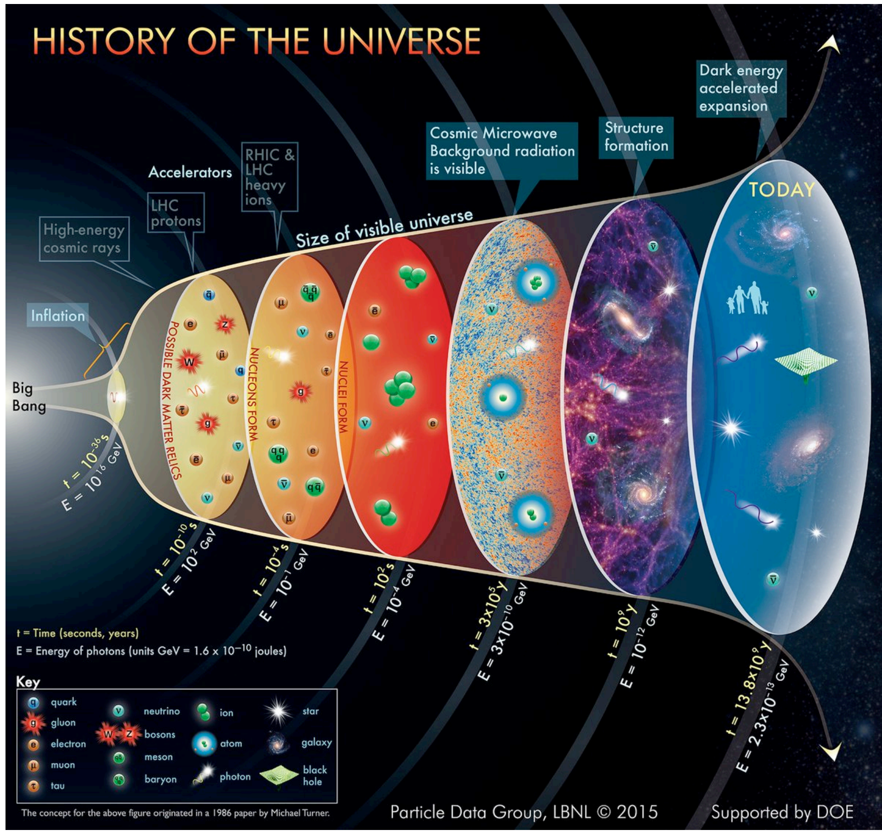

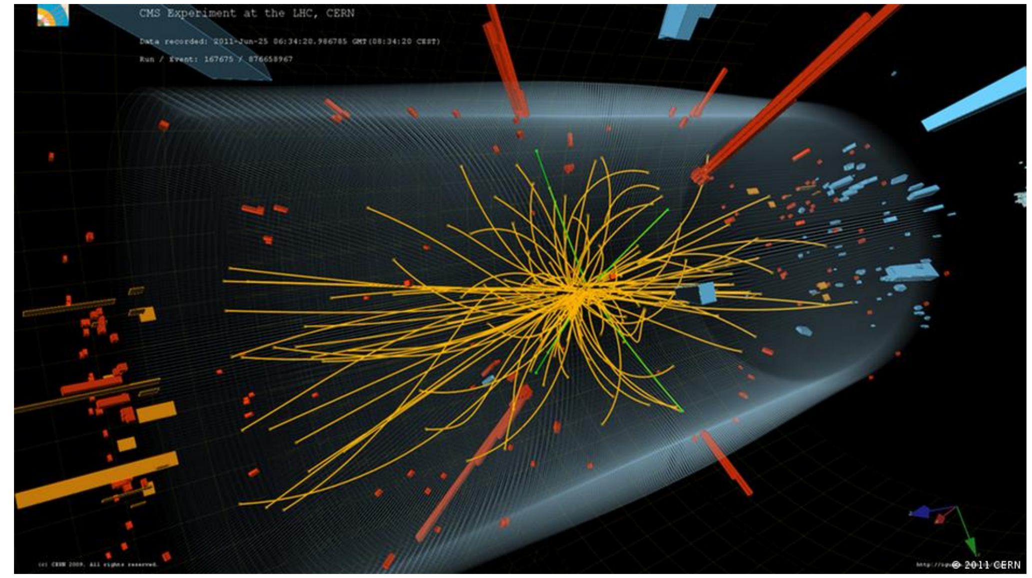

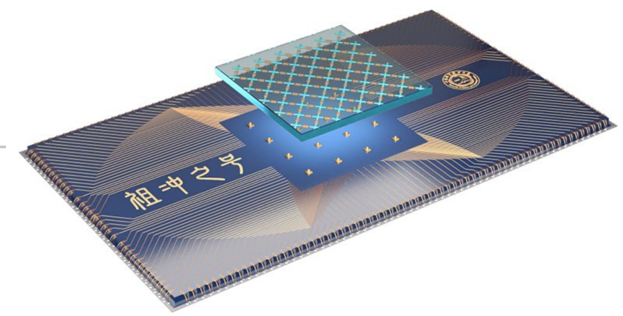

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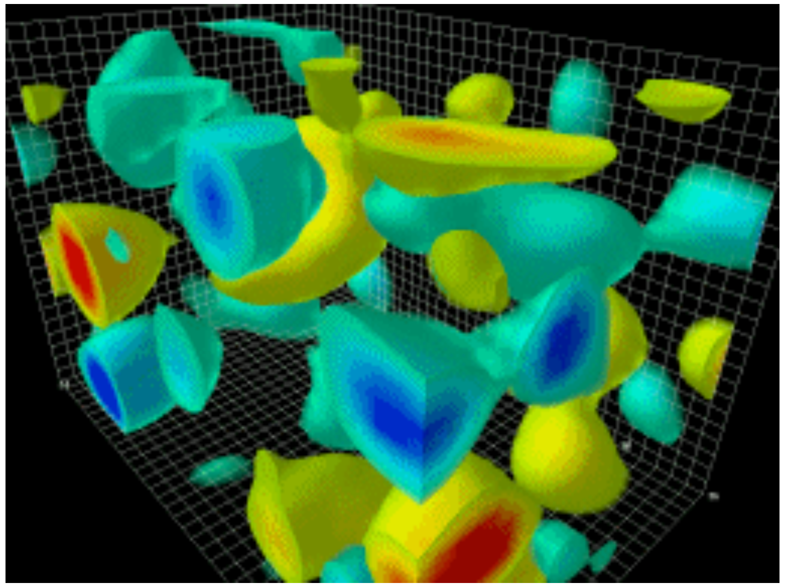
李英英

arXiv:2402.16780, PRL.129, 051601
in collaboration with
Marcela Carena, Henry Lamm, Wanqiang Liu

Keep Gauge Redundancies on Quantum Computer

First Principle Calculations- Lattice QCD

Euclidean Spacetime



field configurations
$\mathcal{C}$ on lattice

Monte Carlo
sampling of lattice
field configurations

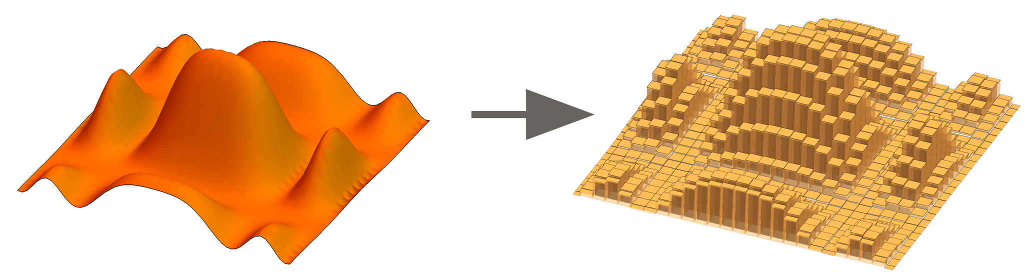
$$W(\mathcal{C}) \sim \exp(-S(\mathcal{C}))$$

$$\langle O \rangle = \frac{\sum_{\mathcal{C}} O(\mathcal{C}) W(\mathcal{C})}{\sum_{\mathcal{C}} W(\mathcal{C})}$$

Real Time

complex $S(\mathcal{C})$

$$\int \mathcal{D}\phi e^{iS} = \langle x| e^{-iHt} |y\rangle$$



$$\dim H \propto |G|^{N_V}$$

exponentially large number
of classical bits in system size

First Principle Calculations- Real Time

``a computing system with qubits”

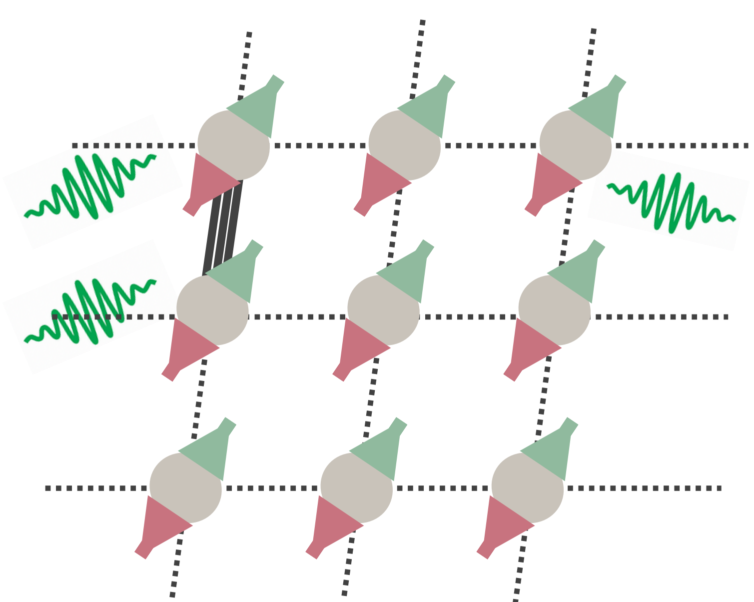
R. P. Feynman - 1982

$$\dim H \propto |G|^{N_V}$$

$$N_q \propto N_V \log |G|$$

The number of qubits required is a polynomial function of the system size

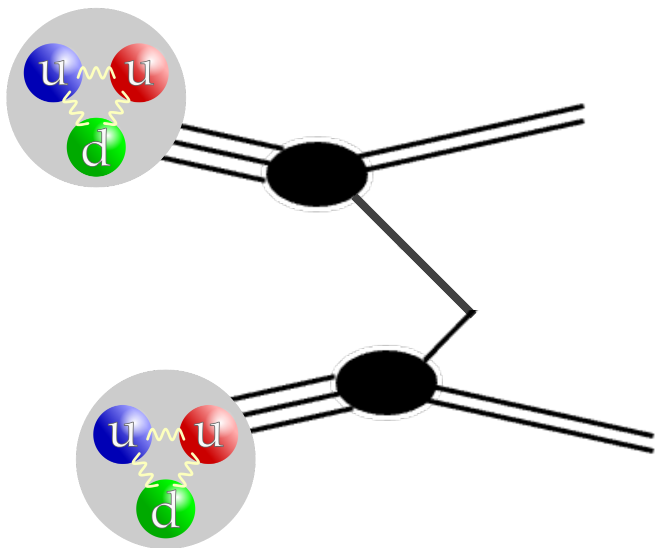
1996 - Seth LIoyd: efficient simulation of LOCAL Hamiltonians



$N($  $) \propto N_q^m$

Quantum Computing for HEP

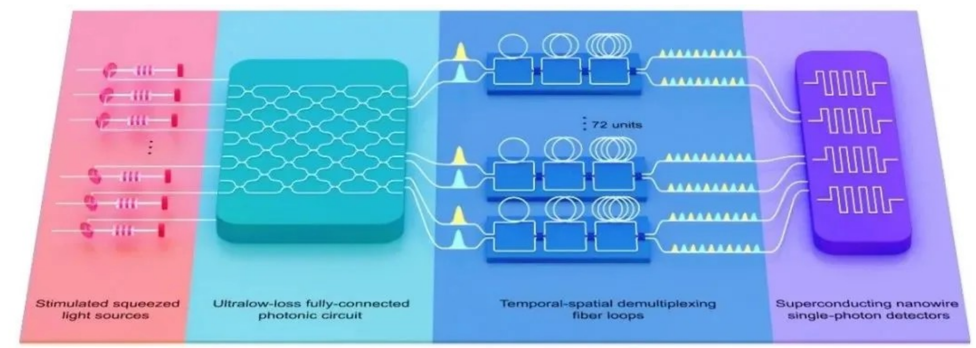
$$\int \mathcal{D}\phi e^{iS} = \langle x| \, e^{-iHt} \, |y\rangle$$

