Multi-Tracer beyond linear theory

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Multi-Tracer analysis

Challenge: extract maximal amount of information from data from LSS.

One possible approach: **Multi-Tracer (MT)** technique.

$$\rho = \rho^A + \rho^B \Longrightarrow \delta = \frac{\overline{n}^A}{\overline{n}}\delta^A + \frac{\overline{n}^B}{\overline{n}}\delta^B$$

E.g. split galaxy sample by mass, color, spin, …

Example: 2 tracers (A and B) +linear order theory + RSD:

CROSS-STOCHASTICITY

(see 1305.2917)

$$\begin{cases} P^{AA} = \langle \delta^A \delta^A \rangle' = (b_1^A + f\mu^2)^2 P_L + \frac{1}{\overline{n}^A}(1 + c_0^{AA}) \\ P^{AB} = \langle \delta^A \delta^B \rangle' = (b_1^A + f\mu^2)(b_1^B + f\mu^2) P_L + \frac{1}{\sqrt{\overline{n}^A \overline{n}^B}} c_0^{AB} \\ P^{BB} = \langle \delta^B \delta^B \rangle' = (b_1^B + f\mu^2)^2 P_L + \frac{1}{\overline{n}^B}(1 + c_0^{BB}) \end{cases}$$

Multi-Tracer analysis

Main advantage in the linear theory: **cosmic variance suppression** (1302.5444 and 2112.01812).

Beyond linear theory: EFTofLSS + bias expansion $\rightarrow$3 new biases: $b_{\delta^2}, b_{\mathcal{G}_2}, b_{\Gamma_3}$.

Now also **break of degeneracies** between parameters (2108.11363, 2306.05474).

Want to understand:

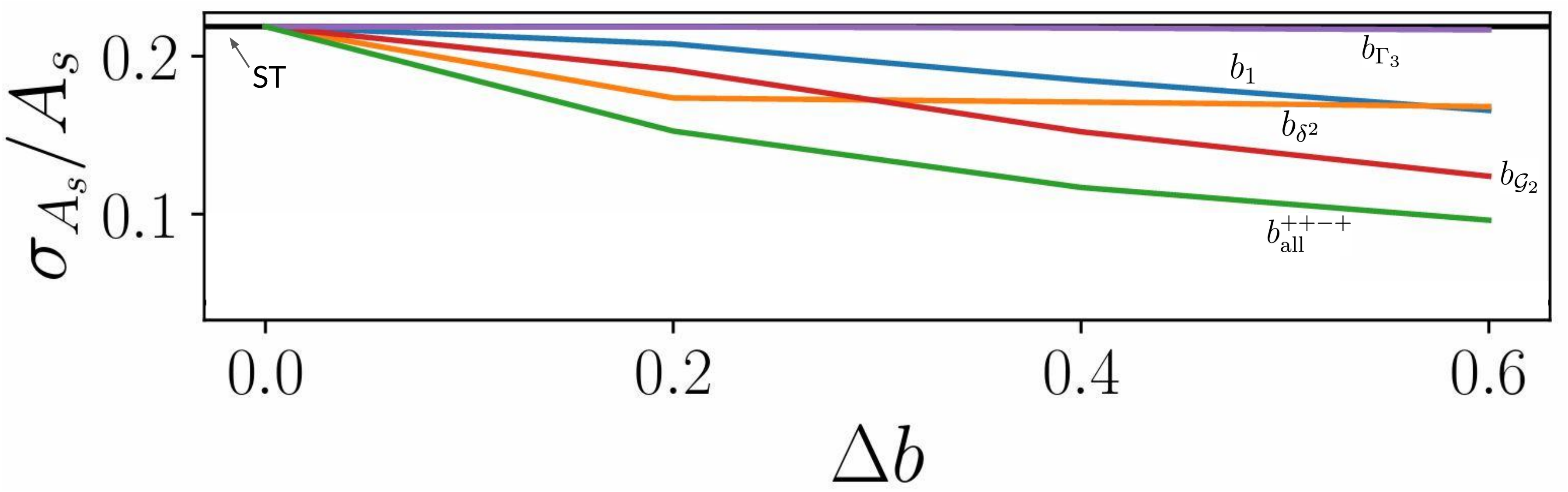
- how to split the dataset more effectively;
- systematically explore information gains.

Use Fisher forecast to compare ST vs MT.

