Ambitwistor-strings, BRST and the Penrose transform

Lionel Mason

The Mathematical Institute, Oxford
lmason@maths.ox.ac.uk

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With David Skinner. arxiv:1311.2564
and much further development w/ Tim Adamo, Eduardo Casali, Yvonne Geyer, Arthur Lipstein, Ricardo Monteiro, Piotr Tourkine ...

Cf. also Cachazo, He, Yuan (CHY) arxiv:1306.2962, 1306.6575, 1307.2199, 1309.0885, ...

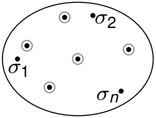
Review: Geyer & M. arxiv:2203.13017.

The scattering equations

Take n null momenta $k_i \in \mathbb{R}^d$, $i = 1, \ldots, n$, $k_i^2 = 0$, $\sum_i k_i = 0$,

- define $P : \mathbb{CP}^1 \to \mathbb{C}^d \otimes \Omega^{1,0}_{\mathbb{CP}^1}$

$$P(\sigma) := \sum_{i=1}^{n} \frac{k_i}{\sigma - \sigma_i} d\sigma \,, \qquad \sigma, \sigma_i \in \mathbb{CP}^1$$



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- Solve for $\sigma_i \in \mathbb{CP}^1$ with the n *scattering equations* [Fairlie 1972]

$$\mathrm{Res}_{\sigma_i}\left(P^2\right) = k_i \cdot P(\sigma_i) = \sum_{j=1}^{n} \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0 \,.$$

$\Rightarrow \; P^2 = 0 \; \forall \sigma$.

- $P \in \mathbb{C}^d \otimes K$, $K = \Omega^{1,0}_{\mathbb{CP}^1} \Rightarrow$ Mobius invariance
- only $n - 3$ scattering equations are independent.
- There are $(n-3)!$ solutions.

Arise in large α' strings [Gross-Mende 1988] & twistor-strings [Roiban, Spradlin, Volovich, Witten 2004].

Proposition (Cachazo, He, Yuan 2013,2014)

Massless tree amplitudes in d-dims are integrals/residue sums:

$$\mathcal{M}_n = \delta^d\left(\sum_i k_i\right) \int_{(\mathbb{CP}^1)^n} \frac{\mathcal{I}^l \mathcal{I}^r \prod_i \bar{\delta}(k_i \cdot P(\sigma_i))}{\text{Vol SL}(2, \mathbb{C}) \times \mathbb{C}^3}$$

where $\mathcal{I}^{l/r} = \mathcal{I}^{l/r}(\epsilon_i^{l/r}, k_i, \sigma_i)$ *depend on the theory.*

- polarizations ϵ_i^l for spin 1, $\epsilon_i^l \otimes \epsilon_i^r$ for spin-2 ($k_i \cdot \epsilon_i = 0 \ldots$).
- Introduce skew $2n \times 2n$ matrices $M = \begin{pmatrix} A & C \\ -C^t & B \end{pmatrix}$,

$$A_{ij} = \frac{k_i \cdot k_j}{\sigma_i - \sigma_j},, \quad B_{ij} = \frac{\epsilon_i \cdot \epsilon_j}{\sigma_i - \sigma_j}, \quad C_{ij} = \frac{k_i \cdot \epsilon_j}{\sigma_i - \sigma_j}, \quad \text{for } i \neq j$$

  and $A_{ii} = B_{ii} = 0$, $C_{ii} = \epsilon_i \cdot P(\sigma_i)$.
- For YM, $\mathcal{I}^l = Pf'(M)$, $\mathcal{I}^r = \prod_i \frac{1}{\sigma_i - \sigma_{i-1}}$.
- For GR $\mathcal{I}^l = Pf'(M^l)$, $\mathcal{I}^r = Pf'(M^r)$.

Worldsheet amplitude formulae

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Geometry of ambitwistor space

Work in complex holomorphic category:

- (M^d, g) is a d-dimensional complex manifold, local coordinates $X^\mu \in \mathbb{C}^d$ with holomorphic metric $g(X)_{\mu\nu}$.
- Ex: complexification of real space-time, small thickening.

