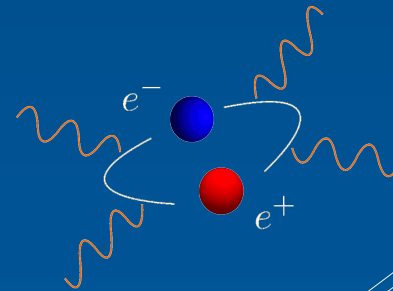


Predicting Optical Quantum Vacuum Signals: Successes and Challenges

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1 Introduction

Classical electrodynamics (ED):

→ accurate description of **macroscopic electromagnetic fields** $\bar{F}^{\mu\nu}$ in laboratory

$$\mathcal{L}_{\text{ED}} = -\frac{1}{4} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} + \bar{J}_\mu \bar{A}^\mu$$

[Maxwell: Philos. Trans. R. Soc. **155** (1865)]



$$\mathcal{L} = \mathcal{L}(\bar{A}^\mu, \bar{F}^{\mu\nu}) \xrightarrow{\text{EOM}} \frac{\partial \mathcal{L}}{\partial \bar{A}_\nu} - 2\partial_\mu \frac{\partial \mathcal{L}}{\partial \bar{F}_{\mu\nu}} = 0$$

$$\partial_\mu \bar{F}^{\mu\nu} = -\bar{J}^\nu$$

→ linear equations of motion (EOM) → superposition principle holds in vacuum

↔ our world is quantum → there should be corrections (depending on $\hbar$)

1 Introduction

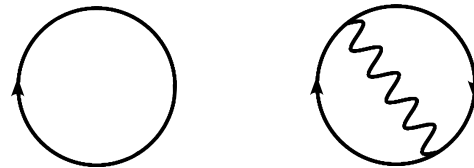
Quantum electrodynamics (QED): ($\hbar = c = 1$; quantum fields $A^\mu, \bar{\psi}, \psi$)

$\hat{=}$ relativistic quantum theory of light and matter, quantum field theory

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + e\bar{\psi}\gamma_\mu\psi A^\mu$$



$\rightarrow$ notion of the “quantum vacuum” $\hat{=}$ diagrams without external lines



$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

1 Introduction

Heisenberg-Euler (HE) effective theory:

[Heisenberg, Euler: Z. Phys. **98** (1936)]

$\hat{=}$ true theory of **macroscopic electromagnetic** $\bar{F}^{\mu\nu}$ in the quantum vacuum

- m is the only dimensionful scale inherited from QED
- gauge invariance: $\mathcal{L}_{\text{HE}}(\bar{F}^{\mu\nu}, \partial^\rho \bar{F}^{\mu\nu}, \dots) \sim \bar{F}^2 \times \text{function}\left(\frac{e\bar{F}}{m^2}, \frac{\partial}{m} \frac{e\bar{F}}{m^2}, \dots\right)$

$$\frac{\hbar}{c^3} \frac{e\mathbf{E}_i}{m^2} = \frac{\mathbf{E}_i}{E_{\text{S}}} = \frac{\mathbf{E}_i}{1.3 \times 10^{18} \text{ V/m}}, \quad \frac{\hbar}{c} \frac{\partial_{\text{x}}}{m} = \lambda_{\text{C}} \partial_{\text{x}} \sim \frac{3.8 \times 10^{-7} \mu\text{m}}{\Delta x}$$

$\rightarrow$ for “weak & slowly varying fields”: $\mathcal{L}_{\text{HE}} \rightarrow \mathcal{L}_{\text{ED}}$

1 Introduction

Diagrammatic representation:

at $\mathcal{O}(\partial^0)$: [Heisenberg, Euler: Z. Phys. **98** (1936)]

[Gies, FK: JHEP **3** (2017)]

$$\mathcal{L}_{\text{HE}}(\bar{F}) = -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

The diagrams are: a double circle, a double circle with a wavy line inside, and two double circles connected by a wavy line.

