QuantHep, Sep 4 2024

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IQOQI

PROBING TOPOLOGICAL ENTANGLEMENT ON LARGE SCALES

Based on: *arXiv 2408.12645*

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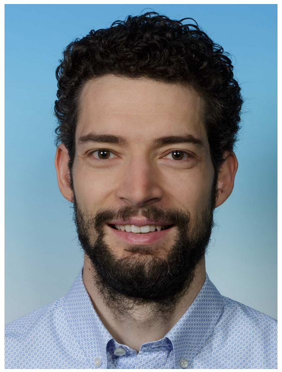
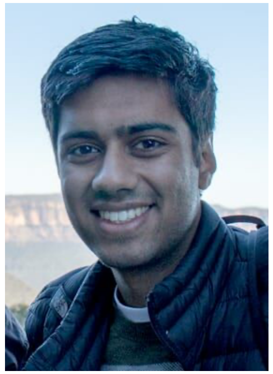
Torsten Zache

Peter Zoller



Hannes Pichler

Harvard:



Nishad Maskara

Mikhail Lukin

Certification in quantum simulation

non-universal properties

- Error detection/mitigation
- Hamiltonian learning
  Ott, Zache, Prüfer, Erne, Tajik, Pichler, Schmiedmayer, Zoller. *arXiv*:2401.01308 (2024).
- ...

universal properties

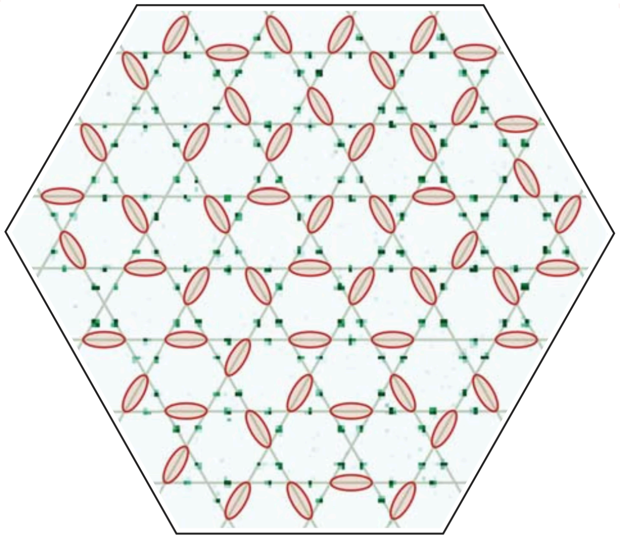
- Phase properties
- Universal exponents
- Large-scale structure of states
- ...

This talk!

Long-range topological entanglement

Condensed matter physics

Kitaev, Preskill. *PRL* **96** (2006)
Levin, Wen. *PRL* **96** (2006)



Semeghini et al. *Science* **374** (2021)

Freedman et al. *Bull.AMS* **40** (2003)
Fowler et al. *PRA* **86** (2012)

High-energy physics

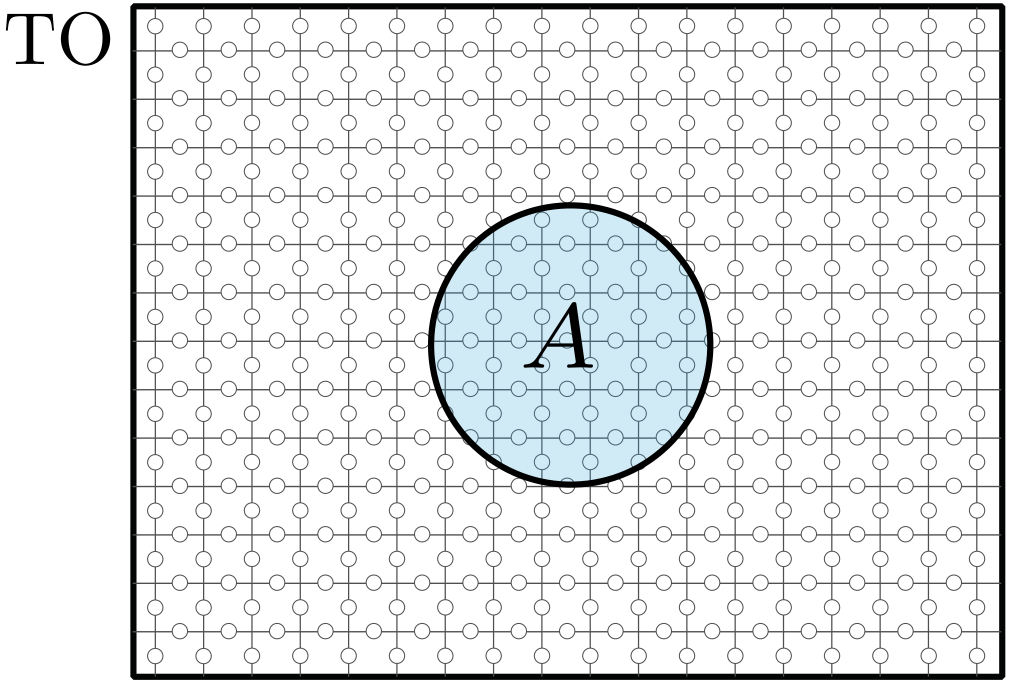
Pretko, Senthil. *PRB* **94** (2016)
Radičević. JHEP **2016.4** (2016)

Experiments:

- Neutral atoms: Semeghini et al. *Science* **374** (2021), Bluvstein et al. *Nature* **626** (2024)
- Trapped ions: Iqbal et al. *Nature* **626** (2024) & *arXiv*:2302.01917 (2023)
- Superconducting qubits: Satzinger et al. *Science* **374** (2021), S. Xu et al. *arXiv*:2404.00091 (2024)

Long-range topological entanglement Kitaev, Preskill. *PRL* **96** (2006)
Levin, Wen. *PRL* **96** (2006)

$$S_A = \alpha L_A - S_{\text{topo}} + \mathcal{O}(1/L_A)$$



$$S_{\text{topo}} = \log(D) \Rightarrow D = \text{total quantum dimension}$$

Topological order and **lattice gauge theories**

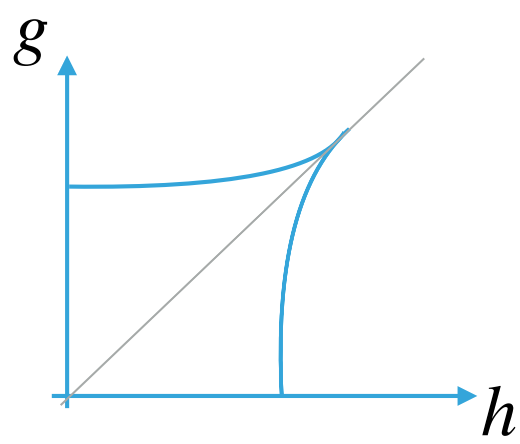
Z_2 LGT

$$H_{Z_2} = H_m + H_E + H_B \qquad G|\psi\rangle = 0 \qquad G = \prod_i Z_i - Q_i$$

Toric code

$$H = -\epsilon_e \sum\nolimits_v A_v - \epsilon_m \sum\nolimits_p B_p + h \sum\nolimits_l Z_l + g \sum\nolimits_l X_l$$

Phase diagram

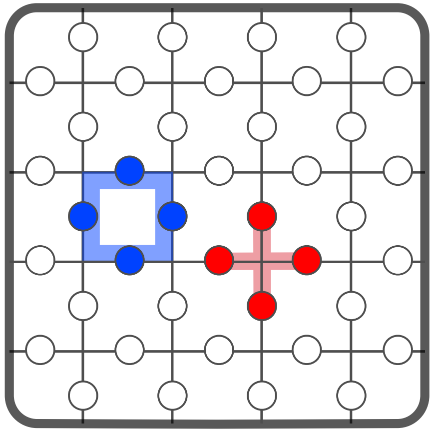


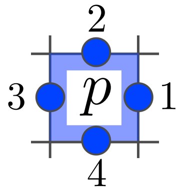
Fradkin. *Field Theory of Condensed Matter Physics* (2006)