Ambitwistor space $\mathbb{A}$:

- $\mathbb{A} :=$ space of complex null geodesics in (M^d, g).
- On T^*M, let P_μ be holomorphic fibre coordinates, and holomorphic symplectic form and potential:

$$\omega = dP_\mu \wedge dx^\mu \,, \quad \theta = P_\mu dx^\mu \,.$$

- Can construct $\mathbb{A}$ as symplectic quotient

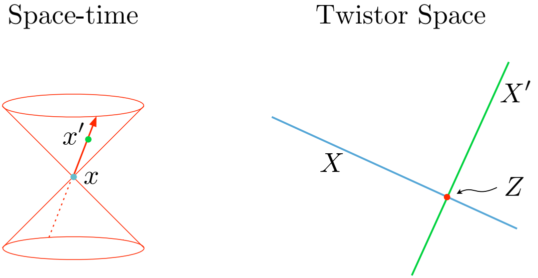
$$\mathbb{A} = T^*_0 M / \{P \cdot \nabla\} \,, \quad T^*_0 M := T^*M|_{P^2=0} \,, \quad P^2 := g^{\mu\nu} P_\mu P_\nu \,.$$

- $P \cdot \nabla =$ Hamiltonian vector field of $P^2 =$ geodesic spray.
- Symplectic potential and form θ, ω descend to $\mathbb{A}$

Correspondence with space-time

Projectivise: $\mathbb{PA}$:= space of *unscaled* complex light rays.

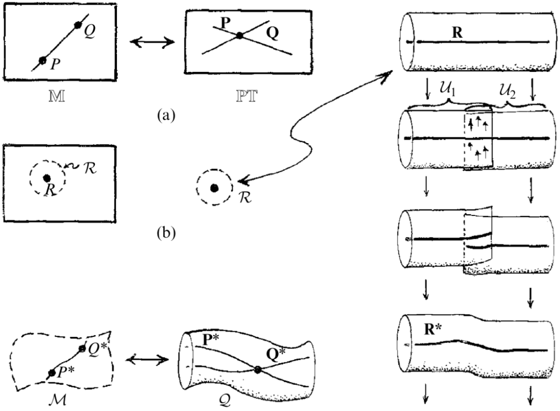
- Points $x \in M$ correspond to $X \subset \mathbb{PA}$,
- $X = \{\text{light rays thru } x\} = \text{projective lightcone} = \mathbb{P}T^*_x M|_{P^2=0}$
- Space-time M = space of such $X \subset \mathbb{PA}$.



Space-time geometry is encoded in complex structure of $\mathbb{A}$.

Theorem (LeBrun 1983 following Penrose 1976)

Complex structure of $\mathbb{PA}$ determines M and conformal metric g. Correspondence is stable under deformations of the complex structure of $\mathbb{PA}$ that preserve symplectic potential $\theta = p_\mu \mathrm{d}x^\mu$.



Preserving $\theta \Rightarrow$ gluing is canonical, generated by Hamiltonians.

For real space-time $(M_{\mathbb{R}}, g_{\mathbb{R}})$ dimension d,

- **Phase space action:** null geodesic γ, $(X, P) : \mathbb{R} \to T^*M_{\mathbb{R}}$

$$S = \int_\gamma (P \cdot dX - eP^2/2) \,,$$

- $e \in \Omega^1(\gamma)$ is 'einbein' and Lagrange multiplier for $P^2 = 0$.
- Flat space gauge freedom $\delta(X, P, e) = (\alpha P, 0, 2d\alpha)$.

Gives space of real null geodesics

$$\mathbb{A}_{\mathbb{R}} := T^*M_{\mathbb{R}}|_{P^2=0}/\{\text{gauge}\}.$$

Complexify: $\gamma \rightsquigarrow \Sigma$, Riemann surface, and $(M_{\mathbb{R}}, g_{\mathbb{R}}) \rightsquigarrow (M, g)$.

- **Ambitwistor string:** $X : \Sigma \to M$, $P \in X^* T^* M \otimes K$

$$S_{\text{Bos}} = \int (P \cdot \bar{\partial}_{\Sigma} X - e\, P^2/2)\,.$$

with $e \in \Omega^{0,1}_{\Sigma} \otimes T^{1,0}_{\Sigma}$, where $K = \Omega^{1,0}_{\Sigma}$.

- e again enforces $P^2 = 0$,
- flat space gauge freedom: $\delta(X, P, e) = (\alpha P, 0, 2\bar{\partial}\alpha)$.

