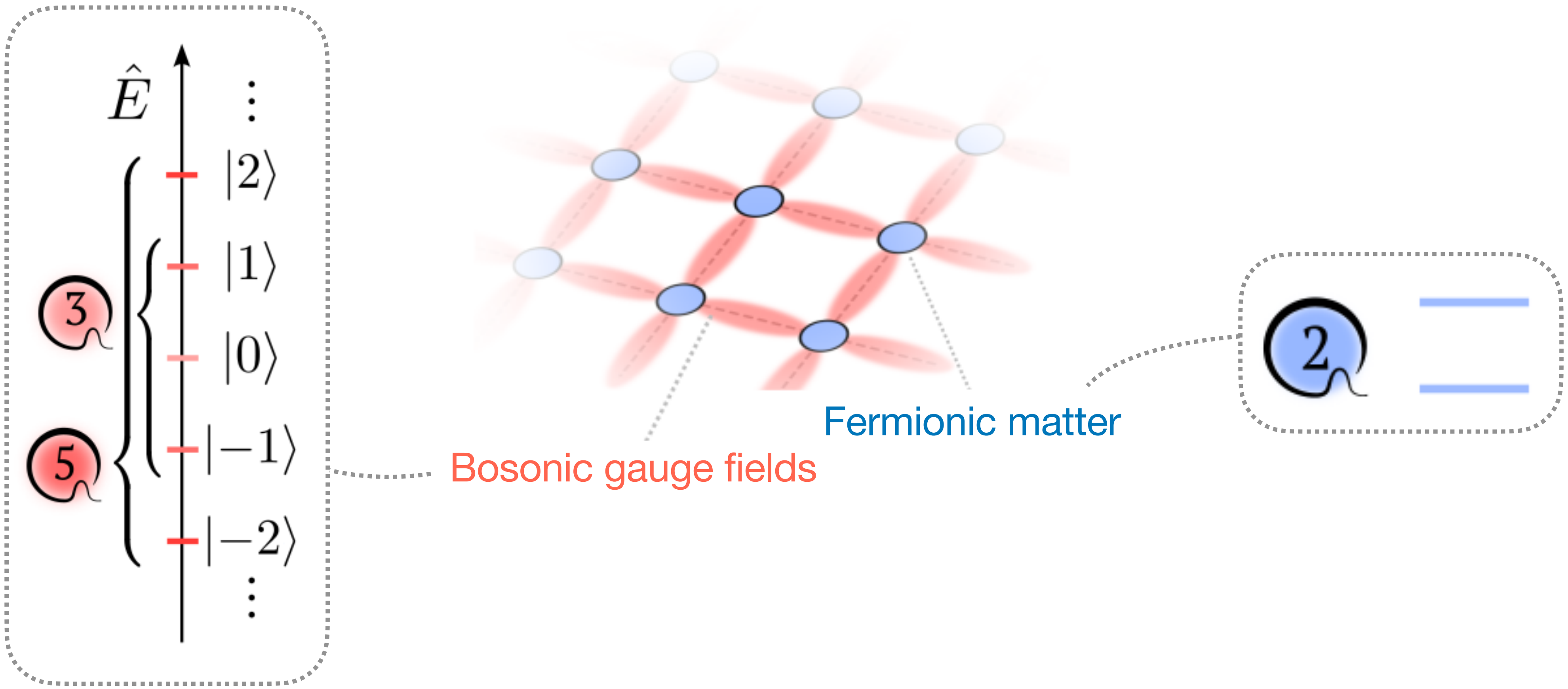


Use Jordan-Wigner transformation to map to two-level qubit

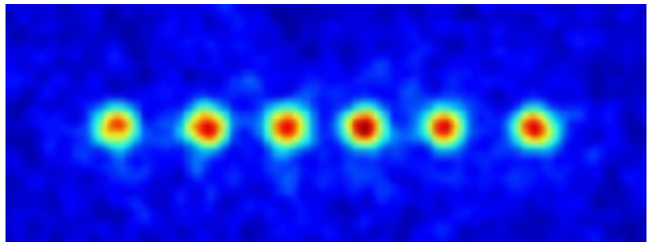
- Approximate U(1) with $\mathbb{Z}_{2L+1}$
- Truncate to $d = 2l + 1$ relevant states to be included in the simulation, we choose $L = l + 1$
- Map the gauge field onto a d-level qudit

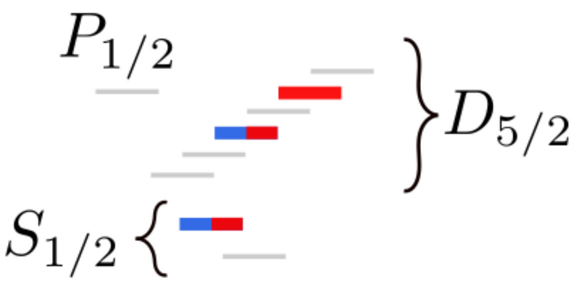
D. Paulson et al., PRX Quantum 2, 030334 (2021)

The model: 2D-QED



Experimental setup





$^{40}Ca^{+}$ level scheme

The model: 2D-QED

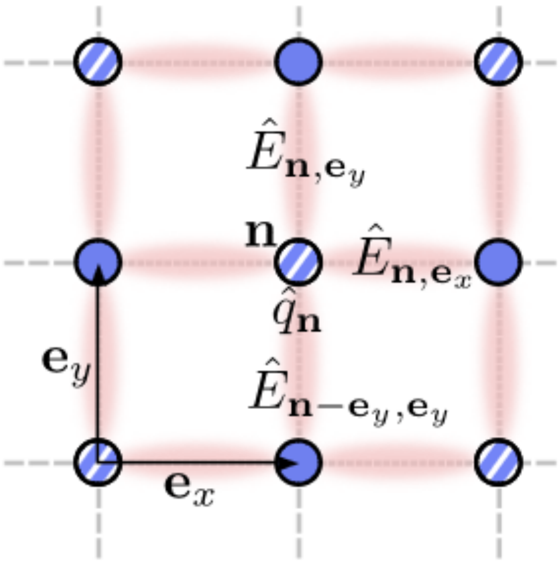
$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2}\hat{H}_B + m\hat{H}_m + \Omega \hat{H}_k$$

$$\hat{H}_E = \frac{1}{2}\sum_{\mathbf{n}}\left(\hat{E}^2_{\mathbf{n},\mathbf{e}_x} + \hat{E}^2_{\mathbf{n},\mathbf{e}_y}\right),$$