Probing long-range topological entanglement

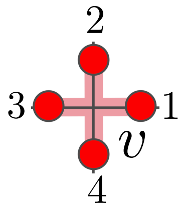
Example: ``Toric code” fixed-point state: $\mathbb{Z}_2$ topological order

$$H = -\epsilon_e \sum_v A_v - \epsilon_m \sum_p B_p$$





$$= B_p = X_1 X_2 X_3 X_4$$



$$= A_v = Z_1 Z_2 Z_3 Z_4$$

Operators are mutually commuting! $[A_v, B_p] = 0 \;\; \forall v, p$ (using that $XZ = -ZX$)

Model is exactly solvable by projecting on +1 eigenspace of all stabilisers

$$P_{p,+1} = \left(\frac{\mathbf{1} + B_p}{2}\right) \longrightarrow \; |\psi\rangle = \prod_p \left(\frac{\mathbf{1} + B_p}{2}\right) |\uparrow\rangle^{\otimes N}$$

Example: ``Toric code” fixed-point state: $\mathbb{Z}_2$ topological order

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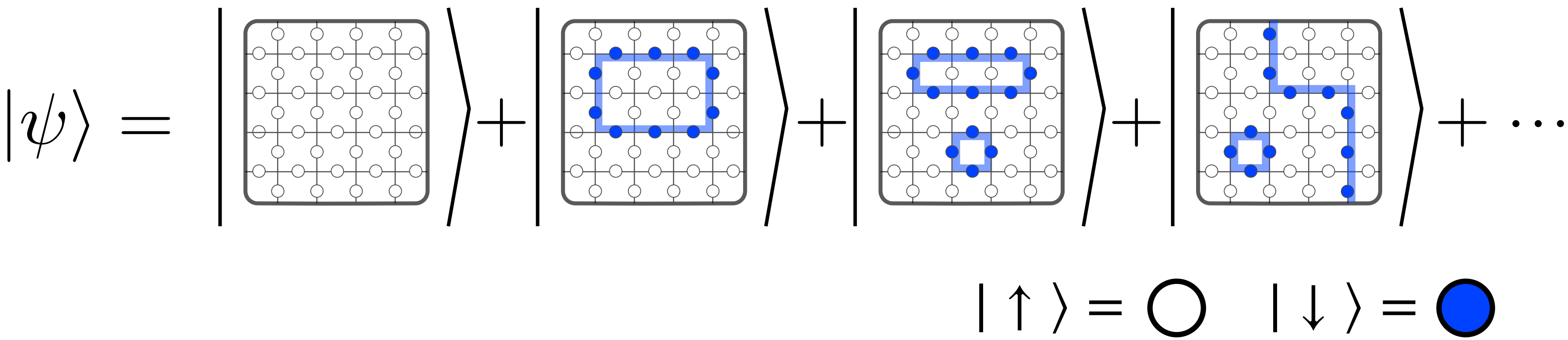
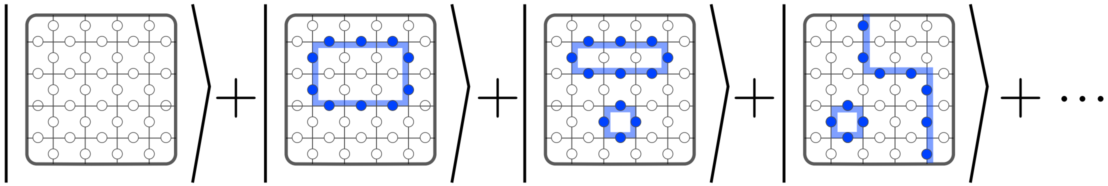
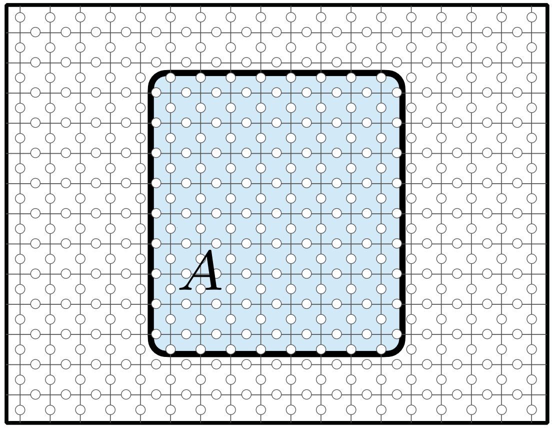
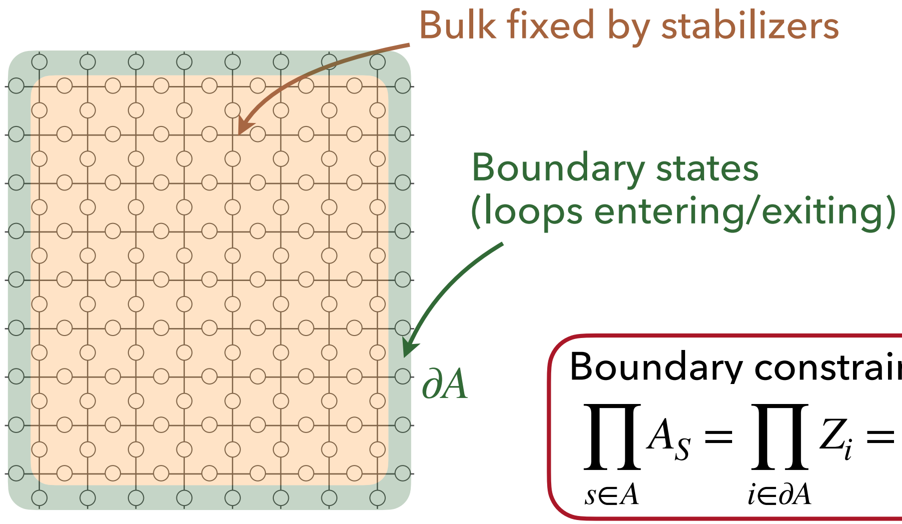
Ground state: equal-weight superposition of all loop/string configurations



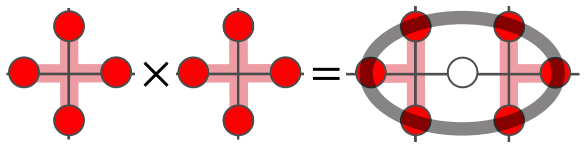
Example: "Toric code" fixed-point state: $\mathbb{Z}_2$ topological order

$|\psi\rangle =$ 



$\rho_A \sim$ 

Boundary constraint:
$$\prod_{s \in A} A_S = \prod_{i \in \partial A} Z_i = 1$$



Entropy: Number of different boundary configurations: $2^{L_A - 1}$

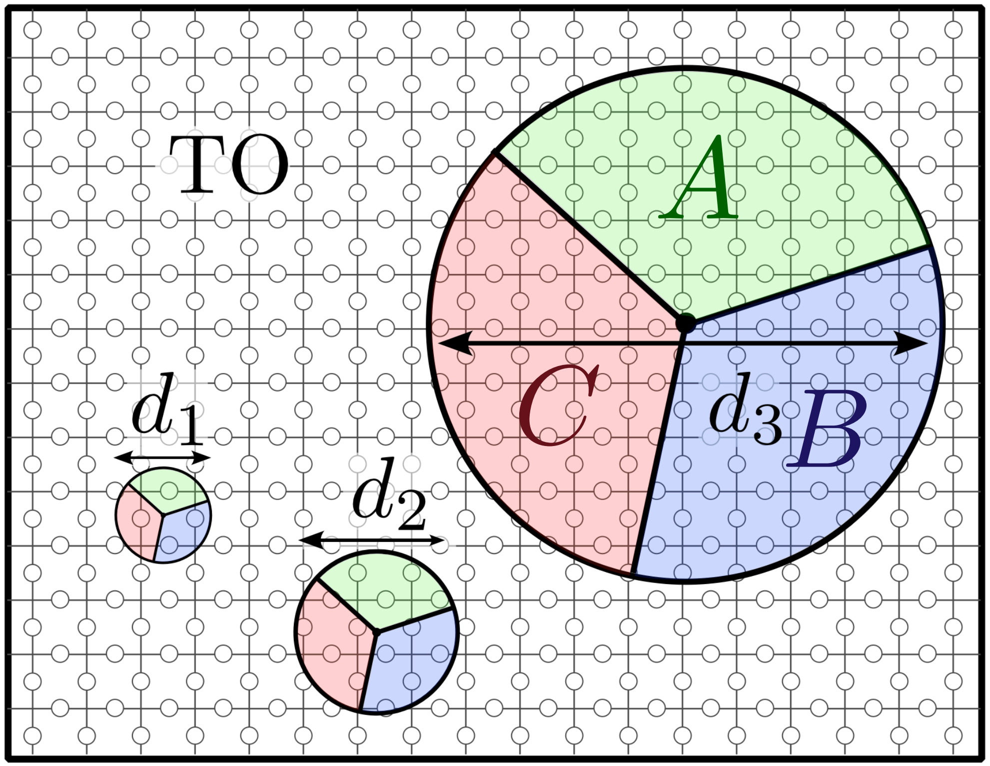
Entanglement entropy: $S_A = \log(2) L_A - \log(2) \;\rightarrow\; S_{\text{topo}} = \log(2)$

