

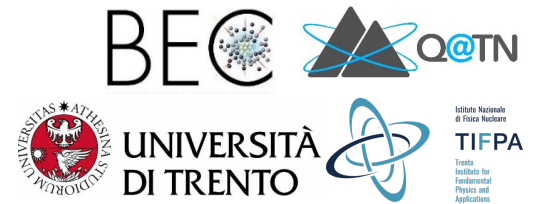


Advances in quantum simulation of lattice gauge theories: multiflavor models, fractional gauge fields, towards 2+1 D

QuantHEP, Munich August 5th, 2024

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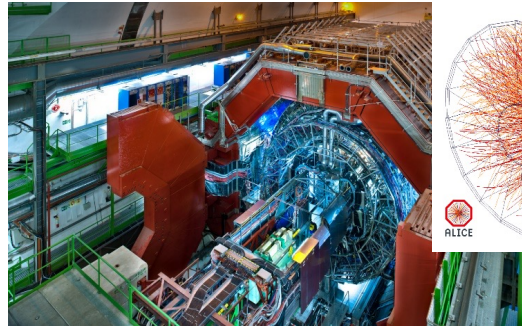


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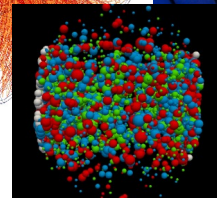
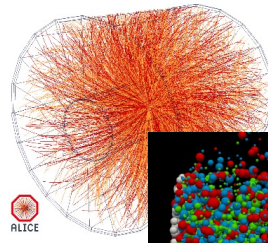


Lattice gauge theories – a corner stone of modern physics

- Subatomic physics

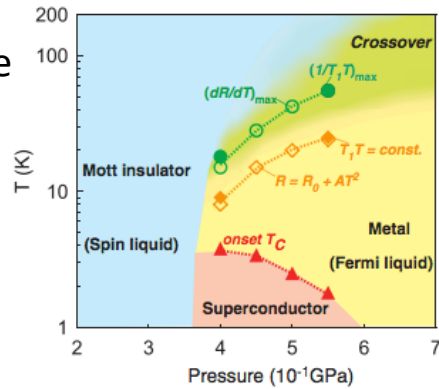


Alice detector @ CERN

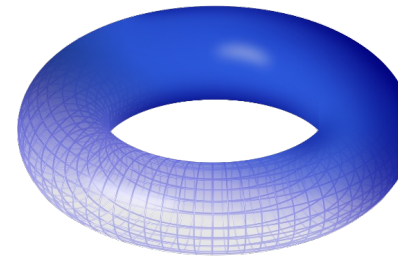


Leonardo Supercomputer @ CINECA

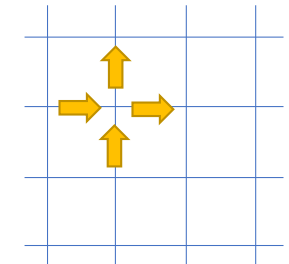
- Solid-state physics



- Quantum-information processing



Topological quantum computing



Lattice gauge theories – a corner stone of modern physics

- Subatomic physics

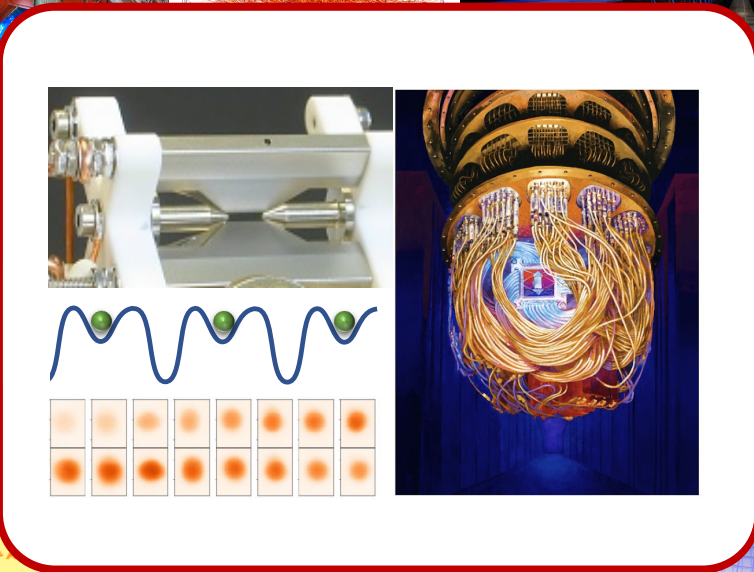
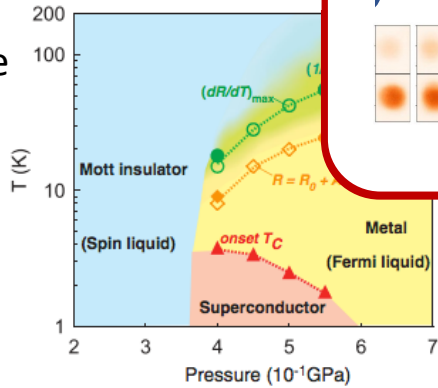


Alice detector @ CERN



Supercomputer @ CINECA

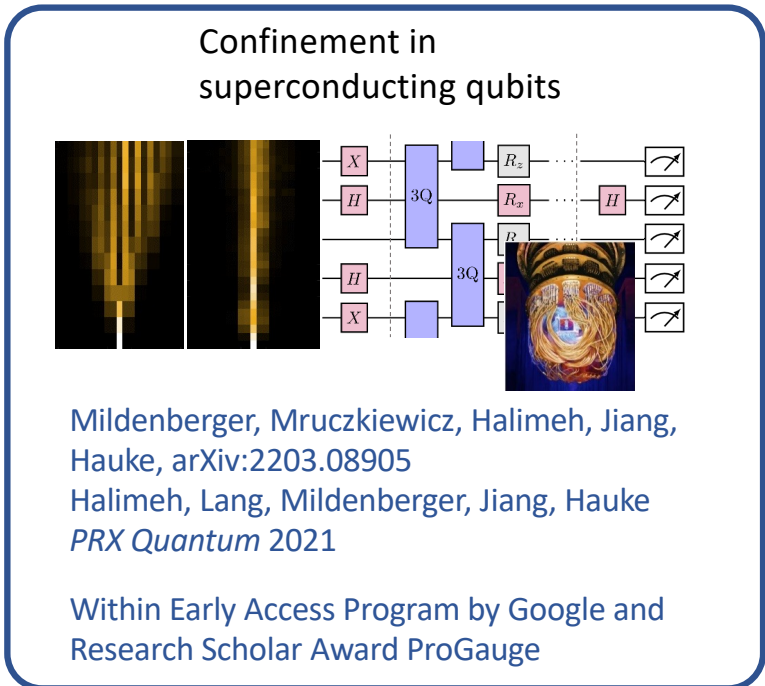
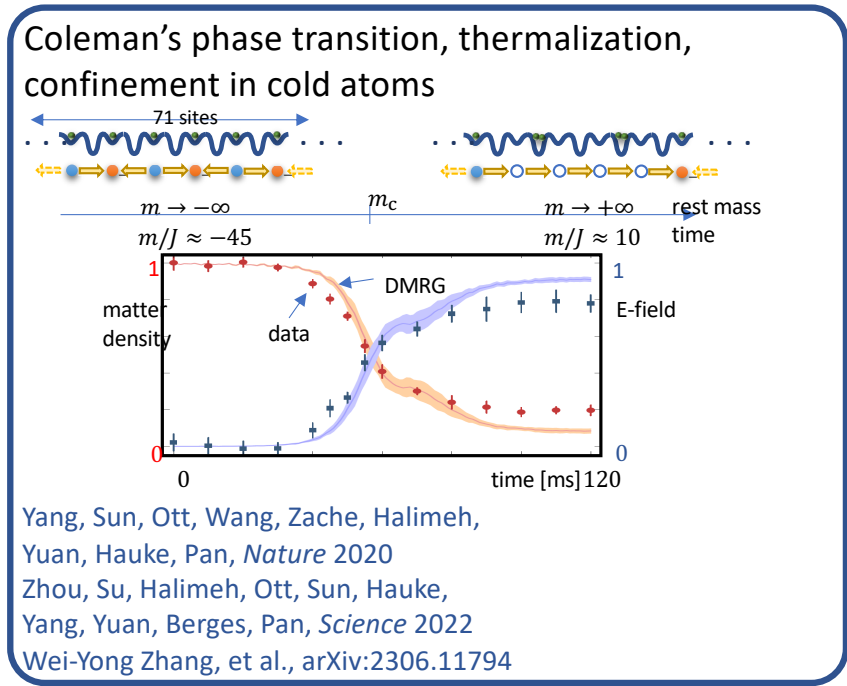
- Solid-state physics



