Probing Parity Violation with Weak Lensing Trispectrum

Sijin Chen

Universitäts-Sternwarte München

With Zhengyangguang (Laurence) Gong and Jiamin Hou

Cambridge-LMU Cosmology Meeting, Sep 17, 2025

Motivation

- Parity violation in cosmology —— potential signature of new physics in large-scale structure
- CMB lensing has been proposed to probe parity violation in the early universe
- Weak lensing traces matter distribution at late times, complementary to CMB

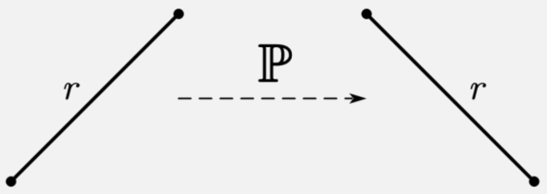
Introduction of parity

Parity transformation:

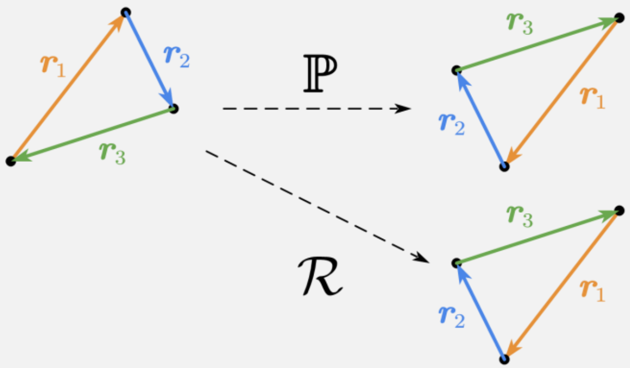
$$\mathbb{P} : \boldsymbol{x} \to -\boldsymbol{x}$$