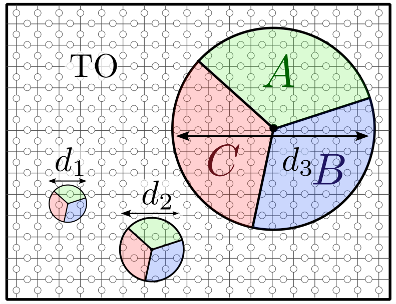
Long-range topological entanglement

Kitaev, Preskill. *PRL* **96** (2006)
Levin, Wen. *PRL* **96** (2006)

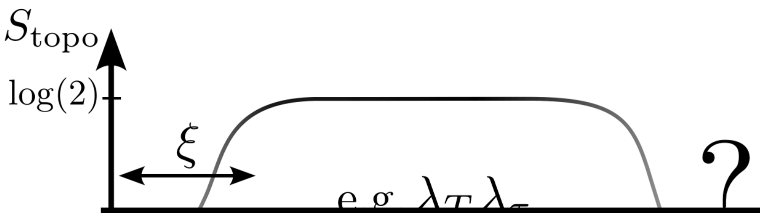
Kitaev-Preskill construction:

$$-S_{\text{topo}} = S_{ABC} - S_{AB} - S_{BC} - S_{AC} + S_A + S_B + S_C$$





Scale-dependence of topological entanglement:

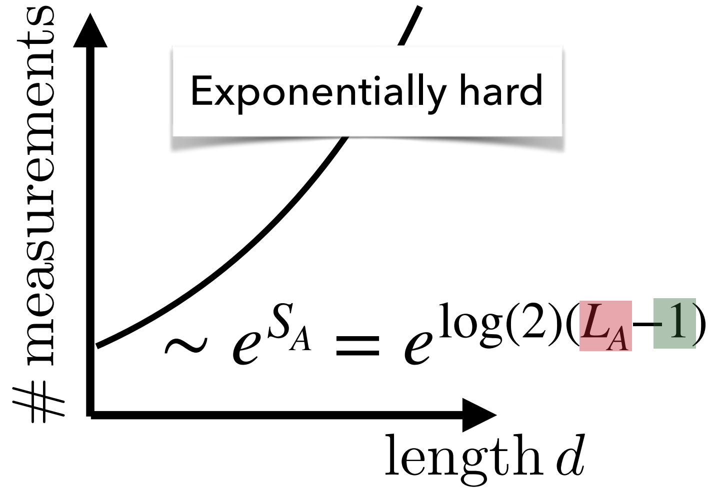


- Short-range correlations ξ
- Thermal excitation scale λ_T

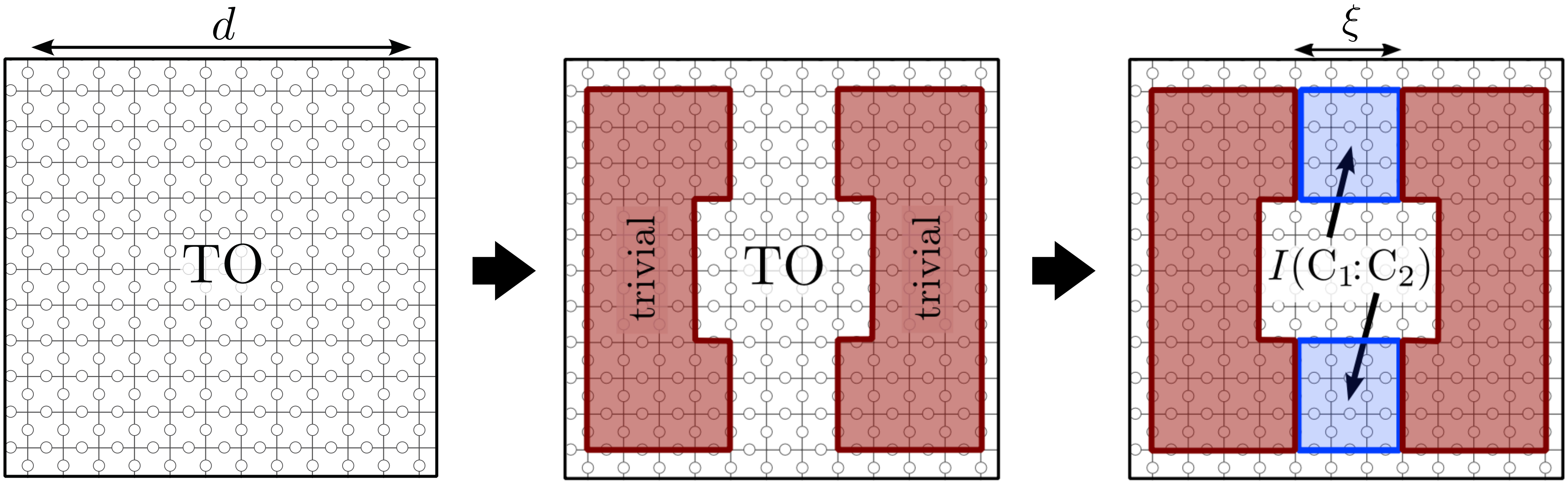
Challenge: Can we efficiently measure length scale over which system looks topologically ordered?

Techniques of **measuring** entanglement measures:

- Tomography — Cramer et al. *Nature comm.* **1.1** (2010)
- Copies & SWAP — Daley et al. *PRL* **109** (2012); Islam et al. Nature **528** (2015); Lukin et al. *Science* **364** (2019)
- Randomized measurements — Elben et al. *Nature Reviews Physics* **5** (2023); Huang et al. *Nature Physics* **16** (2020); KJ Satzinger et al. Science **374** (2021)



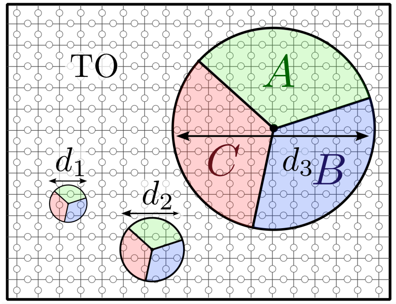
Protocol to measure topological entanglement at large scales (for **gapped** TO phases)