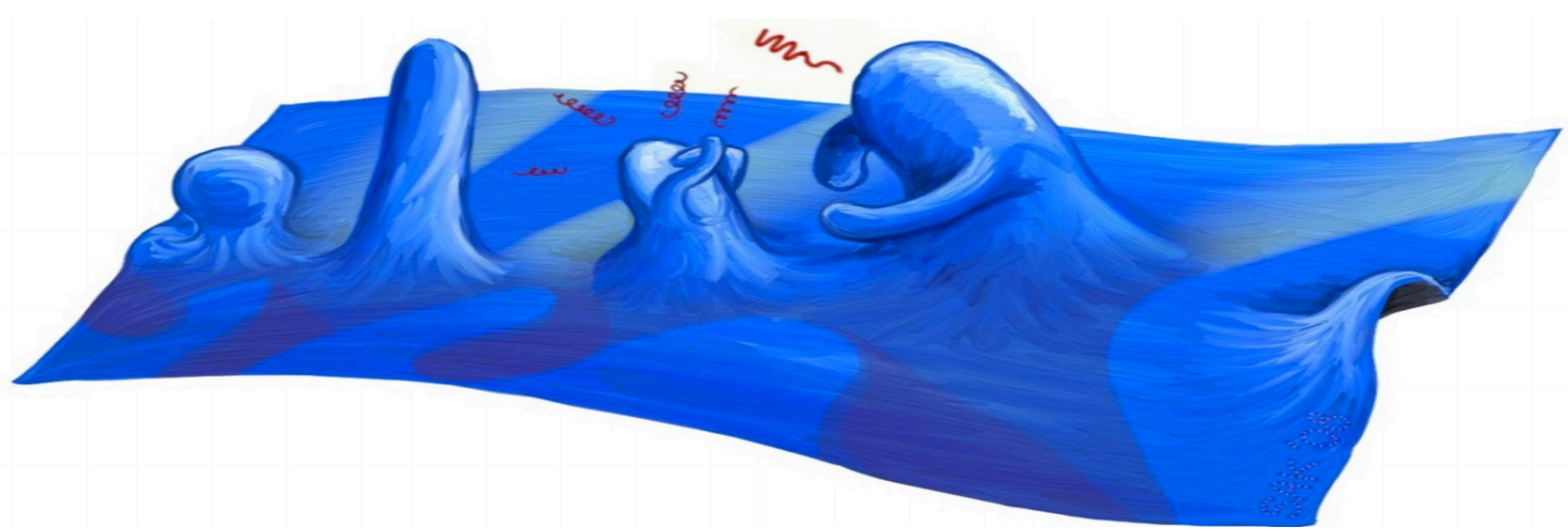


mapping

DOF to qubits

time evolution to quantum gates



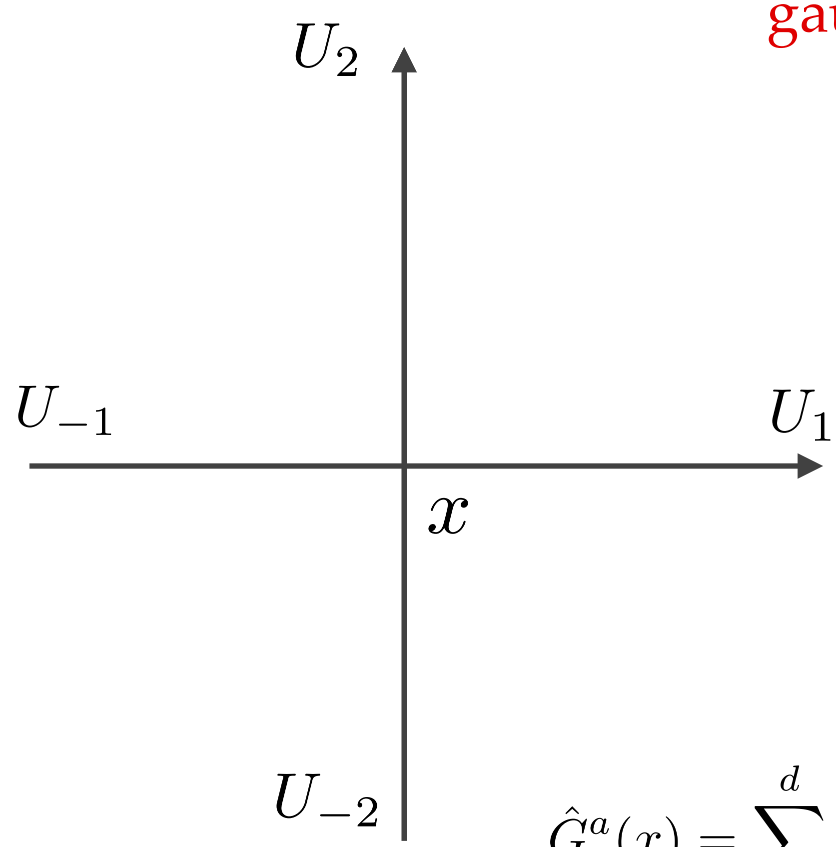


non-trivial vacuum,
composite initial state,
bosonic and fermionic DOF,
symmetries, …

Gauge Symmetries in Quantum Simulations

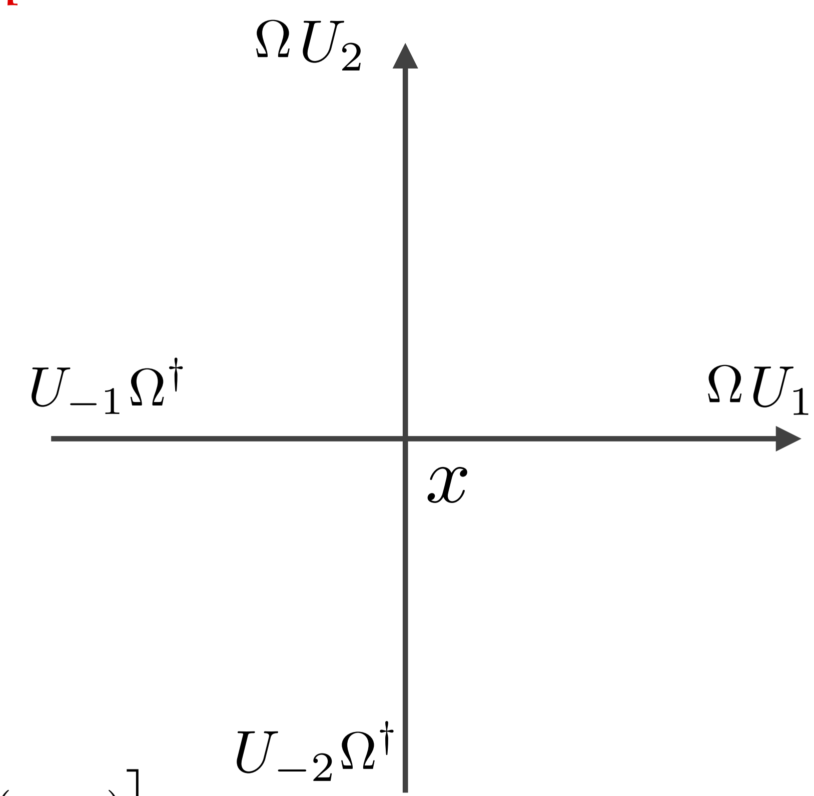
- Gauge transformation
  - Hamiltonians
  - redundant Hilbert space
- Gauge redundancy utilized for error corrections
- Error threshold for gauge redundant encodings
- Time-evolution with gauge redundant encodings

Gauge transformations



$$\hat{\Theta}_\Omega(x) = \exp(i\phi(\Omega)\hat{G}^a(x))$$

gauge transformation



$$\hat{G}^a(x) = \sum_{i=1}^{d} \left[\hat{E}^a_R(x - e_i, e_i) - \hat{E}^a_L(x, e_i) \right]$$

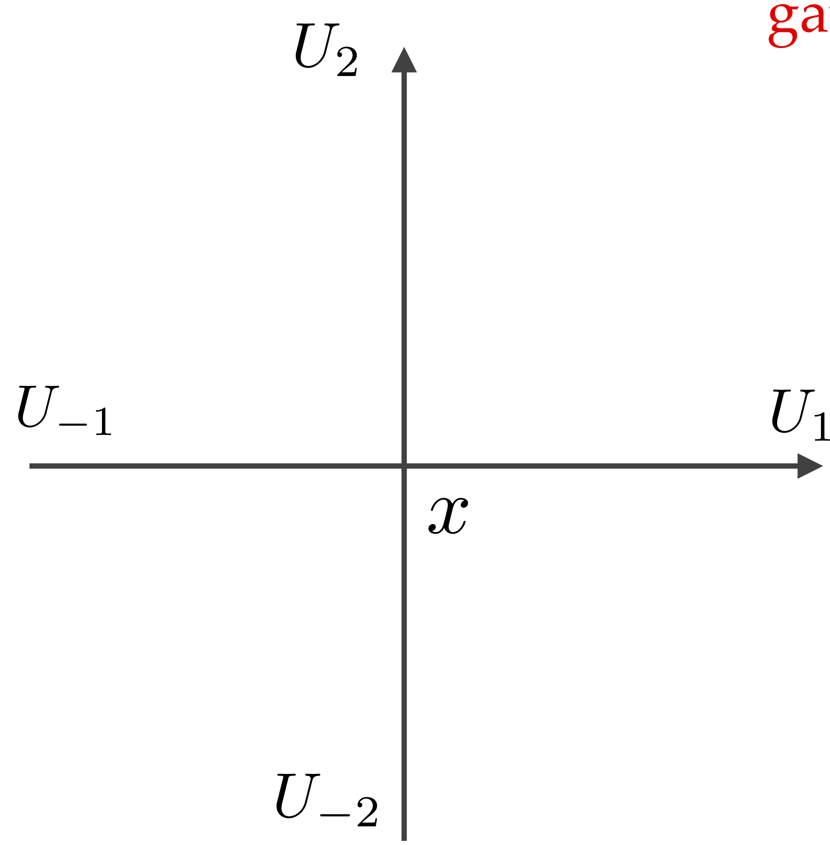
lattice analog of covariant
divergence of chromo-electric field

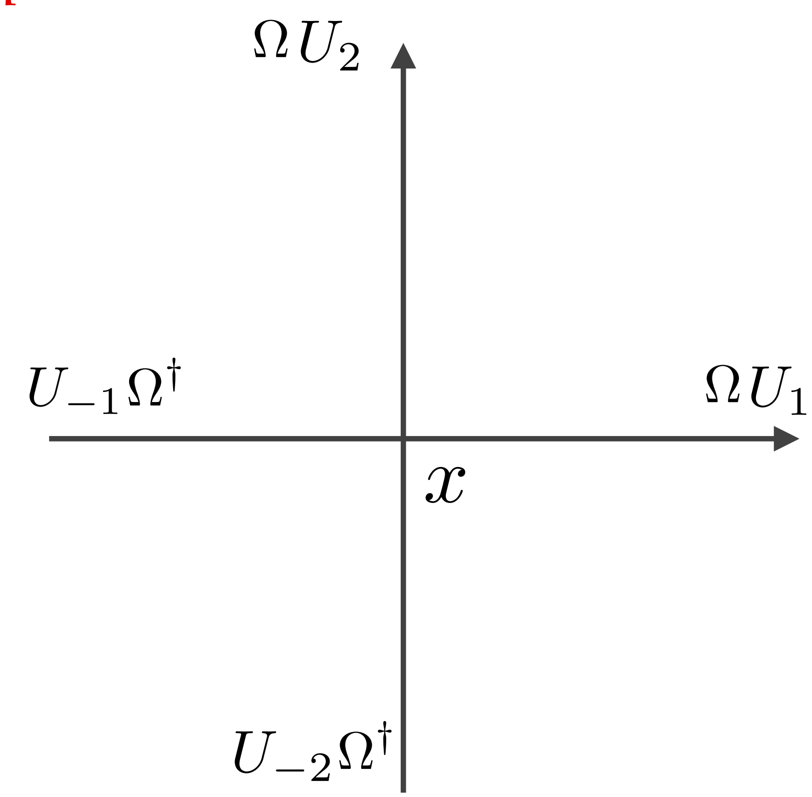
$$\text{quadratic Casmir}: \quad \hat{E}^2 \left| j m_L m_R \right\rangle = j(j+1) \left| j m_L m_R \right\rangle \qquad \left| j m_L m_R \right\rangle \overset{\text{FT}}{\leftrightarrow} \left| U \right\rangle$$