Finding the optimal tracer split: bias difference

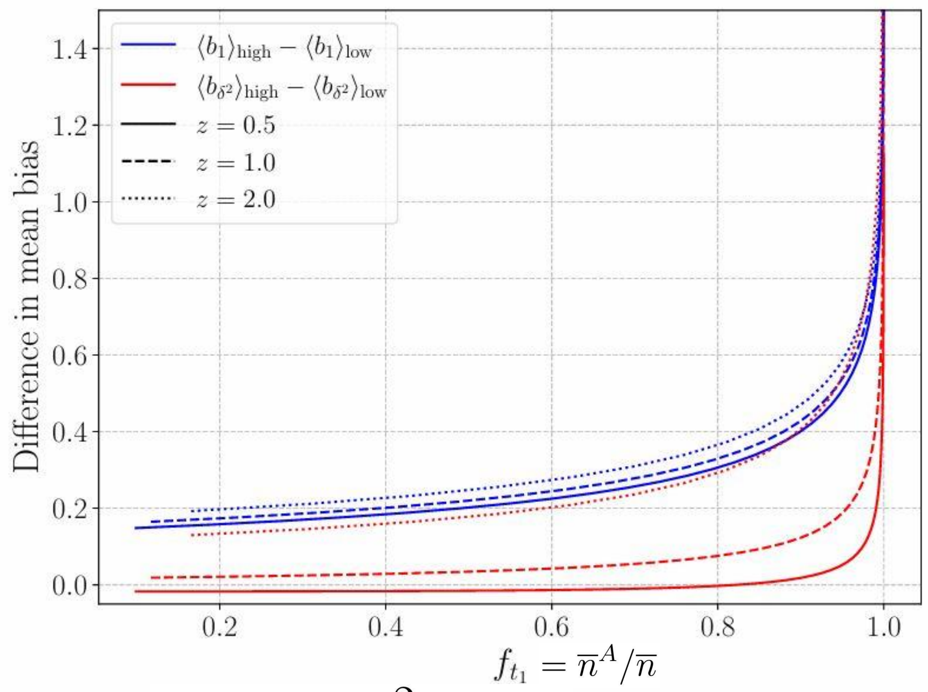
Tracer split → different bias parameters:

$$b_O^A = b_O^{\rm ST} \mp \frac{\Delta b}{2} \quad \text{and} \quad b_O^B = b_O^{\rm ST} \pm \frac{\Delta b}{2}$$



Is it easy to find splits with different biases? PESSIMISTIC SCENARIO

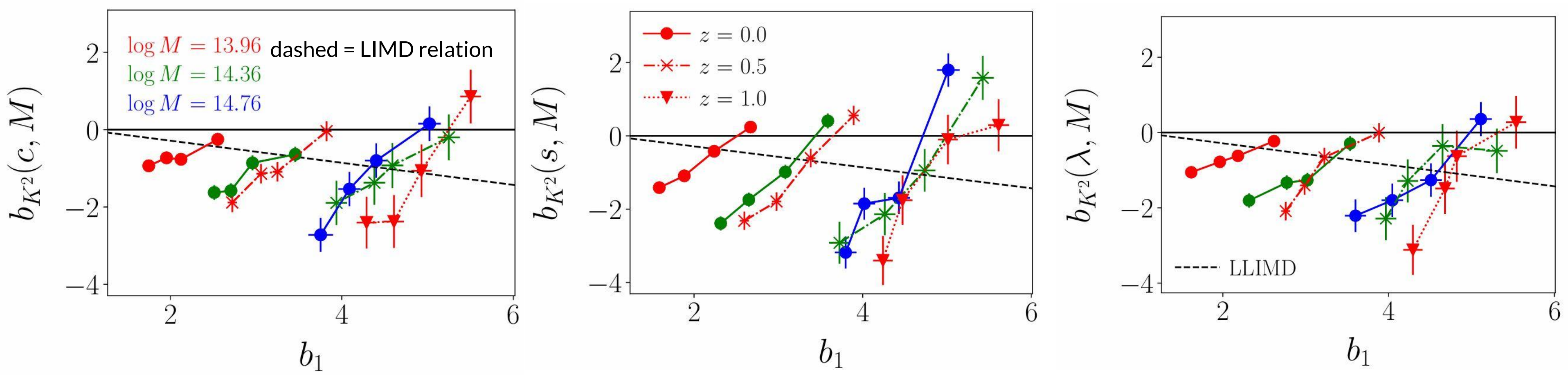
Toy model with DM halos: Tinker's mass function + fit for non-linear bias (1511.01096).



Assuming LIMD relations -> $b_{\mathcal{G}_2}(b_1) = -\frac{2}{7}(b_1 - 1)$ and $b_{\Gamma_3}(b_1) = -\frac{23}{42}(b_1 - 1)$

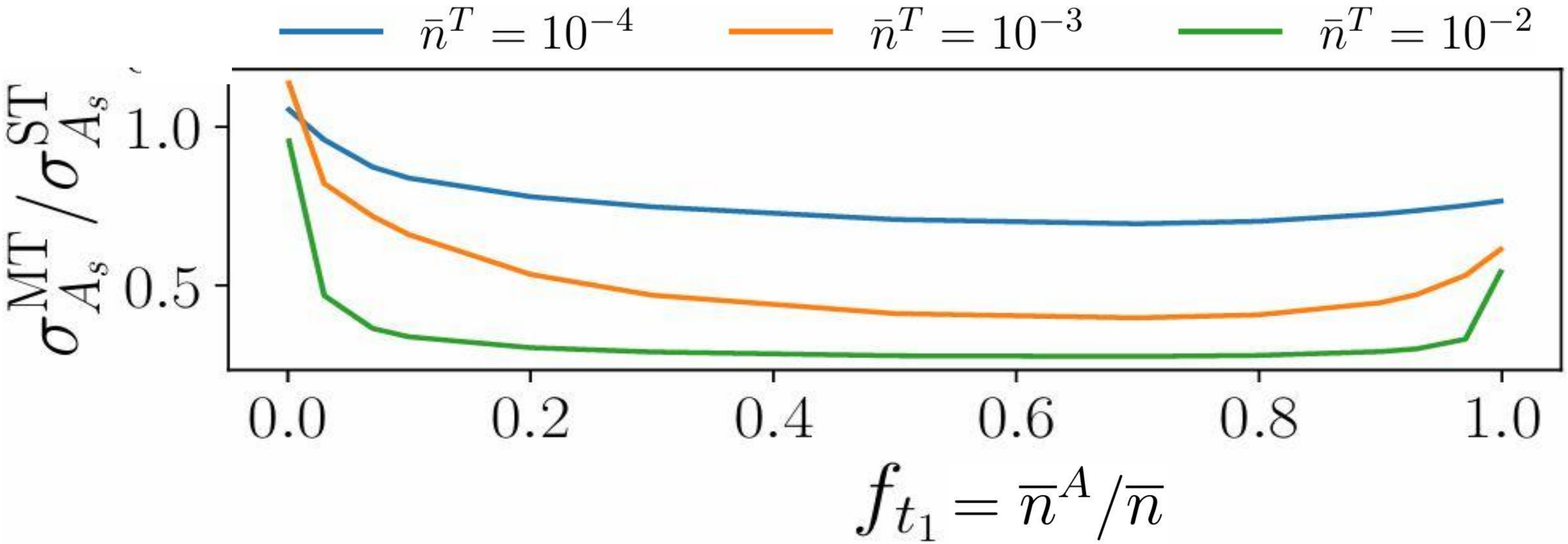
Is it easy to find splits with different biases? MORE REALISTIC SCENARIO