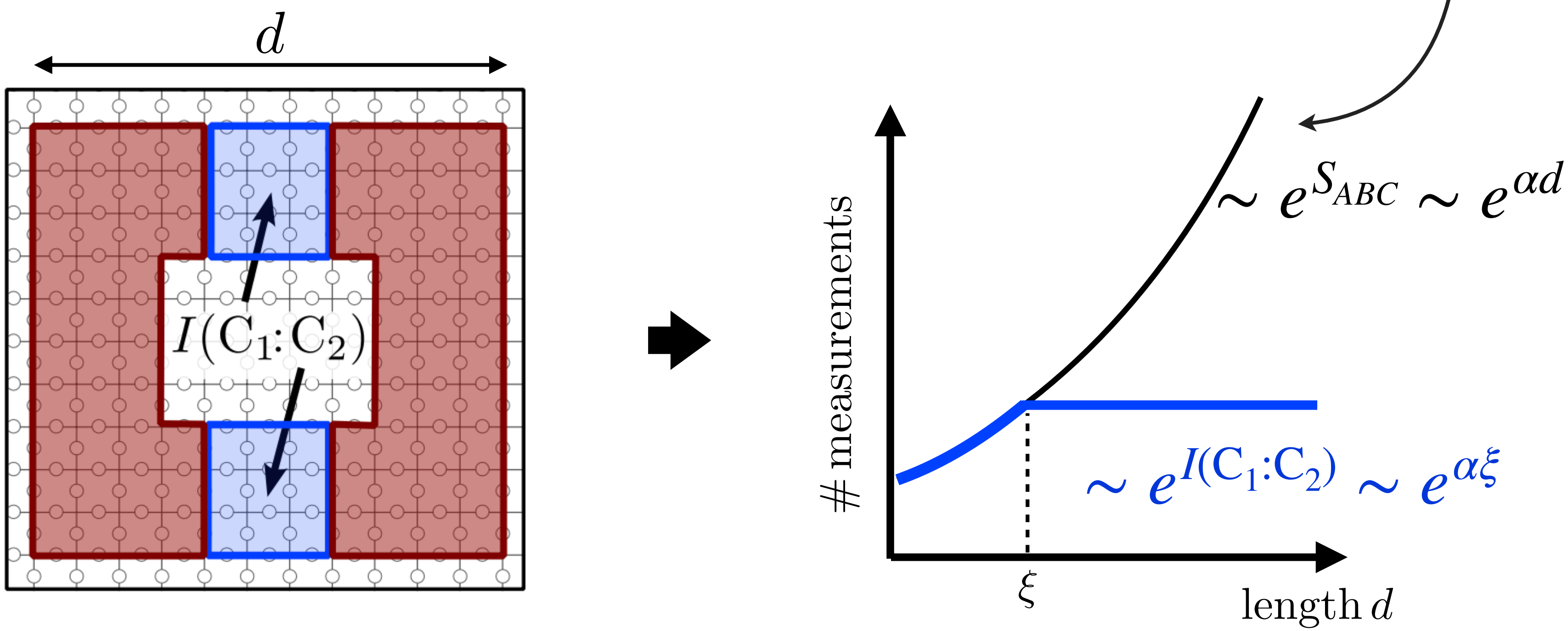


- Probing topological entanglement at scale d
- Adiabatically deform system (trivializing two regions)
- Measure mutual information of ``openings" of size ξ

➡ **Claim**: $I(\mathrm{C}_1 : \mathrm{C}_2) = \log(D)$ (abelian string-net models)

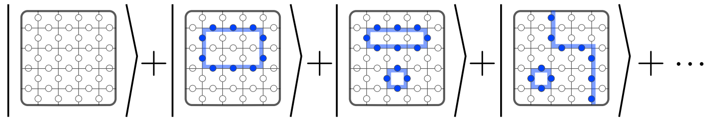
Measurement budget to extract topological entropy:





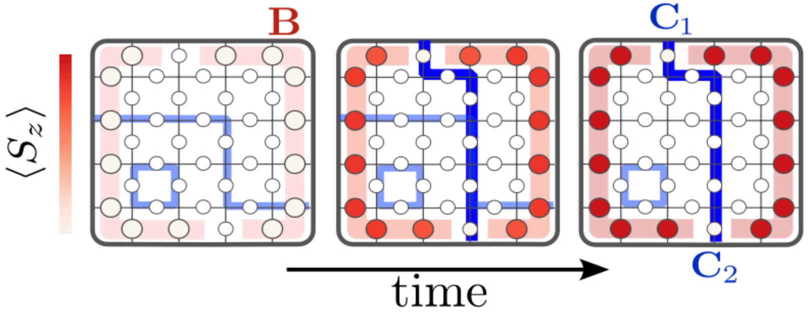
➡ Protocol drives TO-to-trivial phase transition on (finite-size) boundary

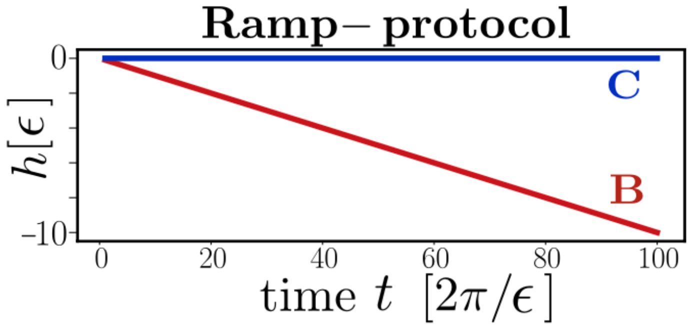
➡ Time scale of protocol determined by nature of transition

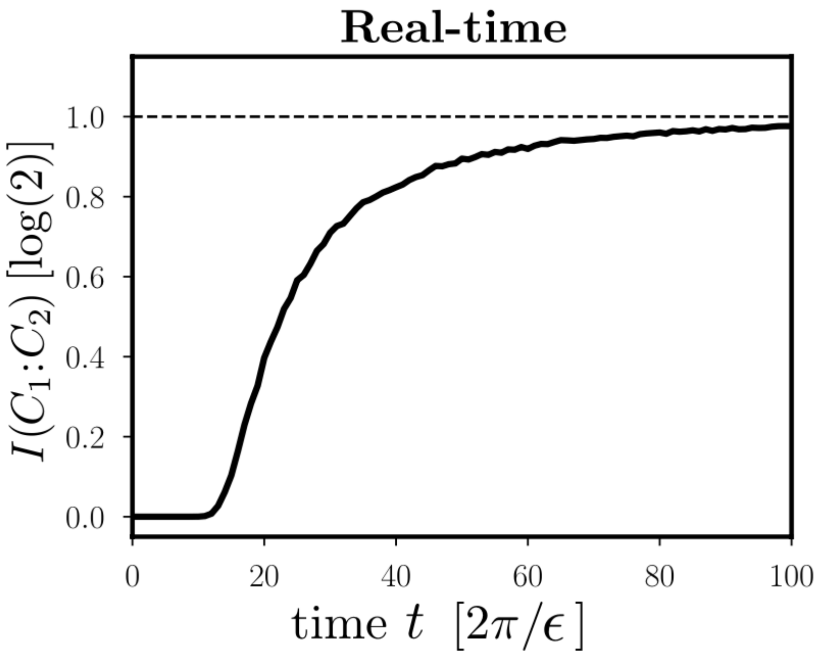
$|\psi\rangle =$ 

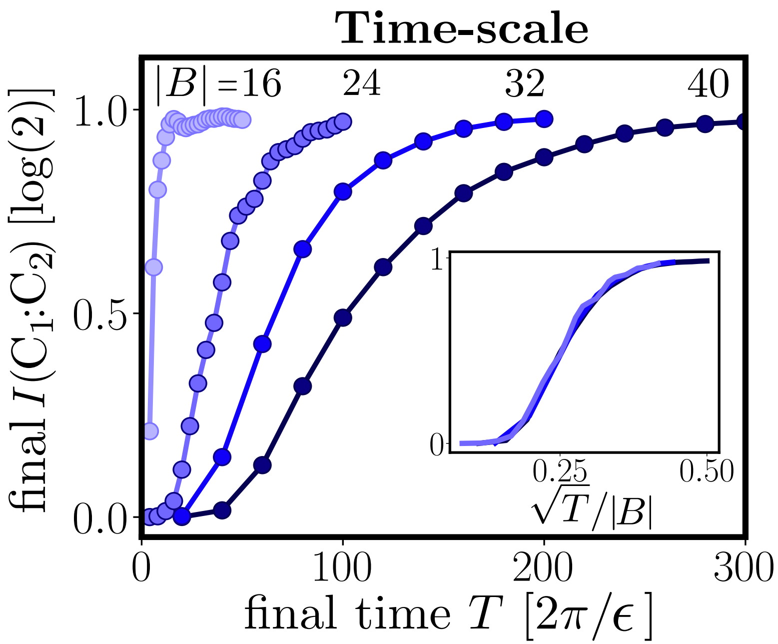
Example: ``Toric code" fixed-point state: $\mathbb{Z}_2$ topological order

