

Monodromic Dark Energy and DESI

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with
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[arXiv:2507.16970](https://arxiv.org/abs/2507.16970)

[arXiv:1709.01544](https://arxiv.org/abs/1709.01544)

Protagonists

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Dark Energy *can* cross phantom divide

- If observations are consistent with $w=-1$, have we proven that $DE=\Lambda$?

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{Pl}}^2 R + p(\phi, X) + \mathcal{L}_m \right]$$

- Canonical scalar field: yes

$$X \equiv -\frac{1}{2} (\partial_\mu \phi)^2$$

$$p(\phi) = X + V(\phi) \quad \Rightarrow \quad w = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$$

- Not true in general: could have equation of state that *varies around* $w=-1$

- **Monodromic k-essence:** $p(\phi, X) = \tilde{V}(\phi) \left[-X/M^4 + (X/M^4)^2 \right]$

$$\tilde{V}(\phi) = C \left(\frac{\phi}{\phi_0} \right)^{-\alpha} [1 - A \sin(\nu H_0 \phi + \delta)].$$

Dark Energy *can* cross phantom divide

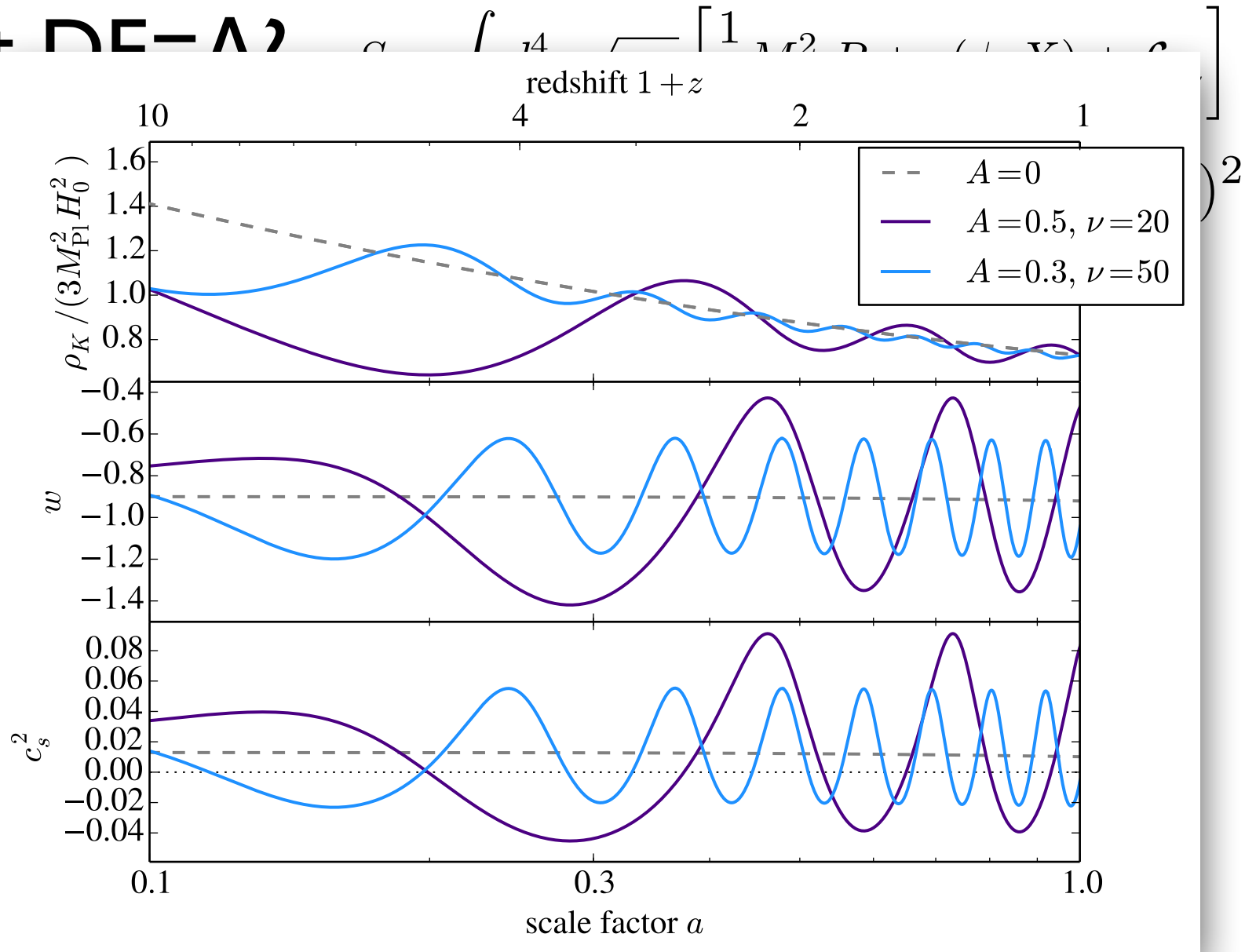
- If observations are consistent with $w=-1$, have we proven that $\rho_{DE} = \Lambda^2$?

