Quantize Kähler-Dirac field

A theoretical explanation for Tenet?!

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September 18, 2025

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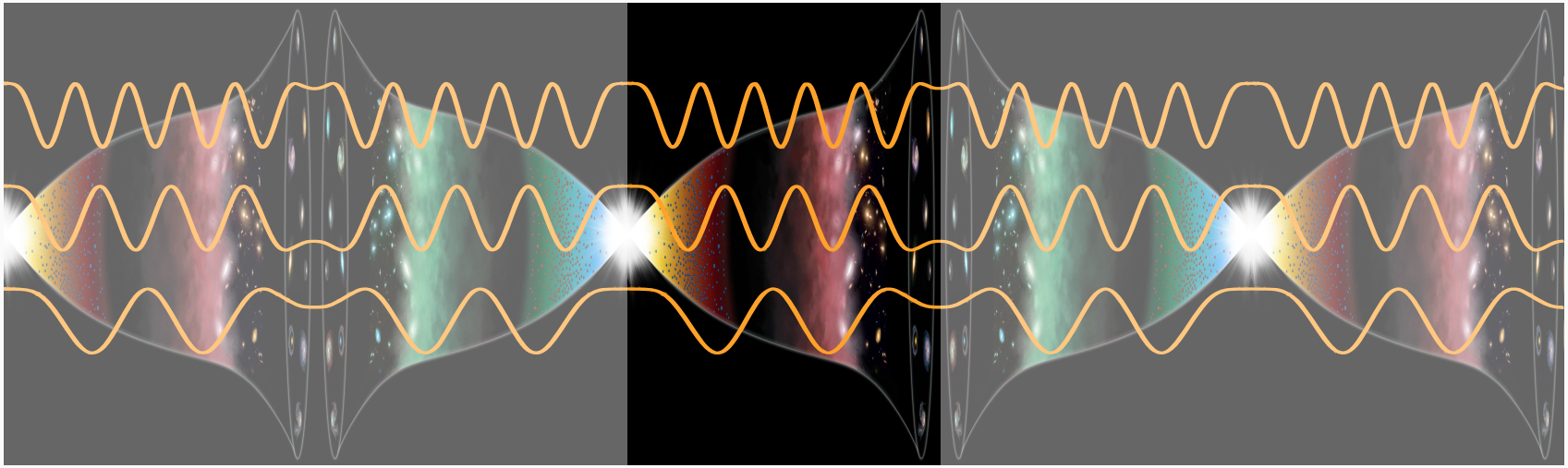
Q1: What does time going backwards universe mean?

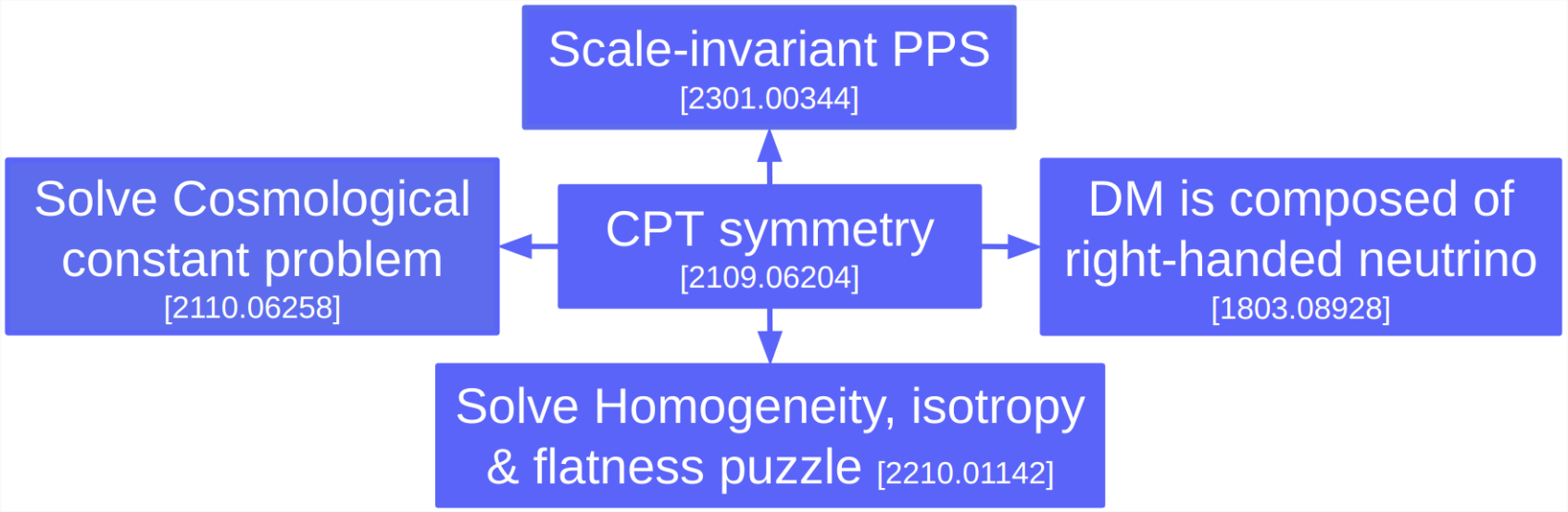
Q2: Can we travel forward in a backward going universe?