$$H(t) = -\epsilon_e \sum_v A_v - \epsilon_m \sum_p B_p + h(t) \sum_i Z_i$$









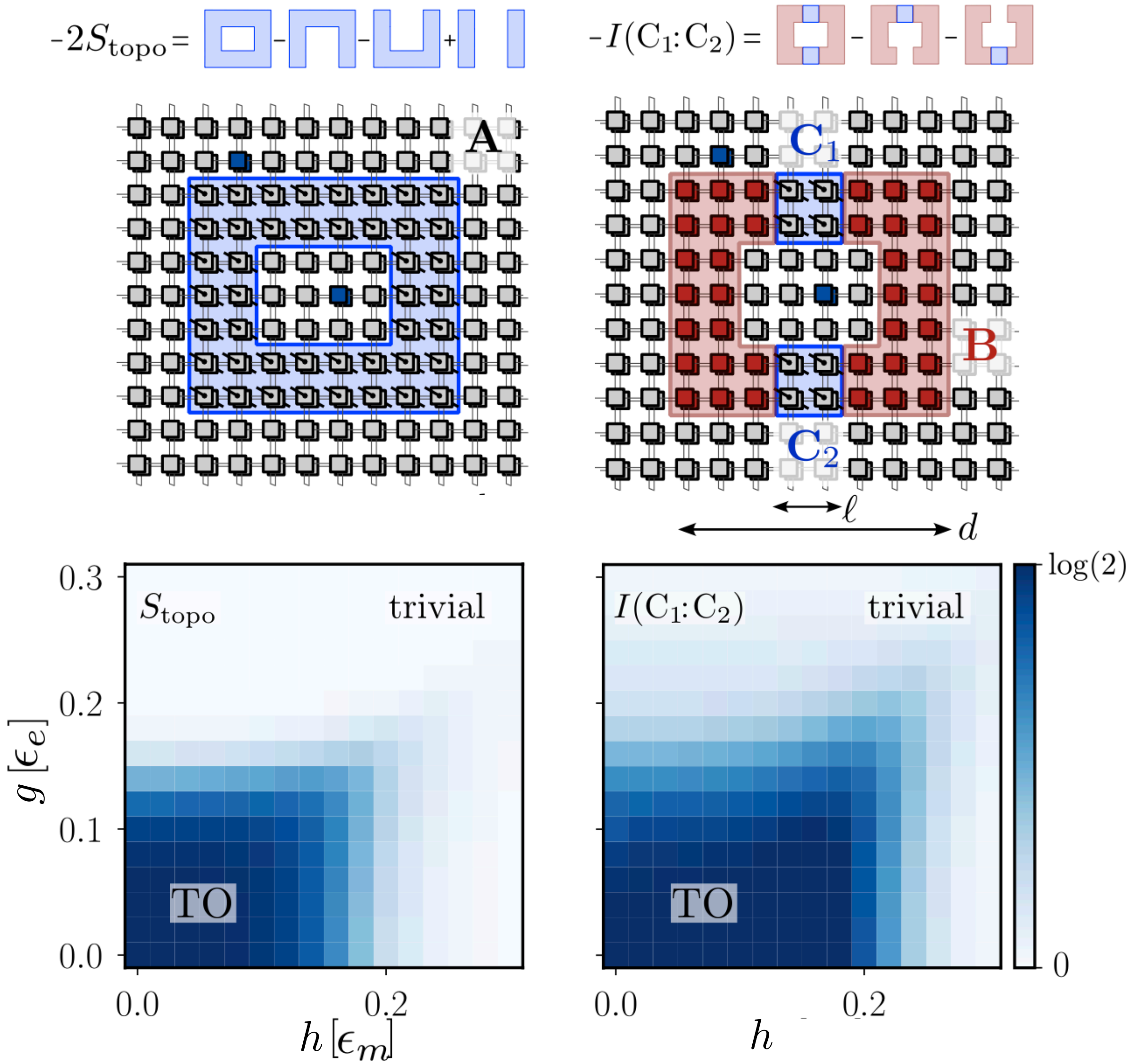
Example: Beyond ``Toric code" fixed-point state

PEPS ansatz:

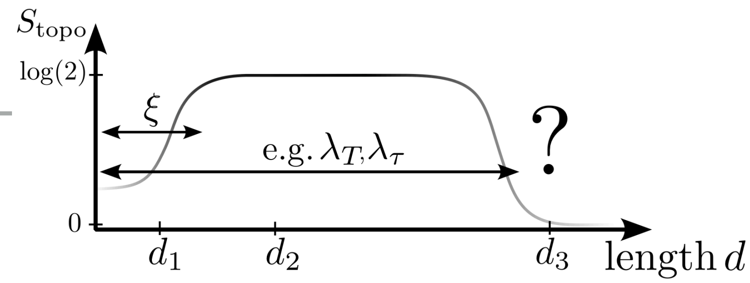
Cong et al. *Nature Comm.* (2024)

$$|\psi\rangle \sim e^{gX+hZ}|\psi_{\mathrm{TC}}\rangle$$

- PEPS with bond dimension 2
- Interpolates between TO- and trivial phase
- Local deformations h realise the trivial boundary region B
- Mutual information recovers topological (Renyi) entropy



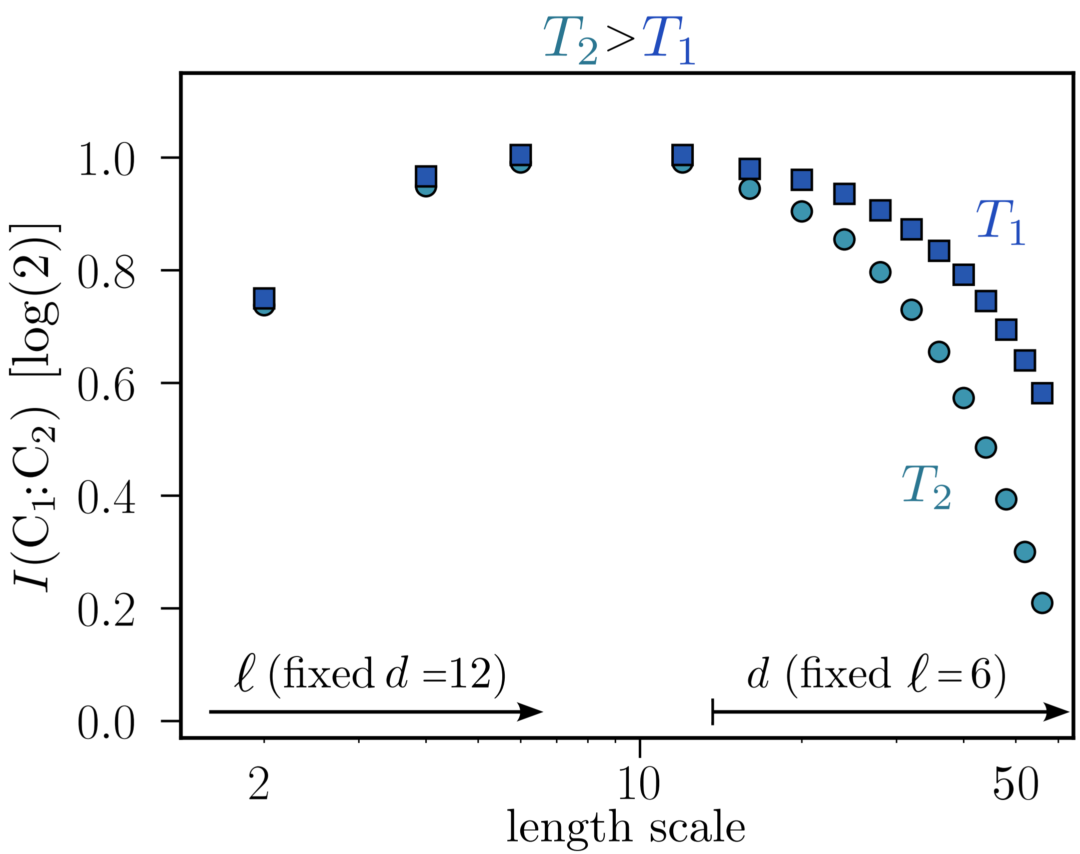
system sizes $[L,d,\ell]$ are [10,4,2]

