Towards higher-loops in LSS

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Based on:
2307.15031, 2404.16929, 2405.21002, 2507.13905
2508.00611

Cambridge-LMU workshop, September 2025

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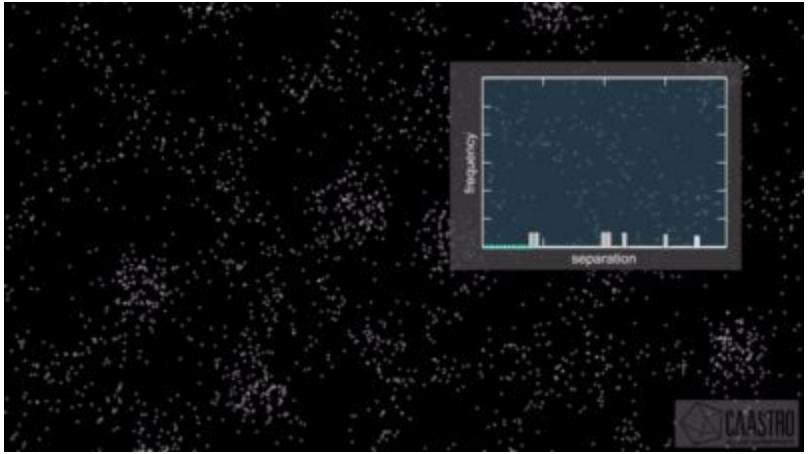
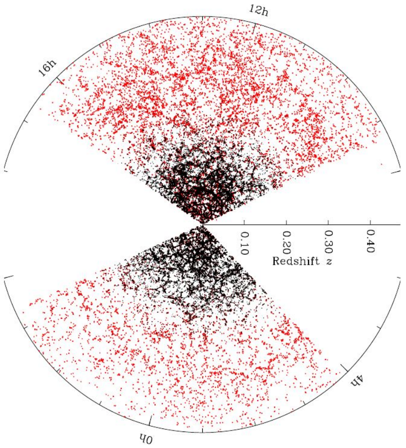
Preamble

Structure formation

CAASTRO



SDSS
collaboration

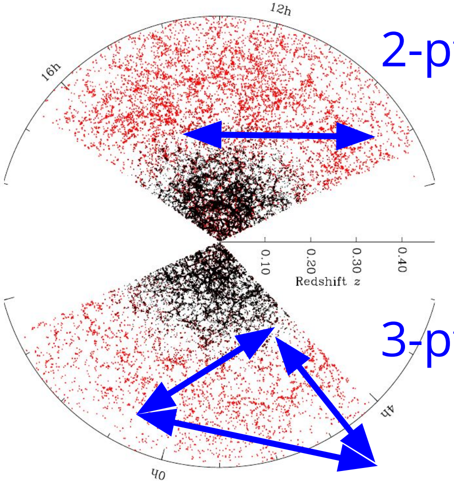


Calculate the galaxy n-point functions

$$\langle \delta_g(k_1)\delta_g(k_2)\ldots\delta_g(k_n)\rangle$$

N-pt functions

Gil-Marin+, 2014, SDSS BOSS data

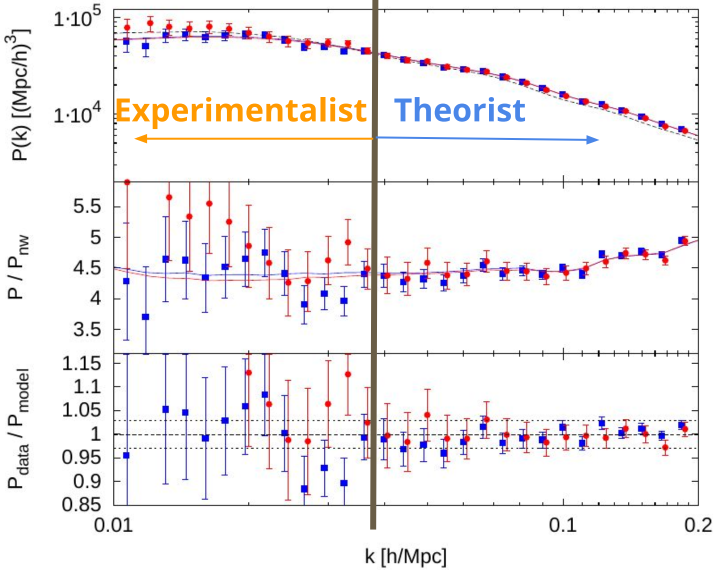


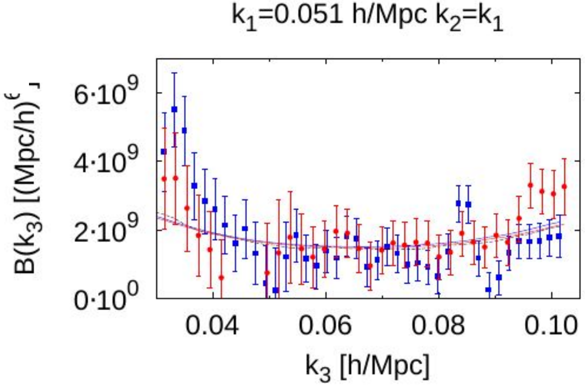
2-pt function

3-pt function

Theory (yes)
Data (no)

Theory (no)
Data (yes)





Is there a way to push towards non-linear scales from 'first principles'?

The (smoothed) EoM

Overview on perturbation theory for LSS

$$\partial_\tau \delta + \theta = -\int_{\mathbf{p}_1}\int_{\mathbf{p}_2} \delta_D\left(\mathbf{p_2} - (\mathbf{k} - \mathbf{p}_1)\right)\theta_{\mathbf{p_1}}\delta_{\mathbf{p_2}}\alpha(\mathbf{p_1}, \mathbf{p_2})$$

$$\partial_\tau \theta + \mathcal{H}\theta + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta = -\int_{\mathbf{p}_1}\int_{\mathbf{p}_2} \delta_D\left(\mathbf{p_2} - (\mathbf{k} - \mathbf{p}_1)\right)\theta_{\mathbf{p_1}}\theta_{\mathbf{p_2}}\beta(\mathbf{p_1}, \mathbf{p_2})$$

Perturbative solution

$$\boxed{\delta(\mathbf{x}, \tau) = \sum_n a^n(\tau)\delta^{(n)}(\mathbf{x})}$$

$$\delta^{(n)}(\mathbf{k}) = \int_{\mathbf{q}_{1\dots n}} \delta_{\rm D}(\mathbf{q}_{1\dots n} - \mathbf{k})\, F^{(n)}(\mathbf{q}_1, \dots, \mathbf{q}_n)\, \delta^{(1)}(\mathbf{q}_1)\dots\delta^{(1)}(\mathbf{q}_n)$$

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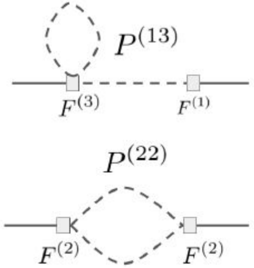
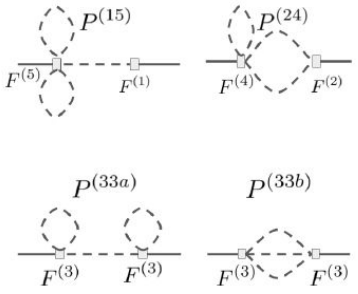
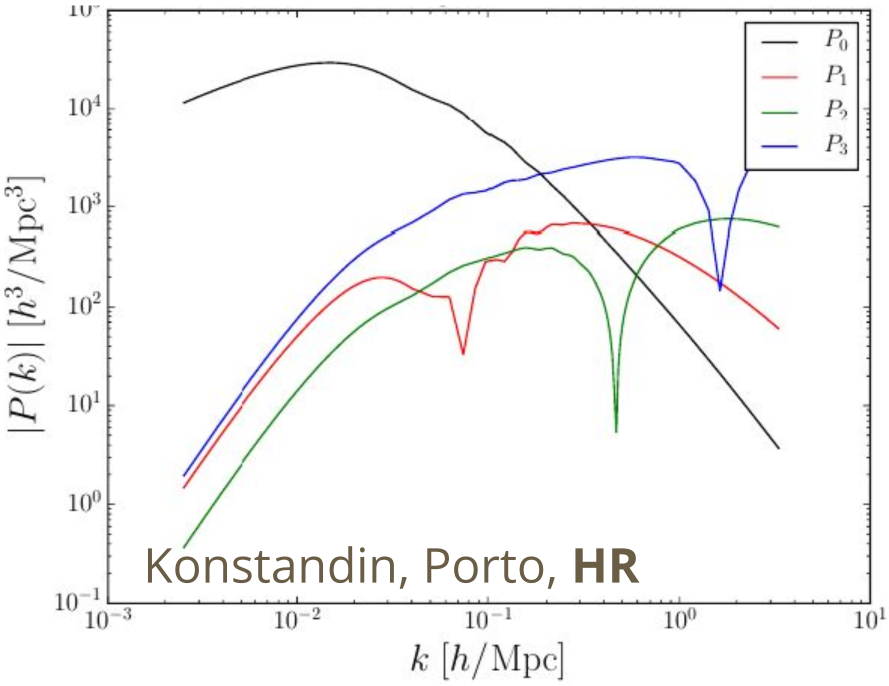
1-loop