$$\mathbb{P} : (x, y, z) \to (-x, -y, -z)$$

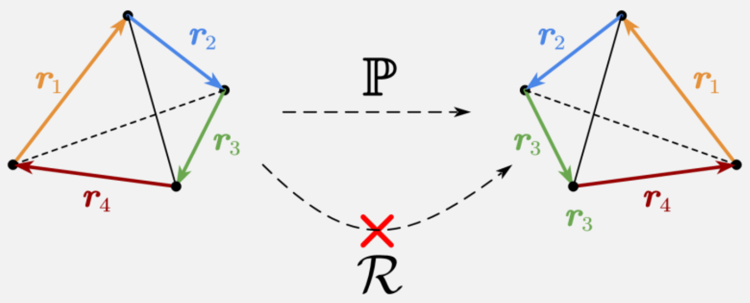
2PCF



3PCF



4PCF



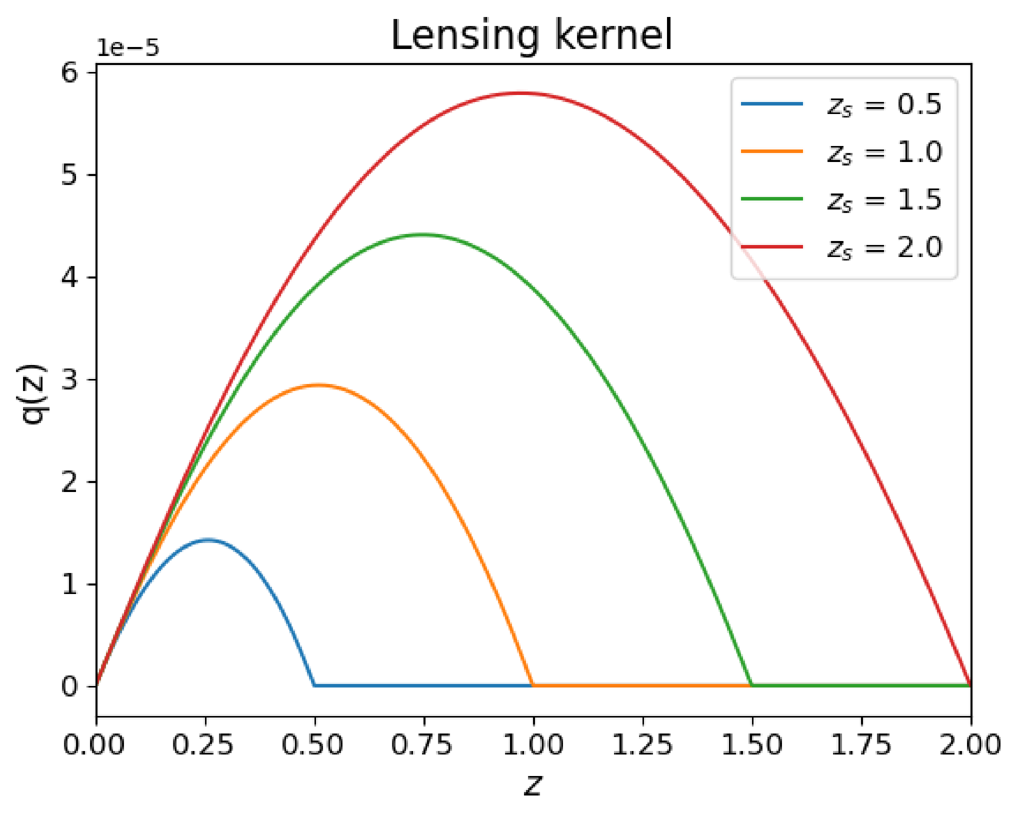
Weak lensing basics

- **Convergence, κ, is the projected matter density contrast along the line of sight**

$$\kappa(\hat{\boldsymbol{n}}) = \int_0^{\chi_H} \mathrm{d}\chi q(\chi) \delta(\chi\hat{\boldsymbol{n}}, \chi)$$

- **Lensing kernel $q(\chi)$ with Dirac delta redshift distribution:**

$$q(\chi) = \frac{3H_0^2\Omega_m}{2c^2} \frac{1}{a(\chi)} \frac{\chi(\chi_s - \chi)}{\chi_s}$$



Weak lensing trispectrum

Weak lensing convergence:

$$\kappa_{\ell m} = 4\pi i^{\ell} \int_0^{\chi_H} \mathrm{d}\chi' q(\chi') \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \tilde{\delta}(\boldsymbol{k}) j_\ell(k\chi') Y^*_{\ell m}(\hat{k})$$

Plane-wave expansion:

$$e^{i\boldsymbol{k}\cdot\boldsymbol{x}} = 4\pi \sum_{L=0}^{\infty} \sum_{M=-L}^{L} i^L j_L(kx) Y_{LM}(\hat{\boldsymbol{k}}) Y^*_{LM}(\hat{\boldsymbol{x}})$$

Angular trispectrum:

$$\langle \kappa_{\ell_1 m_1} \kappa_{\ell_2 m_2} \kappa_{\ell_3 m_3} \kappa_{\ell_4 m_4} \rangle = (2\pi)^3 (4\pi)^4 i^{\ell_1+\ell_2+\ell_3+\ell_4} \int_0^{\chi_{H_1}} \mathrm{d}\chi'_1 \ldots \int_0^{\chi_{H_4}} \mathrm{d}\chi'_4\, q(\chi'_1) \ldots q(\chi'_4)$$
$$\times \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} \ldots \int \frac{\mathrm{d}^3 k_4}{(2\pi)^3}\, \delta_D^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4) T(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)$$
$$\times j_{\ell_1}(k_1\chi'_1)\, j_{\ell_2}(k_2\chi'_2)\, j_{\ell_3}(k_3\chi'_3)\, j_{\ell_4}(k_4\chi'_4)$$
$$\times Y^*_{\ell_1 m_1}(\hat{k}_1)\, Y^*_{\ell_2 m_2}(\hat{k}_2)\, Y^*_{\ell_3 m_3}(\hat{k}_3)\, Y^*_{\ell_4 m_4}(\hat{k}_4)$$

