# Plasma Eyepiece for Petawatt Laser Wakefield Accelerators

Ming Zeng, Alberto Martinez de la Ossa, Kristjan Poder and Jens Osterhoff

Deutsches Elektronen-Synchrotron DESY, 22607 Hamburg, Germany

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## Background and Purposes

- Petawatt class lasers ($1 \sim 100$ PW) are useful for high-flux and high-energy laser wakefield accelerators (LWFA).
- Limited by mirror damage threshold, petawatt lasers have $\sim 1$ m diameter. For LWFAs driven by $1/10/100\ \mathrm{PW}$ lasers, $f \sim 10/100/1000\ \mathrm{m}$.
- LWFA also requires flexible $w_0$ for matching different plasma densities with the matching condition $k_p w_0 = 2\sqrt{a_0}$ [W. Lu et al., Phys. Rev. ST/AB 10, 061301 (2007)]. But changing the focusing system for different $w_0$ is costly.
- Goal: a flexible way for changing $w_0$ without replacing the focusing system, and reduce $f$ for large $w_0$.


Figure 1: A typical OAP for petawatt laser costs $\sim 1$ million Euros.

# Former Active Plasma Lens Studies (courtesy D. Gordon)

- Parabolic channel $n(r) = n_0 + \frac{\Omega^2}{c^2} r^2$ leads to ray equation $\frac{d^2 r}{dt^2} + \Omega^2 r = 0$, where $\Omega^2 = \frac{c^2}{r_{\mathrm{ch}}^2} \frac{\Delta n}{n_0} \frac{\omega_p^2}{\omega^2}$ [R. Hubbard et al., Phys. Plasmas 9, 1431 (2002)].
- Ideal plasma lens which eliminates spherical aberrations [D.F. Gordon et al., Phys. Plasmas 25, 063101 (2018)].

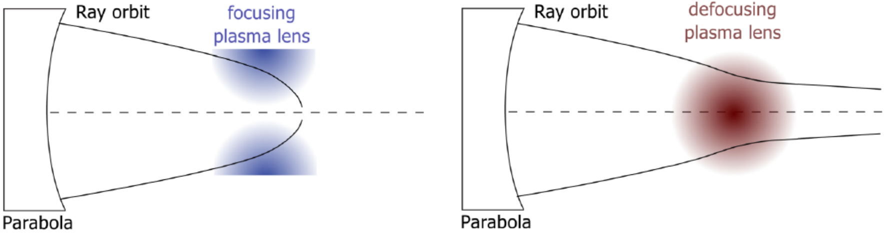



Figure 2: Schematic view of the plasma lenses [J.P. Palastro et al., Phys. Plasmas 22, 123101 (2015)].

# Our idea: A Plasma Eyepiece in a Telescope System

- Use a fixed small f-number focusing system to focus the laser beam in vacuum at $z_0$. The laser beam enters the plasma at $z_1$ and reaching a local maximum beam size at $z_2$.
- The plasma acts as an eyepiece in a telescope. Adjust $d \equiv z_1 - z_0$ and plasma density to change the effective laser focal size $w_2$ in this telescope system. $l \equiv z_2 - z_1$ is the plasma lens thickness.

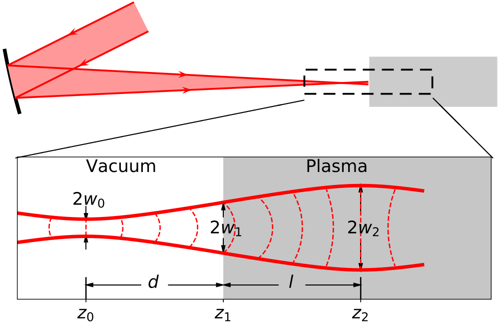
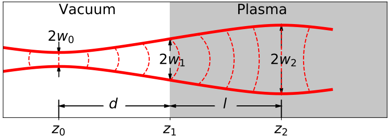



Figure 3: Schematic view of the plasma eyepiece.

