QuantHEP, Sept 05 2024

Spin-exchange enabled quantum simulator for large-scale non-Abelian gauge theories

J. C. Halimeh*, L. Homeier*, A. Bohrdt, F. Grusdt

arXiv:2305.06373 (accepted in PRX Quantum)

Lukas Homeier
Ludwig-Maximilians-Universität Munich
JILA, University of Colorado, Boulder

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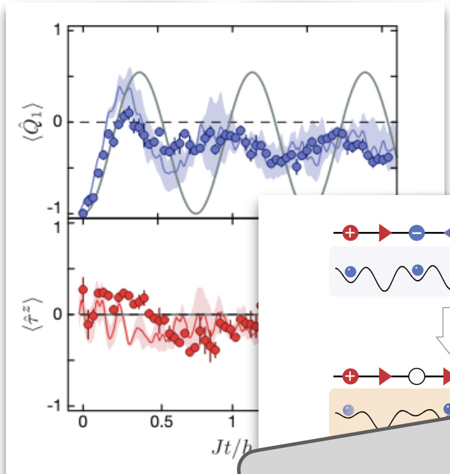
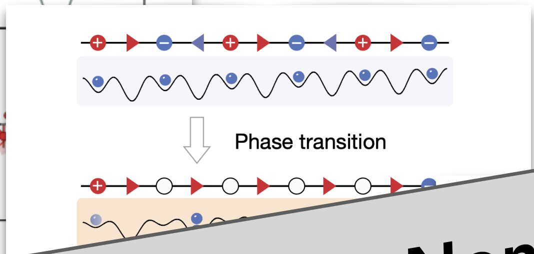
Quantum simulation of gauge theories



from: C. Schw… Nat. Phys. 1…

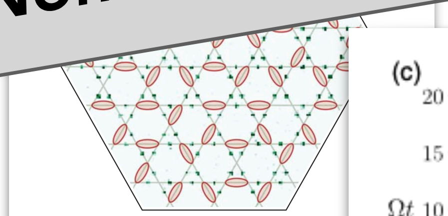


Building blocks

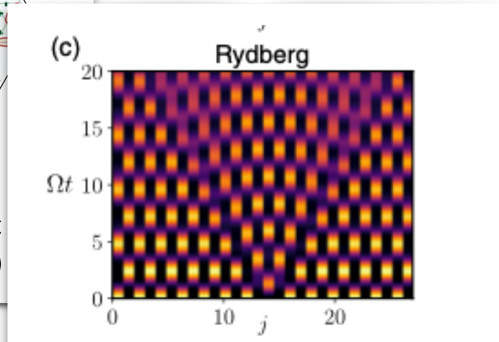
Gauge protection

…gent gauge structures

Next step: Non-Abelian lattice gauge theories



from: G. Semeghini et … Science **374** (2021)



from: Surace et al., PRX **10** (2022)

Efficient Hilbert space

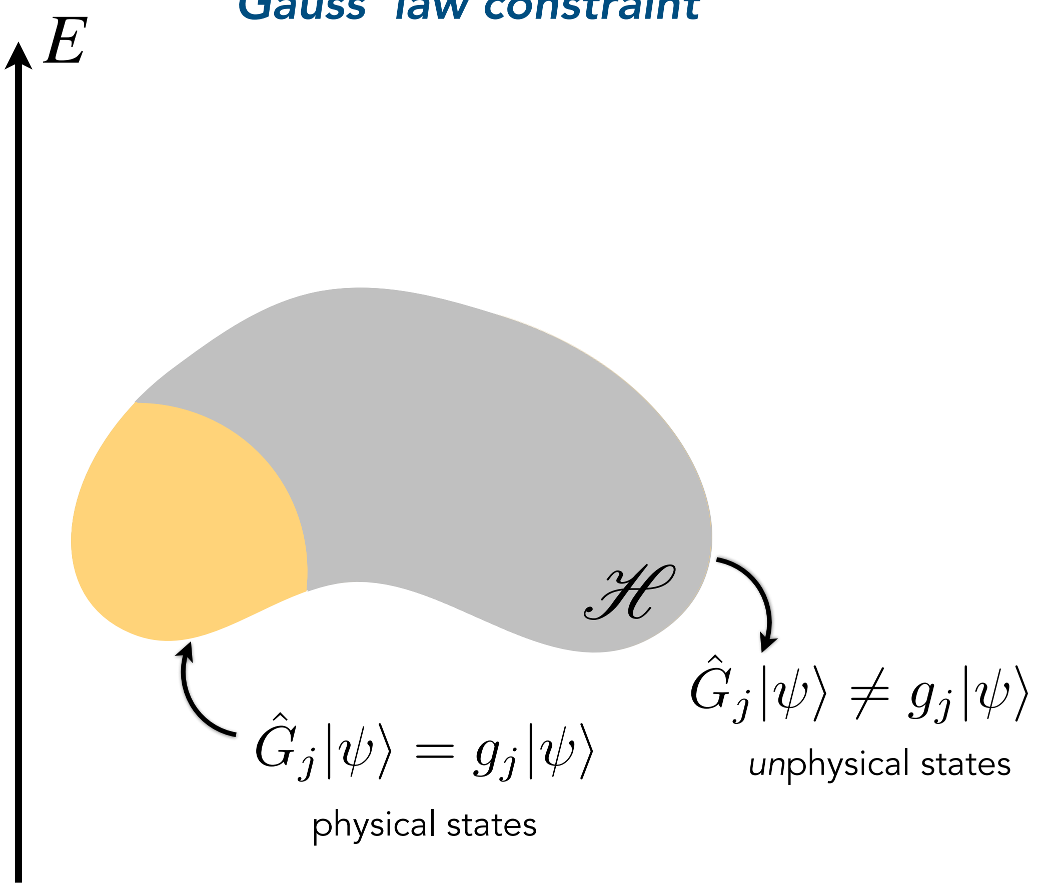
etc...

see e.g.: D. Banerjee et al., PRL **110** (2013), G. Calajò et al., arXiv:2402.07987, ...

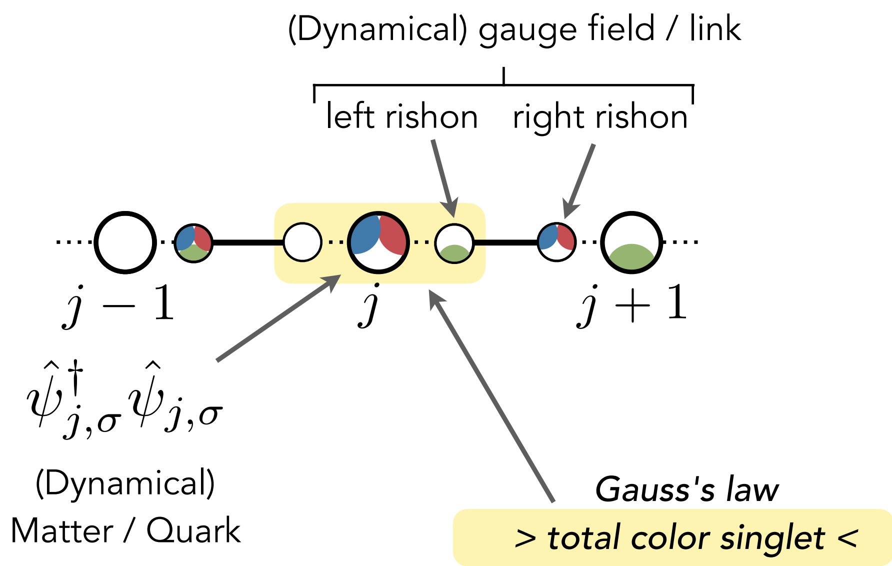
Rishon formulation of lattice gauge theories

[*Review:* U.J. Wiese, Ann. Phy. 525 (2013)]

Gauss' law constraint



Non-Abelian gauge theory



Paths towards (large-scale) quantum simulation

(1) Integrate out degrees-of-freedom

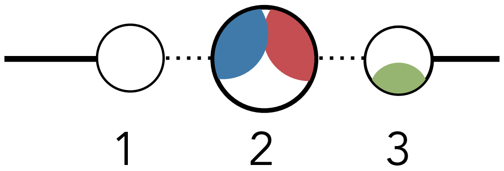
[Surace et al., *PRX (2020)*]

(2) Enforce constraints energetically

[Yang et al., Nature (2020); Semeghini et al., *Science (2021)*]

Gauge protection scheme

Single vertex



Ga

$\hat{\mathbf{G}} = \hat{\mathbf{S}}$

$\hat{\mathbf{G}}|\psi\rangle = 0$

$$\hat{\mathbf{S}}_j = \sum_{\alpha,\beta=1}^{N} \hat{c}^\dagger_{j,\alpha}\hat{\mathbf{T}}^{\alpha\beta}\hat{c}_{j,\beta}$$

Gauge protection

non-Abelian stabilizer

$\hat{H}_J = \frac{J}{2}\sum \hat{G}^2$

enberg

$_3 + \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{S}}_3$

$$+\frac{1}{2}\left(\hat{\mathbf{S}}_1^2 + \hat{\mathbf{S}}_2^2 + \hat{\mathbf{S}}_3^2\right)$$

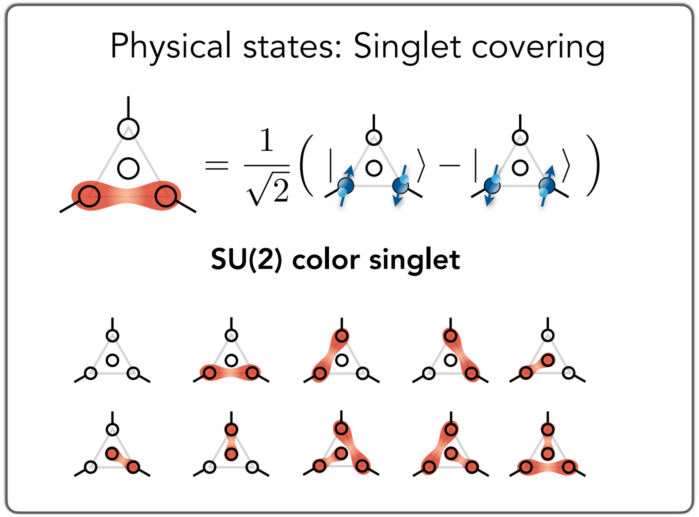
$$\hat{\mathbf{S}}_j^2 = \sum_\alpha \left(\frac{N^2-1}{2N}\hat{n}_{j\alpha} - \frac{1-\xi N}{N}\sum_{\alpha<\beta}\hat{n}_{j\alpha}\hat{n}_{j\beta}\right)$$