introduction: CPT symmetric universe



1. Periodic solution to the Friedmann eq. in conformal time.
2. Physical time goes backwards in anti-universe, since the scale factor a is negative.
3. CPT-symmetric universe assumes not just the scale factor, but all fields should be periodic, following CPT-symmetric at the Bang.

Introduction: CPT symmetric universe



Introduction: conventional Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

1. Describe how fermions evolve.
2. **Problem 1:** in curved spacetime, a spin connection term should be added.
3. **Problem 2:** fermion doubling problem $\rightarrow$ prevent people from numerically simulate fermions on a lattice.

Introduction: conventional Dirac equation

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Q:Is there any other way to write down Dirac equation?

Introduction: Kähler-Dirac equation

Original formulation:

$$(K - m)\Phi = 0, \quad K = d - d^\dagger, \quad \Phi = (\phi, \phi_\mu, \phi_{\mu\nu}, \dots)$$

Spinor formulation:

Ψ is a 4×4 spinor with linear combination of gamma matrices.

$$(i\gamma^\mu \partial_\mu - m)\Psi = 0, \quad \Psi = \sum_{p=0}^{4} \frac{1}{p!} \gamma^{\mu_1 \dots \mu_p} \phi_{\mu_1 \dots \mu_p}$$

1. Written in differential forms, easy to extend to curved spacetime.
2. Consistent with geometry on a lattice (0-form $\leftrightarrow$ vortices, 1-form $\leftrightarrow$ edges) $\Rightarrow$ could naturally be put on a lattice.

Additional γ^0 in Lagrangian

Lorentzian signature is important for understanding underlining physics.

Kähler-Dirac 4x4 spinor:

- Lorentz transform: $\Psi' = U_\Lambda \Psi U_\Lambda^{-1}, \quad U_\Lambda^\dagger = \gamma^0 U_\Lambda^{-1} \gamma^0$
- Scalar should be Lorentz invariant:
  $\mathrm{Tr}\left(\bar{\Psi}'\Psi'\right) = \mathrm{Tr}\left(\bar{\Psi}\Psi\right) \rightarrow \bar{\Psi} = \gamma^0 \Psi^\dagger \gamma^0$
- Lagrangian:
  $\mathcal{L} = \mathrm{Tr}\left[\bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi\right] = \mathrm{Tr}\left[\gamma^0 \Psi^\dagger \gamma^0 (i\gamma^\mu \partial_\mu - m)\Psi\right]$

Lorentz invariance implies an additional γ^0 in the Lagrangian!

Quantize Kähler-Dirac equation

1. The spinor could be separated into even and odd forms part:

$$\Psi = \begin{pmatrix} E & O' \\ O & E' \end{pmatrix} = \Psi_+ + \Psi_-, \quad \Psi_+ = \begin{pmatrix} E & 0 \\ 0 & E' \end{pmatrix}, \quad \Psi_- = \begin{pmatrix} 0 & O' \\ O & 0 \end{pmatrix}$$

2. Quantize the field:

$$\Psi_\pm(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{E_p/m}} \sum_s (b_{s\pm}(\vec{p}) u_{s\pm} e^{-ip\cdot x} + d^\dagger_{s\pm}(\vec{p}) v_{s\pm} e^{ip\cdot x})$$

3. To satisfy anti-commutation relation:
$\{\Psi_\pm(\vec{x}, 0), \Pi_\mp(\vec{y}, 0)\} = \delta^{(3)}(\vec{x} - \vec{y})$ a sign should be introduced (ε_s takes values ± 1):

$$\{b_{s\pm}(\vec{p}), b^\dagger_{s'\mp}(\vec{p}')\} = \varepsilon_s \delta^{(3)}(\vec{p} - \vec{p}')\delta_{ss'}$$
$$\{d^\dagger_{s\pm}(\vec{p}), d_{s'\mp}(\vec{p}')\} = \varepsilon_s \delta^{(3)}(\vec{p} - \vec{p}')\delta_{ss'}$$

The sign ε_s causes negative energy and norm!!!