# Theory of relativistic self-refocusing




- In weakly relativistic regime so that the perturbation of plasma density is negligible, one may write down the transverse profile functions

$$aw = a_0 w_0, \tag{1}$$

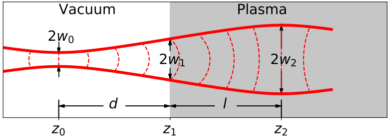
$$\frac{d^2 w}{dz^2} = \frac{4}{k^2 w^3}\left(1 - \frac{a_0^2 w_0^2}{32}\right). \tag{2}$$

- In our case, the initial conditions are

$$w_1 \equiv w\big|_{z_1} = w_0\sqrt{1 + \frac{d^2}{z_R^2}}, \tag{3}$$

$$\left.\frac{dw}{dz}\right|_{z_1} = \frac{w_0^2 d}{z_R^2 w_1}. \tag{4}$$

# Theory of Relativistic Self-refocusing




- These lead to the solution for $\left.\frac{dw}{dz}\right|_{z_2} = 0$

$$w_2 \equiv w|_{z_2} = w_0 \sqrt{1 + \frac{d^2}{z_R^2} \cdot \frac{1}{1 - \left(1 + \frac{d^2}{z_R^2}\right) \frac{32}{a_0^2 w_0^2}}} \tag{5}$$

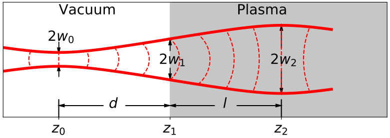
$$l \equiv z_2 - z_1 = \frac{d}{\frac{a_0^2 w_0^2}{32}\left(1 + \frac{d^2}{z_R^2}\right)^{-1} - 1}, \tag{6}$$

with the condition

$$d < z_R \sqrt{\frac{a_0^2 w_0^2}{32} - 1} \equiv d_{\mathrm{M}}. \tag{7}$$

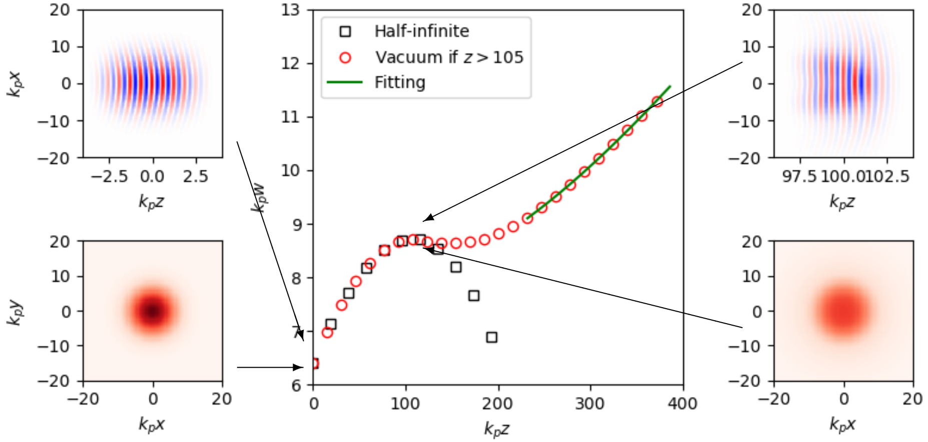
- Eqs. (5) (6) (7) are obtained without the perturbation of the plasma density. Do they still hold in the blowout regime?