Plots from 2106.14713, with $b_{\mathcal{G}_2} = b_{K^2}$.



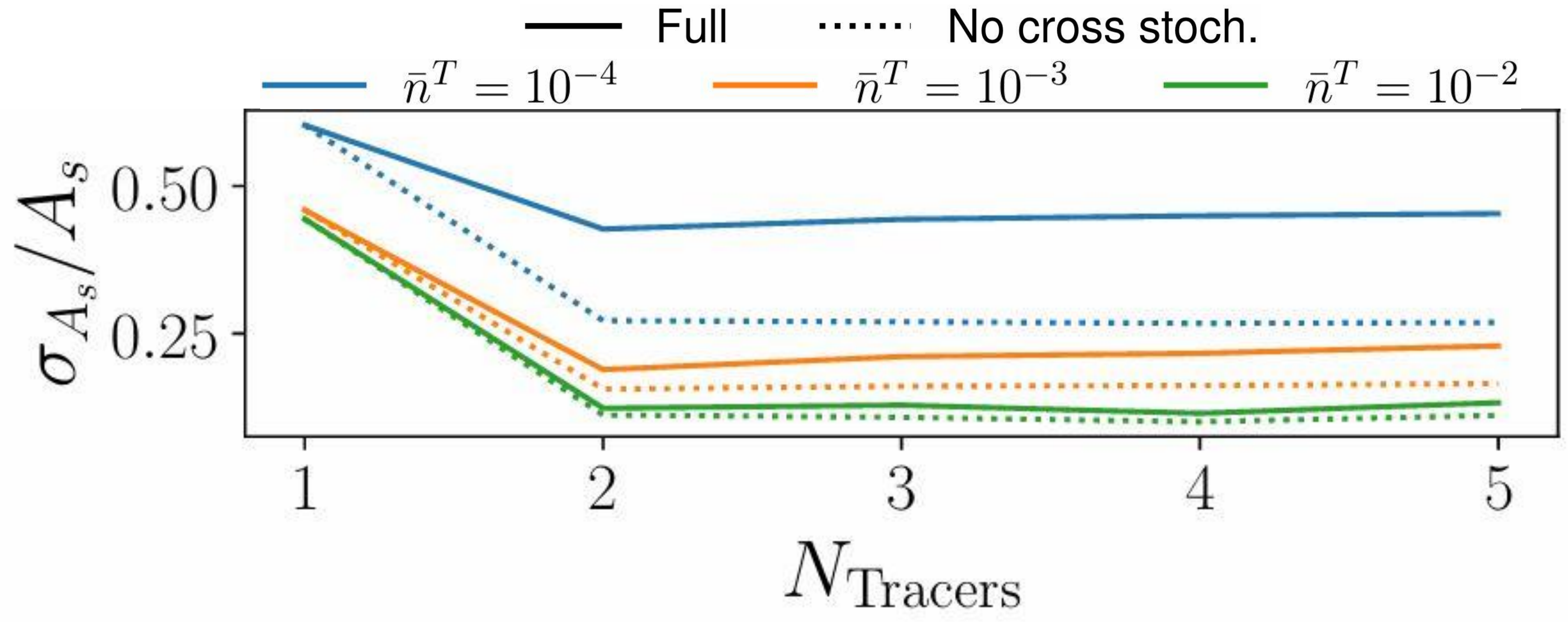
Assembly bias -> easy to find big differences in **non linear** biases!

Relative number density



Good news! It’s easier to find big bias differences!