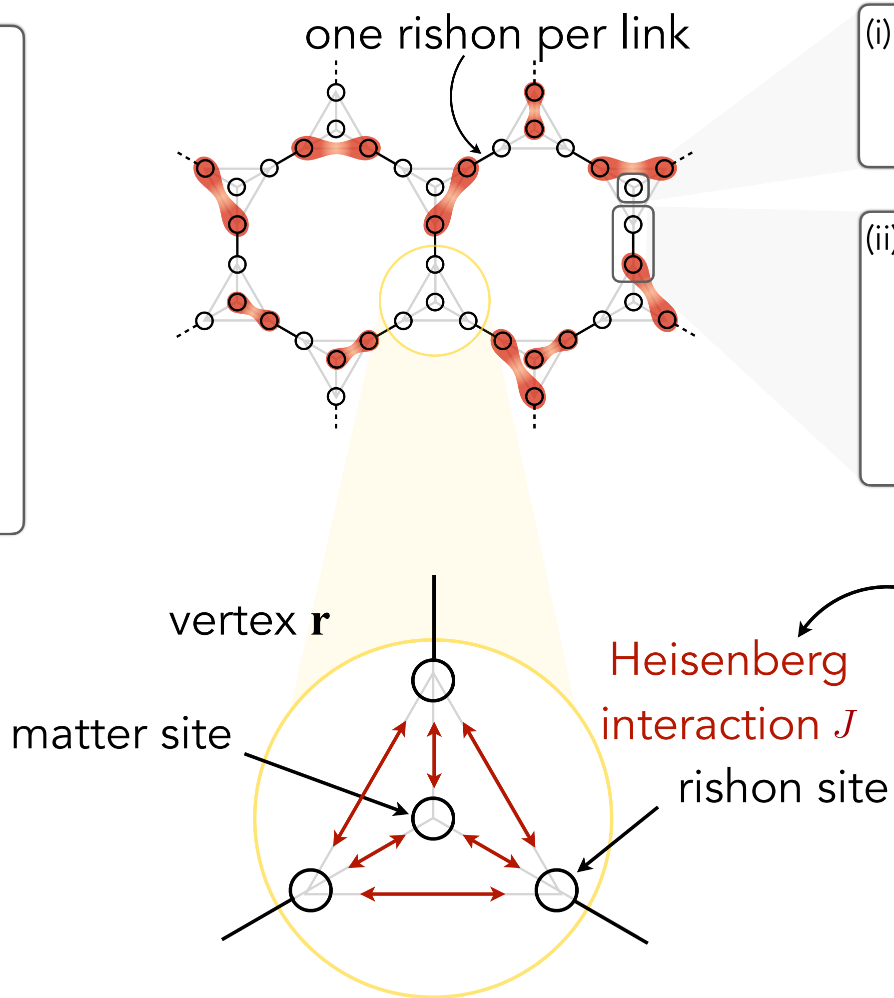
on-site Hubbard

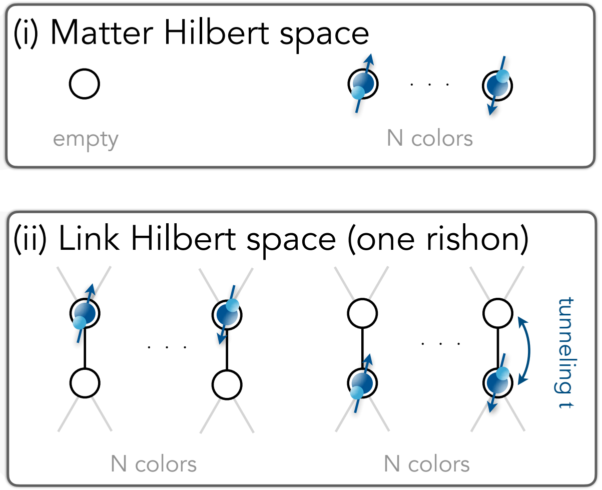
One-rishon / hardcore matter → Heisenberg

Multiparticle models → Heisenberg + Hubbard

SU(2) quantum link model







$$\hat{H}_J = J \sum_{\langle i,j\rangle \in V} \hat{\vec{S}}_i \cdot \hat{\vec{S}}_j$$

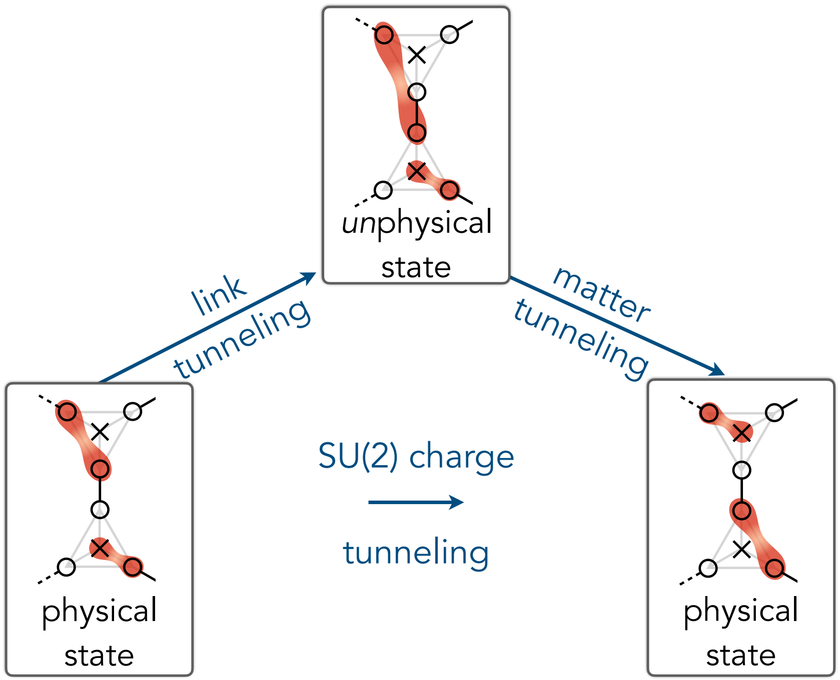
Next: Induce dynamics within constrained Hilbert space

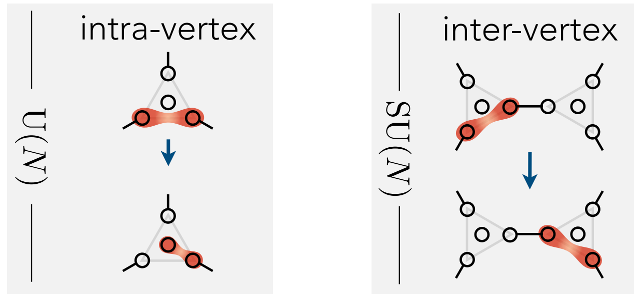
Emergent gauge theory

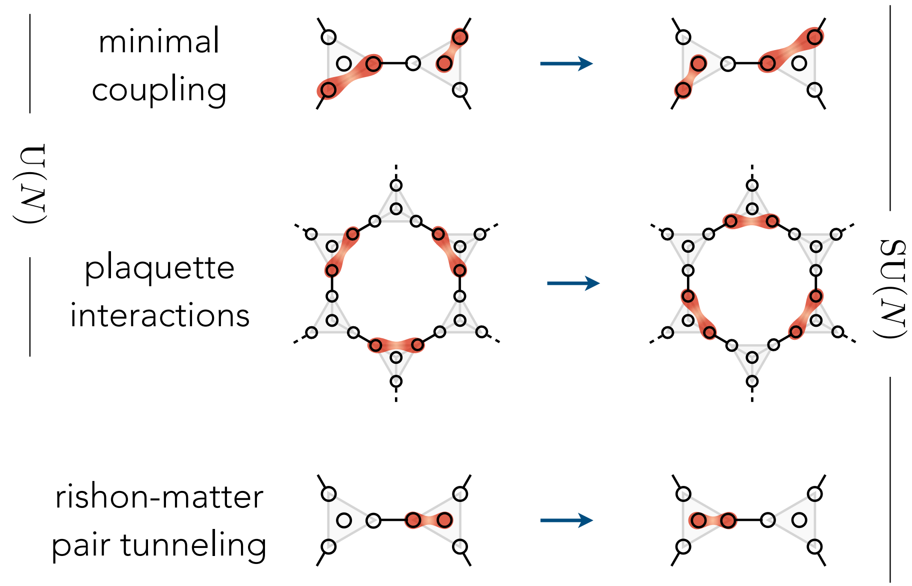
see also: Banerjee et al., PRL **110** (2013)

$$\hat{H}_{\text{mic}} = \hat{H}_J + \hat{H}_t \xrightarrow{t \ll J} \hat{H}_{\text{eff}}$$

