For RN black holes

$$ds^2 = -V(r)\,dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_2, \qquad V = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

$$A_t = -\frac{Q}{r} + \Phi \qquad \Phi = \frac{Q}{r_+} \qquad \text{so } A_t(r=r_+) = 0$$

$$\mathcal{A}_H = 4\pi r_+^2 \qquad r_\pm = M \pm \sqrt{M^2 - Q^2}$$

$$\kappa = \frac{\sqrt{M^2-Q^2}}{r_+^2}$$

$$\boxed{dM = \frac{1}{8\pi}\kappa\, d\mathcal{A}_H + \Phi\, dQ} \qquad 1^{st} \text{ law}$$

We may use r_+, r_- as parameters

$$Q^2 = r_+ r_-$$

$$M = \frac{r_+ + r_-}{2} = \frac{r_+ + Q^2/r_+}{2} = \frac{r_- + Q^2/r_-}{2}$$

$$\left(\frac{\partial M}{\partial A_+}\right)_Q = \left(\frac{\partial M}{\partial r_+}\right)_Q \frac{\partial r_+}{\partial A_+} + \left(\frac{\partial M}{\partial r_-}\right)_Q \overset{=0}{\frac{\partial r_-}{\partial A_+}} = \frac{1}{2}\left(1 - \frac{Q^2}{r_+}\right)\frac{1}{2\sqrt{4\pi A_+}}$$

$$= \frac{1}{16\pi}\frac{r_+^2 - Q^2}{r_+^3} = \frac{1}{8\pi}\frac{r_+ - r_-}{2r_+^2} = \frac{\kappa_+}{8\pi}$$

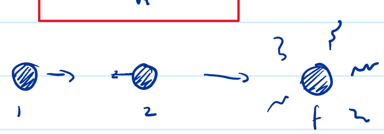
$$\left(\frac{\partial M}{\partial Q}\right)_{A_+} = \left(\frac{\partial M}{\partial r_-}\right)_{A_+}\left(\frac{\partial r_-}{\partial Q}\right)_{A_+} = \frac{1}{2}\,\frac{2Q}{r_+} = \frac{Q}{r_+} = \Phi_+$$

2^{nd} Law (Hawking)

The area of the horizon(s) cannot decrease in time

$$\boxed{\Delta \mathcal{A}_H \geq 0}$$

eg



$$A_1 + A_2 < A_f$$

$$A_1 + A_2 \leq A_f$$

0th Law : $\boxed{\kappa \text{ is uniform over horizon}}$

Striking analogy:

Laws	Thermodynamics	BH mechanics
0th	$T = \text{const}$ at equilibrium	$\kappa = \text{const}$ over stationary horizon
1st	$dE = TdS - pdV + \Omega dJ + \Phi dQ$	$dM = \frac{\kappa}{8\pi G} dA_H + \Omega dJ + \Phi dQ + \dots$
2nd	$\Delta S \geq 0$	$\Delta A_H \geq 0$

But their derivations couldn't be more different!

Carnot cycles — Null geodesics

Heat reservoirs — Killing vectors, surface integrals

Thermodynamics in the presence of black holes looks bizarre.
Could dump entropy into a bh and lose it from sight, entirely and forever!

[Could also extract the energy of a box w/ radiation (Geroch process) and then dump the entropy into the black hole]

Bekenstein: The area of a black hole is a form of entropy

$$S_{BH} = C \frac{A_H}{G\hbar} \sim \frac{A_H}{\ell_{Pl}^2} \quad \sim 1 \text{ bit per unit Planck area}$$

Generalized 2nd law: $\Delta(S_{matter} + S_{BH}) \geq 0$

There should be a temperature $T_H = \frac{\hbar \kappa}{8\pi C}$ $\qquad \frac{\kappa}{8\pi G} A_H = TS$

Puzzle:

If black holes have entropy, they should have a temperature. If they have a temperature, then they should be able to emit thermal radiation.

But black holes do not emit anything at all. They're at absolute zero

a temperature, then they should be able to emit thermal radiation.
But black holes do not emit anything <u>at all</u>. They're at absolute zero temperature! Or are they?

Hawking set out to prove that Bekenstein was wrong: a quantum system in an equilibrium black hole spacetime should be absolutely cold.

But he actually proved the opposite! When considering a collapsing system, initially a quantum field is parametrically excited by the dynamical geometry. But when it settles down into a stationary equilibrium black hole, the field continues to be excited, and actually there's a continuous outflux of radiation with a thermal spectrum with a temperature

$$T_H = \frac{\hbar \kappa}{2\pi}$$

This determines $C = \frac{1}{4}$ and therefore fixes

$$S_{BH} = \frac{A_H}{4G\hbar} \quad : \text{Bekenstein-Hawking entropy}$$

This formula combines gravitation, quantum theory, and thermodynamics in a deep and mysterious way. It's our first formula for quantum gravity, and our most important clue into a quantum theory of gravity.