- Canonical scalar field

$$p(\phi) = X + V(\phi)$$

- Not true in general: state that *varies around*

- *Monodromic k-essence*



On tachyonic instabilities

- Fine at the background level, but DE perturbations suffer tachyonic instabilities if $c_s^2 < 0$
- k-essence case naturally has $c_s^2 \ll 1$; in fact, $c_s^2 \sim (1+w)$ in $1+w \rightarrow 0$ limit, leading to tachyonic instabilities as $1+w < 0$
- These can be dealt with consistently if

- Higher-derivative contributions are present:

$$\delta\ddot{\phi} \sim -c_s^2 k^2 \delta\phi + \frac{k^4}{\bar{M}^2} \delta\phi + \dots$$

e.g., from

$$\Delta\mathcal{L}_{\text{DE,h.deriv.}} = -\frac{\bar{M}^2}{2} [\Box\phi + 3H(\phi)]^2$$

- c_s^2 stays infinitesimally below 0
- Lowers cutoff of the theory, but not ruled out.

Dark Energy *can* cross phantom divide

- An example viable model (due to Marco Celoria):

$$p(\phi, X) = \frac{\bar{M}^4}{2}(2X - 1)^2 - F(\phi) + G(\phi)(2X + 1)$$

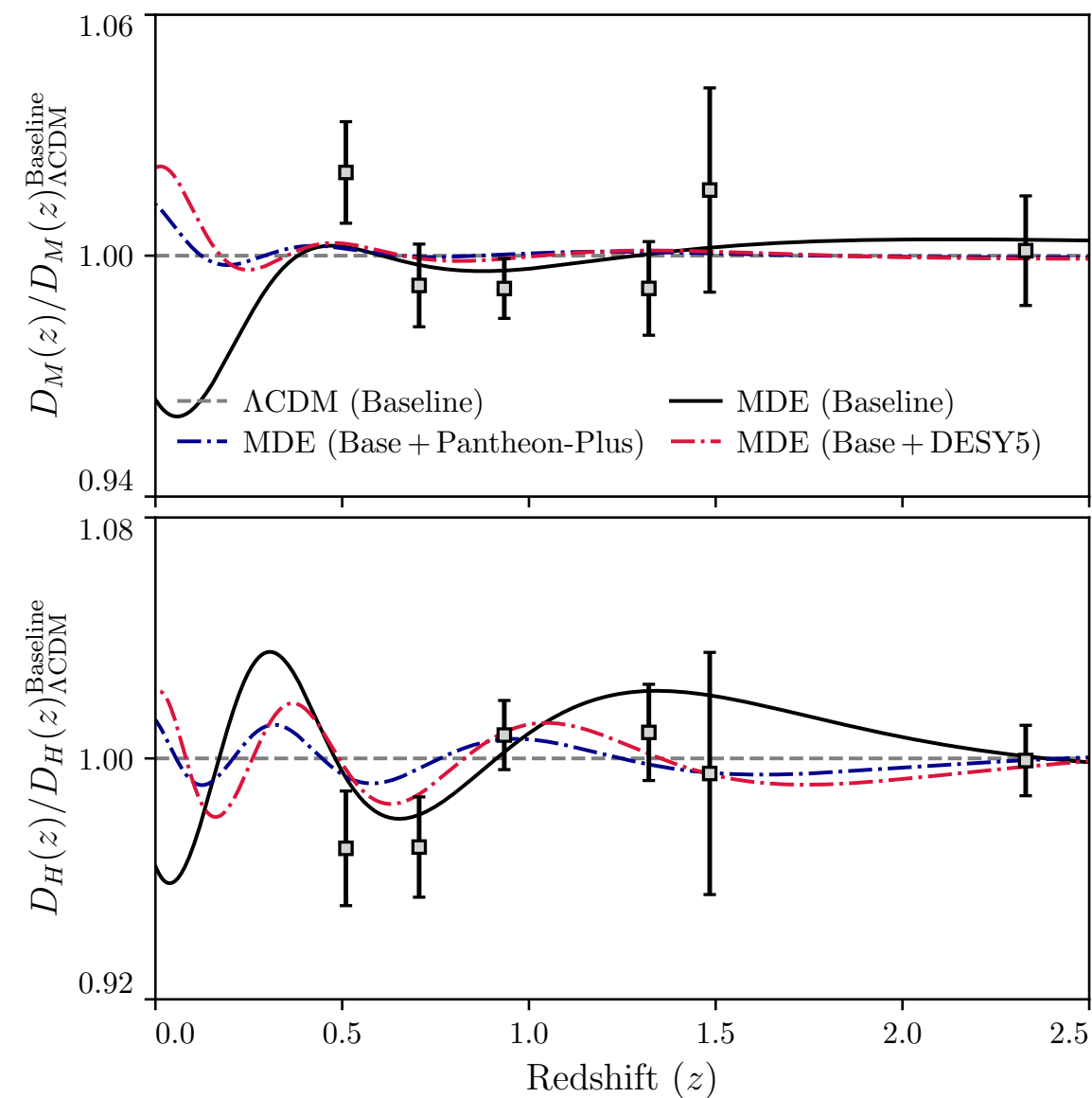
$$F(\phi) = V_0 \left[1 - \tilde{A} \sin(\tilde{\nu} H_0 \phi) \right]$$

$$G(\phi) = V_0 \tilde{A} \tilde{\nu} H_0 \cos(\tilde{\nu} H_0 \phi).$$

- Oscillations with amplitude $\Delta w \sim 0.1$ around $w = -1$ easily possible while satisfying constraints on instabilities and having cutoff $> \text{eV}$ scale.