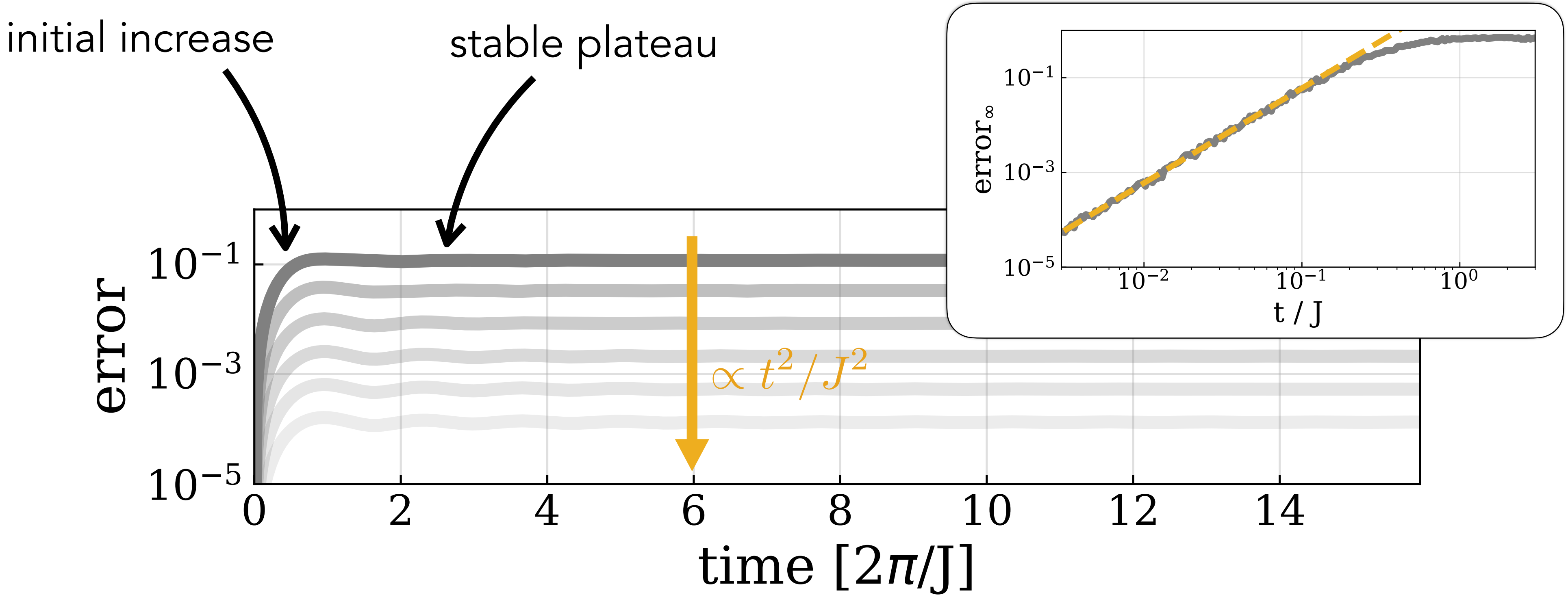
magnetic interaction ($\hat{H}_J$), tunneling ($\hat{H}_t$)





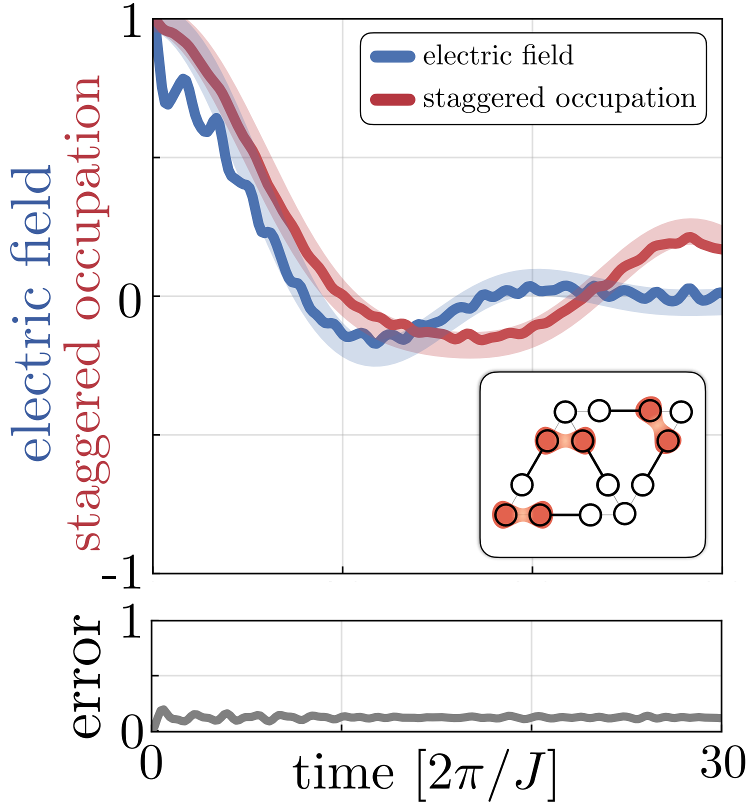


Controllable gauge protection



Emergent gauge-invariant dynamics

Microscopic versus effective Hamiltonian

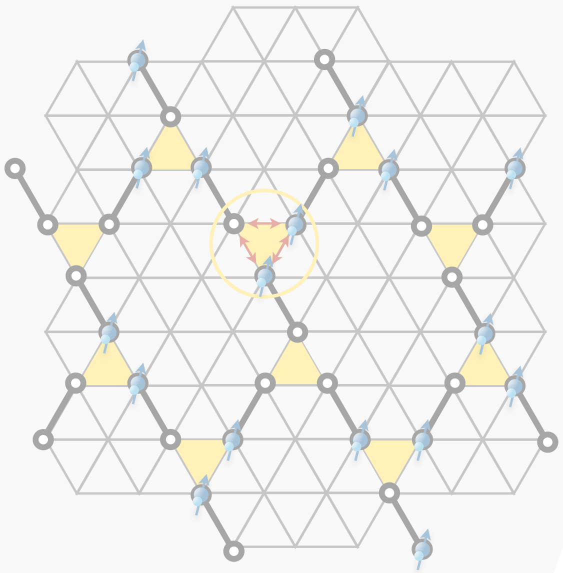


- Emergent non-Abelian gauge theory *
- Scalable and gauge-invariant by construction
- Two-body interactions
- Realistic timescale

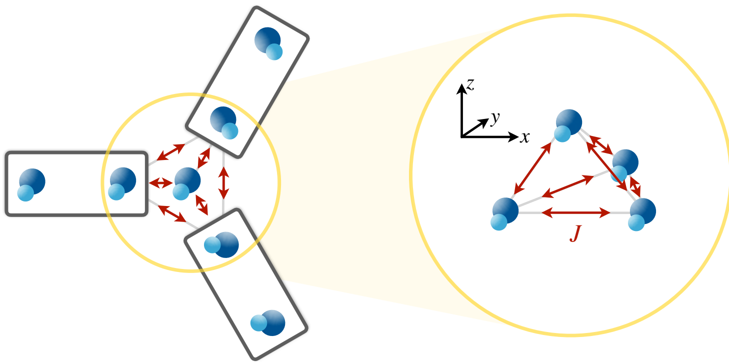
*This example: SU(2) gauge theory with hardcore bosonic matter in (2+1)D

Experimental proposals

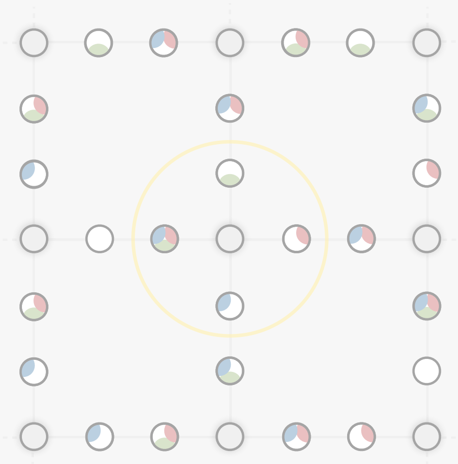
Pure gauge theory: molecules in optical lattice



see also: L. Christakis et al., Nature **614** (2023)

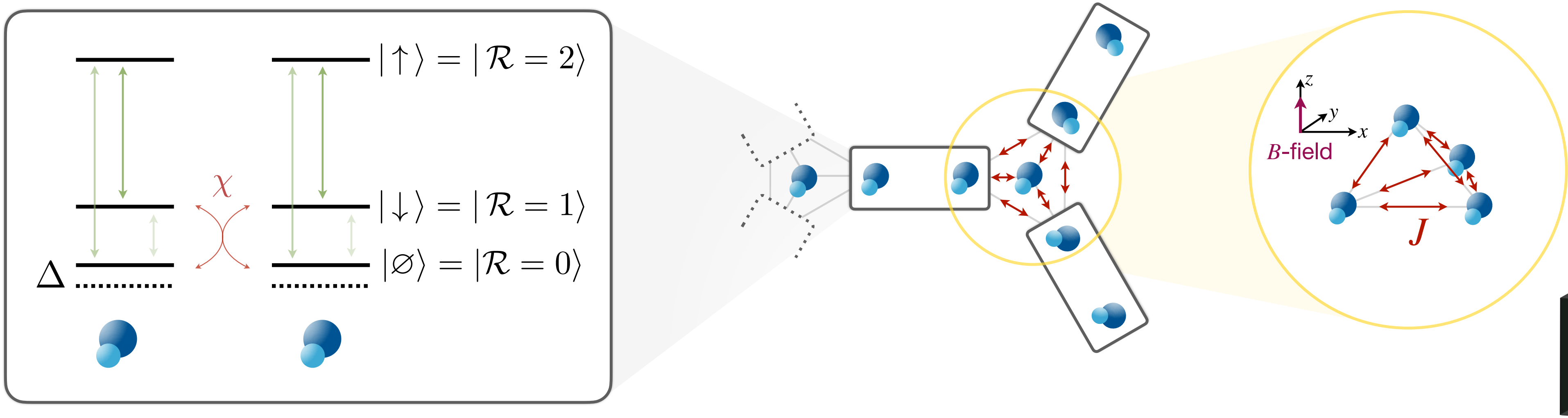


Gauge theory with matter: molecules in optical tweezers

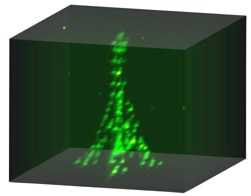


Multirishon models with fermionic matter

Proposal for cold molecules

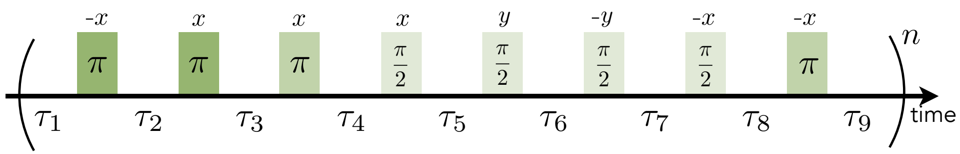


magnetic interaction



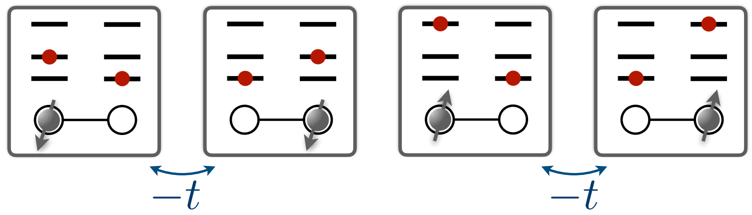
from: D. Barredo et al., Nature **561** (2018)