# PIC Simulation Configuration



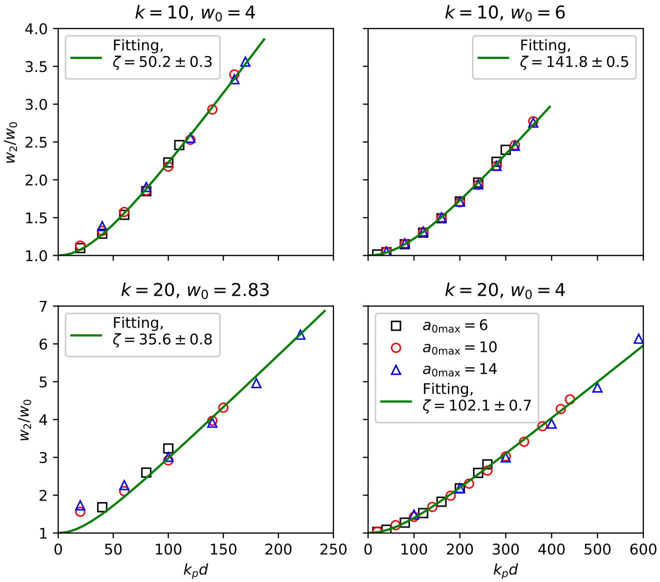

- In the simulations using the code OSIRIS, the time is normalized to the inverse of the plasma frequency $\omega_p^{-1}$, length to the plasma skin depth $c/\omega_p$, or simply $k_p^{-1}$.
- We use a step-function plasma density profile. There are 5 key parameters: $a_{0\,\mathrm{max}}$ $(= a_0(z-ct)|_{\mathrm{max}})$, $w_0$, $\omega$ (or $k$), $d$ and $\tau$ (pulse duration).
- We firstly keep $\tau = 4$ and do parameter scan for the other 4 parameters.

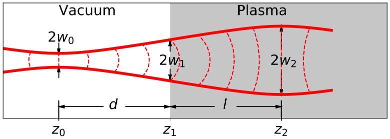
# Example Simulations




- Example simulations with $k = 10$, $w_0 = 4$, $a_{0\,\max} = 10$, $d = 100$ and $\tau = 4$.
- In (a), black square is result from a half-infinite plasma, and red circle is from a simulation with the same parameters but for $z > 105$ it is vacuum.
- In this case $w_2 = 8.7$, and red circle in vacuum region gives $w_{0\mathrm{eff}} = 7.9$ which has only 10% error.

# $w_2/w_0$ vs. $d$




- With a considerable amount of simulations, we found that in most of the cases $a_{0\,\max}$ does not change the $w_2/w_0$ vs. $d$ curve (it only changes the upper-limit for $d$).
- For different $k$ and $w_0$, we plot $w_2/w_0$ vs. $d$ and fit with

$$\frac{w_2}{w_0} = \sqrt{1 + \frac{d^2}{\zeta^2}}, \tag{8}$$

where $\zeta$ is a function of $k$ and $w_0$.

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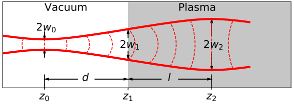
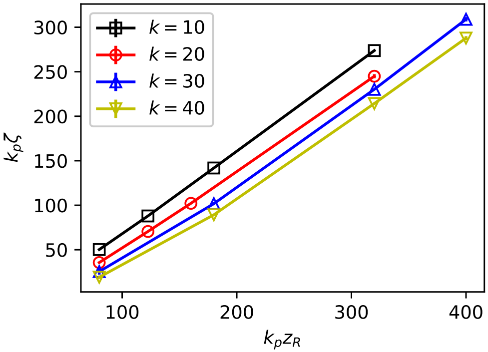
## $\zeta$ vs. $z_R$



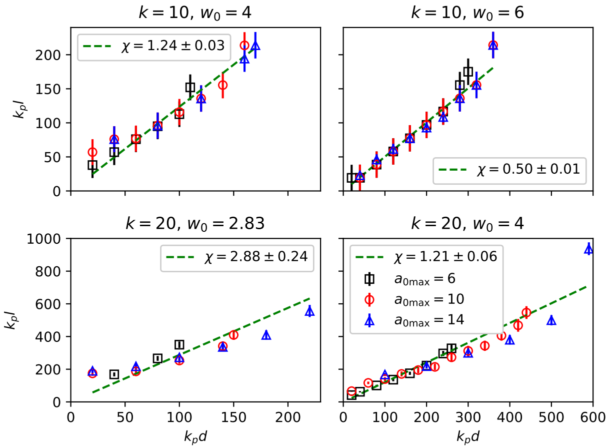



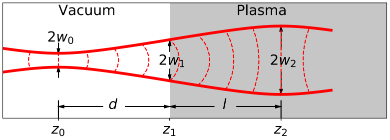

- We found that $\zeta$ almost linearly depends on $z_R \equiv \frac{k}{2} w_0^2$, if $k$ is fixed.
- With linear fittings we finally found