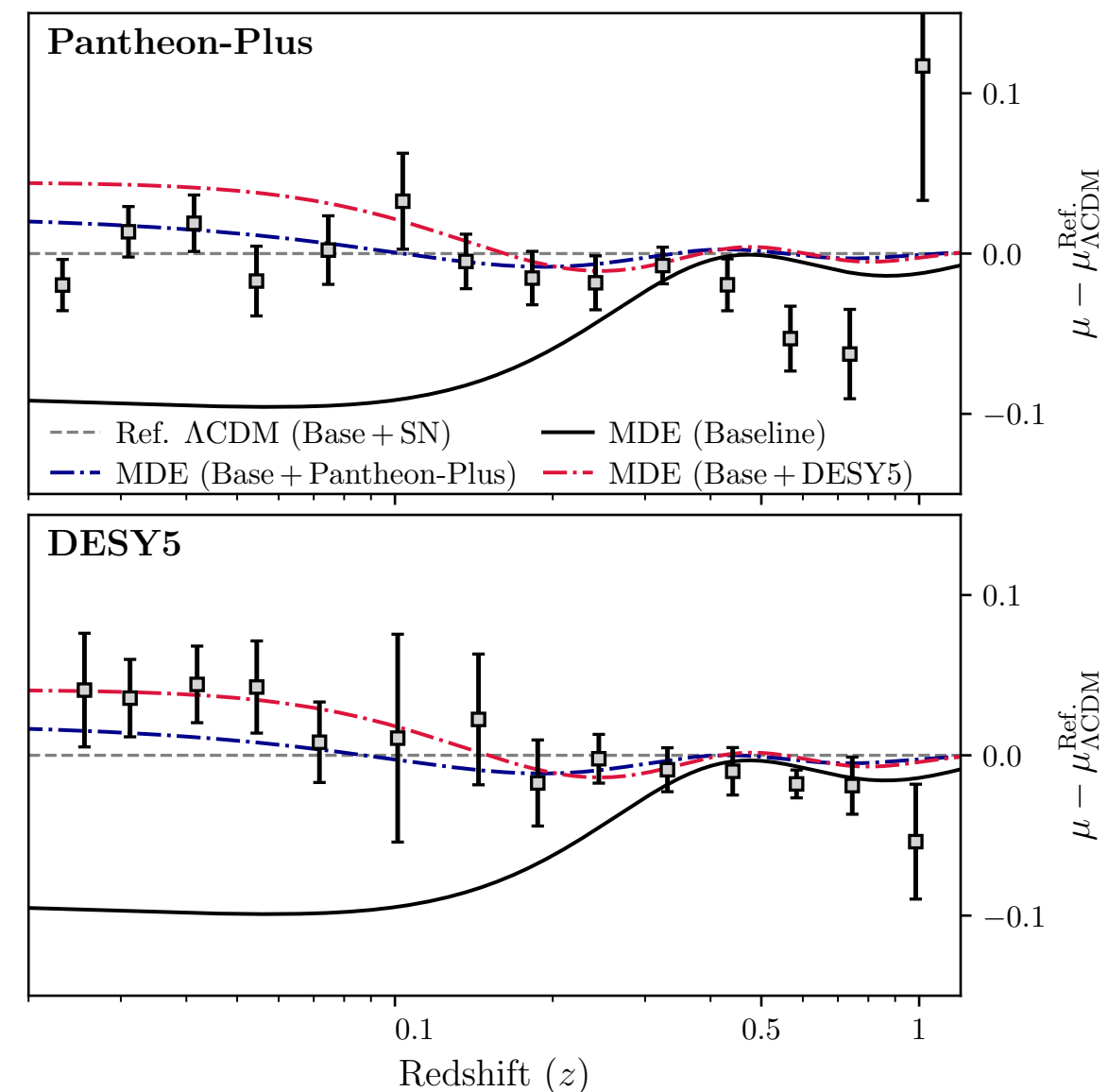
Monodromic k-essence and DESI

- 3 free parameters (FS 2017 model) in addition to Ω_{de} , potential tilt $\alpha \Leftrightarrow$ mean w :
 - amplitude, frequency, phase of oscillations
- Exclude all observables sensitive to perturbations here