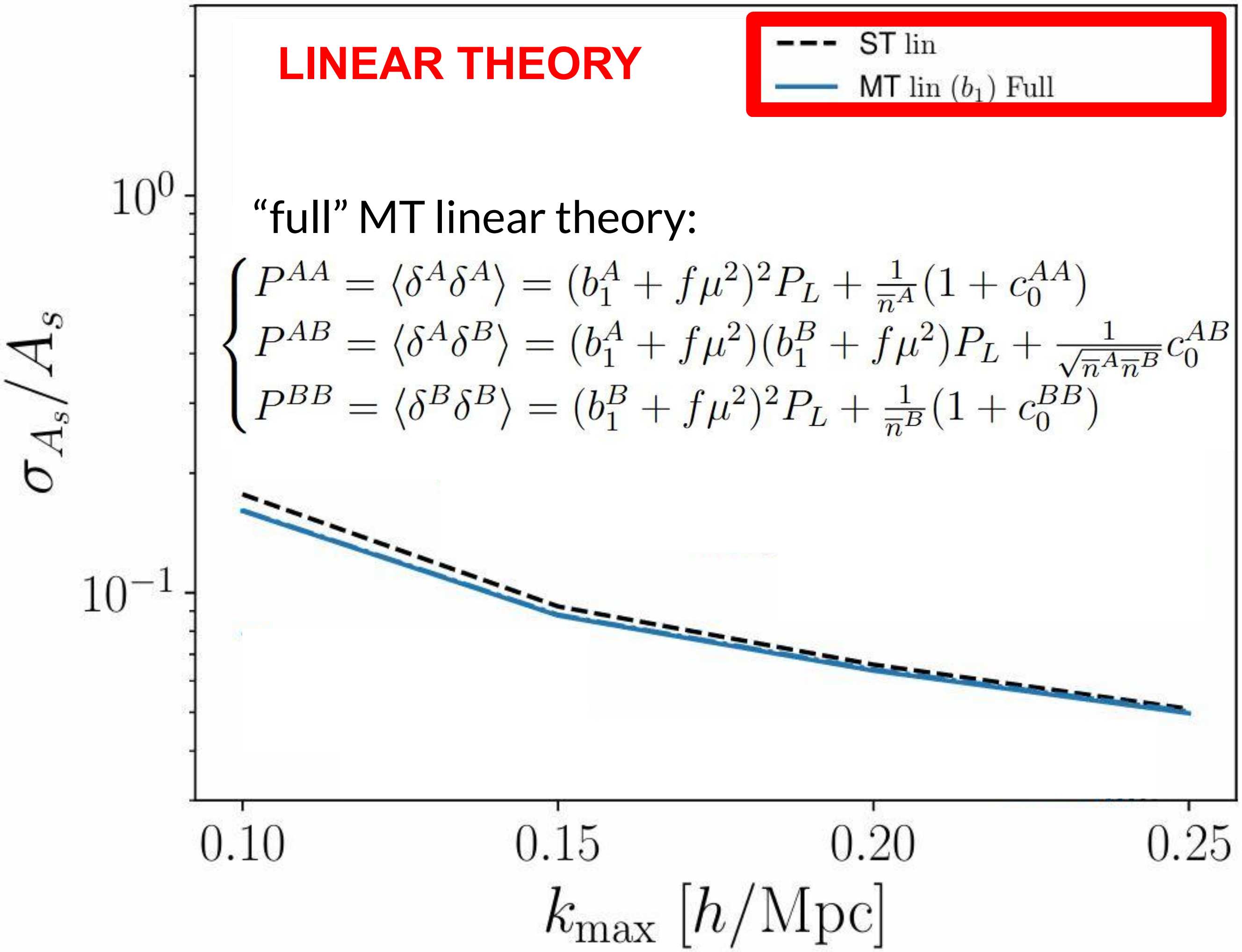
More tracers?



LINEAR THEORY

"full" MT linear theory:

$$
\begin{cases}
P^{AA} = \langle \delta^A \delta^A \rangle = (b_1^A + f\mu^2)^2 P_L + \frac{1}{\bar{n}^A}(1 + c_0^{AA}) \\
P^{AB} = \langle \delta^A \delta^B \rangle = (b_1^A + f\mu^2)(b_1^B + f\mu^2) P_L + \frac{1}{\sqrt{\bar{n}^A \bar{n}^B}} c_0^{AB} \\
P^{BB} = \langle \delta^B \delta^B \rangle = (b_1^B + f\mu^2)^2 P_L + \frac{1}{\bar{n}^B}(1 + c_0^{BB})
\end{cases}
$$