Gauge transformations

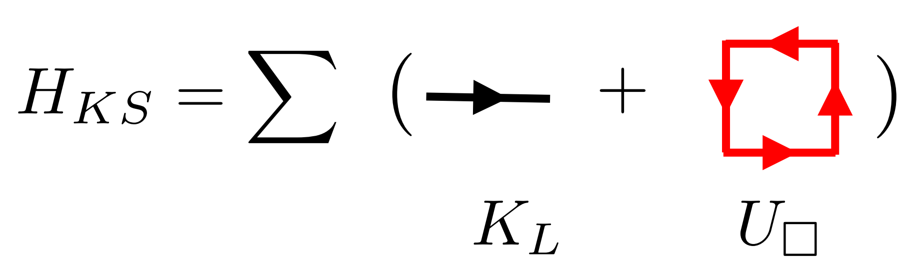
$$\hat{\Theta}_{\Omega}(x) = \exp(i\phi(\Omega)\hat{G}^a(x))$$

gauge transformation



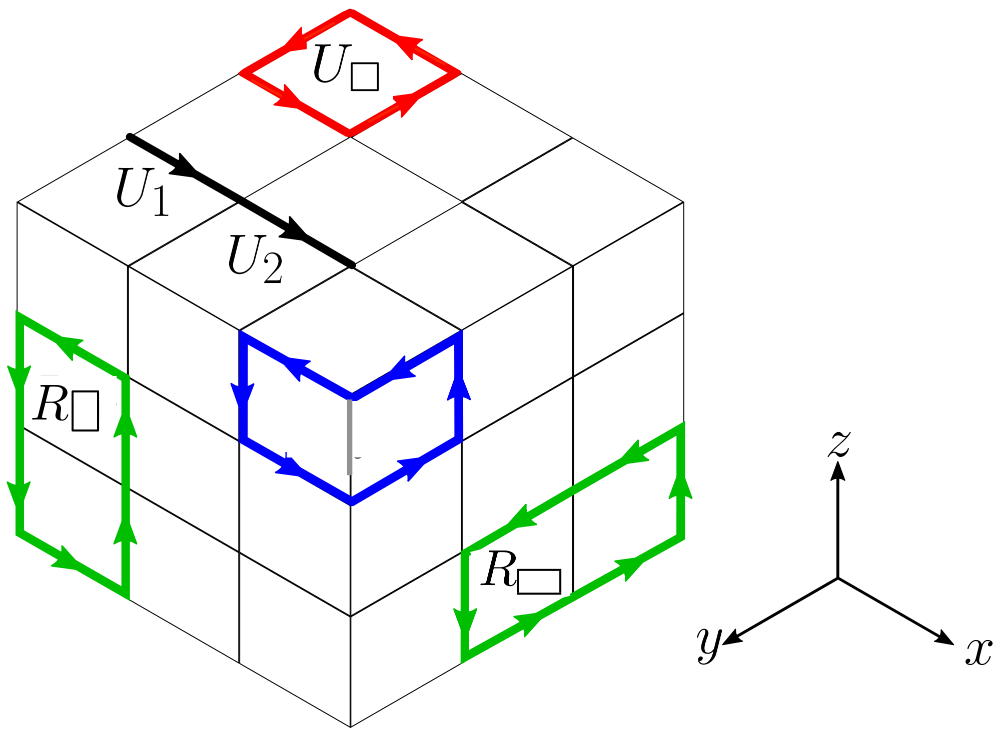


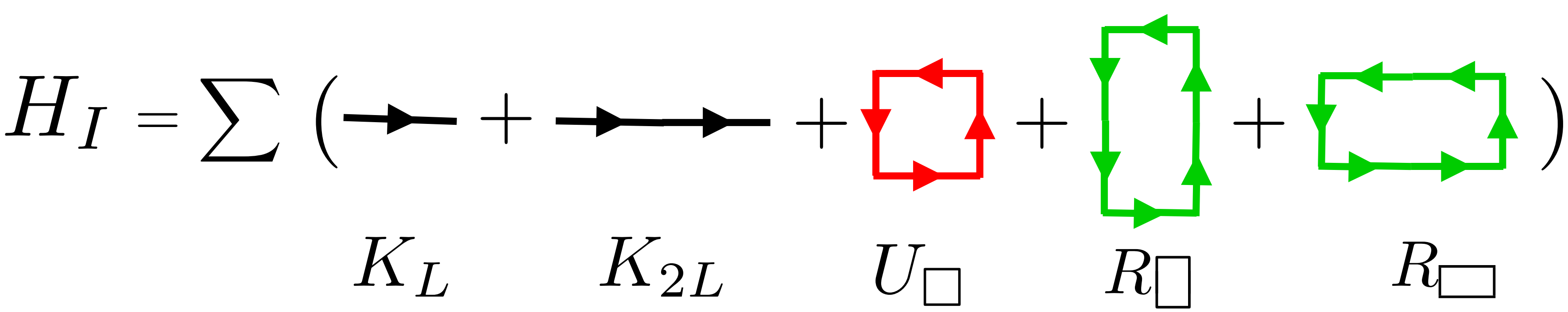
gauge invariant Hamiltonian



quadratic Casimir

Gauge transformations





improved Hamiltonian

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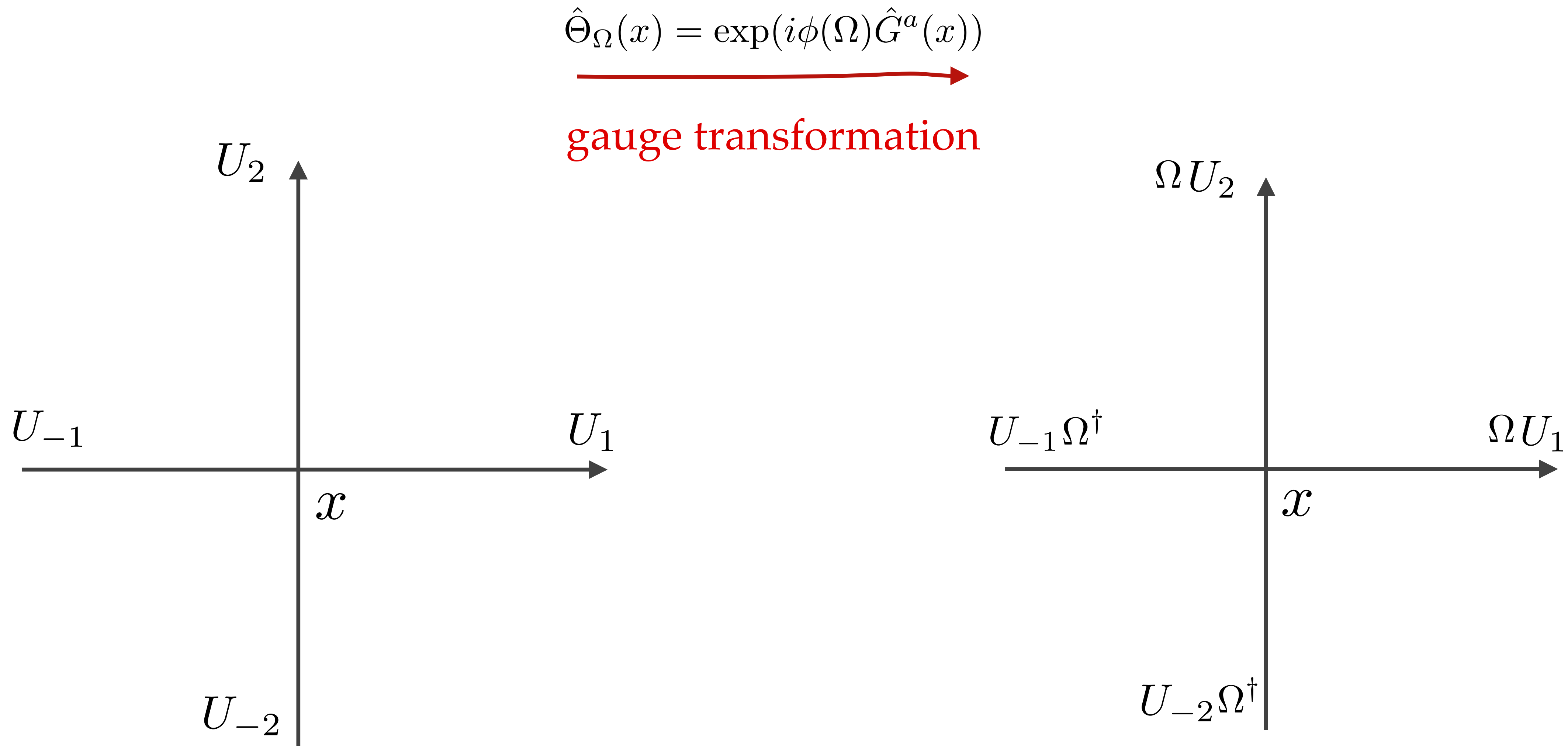
Gauge transformations



gauge equivalent states

$$\hat{\Theta}_\Omega(x)\,|U_{-1}U_1U_{-2}U_2\rangle = \left|U'_{-1}U'_1U'_{-2}U'_2\right\rangle$$

Gauge transformations

gauge invariant states

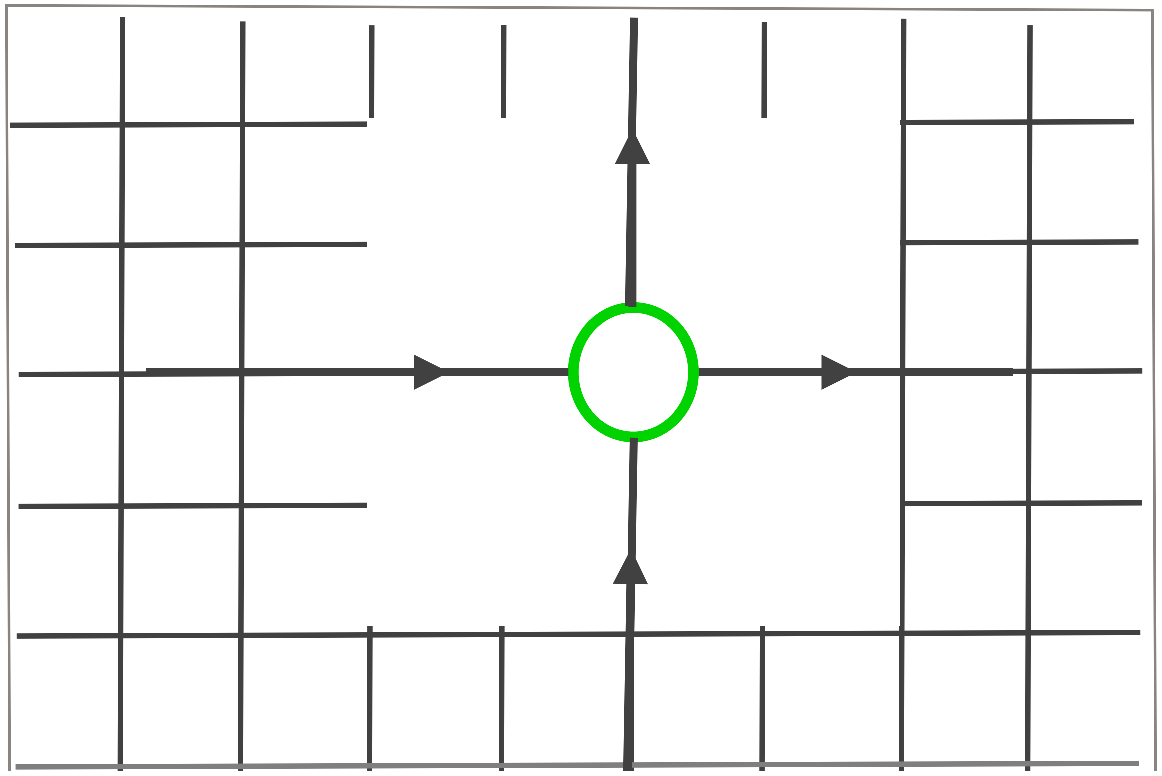
$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