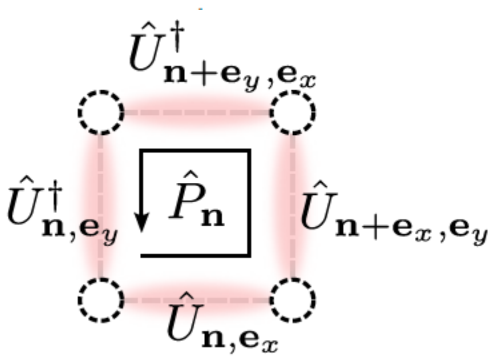
$$\hat{H}_B = -\frac{1}{2}\sum_{\mathbf{n}}\left(\hat{P}_{\mathbf{n}} + \hat{P}^\dagger_{\mathbf{n}}\right),$$

$$\hat{H}_m = \sum_{\mathbf{n}}(-1)^{n_x+n_y}\hat{\phi}^\dagger_{\mathbf{n}}\hat{\phi}_{\mathbf{n}},$$

$$\hat{H}_k = \sum_{\mathbf{n}}\Big(\hat{\phi}_{\mathbf{n}}\hat{U}^\dagger_{\mathbf{n},\mathbf{e}_x}\hat{\phi}^\dagger_{\mathbf{n}+\mathbf{e}_x} + (-1)^{n_x}\hat{\phi}_{\mathbf{n}}\hat{U}^\dagger_{\mathbf{n},\mathbf{e}_y}\hat{\phi}^\dagger_{\mathbf{n}+\mathbf{e}_y} + \text{H.c.}\Big).$$



$$\hat{P}_{\mathbf{n}} = \hat{U}_{\mathbf{n},\mathbf{e}_x}\hat{U}_{\mathbf{n}+\mathbf{e}_x,\mathbf{e}_y}\hat{U}^\dagger_{\mathbf{n}+\mathbf{e}_y,\mathbf{e}_x}\hat{U}^\dagger_{\mathbf{n},\mathbf{e}_y}$$

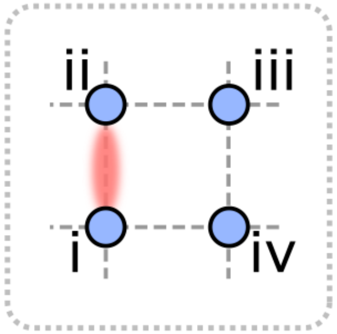


Gauss' law: $\hat{G}_{\mathbf{n}}|\Psi_{\text{phys}}\rangle = 0,$

$$\hat{G}_{\mathbf{n}} = \sum_{\mu}(\hat{E}_{\mathbf{n},\mathbf{e}_\mu} - \hat{E}_{\mathbf{n}-\mathbf{e}_\mu,\mathbf{e}_\mu}) - \hat{q}_{\mathbf{n}}$$

D. Paulson et al., PRX Quantum 2, 030334 (2021)

Hybrid qubit-qudit circuit



- Gauss Law can be used to eliminate three gauge field degrees of freedom

D. Paulson et al., PRX Quantum 2, 030334 (2021)

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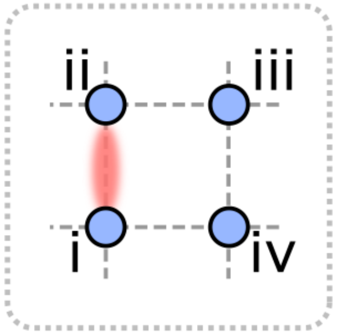
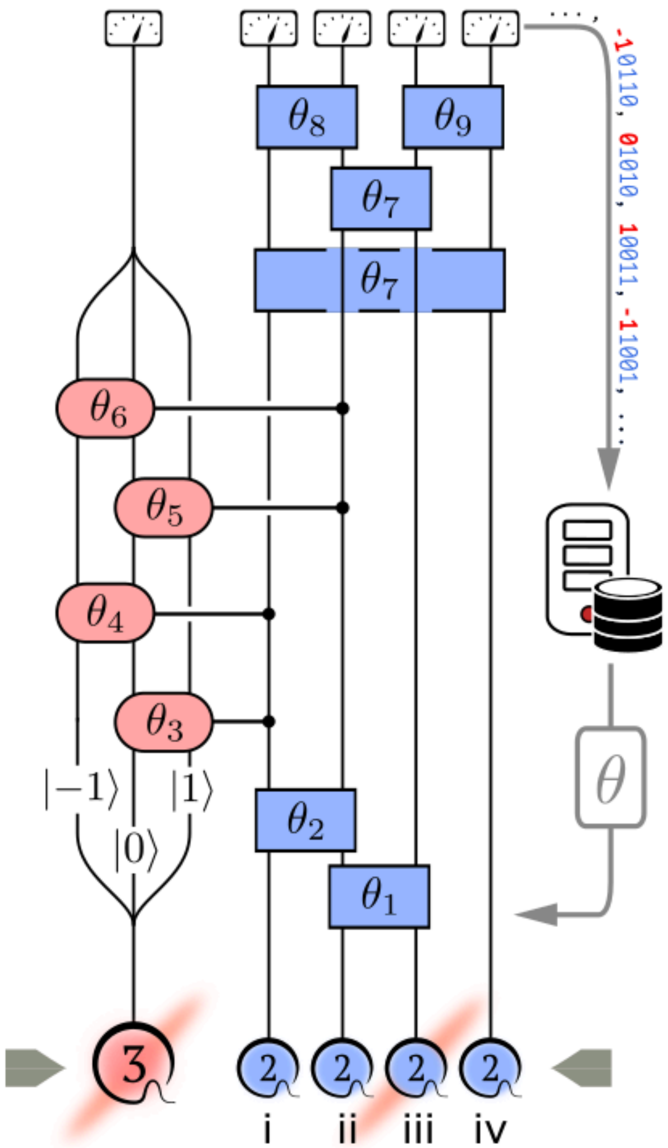