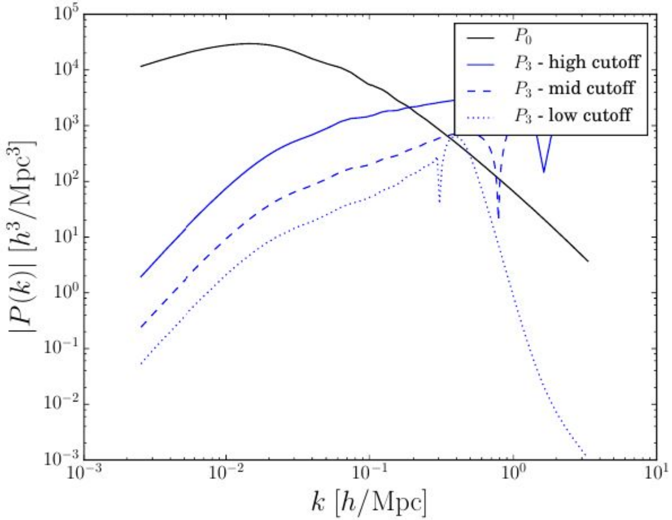


2-loop





Konstandin, Porto, **HR**



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Overview on perturbation theory for LSS

$$\partial_\tau \delta + \theta = -\int_{\mathbf{p}_1}^{\Lambda}\int_{\mathbf{p}_2} \delta_D\left(\mathbf{p_2} - (\mathbf{k} - \mathbf{p}_1)\right)\theta_{\mathbf{p_1}}\delta_{\mathbf{p_2}}\alpha(\mathbf{p_1}, \mathbf{p_2})$$

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\+ small-scale contributions

Perturbative solution

$$\boxed{\delta(\mathbf{x}, \tau) = \sum_n a^n(\tau)\delta^{(n)}(\mathbf{x})}$$

\+ counter-terms (Λ)

$$\delta^{(n)}(\mathbf{k}) = \int_{\mathbf{q}_{1\ldots n}}^{\Lambda} \delta_{\rm D}(\mathbf{q}_{1\ldots n} - \mathbf{k})\, F^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n)\, \delta^{(1)}(\mathbf{q}_1)\ldots\delta^{(1)}(\mathbf{q}_n)$$

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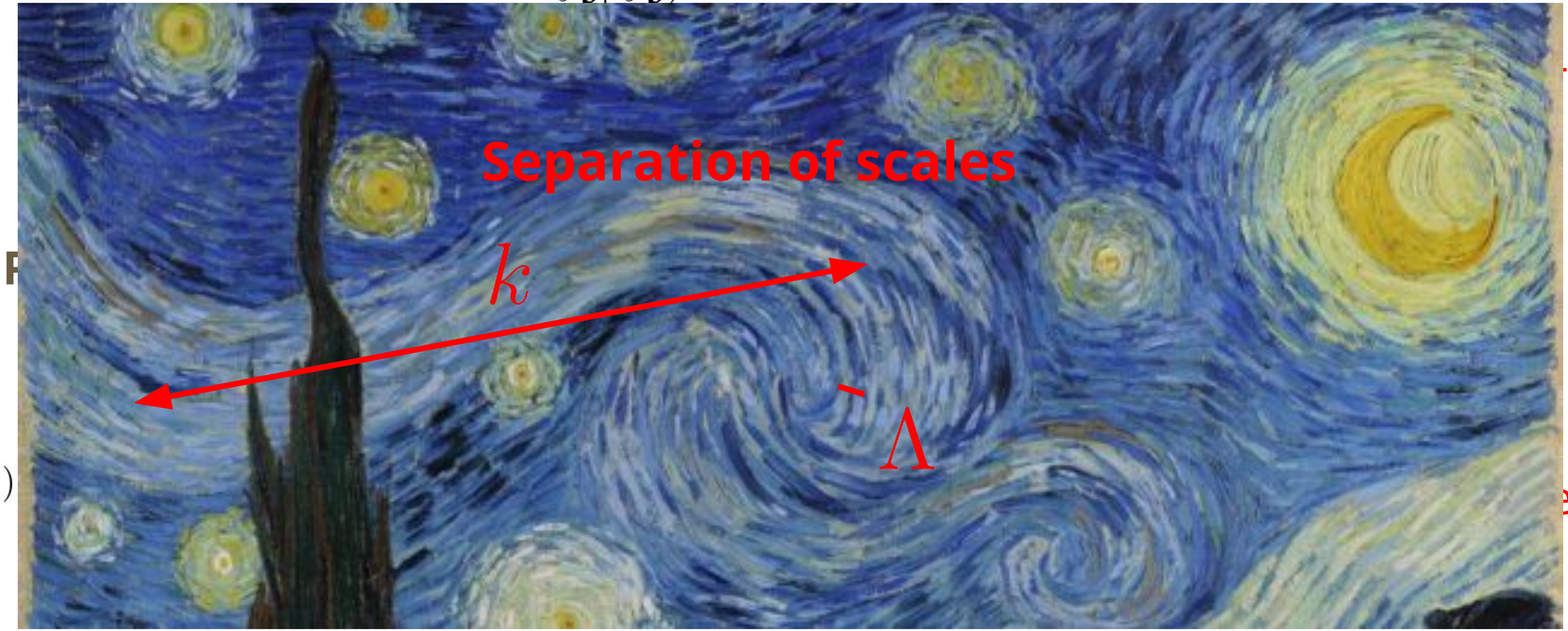


-scale
butions

F

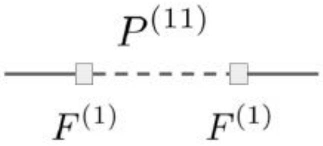
(Λ)

$\delta^{(n)}$

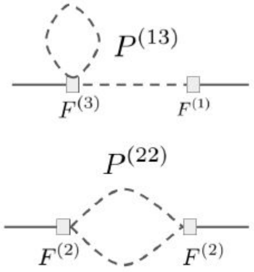
er-terms
(Λ)

Overview on perturbation theory for LSS

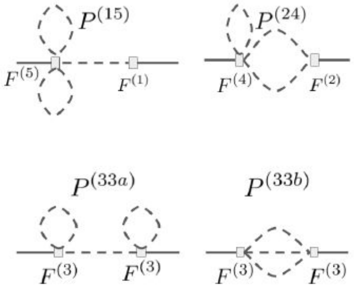
Linear:



1-loop



2-loop



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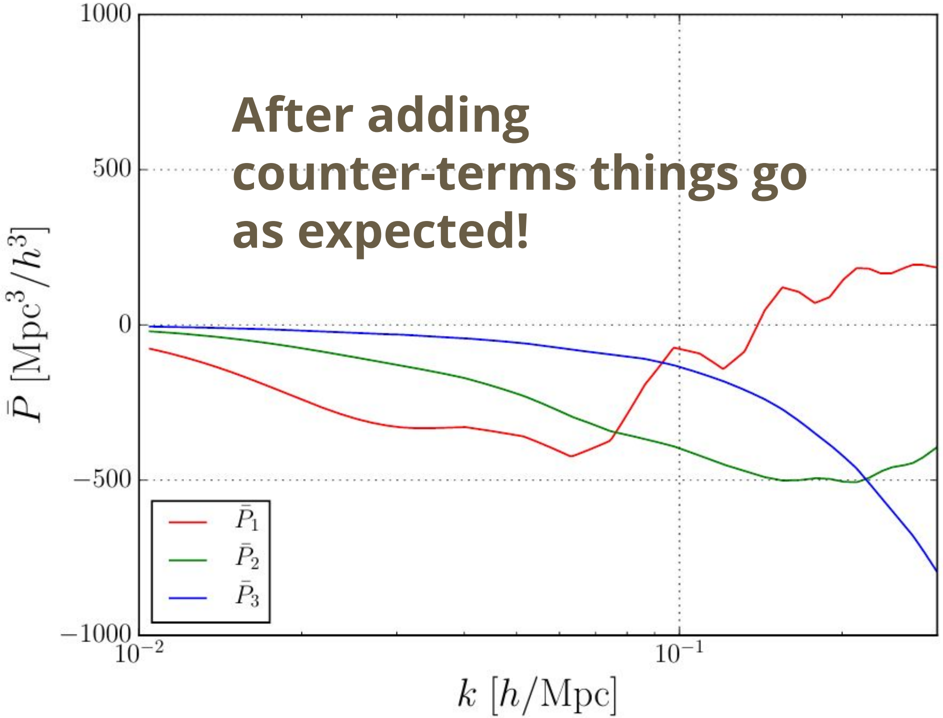


After adding counter-terms things go as expected!

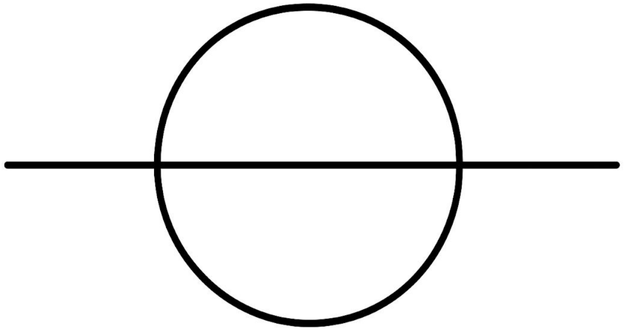
State of the art

1-loop widely applied to data (see e.g. newest DESI results)

Challenges to going to higher-loops:

- computational time
- structure of bias parameters and counter-terms gets complicated

Part I:
Towards two-loop EFT



Fast two-loop evaluation

1-Loop: 2dim integral (~seconds)
2-Loop: 5dim integral (~minutes)
3-Loop: 8dim integral (~week)

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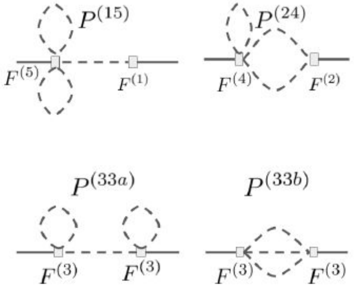


