

# BRST quantization of constrained Hamiltonian systems: the example of asymptotically flat gravity

Marc Henneaux  
(Université Libre de Bruxelles and Collège de France)

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Marc Henneaux  
(Université Libre  
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According to Dirac, systems with a gauge invariance are characterized by (first class) constraints, which we denote by  $\mathcal{G}_\alpha \approx 0$ , and which fulfill

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What are the canonical generators  $G[\epsilon^\alpha]$  of the gauge transformations?

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Again according to Dirac, these generators are proportional to the (first class) constraints functions  $\mathcal{G}_\alpha$ ,

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$$G[\epsilon^\alpha] = \epsilon^\alpha \mathcal{G}_\alpha,$$

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Again according to Dirac, these generators are proportional to the (first class) constraints functions  $\mathcal{G}_\alpha$ ,

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and so weakly vanish.

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In a field theory, the constraints depend on the (spatial)  
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and the generator takes the form

$$G[\epsilon^\alpha(x)] = \int d^3x \epsilon^\alpha(x) \mathcal{G}_\alpha(x),$$

which is valid if there is no boundary, or, if there is a boundary  
(which can be at infinity), when the gauge parameters go to zero  
at the boundary (at infinity).

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that is necessary to make  $G[\epsilon^\alpha]$  a well-defined Hamiltonian generator (Regge-Teitelboim 1974).

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The corresponding transformation is called “a proper gauge transformation”. It gives a true redundancy in the description of the system (Benguria-Cordero-Teitelboim 1977).

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If the boundary term  $B_{\epsilon_\infty}^\alpha$  does not vanish,  $G[\epsilon^\alpha]$  generates "an improper gauge transformation" with physical content,

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If the boundary term  $B_{\epsilon_\infty}^\alpha$  does not vanish,  $G[\epsilon^\alpha]$  generates "an improper gauge transformation" with physical content, and should not be factored out.

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In the quantum theory, the physical states should be annihilated by the constraints, i.e., should be invariant under proper gauge transformations.

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But this raises two questions :

- 1 How should the physical states be normalized (naive scalar product generically diverges)?

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But this raises two questions :

- 1 How should the physical states be normalized (naive scalar product generically diverges)?
- 2 What about improper gauge transformations?

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The difficulty with the scalar product is that it diverges for non-compact gauge group, since the gauge orbits along which the physical states are constant have then infinite volume.

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Different proposals to handle this question have been suggested, from inserting gauge conditions

to adopting the “co-isotropic” / “group average” quantization method, in which the physical states are not requested to be annihilated by the constraints, but one identifies two states that differ by a combination of constraints,  $|\psi\rangle \simeq |\psi\rangle + \epsilon^\alpha \mathcal{G}_\alpha |\chi\rangle$ ,

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These methods are in fact encompassed by the BRST approach (developed before), which frames the discussion on a more satisfactory basis (gauge independence manifest etc).

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The difficulty with the scalar product is that it diverges for non-compact gauge group, since the gauge orbits along which the physical states are constant have then infinite volume. Different proposals to handle this question have been suggested, from inserting gauge conditions to adopting the “co-isotropic”/“group average” quantization method, in which the physical states are not requested to be annihilated by the constraints, but one identifies two states that differ by a combination of constraints,  $|\psi\rangle \simeq |\psi\rangle + \epsilon^\alpha \mathcal{G}_\alpha |\chi\rangle$ , to adopting a Gupta-Bleuler approach somehow mid-way between these two proposals.

These methods are in fact encompassed by the BRST approach (developed before), which frames the discussion on a more satisfactory basis (gauge independence manifest etc).

The BRST method, and the treatment of improper gauge symmetries, will be explained in the context of gravity in the asymptotically flat case, where the constraints yield the Wheeler-DeWitt equation.

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It raises many conceptual questions related to diffeomorphism invariance that any theory of gravity will have to answer.

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(Raju 2022, Witten 2023, Held and Maxfield 2025, Berezhiani, Dvali and Sakhelashvili 2025 etc).

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This recent work mostly deals with asymptotically anti-de Sitter spaces.

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How does it extend to asymptotically flat spaces, where the asymptotic symmetry is infinite-dimensional and described by the BMS group?

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How does it extend to asymptotically flat spaces, where the asymptotic symmetry is infinite-dimensional and described by the BMS group?

Answering this question is one of the purposes of this talk, based on MH Phys. Rev. Lett. **135** (2025) no.6, 061501 [arXiv :2506.02240 [hep-th]].