[Ritus: JETP **42** (1975)]

where $\Rightarrow = \rightarrow + \text{[Diagram 4]} + \text{[Diagram 5]} + \dots$

The diagrams are: a wavy line with an 'X' at the top, and two wavy lines with 'X' at the top.

$$\rightarrow \mathcal{L}_{\text{HE}}(\bar{F}) = -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \mathcal{L}_{\text{int}}(\bar{F})$$

$$\begin{array}{c} \alpha \\ \nearrow \frac{\partial^\mu}{m} \\ \searrow \frac{e\bar{F}^{\mu\nu}}{m^2} \end{array}$$

1 Introduction

Leading low-energy quantum vacuum nonlinearity:

$$\mathcal{L}_{\text{int}}(\bar{F}) \simeq \frac{m^4}{5760\pi^2} \left(\frac{e}{m^2}\right)^4 \left[\mathbf{a} (\bar{F}_{\mu\nu} \bar{F}^{\mu\nu})^2 + \mathbf{b} (\bar{F}_{\mu\nu} {}^* \bar{F}^{\mu\nu})^2 \right]$$

$\uparrow \qquad \qquad \qquad \uparrow$

$\sim (\vec{B}^2 - \vec{E}^2)^2 \qquad \sim (\vec{B} \cdot \vec{E})^2$

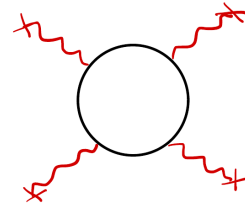
→ QED, up to two loops:

$$\mathbf{a} = 4 \left(1 + \frac{40}{9} \frac{\alpha}{\pi} + \dots \right)$$

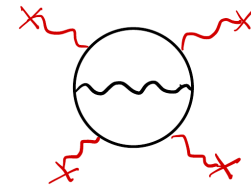
$$\mathbf{b} = 7 \left(1 + \frac{1315}{252} \frac{\alpha}{\pi} + \dots \right)$$



corrections on 1% level



[Euler: Ann. Phys. **418** (1936)]



[Ritus: JETP **42** (1975)]

2 Our formalism

Optical quantum vacuum signals:

[FK, Shaisultanov: Phys. Rev. D **91** (2015)]

→ applied **macroscopic electromagnetic fields** $\bar{F}^{\mu\nu}$ induce signals

- $(0 \rightarrow 1)$ **vacuum emission** amplitude: $\mathcal{S}_p(\vec{k}) = \langle \gamma_p(\vec{k}) | e^{i \int d^4x \mathcal{L}_{\text{int}}(\bar{F}+f)} | 0 \rangle$
- differential signal photon number: $d^3 N_p = \frac{d^3 k}{(2\pi)^3} |\mathcal{S}_p(\vec{k})|^2$

inhomogeneous
strong field



signal photons

- energy $|\vec{k}|$,
- emission direction $\vec{k}/|\vec{k}|$,
- polarization p .

2 Our formalism

→ accurate analytic scalings ✓

PHYSICAL REVIEW D **104**, 013006 (2021)

Vacuum birefringence and diffraction at an x-ray free-electron laser: From analytical estimates to optimal parameters

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We study vacuum birefringence and x-ray photon scattering in the head-on collision of x-ray free electron and high-intensity laser pulses. Resorting to analytical approximations for the numbers of attainable signal photons, we analyze the behavior of the phenomenon under the variation of various experimental key-parameters and provide new analytical scalings. Our optimized approximations allow for quantitatively accurate results on the one-percent level. We in particular demonstrate that an appropriate choice of the x-ray focus and pulse duration can significantly improve the signal for given laser parameters, using the experimental parameters to be available at the Helmholtz International Beamline for Extreme Fields at the European XFEL as example. Our results are essential for the identification of the optimal choice of parameters in a discovery experiment of vacuum birefringence at the high-intensity frontier.

2 Our formalism

→ first-principles numerical studies ✓

PHYSICAL REVIEW D **99**, 016006 (2019)

All-optical signatures of quantum vacuum nonlinearities in generic laser fields

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All-optical experiments at the high-intensity frontier offer a promising route to unprecedented precision tests of quantum electrodynamics in strong macroscopic electromagnetic fields. So far, most theoretical studies of all-optical signatures of quantum vacuum nonlinearity are based on simplifying approximations of the beam profiles and pulse shapes of the driving laser fields. Since precision tests require accurate quantitative theoretical predictions, we introduce an efficient numerical tool facilitating the quantitative theoretical study of all-optical signatures of quantum vacuum nonlinearity in generic laser fields. Our approach is based on the vacuum emission picture, and makes use of the fact that the dynamics of the driving laser fields are to an excellent approximation governed by classical Maxwell theory in vacuum. In combination with a Maxwell solver, which self-consistently propagates any given laser field configuration, this allows for accurate theoretical predictions of photonic signatures of vacuum nonlinearity in high-intensity laser experiments from first principles.