Floquet driving



from: LH et al., PRL **132** (2024)

Rishon link

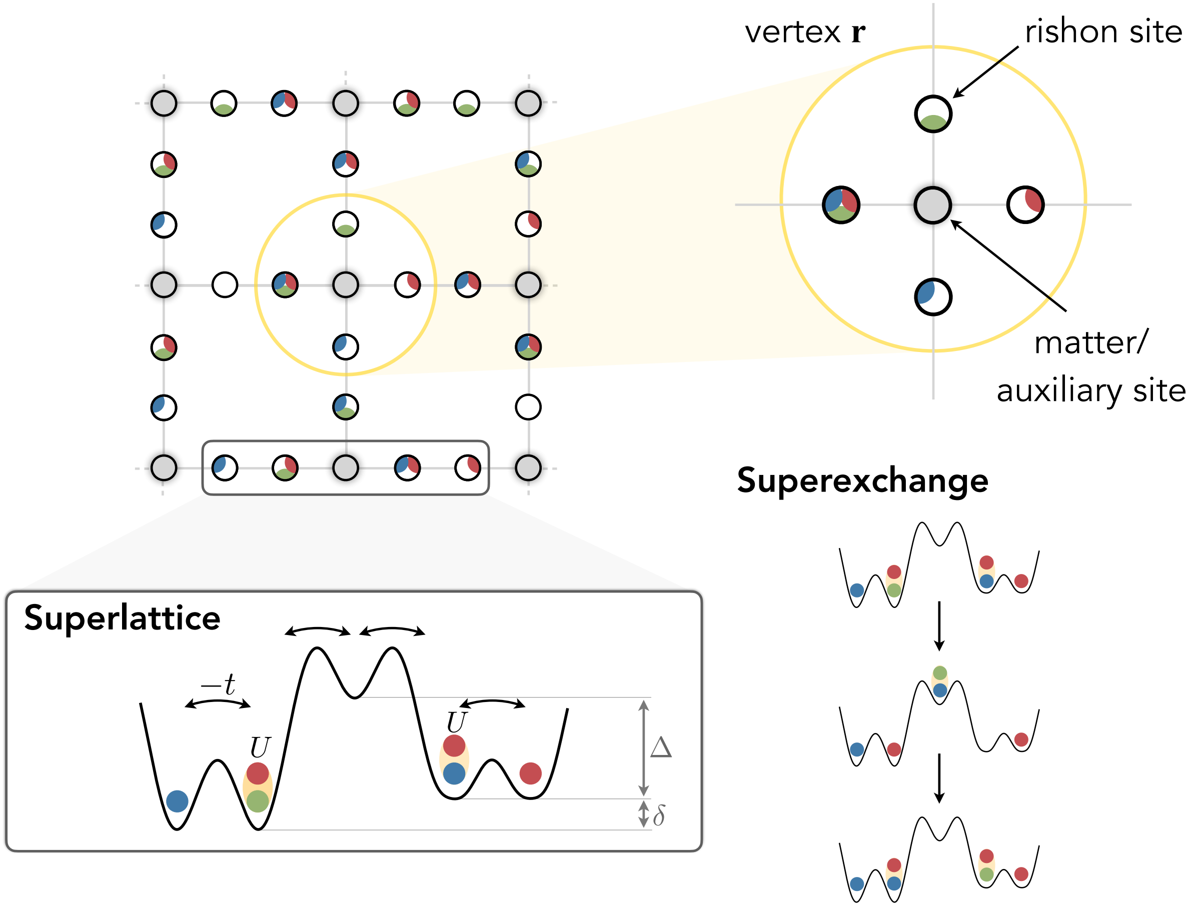


tunneling

2-body XY + Floquet

from: J.C.Halimeh*, LH* et al., arXiv:2305.06373
see also: L. Christakis et al., Nature **614** (2023)

Multi-rishon model: towards lattice QCD



Generic protection scheme for SU(N) fermionic LGTs

attractive Hubbard interaction

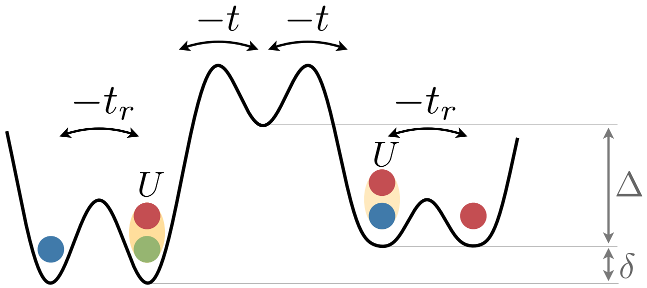
+

potential landscape

+

tunneling

Multi-rishon gauge protection



$t = 1$

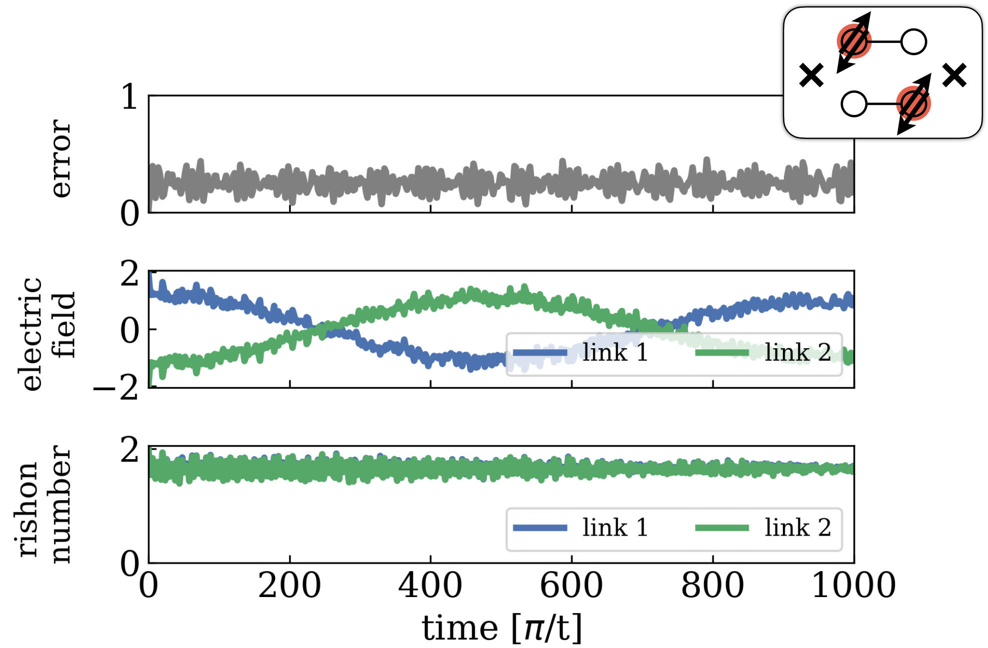
$\Delta/t = -10.35$

$\delta/t = 20$

$U/t = -0.25$

protection strength
$J/t \approx 0.32$

perturbation
$t_r/J = 0.125$



Acknowledgements

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University of Regensburg

Annabelle Bohrdt

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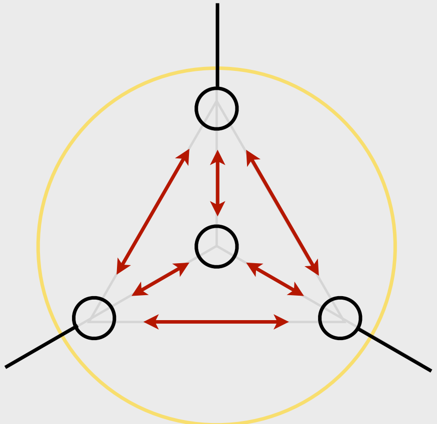
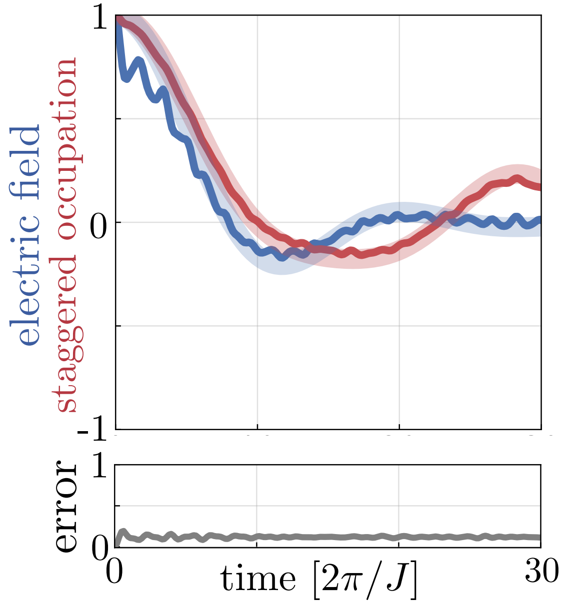
Summary