1. **Hamiltonian:** create particles with negative energy! – **ghost!!**

$$H = \int d^3p \sum_s E_p \varepsilon_s (b^\dagger_{s+}(\vec{p}) b_{s-}(\vec{p}) + b^\dagger_{s-}(\vec{p}) b_{s+}(\vec{p}))$$

$$+ \sum_s E_p \varepsilon_s (d^\dagger_{s+}(\vec{p}) d_{s-}(\vec{p}) + d^\dagger_{s-}(\vec{p}) d_{s+}(\vec{p})) - \sum_s \delta^{(3)}(0) \int d^3p E_p$$

2. **Norm:** states of particles have negative probability! Nonsense!!

$$\langle 1_{s\pm}(\vec{p}) | 1_{s'\pm}(\vec{p}') \rangle = \varepsilon_s \delta_{ss'} \delta^{(3)}(\vec{p} - \vec{p}') \langle 0|0 \rangle$$

Solution: particles/antiparticles created/annihilated together

Apply the method used by Gupta-Bleuler quantization in QED.

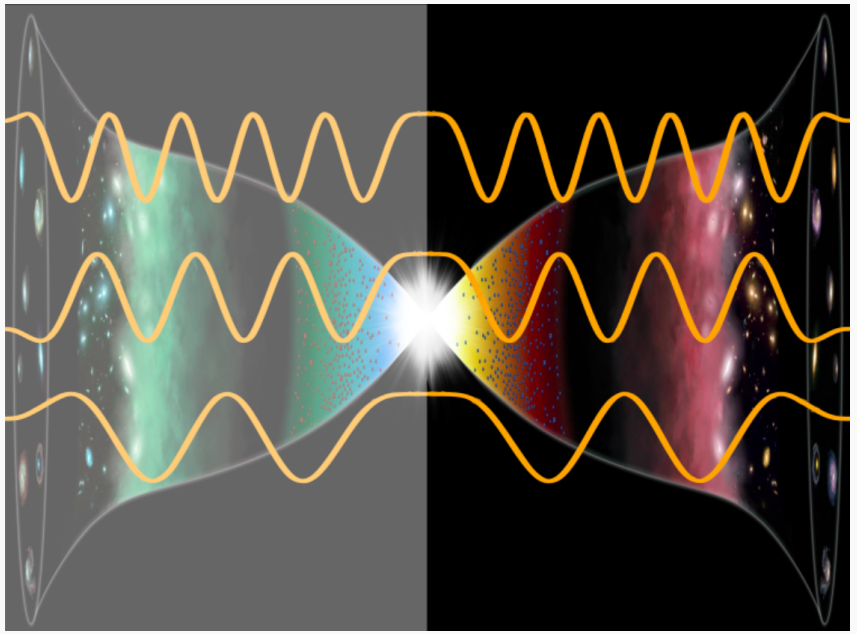
1. Physical states $|\phi\rangle$ should satisfy the relation: that is, particles have positive and negative energy should be created and annihilated together in pairs.

$$(a^{(-)}_{\text{type},\varepsilon_s=1} - a^{(-)}_{\text{type}',\varepsilon_s=-1})|\phi\rangle = 0$$
$$\varepsilon_s = 1 : \phi \leftrightarrow \phi_t, \quad \tilde{\phi}_{ti} \leftrightarrow \tilde{\phi}_i$$
$$\varepsilon_s = -1 : \tilde{\phi} \leftrightarrow \tilde{\phi}_t, \quad \phi_{ti} \leftrightarrow \phi_i$$

2. The positive and negative terms cancel out to form zero energy and norm.

Physical meaning: CPT symmetric universe

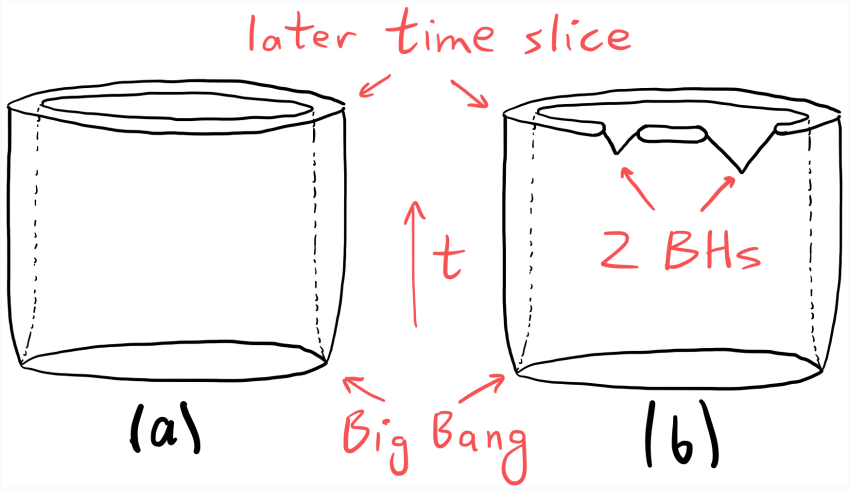
1. Particles with positive and negative energy are created in universe & anti-universe, respectively.
2. Since time goes backward in anti-universe it is natural to have negative energy (in our view).
3. Since they follow the same equation, their evolution after creation are the same.