- Variational circuit inspired by the form of the Hamiltonian



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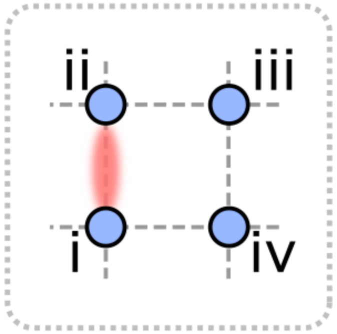
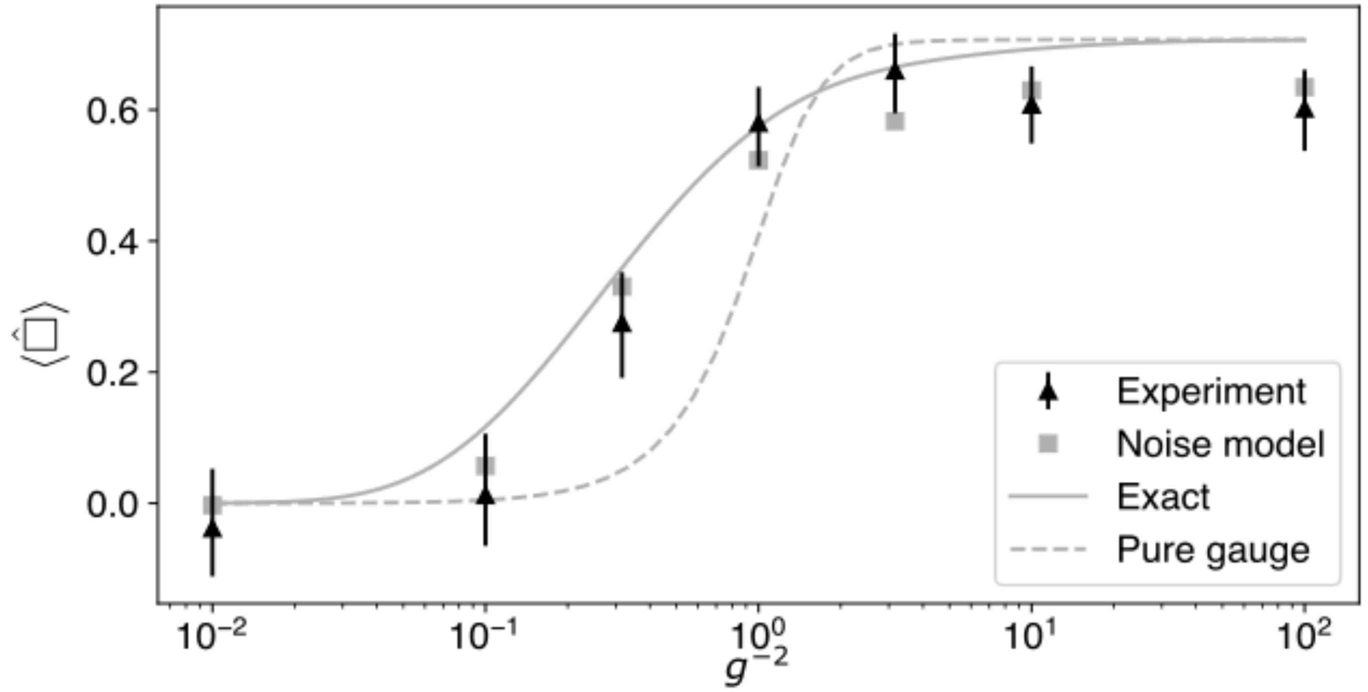
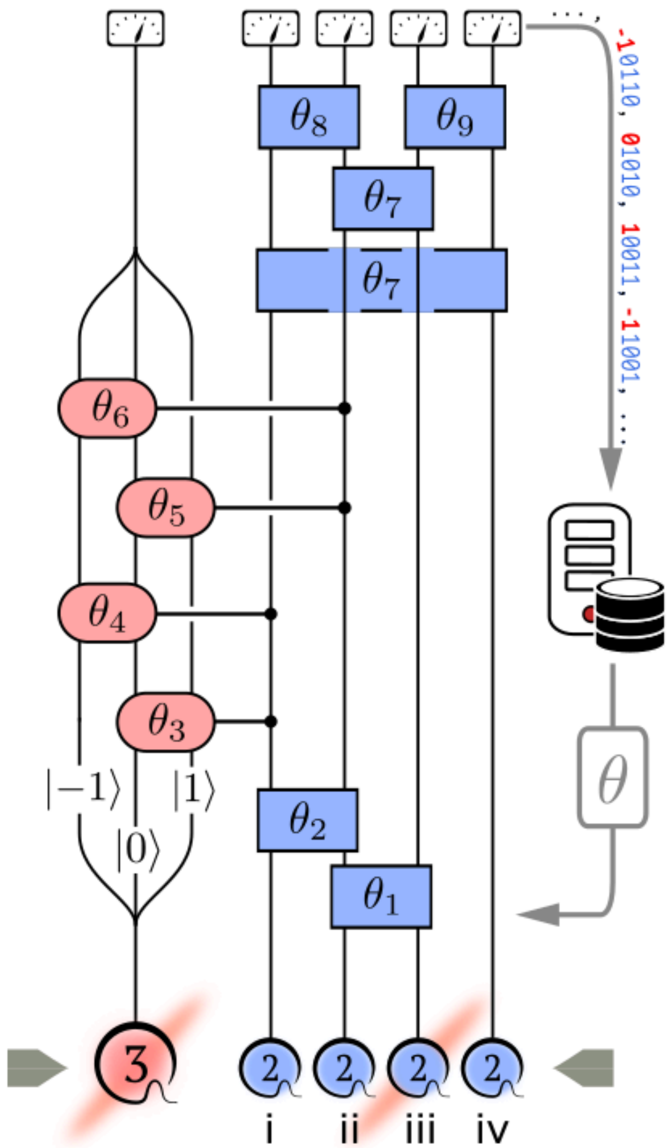