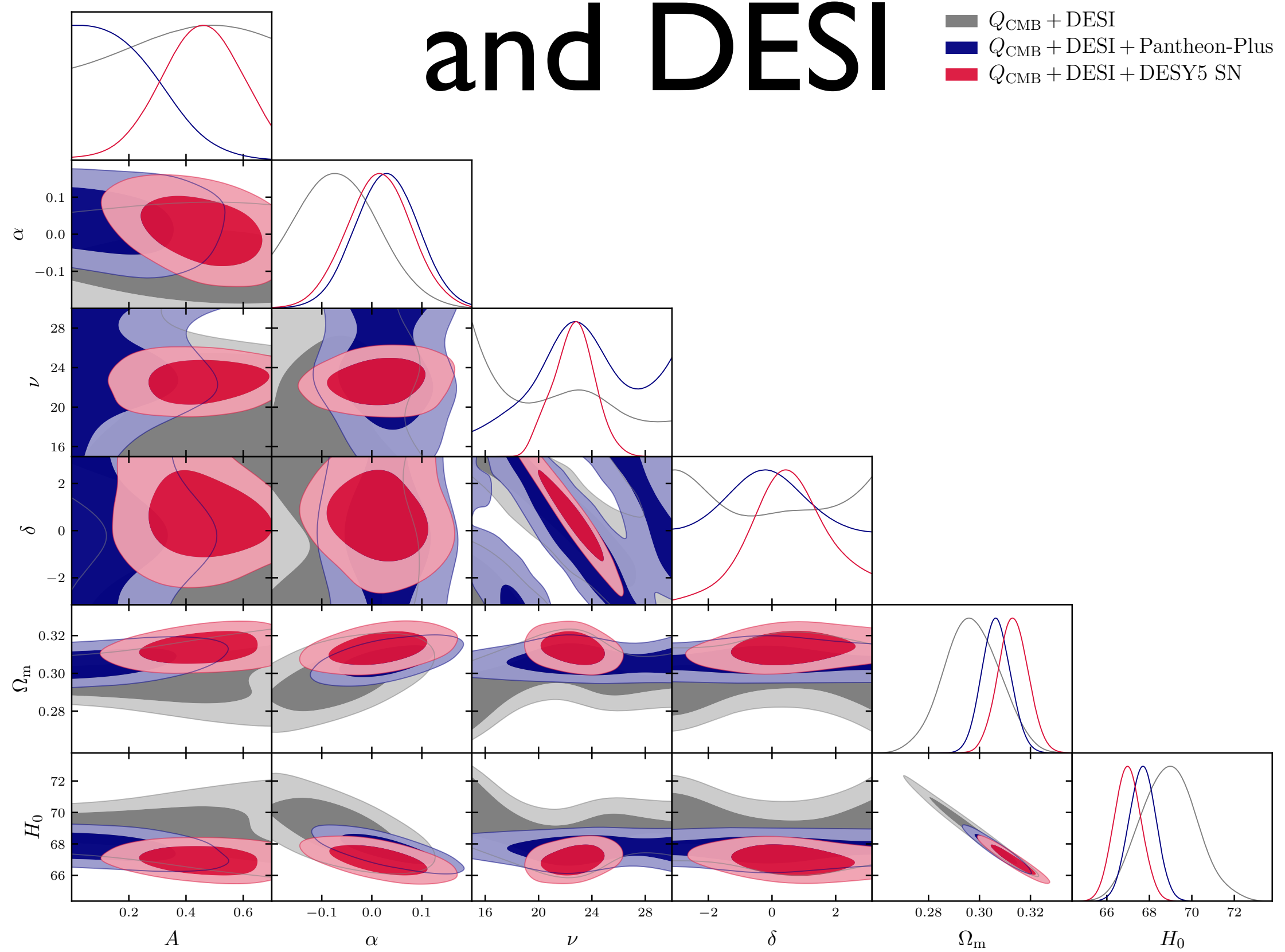


Monodromic k-essence and DESI

- 3 free parameters (FS 2017 model) in addition to Ω_{de} , potential tilt $\alpha \Leftrightarrow$ mean w :
 - amplitude, frequency, phase of oscillations
- Exclude all observables sensitive to perturbations here
- Similar fit quality to DESI BAO + SN as w_0, w_a
- Mean w consistent with -1 (motivated by theory as well); then, only 1 more free parameter than w_0, w_a !