Idea: PCA expand the linear spec

$$P_L^{\Theta}(k) = \sum_{i=1}^{N_b} w_i(\Theta) v_i(k)$$

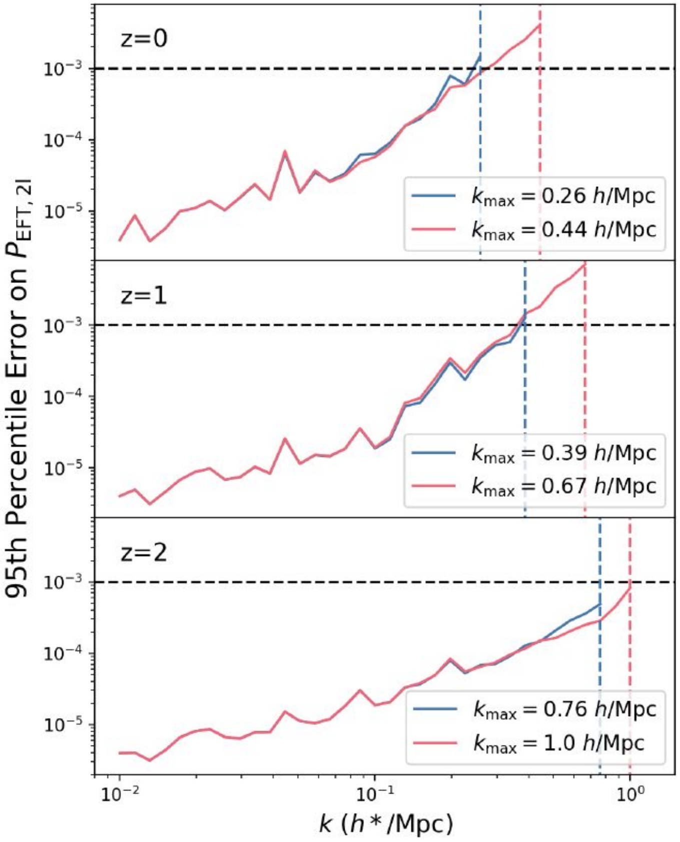
↓

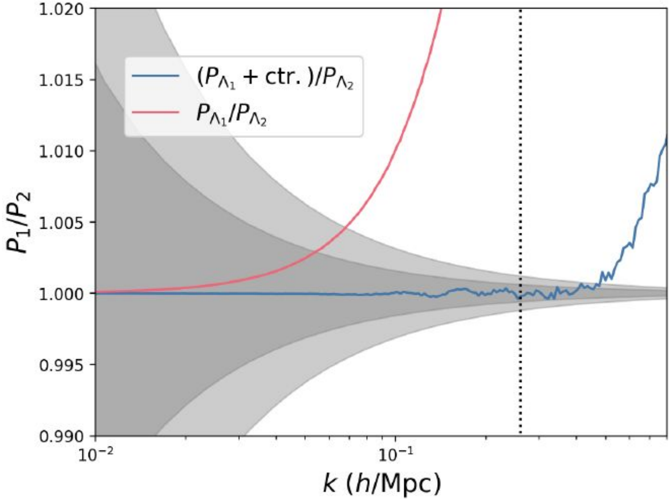
$$P_{\text{1-loop}}^{\Theta}(k,\mu) = \text{const.}(k,\mu) + \mathcal{S}_i^l(k,\mu) w_i(\Theta) + \mathcal{S}_{ij}^q(k,\mu) w_i(\Theta) w_j(\Theta)$$

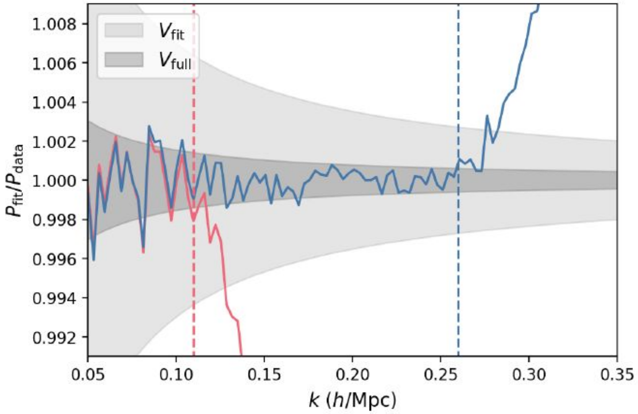
	Default	
Θ	Emulation range for $w_i(\Theta)$	Grid size for SVD
ω_c	[0.095,0.145]	30
ω_b	[0.0202,0.0238]	15
n_s	[0.91,1.01]	15
$10^9 A_s$	-	$10^9 A_s^* = 2$
h	[0.55,0.8]	$h^* = 0.7$
z	-	$z^* = 0$

Bakx, **HR**, Chisari, Vlah 2025;

Fast two-loop evaluation







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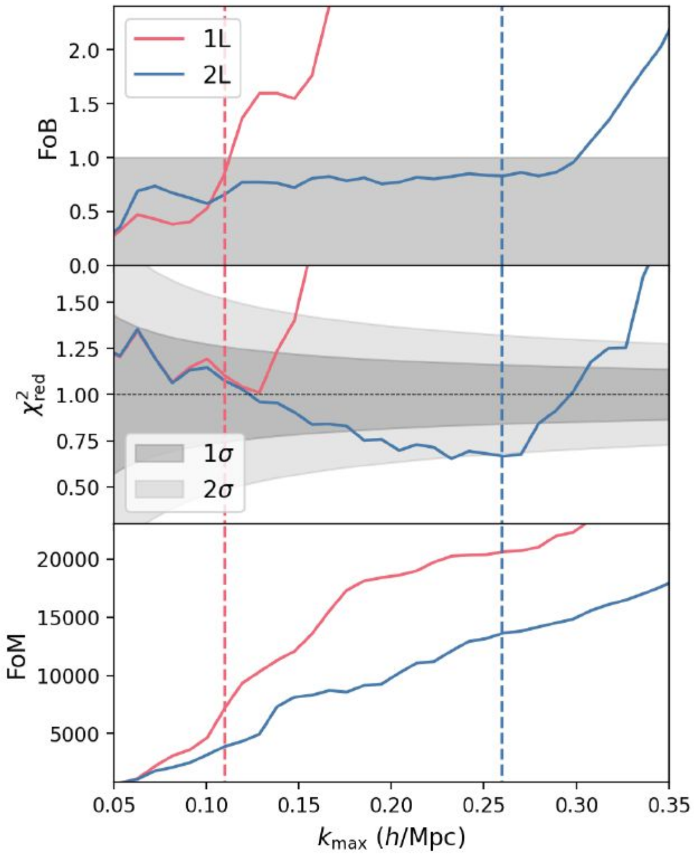
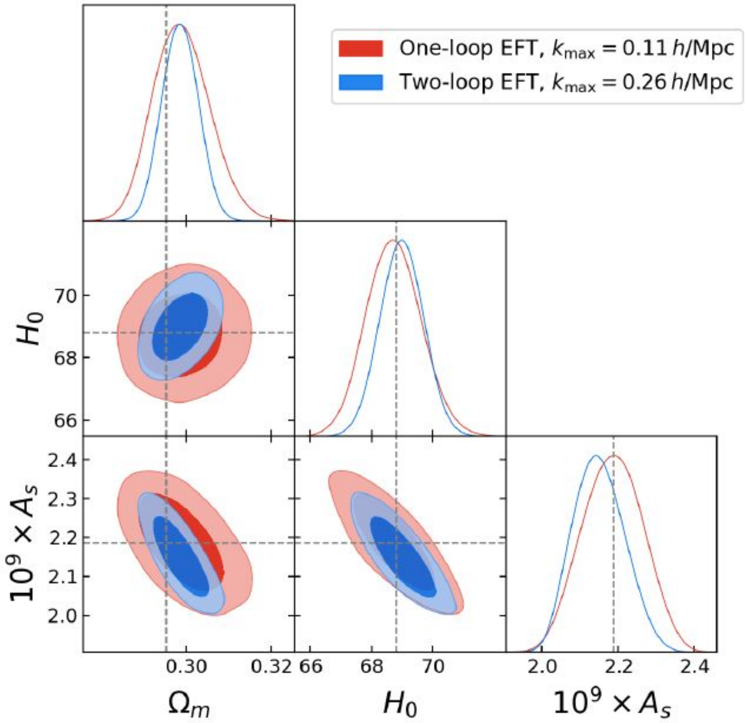




Bakx, **HR**, Chisari, Vlah 2025;

Conclusion I

- With two-loop we can go from k~0.11h/Mpc (one-loop) to k~0.26h/Mpc

- Factor 2 in FoM

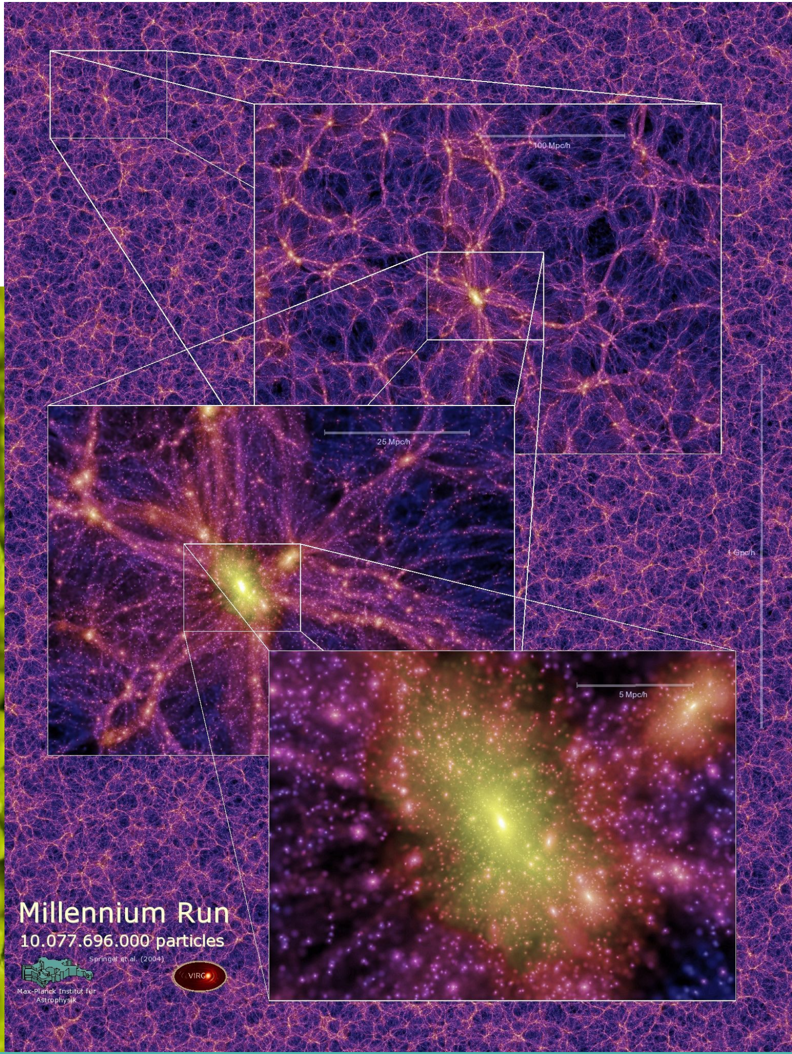
- 20 to 30% gain in Cosmological parameters 'for free'

Part II: How things change with scale?