- Gauss Law can be used to eliminate three gauge field degrees of freedom

- Variational circuit inspired by the form of the Hamiltonian

- Study expectation value of $\hat{\Box} = -\frac{1}{V}\hat{H}_B$





D. Paulson et al., PRX Quantum 2, 030334 (2021)

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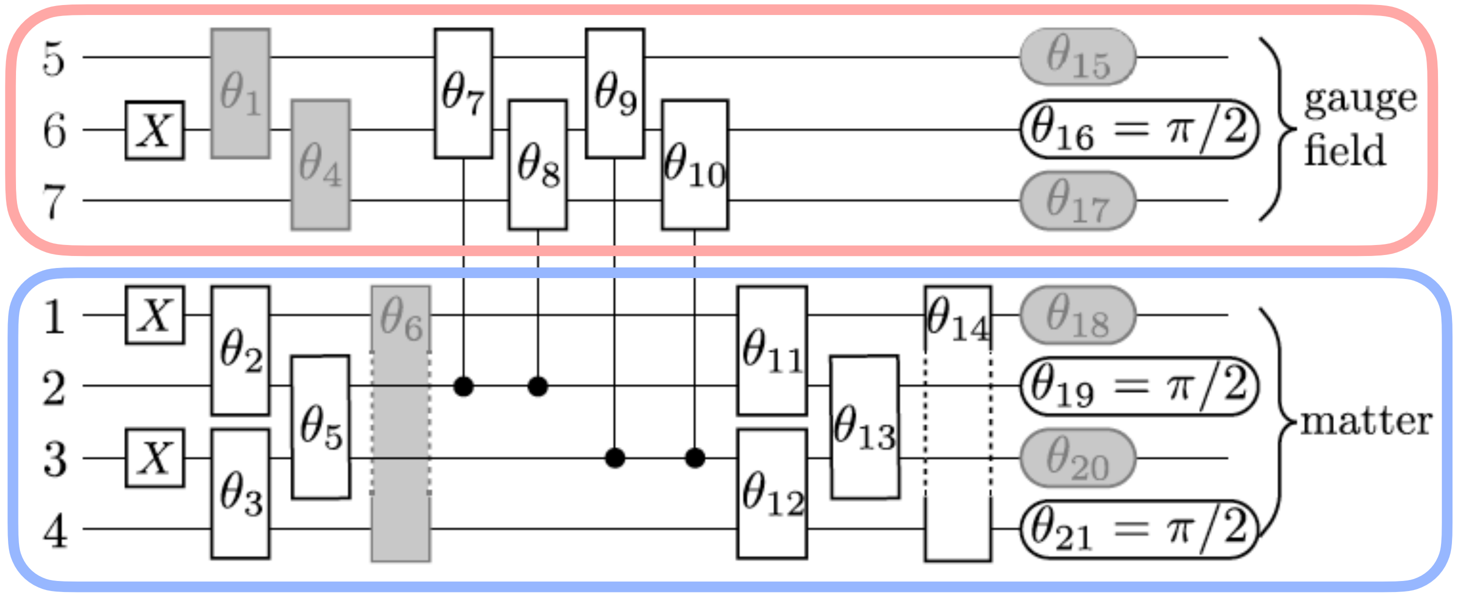
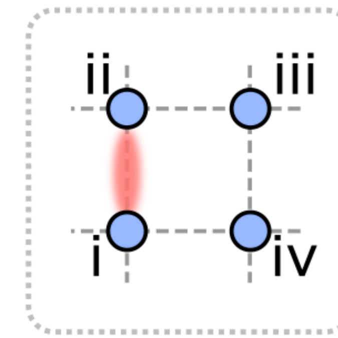
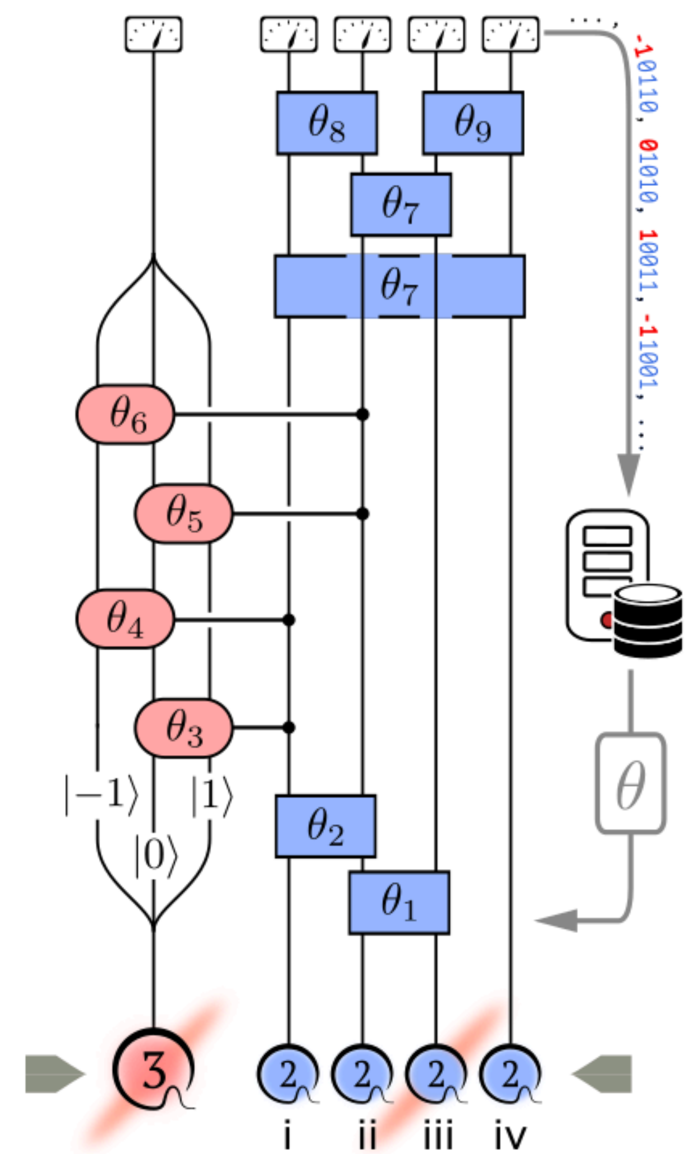




One-hot encoding that uses d qubits for a d-level system

$$|0,\ldots,010,\ldots,0\rangle$$



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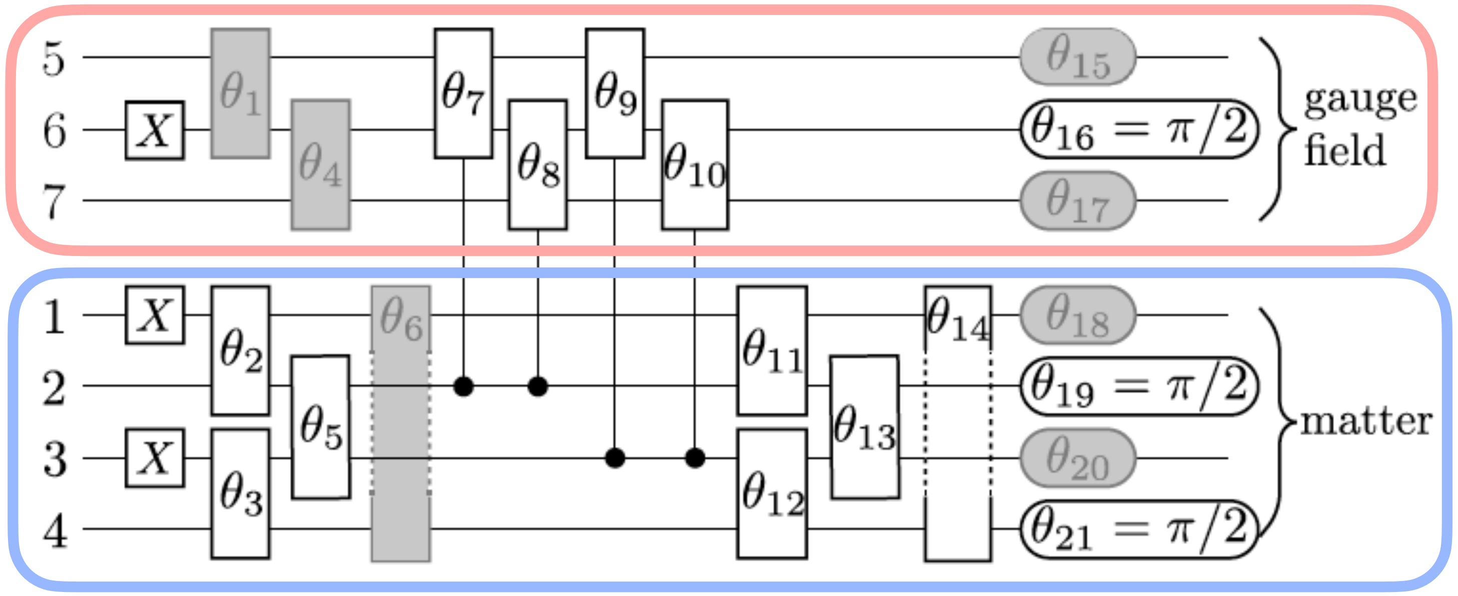