Non-Abelian gauge protection



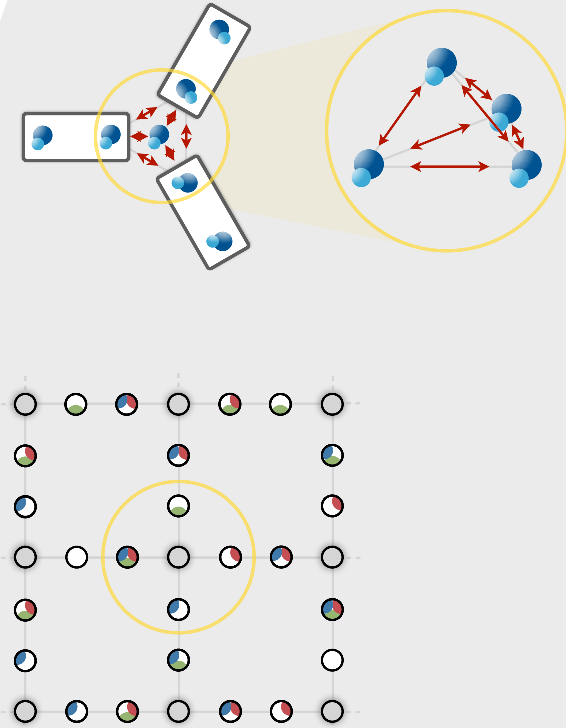
Hubbard
+
AFM Heisenberg

Emergent gauge theory



Thank you
for your attention!

Experimental proposals



Additional slides

non-Abelian LGT

Method

Bottom-up

> engineering gauge-invariant Hamiltonian $\hat{H}_{SU(2)}$

> experimental errors break gauge constraint

[D. Banerjee et al., PRL 110 (2013),
E. Zohar et al., PRL 110 (2013),
L. Tagliacozzo et al., Nat. Comm. 4 (2016),
Review: M. Aidelsburger et al., C. R. Phys. 19 (2018)]

Top-down

> experimentally imposing gauge constraints

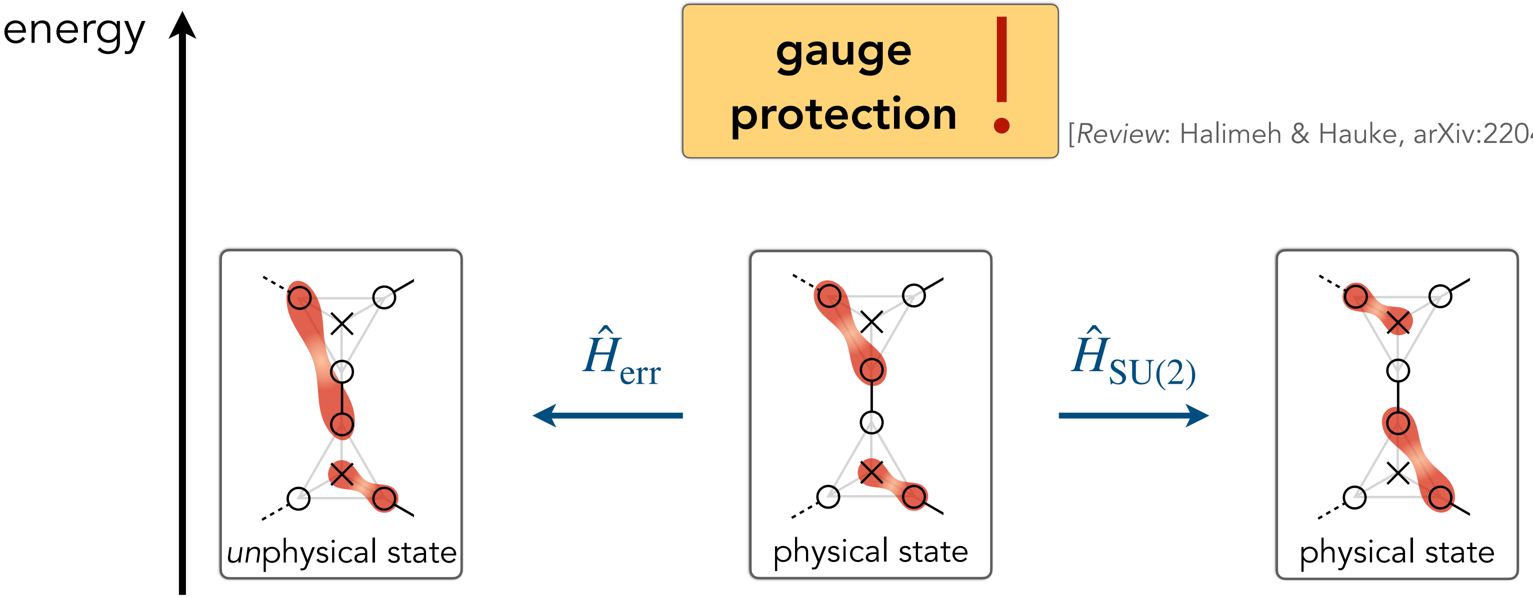
> dynamics induced perturbatively

> enables **large-scale** quantum simulation

[Z2 w/o matter (2+1)D: G. Semeghini et al., Science 374 (2021),
U(1) QLM w/ matter (1+1)D: B. Yang et al., Nature 587 (2020),
U(1) QLM w/ matter (1+1)D: F. Surace et al., PRX 10 (2020)]

gauge protection !

[*Review*: Halimeh & Hauke, arXiv:2204.13709]



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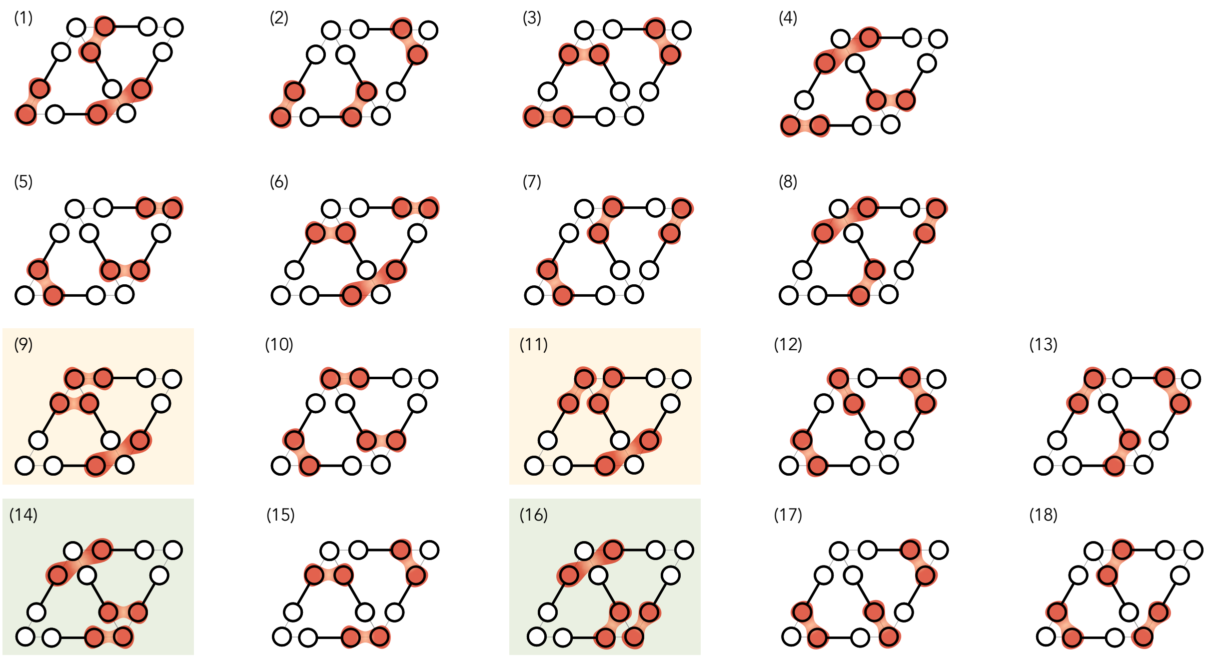
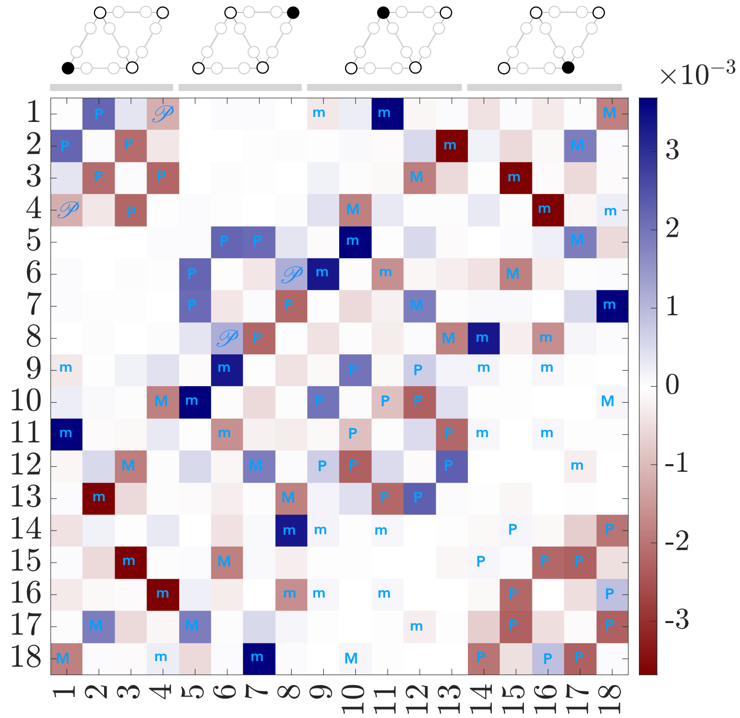
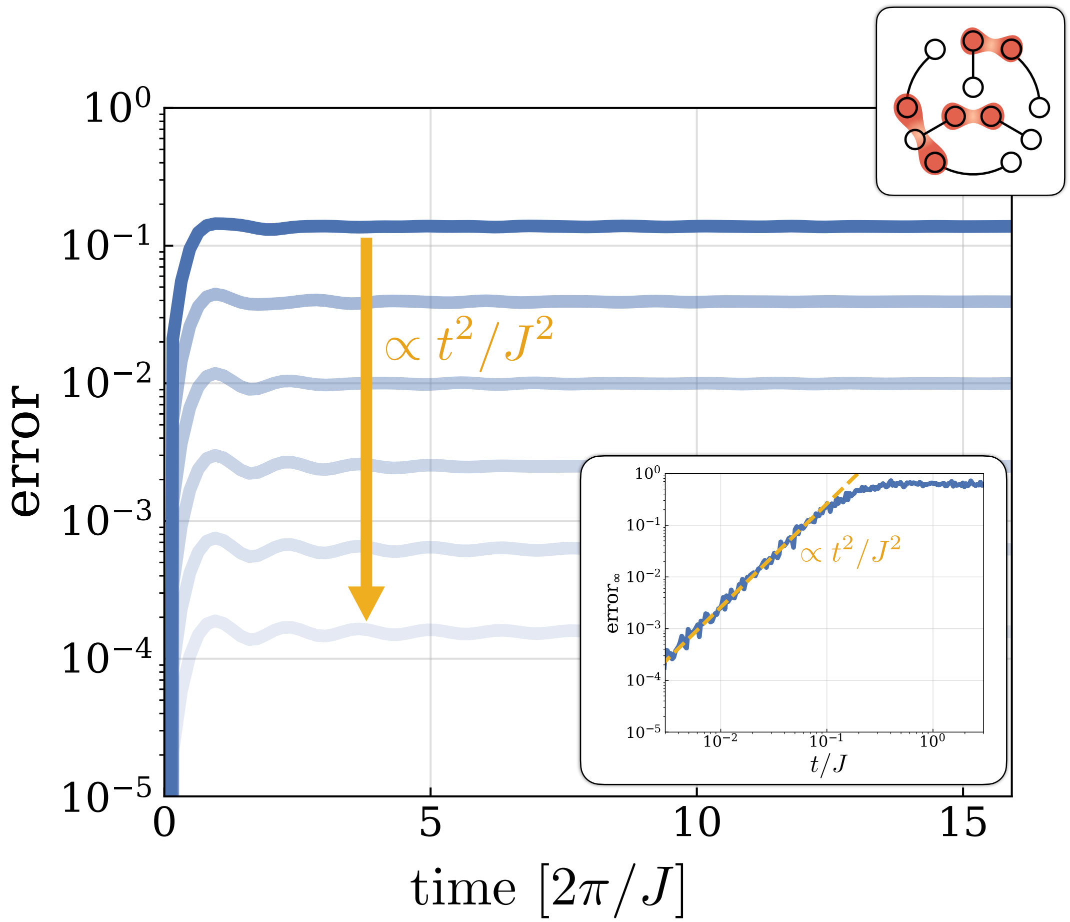
Effective Hamiltonian