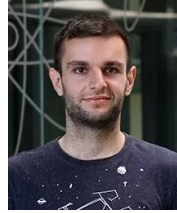
Topological quantum computing

Many strides forward in quantum simulation of LGTs have been made

Reviews, e.g., Zohar, Cirac, Reznik, *RoPP* 2015; Dalmonte, Montangero, *Contemp. Phys.* 2016; Banuls et al., *EPJD* 2020; Aidelburger et al., *Phil. Trans. R. Soc. A* 2022; Zohar, *Phil. Trans. R. Soc. A* 2022; Bauer, *PRX Quantum* 2023; Di Meglio et al., arXiv:2307.03236; Halimeh, Aidelburger, Grusdt, Hauke, Yang, arXiv:2310.12201



Nice, but new challenge: proceed beyond single-flavor Abelian 1+1D



Pavel Popov



Paolo Stornati



Valentin Kasper



Erez Zohar



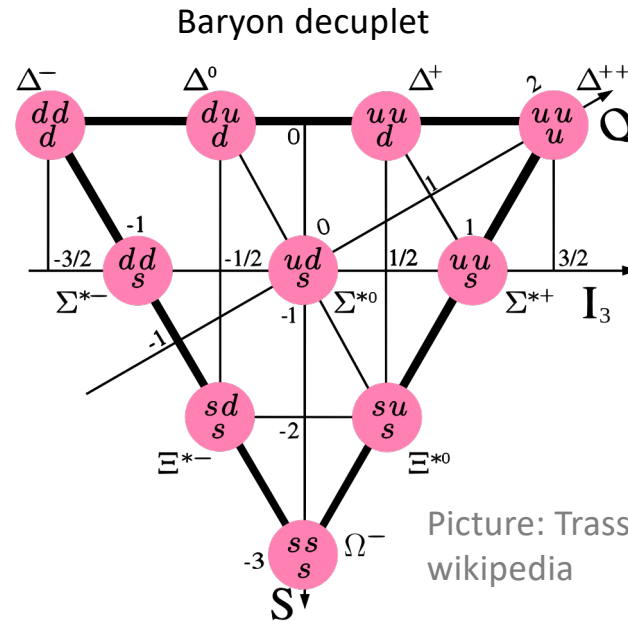
Maciej Lewenstein

Fractons in the multi-flavour Schwinger model

Pavel Popov, Valentin Kasper, Maciej Lewenstein, Erez Zohar,
Paolo Stornati, and Philipp Hauke, arXiv:2405.00745

Multiflavor models hosts rich physics

Isospin,
flavor symmetry

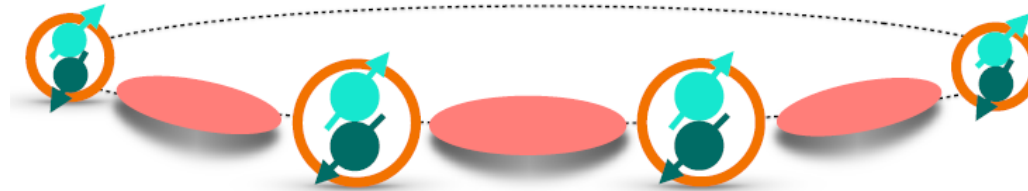


$N=3$ severe sign problem

Non-trivial extension, but easier than colour

In context of quantum simulation & tensor networks, e.g., Funcke, Hartung, Jansen, Kühn, Pleinert, PoS LATTICE2022 (2023) 020, Bañuls, Cichy, Cirac, Jansen, Kühn, Saito, arXiv:1611.00705

Multiflavor-flavor Schwinger model



Continuum

$$H_{\text{QED}} = \sum_{\sigma=1}^{N_f} \int dx \left[(\bar{\psi}_{\sigma}^{\dagger}(x) \gamma_0 i \gamma_1 (\partial_x + ie \hat{A}(x)) \bar{\psi}_{\sigma}(x) + \text{h.c.}) + m_{\sigma} \bar{\psi}_{\sigma}^{\dagger}(x) \gamma_0 \bar{\psi}_{\sigma}(x) + \frac{1}{2} \hat{E}(x)^2 \right]$$

Gauss' law $\nabla E(x) - e \sum_{\sigma} \bar{\psi}_{\sigma}^{\dagger}(x) \gamma_0 \bar{\psi}_{\sigma}(x) = 0$

Lattice

$$H = J \sum_{\sigma=1}^{N_f} \sum_x (\hat{\psi}_{\sigma,x}^{\dagger} \hat{S}_{x,x+1}^{+} \hat{\psi}_{\sigma,x+1} + \text{h.c.}) + m_{\sigma} \sum_x (-1)^x \hat{\psi}_{\sigma,x}^{\dagger} \hat{\psi}_{\sigma,x} + g^2 \sum_x (S_{x,x+1}^z)^2$$

$$\frac{S_{i,i+1}^z}{2} - \frac{S_{i-1,i}^z}{2} - e \sum_{\sigma} \hat{\psi}_{\sigma,x}^{\dagger} \hat{\psi}_{\sigma,x} - \frac{1+(-1)^i}{2} = 0$$

(How much of the physics does the lattice model retain?)
(cutoff Λ , lattice spacing a)

Multiflavor Schwinger model as prototype model for topological gauge-theory phenomena

Shifman, Smilga, PRD 1994

SuSy Yang-Mills has non-zero gluino condensate
(λ = Majorana field, superpartner of gluon)


$$\langle \bar{\lambda} \lambda \rangle \neq 0$$

But: path-integral predicts (at small mass)

$$\langle \bar{\lambda} \lambda \rangle = -\partial_m \ln Z \Big|_{m=0}$$

topological sectors
common lore:
 ν integer

$$\text{with } Z = \sum_{\nu} Z_{\nu} \\ Z_{\nu} = z_{\nu} m^{\nu N_c}$$

thus $\langle \bar{\lambda} \lambda \rangle = 0$ 

How to reconcile? \rightarrow need to admit for presence of **fractional topological sectors**

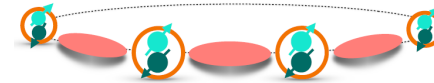
$$\langle \bar{\lambda} \lambda \rangle = -\lim_{m \rightarrow 0} \frac{Z_{1/N_c}}{Z_0}$$

Can we probe such fractons in a simpler theory (and on a quantum simulator)?