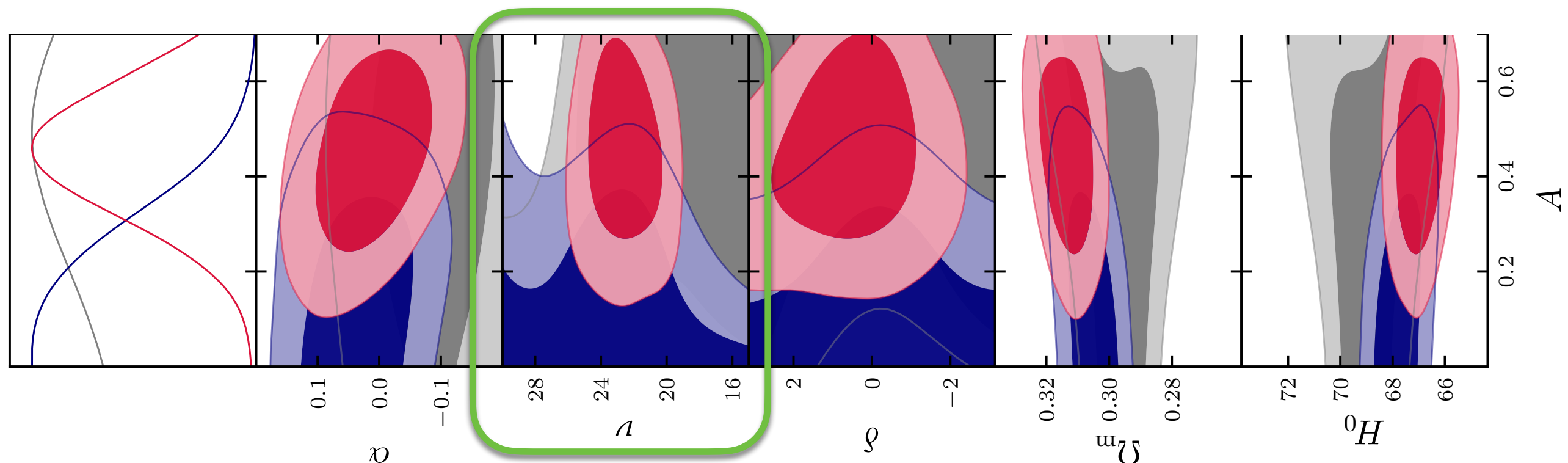


Monodromic k-essence and DESI



Monodromic k-essence and DESI