... Or on how to use a one-loop (renormalization group) to get information about higher-loop terms 'for free'

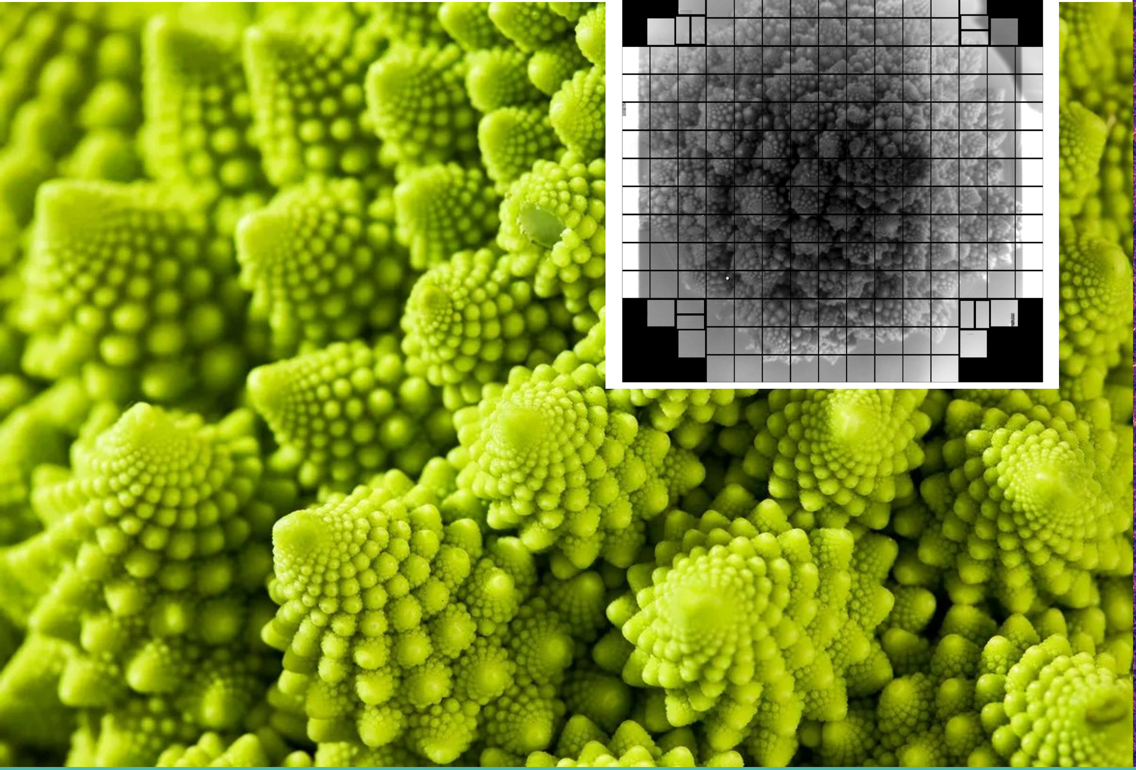
Intuition: (1loop)^n ~ n-loop
(for some part of the integrals domain)

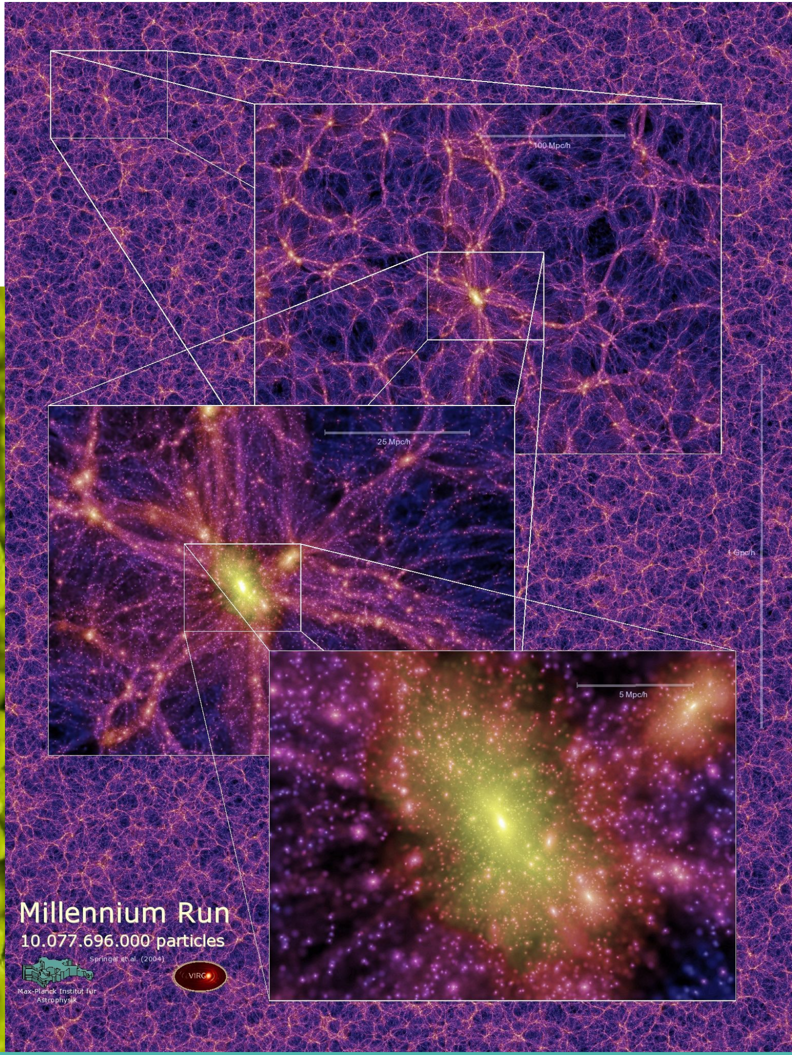
Part II: How things change with scale?



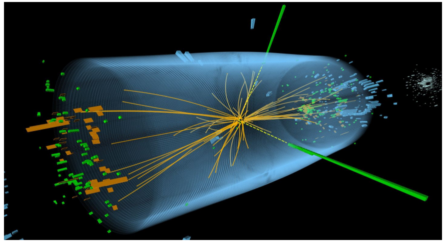
Part II: How things change with scale?

First images of Rubin

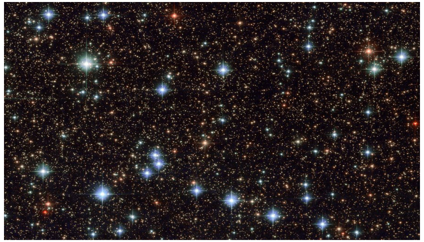


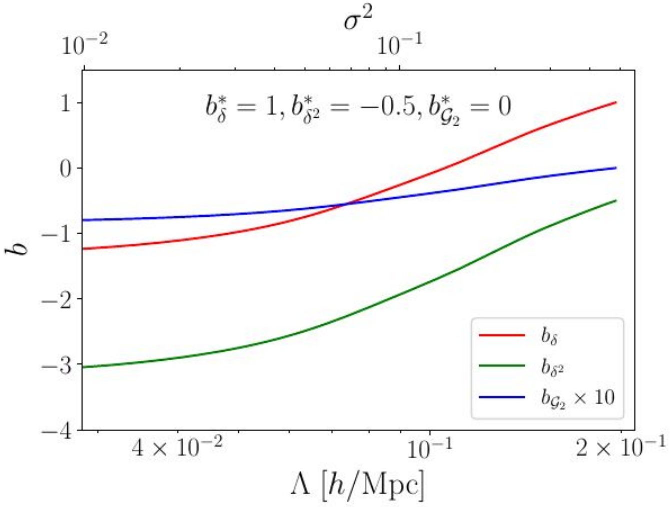


Message to take home



$\sim$



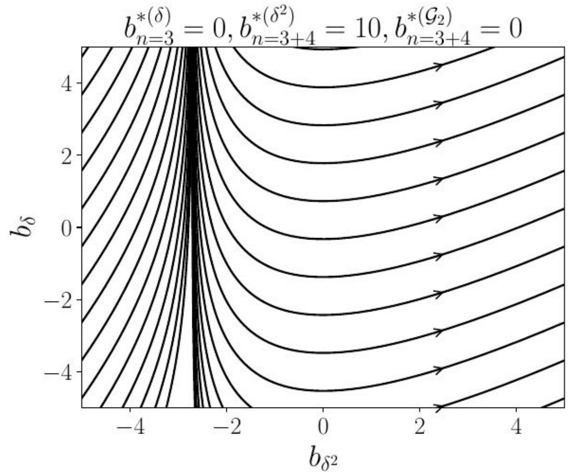


$$\frac{\partial g}{\partial \ln \mu} = \beta(g) \quad \sim$$

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2} + 3 b^*_{\delta^3} - \frac{4}{3} b^*_{\mathcal{G}_2\delta}\right] \frac{d\sigma^2_\Lambda}{d\Lambda},$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205} b_{\delta^2} + \frac{17}{7} b^*_{\delta^3} - \frac{376}{105} b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right] \frac{d\sigma^2_\Lambda}{d\Lambda},$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205} b_{\delta^2} + \frac{116}{105} b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}.$$



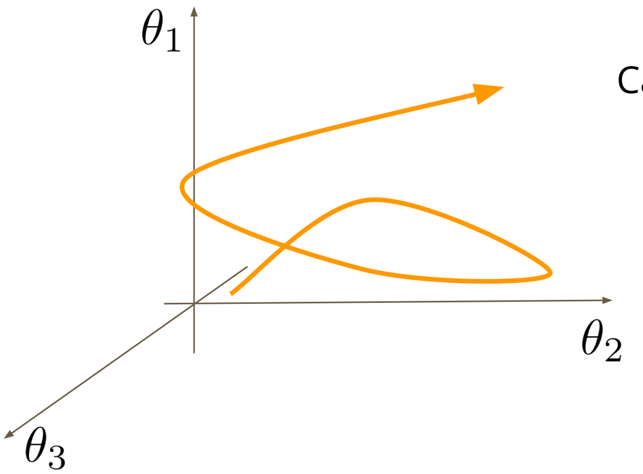
Many things to explore:

- Systematic construction of operator basis,
- Systematic renormalization,
- Cross-checks,
- More information from galaxy clustering (TBD)

QFT101

Coupling constants evolve "flow" with the cutoff

Observables don't depend on the cutoff!