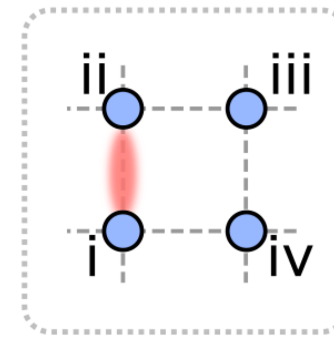

One-hot encoding that uses d qubits for a d-level system

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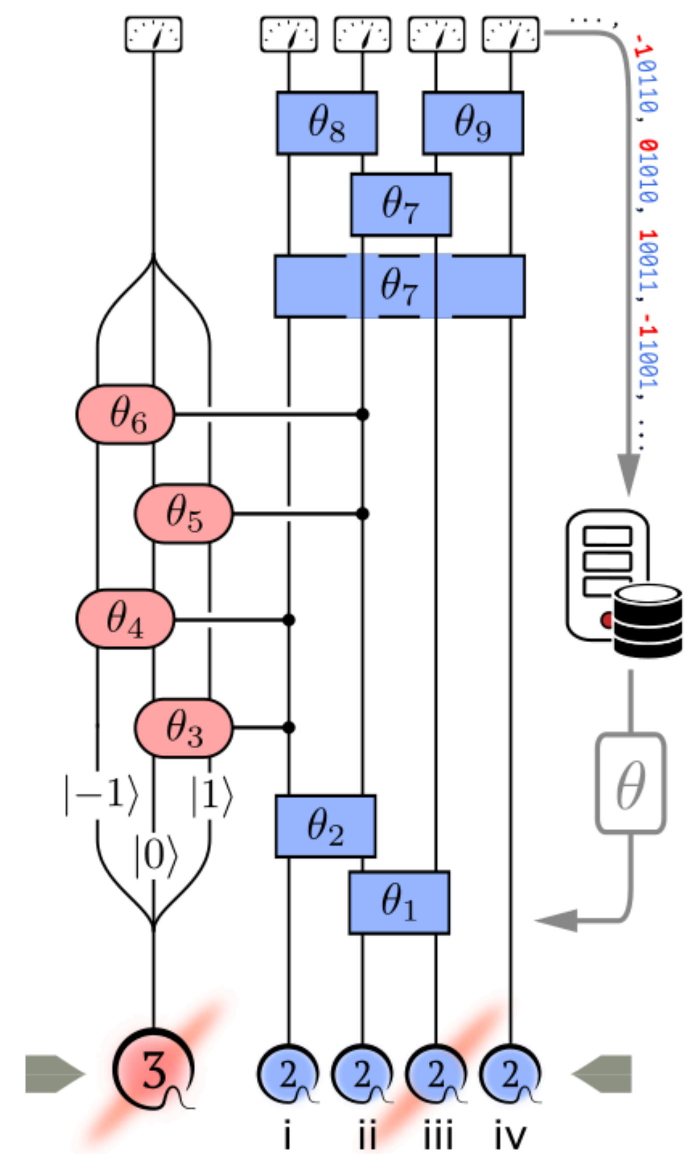
	Qudit encoding			Qubit encoding		
dimension d	3	5	7	3	5	7
register size	5	5	5	7	9	11
CNOT count	26	34	42	90	162	234
CNOT fidelity	99%					
approx. circ. fid.	77%	71%	66%	40%	20%	10%
CNOT fidelity	99.5%					
approx. circ. fid.	88%	84%	81%	64%	44%	31%



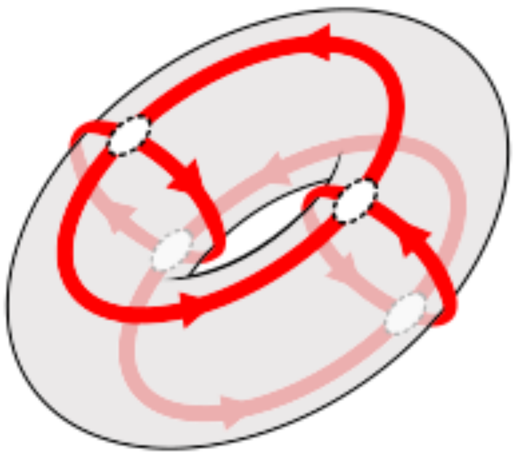
D. Paulson et al., PRX Quantum 2, 030334 (2021)

Can we see experimentally the improvements with refining the discretisation?

Pure gauge U(1) with PBC

Pure gauge system with periodic boundary condition

$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2} \hat{H}_B$$



J. F. Haase et al., Quantum 5, 393 (2021)

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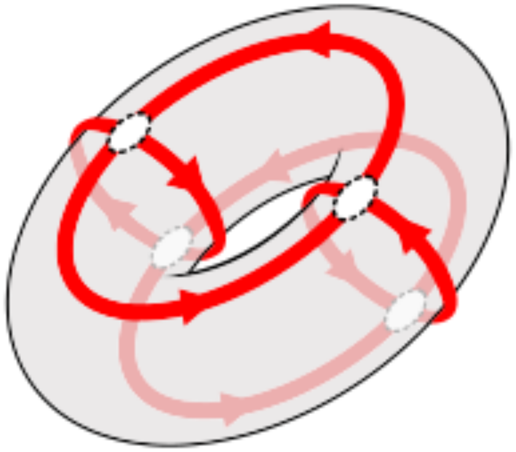


- 8 gauge fields
- 3 can be eliminated with Gauss Law
- 2 dof are zero for the ground state
- 3 independent rotors describe the system