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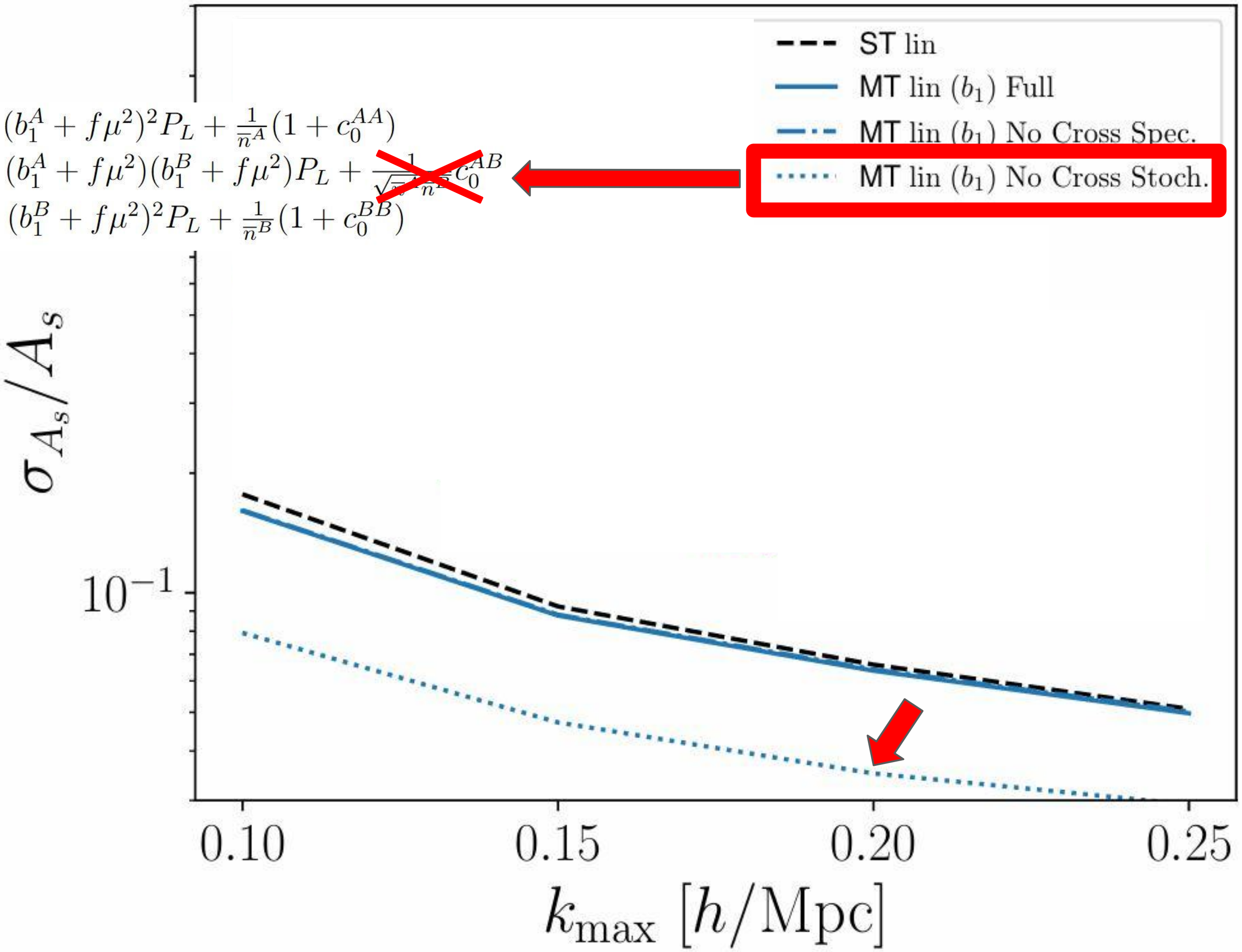