Callan-Symanzik eq:

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

For the fine-structure constant (QED):

$$\frac{d\alpha}{d \ln \mu} = \beta_{1\mathrm{L}}\alpha^2 + \beta_{2\mathrm{L}}\alpha^3 + O(\alpha^4)$$

$$\beta_{1\mathrm{L}} = 2/(3\pi)$$

$$\beta_{2\mathrm{L}} = 1/(4\pi^2)$$

Solution to the RG

$$\alpha(\mu)\big|_{\mathrm{LL}} = \frac{\alpha}{1 - \beta_{1\mathrm{L}}\alpha \ln(\mu/\mu_*)}$$

$$= \alpha\left[1 + \beta_{1\mathrm{L}}\alpha \ln(\mu/\mu_*) - \beta_{1\mathrm{L}}^2\alpha^2 \ln^2(\mu/\mu_*) + \ldots\right]$$

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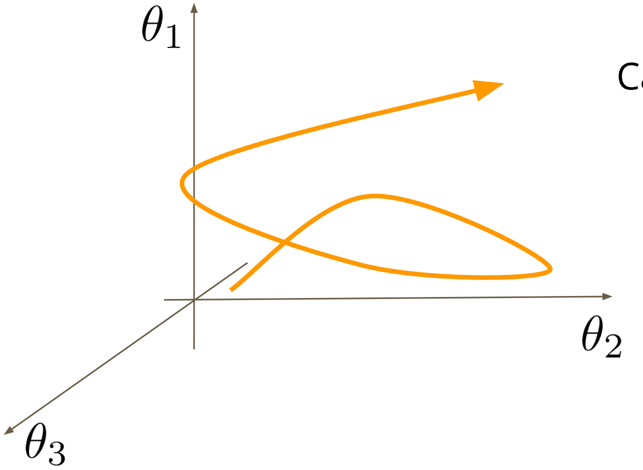

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Suppose you have an amplitude

$$\frac{\sigma_{\ell\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^\ell\left[c^{(\ell,\ell)}\ln^\ell(\mu/\mu_*) + c^{(\ell,\ell-1)}\ln^{\ell-1}(\mu/\mu_*) + \dots\right]$$

$$\frac{\sigma_{\mathrm{tree}}}{\sigma_{\mathrm{tree}}} = \alpha^0[c^{(0,0)}\ln^0]$$

$$\frac{\sigma_{1\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^1[c^{(1,1)}\ln^1 + c^{(1,0)}\ln^0]$$

$$\frac{\sigma_{2\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^2[c^{(2,2)}\ln^2 + c^{(2,1)}\ln^1 + c^{(2,0)}\ln^0]$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

QFT101

Coupling constants evolve "flow" with the cutoff

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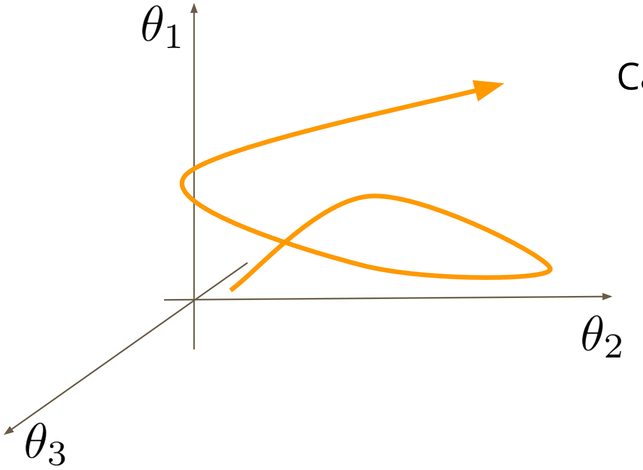

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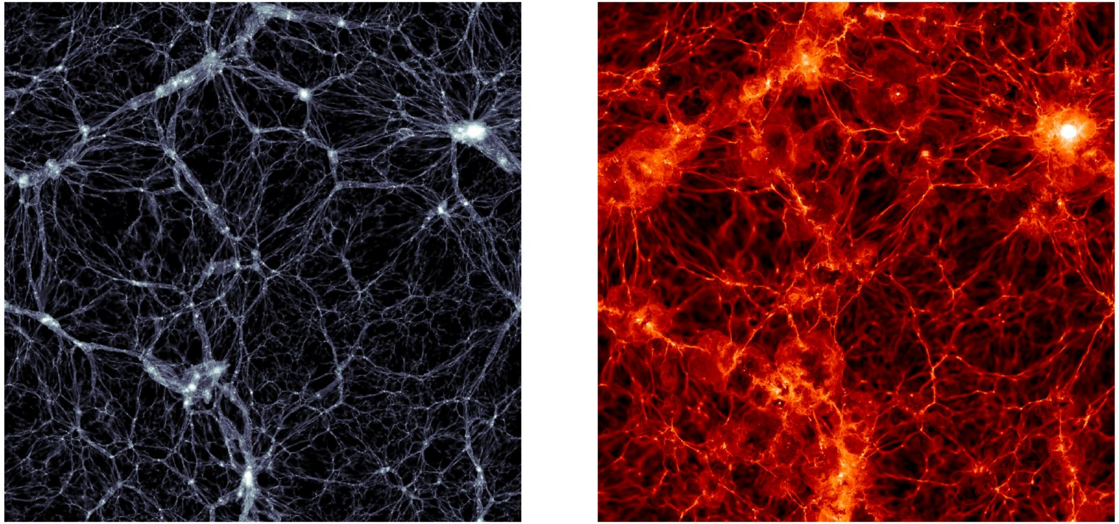
	LL (1loop RG)	NLL (2loop RG)	N^2LL (3loop RG)
$\frac{\sigma_{\mathrm{tree}}}{\sigma_{\mathrm{tree}}} = \alpha^0[$	$c^{(0,0)}\ln^0]$		
$\frac{\sigma_{1\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^1[$	$c^{(1,1)}\ln^1$	$+c^{(1,0)}\ln^0]$	
$\frac{\sigma_{2\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^2[$	$c^{(2,2)}\ln^2$	$+c^{(2,1)}\ln^1$	$+c^{(2,0)}\ln^0]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

The galaxy bias expansion



(a) dark matter (b) baryons

From Illustris simulation,
Haiden, Steinhauser, Vogelsberger,
Genel, Springel, Torrey, Hernquist, 15

Stochastic field

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

Bias

Bias review: Desjacques, Jeong, Schmidt

Renormalizing the bias parameters

Important: those are the same parameters for all n-pt functions

In a nutshell, it is an **Operator Product Expansion (OPE)**

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

$$\begin{aligned} \text{First order:} \quad & \delta\,; \\ \text{Second order:} \quad & \delta^2,\ \mathcal{G}_2\,; \\ \text{Third order:} \quad & \delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,; \end{aligned}$$

Contribution from arbitrarily small scales!

Renormalizing the bias parameters

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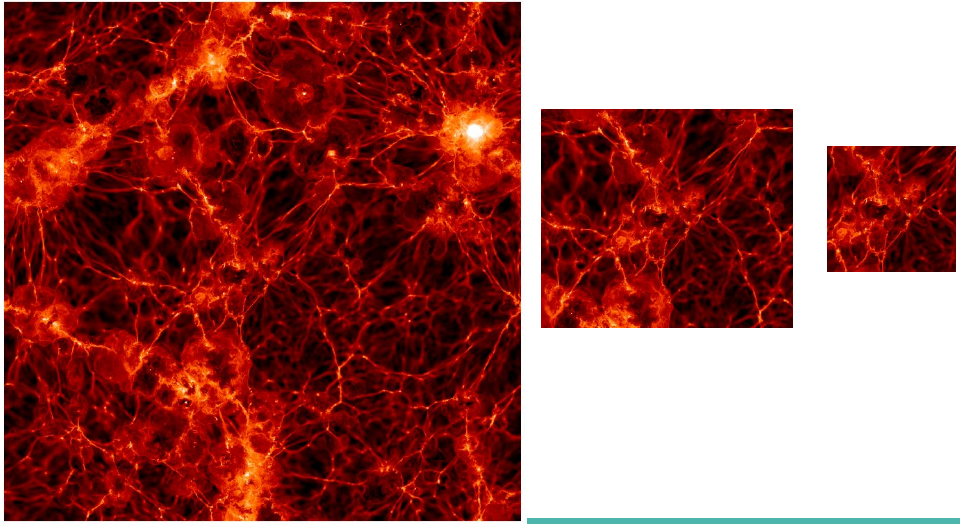
$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[\overset{\Lambda}{b_O}(\tau) + \overset{\Lambda}{c_{\epsilon,O}}(\tau)\overset{\Lambda}{\epsilon}(\boldsymbol{x},\tau)\right] \overset{\Lambda}{O}(\boldsymbol{x},\tau) + \overset{\Lambda}{\epsilon}(\boldsymbol{x},\tau)$$

+ counter-terms (Λ)



First order: δ ;

Second order: $\delta^2, \ \mathcal{G}_2$;

Third order: $\delta^3, \ \delta\,\mathcal{G}_2, \ \Gamma_3, \ \mathcal{G}_3$;

Contribution from arbitrarily small scales!