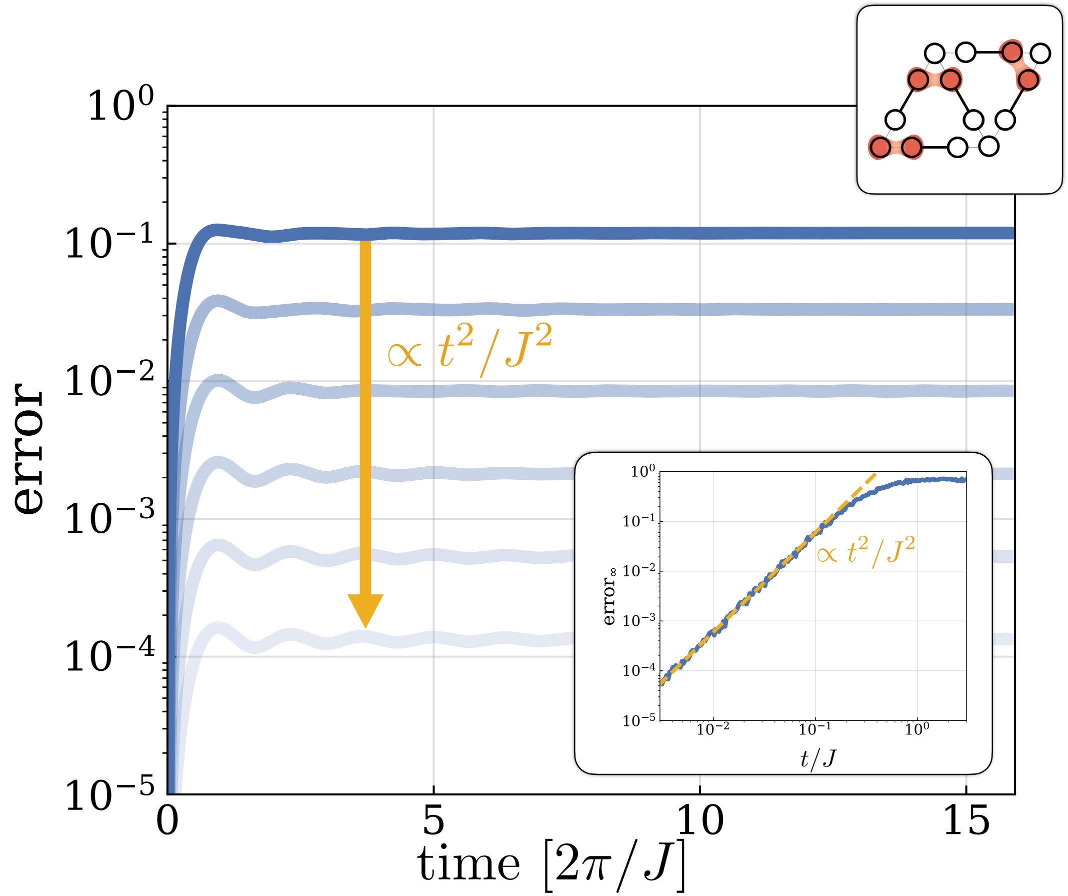
Construct effective Hamiltonian in gauge-invariant subspace: $\hat{\mathcal{P}}_G e^{-i\delta t \hat{H}_{\mathrm{mic}}} |\psi\rangle \approx (1 - i\delta t \hat{H}_{\mathrm{eff}}) |\psi\rangle$



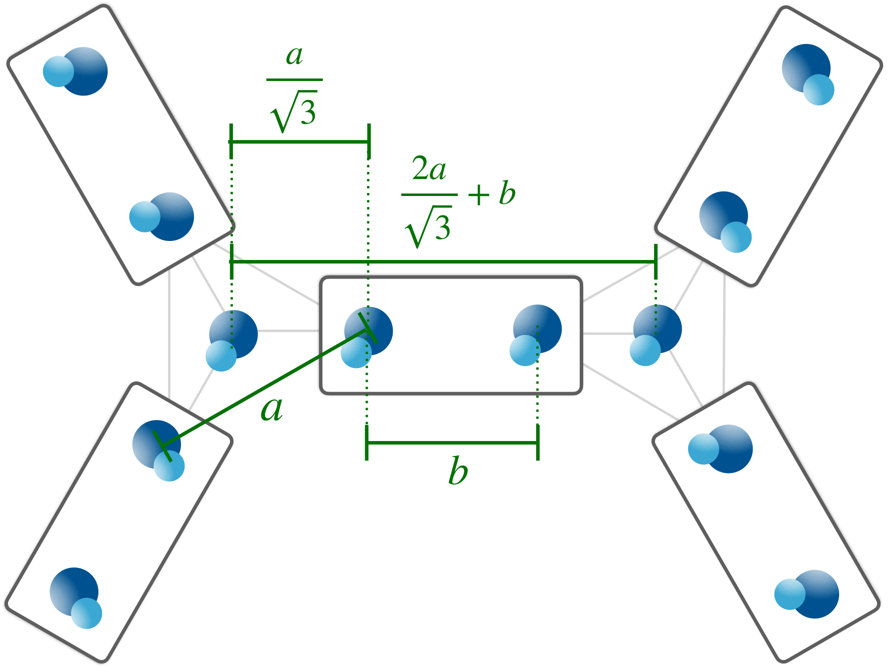


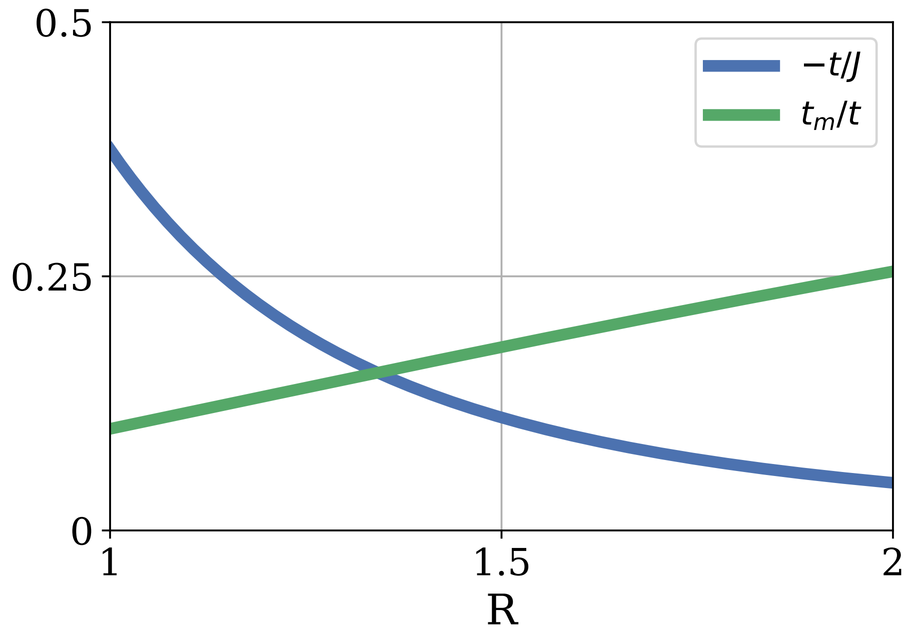
Gauge violation





Tweezer geometry





Rishon mapping

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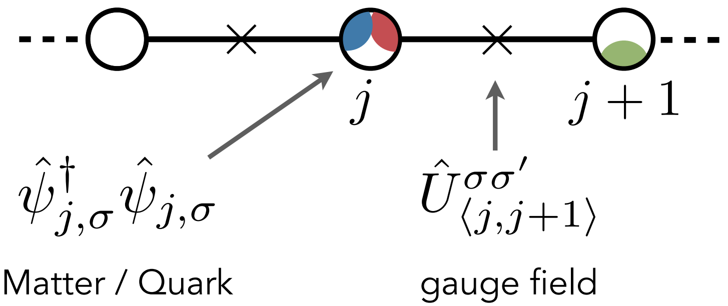
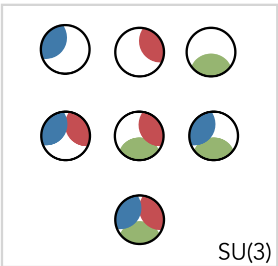
Wilson's lattice gauge theory