Gives ambitwistor space

$$\mathbb{A} := T^* M|_{P^2=0} / \{\text{complex gauge}\}.$$

BRST Quantization of bosonic ambitwistor string

To quantize, gauge fix $S_{\mathrm{Bos}} = \int (P \cdot \bar\partial_\Sigma X - e\, P^2/2)$ by setting

- $\bar\partial_\Sigma = \bar\partial_0$ on $\Sigma \rightsquigarrow$ usual $(b, c) \in (K^2, T_\Sigma^{1,0})$ ghosts for diffeos.
- $e = 0 \rightsquigarrow$ ghosts $(\tilde b, \tilde c) \in (K^2, T_\Sigma^{1,0})$ for translation by $P \cdot \nabla$.

Gives quadratic gauge fixed action

$$S_{GF} = S_{\mathrm{Bos}}|_{e=0, \bar\partial_\Sigma = \bar\partial_0} + S_{\mathrm{ghost}} = \int P \cdot \bar\partial X + b\bar\partial c + \tilde b \bar\partial \tilde c\,.$$

But with BRST operator

$$\mathcal{Q} = \oint cP \cdot \partial X + \tilde c P^2\,.$$

In flat space, computing $\mathcal{Q}^2$ we have central charge

$$C = 2d - 26 - 26$$

so to quantize consistently $\mathcal{Q}^2 = 0 \Rightarrow d = 26$.

Vertex operators and descent

Physical states $\leftrightarrow$ vertex operators $\leftrightarrow \mathcal{D} = \mathcal{Q} + d_\Sigma$-cohomology

$$V^{g,p} \in \Omega^p_\Sigma|_{\text{Ghost }\#=g}$$

- Naturally have $\mathcal{Q} = Q + \tilde{Q}$, $d = \partial + \bar{\partial}$, $\mathcal{D} = D + \tilde{D}$.
- (b, c)-system $\leftrightarrow$ standard hol. diffeos $\leftrightarrow \partial : \Omega^{p,q}_\Sigma \to \Omega^{p+1,q}_\Sigma$.
- Focus on novel $(\tilde{b}, \tilde{c}) \leftrightarrow P \cdot \nabla$ but $\Omega^{*,q} \to \Omega^{*,q+1}$.
- $\tilde{D} = \tilde{Q} + \bar{\partial}$-closure gives tilde-vertex operator descent

$$\bar{\partial} V^{g+1,q} + \tilde{Q} V^{g,q+1} = 0\,, \qquad V^{g,q} \in (\Omega^{1,0}_\Sigma)^p \otimes \Omega^{0,q}|_{\text{Ghost}\#=g}\,.$$

- Claim: Descent implements Penrose transform for,

$$V^{1,0} = \phi(x)_{\mu_1 \ldots \mu_p} P^{\mu_1} \ldots P^{\mu_p}\,, \qquad V^{0,1} \in H^1_{\bar{\partial}}(\mathbb{PA}, \mathcal{O}(n-1))$$

$\mathcal{O}(n)$ = hgs fns degree n in P_μ on $\mathbb{P}T^*M$, descending to $\mathbb{PA}$.

Theorem (Baston & M.1988)

Trace-free symmetric fields on M denoted $(\ldots)_0$*, are given by*

$$H^1(\mathbb{PA}, \mathcal{O}(n-1)) \simeq \{\phi_{(\mu_1\ldots\mu_n)_0}\}/\{\nabla_{(\mu_1}\gamma_{\mu_2\ldots\mu_n)_0}\} .$$

- Define $T^*_0 M := T^*M|_{P^2=0}$, and double fibration,

$$\begin{array}{ccc} & \mathbb{P}T^*_0 M & \\ \pi_1 \swarrow P\cdot\nabla & & \searrow \pi_2 \\ \mathbb{PA} & & M . \end{array}$$

- De Rham up fibres of π_1 give short exact sequence

$$0 \to \mathcal{O}(n-1)_{\mathbb{PA}} \to \mathcal{O}(n-1)_{\mathbb{P}T^*_0 M} \stackrel{P\cdot\nabla}{\to} \mathcal{O}(n)_{\mathbb{P}T^*_0 M} \to 0$$

- Gives long exact sequence connecting hom. $\delta =$ descent,

$$\to H^0(\mathbb{P}T^*_0 M, \mathcal{O}(n-1)) \stackrel{P\cdot\nabla}{\to} H^0(\mathbb{P}T^*_0 M, \mathcal{O}(n)) \stackrel{\delta}{\to} H^1(\mathbb{PA}, \mathcal{O}(n-1)) \to 0$$