Fractons in multiflavor Schwinger model

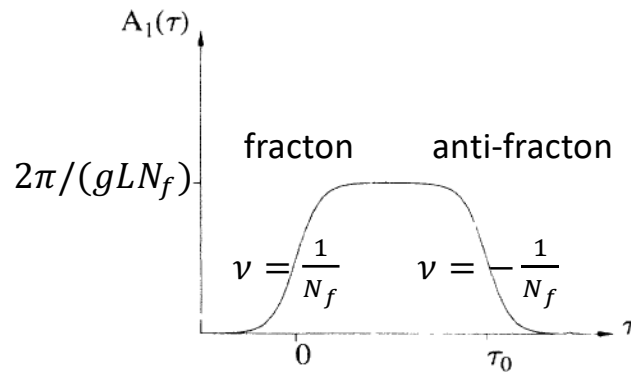
Shifman, Smilga, PRD 1994

Similarly, for $e^{ig \int A_1 dx} = \text{const}$, A_1 defined up to different windings



of windings of A define topological Pontryagin charge $\nu_2 = \frac{e}{4\pi} \int d^2x \epsilon_{\mu\nu} F_{\mu\nu}$

Existence of fractional windings is known,
but usually they are confined



Topological charge:

$$\nu = \frac{gL}{2\pi} \int_{-\infty}^{\infty} \dot{A}_1 d\tau = \frac{gL}{2\pi} [A_1(\infty) - A_1(-\infty)] = 0.$$

How to reveal them?

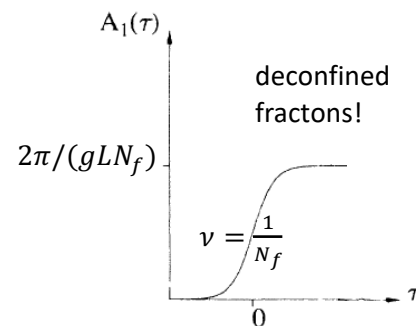
Fractional gauge fields through chiral condensate

Shifman, Smilga, PRD 1994

Path-integral solution for chiral condensate $\langle \bar{\psi}\psi \rangle = -\partial_m \ln Z \Big|_{m=0}$ $Z = \sum_{\nu} Z_{\nu}$ with $Z_{\nu} = z_{\nu} m^{2\nu}$

If ν only integer $\langle \bar{\psi}\psi \rangle \xrightarrow{m \rightarrow 0} -\frac{z_1 m}{z_0} \Big|_{m=0} = 0$
 (no individual fractional gauge-field configurations)

Conversely $\langle \bar{\psi}\psi \rangle \neq 0$ implies existence of $\nu = 1/2$ $(\langle \bar{\psi}\psi \rangle \xrightarrow{m \rightarrow 0} -\frac{z_{1/2}}{z_0})$



Topological charge:

$$\nu = \frac{gL}{2\pi} \int_{-\infty}^{\infty} \dot{A}_1 d\tau = \frac{gL}{2\pi} [A_1(\infty) - A_1(-\infty)] = \frac{1}{N_f}$$

Do these fractons become relevant?

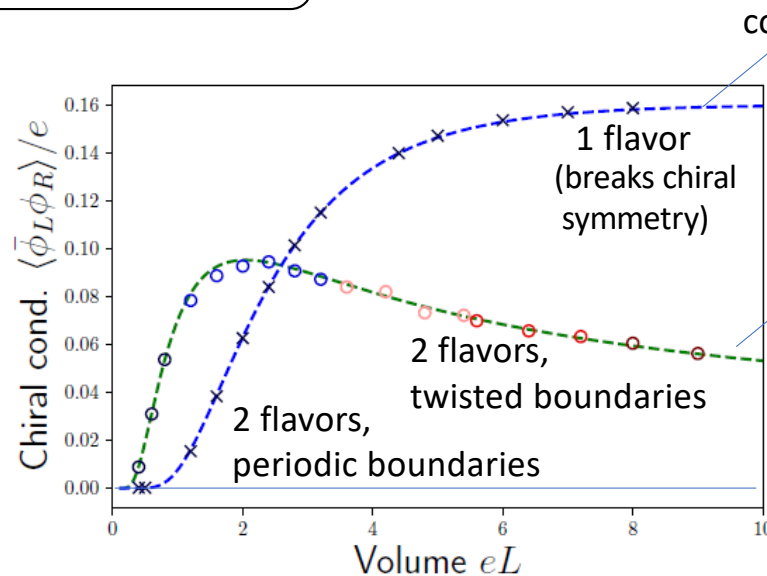
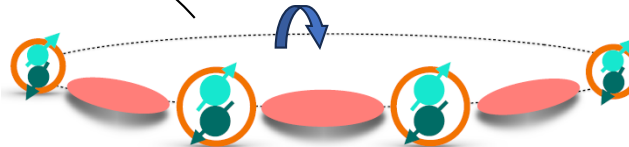
Fracton field configurations become visible in chiral condensate

Popov, Kasper, Lewenstein, Zohar, Stornati, and Hauke, arXiv:2405.00745

flavor twisted boundary conditions

“torons” $\psi(x=L) = \psi(x=0),$
 $\chi(x=L) = -\chi(x=0)$

breaks chiral symmetry



Shifman, Smilga, PRD 1994

$$\frac{1}{NL} \exp \left\{ -\frac{\pi}{N\mu L} \right\} \quad \mu^2 = Ng^2/\pi$$

(at small L)

Note: in QED (in contrast to SuSy YM) finite volume effect (but slow algebraic decrease)
 Formulated positively: Finite volume reveals fractons

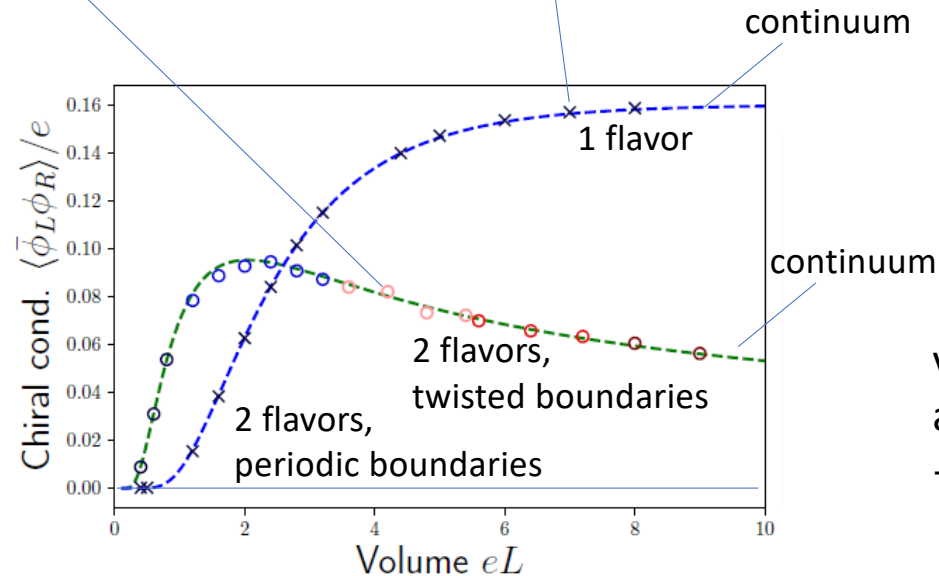
→ drastically different behavior for 1-flavor and 2-flavor model!

Can we probe that on a quantum simulator?