The pair universes are symmetric not just globally, but also locally!

Black Hole mirror

1. Black hole interior problem.
2. Latham Boyle: BH has no interior, the event horizon acts as a mirror between universe & anti-universe!
3. It requires the pair universe to be **locally symmetric**, such that particles and antiparticles falling into the BH annihilate at the horizon.
4. Pair particles falling into BH from both sides = particles fall into BH and come out of a white hole.



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Theoretical explanation for Tenet?!

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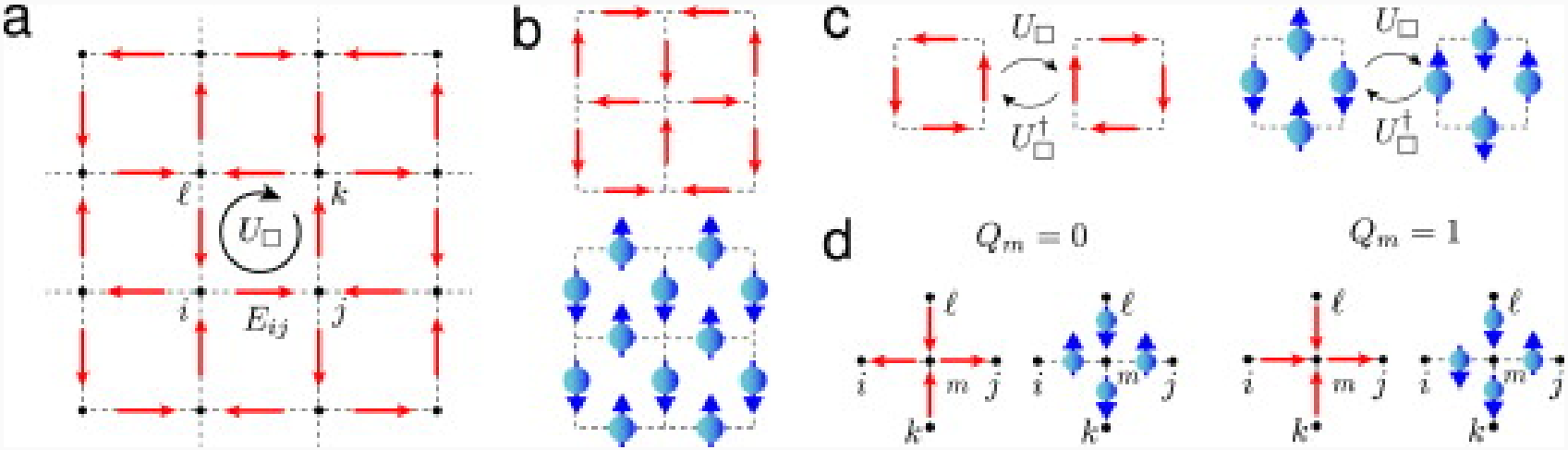
1. **Same:**
   - 1.1 Time goes forward in universe and backward in the anti-universe, where there are you and anti-you, respectively.
   - 1.2 The gate connecting the pair universes are black holes.
2. **Difference:**
   - 2.1 You annihilate with anti-you at the transition gate without passing through it.
   - 2.2 In our universe's view, you'd emerge from a white hole and found time going backward, but you wouldn't be able to alter the past—only relive it.

Application on lattice?

Kähler-Dirac field is famous for putting fermions on a lattice.

1. Additional γ^0 makes the Lagrangian Hermitian and chiral decoupled in Euclidean space.
2. To build an anomaly-free theory, Latham Boyle found two-sheeted is needed, as we found in the work.
3. Numerical tool for understanding non-perturbative phenomenon in chiral gauge theory.



Remaining Questions

1. There is another solution: even and odd forms created and annihilated together. Is it a physical solution?
2. As a 4x4 spinor, Kahler-Dirac field has four fermions instead of one. Are they all physical or duplicated fermions? ex: Catterall shows that 4 stagger fermions cuold leads to Pati-Salam GUT (Catterall, 2024).
3. Unify theory: Kahler-Dirac equation can explain both bosons' and fermions' evolution. Does it has further physical implication?
4. Can we test the theory by observation? Fermion propagating in early universe or near BH and neutron stars.

Summary

1. CPT-symmetric universe, assuming CPT-symmetry between universe pairs at the Bang, can solve lots of problems elegantly.
2. The Kähler-Dirac field, a powerful tool for cosmology and lattice theory, suffers from negative-energy 'ghosts' when quantized.
3. Solution: particles with positive and negative energy should be created in pairs, in universe & anti-universe, respectively.
4. Application: put fermions on lattice to understand chiral gauge theory.

Thanks for Listening. Questions?

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