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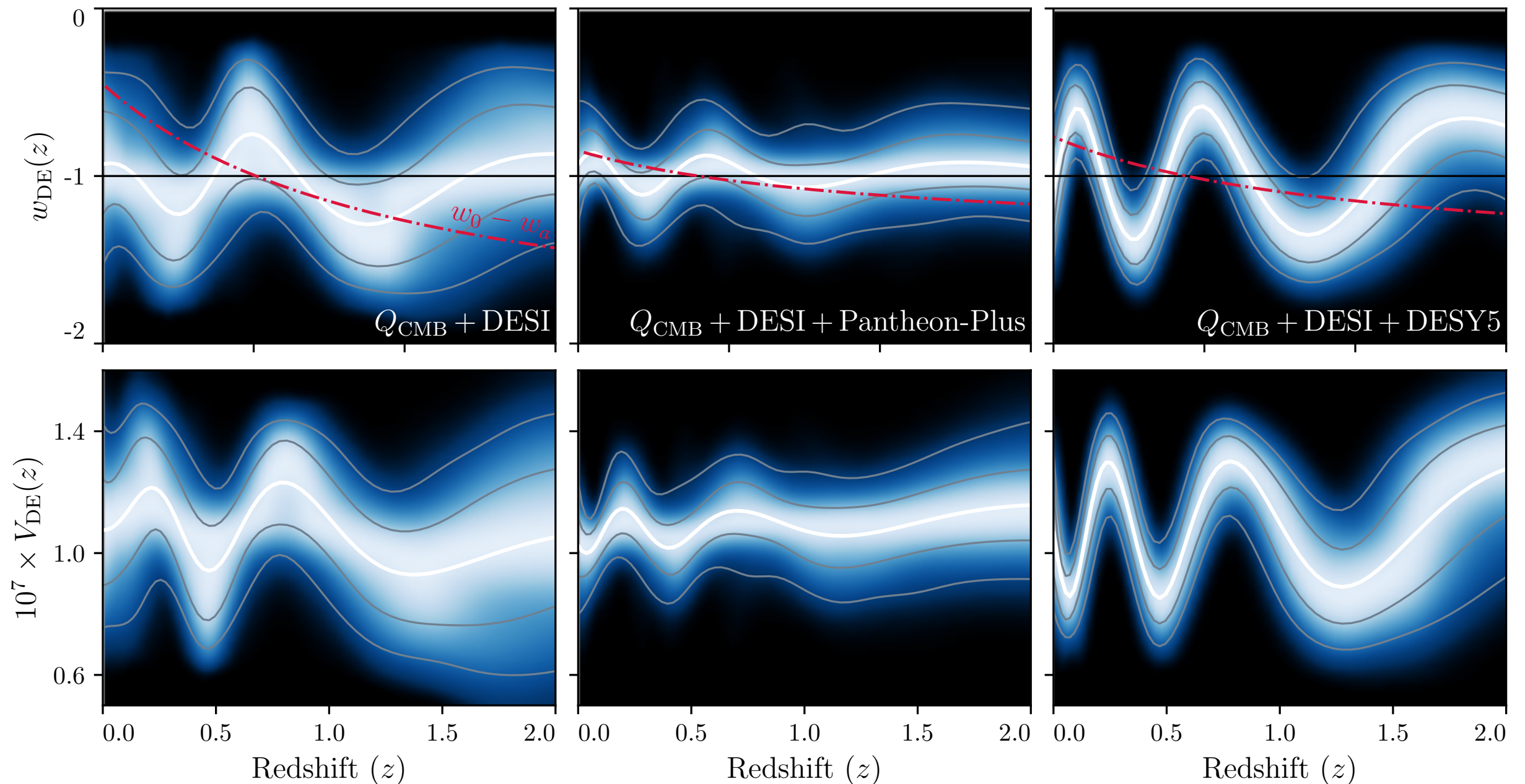
Monodromic k-essence and DESI