3 Successes and challenges

Collide ≥ 2 intense lasers: $\bar{F}^{\mu\nu} \rightarrow 1 \text{ pump} + 1 \text{ probe field}$

$\rightarrow$ quasi-elastic signal $\sim$ probe's forward cone dominates: [FK, Sundqvist: Phys. Rev. D **94** (2016)]

$$N_{\parallel/\perp,\text{sig}} \sim c_{\parallel,\perp} (1 - \cos \vartheta_{\text{coll}})^4 \left(\frac{I_{0,\text{pump}}}{I_S} \frac{\omega_{\text{probe}}}{mc^2} \right)^2 N_{\text{probe}} \quad \text{with} \quad I_S \simeq 10^{29} \frac{\text{W}}{\text{cm}^2}$$

$\gtrsim 1/\text{shot}$ for tightly focused PW class lasers

Signal-to-background separation?

$\rightarrow$ measure $\perp$ -signal (at an XFEL)! [Aleksandrov, Anselm, Moskalev: JETP **62** (1985)], ...

$$N_{\perp,\text{bgr}} = \mathcal{P} N_{\text{probe}} \quad \text{with} \quad \mathcal{P} \sim 10^{-11} \quad @ \quad \omega_{\text{probe}} \sim 10 \text{ keV} \quad \checkmark \quad [\text{Schulze, et al.: Phys. Rev. Res. 4 (2022)}]$$

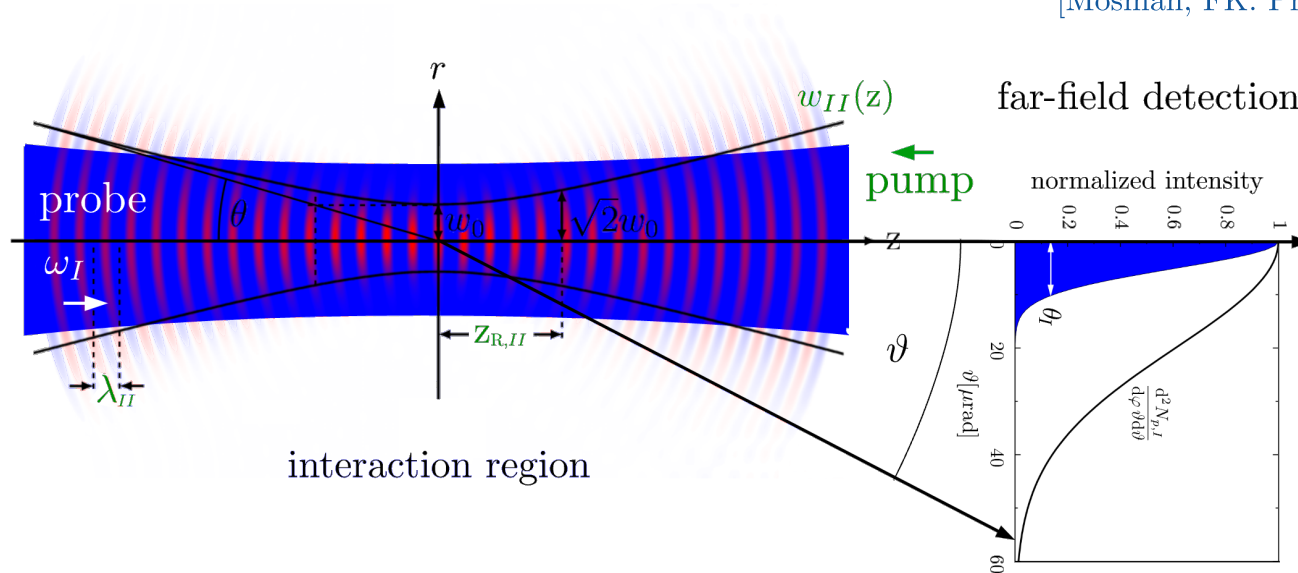
$\leftrightarrow$ signals are background dominated for available parameters ☹

3 Successes and challenges

Signal enhancement strategies (in the collision of two laser beams)?