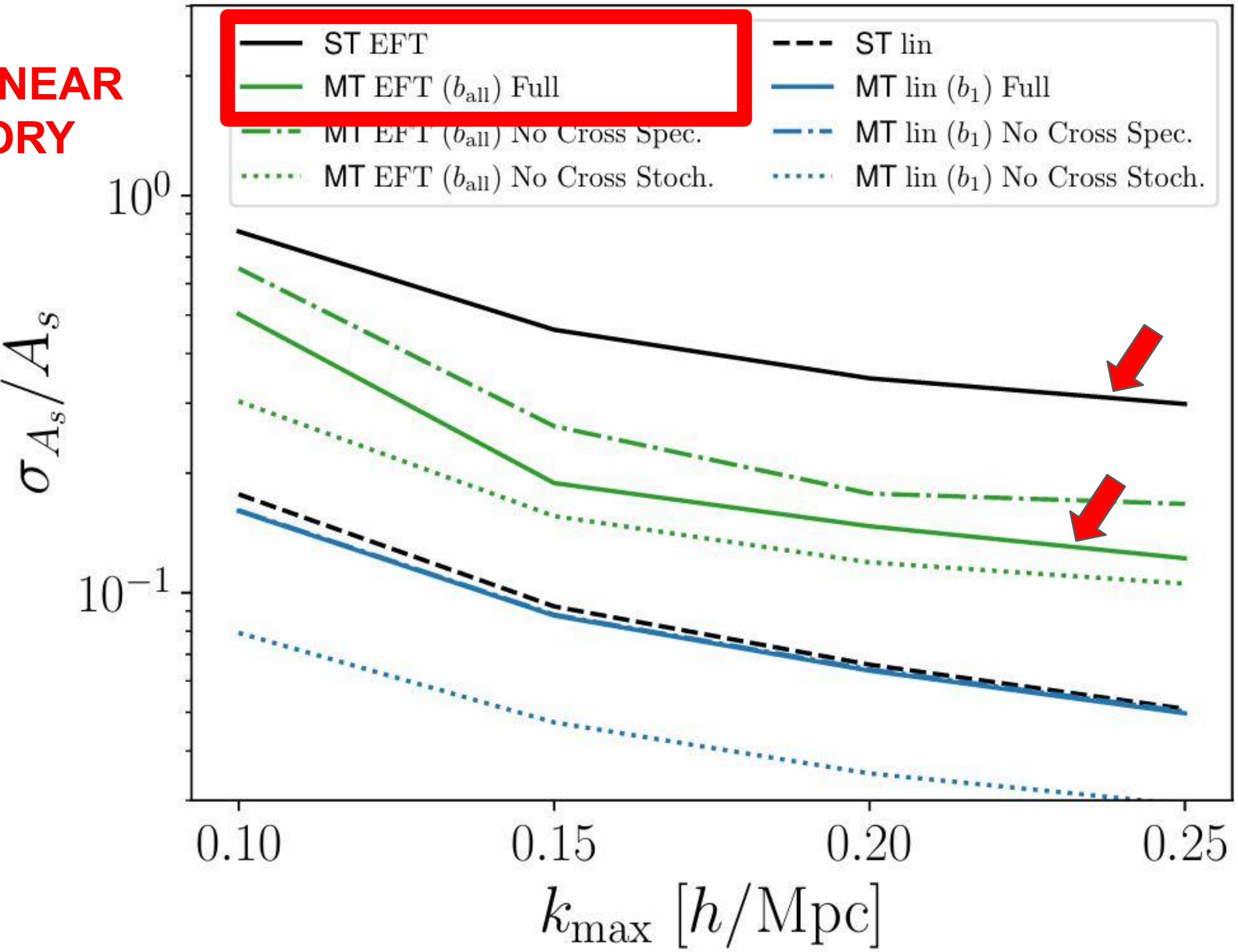
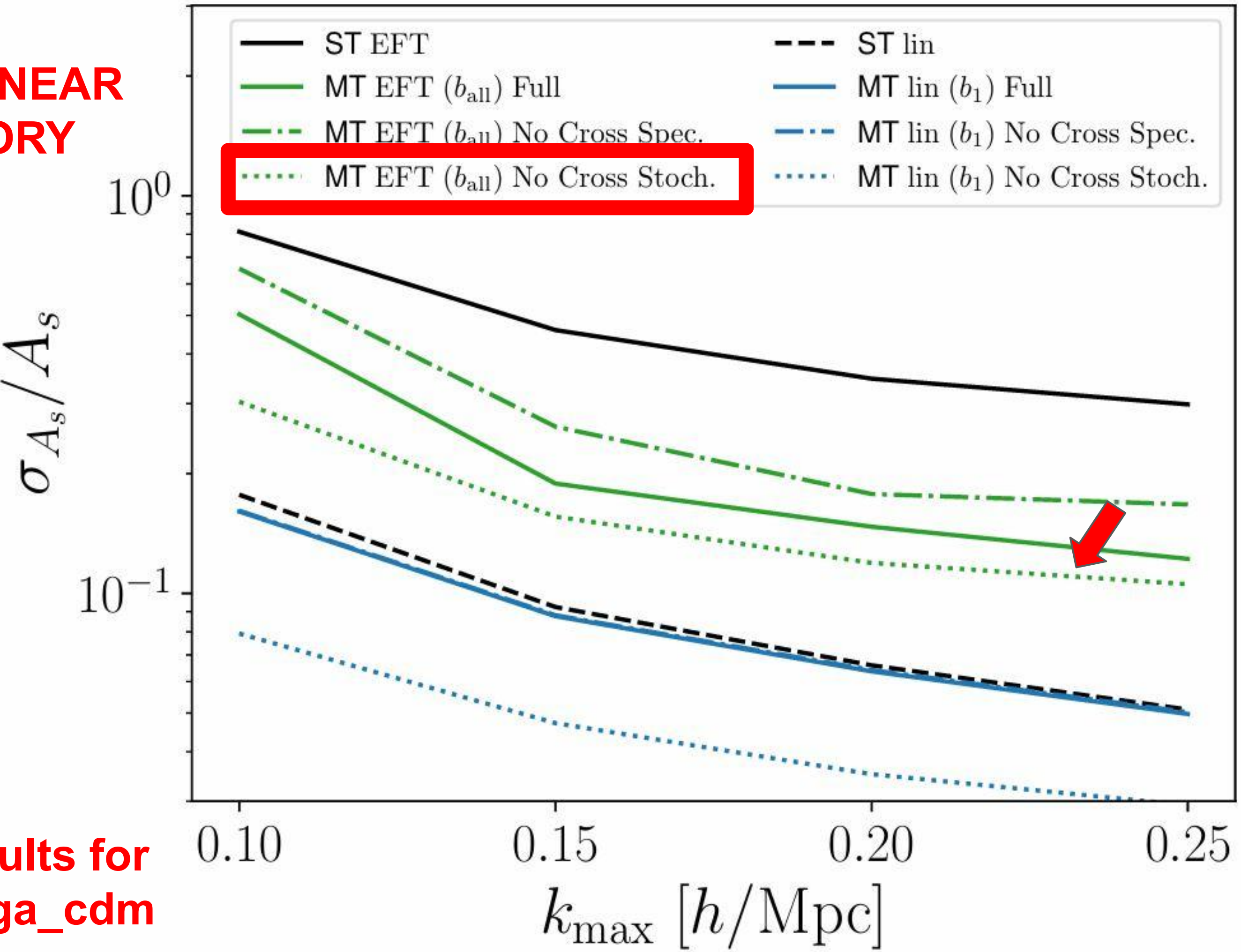