Key example $n = 2$ for linear gravity

On $\mathbb{PA}$, $\theta \in \Omega^1_{P_{\mathbb{A}}} \otimes \mathcal{O}(1)$ is a holomorphic contact structure.
θ gives complex structure on $\mathbb{PA}$ via $\theta \wedge d\theta^{d-2}$. So:

$$\text{Deformations of complex structure } \leftrightarrow [\delta\theta] \in H^1_{\bar\partial}(\mathbb{PA}, \mathcal{O}(1))\,.$$

- $H^1(\mathbb{P}T^*_0 M, \mathcal{O}(1)) = 0$ so $\pi_1^*\delta\theta = \bar\partial j$ for some j mod $P \cdot V(x)$.
- $\bar\partial P \cdot \nabla j = P \cdot \nabla \pi_1^* \delta\theta = 0$ so $P \cdot \nabla j$ is holomorphic in (P, X), so
- $P \cdot \nabla j = \delta g_{\mu\nu}(X) P^\mu P^\nu$ for some variation in the metric δg.

On flat space-time, set $\delta g_{\mu\nu} = \mathrm{e}^{ik\cdot x}\epsilon_\mu \epsilon_\nu$ then

$$j = \mathrm{e}^{ik\cdot X}\frac{(\epsilon \cdot P)^2}{k \cdot P}\,, \qquad \delta\theta = \bar\partial j = \bar\delta(k\cdot P)\mathrm{e}^{ik\cdot X}(\epsilon\cdot P)^2\,.$$

Delta-function support on $k \cdot P = 0 \Rightarrow$ the scattering equations.

Vertex operators and amplitudes

- Integrated vertex ops $\mathcal{V} = \int_\Sigma V^{0,1} = \delta(S_{\rm Bos}) \leftrightarrow \delta g$.
- Action is $\int \theta = \int P \cdot \bar\partial X$ so integrated vertex operator is

$$\mathcal{V}_i^{0,1} := \int_\Sigma \delta\theta(\sigma_i) = \int_\Sigma \bar\delta(k_i \cdot P(\sigma_i))\, \mathrm{e}^{ik\cdot X(\sigma_i)}(\epsilon_i \cdot P(\sigma_i))^2 \,.$$

- Fixed vertex ops $\mathcal{V}^{1,0} := c(\sigma_i) V_i^{1,0}(\sigma_i)$ where

$$V_i^{1,0} = \tilde{c}\mathrm{e}^{ik_i\cdot X(\sigma_i)}(\epsilon \cdot P(\sigma_i))^2 \,.$$

- Quantum consistency implies field equations:

$$\{Q, \mathcal{V}_i\} = 0 \quad \Leftrightarrow \quad k^2 = 0, \quad k^\mu \epsilon_\mu = 0 \,.$$

Need 3 fixed vertex ops to fix residual gauge $\rightsquigarrow$ amplitude

$$\mathcal{M}(1, \dots, n) = \int D[X, P, \dots]\; \mathcal{V}_1^{1,0}\mathcal{V}_2^{1,0}\mathcal{V}_3^{1,0}\, \mathcal{V}_4^{0,1} \dots \mathcal{V}_n^{0,1}\; \mathrm{e}^{iS} \,.$$

Evaluation of amplitude

- Take $\mathrm{e}^{ik_i\cdot X(\sigma_i)}$ factors into action to give

$$S=\frac{1}{2\pi}\int_\Sigma P\cdot\bar\partial X+2\pi\sum_i ik\cdot X(\sigma_i)\,.$$

- Gives field equations $\bar\partial X=0\,,\quad \bar\partial P=2\pi\sum_i ik\delta^2(\sigma-\sigma_i)\,.$
- Solutions $X(\sigma)=X=\text{const.}\,,\quad P(\sigma)=\sum_i\frac{k_i}{\sigma-\sigma_i}d\sigma\,.$

Thus path-integral reduces to

$$\mathcal{M}(1,\ldots,n)=\int_{\mathbb{R}^d\times\mathcal{M}_{0,n}} d^dX\,\mathrm{e}^{i\sum_j k_j\cdot X}\,\frac{\sigma_{12}\sigma_{13}\sigma_{23}\prod_{i=4}^n\bar\delta(k_i\cdot P)\,(\epsilon_i\cdot P(\sigma_i))^2}{\text{Vol Mobius}}$$

$$=\delta^d(\sum_i k_i)\int_{\mathcal{M}_{0,n}}\frac{\prod_{i=1}^n\bar\delta(k_i\cdot P)\,(\epsilon_i\cdot P(\sigma_i))^2}{(\text{Vol Mobius})^2}$$

where $c,\tilde c$-ghosts $\rightsquigarrow$ usual Fadeev-Popov for residual gauge.