Weak lensing reduced trispectrum

Formula of weak lensing angular reduced trispectrum:

$$Q^{\ell_1\ell_2}_{\ell_3\ell_4}(L) = (2L+1)\sum_{m_1m_2m_3m_4M}(-1)^M\begin{pmatrix}\ell_1 & \ell_2 & L\\ m_1 & m_2 & M\end{pmatrix}\begin{pmatrix}\ell_3 & \ell_4 & L\\ m_3 & m_4 & -M\end{pmatrix}\langle\kappa_{\ell_1m_1}\kappa_{\ell_2m_2}\kappa_{\ell_3m_3}\kappa_{\ell_4m_4}\rangle$$

$$= (2L+1)\times i^{\ell_1+\ell_2+\ell_3}$$

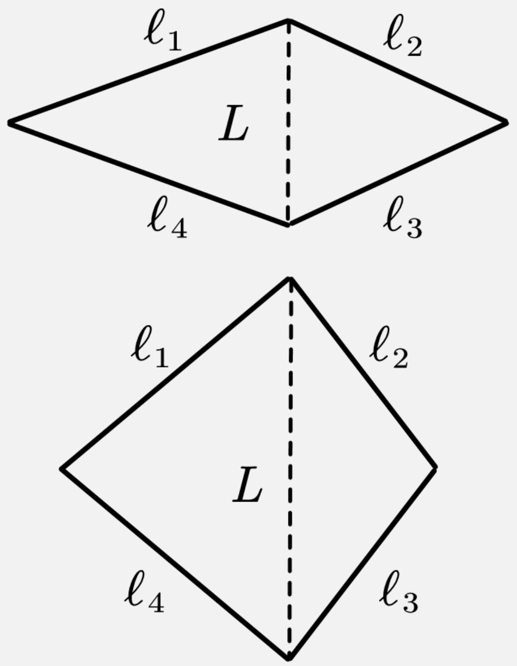
$$\times\sum_{L_1L_2L_3}\sum_{L'}\mathcal{F}_{L_1L_2L'}\mathcal{F}_{L_3\ell_4L'}\times i^{L_1+L_2+L_3}\sum_{\ell_1'\ell_2'\ell_3'}\times\mathcal{F}_{L_1\ell_1'\ell_1}\mathcal{F}_{L_2\ell_2'\ell_2}\mathcal{F}_{L_3\ell_3'\ell_3}$$

$$\times(-1)^{\ell_1'+\ell_2'}\begin{Bmatrix}\ell_3 & \ell_4 & L\\ L' & \ell_3' & L_3\end{Bmatrix}\begin{Bmatrix}L & L' & \ell_3'\\ \ell_2 & L_2 & \ell_2'\\ \ell_1 & L_1 & \ell_1'\end{Bmatrix}$$

Geometry part

$$\times\left[\frac{2}{5}\frac{1}{\Omega_{m,0}H_0^2}\right]^4\int_0^\infty\frac{\mathrm{d}\chi}{\chi^{14}}\prod_{n=1}^{4}\left[q_n(\chi)D(\chi)\mathcal{T}_\delta\left(\frac{\ell_n}{\chi}\right)\ell_n^2\right]T^{\mathcal{R}}_{\ell_1'\ell_2'\ell_3'}\left(\frac{\ell_1}{\chi},\frac{\ell_2}{\chi},\frac{\ell_3}{\chi},\frac{\ell_4}{\chi}\right)$$

Projection part



where $\mathcal{F}$ is: explain the projection, geometry and finally the total reduced trispectrum.

$$\mathcal{F}_{\ell_1\ell_2\ell_3} = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ 0 & 0 & 0\end{pmatrix}$$

Weak lensing trispectrum projection part

Projection part:

$$\mathcal{I}^{\ell_1'\ell_2'\ell_3'}_{\ell_1\ell_2\ell_3\ell_4} = \left[\frac{2}{5}\frac{1}{\Omega_{m,0}H_0^2}\right]^4 \int_0^\infty \frac{\mathrm{d}\chi}{\chi^{14}} \prod_{n=1}^{4} \left[q_n(\chi) D(\chi) \mathcal{T}_\delta\left(\frac{\ell_n}{\chi}\right)\ell_n^2\right] T^{\mathcal{R}}_{\ell_1'\ell_2'\ell_3'}\left(\frac{\ell_1}{\chi},\frac{\ell_2}{\chi},\frac{\ell_3}{\chi},\frac{\ell_4}{\chi}\right)$$