NON-LINEAR THEORY



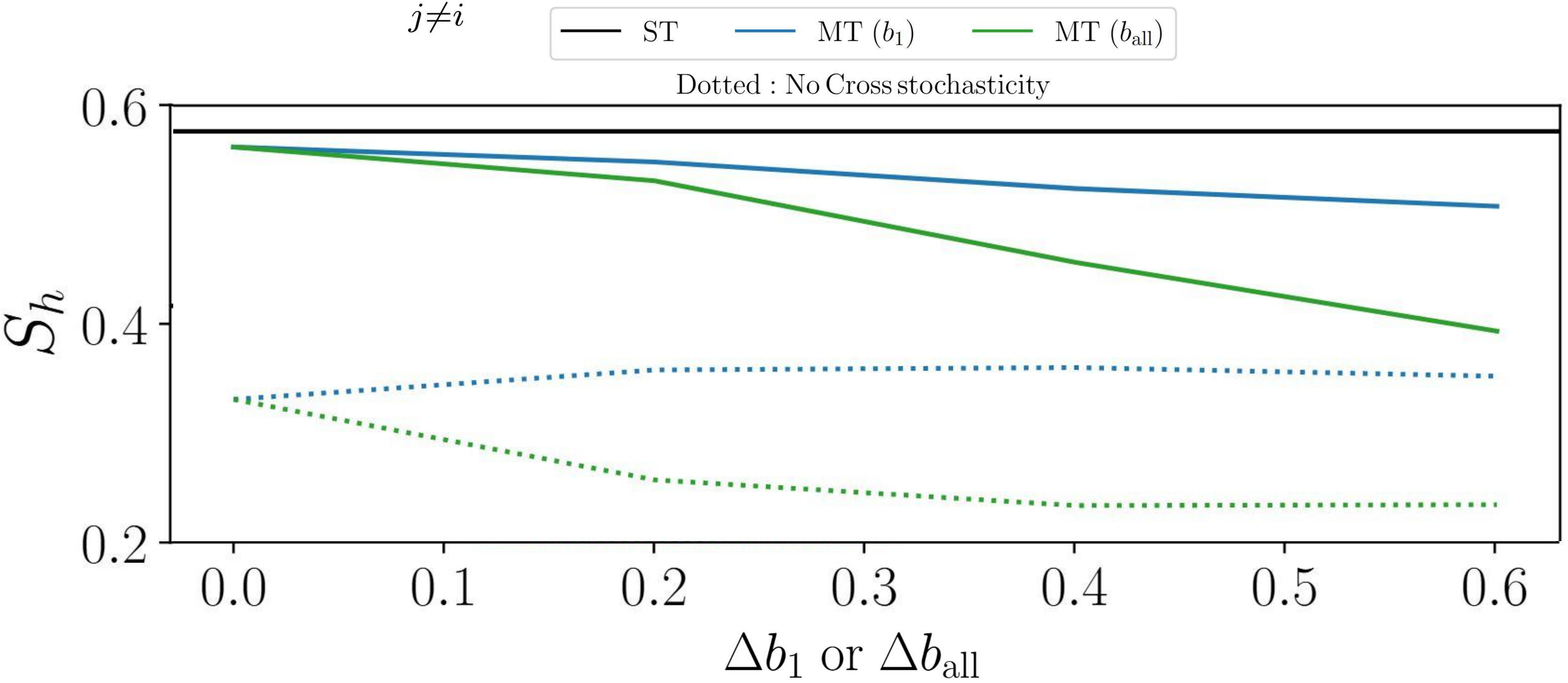
NON-LINEAR THEORY



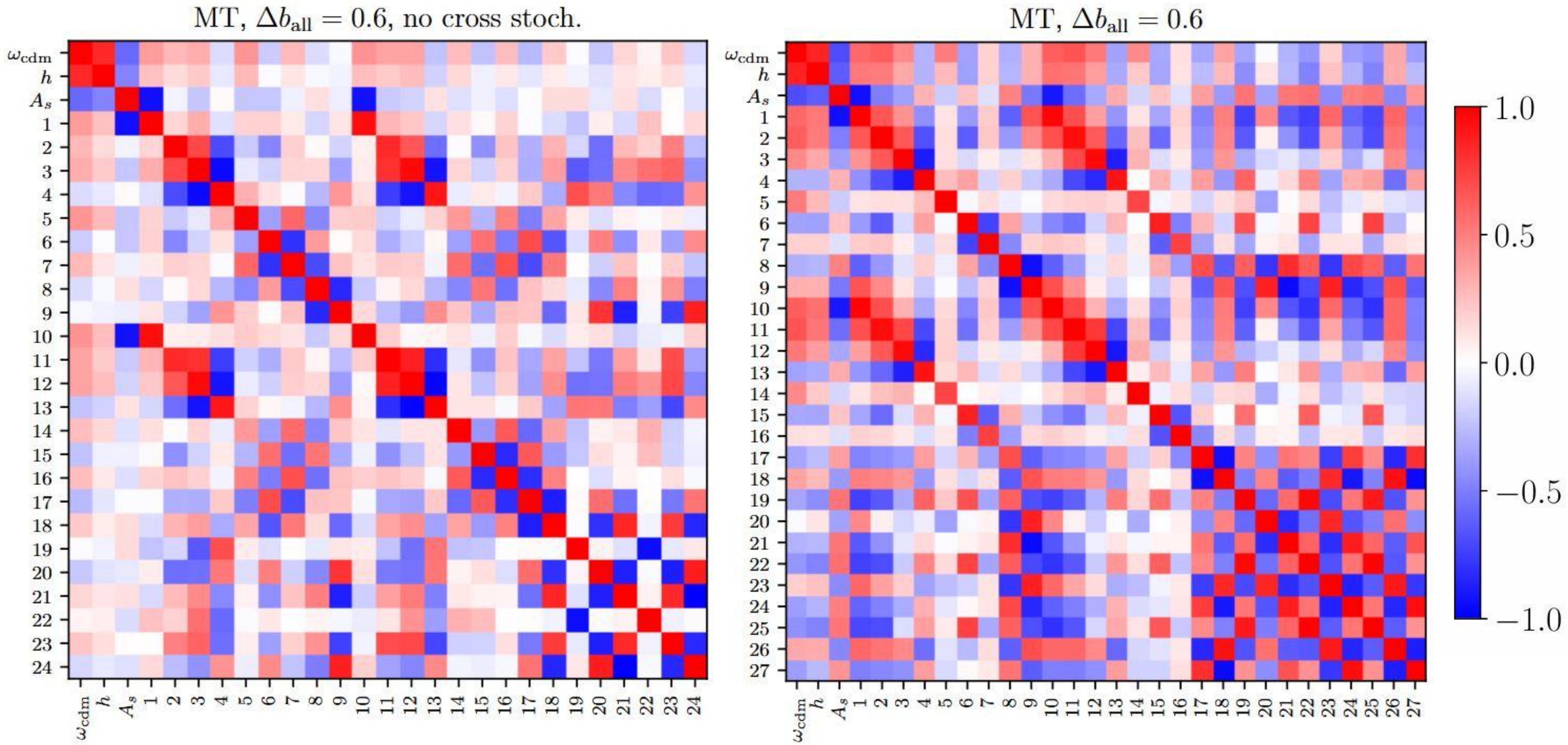
Similar results for h and omega_cdm

How much are the parameters correlated?

Score $S^2_{\theta_i} = \frac{1}{N-1} \sum_{\substack{j=1 \\ j \neq i}}^{N} r^2_{ij}.$ (Pearson) correlation matrix: $r_{ij} = \frac{(F^{-1})_{ij}}{\sqrt{(F^{-1})_{ii}(F^{-1})_{jj}}}.$



How much are the parameters correlated?



Summary

- **Big non-linear Δb** enhances the constraints.

  Hard to find big Δb_1 (high redshift, very unbalanced split), but **easy for non-linear biases**!

  Even with unbalanced splits, good constraints!