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Spinning light rays and super ambitwistor space

Pfaffians arise as correlators of RNS fermions $\Psi_r^\mu \in \Omega_\Sigma^{\frac{1}{2},0}$:

$$S[X, P, \Psi] = \int P \cdot \mathrm{d}X - \frac{e}{2}P^2 + \sum_{r=1}^{\mathcal{N}} \Psi_r \cdot \mathrm{d}\Psi_r - \chi_r P \cdot \Psi_r$$

$\chi_r \rightsquigarrow$ constraints $P \cdot \Psi_r = 0$ that generate worldline $N = 2$ susy

$$D_r = \Psi_r \cdot \frac{\partial}{\partial X} + P \cdot \frac{\partial}{\partial \Psi_r}\,, \qquad \{D_r, D_s\} = \delta_{rs} P \cdot \nabla\,.$$

Super ambitwistor space: $\mathcal{N} = 1, 2$

$\mathbb{A}_s^{\mathcal{N}} =$ symplectic quotient of $\mathbb{C}^{2d|d\mathcal{N}} \ni (X, P, \Psi_r)$ by $P^2, P \cdot \Psi_r$.

Symplectic potential: $\theta = P \cdot dX + \Psi_r \cdot d\Psi_r$

Super ambitwistor correspondence holds with perturbations

$$\delta\theta = \mathrm{e}^{ik\cdot X}\, \bar{\delta}(k \cdot P) \prod_{r=1}^{2} (\epsilon_r \cdot P + \epsilon_r \cdot \Psi_r\, k \cdot \Psi_r)\,.$$

Note: polarization states $\epsilon_{1\mu}\epsilon_{2\nu} \rightsquigarrow$ NS sector of type II sugra.

Use chiral RNS-like action

$$S[X,P,\Psi] = \int_\Sigma P\cdot\bar\partial X - \frac{e}{2}P^2 + \sum_{r=1}^{2}\Psi_r\cdot\bar\partial\Psi_r + \chi_r P\cdot\Psi_r$$

with $N=2$ susy (degenerate).

- To quantize, gauge fix $\chi_r = 0 \rightsquigarrow$ bosonic ghosts (β_r,γ_r) in $(\Omega_\Sigma^{\frac{3}{2},0}, T^{\frac{1}{2},0})$ for fermionic symmetry (and (b,c), $(\tilde b,\tilde c)$).
- We obtain BRST operator

$$\mathcal{Q} = \int cT + \tilde c P^2 + \gamma_r P\cdot\Psi_r\,.$$

- For $\mathcal{Q}^2 = 0$ central charge C must vanish

$$C = 2d + \frac{d}{2} + \frac{d}{2} - 26 + 11 - 26 + 11 = 3(D-10)$$

- So critical in $d = 10$ dimensions.

- Integrated vertex operator

$$\mathcal{V}_i = \int_\Sigma \mathrm{e}^{ik\cdot X(\sigma_i)}\,\bar\delta(k\cdot P(\sigma_i))\prod_{r=1}^{2}(\epsilon_r\cdot P(\sigma_i)+\epsilon_r\cdot\Psi_r(\sigma_i)k\cdot\Psi_r(\sigma_i))$$

- need two fixed operators for γ_r zero modes

$$U_i = \mathrm{e}^{k_i\cdot X(\sigma_i)}\prod_r \epsilon_r\cdot\Psi_r(\sigma_i)$$

- and an extra fixed one to fix 3rd c and $\tilde c$ zero modes

$$V_i = \prod_{r=1}^{2}\epsilon_r\cdot(P(\sigma_i)+\Psi_r(\sigma_i)k\cdot\Psi_r(\sigma_i))$$

So amplitudes are given by

$$\mathcal{M}(1,\ldots,n) = \left\langle c_1\tilde c_1\prod_r \gamma_{r1}U_1 c_2\tilde c_2\prod_r\gamma_{r2}U_2 c_3\tilde c_3 V_3\mathcal{V}_4\ldots\mathcal{V}_n\right\rangle .$$