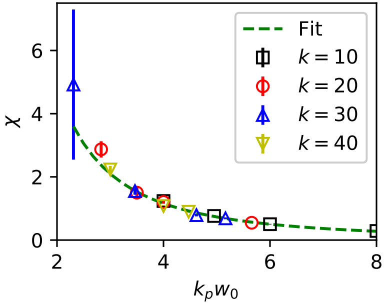
$$\zeta \approx 0.95 z_R - 1.2k - 13. \tag{9}$$

which can be put back to $\frac{w_2}{w_0} = \sqrt{1 + \frac{d^2}{\zeta^2}}$ for $w_2$.

# $l$ vs. $d$



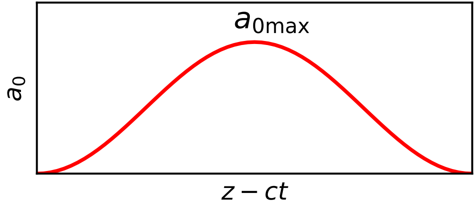




- We also found that $l$ is almost linearly depends on $d$, if $k$ and $w_0$ are fixed, i. e. $l = \chi d$.
- We plot $\chi$ vs. $w_0$ and found $k$ only has minor influence on the curve. Thus we do one fitting for all the $\chi$ vs. $w_0$ data and write the empirical formula

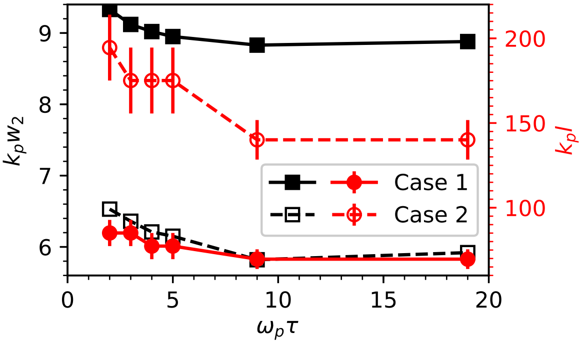
$$\chi = 21.0 w_0^{-2.08}. \tag{10}$$

# Why not depend on $a_{0\,\mathrm{max}}$?




- $a_0$ is a function of $z - ct$. There can be other regimes in which the self-focusing strongly depends on $a_0$. In those cases, the laser front where $a_0$ is smaller has totally different self-focusing behavior compared to the laser central part, and the laser cannot be self-refocused as a whole.
- Only in the regime that the self-focusing weakly depends on $a_0$, the laser can self-focus uniformly.

# What if initial laser pulse duration changes?




- Two sets of examples with $k = 10$, $w_0 = 6$, $a_{0\,\max} = 14$, $d = 160$ (top) and $k = 20$, $w_0 = 3.5$, $a_{0\,\max} = 10$, $d = 100$ (bottom).
- The laser is less guided at smaller $\tau$, thus has larger $w_2$ and $l$. At larger $\tau$ there is saturation. Only $\sim 10\%$ differences are observed while $\tau$ changes.

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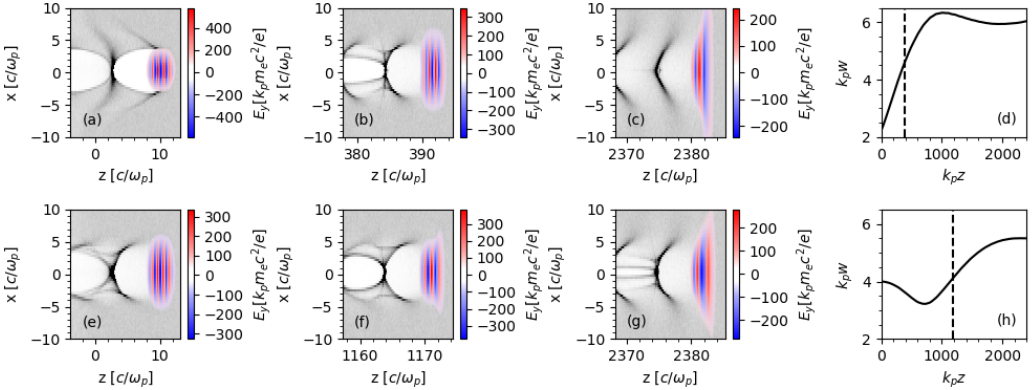
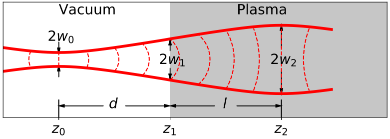
# Full 3D LWFA simulations with/without plasma eyepiece