	A	α	ν	δ	Ω_m	H_0 [km/s/Mpc]	$\Delta\chi^2$
Base: $Q_{\text{CMB}}+\text{DESI}$	— (0.58)	$-0.06^{+0.06}_{-0.09}$ (−0.08)	— (15.2)	— (−2.2)	0.297 ± 0.011 (0.281)	68.9 ± 1.3 (71.0)	-7
Base+Pantheon-Plus	< 0.44 (0.25)	0.03 ± 0.06 (0.01)	— (22.1)	— (0.2)	0.307 ± 0.006 (0.308)	67.7 ± 0.6 (67.5)	-5
Base+DESY5 SN	$0.44^{+0.15}_{-0.12}$ (0.47)	0.01 ± 0.06 (0.00)	$22.6^{+1.6}_{-1.5}$ (22.8)	0.5 ± 1.2 (0.5)	0.313 ± 0.006 (0.314)	67.0 ± 0.6 (66.9)	-16

TABLE I. Marginalized constraints on monodromic dark energy and other cosmological parameters for the datasets considered in this work. For each dataset, we report the posterior mean and 68% two-tailed C.L. for all parameters that are detected at $> 2\sigma$, otherwise we report the one-tailed 95% C.L.; “—” denotes parameters that are entirely constrained by the prior at 2σ . Maximum *a posteriori* values are shown in parentheses. The final column reports the $\Delta\chi^2$ values computed with respect to a Λ CDM cosmology.

Monodromic k-essence and DESI

- Reconstruction of $w(z)$ and k-essence “potential”



Outlook

- **Monodromic k-essence: physically viable model that crosses phantom divide** and provides reasonable improvement over Λ CDM to DESI+SN
 - Would be interesting to combine with more high-z/low-z data
- Key next step: calculate evolution of perturbations
 - Expect interesting constraints from growth rate
 - Especially if extending to higher frequencies
- $c_s=0$: Dark energy clusters, and perturbations become nonlinear on small scales
 - Need N-body simulations that jointly solve for DE (**ongoing work with Linda Blot**)