- **Linear theory**: MT gains are based on the **absence** of $c_{\rm st}^{AB}$.

  Non-linear theory: always **better performances**, also due to **more diagonal** basis.

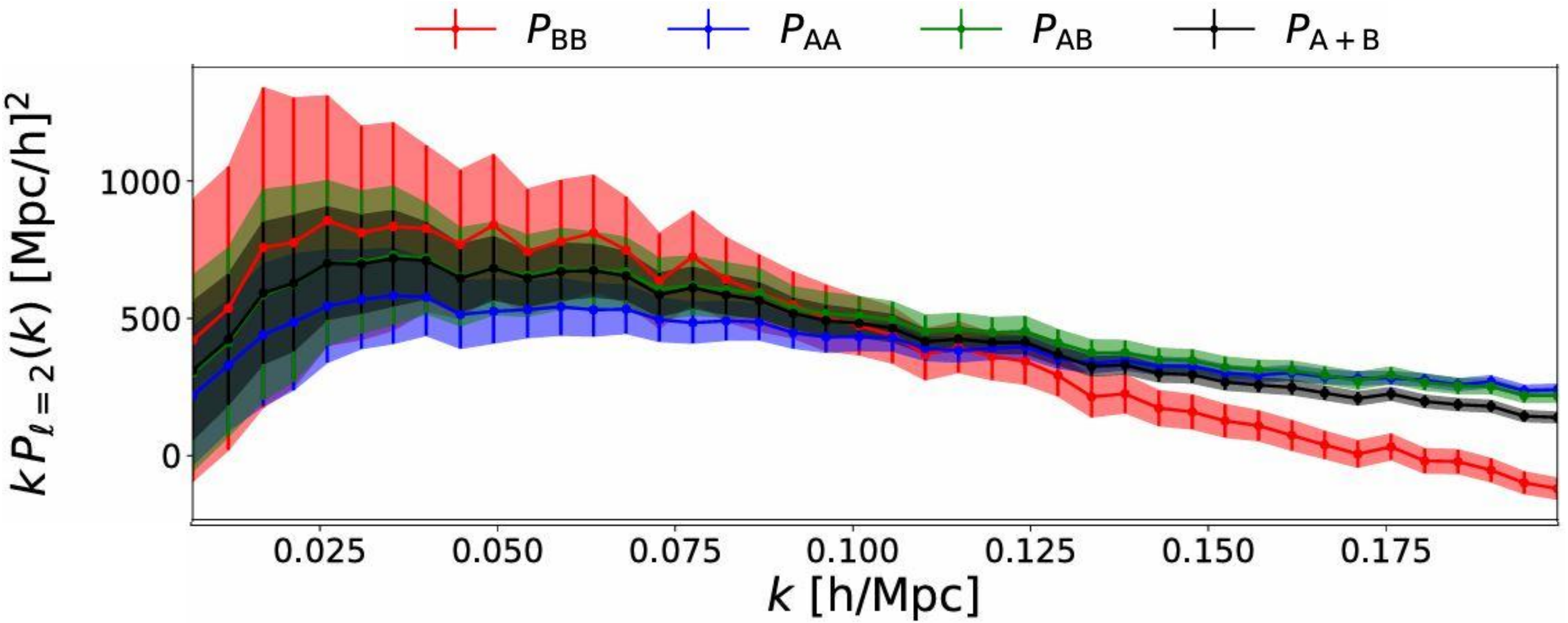
  Would like: tracers with physically motivated $c_{\rm st}^{AB} = 0$.

- N>2 tracers doesn't seem useful!

THANK YOU!

FoG and different k_max

2501.10587 -> use quadrupole to understand when FoG become non perturbative.



Plot from 202306.05474

FoG and different k_max

2501.10587 -> use quadrupole to understand when FoG become non perturbative.
Idea: use different k_max for different tracers