neutral charge

$$\hat{\Theta}_\Omega(x) = \exp(i\phi(\Omega)\hat{G}^a(x))$$

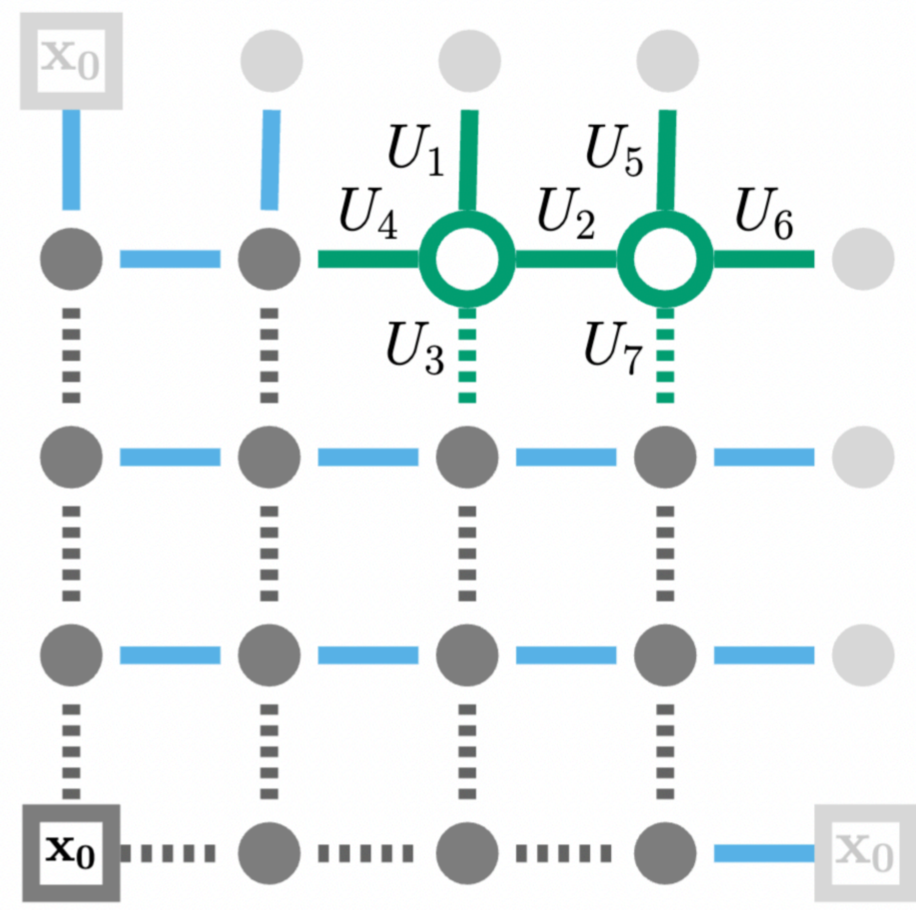
$$\hat{G}^a(x) = \sum_{i=1}^{d}\left[\hat{E}^a_R(x-e_i,e_i) - \hat{E}^a_L(x,e_i)\right]$$

$$\hat{G}^a(x)\,|\psi_{\text{phys}}\rangle = 0$$



Gauge transformations

See also Dorota Grabowska's talk



$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L}$$

$$\hat{\Theta}_\Omega(x)\,|\psi\rangle = |\psi'\rangle$$

gauge redundant

$$\hat{G}^a(x)\,|\psi\rangle = 0$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L - N_V + 1}$$

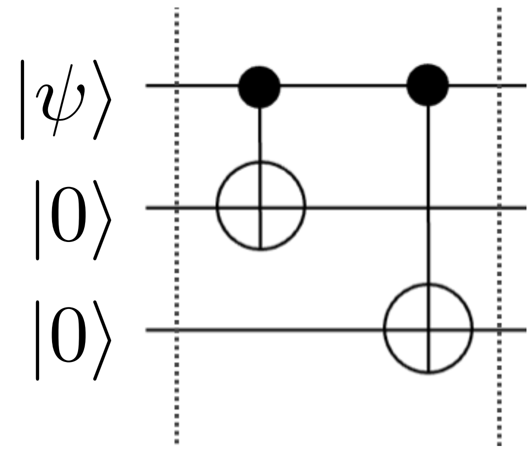
$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

Gauge redundancy utilized for error corrections

quantum error corrections

$|\psi\rangle$

$\alpha|0\rangle + \beta|1\rangle$



$$|\Psi\rangle = \alpha|000\rangle + \beta|111\rangle$$

undetectable errors

$\mathcal{H}_{\text{full}}:$

$|001\rangle, |010\rangle, |100\rangle$

$|011\rangle, |110\rangle, |101\rangle$

quantum errors

detectable errors

$\mathcal{H}_{\text{code}}: |111\rangle, |000\rangle$

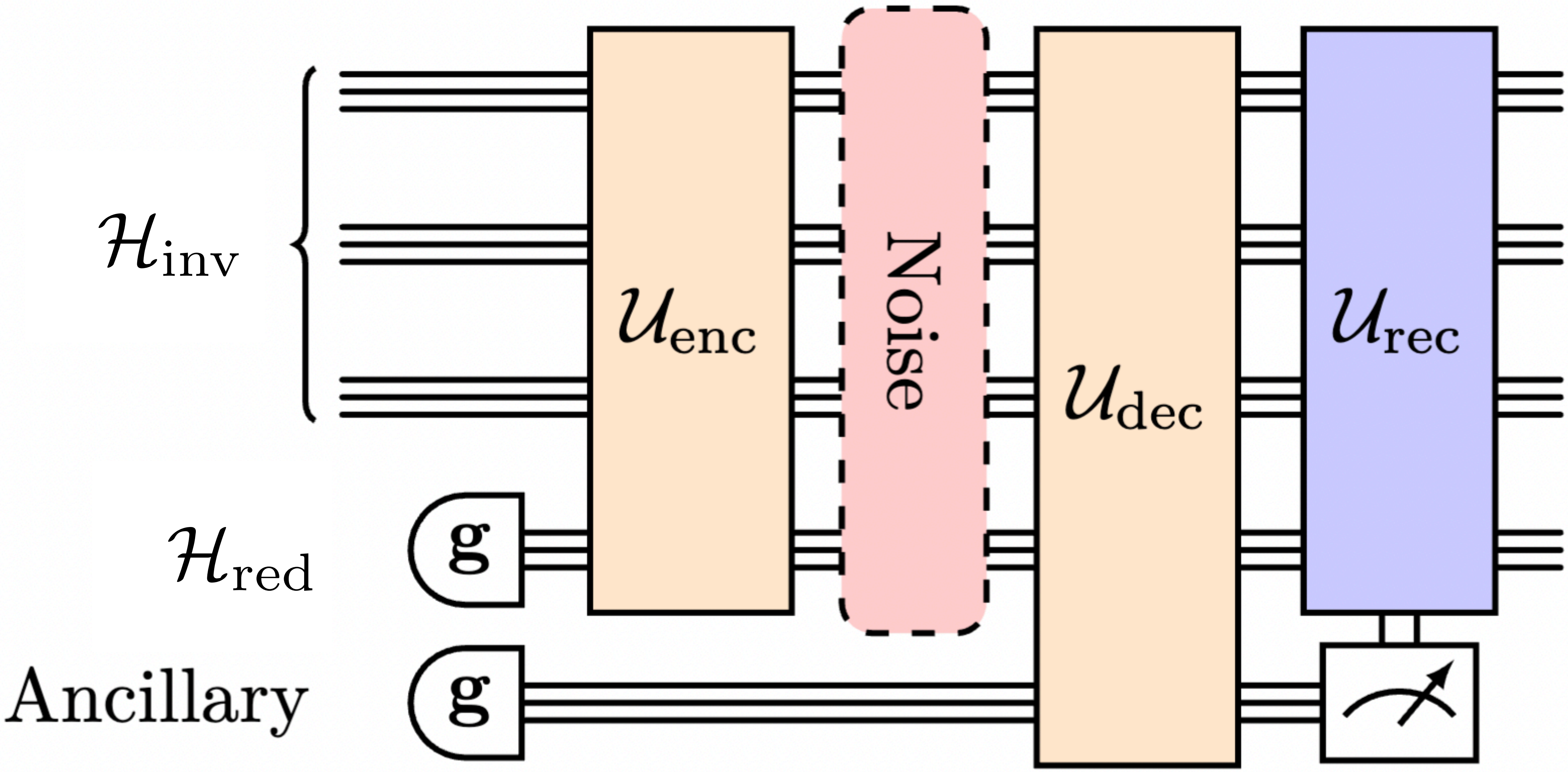
$$\hat{\Theta}_{\mathcal{S}}(x)|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$\hat{\Theta}_{\mathcal{S}} = \{I, Z_1Z_2, Z_2Z_3, Z_1Z_3\}$$

Gauge redundancy utilized for error corrections

state preparation:
encoding Gauss's law

charge
computation



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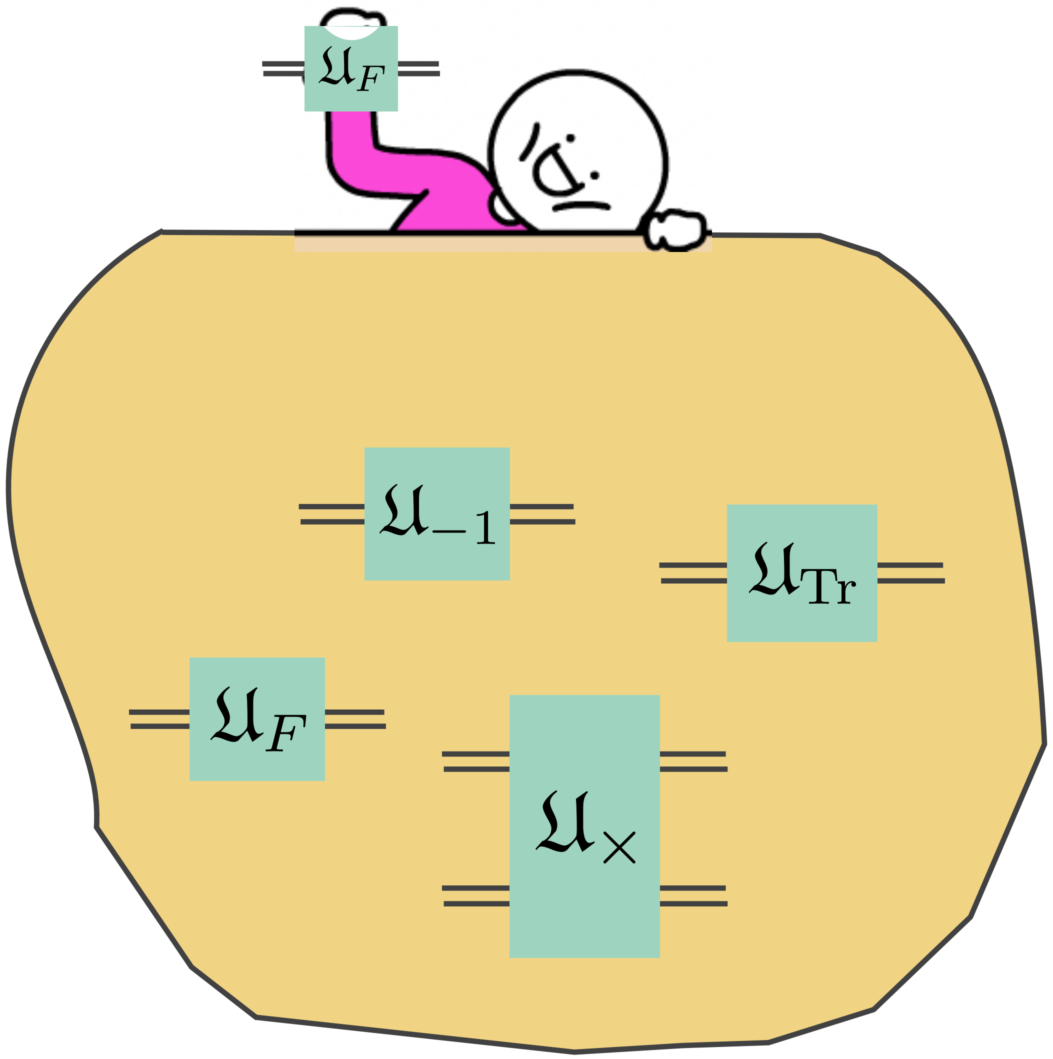
Gauge redundancy utilized for error corrections