From Λ -independence to bias running

$$0 = \frac{d}{d\Lambda}\delta_g(\boldsymbol{x}) = \frac{db_a}{d\Lambda}\mathcal{O}_a(\boldsymbol{x}) + b_a\frac{d\mathcal{O}_a(\boldsymbol{x})}{d\Lambda}$$

Then we expand...

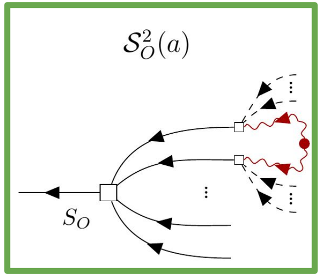
$$\frac{db_a}{d\Lambda} = \frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}} + \frac{db_a}{d\Lambda}\Big|_{2\mathrm{L}} + \ldots$$

From Λ -independence to bias running

$$0 = \frac{d}{d\Lambda}\delta_g(\boldsymbol{x}) = \frac{db_a}{d\Lambda}\mathcal{O}_a(\boldsymbol{x}) + b_a\frac{d\mathcal{O}_a(\boldsymbol{x})}{d\Lambda}$$

Then we expand...

$$\frac{db_a}{d\Lambda} = \frac{db_a}{d\Lambda}\Big|_{1L} + \frac{db_a}{d\Lambda}\Big|_{2L} + \dots$$



one-loop: $$\frac{db_a}{d\Lambda}\Big|_{1L} = -b_b s^{1L}_{ba}\frac{d\sigma^2_\Lambda}{d\Lambda}$$ **HR**, Schmidt, 23

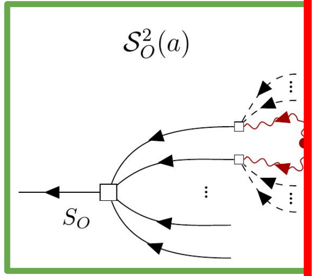
$s_{O'}^{O}$	δ	δ^2	$\mathcal{G}_2$	δ^3	$\mathcal{G}_3$	Γ_3	$\delta\mathcal{G}_2$
$\mathbb{1}$	-	-	-	-	-	-	-
δ	-	$68/21$	-	3	-	-	$-4/3$
δ^2	-	$8126/2205$	-	$68/7$	-	-	$-376/105$
$\mathcal{G}_2$	-	$254/2205$	-	-	-	-	$116/105$

From Λ -independence to bias running

$$0 = \frac{d}{d\Lambda}\delta_g(\boldsymbol{x}) = \frac{db_a}{d\Lambda}\mathcal{O}_a(\boldsymbol{x}) + b_a\frac{d\mathcal{O}_a(\boldsymbol{x})}{d\Lambda}$$

Then we expand...

$$\frac{db_a}{d\Lambda} = \frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}} + \frac{db_a}{d\Lambda}\Big|_{2\mathrm{L}}$$



a	$c^{(3)}_{a\delta}$	$d^{(5)}_a$	$\tilde{d}^{(5)}_a$
$\mathrm{tr}[\Pi^{[1]}]$	0	0	0
$\mathrm{tr}[(\Pi^{[1]})^2]$	$\frac{68}{63}$	$\frac{862}{1575}$	$\frac{376}{6615}$
$(\mathrm{tr}[\Pi^{[1]}])^2$	$\frac{68}{63}$	$\frac{862}{1575}$	$\frac{376}{6615}$
$(\mathrm{tr}[\Pi^{[1]}])^3$	1	$\frac{70739}{33075}$	$\frac{4}{105}$
$\mathrm{tr}[(\Pi^{[1]})^2]\mathrm{tr}[\Pi^{[1]}]$	$\frac{5}{9}$	$\frac{2917}{2205}$	$\frac{716}{1323}$
$\mathrm{tr}[(\Pi^{[1]})^3]$	$\frac{1}{3}$	$\frac{30263}{33075}$	$\frac{1748}{2205}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	$\frac{41}{63}$	$\frac{134957}{99225}$	$\frac{148}{441}$
$(\mathrm{tr}[\Pi^{[1]}])^4$	0	$\frac{272}{105}$	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[\Pi^{[1]}]$	0	$\frac{82}{105}$	$\frac{5}{21}$
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^2$	0	$\frac{6352}{4725}$	$\frac{4}{21}$
$(\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	$\frac{592}{675}$	$\frac{8}{63}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{16112}{19845}$	$\frac{3706}{6615}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{18166}{14175}$	$\frac{373}{735}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{12814}{11025}$	$\frac{401}{2205}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{27784}{19845}$	$\frac{232}{315}$
$(\mathrm{tr}[\Pi^{[1]}])^5$	0	1	0
$\mathrm{tr}[(\Pi^{[1]})^3](\mathrm{tr}[\Pi^{[1]}])^2$	0	$\frac{11}{45}$	0
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^3$	0	$\frac{7}{15}$	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[(\Pi^{[1]})^2]$	0	$\frac{31}{225}$	0
$\mathrm{tr}[\Pi^{[1]}](\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	$\frac{163}{675}$	0
$(\mathrm{tr}[\Pi^{[1]}])^2\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{47}{105}$	$\frac{2}{21}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{173}{675}$	$\frac{2}{63}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{391}{1575}$	$\frac{5}{63}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{197}{735}$	$\frac{82}{441}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{5057}{11025}$	$\frac{110}{441}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{643}{2835}$	$\frac{104}{945}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{5477}{14175}$	$\frac{4}{35}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[3]}]$	0	$\frac{8629}{19845}$	$\frac{491}{2205}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[4]}]$	0	$\frac{33403}{198450}$	$\frac{74}{6615}$

one-loop:

$$\frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}} = -b_b s^{1\mathrm{L}}_{ba}\frac{d\sigma^2_\Lambda}{d\Lambda}$$

HR, Schmidt, 23

$s^O_{O'}$	δ	δ^2	$\mathcal{G}_2$	δ^3	$\mathcal{G}_3$	Γ_3	$\delta\mathcal{G}_2$
$\mathbb{1}$	-	-	-	-	-	-	-
δ	-	68/21	-	3	-	-	$-4/3$
δ^2	-	8126/2205	-	68/7	-	-	$-376/105$
$\mathcal{G}_2$	-	254/2205	-	-	-	-	116/105

two-loop:

$$\frac{db_\delta}{d\Lambda}\Big|_{2\mathrm{L}} = -30 b_b \tilde{d}^{(5)}_b \frac{d\sigma^2_\Lambda}{d\Lambda}\int_0^\Lambda dq \frac{q^2 P^{\mathrm{lin}}(q)}{2\pi^2} g(q/\Lambda)\,,$$

Bakx, Garny, **HR**, Vlah

Solutions

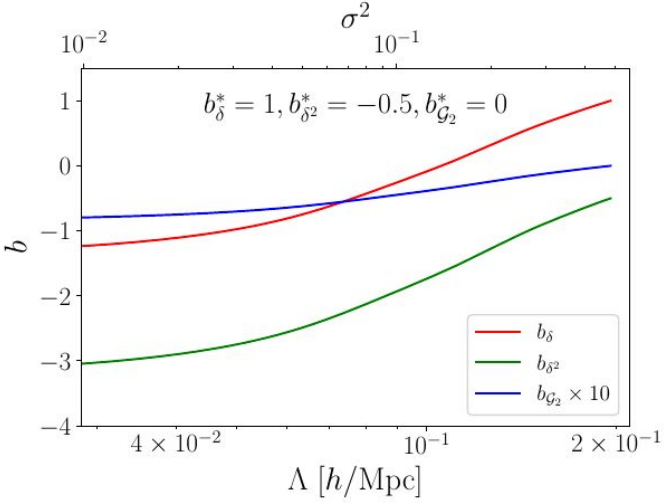
Wilson-Polchinski RG-equations

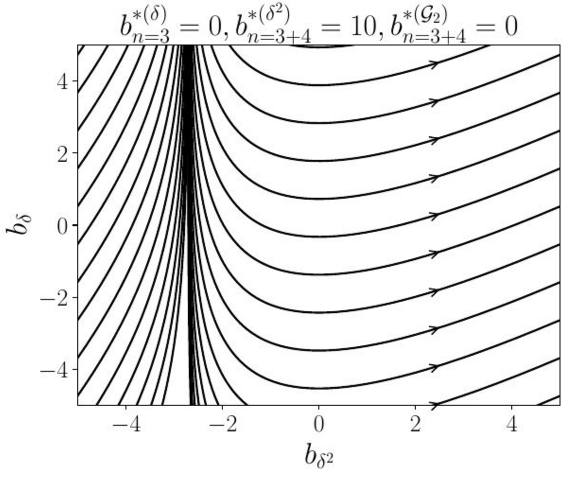
HR, Schmidt 23

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3} b^*_{\mathcal{G}_2\delta}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205} b_{\delta^2} + \frac{17}{7} b^*_{\delta^3} - \frac{376}{105} b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205} b_{\delta^2} + \frac{116}{105} b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$

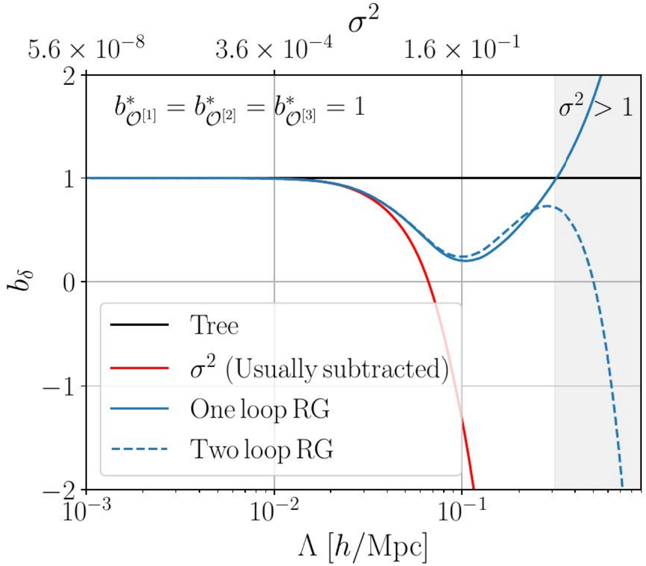




Solutions (two-loop)

Bakx, Garny, **HR**, Vlah

Smallness of two-loop tell us that one-loop RG is able to absorb important part of higher-loop terms



So the 2Loop is small. Why should you care?