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Shall focus on general framework and questions of principles

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The Wheeler-DeWitt equation expresses that the states  $|\psi\rangle$  of the gravitational field should be annihilated by the gravitational quantum constraint operators  $\hat{\mathcal{H}}$  and  $\hat{\mathcal{H}}_k$

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$$\text{i.e., } \int d^3x (\zeta^\perp \hat{\mathcal{H}} + \zeta^k \hat{\mathcal{H}}_k) |\psi\rangle = 0,$$

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where  $\hat{\mathcal{H}}$  and  $\hat{\mathcal{H}}_k$  are the quantum versions of the Hamiltonian and momentum constraint functions  $\mathcal{H}$  and  $\mathcal{H}_k$

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$$\mathcal{H} = G_{ijmn} \pi^{ij} \pi^{mn} - R \sqrt{g} \approx 0, \quad \mathcal{H}_k = -2 \nabla_m \pi_k^m \approx 0,$$

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and where  $(\zeta^\perp, \zeta^k)$  are arbitrary vector fields required to vanish at infinity.

( $c = 1$  and  $16\pi G = 1$ ,  $G_{ijmn} = \frac{1}{2} g^{-\frac{1}{2}} (g_{im} g_{jn} + g_{in} g_{jm} - g_{ij} g_{mn})$  is the inverse of the DeWitt supermetric,  $\pi^{ij}$  are the conjugate momenta to the spatial metric  $g_{ij}$ , geometric objects are 3-dimensional (spatial))

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The quantum constraints were originally written in the metric representation where  $\Psi = \Psi[g_{ij}(x)]$  and  $\pi^{ij}(x) = \frac{\hbar}{i} \frac{\delta}{\delta g_{ij}(x)}$ ,

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leading to second order functional differential equations with respect to the metric :

$$\hbar^2 G_{ijmn} \frac{\delta}{\delta g_{ij}} \left( \frac{\delta \Psi}{\delta g_{mn}} \right) + R \sqrt{g} \Psi = 0 \quad , \quad \left( \frac{\delta \Psi}{\delta g_{mn}} \right)_{|m} = 0$$

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The second equation expresses invariance of  $\Psi$  under spatial diffeomorphisms, i.e., the fact that it depends only on the 3-geometry,  $\Psi = \Psi[({}^3\mathcal{G})]$ , and not on how this 3-geometry is described in coordinates.

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The first one constrains then the dependence on the 3-geometry (and it is sometimes that equation that is called the WDW equation, but we shall use this terminology to refer to the gravity quantum constraint equations (independently of the representation)).

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For asymptotically flat spaces, the Wheeler-DeWitt equation must be supplemented by a Schrödinger-like equation which gives the change of  $|\psi\rangle$  when the vector field  $(\xi^\perp, \xi^k)$  does not go to zero asymptotically but rather approaches an asymptotic time translation,

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$$i\frac{\partial |\Psi\rangle}{\partial T} = \hat{H}|\psi\rangle \quad (\hbar = 1)$$

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$$i\frac{\partial |\Psi\rangle}{\partial T} = \hat{H}|\psi\rangle \quad (\hbar = 1)$$

where  $\xi^\perp \rightarrow 1$ ,  $\xi^k \rightarrow 0$  for  $r \rightarrow \infty$  and  $\hat{H}$  is the Hamiltonian operator corresponding to the classical Hamiltonian

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$$i \frac{\partial |\Psi\rangle}{\partial T} = \hat{H} |\psi\rangle \quad (\hbar = 1)$$

where  $\xi^\perp \rightarrow 1$ ,  $\xi^k \rightarrow 0$  for  $r \rightarrow \infty$  and  $\hat{H}$  is the Hamiltonian operator corresponding to the classical Hamiltonian

$$H = \int d^3x (\xi^\perp \mathcal{H} + \xi^k \mathcal{H}_k) + \oint_{S_\infty} d^2S_i (g_{ik,k} - g_{kk,i}) .$$

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For asymptotically flat spaces, the Wheeler-DeWitt equation must be supplemented by a Schrödinger-like equation which gives the change of  $|\psi\rangle$  when the vector field  $(\xi^\perp, \xi^k)$  does not go to zero asymptotically but rather approaches an asymptotic time translation,

$$i\frac{\partial |\Psi\rangle}{\partial T} = \hat{H}|\psi\rangle \quad (\hbar = 1)$$

where  $\xi^\perp \rightarrow 1$ ,  $\xi^k \rightarrow 0$  for  $r \rightarrow \infty$  and  $\hat{H}$  is the Hamiltonian operator corresponding to the classical Hamiltonian

$$H = \int d^3x (\xi^\perp \mathcal{H} + \xi^k \mathcal{H}_k) + \oint_{S_\infty} d^2S_i (g_{ik,k} - g_{kk,i}).$$

**Key fact : The (ADM) energy reduces to a surface integral for physical states ("holography")**

# Hamiltonian form of the BMS symmetry

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Asymptotic time translations form only a subgroup of the asymptotic symmetry group of gravity in the asymptotic flat context.