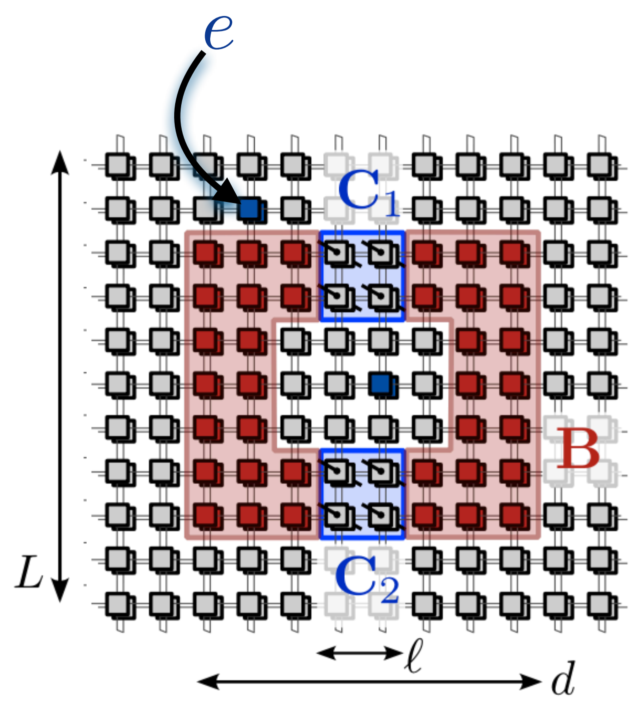


Example: Thermal/incoherent excitations

$$\rho \propto e^{\sum_\ell g_\ell^X X_\ell} e^{-\beta H_{\rm TC}} e^{\sum_\ell g_\ell^X X_\ell}$$

gives rise to **length scale** of topological entanglement



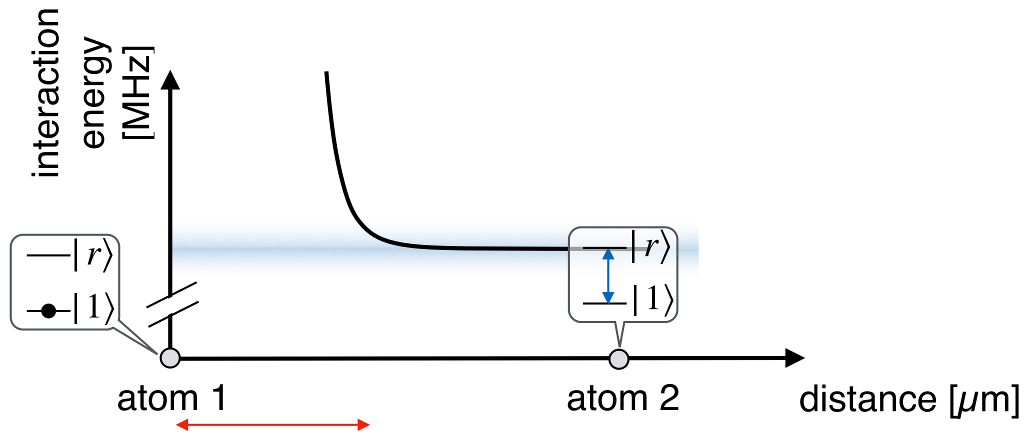


Two driven atoms: Rydberg Blockade

- **Hamiltonian**

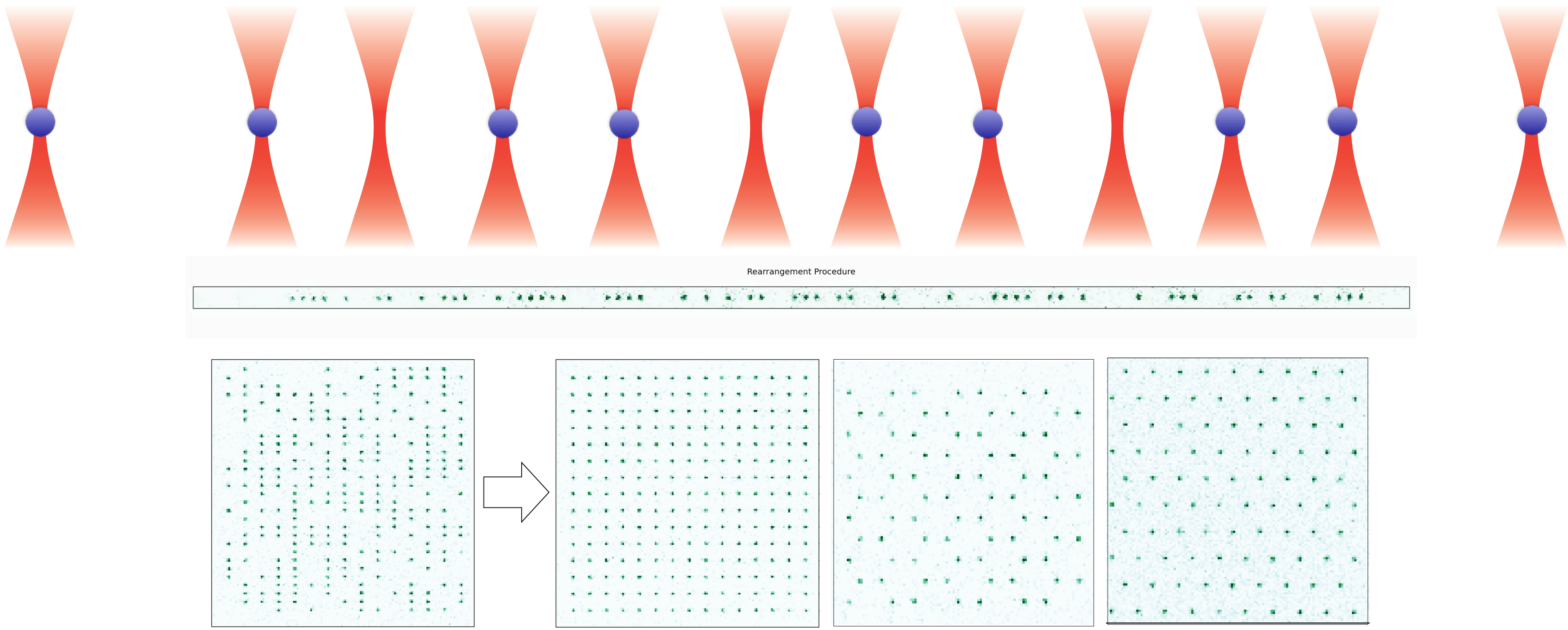
laser drive | interactions

$$H = \sum_{i=1,2} \left(\frac{\Omega}{2} \left(|r\rangle_i \langle 1| + |1\rangle_i \langle r| \right) - \Delta |r\rangle_i \langle r| \right) + \frac{C_6}{R^6} |r\rangle_1 \langle r| \otimes |r\rangle_2 \langle r|$$



Blockade radius R_B [1-10μm]

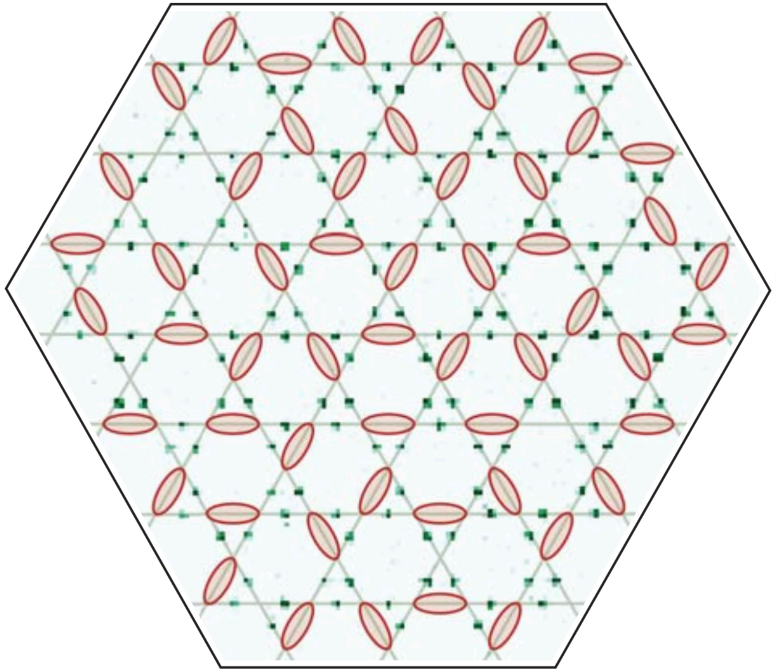
Atom-by-atom approach for building quantum matter



Harvard: Lukin group

Experiments: Wisconsin, Harvard/MIT, Paris, Boulder, Princeton, Caltech, KAIST…

Atom-by-atom approach for building quantum matter



Semeghini et al. *Science* ***374*** (2021)