(a) Diffraction assisted measurement:

[FK, Gies, Reuter, Zepf: Phys. Rev. D **92** (2015)],
 [FK, Sundqvist: Phys. Rev. D **94** (2016)],
 [FK: Phys. Rev. D **98** (2018)],
 [Mosman, FK: Phys. Rev. D **104** (2021)]



- attempts to measure $N_{\parallel, \text{sig}}$ with this schema at SACLA, Japan

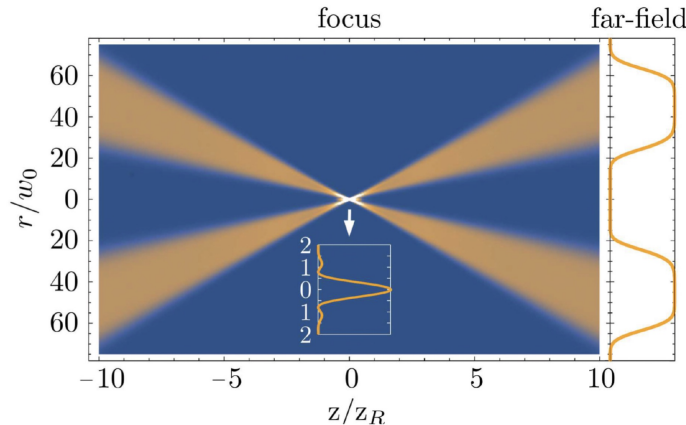
[Inada, *et al.*: Appl. Sci. **7** (2017)],
[Seino, *et al.*: PTEP **7** (2020)]

3 Successes and challenges

(b) Annular probe / dark-field approach:

[FK, Ullmann, Mosman, Zepf: Phys. Rev. Lett. **129** (2022)]

→ modify probe to make forward $\perp$ and $\parallel$ signals accessible:



[Peatross, *et al.*: Opt. Lett. **19** (1994)],
[Zepf, *et al.*: Phys. Rev. E **58** (1998)]

→ in the focus deviations from Gaussian far-field profile are encoded in Airy pattern

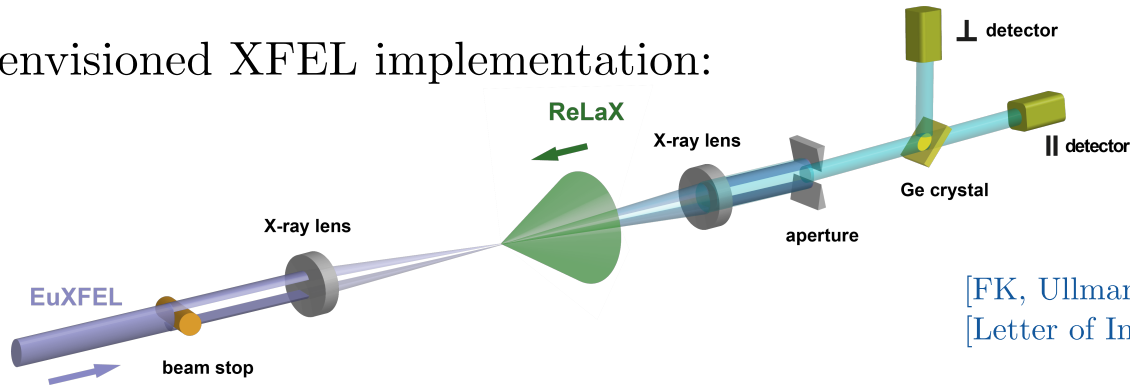
↔ self-consistent beam model ✓

[FK, Mosman: Phys. Rev. D **101** (2020)]

→ laser fluctuation independent ratio $\left. \frac{N_{\perp, \text{sig}}}{N_{\parallel, \text{sig}}} \right|_{\phi=\frac{\pi}{4}} \simeq \left(\frac{a-b}{a+b} \right)^2 \xrightarrow{\text{QED}} \frac{9}{121} \left(1 + \frac{\alpha}{\pi} \frac{260}{99} + \dots \right)$