Popov, Kasper, Lewenstein, Zohar, Stornati, and Hauke, arXiv:2405.00745

numerics
 $a = 0.1 - 0.5, m = 0$
 E-field cutoff = ± 3

numerics
 E-field cutoff = $\pm 3, a = 0.05 - 0.5, m = -1/4$



Shifman, Smilga, PRD 1994

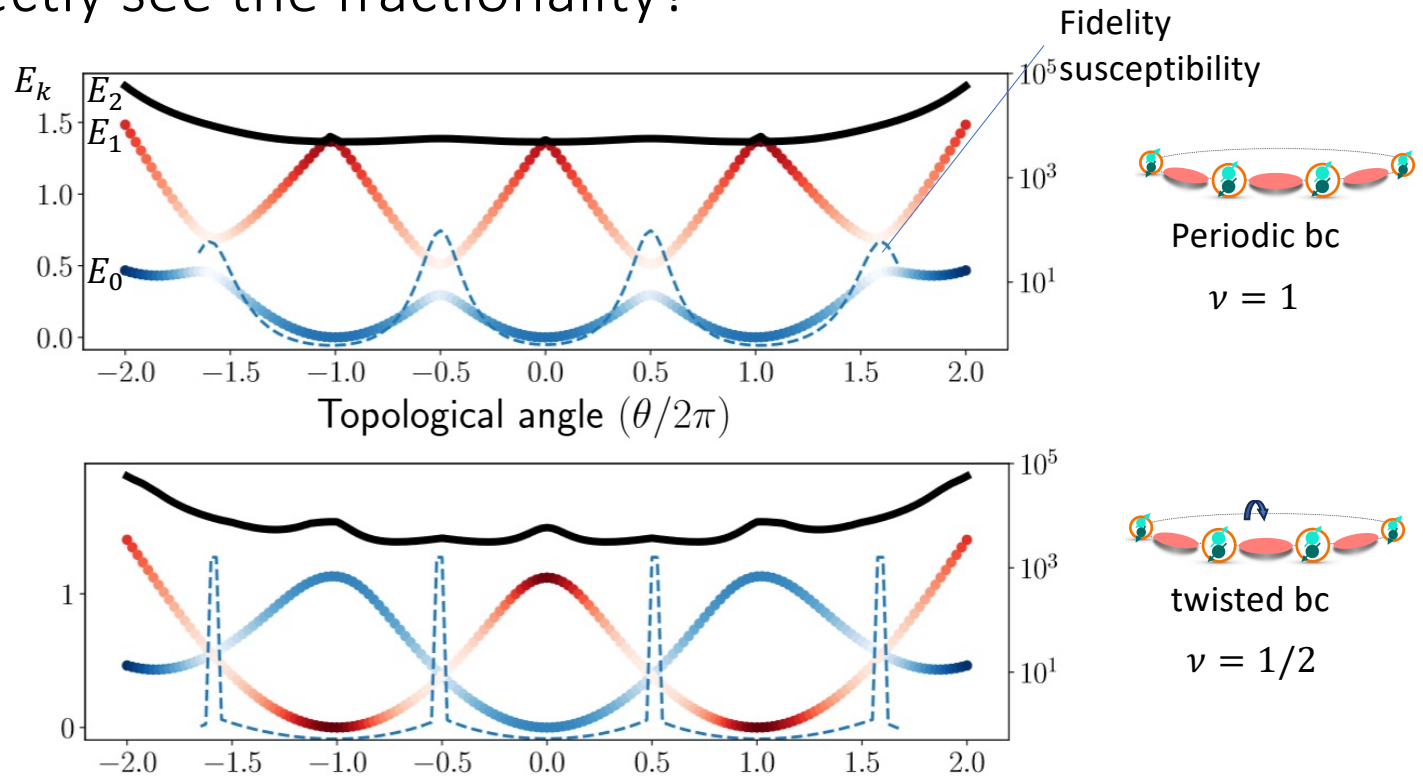
$$\frac{1}{NL} \exp \left\{ -\frac{\pi}{N\mu L} \right\} \quad \mu^2 = Ng^2/\pi$$

visible already in very coarse and highly truncated system!
 → perfect playground for quantum simulators!

Can we directly see the fractionality?

$$Z = \sum_{\nu} e^{i\nu\theta} Z_{\nu}$$

Periodic under
 $\theta \rightarrow \theta + 2\pi/\nu$



Lattice result

$$L = (4 \text{ sites} + 4 \text{ links})$$

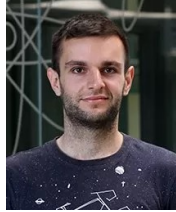
$$S = 2, ea = 1$$

$$\frac{m}{e} = 0.4$$

- Consistent with small m continuum prediction [Misumi, Tanizaki, Ünsal, JHEP 2019](#)

$$E_k(\theta) = -2m \exp\left(-\frac{\pi}{N\mu e L}\right) \cos\left(\frac{\theta + 2\pi k}{N}\right)$$

- **Physics retained, even in non-perturbative regime** ($\frac{m}{e} \sim 1$)
- **Small sizes and cutoffs ideal for quantum simulators!**



Pavel Popov



Valentin Kasper



Erez Zohar



Maciej Lewenstein



Michael Meth



Martin Ringbauer

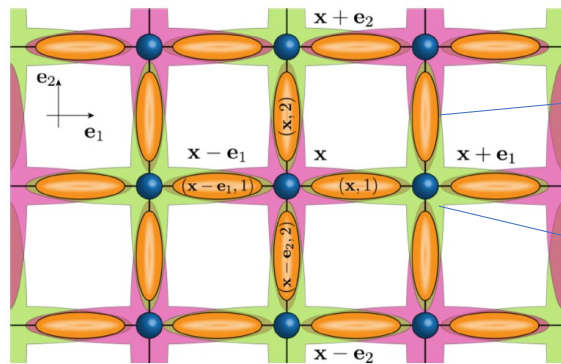
Implementing gauge theories with qudits

Pavel P Popov, Meth, Lewenstein, Hauke, Ringbauer, Zohar, Kasper,
Phys. Rev. Research 6, 013202 (2024)

Starting point: Qudits with cold atoms Kasper, González-Cuadra, Hegde, Xia, Dauphin, Huber, Tiemann, Lewenstein,
Jendrzejewski, Hauke, Quantum Sci. Technol. 7 015008 (2022)

For LGTs, see also, e.g., González-Cuadra, Zache, Carrasco, Kraus, Zoller, Phys. Rev. Lett. 129, 160501 (2022)

Lattice gauge theories require lots of degrees of freedom



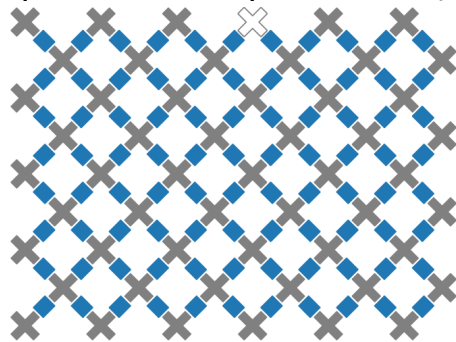
gauge-field (from $-$ cutoff to $+$ cutoff)

matter

could have

- spin
- particle/antiparticle
- Flavour
- colour

But: quantum computers are (conventionally) lattices of qubits



Google Quantum AI

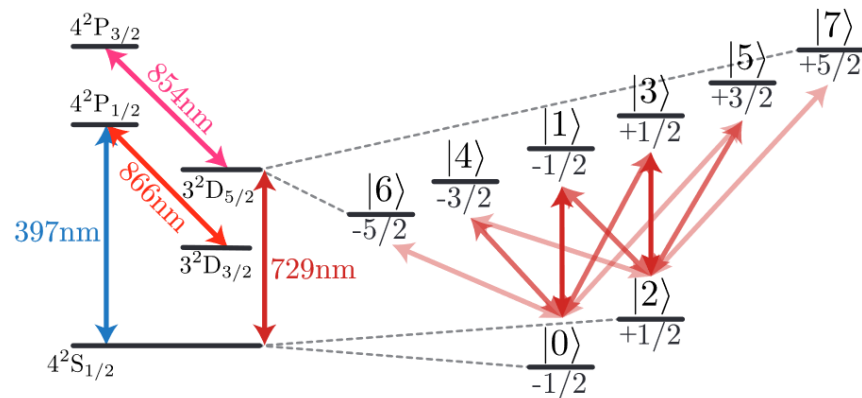
Solution 1: find minimal models that contain interesting physics

Solution 2: go analog and use additional degrees of freedom

Solution 3: compress quantum information

New development in quantum computing: qudit systems are now available

$^{40}\text{Ca}^+$ ions: 7 levels free for universal computation



A universal qudit quantum processor with trapped ions
Ringbauer, Meth, Postler, Stricker, Blatt, Schindler, Monz
Nature Physics 18, 1053 (2022)

Note: qudits developed also, e.g., for SC circuits, Rydbergs, ...