- By using the laser-plasma matching condition $k_p w_2 = 2\sqrt{a_2}$ and our empirical formulas, we can write down a set of parameters for a 1 PW, 800 nm laser pulse:
  $a_{0\,\max} = 8$, $w_0 = 2k_p^{-1} = 21.6\ \mu\mathrm{m}$, $k/k_p = 84.7$ ($k_p^{-1} = 10.8\ \mu\mathrm{m}$), $\tau = 3\omega_p^{-1} = 108\ \mathrm{fs}$,
  $d = 80k_p^{-1} = 864\ \mu\mathrm{m}$,
  so that $a_2 = 4$, $w_2 = 4k_p^{-1} = 43.2\ \mu\mathrm{m}$ and $l = 397k_p^{-1} = 4287.6\ \mu\mathrm{m}$.
- To achieve a similar effective spot size with plasma eyepiece, focal length is reduced from $\sim 20\ \mathrm{m}$ to $\sim 10\ \mathrm{m}$.



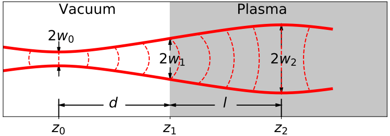
# Estimations for 10 and 100 PW LWFA design




| $P$ [PW] | 10 | 10 | 100 | 100 |
|---|---|---|---|---|
| $a_2$ | 4 | 4 | 4 | 4 |
| $n_p$ $[\mathrm{cm}^{-3}]$ | $2.4 \times 10^{16}$ | $2.4 \times 10^{16}$ | $2.4 \times 10^{15}$ | $2.4 \times 10^{15}$ |
| $w_0$ [$\mu$m] | 30 | 40 | 60 | 70 |
| $w_2$ [$\mu$m] | 136 | 136 | 431 | 431 |
| $d$ [mm] | 35.6 | 17.7 | 694 | 563 |
| $l$ [m] | 0.98 | 0.27 | 49 | 29 |
| $L_d$ [m] | 6.52 | 6.52 | 206 | 206 |
| $\tau_{\mathrm{opt}}$ [fs] | 303 | 303 | 958 | 958 |
| $\Delta W$ [GeV] | 97.7 | 97.7 | 977 | 977 |

Table 1: Plasma eyepiece parameters for 10 PW and 100 PW laser driven LWFAs. The dephasing length $L_d$, the optimal pulse duration $\tau_{\mathrm{opt}}$ for matching the pump depletion length with the dephasing length and the energy gain $\Delta W$ for LWFAs according to Lu et al. [Phys. Rev. ST Accel. Beams 10, 061301 (2007)] are also shown.

# Conclusions




- A plasma lens for laser, like an eyepiece in a telescope, greatly reduces the focal length for petawatt level LWFA applications. It also makes the laser spot size easily adjustable (by changing $d$).
- The empirical formula for the effective laser spot size is found to be $w_2 = w_0\sqrt{1+(d/\zeta)^2}$, where $k_p\zeta \approx 0.95k_pz_R - 1.2k/k_p - 13$.
- The empirical formula for the thickness of the plasma lens is found to be $l \approx 21.0d/(k_pw_0)^{2.08}$.
- Scanning of $d$ around the predicted value is still necessary in real experiments, because of the errors from the fit parameters, and the non-sharpened vacuum-plasma transition.