Amplitude formulae with Pfaffians

- Scattering equs arise as before $\rightsquigarrow \delta^d(\sum_i k_i) \prod_i{}' \bar{\delta}(k_i \cdot P)$.
- Correlator of Ψ_1, Ψ_2 independent, $\rightsquigarrow$ two factors.
- Contractions give for example

$$A_{ij} := \langle k_i \cdot \Psi_i k_j \cdot \Psi_j \rangle = \frac{k_i \cdot k_j}{\sigma_i - \sigma_j}, \quad B_{ij} := \langle \epsilon_i \cdot \Psi_i \, \epsilon_j \Psi_j \rangle = \frac{\epsilon_i \cdot \epsilon_j}{\sigma_i - \sigma_j}.$$

Proposition

We obtain CHY formula

$$\mathcal{M}(1, \dots, n) = \delta^d \left(\sum_i k_i \right) \int_{\mathbb{CP}^{1\,n}} \frac{Pf'(M_1) Pf'(M_2)}{\mathrm{Vol\ SL}(2, \mathbb{C})} \prod_i{}' \bar{\delta}(k_i \cdot P(\sigma_i)) d\sigma_i$$

- Can include current algebra etc. $\rightsquigarrow$ Parke-Taylor factors
- **Heterotic model:** as above but $r = 1$ and current algebra (SO(32) or $E_8 \times E_8$ for $Q^2 = 0$) $\rightsquigarrow$ CHY Yang-Mills formula.
- Bosonic case $+2$ current algebras $\rightsquigarrow$ CHY scalar formula.
- Extends CHY formulae to loops & Ramond sectors.

Algorithm: null (super-) particle $\rightsquigarrow$ (super-) ambitwistor string:

- 4d Ferber's supertwistor massless super-particle $\rightsquigarrow$ Witten twistor-string and new ambidextrous model [geyer,M.,Lipstein, 2014].
- Green-Schwarz version:

$$S = \int P \cdot \bar{\partial} X + P_\mu \gamma^\mu_{\alpha\beta} \theta^\alpha \bar{\partial} \theta^\beta \, .$$

- Berkovits pure spinor $S = \int P \cdot \bar{\partial} X + p_\alpha \bar{\partial} \theta^\alpha + \ldots$, cf also w/ Guillen & M other pure spinor models in 10 & 11d.
- M+Geyer: twistor models in dims 4, 6, 10, 11, & massive.
- New models $\rightsquigarrow$ new worldsheet amplitude formulae.

For the RNS models

- classically $\mathcal{Q}^2 = 0$ trivially.
- Quantum mechanically $\mathcal{Q}^2 = 0 \Leftrightarrow$ field equs,
- e.g., type II RNS models $\rightsquigarrow$ field equs of type II SUGRA.

  [Adamo,Casali,Skinner 2014].

The Type II example:

- With $\Psi^\mu = \Psi_1^\mu + i\Psi_2^\mu$, $\tilde{\Psi}_\mu = \Psi_{1\mu} - i\Psi_{2\mu}$ gauge-fixed action is quadratic in curved space!
- However, BRST is generated by operators

$$H = g^{\mu\nu}(X) P_\mu P_\nu \,, \qquad G = P_\mu \Psi^\mu \,, \qquad \tilde{G} = g^{\mu\nu}(X)\tilde{P}_\mu \Psi_\nu \,.$$

- Field equations are quantum anomalies that violate algebra

$$\{G, G\} = 0, \quad \{\tilde{G}, \tilde{G}\} = 0 \,, \quad \{G, \tilde{G}\} = H \,, \quad [H, H] = 0 \,.$$

- Ambitwistor chiral algebra's Quantum consistency $\Leftrightarrow$ field equs!

Summary

- Construction of ambitwistor space as symplectic quotient is realized via BRST.
- Vertex operator descent implements the Penrose transform for perturbations.
- Quantum consistency $\mathcal{Q}^2 = 0$ encodes field equations.

Oulook:

- Are other Penrose transforms implemented by BRST?
- Can we find ambitwistor string field theory?
- e.g., as a Costello-Li twisted supergravity in $\mathbb{A}$?
- Twisted holography in ambitwistor space?

Thank You