Also for quantum optimization, e.g.:

Deller, Schmitt, Lewenstein, Lenk, Federer, Jendrzejewski,
Hauke, Kasper, PRA, 2023; Garcia de Andoin, Bottarelli, Schmitt,
Oregi, Hauke, Sanz, arXiv:2306.04414 (2023)

Our model system: Abelian 2+1D

$E_{x,i}$: (from $-$ cutoff to $+$ cutoff)

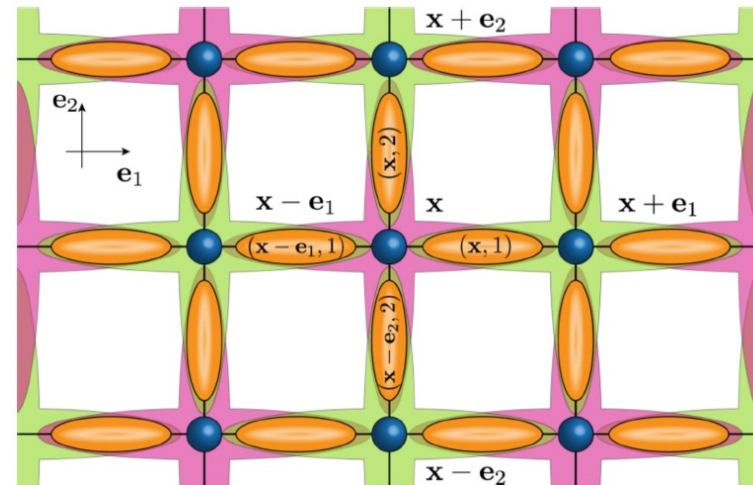
$$H_G = \frac{g^2}{2} \sum_{\mathbf{x},i} E_{\mathbf{x},i}^2 + \frac{1}{2g^2} \sum_{\mathbf{p}} \left[U_{\mathbf{p}} + U_{\mathbf{p}}^\dagger \right]$$

$$U_{\mathbf{p}} = U_{\mathbf{x},1} U_{\mathbf{x}+\mathbf{e}_1,2} U_{\mathbf{x}+\mathbf{e}_2,1}^\dagger U_{\mathbf{x},2}^\dagger$$

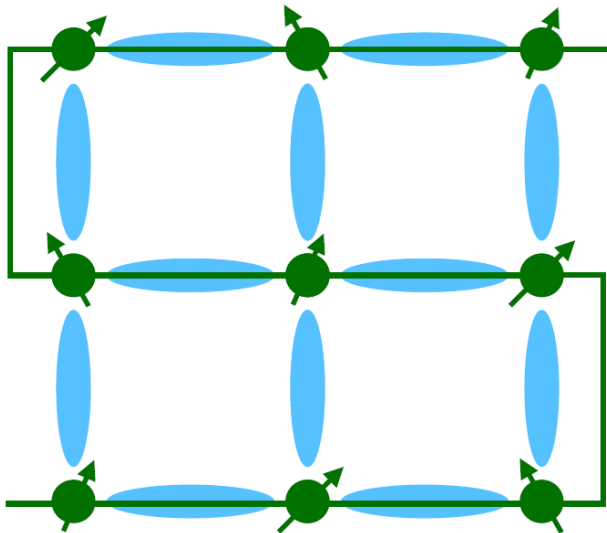
$$H_M = M \sum_{\mathbf{x}} (-1)^{s_{\mathbf{x}}} \psi_{\mathbf{x}}^\dagger \psi_{\mathbf{x}}$$

$$H_{GM} = \frac{1}{2} \sum_{\mathbf{x},i} \left[i \psi_{\mathbf{x}}^\dagger U_{\mathbf{x},i} \psi_{\mathbf{x}+\mathbf{e}_i} + h.c. \right]$$

$$G_{\mathbf{x}} = \sum_i \left[E_{\mathbf{x},i} - E_{\mathbf{x}-\mathbf{e}_i,i} \right] - \underbrace{\psi_{\mathbf{x}}^\dagger \psi_{\mathbf{x}} + \frac{1}{2} [1 - (-1)^{\mathbf{x}}]}_{=:s_{\mathbf{x}}} \quad G_{\mathbf{x}} |\psi\rangle = 0 \quad \forall \mathbf{x}$$



Challenge for quantum simulation in 2+1D: fermionic statistics



Problem: Jordan-Wigner strings do not vanish in higher spatial dimensions \rightarrow highly non-local interactions

Solution: Absorb fermionic statistics into gauge fields
[E. Zohar and JI. Cirac, Physical Review B 98 \(7\), 075119 \(2018\)](#)

\rightarrow introduce hard-core bosons η_x , plus auxiliary fields

... some unitary transformations later

[Popov, Meth, Lewenstein, Hauke, Ringbauer, Zohar, Kasper, PRR 2024](#)

Unitarily equivalent theory

$$H = \frac{g^2}{2} \sum_{\mathbf{x},i} E_{\mathbf{x},i}^2 - \frac{1}{2g^2} \sum_{\mathbf{p}} \left[\tilde{U}_{\mathbf{p}} + \tilde{U}_{\mathbf{p}}^\dagger \right] + M \sum_{\mathbf{x}} (-1)^{s_{\mathbf{x}}} \eta_{\mathbf{x}}^\dagger \eta_{\mathbf{x}} + \frac{1}{2} \sum_{\mathbf{x},i} \left(f_{\mathbf{x},i}(E_{\cdot,\cdot}) \eta_{\mathbf{x}}^\dagger U_{\mathbf{x},i} \eta_{\mathbf{x}+\mathbf{e}_i} + \text{h.c.} \right)$$

$$\tilde{U}_{\mathbf{p}} = \exp[i\pi(E_{\mathbf{x},1} + E_{\mathbf{x}+\mathbf{e}_1,2} + E_{\mathbf{x}+\mathbf{e}_2,2} + E_{\mathbf{x}+\mathbf{e}_2-\mathbf{e}_1,1})] U_{\mathbf{x},1} U_{\mathbf{x}+\mathbf{e}_1,2} U_{\mathbf{x}+\mathbf{e}_2,1}^\dagger U_{\mathbf{x},2}^\dagger$$

In 2+1D $f_{\mathbf{x},1}(E_{\cdot,\cdot}) = (-1)^{E_{\mathbf{x}+\mathbf{e}_1,1} + E_{\mathbf{x}+\mathbf{e}_1,2}} ; f_{\mathbf{x},2}(E_{\cdot,\cdot}) = (-1)^{E_{\mathbf{x},1} + E_{\mathbf{x}-\mathbf{e}_1+\mathbf{e}_2,1} + E_{\mathbf{x}+\mathbf{e}_2,2} + E_{\mathbf{x}+\mathbf{e}_2,1}}$

In 1+1D $f_{\mathbf{x}}(E_{\cdot,\cdot}) = (-1)^{E_{\mathbf{x}+1}}$

Local theory

Perfectly fine gauge theory

Fulfills the Gauss' law for the bosons

$$G_{\mathbf{x}} = \sum_i \left[E_{\mathbf{x},i} - E_{\mathbf{x}-\mathbf{e}_i,i} \right] - \eta_{\mathbf{x}}^\dagger \eta_{\mathbf{x}} + s_{\mathbf{x}}$$

Unitarily equivalent to original theory

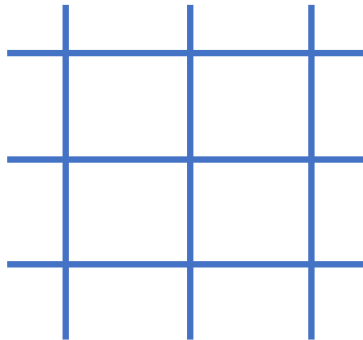
Further reduce degrees of freedom: integrate out matter fields