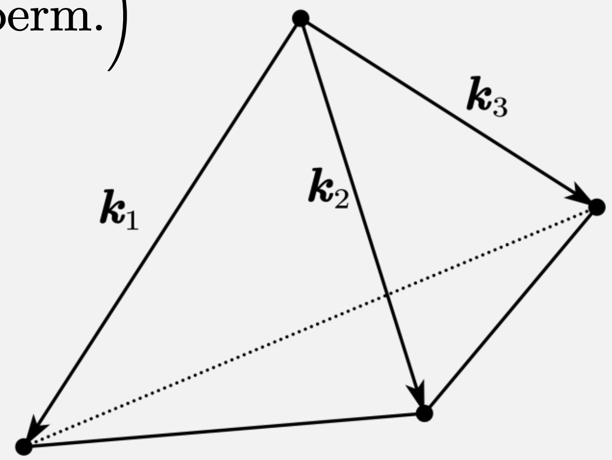
- $D(\chi)$: linear growth factor
- $\mathcal{T}_\delta\left(\frac{\ell_n}{\chi}\right)$: linear transfer function

Primordial curvature trispectrum template:

$$T^{\mathcal{R}}_{-}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \equiv [\boldsymbol{k}_1 \cdot (\boldsymbol{k}_2 \times \boldsymbol{k}_3)]\, g_- [2\pi^2 A_s]^3 \left(\frac{k_1^{-2}k_2^{-1}k_3^0k_4^0}{k_1^3k_2^3k_3^3k_4^0} \mp 23\,\text{perm.}\right)$$