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**The complete asymptotic symmetry group is the BMS group and is infinite-dimensional.**

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ADM-like expressions for all the BMS generators have been given some time ago and define also boundary observables.

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(M.H + C. Troessaert, JHEP **07** (2018), 171 [arXiv :1805.11288 [gr-qc]], as well as subsequent work with O. Fuentealba and C. Troessaert).

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The generator of the most general BMS transformation reads

$$\mathring{P}_{\xi^\perp, \xi^k} = \int d^3x (\xi^\perp \mathcal{H} + \xi^k \mathcal{H}_k) + \frac{1}{2} \beta^{\mu\nu} M_{\mu\nu} + \mathcal{B}_{\{T, W\}}^{grav}$$

where the surface integrals are explicitly given in the  
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where the surface integrals are explicitly given in the above-mentioned literature.

The vector field  $(\xi^\perp, \xi^k)$  behaves asymptotically as an infinitesimal BMS transformation  $\xi^\perp = \beta^\perp{}^i x_i + T(\theta, \varphi) + \mathcal{O}\left(\frac{1}{r}\right)$ ,  $\xi^k = \beta^{ki} x_i + \partial^k (rW(\theta, \varphi)) + \mathcal{O}\left(\frac{1}{r}\right)$ ,  $M_{\mu\nu}$  and  $\mathcal{B}_{\{T, W\}}^{grav}$  are the corresponding (surface integral) charges.

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The generator of the most general BMS transformation reads

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$T$  and  $W$  parametrize the supertranslations in the Hamiltonian formulation;  $T$  is an even function on the sphere,  $W$  is an odd function on the sphere.

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$T$  and  $W$  parametrize the supertranslations in the Hamiltonian formulation;  $T$  is an even function on the sphere,  $W$  is an odd function on the sphere.

All these symmetry generators reduce to surface integrals for physical states, as the energy.

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Explicitly, for the supertranslations,

$$\mathcal{B}_{\{T,W\}}^{grav} = \oint \dot{G}^{ijkl} T \left( (h_{ij}^{(1)})^{even} \right)_{,k} d^2 S_l + \oint \left( 2\partial_k (rW) (\Pi^{(2)kl})^{odd} \right) d^2 S_l$$

.

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Here,  $h_{ij}^{(1)}$  and  $\Pi^{(2)kl}$  are the leading deviations from Minkowski.

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Note that  $\mathcal{B}_{\{T,W\}}^{grav} = 0$  for Minkowski space, which has  $h_{ij} = \pi^{ij} = 0$ .

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The BMS generators are first class (gauge invariant under proper diffeomorphisms) and real. They define therefore boundary observables (as announced).

# Quantum questions

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Provided these properties are preserved quantum-mechanically, their quantum versions unitarily map solutions of the Wheeler-DeWitt equations on solutions of the Wheeler-DeWitt equations.

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But what does unitarity mean?

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If one has two solutions of the Wheeler-DeWitt equation, how should one compute their scalar product?

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But what does unitarity mean?

If one has two solutions of the Wheeler-DeWitt equation, how should one compute their scalar product?

How should one compute expectation values of gauge-invariant operators such as the BMS generators?

# BRST reformulation

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A difficulty is that the naive scalar product among solutions of the constraint equations typically diverges because of the integration along the gauge directions.

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A difficulty is that the naive scalar product among solutions of the constraint equations typically diverges because of the integration along the gauge directions.

In order to be able to compute physical amplitudes and to talk about unitary action in the physical subspace, one needs a prescription for computing scalar products among solutions of the Wheeler-DeWitt equation.

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In order to be able to compute physical amplitudes and to talk about unitary action in the physical subspace, one needs a prescription for computing scalar products among solutions of the Wheeler-DeWitt equation.

The BRST method is the most powerful method for achieving this task.

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In order to be able to compute physical amplitudes and to talk about unitary action in the physical subspace, one needs a prescription for computing scalar products among solutions of the Wheeler-DeWitt equation.