Note: in 1+1D can integrate out gauge fields

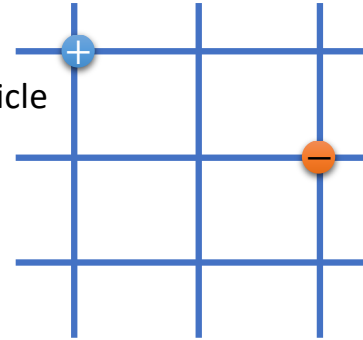
$$E_x = E_{x-1} + n_x + s_x$$

In 2+1D: charges do no longer uniquely define gauge fields

vacuum



Particle-
antiparticle



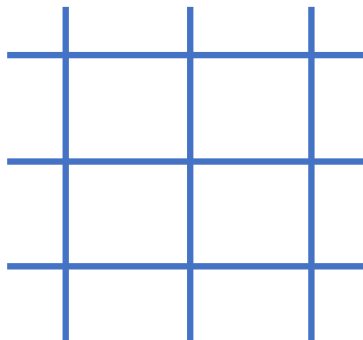
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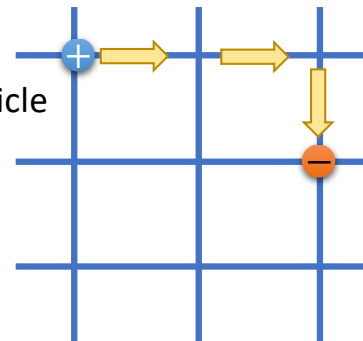
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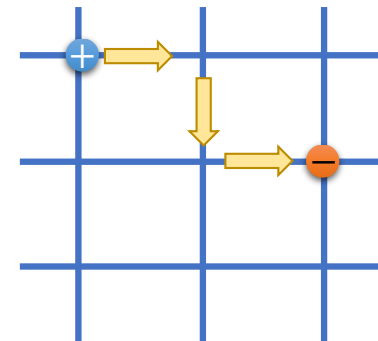
vacuum



Particle-
antiparticle



or



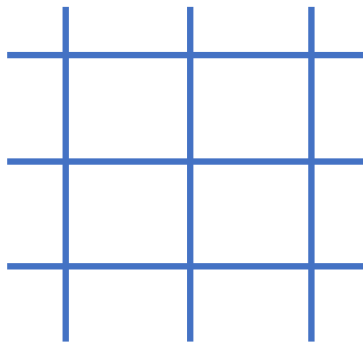
?

Further reduce degrees of freedom: integrate out matter fields

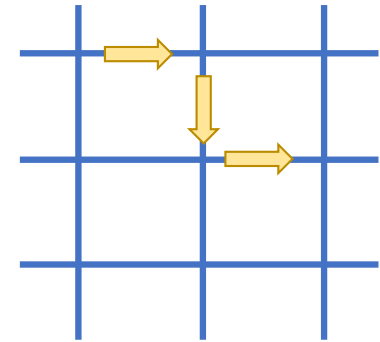
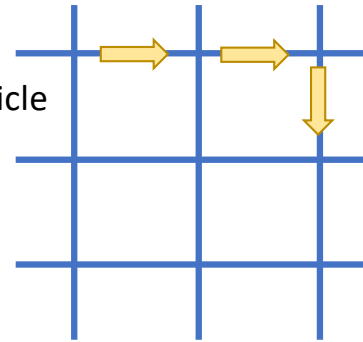
But: gauge fields uniquely define charges

$$n_x = E_x - E_{x-1} - s_x$$

vacuum



Particle-
antiparticle



Continuity: what goes in comes out

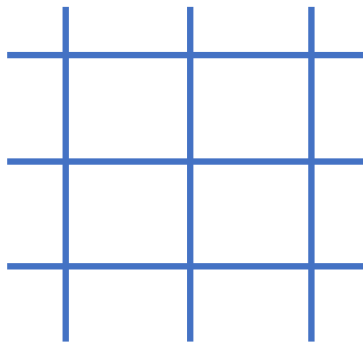
⇒ no charges present

Further reduce degrees of freedom: integrate out matter fields

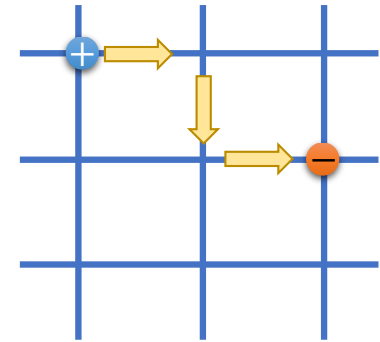
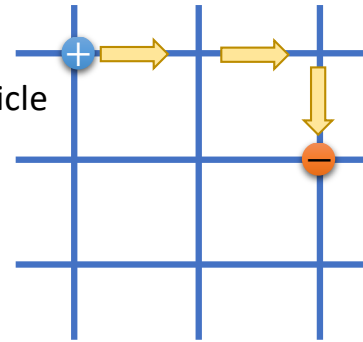
But: gauge fields uniquely define charges

$$n_x = E_x - E_{x-1} - s_x$$

vacuum



Particle-
antiparticle



Continuity: what goes in comes out

⇒ no charges present

⇒ two charges present

Use this to remove charges from game

Unitary transformation $\mathcal{U} = \prod_{\mathbf{x}} (\eta_{\mathbf{x}} + \eta_{\mathbf{x}}^\dagger)^{g_{\mathbf{x}}}$ Physical states $|\tilde{\psi}\rangle = \mathcal{U} |\psi\rangle$ obey $\eta_{\mathbf{x}}^\dagger \eta_{\mathbf{x}} |\tilde{\psi}\rangle = 0$

$$\begin{aligned}
 \text{2+1D} \quad H = & \frac{g^2}{2} \sum_{\mathbf{x},i} E_{\mathbf{x},i}^2 - \frac{1}{2g^2} \sum_{\mathbf{p}} \left[\tilde{U}_{\mathbf{p}} + \tilde{U}_{\mathbf{p}}^\dagger \right] + 2M \sum_{\mathbf{x},i} (-1)^{\mathbf{x}} E_{\mathbf{x},i} \\
 & + \frac{1}{2} \sum_{\mathbf{x}} \left(P_{1,\mathbf{x}} (-1)^{E_{\mathbf{x},1} + E_{\mathbf{x}+\mathbf{e}_1 - \mathbf{e}_2, 2} - s_{\mathbf{x}+\mathbf{e}_1}} U_{\mathbf{x},1} P_{1,\mathbf{x}+\mathbf{e}_1} - P_{1,\mathbf{x}} (-1)^{E_{\mathbf{x},1} + E_{\mathbf{x},2} - s_{\mathbf{x}+\mathbf{e}_2}} U_{\mathbf{x},2} P_{1,\mathbf{x}+\mathbf{e}_2} + \text{h.c.} \right)
 \end{aligned}$$

$$\text{1+1D} \quad H = \frac{g^2}{2} \sum_{\mathbf{x}} E_{\mathbf{x}}^2 + 2M \sum_{\mathbf{x}} (-1)^{\mathbf{x}} E_{\mathbf{x}} + \sum_{\mathbf{x}} \left[P_{1,\mathbf{x}} (-1)^{E_{\mathbf{x}+1}} U_{\mathbf{x}} P_{1,\mathbf{x}+1} + \text{h.c.} \right]$$

$P_{1,\mathbf{x}}$: projector onto E-field configurations
corresponding to physical subspace

flips E-field

But is only allowed to do so, if underlying
is the correct charge configuration!

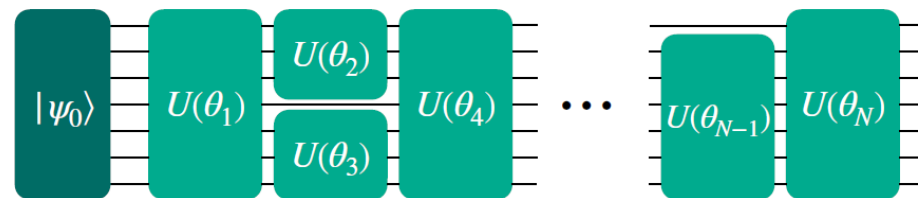
(in 1+1D: "complicated way to get PXP Hamiltonian")

Result: local Hamiltonian of only gauge fields,
values from -cutoff to +cutoff \rightarrow qudits!