- keep indices position fixed
- odd permutation → −
- even permutation → +

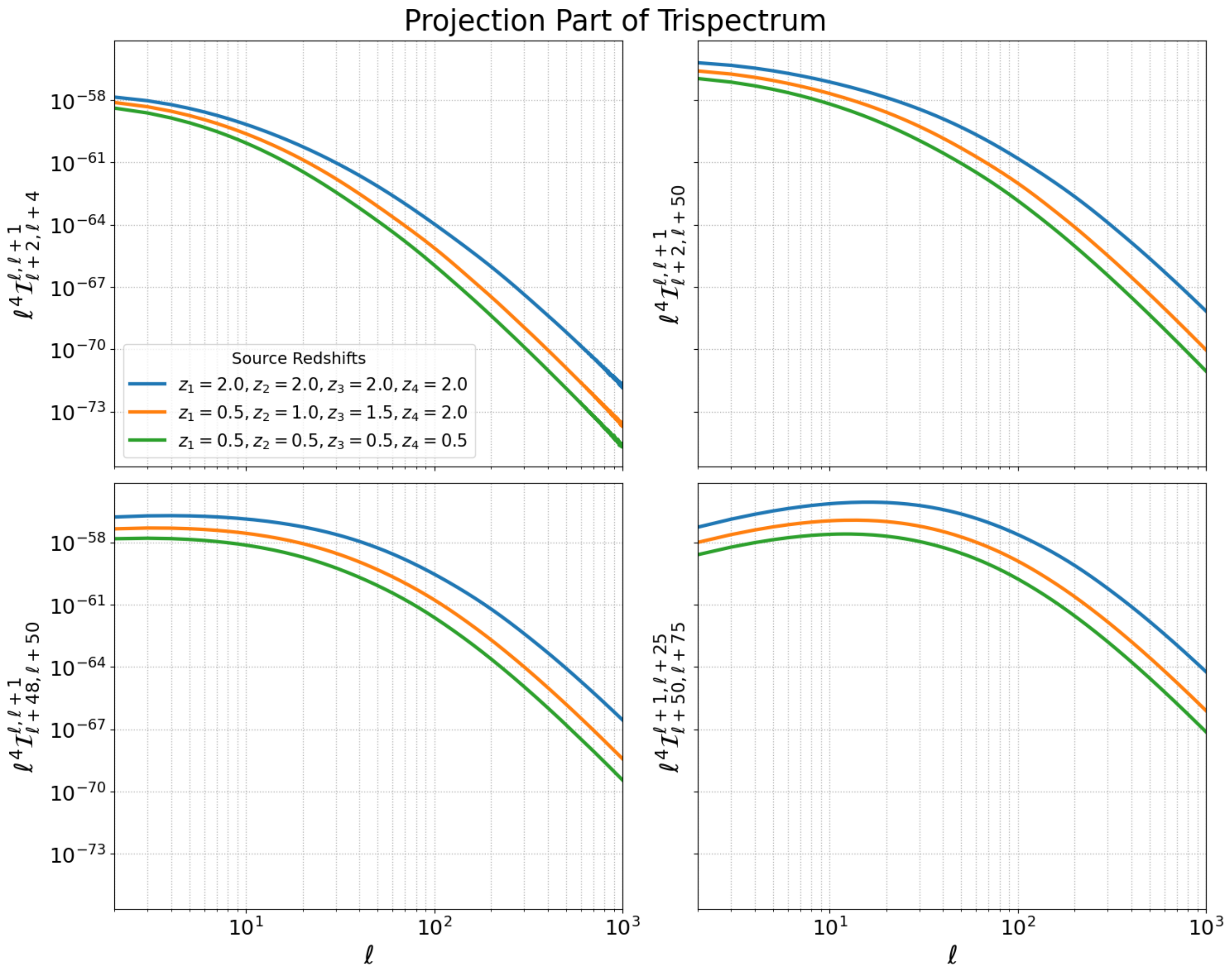
Coulton et al. 2024



Weak lensing trispectrum projection part

- **Projection part:**

$$\mathcal{I}^{\ell_1'\ell_2'\ell_3'}_{\ell_1\ell_2\ell_3\ell_4} = \left[\frac{2}{5}\frac{1}{\Omega_{m,0}H_0^2}\right]^4 \int_0^\infty \frac{\mathrm{d}\chi}{\chi^{14}} \times \prod_{n=1}^{4} \left[q_n(\chi) D(\chi) \mathcal{T}_\delta\left(\frac{\ell_n}{\chi}\right)\ell_n^2\right] \times T^{\mathcal{R}}_{\ell_1'\ell_2'\ell_3'}\left(\frac{\ell_1}{\chi},\frac{\ell_2}{\chi},\frac{\ell_3}{\chi},\frac{\ell_4}{\chi}\right)$$