J. F. Haase et al., Quantum 5, 393 (2021)

Electric and magnetic representations

$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2} \hat{H}_B$$

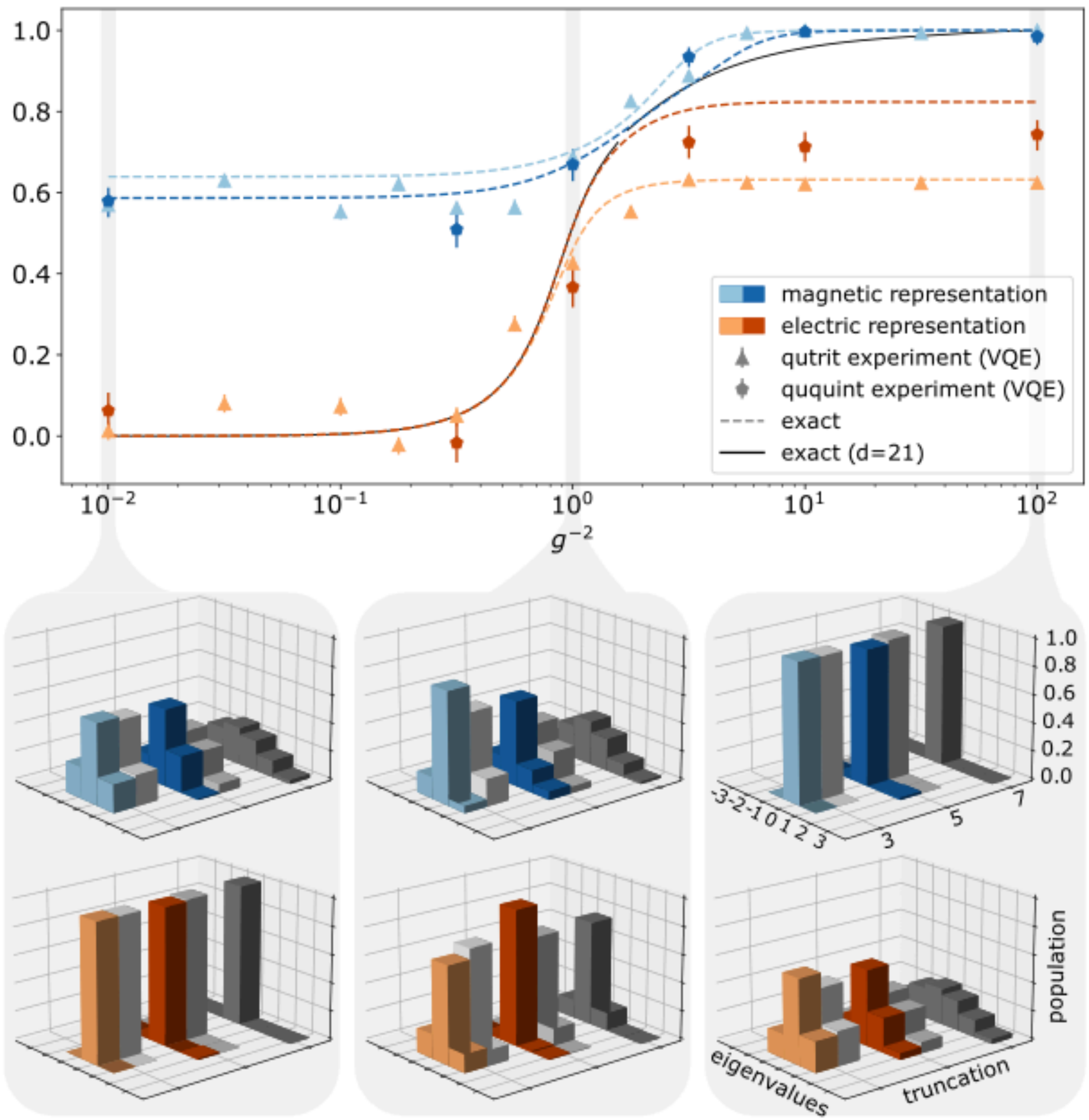
Two representation are used:

- electric basis
- magnetic basis

Each more efficient in different regimes of g^{-2}, but more truncation error in the opposite regime

J. F. Haase et al., Quantum 5, 393 (2021)

Hybrid qubit-qudit circuit



- ✦ Experimental VQE solution for d=3 and d=5
- ✦ By changing the qudit dimension we can study systematically truncation effects

Summary

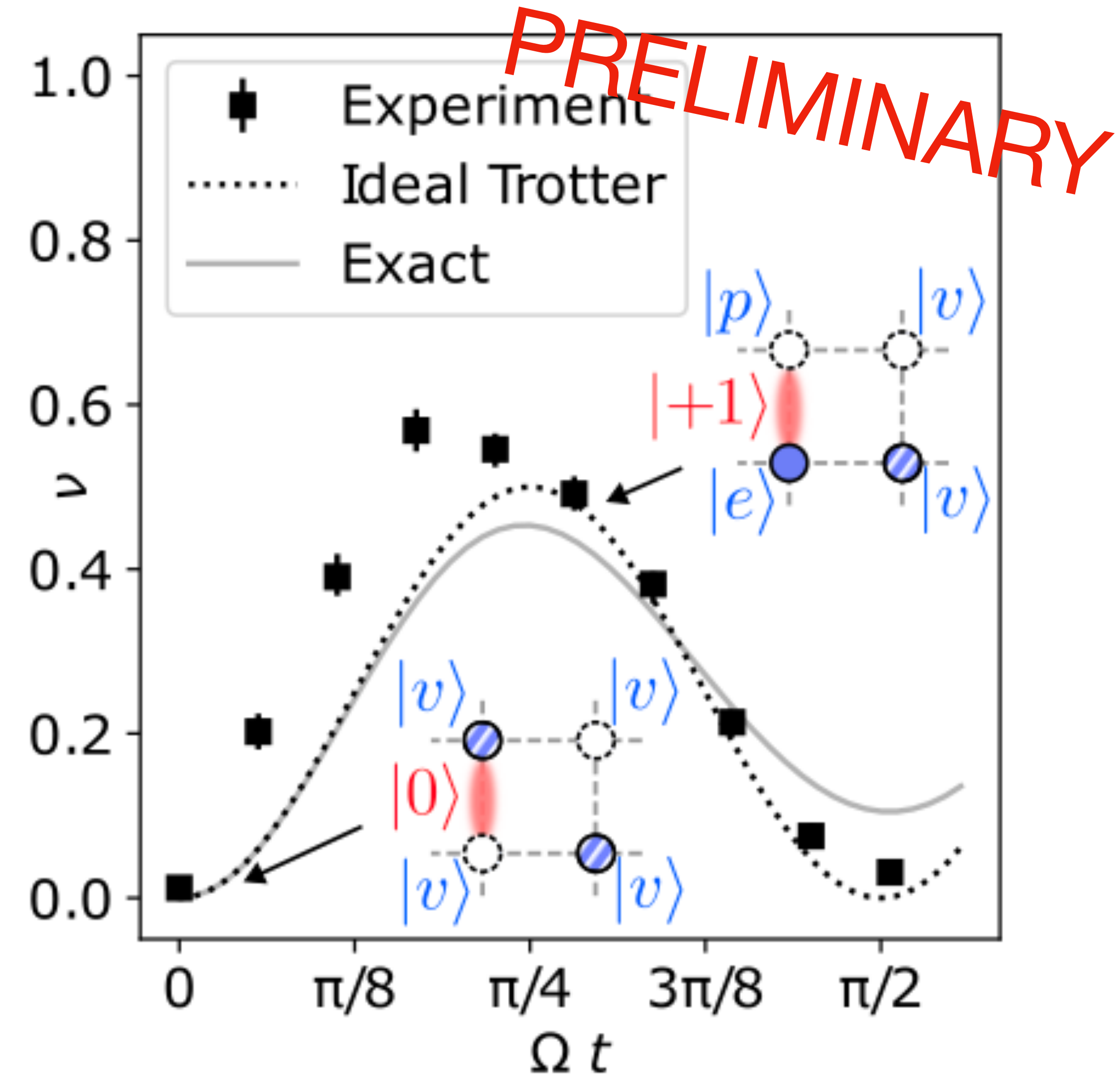
- Successful experimental VQE with qutrits and ququints with trapped ions
- Qudits can be a natural implementation of LGT degrees of freedom
- Flexibility in realising systems with mixed dimensions, and in adjusting truncation