Bernien et al.,
Surace et al., etc

Since interactions look complicated, we look at a variational algorithm

Trial state of the form $|\psi(\boldsymbol{\theta})\rangle = U_N(\theta_N) \cdots U_k(\theta_k) \cdots U_1(\theta_1) |\psi_0\rangle$



Evolution within variational manifold:

Evolution equation for $|\psi(t)\rangle$ is traded for evolution equation for $\boldsymbol{\theta}(t)$

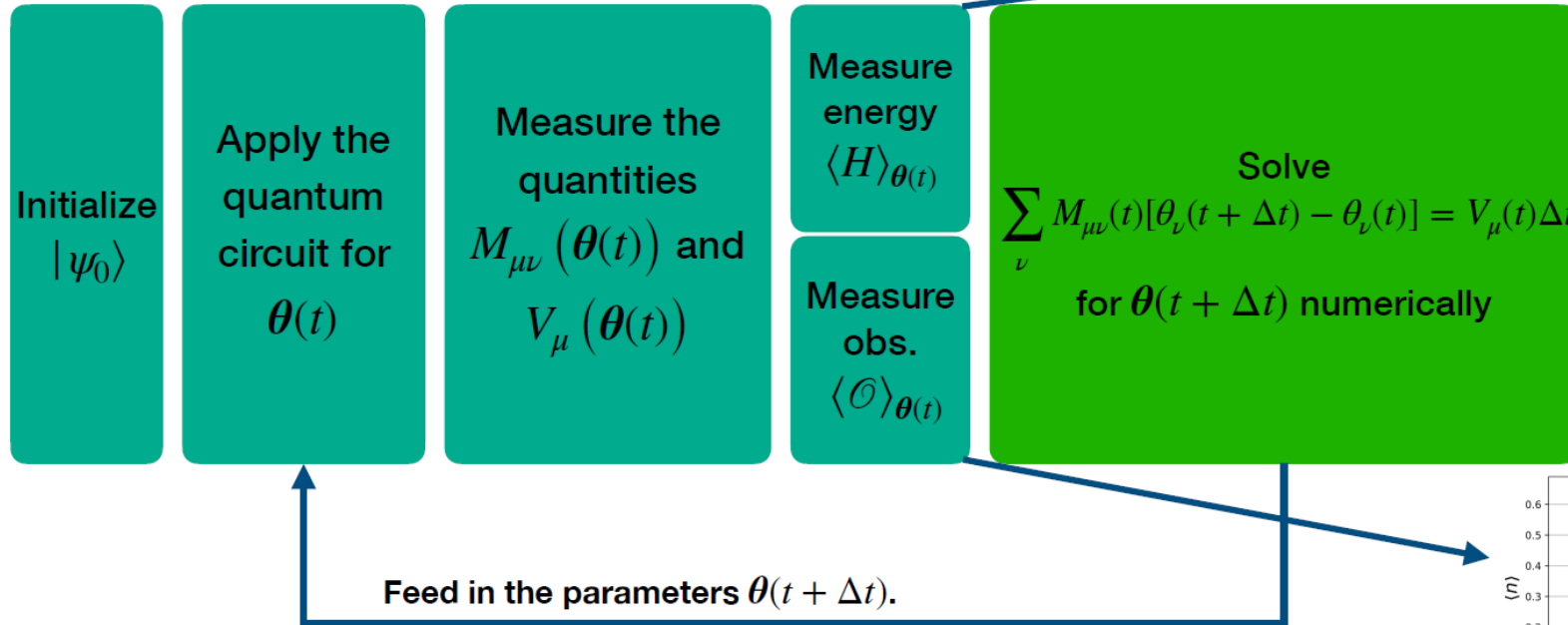
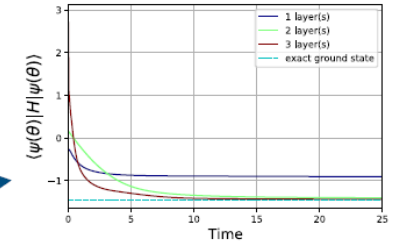
$$\sum_{\nu} M_{\mu\nu} \dot{\theta}_{\nu}(t) = V_{\mu}$$

$$M_{\mu\nu}(\boldsymbol{\theta}) = \text{Tr} \left[\frac{\partial \rho}{\partial \theta_{\mu}} \frac{\partial \rho}{\partial \theta_{\nu}} \right] \text{ and } V_{\mu}(\boldsymbol{\theta}) = \text{Tr} \left[\frac{\partial \rho}{\partial \theta_{\mu}} \mathcal{L}[\rho] \right]$$

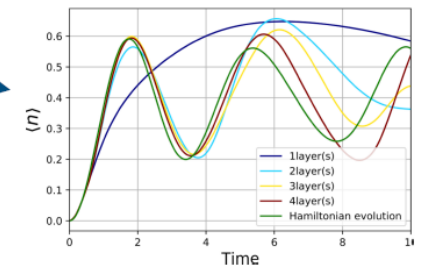
$$\rho(\boldsymbol{\theta}) := |\psi(\boldsymbol{\theta})\rangle \langle \psi(\boldsymbol{\theta})|$$

Execution of the variational algorithm

Imaginary-time evolution



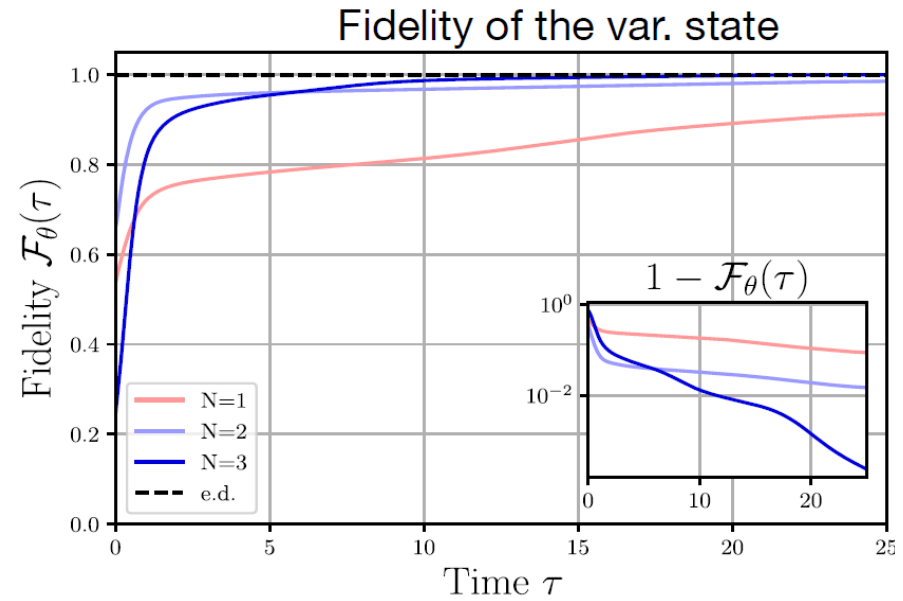
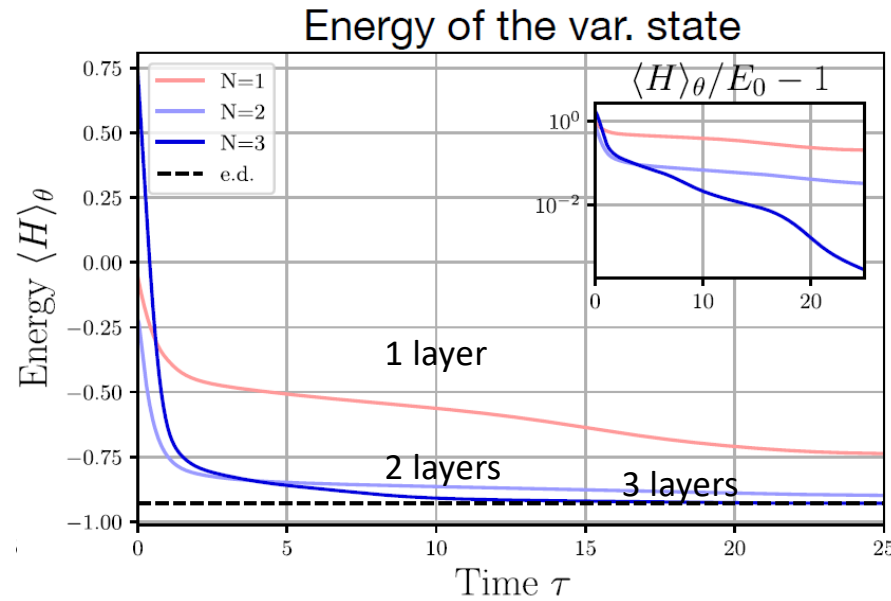
Real-time evolution



Numerical benchmarks

2+1D, 4 qutrits in a plaquette; for each layer, 4 MS gates.