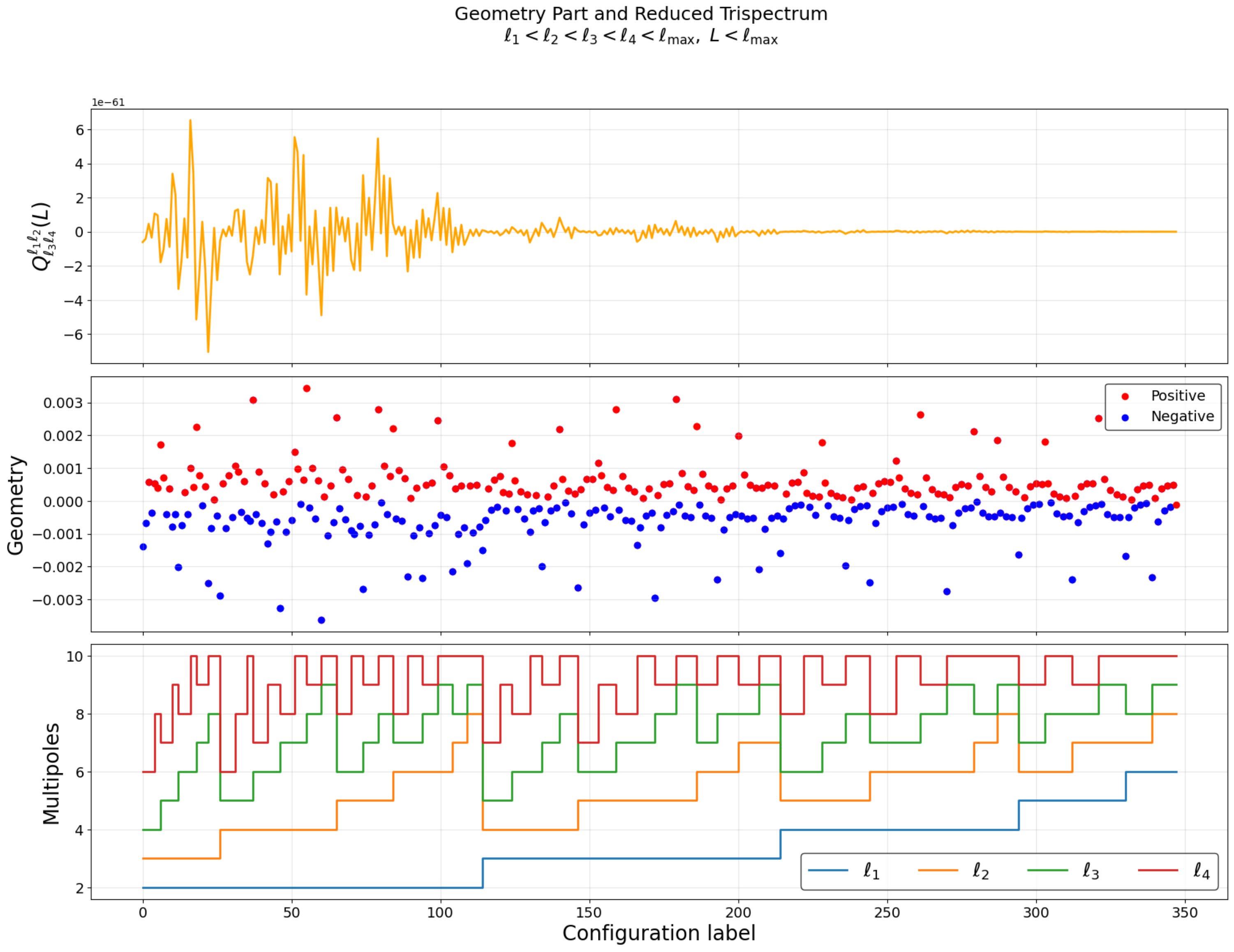


Weak lensing trispectrum geometry part

- **Geometry part:**

$$
\begin{aligned}
\mathcal{G} = &(2L+1) \times i^{\ell_1+\ell_2+\ell_3} \\
&\times \sum_{L_1 L_2 L_3} \sum_{L'} \mathcal{F}_{L_1 L_2 L'} \mathcal{F}_{L_3 \ell_4 L'} \times i^{L_1+L_2+L_3} \\
&\times \sum_{\ell_1' \ell_2' \ell_3'} \mathcal{F}_{L_1 \ell_1' \ell_1} \mathcal{F}_{L_2 \ell_2' \ell_2} \mathcal{F}_{L_3 \ell_3' \ell_3} \\
&\times (-1)^{\ell_1'+\ell_2'} \begin{Bmatrix} \ell_3 & \ell_4 & L \\ L' & \ell_3' & L_3 \end{Bmatrix} \begin{Bmatrix} L & L' & \ell_3' \\ \ell_2 & L_2 & \ell_2' \\ \ell_1 & L_1 & \ell_1' \end{Bmatrix}
\end{aligned}
$$