Outlook

- Realize time evolution

Outlook

- Realize time evolution



Outlook

- Realize time evolution
- Study higher dimensions or non-Abelian theory
- Qudits also available on other platforms (Rydberg atoms, microwave photons, superconducting circuits, ultracold atoms…)

M Rambach, et al. PRL 10, 100402 (2021)
N. Goss et al. arXiv:2206.07216126
P.J. Low et al. arXiv:2306.03340
M. Subramanian et al. PRA 108, 062616 (2023)
V.V. Sivak et al. Nature 616 50–55 (2023)
P. Liu et al., PRX 13, 021028 (2023)
V. Tripath et al., arXiv:2407.04893
…

- Extension of e.g. error mitigation techniques to qudit systems, efficient measurement protocols,…
- Applications of qudit to other field theory, quantum chemistry, etc.

H. de Guise et al. PRA 97 022328 (2018)
Y. Wang et al. Front Phys (2020)
M. A. Yurtalan PRL 125, 180504 (2020)
P. Roy et al. arXiv:2307.10095
…

Outlook

Thank you for your attention!

universität innsbruck

Michael Meth

Martin Ringbauer

TQT Transformative Quantum Technologies

CANADA FIRST RESEARCH EXCELLENCE FUND

NSERC CRSNG

CIFAR

European Commission | HORIZON EUROPE 2021-2027

FWF Austrian Science Fund

IQI INFORMATION QUALITY INSTITUTE GMBH

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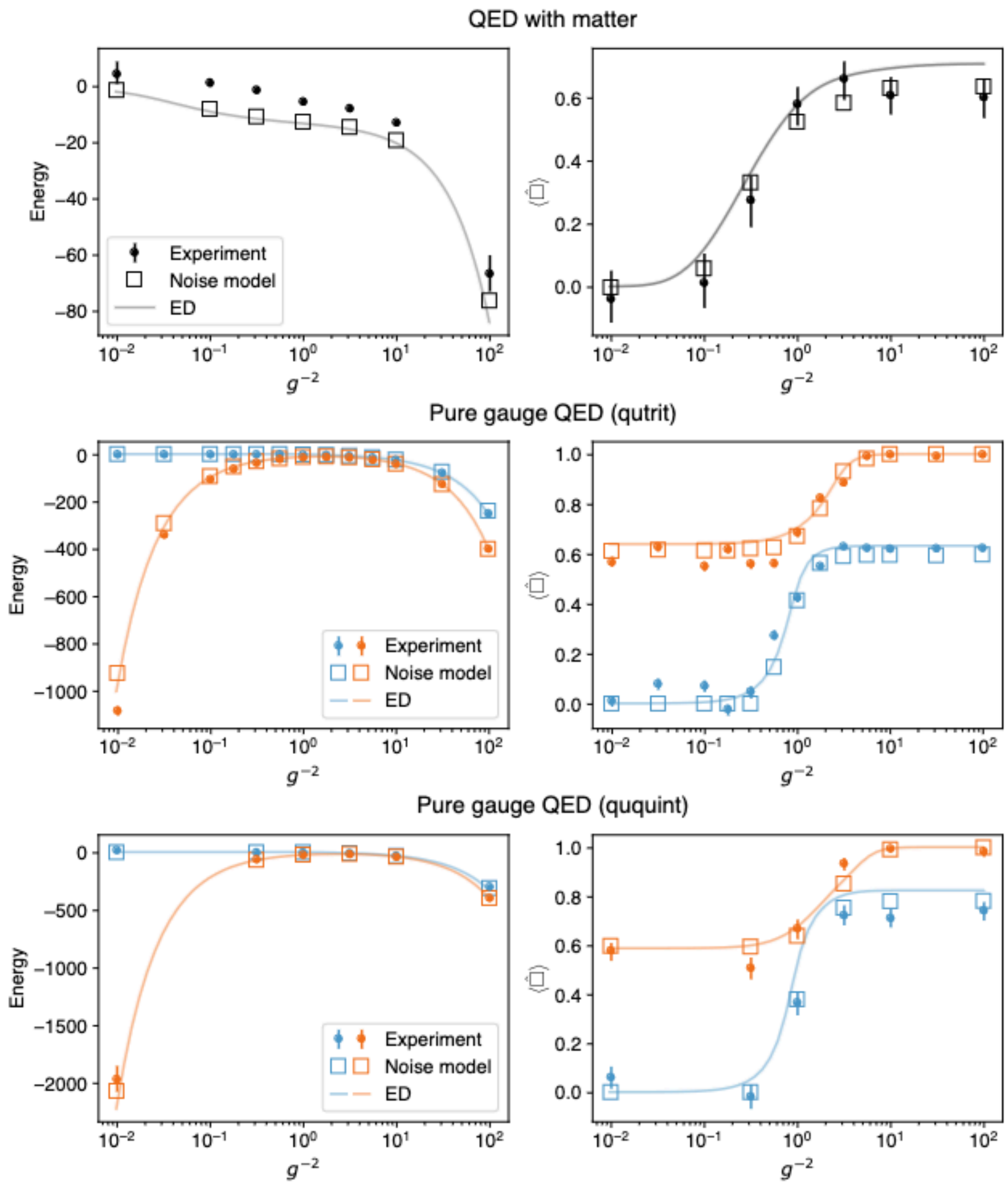
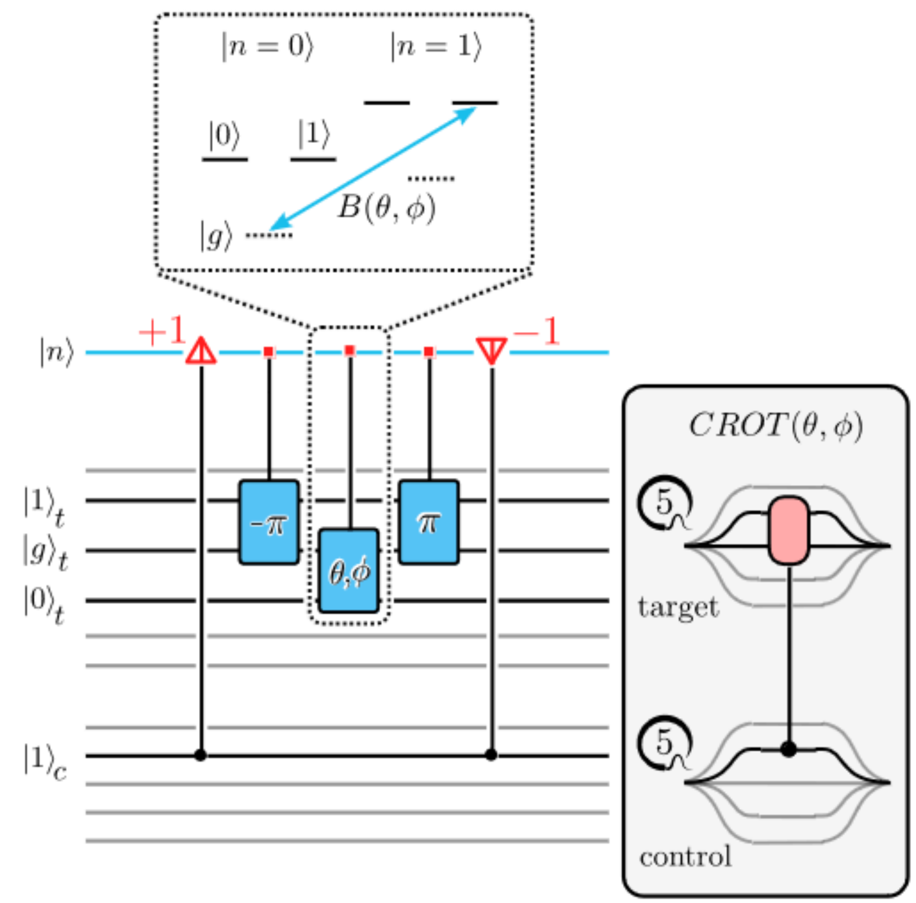
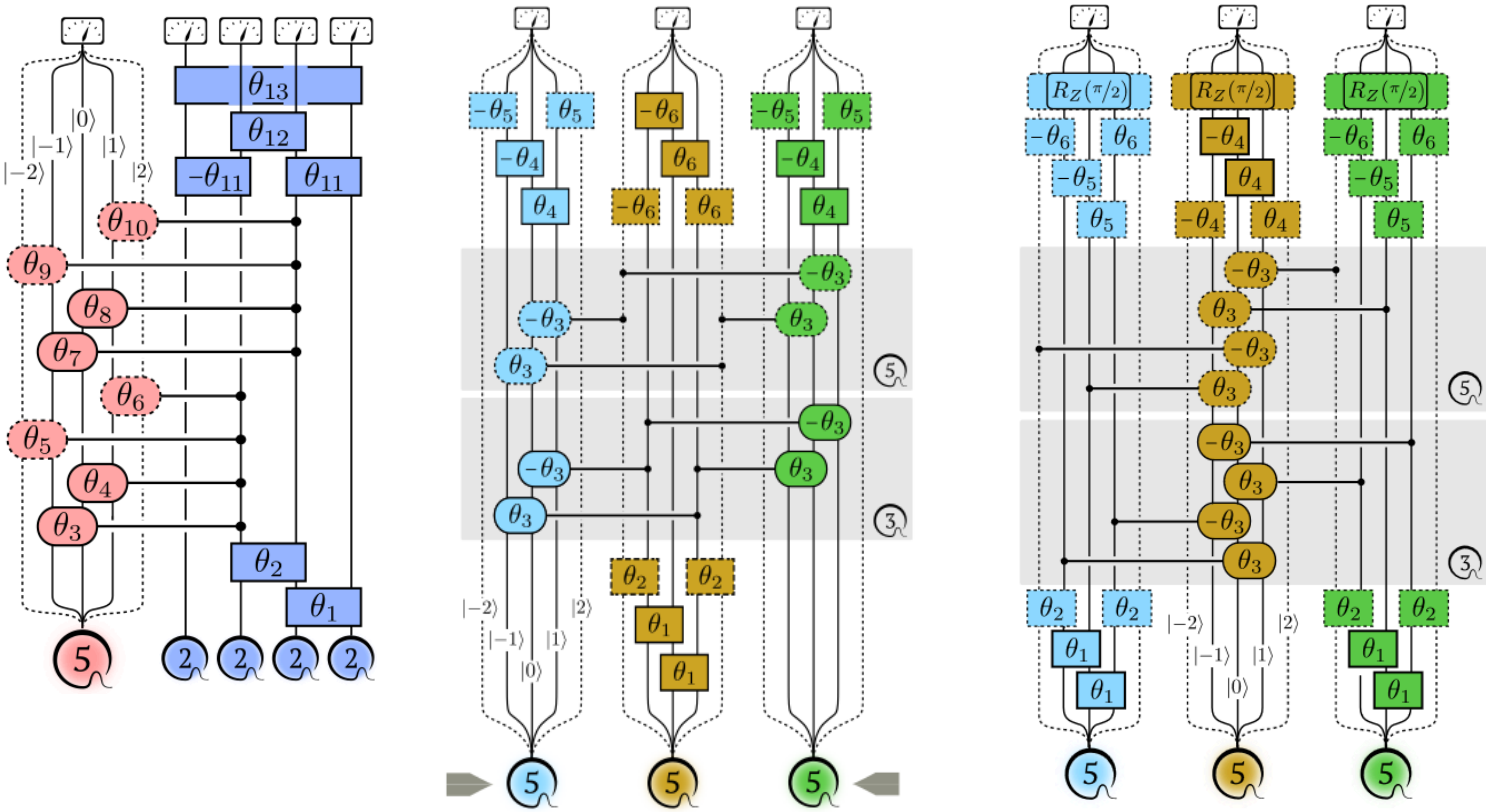
FFG Promoting Innovation.



- Amplitude fluctuations: error in the variational parameter
- Phase error for qudit entangling gates:
  - Imperfect compensation of AC stark shifts on spectator states



$$\hat{H}_j = i\eta\Omega_j \left(\hat{a}^\dagger \hat{\sigma}_j^+ + \hat{a}\hat{\sigma}_j^-\right)^{|g\rangle \longleftrightarrow |k\rangle}$$



$$\hat{H}^{(\mathrm{e})} = g^2 \hat{H}_E^{(\mathrm{e})} + \frac{1}{g^2} \hat{H}_B^{(\mathrm{e})},$$

$$\hat{H}_E^{(\mathrm{e})} = 2\left[\hat{R}_1^2 + \hat{R}_2^2 + \hat{R}_3^2 - \hat{R}_2\left(\hat{R}_1 + \hat{R}_3\right)\right],$$

$$\hat{H}_B^{(\mathrm{e})} = -\frac{1}{2}\left(\hat{P}_1 + \hat{P}_2 + \hat{P}_3 + \hat{P}_1\hat{P}_2\hat{P}_3 + \mathrm{H.c.}\right),$$