We can write EFT loops as:

$$\left.\frac{P_{ab}^{\ell\mathrm{L}}(k)}{P^{\mathrm{lin}}(k)}\right|_{k\ll\Lambda} = (\Delta_\Lambda^2)^\ell \times \left[c_{ab}^{(\ell,\ell)}\left(\frac{1}{n+3}\right)^\ell + c_{ab}^{(\ell,\ell-1)}\left(\frac{1}{n+3}\right)^{\ell-1} + \dots\right]$$

$$\frac{P_{ab}^{\mathrm{lin}}}{P^{\mathrm{lin}}} = (\Delta_\Lambda^2)^0\left[c_{ab}^{(0,0)}\left(\frac{1}{n+3}\right)^0\right]$$

$$\frac{P_{ab}^{1\mathrm{L}}}{P^{\mathrm{lin}}} = (\Delta_\Lambda^2)^1\left[c_{ab}^{(1,1)}\left(\frac{1}{n+3}\right)^1 + c_{ab}^{(1,0)}\left(\frac{1}{n+3}\right)^0\right]$$

$$\frac{P_{ab}^{2\mathrm{L}}}{P^{\mathrm{lin}}} = (\Delta_\Lambda^2)^2\left[c_{ab}^{(2,2)}\left(\frac{1}{n+3}\right)^2 + c_{ab}^{(2,1)}\left(\frac{1}{n+3}\right)^1 + c_{ab}^{(2,0)}\left(\frac{1}{n+3}\right)^0\right]$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

So the 2Loop is small. Why should you care?

We can write EFT loops as:

$$\left.\frac{P^{\ell\mathrm{L}}_{ab}(k)}{P^{\mathrm{lin}}(k)}\right|_{k\ll\Lambda} = (\Delta^2_\Lambda)^\ell \times \left[c^{(\ell,\ell)}_{ab}\left(\frac{1}{n+3}\right)^{\ell} + c^{(\ell,\ell-1)}_{ab}\left(\frac{1}{n+3}\right)^{\ell-1} + \dots\right]$$

	LL (1loop RG)	NLL (2loop RG)	N^{2}LL (3loop RG)
$\frac{P^{\mathrm{lin}}_{ab}}{P^{\mathrm{lin}}} = (\Delta^2_\Lambda)^0$	$\left[c^{(0,0)}_{ab}\left(\frac{1}{n+3}\right)^0\right]$		
$\frac{P^{1\mathrm{L}}_{ab}}{P^{\mathrm{lin}}} = (\Delta^2_\Lambda)^1$	$\left[c^{(1,1)}_{ab}\left(\frac{1}{n+3}\right)^1\right.$	$\left.+c^{(1,0)}_{ab}\left(\frac{1}{n+3}\right)^0\right]$	
$\frac{P^{2\mathrm{L}}_{ab}}{P^{\mathrm{lin}}} = (\Delta^2_\Lambda)^2$	$\left[c^{(2,2)}_{ab}\left(\frac{1}{n+3}\right)^2\right.$	$+c^{(2,1)}_{ab}\left(\frac{1}{n+3}\right)^1$	$\left.+c^{(2,0)}_{ab}\left(\frac{1}{n+3}\right)^0\right]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

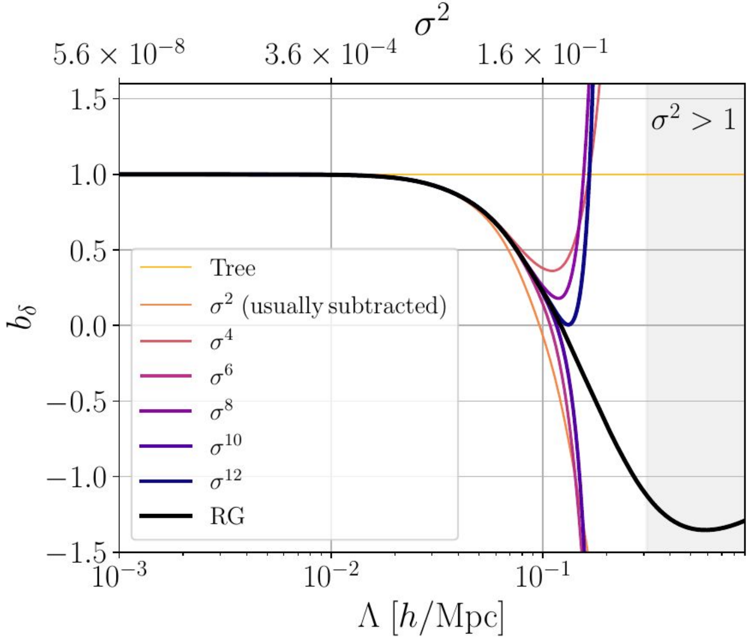
	LL (1loop RG)	NLL (2loop RG)	N^{2}LL (3loop RG)
$\frac{\sigma_{\mathrm{tree}}}{\sigma_{\mathrm{tree}}} = \alpha^0$	$\left[c^{(0,0)}\ln^0\right]$		
$\frac{\sigma_{1\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^1$	$\left[c^{(1,1)}\ln^1\right.$	$\left.+c^{(1,0)}\ln^0\right]$	
$\frac{\sigma_{2\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^2$	$\left[c^{(2,2)}\ln^2\right.$	$+c^{(2,1)}\ln^1$	$\left.+c^{(2,0)}\ln^0\right]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Resumming terms with the RG equations

Bakx, Garny, **HR**, Vlah

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1L}_{ac} b_c$$



Solution

$$b_a(\sigma^2) = \left[e^{-\bar{s}^{1L}\times(\sigma^2-\sigma^2_*)}\right]_{ac} b^*_c$$

RG resums the series!

$$= b^*_a - (\sigma^2 - \sigma^2_*)\bar{s}^{1L}_{ac} b^*_c + \frac{1}{2}(\sigma^2 - \sigma^2_*)^2 \bar{s}^{1L}_{ab}\bar{s}^{1L}_{bc} b^*_c - \frac{1}{6}(\sigma^2 - \sigma^2_*)^3 \bar{s}^{1L}_{ab}\bar{s}^{1L}_{bd}\bar{s}^{1L}_{dc} b^*_c + \ldots$$

What do the solutions of the RG tell us?

Bakx, Garny, **HR**, Vlah

We can always diagonalize the bias basis

$$\frac{db_i^{\text{diag}}}{d\sigma^2} = \lambda_i b_i^{\text{diag}}$$

$$b_a(\sigma^2) = p_{ai} e^{\lambda_i(\sigma^2 - \sigma_*^2)} c_i$$

If we stop at second-order, we find:

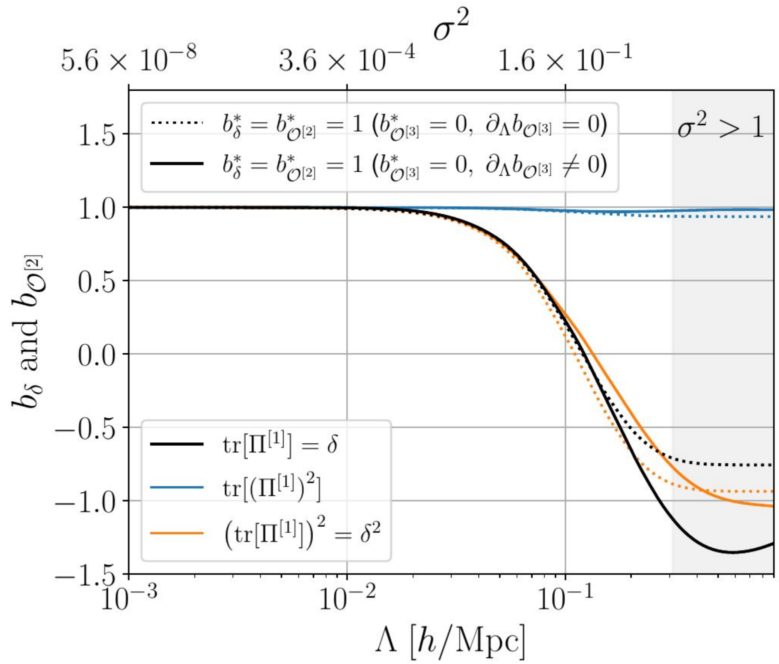
$$\{\lambda_1, \lambda_2, \lambda_3\} \simeq \{0, 0, -3.69\}$$

Marginal *Relevant*

Extending to third-order:

Irrelevant

$$\{0, 0, 0, -12.6, -3.44, -2.01, 0.220\}$$



PNGs

Free term

New interaction

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2}(\Lambda) + b^{*\{\delta\}\mathrm{G}}_{n=3}\right]\frac{d\sigma^2_\Lambda}{d\Lambda} - a_0 f_{\mathrm{NL}}\left[-\frac{13}{21}b_\Psi + \frac{13}{21}b_{\Psi\delta} + b^{*\{\delta\}\mathrm{NG}}_{n=3}\right]\left(\frac{H_0}{\Lambda}\right)^2 \frac{3\,\Omega_m}{2\,T(\Lambda)}\frac{d\sigma^2_\Lambda}{d\Lambda};$$

Now a coupled set of ODEs

$$\frac{db_\Psi}{d\Lambda} = -a_0 f_{\mathrm{NL}} b^{*\{\Psi\}\mathrm{NG}}_{n=3}\frac{d\sigma^2_\Lambda}{d\Lambda} - 4a_0 f_{\mathrm{NL}} b_{\delta^2}\frac{d\sigma^2_\Lambda}{d\Lambda},$$