$$G\text{−register} : |U\rangle \quad =\!=$$



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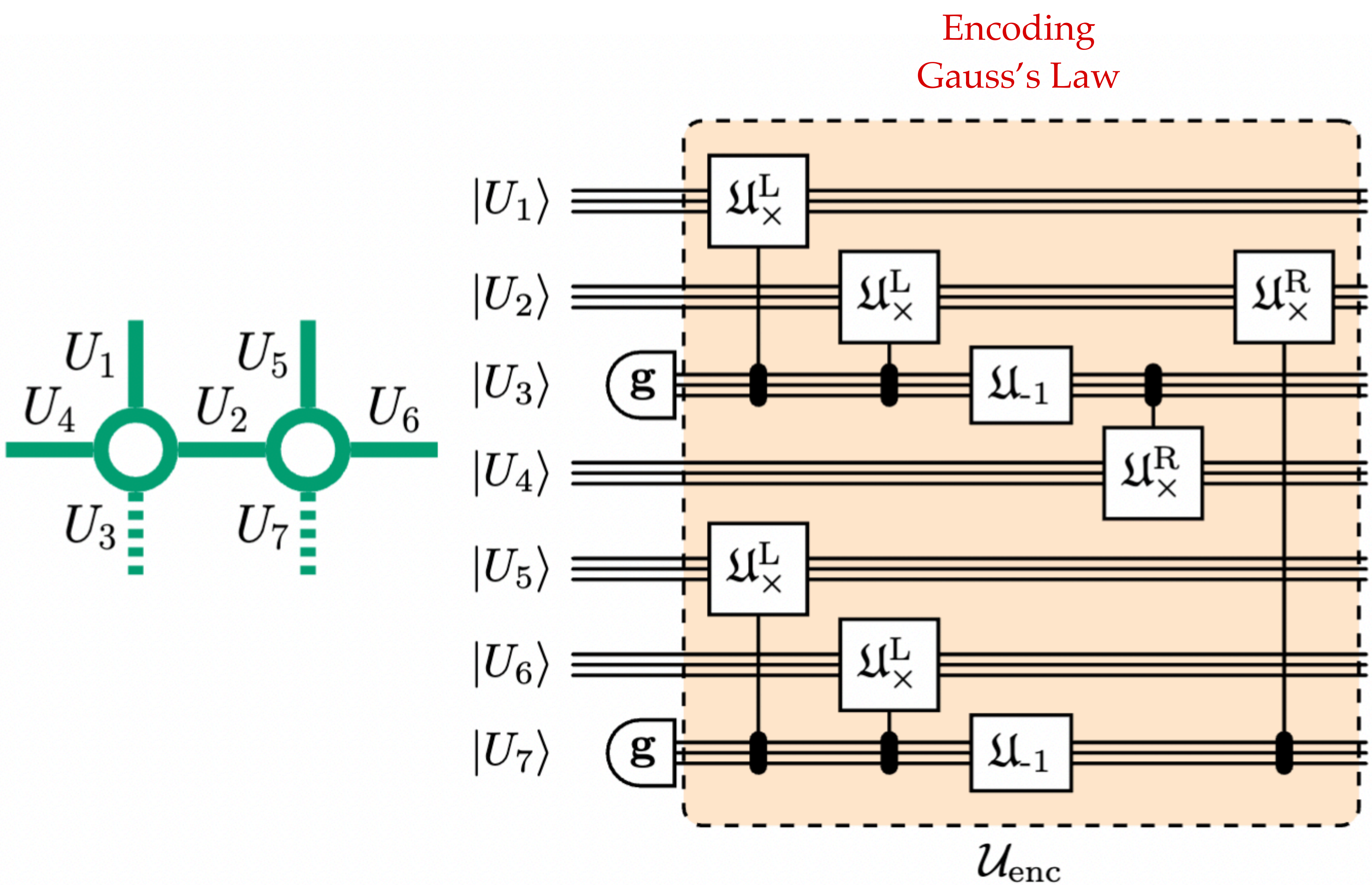
Gauge redundancy utilized for error corrections



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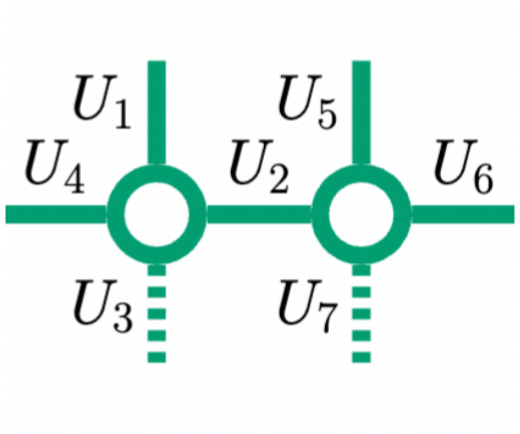
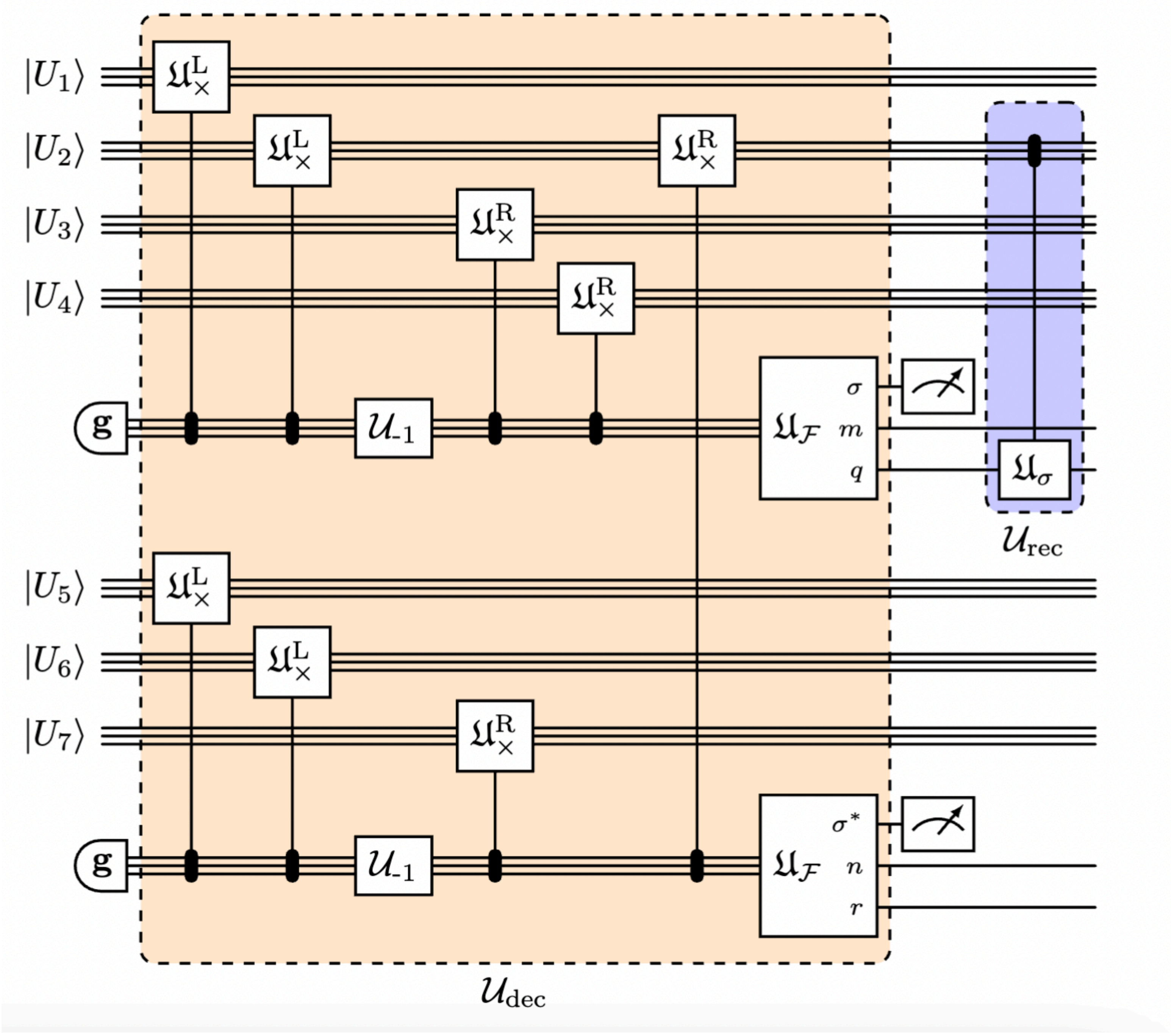
Gauge redundancy utilized for error corrections