The BRST method is the most powerful method for achieving this task.

It formally guarantees gauge independence and provides in a controlled way a scalar product that does not involve infinities related to integrations over the gauge directions.

# BRST construction

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One extends the original phase space by introducing anticommuting variables, which are the real ghosts ( $C^\perp, C^k$ ) (ghost number one) and antighosts ( $\bar{C}_\perp, \bar{C}_k$ ) (ghost number minus one) as well as their imaginary ghost momenta ( $\bar{\mathcal{P}}_\perp, \bar{\mathcal{P}}_k$ ) and ( $\mathcal{P}^\perp, \mathcal{P}^k$ ).

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One also brings in explicitly the Lagrange multipliers  $\lambda^\perp$  and  $\lambda^k$  for the constraints (the lapse and the shift) together with their conjugate momenta  $b_\perp$  and  $b_k$ , which are constrained to vanish,  $b_\perp \approx 0$ ,  $b_k \approx 0$ .

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The new fields all vanish at infinity, corresponding to “proper gauge symmetries”.

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The (real) BRST charge reads (Batalin-Fradkin-Vilkovisky) :

$$\Omega = \Omega^{\text{Min}} + i \int d^3x (b_{\perp} \mathcal{P}^{\perp} + b_k \mathcal{P}^k)$$

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The (real) BRST charge reads (Batalin-Fradkin-Vilkovisky) :

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with  $\Omega^{\perp} = C^{\perp}_{,k} C^k$  and  $\Omega^k = -g^{km} C^{\perp}_{,m} - C^m C^k_{,m}$ .

It fulfills  $\{\Omega, \Omega\} = 0$ , which implies  $\hat{\Omega}^2 = 0$  in the quantum theory.

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In particular, the generators of the BMS symmetry possess BRST extensions,

$$P_{\xi^\perp, \xi^k} = \int d^3x \left[ \xi^\perp (\mathcal{H}_\perp + \Delta_\perp) + \xi^k (\mathcal{H}_k + \Delta_k) \right] + \frac{1}{2} \beta^{\mu\nu} M_{\mu\nu} + \mathcal{B}_{\{T, W\}}^{grav}$$
$$[P_{\xi^\perp, \xi^k}, \Omega] = 0$$

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$$[P_{\xi^\perp, \xi^k}, \Omega] = 0$$

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and  $\Delta_\perp = (\overline{\mathcal{D}}_\perp C^k)_{,k} + \overline{\mathcal{D}}_k C_{,m}^\perp g^{km} + (\overline{\mathcal{D}}_k C^\perp g^{km})_{,m}$  and  
 $\Delta_k = \overline{\mathcal{D}}_\perp C^\perp_{,k} + \overline{\mathcal{D}}_m C^m_{,k} + (\overline{\mathcal{D}}_k C^m)_{,m}$ .

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Because of the non-trivial behaviour of  $(\xi^\perp, \xi^k)$  at infinity and of the presence of the surface terms, the BMS generators are not BRST-exact,  $P_{\xi^\perp, \xi^k} \neq [K, \Omega]$ .

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Proper gauge symmetries have BRST-exact generators but not improper ones.

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Of course, different continuations in the bulk of vector fields  $(\xi^\perp, \xi^k)$  with the same asymptotics differ by a BRST-exact term, as it should be the case,

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The BRST extensions form a representation of the BMS algebra (up to trivial terms).

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Physical states are defined by the BRST state cohomology at ghost number zero ( $\Omega |\psi\rangle = 0$ ,  $|\psi\rangle \sim |\psi\rangle + \Omega |\chi\rangle$  + ghost number zero condition).

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We thus write

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**where  $|\uparrow\rangle$  is the maximally filled state annihilated by the ghosts.**

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The BRST invariance condition imposes  $\frac{\delta\Psi}{\delta\lambda^\mu(x)} = 0$ , i.e., the physical states should not depend on the Lagrange multipliers, but their dependence on the metric is completely unrestricted.

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Although  $\Psi[g_{ij}]$  is subject to no equation, one recovers the Wheeler-DeWitt equation through the quotient relation  $|\psi\rangle \sim |\psi\rangle + \hat{\Omega}|\chi\rangle$ .

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$$|\psi\rangle = \Psi[g_{ij}] |0\rangle |\uparrow\rangle$$

where  $|0\rangle$  stands for the state  $|b_\mu = 0\rangle$  with zero eigenvalue for the momenta conjugate to the Lagrange multipliers, which is indeed such that  $\langle \lambda^\mu | 0 \rangle$  is independent of  $\lambda^\mu$ .