Harvard: Lukin group

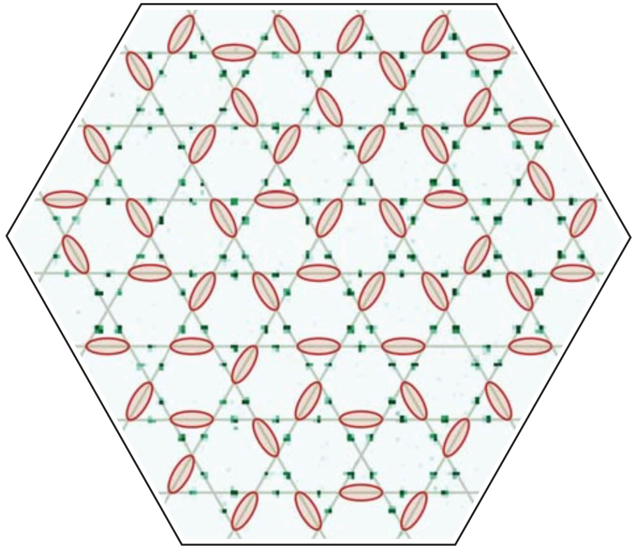
Experiments: Wisconsin, Harvard/MIT, Paris, Boulder, Princeton, Caltech, KAIST…

Experimental application: Rydberg atom arrays

PXP Hamiltonian in Kagome lattice (Rydberg blockade)

➡ $\mathbb{Z}_2$ topological order

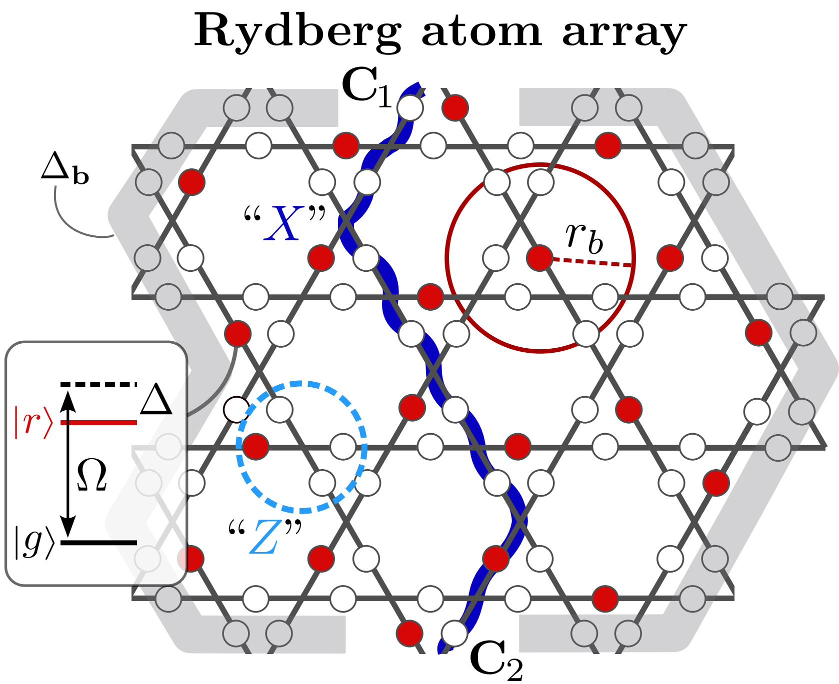
Verresen et al. *PRX* ***11*** (2021)
Semeghini et al. *Science* ***374*** (2021)

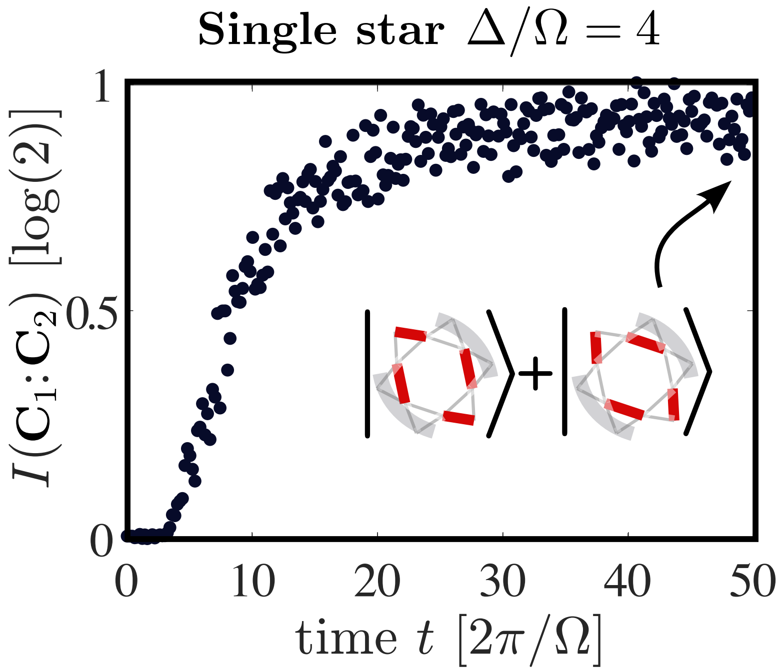


Semeghini et al. *Science* ***374*** (2021)

Consider additional local detuning on boundary:

$$H(t) = H_{\mathrm{PXP}} - \Delta_{\mathbf{b}}(t) \sum_{i \in \mathbf{b}} n_i$$





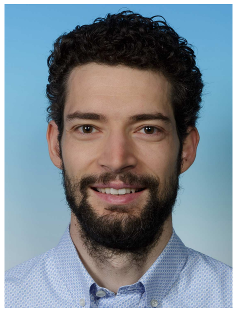
Discussion & Outlook

- Scalable protocol to probe topological entanglement at large scales
- Protocol time determined by nature of TO-to-trivial phase transition
- Applicable to various classes of TO phases
- Requires local control over single-body terms

Innsbruck:

Torsten Zache

Peter Zoller



Hannes Pichler

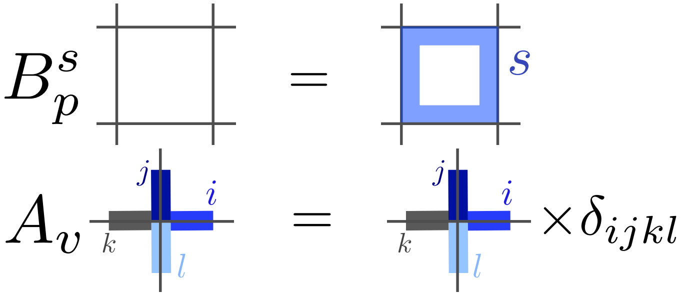
Harvard:

Nishad Maskara

Mikhail Lukin

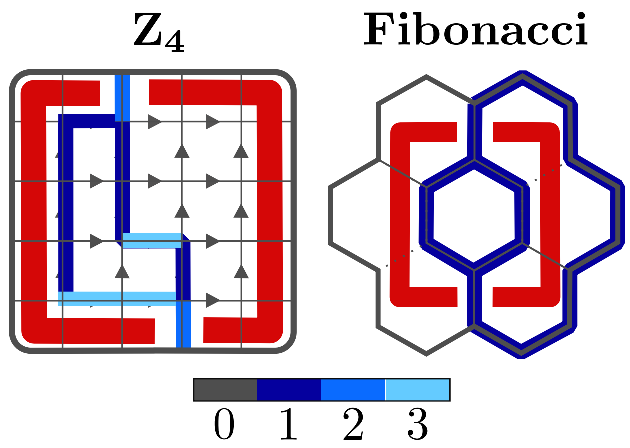
Generalizations: Abelian and Non-abelian string-net models Levin, Wen. *PRB* **71** (2005)

$$H = -\sum_v A_v - \sum_{p,s} a_s B_p^s$$



$$\delta_{ijkl} = \begin{cases} 1 & \text{fusion constraints fulfilled} \\ 0 & \text{else} \end{cases}$$

Examples:

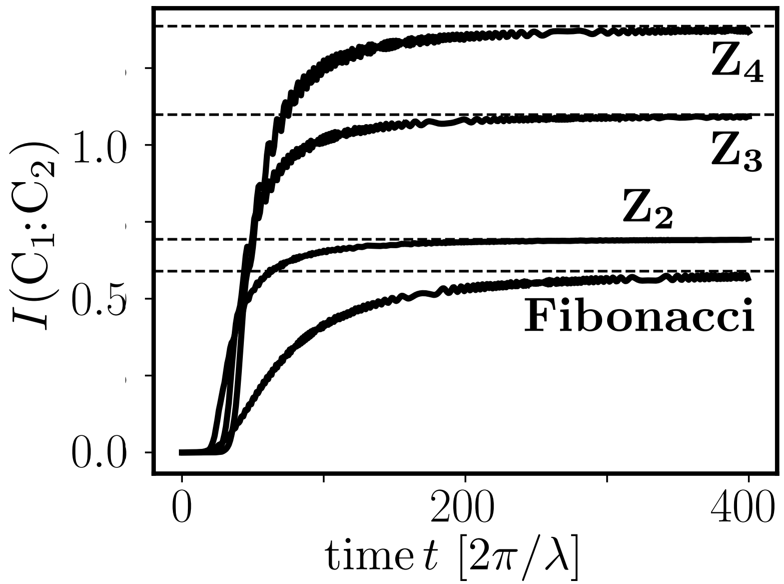


- Abelian: $I(\mathrm{C}_1 : \mathrm{C}_2) = \log(D)$
- Non-abelian:

$$I(\mathrm{C}_1 : \mathrm{C}_2) = -\log(D) + 2\sum_k \frac{d_k^2}{D}\log(\frac{d_k}{D})$$

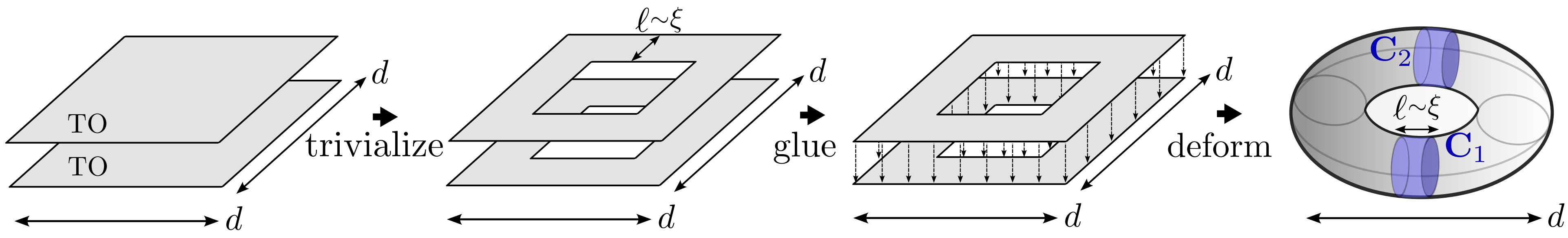
d_k : quantum dimension

$D = \sum_k d_k^2$: total quantum dimension



Extension to (undoubled) non-time-reversal-symmetric phases
Example: fractional quantum Hall states

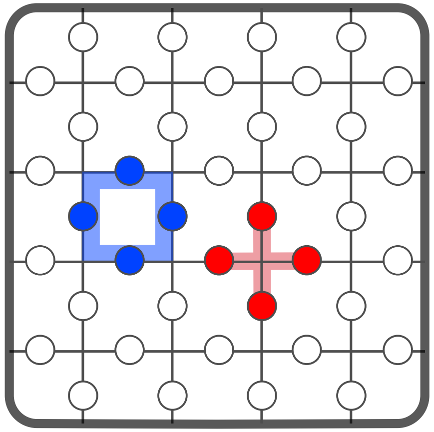
Glue and deform two TO sheets (time-reversals of each other):

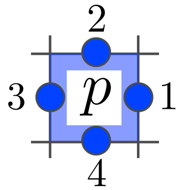


- Torus has no internal boundaries
- Mutual information extracted from regions around openings yields topological entanglement 2log(2)
- Extension to time-reversal-breaking phases e.g. fractional quantum hall states.

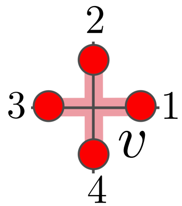
Example: ``Toric code” fixed-point state: $\mathbb{Z}_2$ topological order

$$H = -\epsilon_e \sum_v A_v - \epsilon_m \sum_p B_p$$





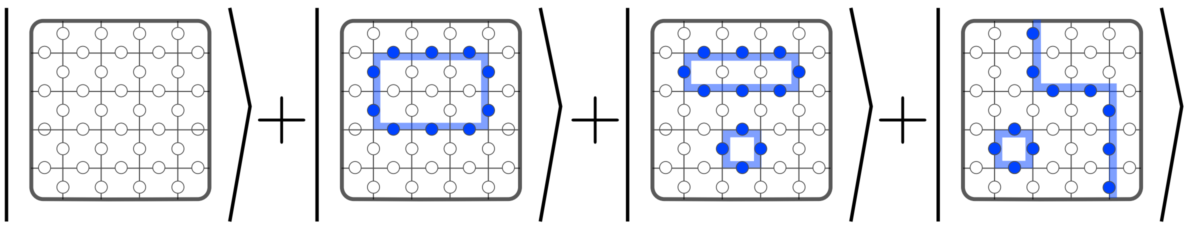
$= B_p = X_1 X_2 X_3 X_4$



$= A_v = Z_1 Z_2 Z_3 Z_4$

Ground state: equal-weight superposition of all loop/string configurations

$|\psi\rangle =$



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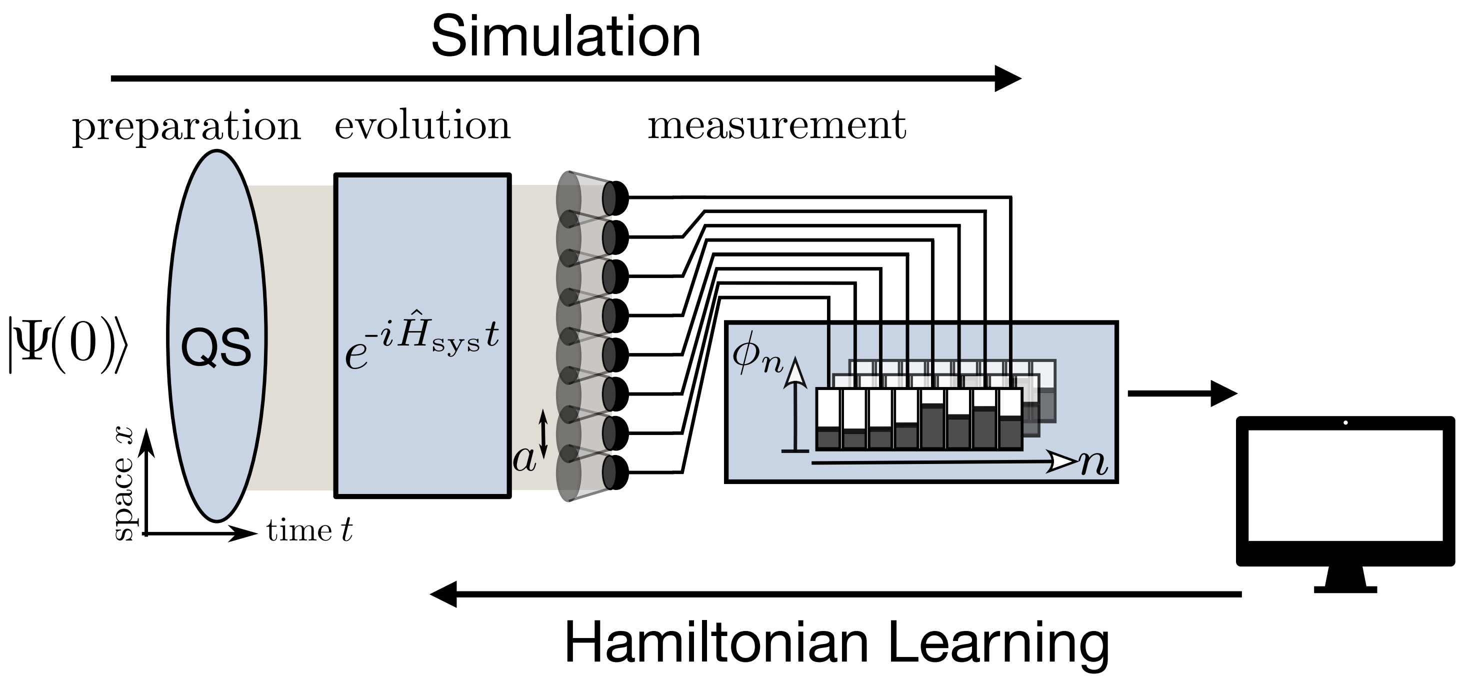

Simulation
preparation
evolution
measurement
$|\Psi(0)\rangle$
QS
$e^{-i\hat{H}_{\rm sys}t}$
a
ϕ_n
n
space x
time t
Hamiltonian Learning