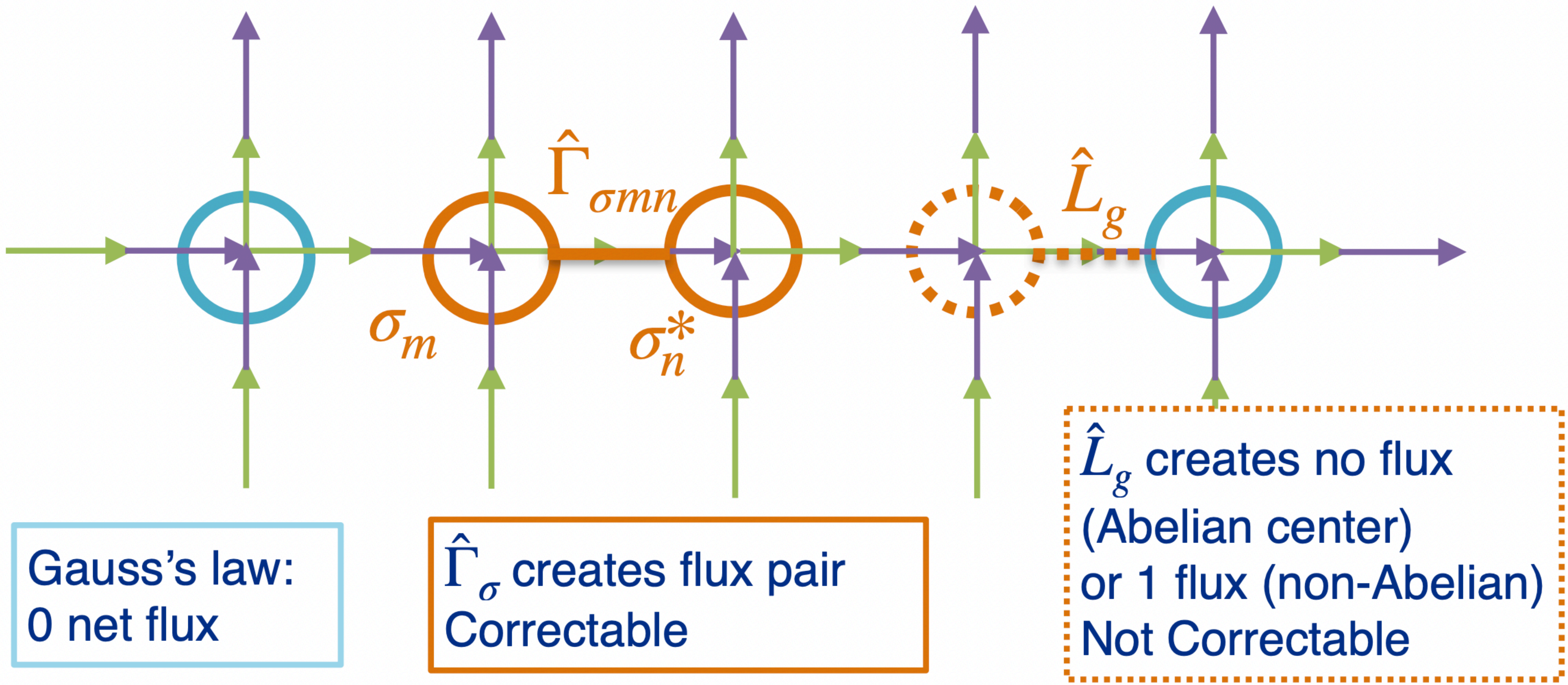
charge
computation



M. Carena, H. Lamm,**YYL**, W. Liu, arXiv:2402.16780

Gauge redundancy utilized for error corrections

correctable errors



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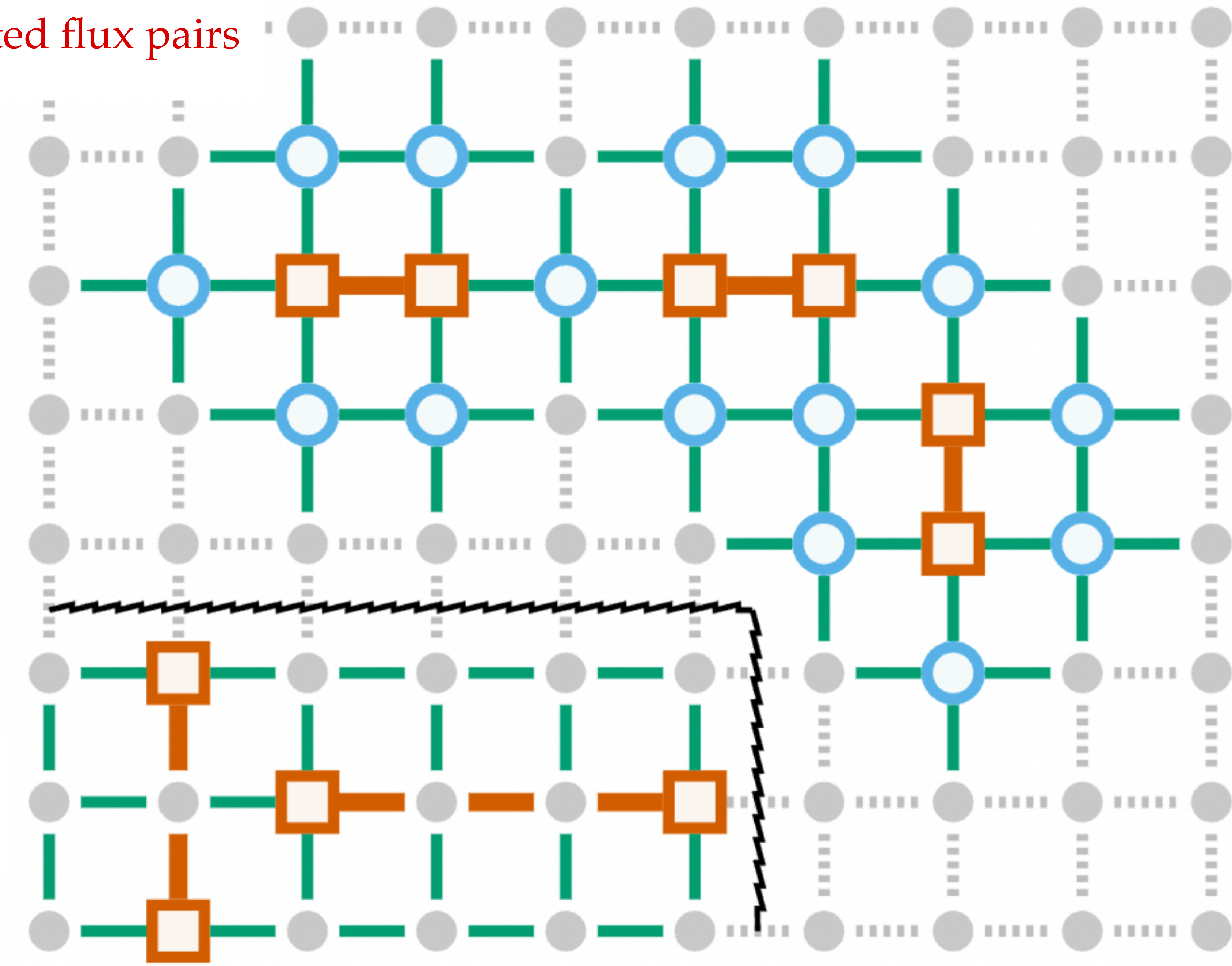
Gauge redundancy utilized for error corrections

correctable errors

isolated flux pairs



KL condition

M. Carena, H. Lamm, **YYL**, W. Liu, arXiv:2402.16780

Error threshold for gauge redundant encodings

worthwhile to keep the redundancy?

resource requirements?

easy implementation or
Hamiltonian complexity?

resilience to errors?

Error threshold for gauge redundant encodings

resource requirements?

$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L}$$

$$N_q = N_L \log|G|$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L - N_V + 1}$$

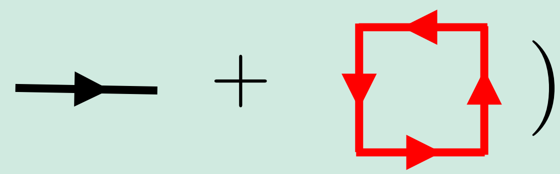
$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$N_q = (N_L - N_V + 1)\log|G|$$

Error threshold for gauge redundant encodings

Hamiltonian complexity?

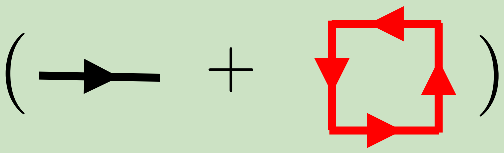
$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L}$$

$$H_{KS} = \sum \Big($$  $$\Big)$$

K_L $\quad U_\Box$

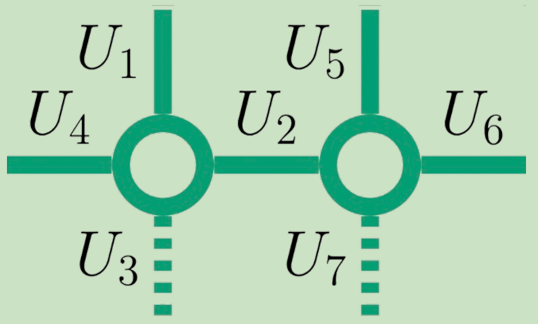
$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L - N_V + 1}$$

$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

$$H_{KS} = \sum \Big($$  $$\Big)$$

K_L $\quad U_\Box$

kinetic terms for U_3, U_7 depend on other links



Error threshold for gauge redundant encodings

resilience to errors?

$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L}$$

+ error correction

single link correctable error rate ϵ_c

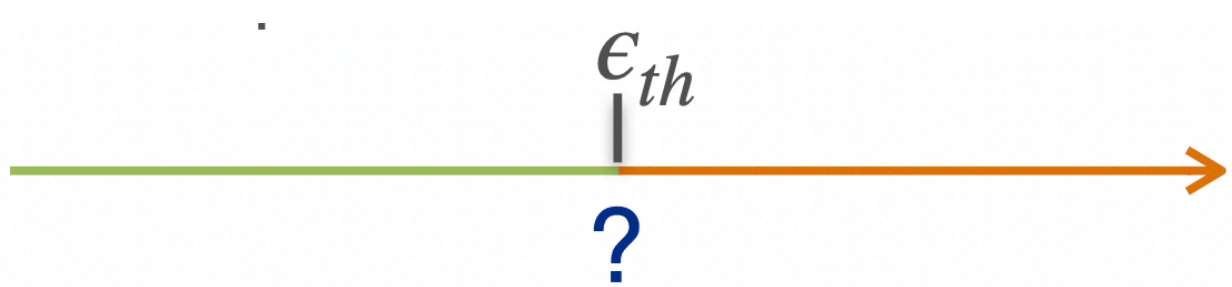
$$F_{\text{restored}} \geq \sum_{n=0}^{N_L} Q_n \epsilon_c^n (1-\epsilon)^{N_L - n}$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle, U \in G\})^{\otimes N_L - N_V + 1}$$

$$\hat{\Theta}_\Omega(x) |\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

not correctable

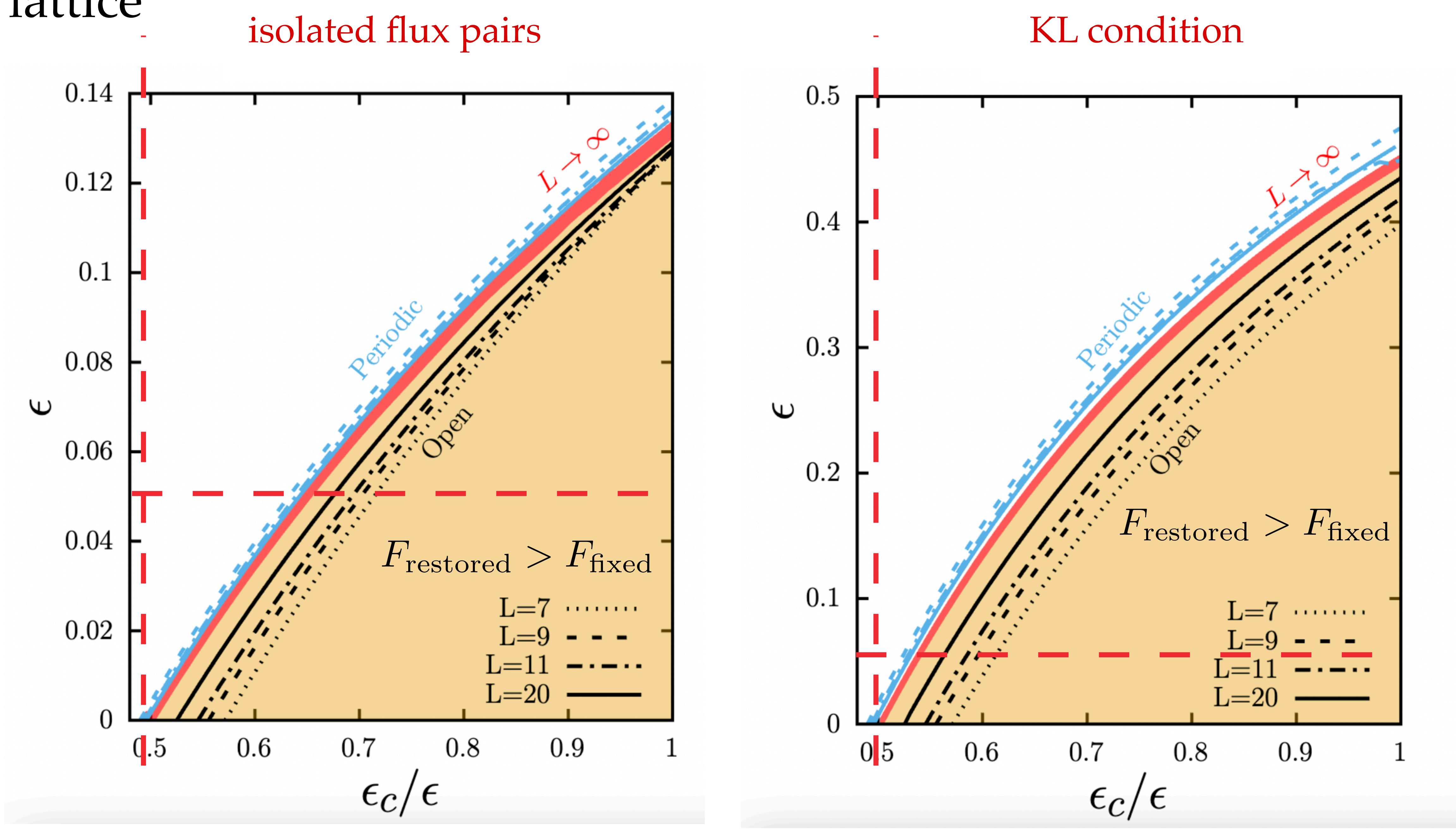
$$F_{\text{inv}} \geq (1-\epsilon)^{N_L - N_V + 1}$$