3 Successes and challenges

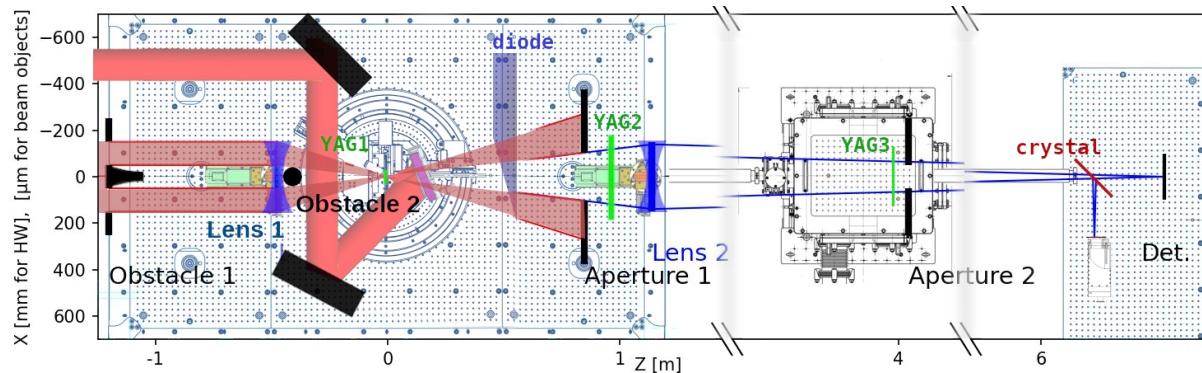
→ envisioned XFEL implementation:



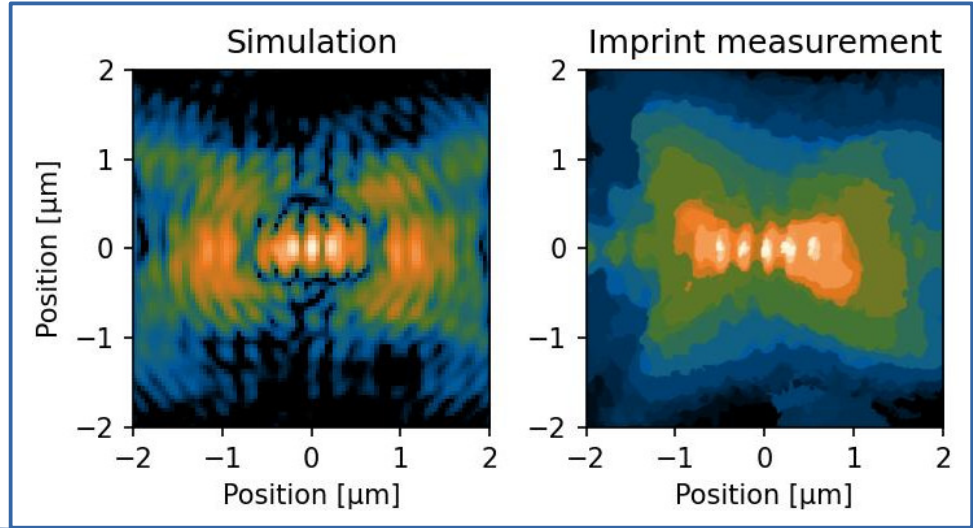
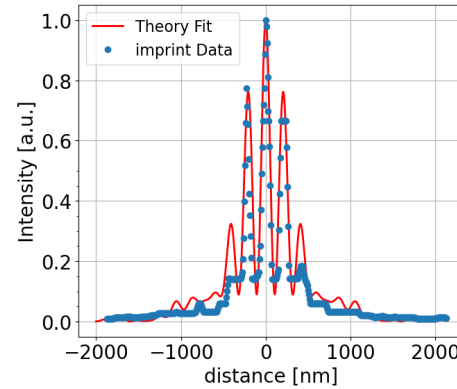
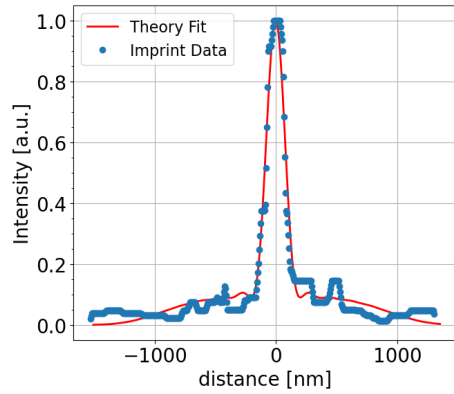
[FK, Ullmann, Mosman, Zepf: Phys. Rev. Lett. **129** (2022)]
[Letter of Intent: High Power Laser Sci. Eng. **13**, e7 (2025)]

→ proof-of-concept experiment @ EuXFEL:

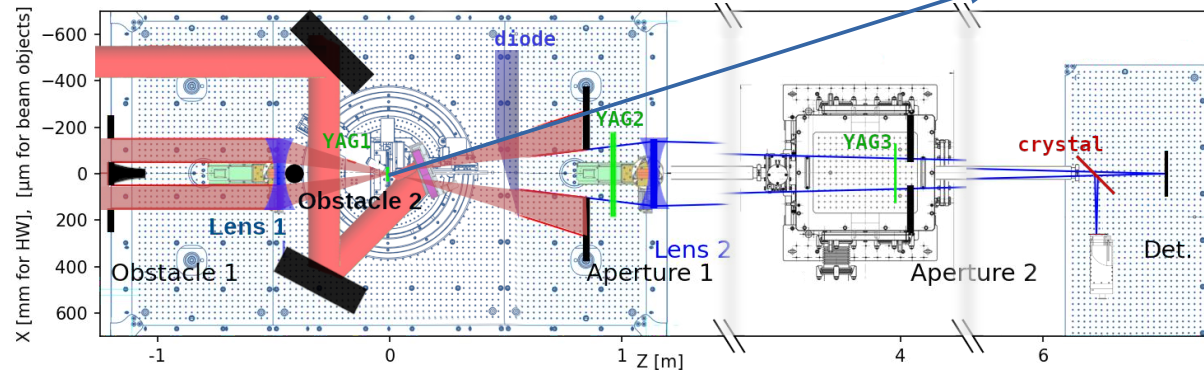
[Šmíd, *et al.*: arXiv:2506.11649 (2025)]



3 Successes and challenges



→ proof-of-concept experiment @ **EuXFEL**:



→ simulations $\hat{=}$ measurements

$\hat{=}$ **even our analytics!**

[FK: Phys. Rev. D **98** (2018)],
[FK, Mosman: Phys. Rev. D **101** (2020)]
[Mosman, FK: Phys. Rev. D **104** (2021)]
[FK: 10.22032/dbt.59618 (2024)]

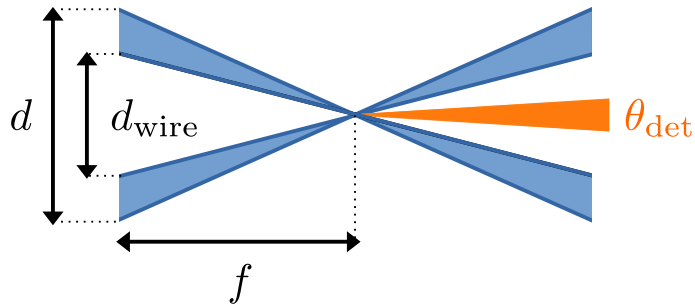
3 Successes and challenges

ReLaX

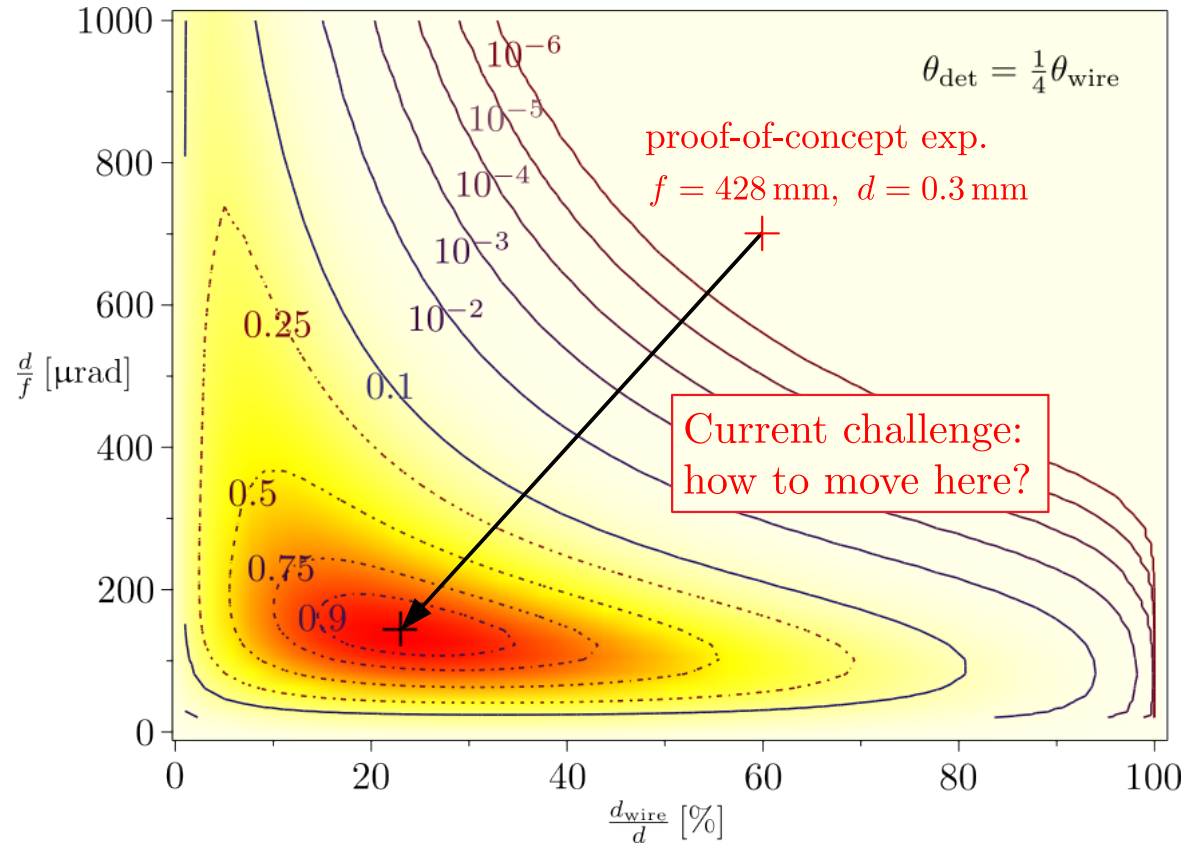
- W = 4.8 J
- τ_{FWHM} = 30 fs
- w_{FWHM} = 1.3 m