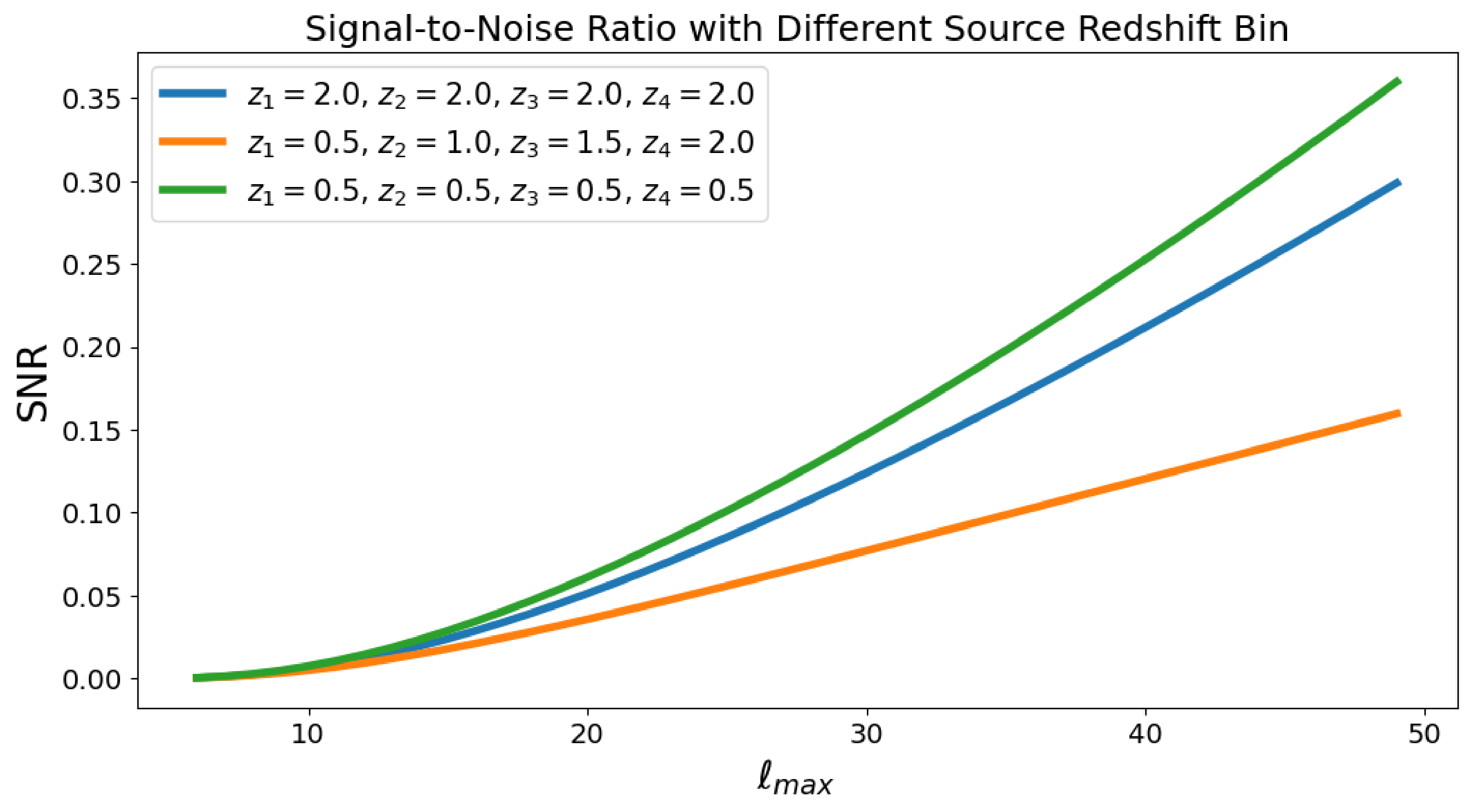
- A fix ordering $\ell_1 < \ell_2 < \ell_3 < \ell_4$
- For each set, vary L from minimum to maximum to generate configurations
- Figure shows the reduced trispectrum for each configuration



Signal-to-Noise Ratio

$$\mathrm{SNR} \approx \sqrt{\sum_{L=2}^{\ell_{\max}} (2L+1)^{-1} \sum_{\ell_1<\ell_2<\ell_3<\ell_4}^{\ell_{\max}} \frac{\left|Q^{\ell_1\ell_2}_{\ell_3\ell_4}(L)\right|^2}{C_{\ell_1}C_{\ell_2}C_{\ell_3}C_{\ell_4}}}$$

- **SNR computed with multipole summation cut as ℓ_{max}**
- **ℓ_{max} is the maximum of different combinations of $\{\ell_1, \ell_2, \ell_3, \ell_4, L\}$.**



Summary

- We applied a toy model primordial curvature parity-odd trispectrum, evolved with linear growth factor and transfer functions and then projected, to obtain the weak lensing trispectrum.

- We estimated its signal-to-noise ratio, suggesting that weak lensing may provide a complementary probe of parity-odd information at late times.