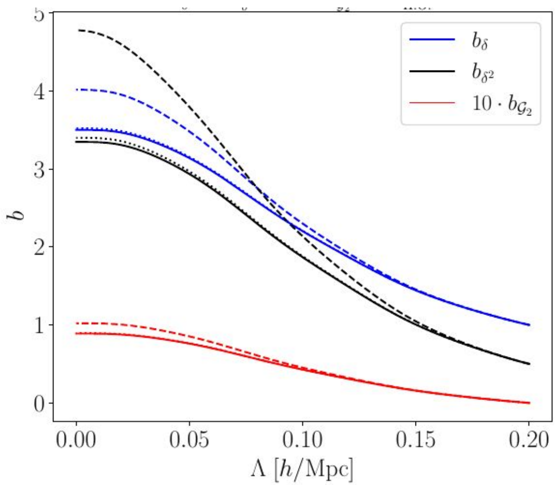
$$\frac{db_{\Psi\delta}}{d\Lambda} = -a_0 f_{\mathrm{NL}}\left[\frac{272}{21}b_{\delta^2} + b^{*\{\Psi\delta\}\mathrm{G}}_{n=3+4} + b^{*\{\Psi\delta\}\mathrm{NG}}_{n=3+4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda},$$

Rederivation of Dalal+ 07 (in an elegant way)

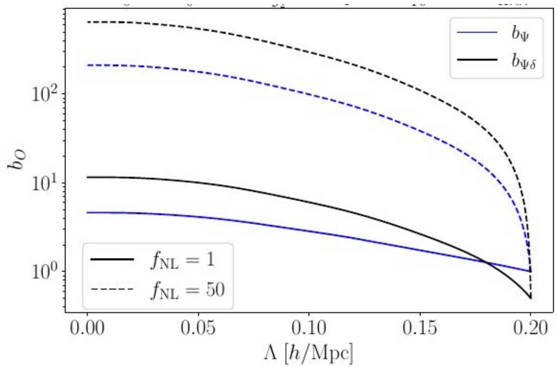
$s^{O}_{O'}$	δ^2	δ^3	$\delta\mathcal{G}_2$	Ψ	$\Psi\delta$	$\Psi\delta^2$	$\Psi\mathcal{G}_2$	$\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\delta\,\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\mathrm{Tr}\,\Psi\Pi^{[2]}$
δ	68/21	3	−4/3	−13/21	13/21	2	−4/3	34/21	1	34/21
δ^2	8126/2205	68/7	−376/105	43/135	478/135	47/21	−31/21	124/315	178/105	14347/6027
$\mathcal{G}_2$	254/2205	-	116/105	−1699/13230	79/2205	-	−1/21	−661/4410	4/35	−241/735
Ψ	4	-	-	-	-	1	-	-	-	-
$\delta\Psi$	272/21	12	−8/3	-	-	68/21	-	-	-	-
$\mathrm{Tr}\,\Psi\Pi^{[1]}$	64/105	-	16/15	-	-	-	-	-	8/105	58/305

Nikolis, **HR**, Schmidt

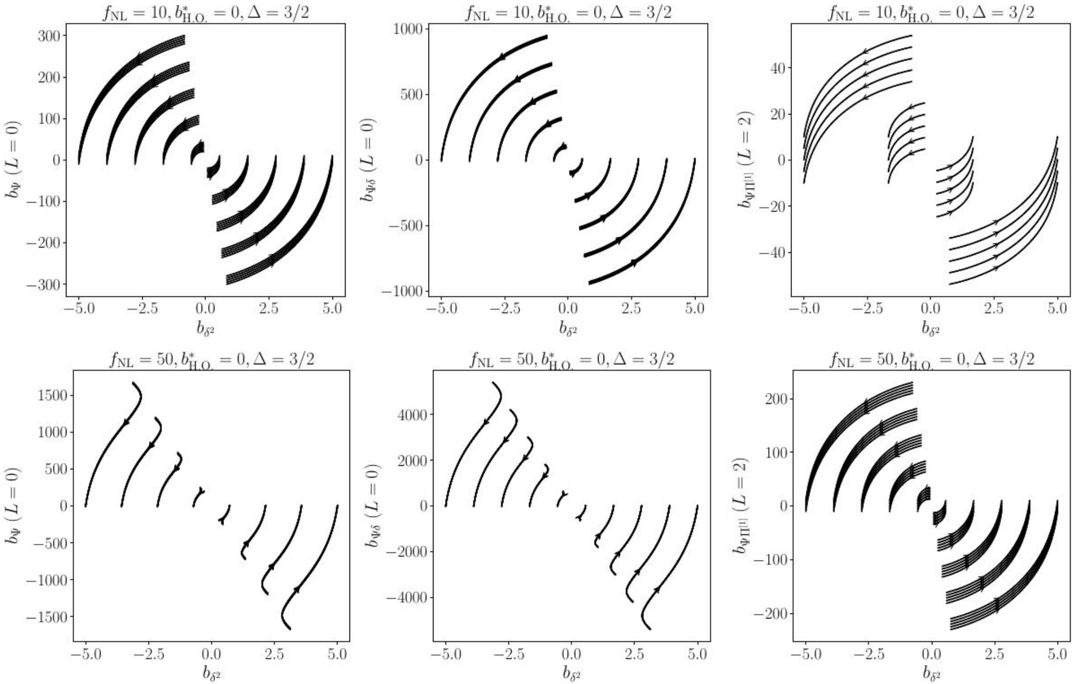
PNGs



b_δ
b_{δ^2}
$10 \cdot b_{\mathcal{G}_2}$
b
Λ [h/Mpc]



b_Ψ
$b_{\Psi\delta}$
$f_{\rm NL} = 1$
$f_{\rm NL} = 50$
b_O
Λ [h/Mpc]

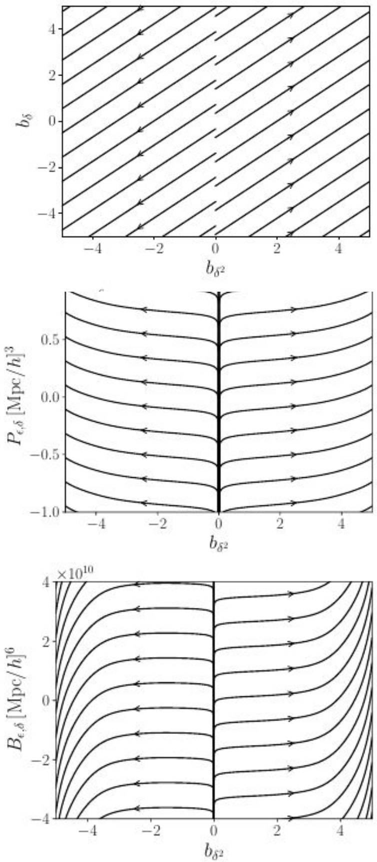


$f_{\rm NL} = 10, b^*_{\rm H.O.} = 0, \Delta = 3/2$
$f_{\rm NL} = 50, b^*_{\rm H.O.} = 0, \Delta = 3/2$
b_Ψ $(L = 0)$
$b_{\Psi\delta}$ $(L = 0)$
$b_{\Psi\Pi^{[1]}}$ $(L = 2)$
b_{δ^2}

Stochasticity

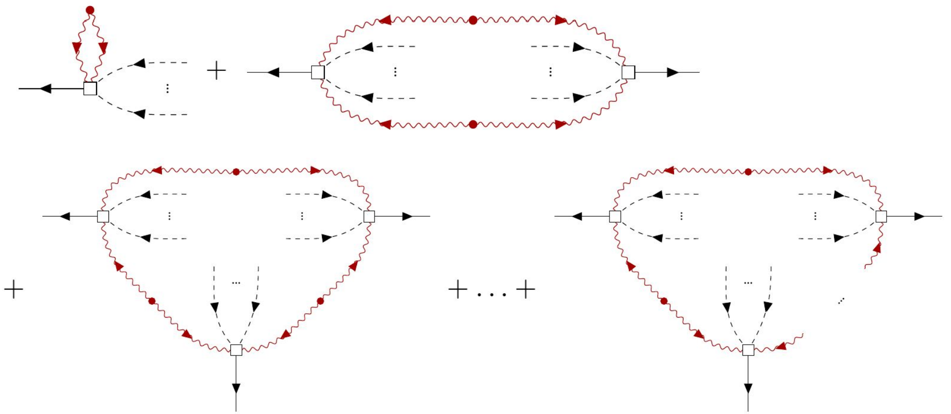
$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

$$\langle \epsilon(\boldsymbol{k}_1)\ldots\epsilon(\boldsymbol{k}_m) O(\boldsymbol{k}_{m+1})\rangle = \hat{\delta}_{\mathrm{D}}(\boldsymbol{k}_{1\ldots m}) C^{(m)}_{\epsilon,O} O(\boldsymbol{k}_{m+1})$$



Simple expression for how stochastic terms talk to each other

$$\frac{d}{d\Lambda} C^{(m)}_O(\Lambda) \propto -\left[P_{\mathrm{L}}(\Lambda)\right]^{p-1} \frac{d\sigma^2_\Lambda}{d\Lambda} \sum_{O_1, O_2, \ldots O_m} s^O_{O_1 O_2 \ldots O_m} C^{(i_1)}_{O_1}(\Lambda) \ldots C^{(i_p)}_{O_p}(\Lambda)$$

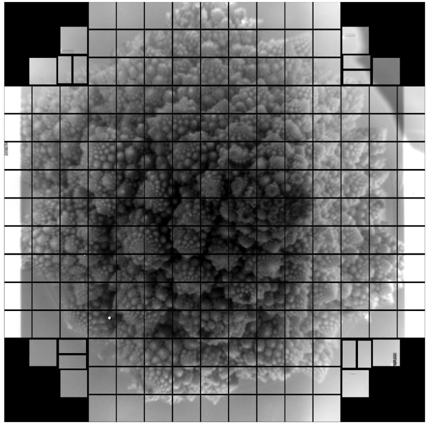


Simple diagrammatic interpretation

HR, Schmidt, 24

Conclusion II

First images of Rubin



- Cross-check for EFT inference;
- Systematic renormalization (+ stochastic +PNG);
- Systematic renormalization of n-point functions. Self-consistent renormalization for P(k), B(k1,k2,k3), ...
- (Unambiguously) Define Priors for EFT analysis in $\Lambda \to 0$
- More information from resummation? TBD!
- Measuring the running in the lattice (Harry's talk)


Thanks a lot!