EuXFEL

- N = 2×10^{11}
- ω = 8766 eV
- τ_{FWHM} = 25 fs



$$N_{\perp, \text{sig}}^{\text{max}} = 3/h, \quad N_{\parallel, \text{sig}}^{\text{max}} = 40/h$$



3 Successes and challenges

Towards an **all-optical** implementation of the **dark-field** approach

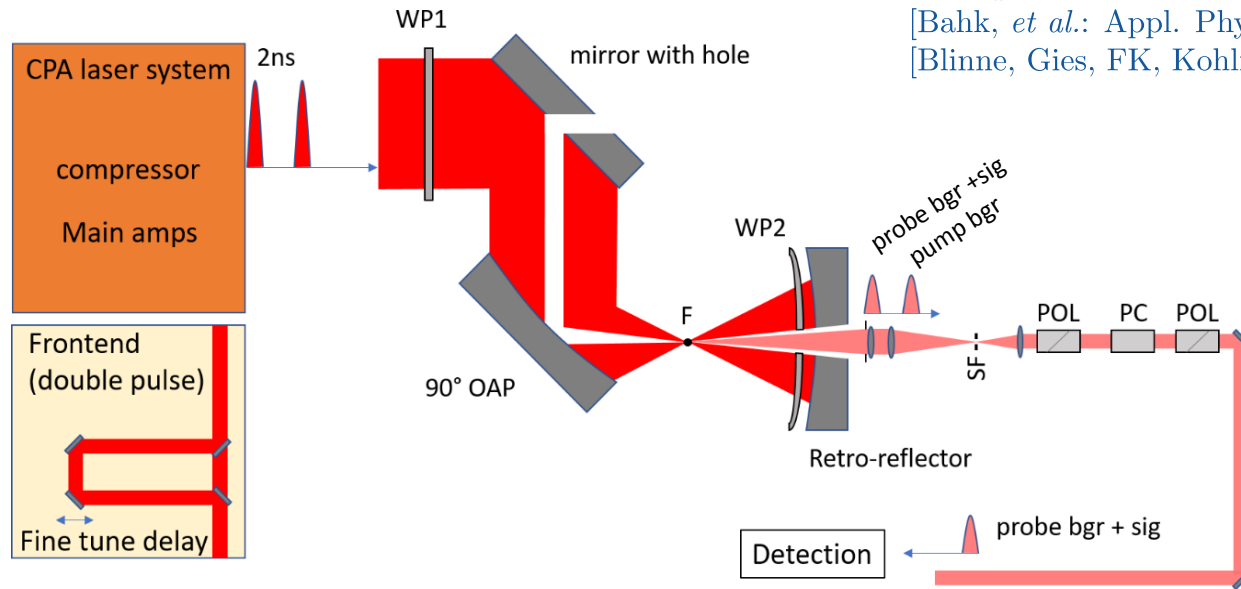
→ schematic layout of envisioned setup:

[Schütze, Doyle, Schreiber, Zepf, FK: Phys. Rev. D **109** (2024)]

Laser pulse model & field evolution:

[Bahk, *et al.*: Appl. Phys. B **80** (2005)],

[Blinne, Gies, FK, Kohlfürst, Zepf: Phys. Rev. D **99** (2019)]

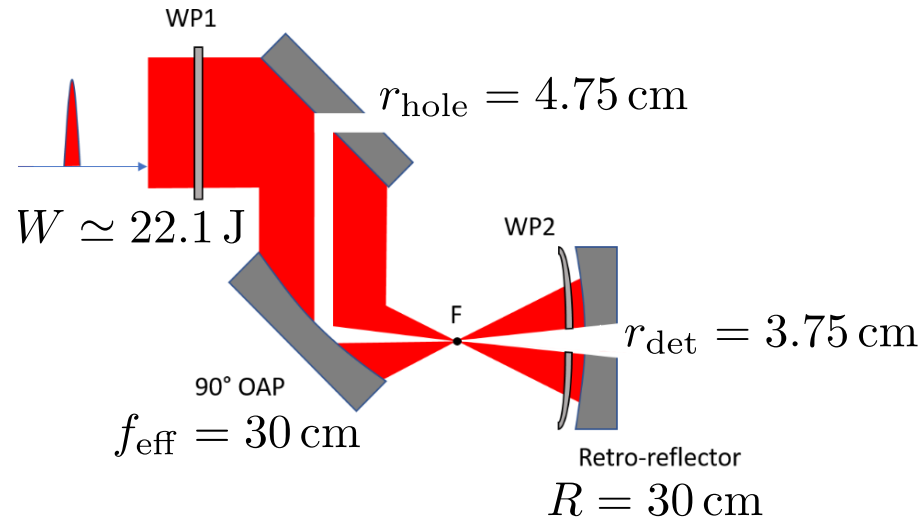


3 Successes and challenges

Predictions for ideal shots:

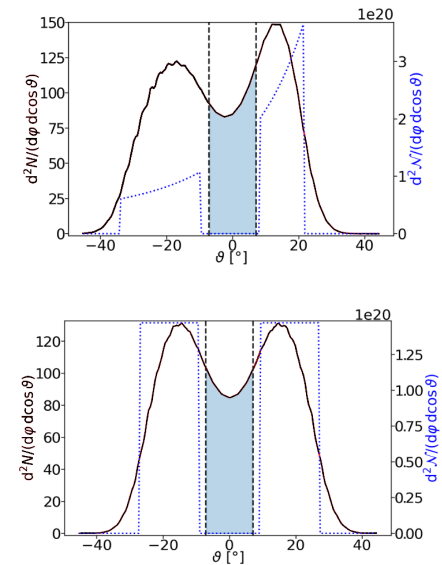
→ laser pulse parameters (ATLAS 3000 @ CALA):

- $W_0 = 25 \text{ J}$
- $\lambda = 800 \text{ nm}$
- $\tau = 30 \text{ fs}$
- $r_{\text{beam}} = 14 \text{ cm}$



→ $N_{\text{sig@det}}^{\text{max}} \simeq 4.3/\text{shot}$, $N_{\perp,\text{sig@det}}^{\text{max}} \simeq 0.2/\text{shot}$

		(A1)	(A2)
Estimate	N_{det}	1.41	2.86
	$N_{\perp,\text{det}}$	0	0.20
Full calculation	N_{det}	1.48	2.88
	$N_{\perp,\text{det}}$	6.71×10^{-4}	0.19



4 Perspectives

I just convinced you that we are ready to **collaborate closely with experiment**

→ model & optimize quantum vacuum signals in realistic laser fields ✓

There is **still a lot to do**:

→ advance simulation capabilities towards realistic XFEL fields

→ “simulate the experiment”: shot-to-shot fluctuations via Monte Carlo, etc.

→ account for / study Beyond the Standard Model signatures: @ axions, etc.

→ study effects at large *quantum nonlinear parameter* χ in more realistic fields

[Marmier, Seegert, FK: Phys. Rev. D **111** (2025)]