$\hat{U}$: (N x N)-matrix

$\mathbb{C}$-valued

$$\hat{U} = \begin{pmatrix} U_{11} & \dots & U_{1N} \\ \vdots & \ddots & \\ U_{N1} & & U_{NN} \end{pmatrix}$$

Gauss's law

$N^2 - 1$ generators of SU(N)

$$\hat{G}^a_j = \hat{\psi}^\dagger_{j,\sigma} T^a_{\sigma\sigma'} \hat{\psi}_{j,\sigma'} + \hat{L}^a_{\langle j-1,j\rangle} + \hat{R}^a_{\langle j,j+1\rangle}$$

$$\sum_{a=1}^{N^2-1} \hat{G}^a_j |\psi\rangle = 0$$

Commutation Relations

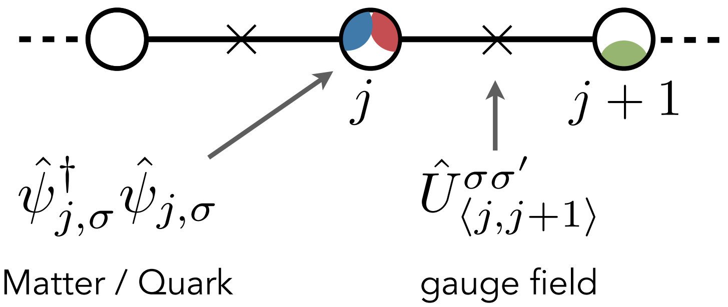
$$[\hat{L}^a, \hat{L}^b] = 2i f_{abc} \hat{L}^c \qquad [\hat{R}^a, \hat{R}^b] = 2i f_{abc} \hat{R}^c$$

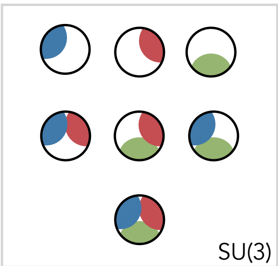
$$[\hat{L}^a, \hat{U}] = -T^a \hat{U} \qquad [\hat{R}^a, \hat{U}] = -\hat{U} T^a$$

link = particle in infinite dimensional group space SU(N)

[*Review:* U.J. Wiese, Ann. Phy. 525 (2013)]

Quantum link formulation





$\hat{U}$: (N x N)-matrix
operator-valued

$$\hat{U} = \begin{pmatrix} \hat{U}_{11} & \cdots & \hat{U}_{1N} \\ \vdots & \ddots & \\ \hat{U}_{N1} & & \hat{U}_{NN} \end{pmatrix}$$

Gauss's law

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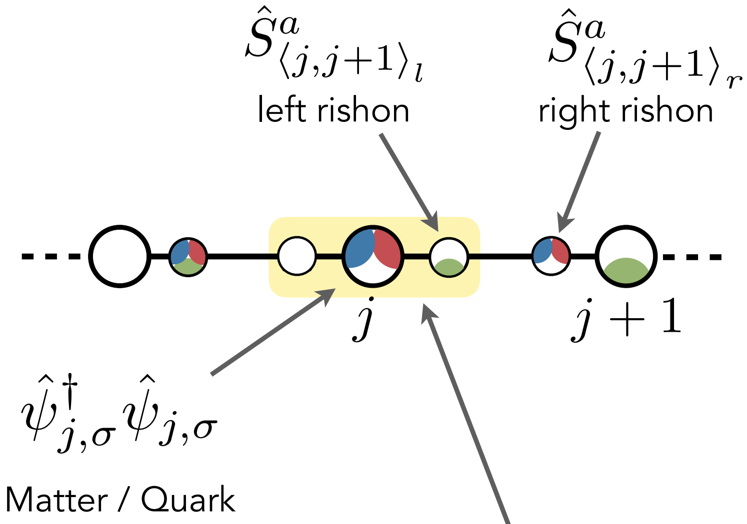
$$[\hat{L}^a, \hat{U}] = -T^a \hat{U} \qquad [\hat{R}^a, \hat{U}] = -\hat{U} T^a$$

Quantum link formulation = discretized link Hilbert space

[*Review:* U.J. Wiese, Ann. Phy. 525 (2013)]

Rishon formulation



Gauss's law

> total color singlet <

Schwinger fermion representation

$$\hat{S}^a_{\langle j,j+1\rangle_r} = \hat{c}^\dagger_{\langle j,j+1\rangle_l,\sigma} T^a_{\sigma\sigma'} \hat{c}_{\langle j,j+1\rangle_l,\sigma'}$$

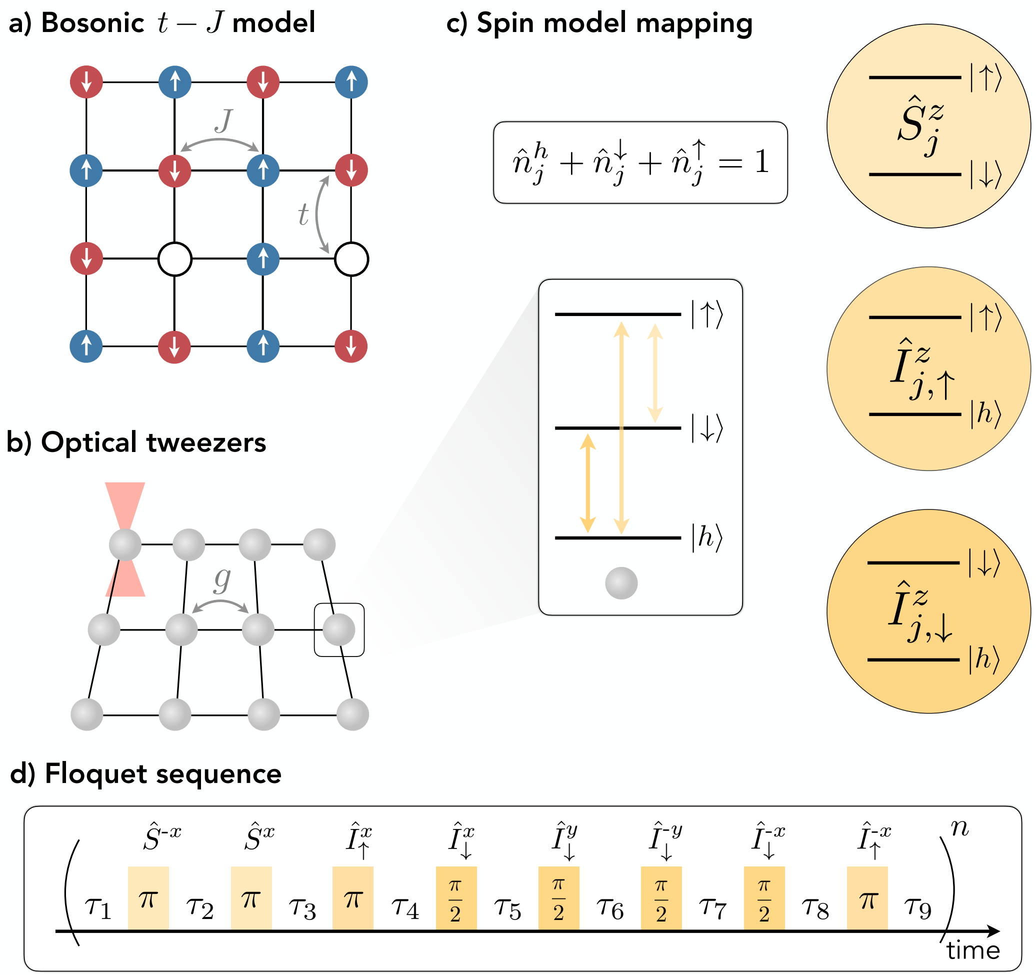
$$\hat{U}^{\sigma\sigma'}_{\langle j,j+1\rangle} = \hat{c}_{\langle j,j+1\rangle_l,\sigma} \hat{c}^\dagger_{\langle j,j+1\rangle_r,\sigma'}$$

$$\mathcal{N} = \sum_\sigma \left(\hat{c}^\dagger_{\langle j,j+1\rangle_l,\sigma} \hat{c}_{\langle j,j+1\rangle_l,\sigma} + \hat{c}^\dagger_{\langle j,j+1\rangle_r,\sigma} \hat{c}_{\langle j,j+1\rangle_r,\sigma} \right)$$

[*Review:* U.J. Wiese, Ann. Phy. 525 (2013)]