ϵ single link error rate

Error threshold for gauge redundant encodings

2d lattice

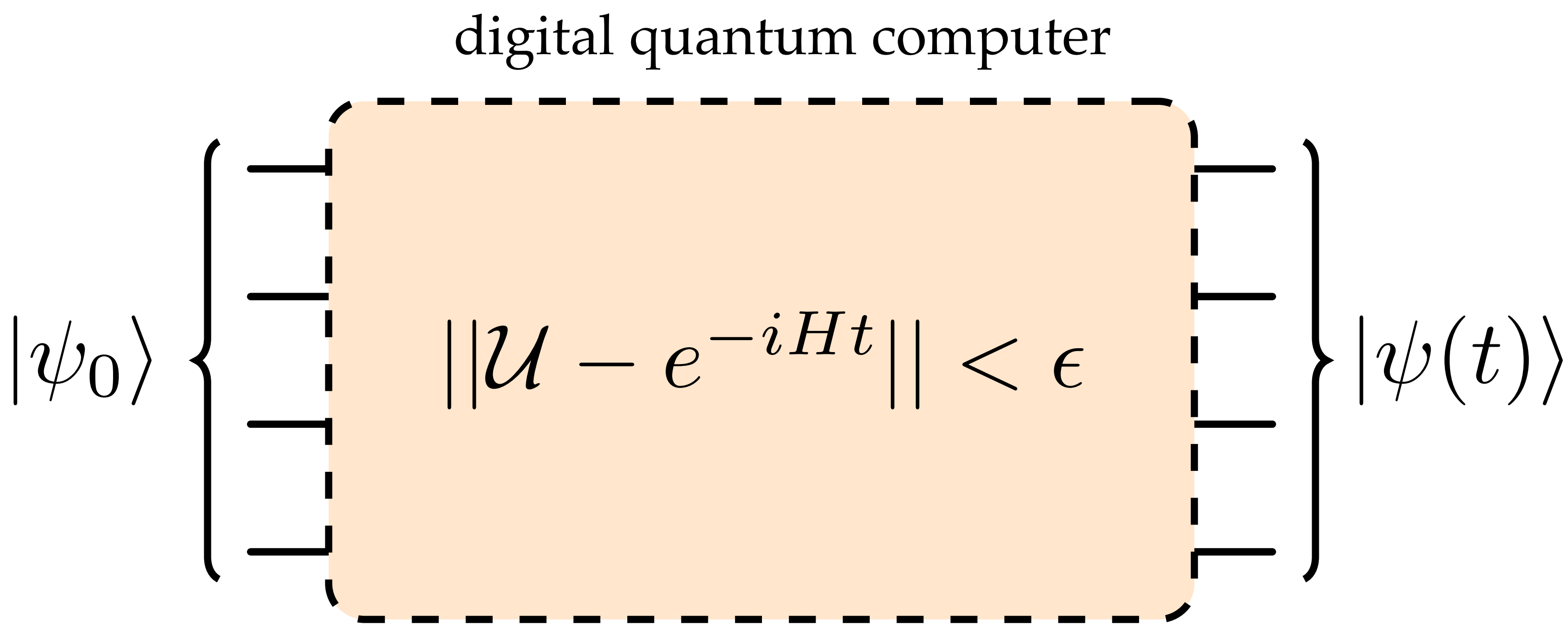


Near-term hardwares are reaching such error threshold!

applying to various methods of
encoding gauge field

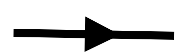
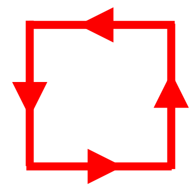
including charged matters

Propagation



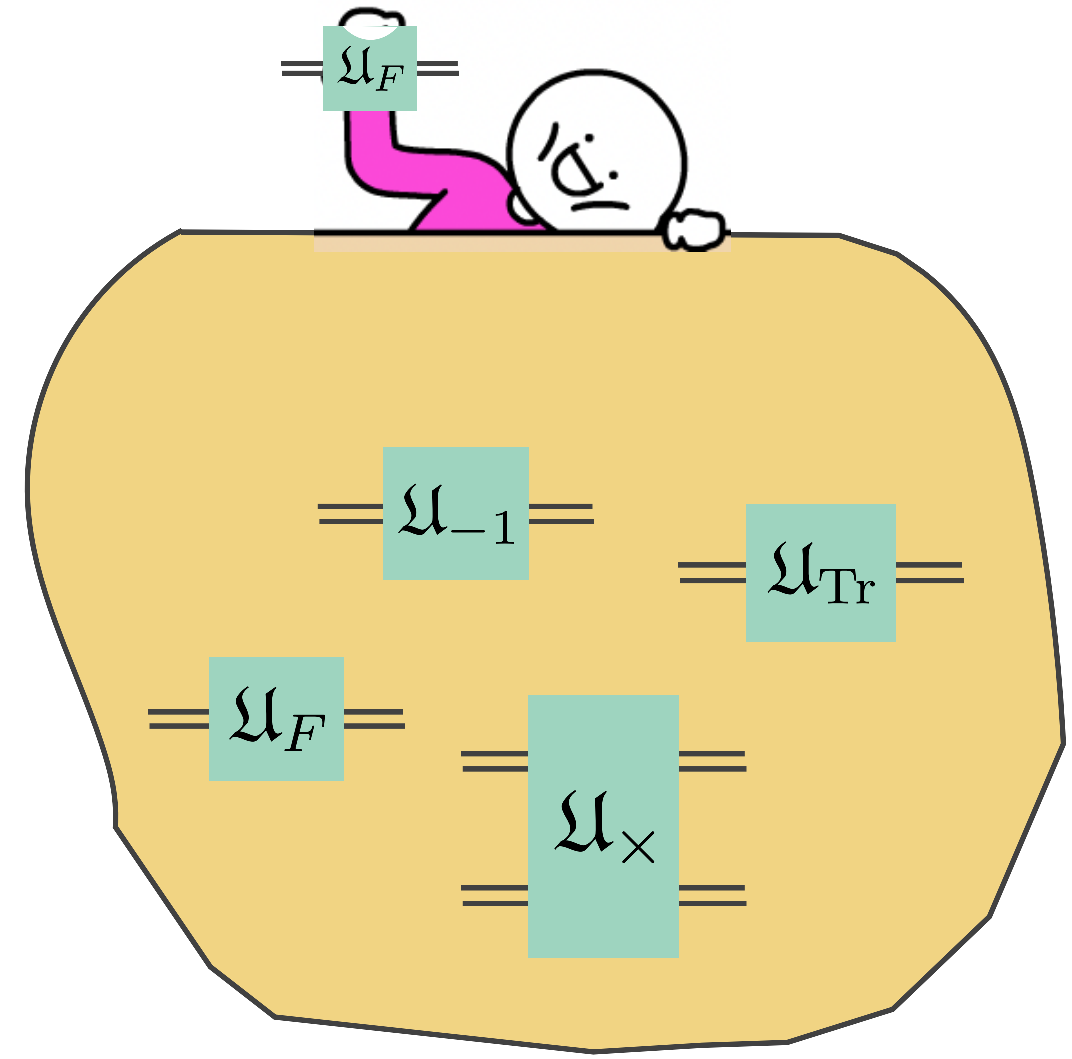
time-evolution with gauge redundant encodings

Propagation with gauge redundant encodings

$H_{KS} = \sum \Big($  $+$  $\Big)$

K_L $\qquad$ $U_\square$

$G-$register : $|U\rangle$ ══

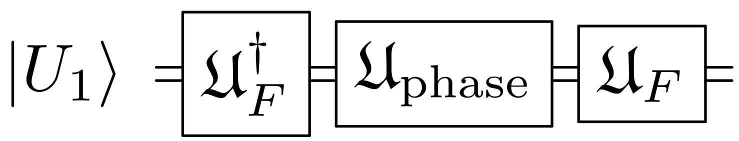


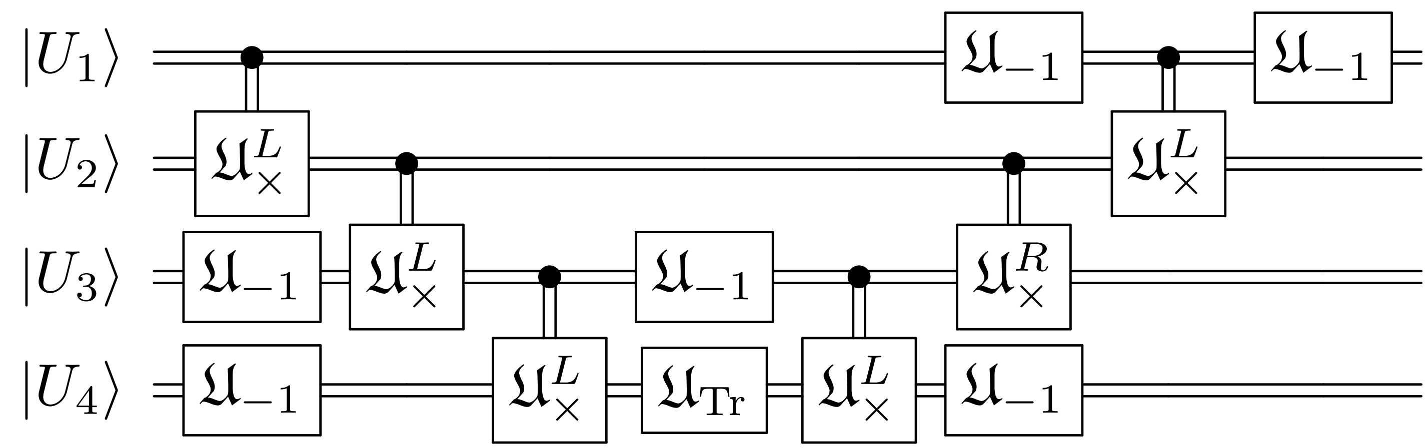
$$\mathcal{U}(t) = e^{-iH_{KS}t}$$
$$\approx \left[e^{-i\delta t K_L} e^{-i\delta t U_\square}\right]^{t/\delta t}$$