Although  $\Psi[g_{ij}]$  is subject to no equation, one recovers the Wheeler-DeWitt equation through the quotient relation  $|\psi\rangle \sim |\psi\rangle + \hat{\Omega} |\chi\rangle$ .

Thus, one can fully capture the physics without having to solve the Wheeler-DeWitt equation '("co-invariant states").

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The above approach is simpler (one does not have to solve the constraints). [Yet another approach : Gupta-Bleuler]

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One gives meaning to this expression by inserting the operator  $\exp [K, \Omega]$ ,

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For physical states, the regularized value does not depend on  $K$  because  $\exp [K, \Omega]$  differs from the identity by a trivial (BRST-exact) operator.

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For physical states, the regularized value does not depend on  $K$  because  $\exp [K, \Omega]$  differs from the identity by a trivial (BRST-exact) operator.

Similarly, if  $A$  is BRST-invariant, one sets

$$\langle \psi | A | \chi \rangle_R = \langle \psi | A \exp [K, \Omega] | \chi \rangle .$$

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If one takes for  $|\psi\rangle$  the state  $|g_{ij}^{(1)}, 0, \uparrow\rangle$  and for  $|\chi\rangle$  the state  $|g_{ij}^{(2)}, 0, \uparrow\rangle$ , one gets the so-called projected kernel of the gauge-invariant operator  $A_0$ ,

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and  $A_0$  is the gauge operator of which  $A$  is the BRST extension.

The projected kernel  $A_0^P[g_{ij}^{(2)}, g_{ij}^{(1)}]$  is the kernel of  $A_0$  in a basis of physical states expressed as functions of the metric. It contains the information about all the scalar products of  $A_0$  between physical states. It fulfills the Wheeler-DeWitt equation at both ends.

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A convenient choice of  $K$  is  $K = \int d^3x (\lambda^\perp \overline{\mathcal{P}}_\perp + \lambda^k \overline{\mathcal{P}}_k)$

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$$[K, \Omega] = \int d^3x (i\lambda^\mu \mathcal{H}_\mu - \overline{\mathcal{P}}_\mu \mathcal{P}^\mu + i\lambda^\mu \Delta_\mu)$$

(( $\mu = \perp, k$ )).

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$$p(\lambda) = i\sigma(\lambda) - \tilde{\mathcal{P}}_\mu \mathcal{P}^\mu, \quad \sigma(\lambda) = \lambda^\mu \mathcal{H}_\mu + \lambda^\mu \Delta_\mu$$

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$$p(\lambda) = i\sigma(\lambda) - \overline{\mathcal{P}}_\mu \mathcal{P}^\mu, \quad \sigma(\lambda) = \lambda^\mu \mathcal{H}_\mu + \lambda^\mu \Delta_\mu$$

(This would be the group average with the Haar measure if the constraints formed a Lie algebra.)

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Hamiltonian  
systems: the  
example of  
asymptotically  
flat gravity

Marc Henneaux  
(Université Libre  
de Bruxelles and  
Collège de  
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One can take  $A = P_{\xi^\perp, \xi^k}$

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in which case one gets the projected kernel of the BMS generator  $\mathring{P}_{\xi^\perp, \xi^k}$  as a “group average”,

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$$\mathring{P}_{\xi^\perp, \xi^k}^P [g_{ij}^{(2)}, g_{ij}^{(1)}] = \int \mathcal{D}\lambda^\mu \langle g_{ij}^{(2)} | \uparrow | P_{\xi^\perp, \xi^k} e^{\int d^3x p(\lambda)} | g_{ij}^{(1)} | \uparrow \rangle$$

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Similar expressions hold for the projected kernel of the BMS group element  $e^{iP_{\xi^\perp, \xi^k}}$ .

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Because of the non-trivial surface terms, the matrix elements of the exponential  $e^{iP_{\xi^\perp, \xi^k}}$  of the improper (BMS) gauge symmetries differ from those of the identity, contrary to what would be the case for the exponentials of proper gauge symmetries.

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One can also show that these expressions give a representation of the BMS algebra/group.

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We have given the BRST reformulation of the Wheeler-DeWitt equation in the asymptotically flat context.

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We have given the BRST reformulation of the Wheeler-DeWitt equation in the asymptotically flat context.

We have given explicit expressions for the generators of the BMS symmetry, which differ from BRST-exact expressions by surface terms. [Note : could be combined in  $\Omega$  with non-trivial asymptotics for the ghosts.]

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THANK YOU!