Bosonic t-J model

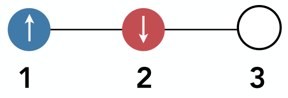
Bosonic t-J model - molecules



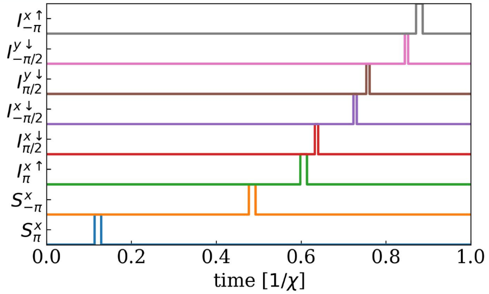
from: LH et al., PRL **132** (2024)

Bosonic t-J model - molecules

a) Initial state



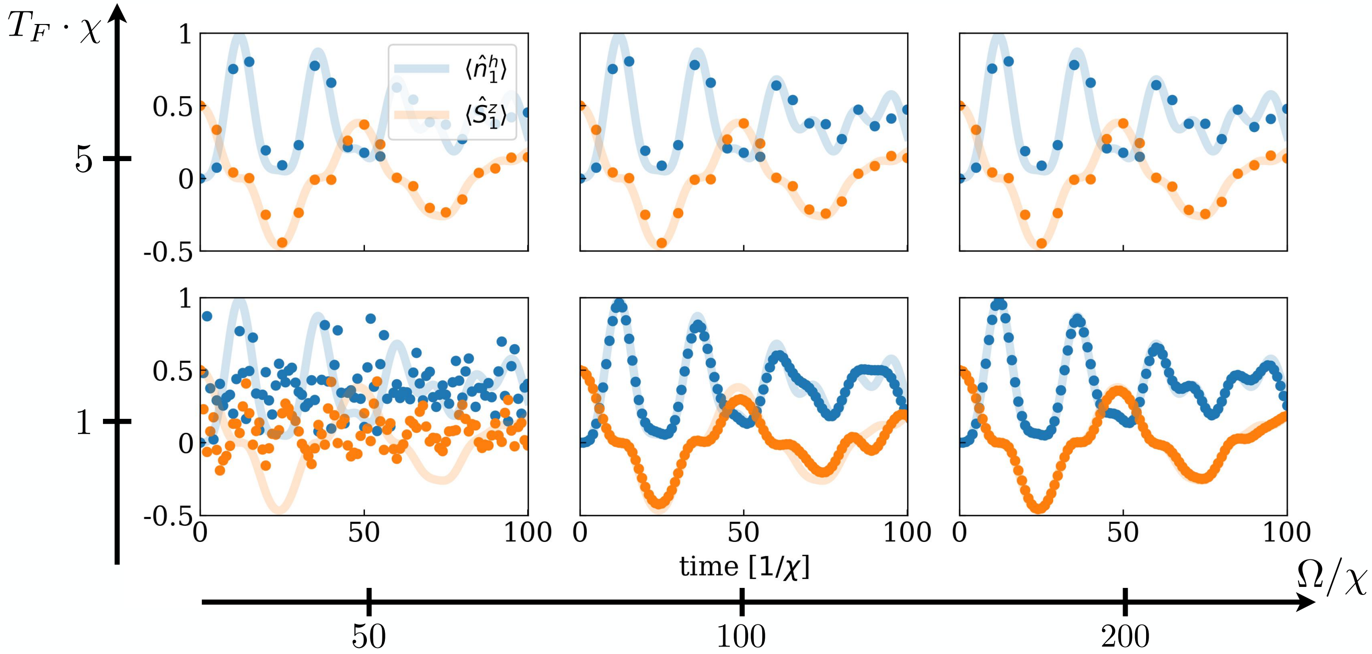
b) Pulse sequence



$T_F \cdot \chi = 1$

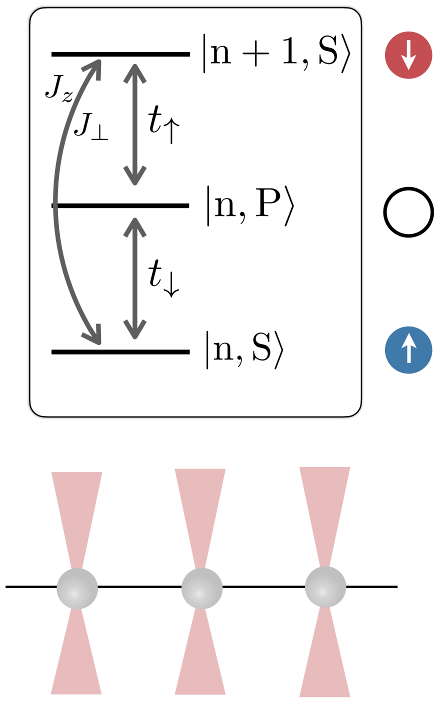
$\Omega/\chi = 200$

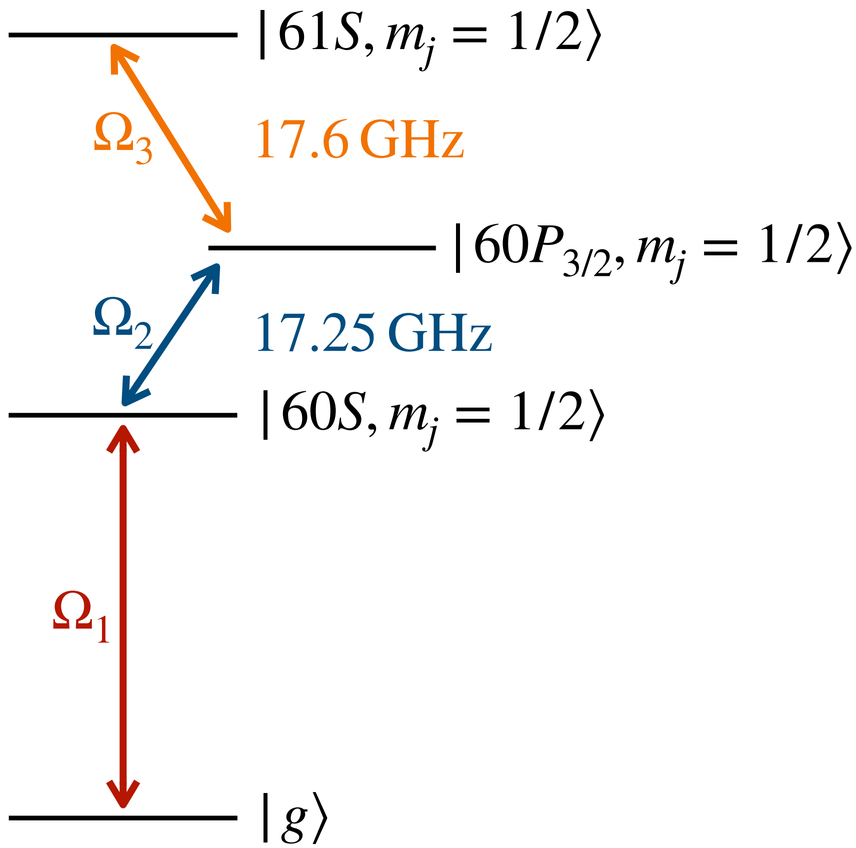
c) Exact vs. Floquet dynamics



from: LH et al., PRL **132** (2024)

Bosonic t-J model - Rydbergs

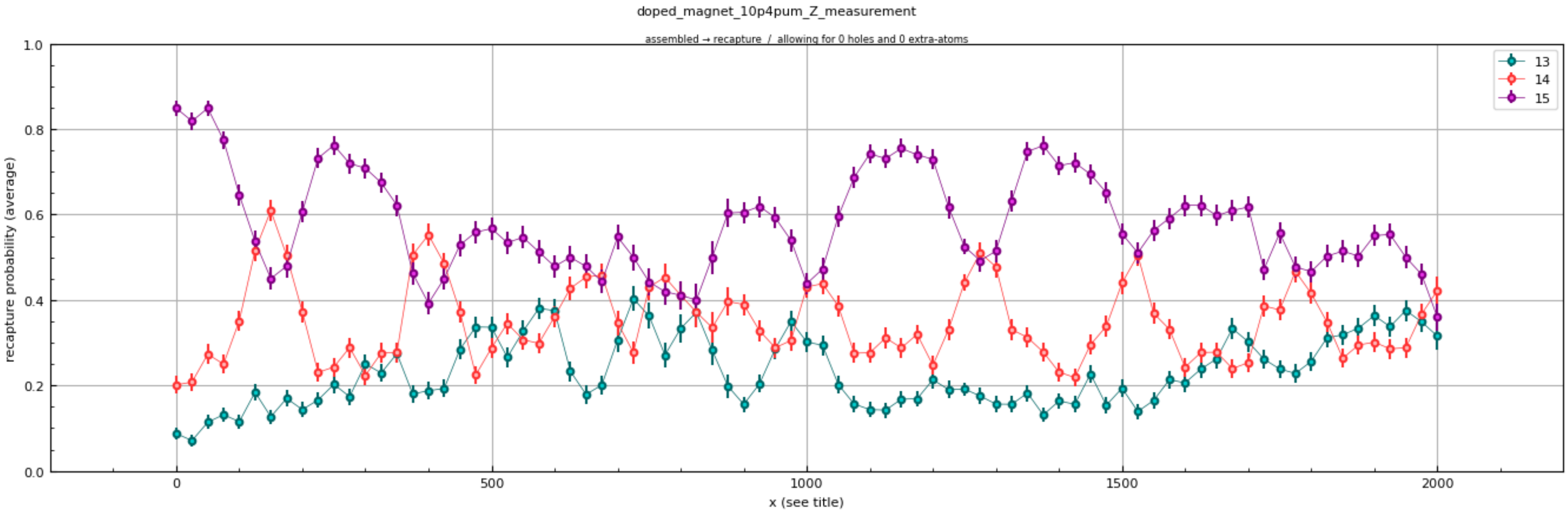




from: LH et al., PRL **132** (2024)

Bosonic t-J model - Rydbergs

Preliminary data: Browaey's group



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