Propagation with gauge redundant encodings

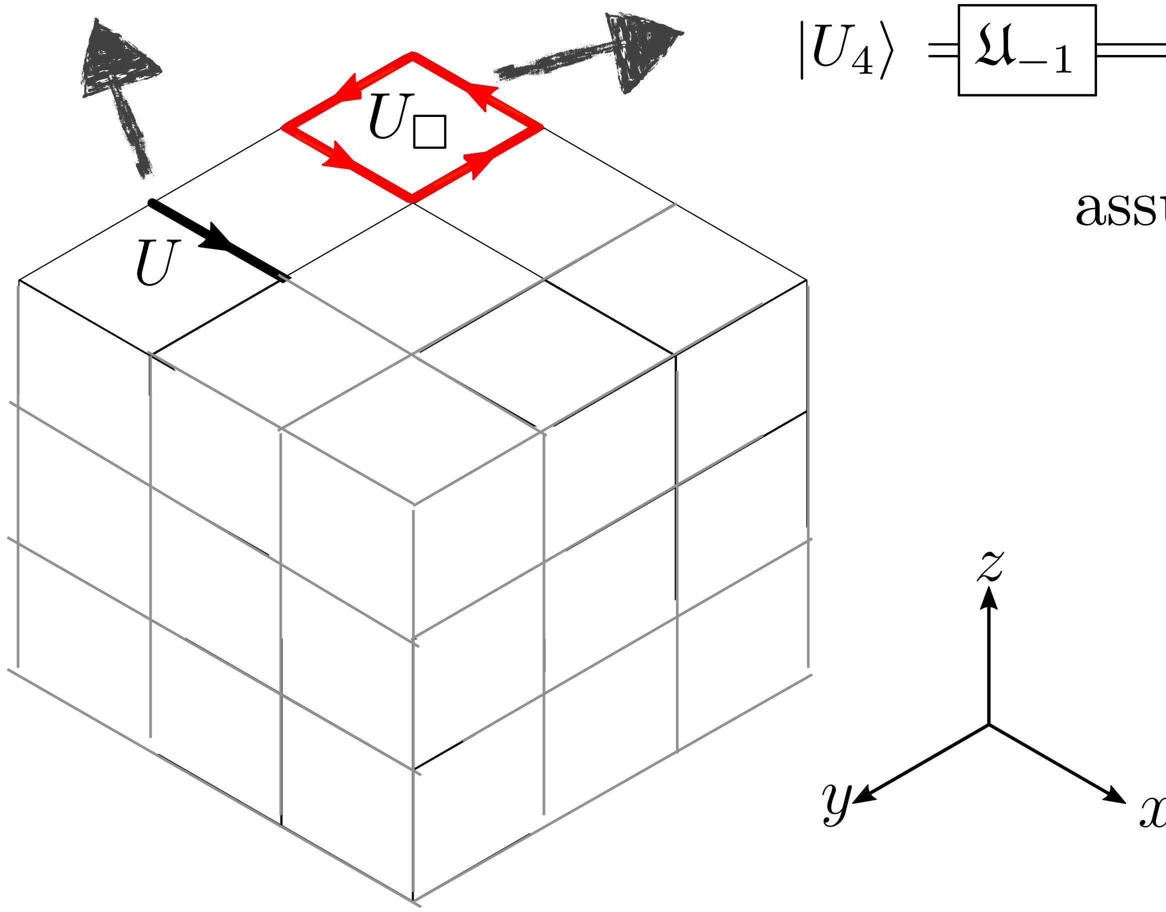
$$e^{-i\delta t U_\square}$$

$$e^{-i\delta t K_L}$$





assuming linear register connectivity



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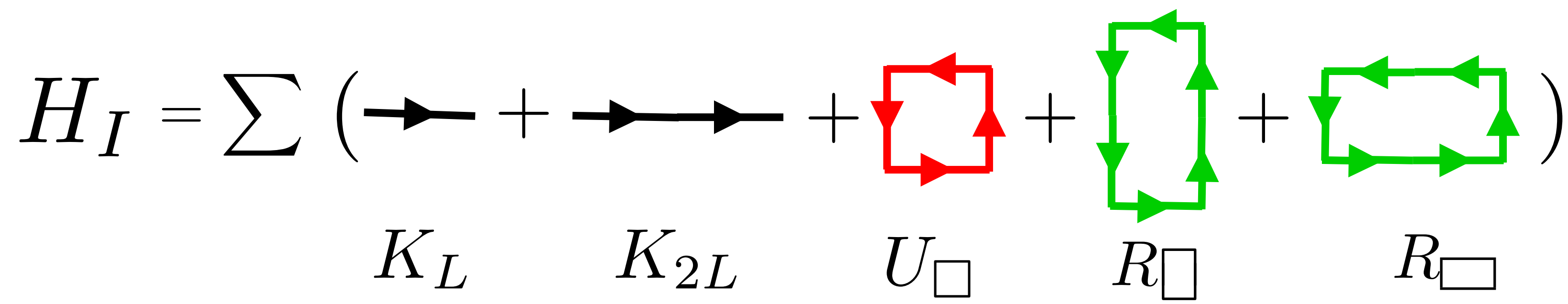
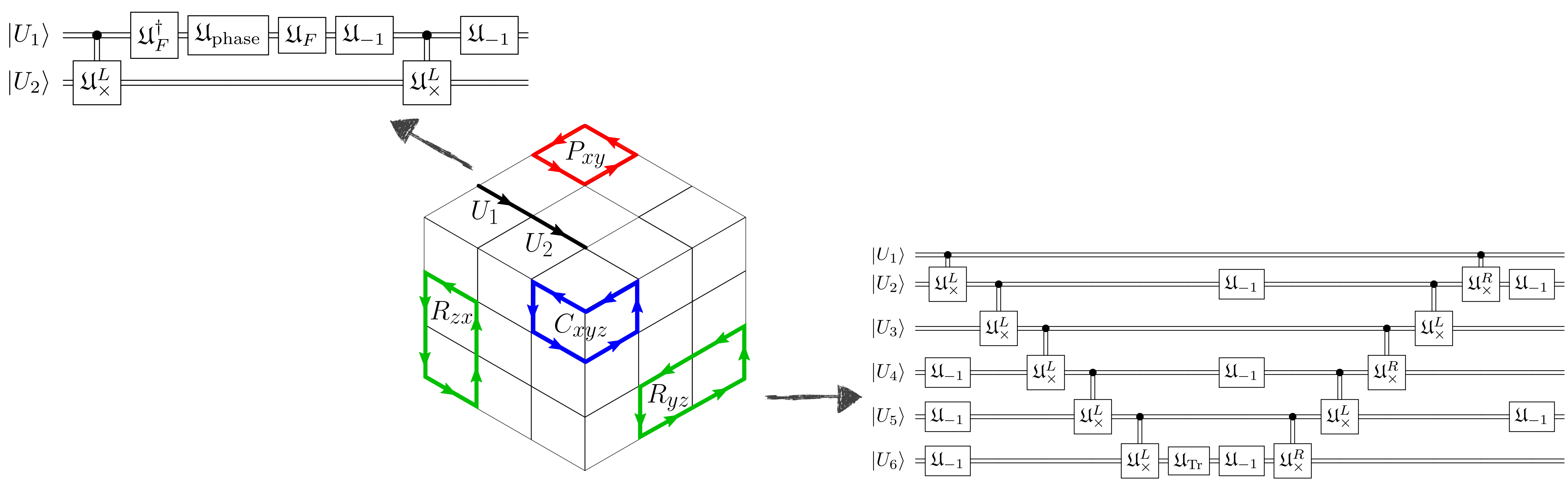
Propagation with gauge redundant encodings



$$\langle U_1', U_2' | \mathcal{U}_{K_{2L}} | U_1, U_2 \rangle = \delta_{U_1' U_2', U_1 U_2} \langle U_1' | e^{i\theta K_{L_1}} | U_1 \rangle$$



Demonstration of improved Hamiltonian is allowed in the near future

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Summary and Outlook

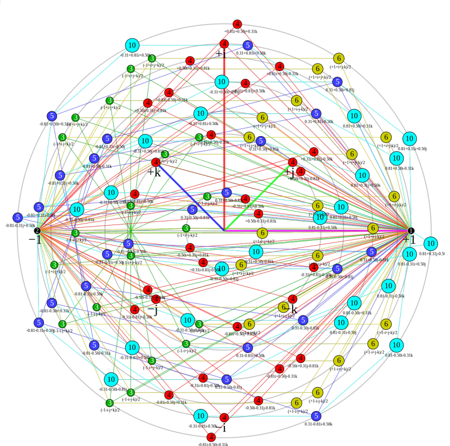
- **Gauge redundancy as quantum error correction codes**

  quantum error threshold for gauge-redundant digitization, with the error rate achievable for near-term quantum devices.

- **Techniques on real-time simulation of lattice field theory**

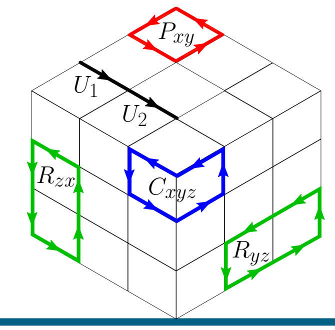
  improved Hamiltonian: matrix elements for the improved terms, circuits designed

discrete subgroup redundancies



improved Hamiltonian demonstration with redundancies

$$|\langle H_{KS}(a) - H\rangle| \sim |\langle H_I(2a) - H\rangle|?$$

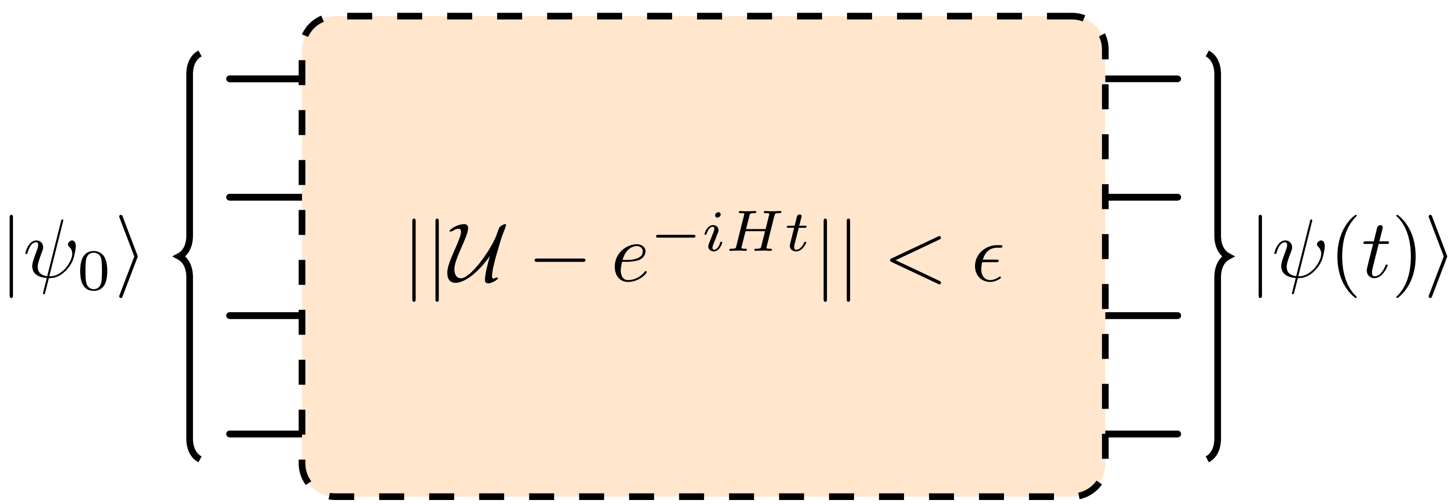


Thank you

BACK UP

Propagation with gauge redundant encodings

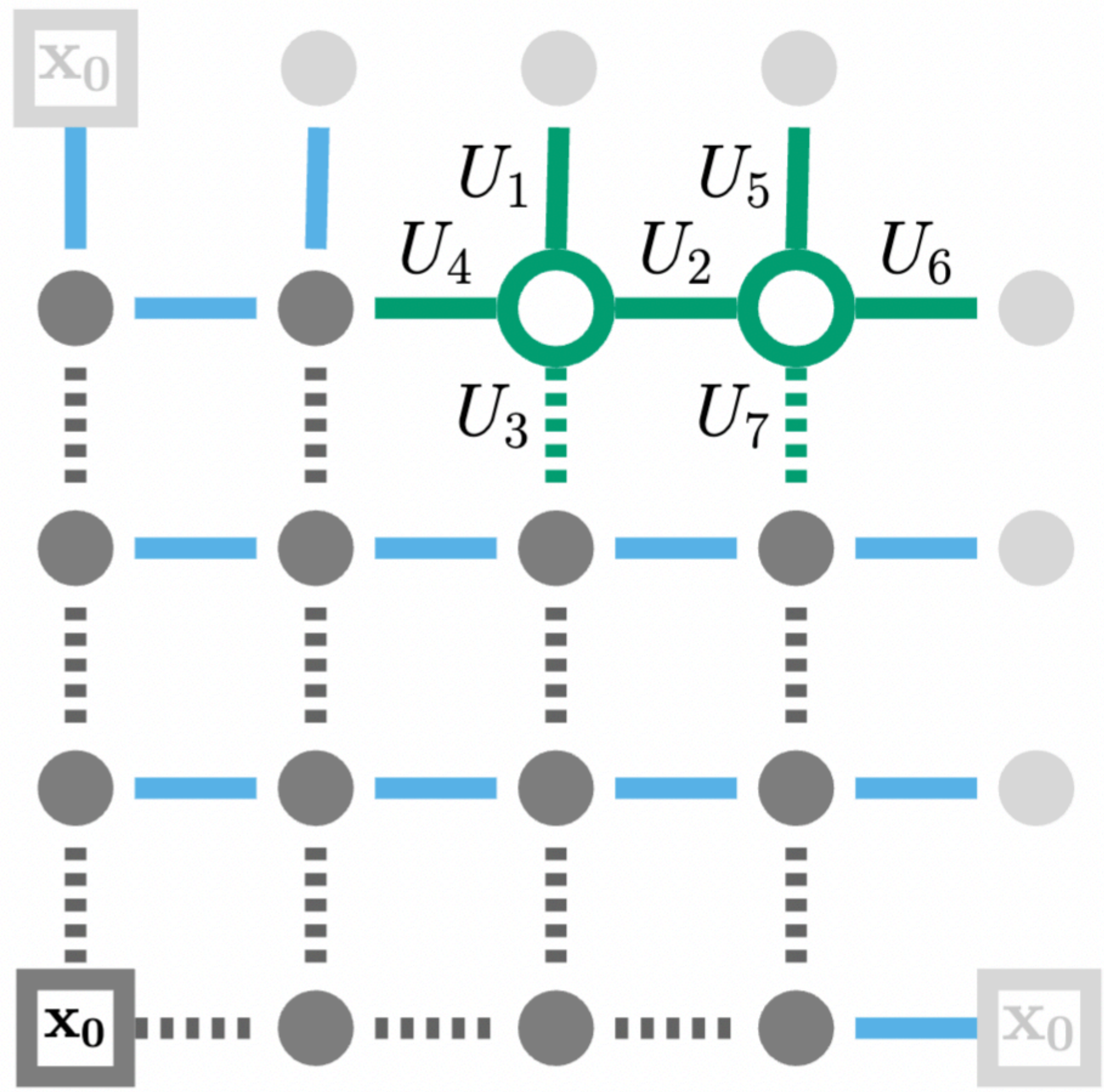
DIGITAL



Operator	Gate(s)	Matrix
Pauli-X (X)	X, ⊕	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)	Y	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)	H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	S	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	T	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)	Z	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Gauge redundancy utilized for error corrections

$\mathcal{H}_{\rm red}$



Maximal Tree Gauge