Imaginary time

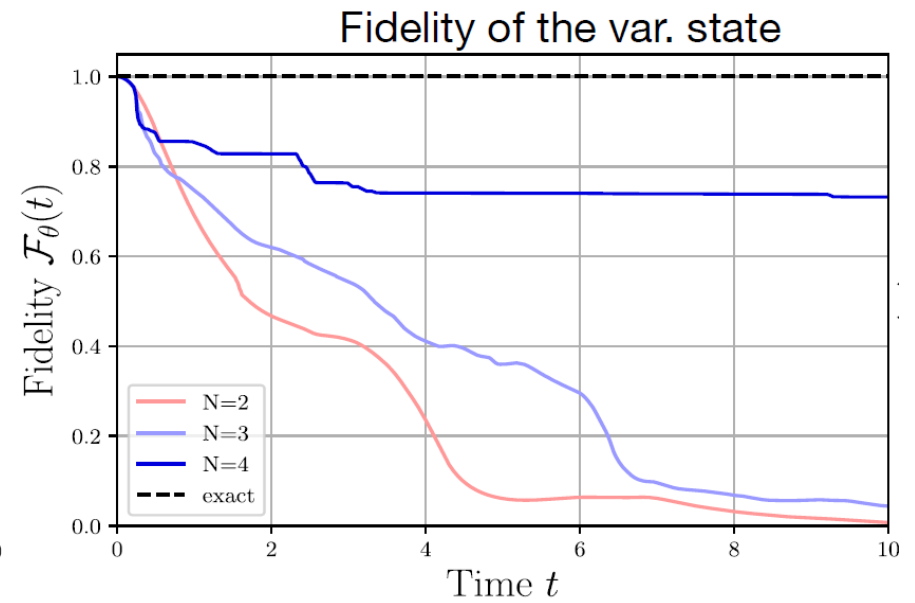
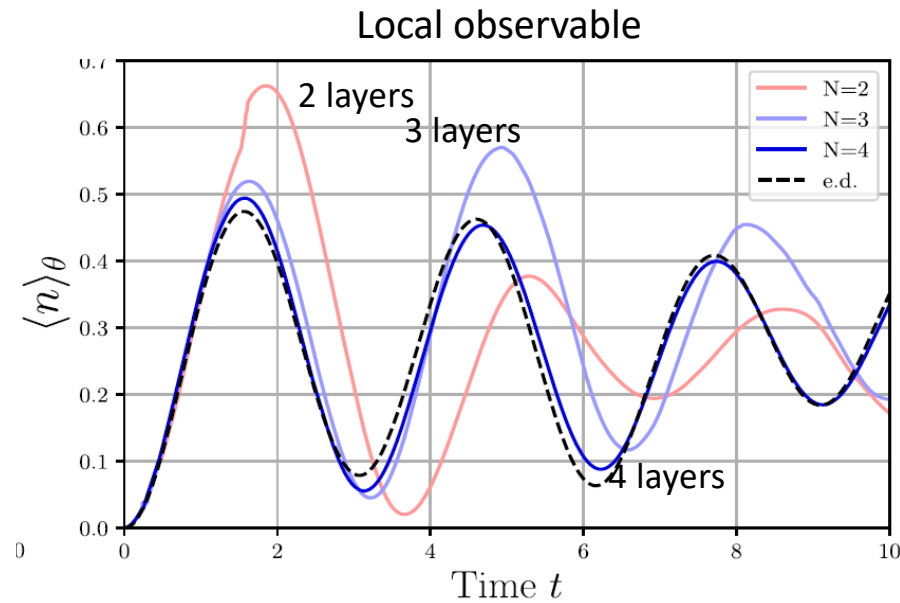


Pavel P Popov, Meth, Lewenstein, Hauke, Ringbauer, Zohar, Kasper
arXiv:2307.15173; PRR 2024

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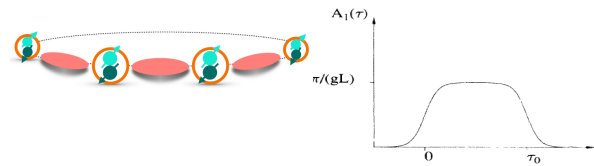


Pavel P Popov, Meth, Lewenstein, Hauke, Ringbauer, Zohar, Kasper
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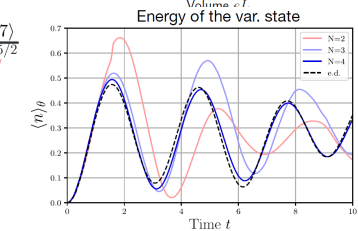
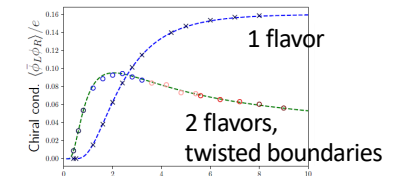
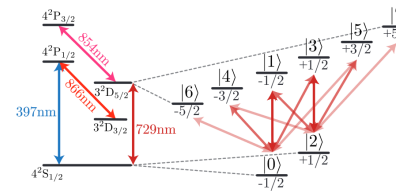
Conclusions

Take away messages

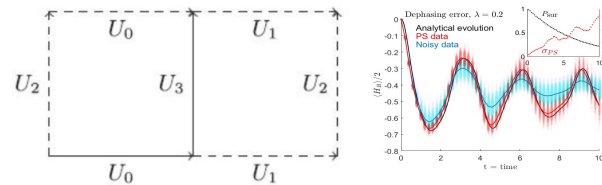
Fascinating target within reach: fracton excitations



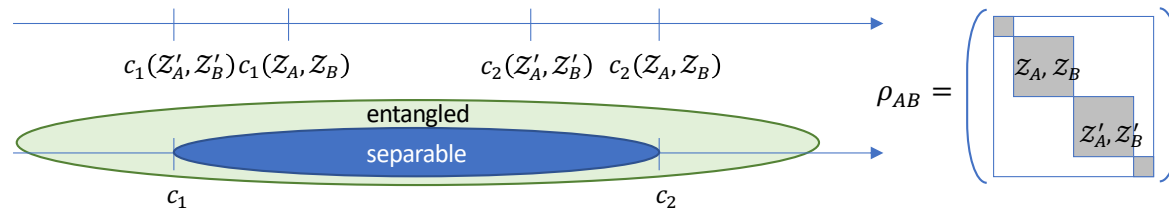
Qudits open new ways for efficient quantum simulation, also in dimensions higher than 1+1D and for multiple flavors



Ongoing work: extensions to non-Abelian



Quantum simulators open ways for new observables: witness entanglement
Panizza, Costa de Almeida, Hauke, JHEP 2022





Team members involved (left to right) Veronica Panizza, Julius Mildenberger, Edoardo Ballini, Emanuele Tirrito, Matteo Wauters.

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Thank you!

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